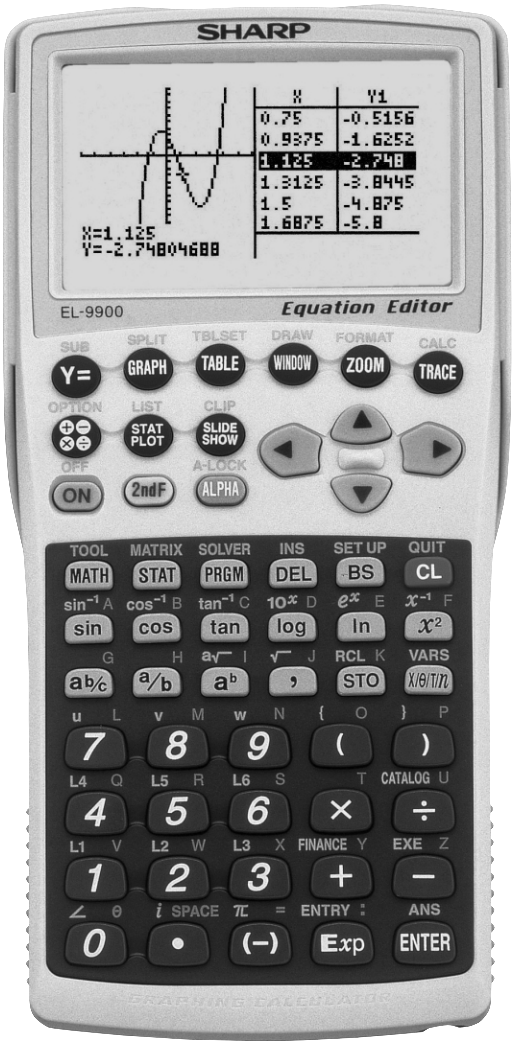
SHARP

Graphing Calculator

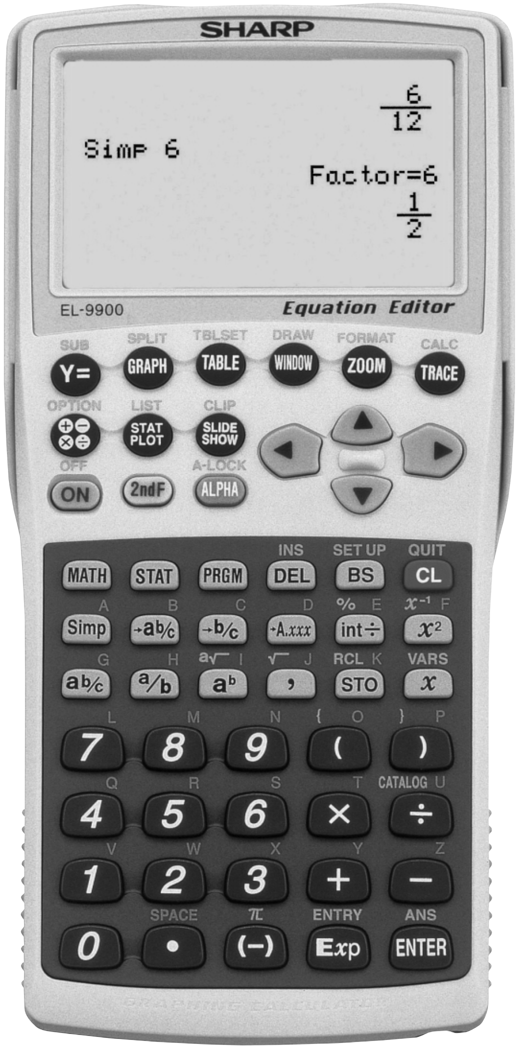
EL-9900

Handbook Vol. 1

Algebra



For Advanced Levels



For Basic Levels

Contents

Read this first

1. Always read "Before Starting"

The key operations of the set up conndition are written in "Before Starting" in each section. It is essential to follow the instructions in order to display the screens as they appear in the handbook.

2. Set Up Condition

As key operations for this handbook are conducted from the initial condition, reset all memories to the initial condition beforehand.

[2nd F] [OPTION] [E] [2] [CL]

Note: Since all memories will be deleted, it is advised to use the CE-LK2 PC link kit (sold separately) to back up any programmes not to be erased, or to return the settings to the initial condition (cf. 3. Initial Settings below) and to erase the data of the function to be used.

- To delete a single data, press [2nd F] [OPTION] [C] and select data to be deleted from the menu.
- Other keys to delete data:

[CL] : to erase equations and remove error displays
[2nd F] [QUIT] : to cancel previous function

3. Initial settings

Initial settings are as follows:

☆ Set up ([2nd F] [SET UP]): Advanced keyboard: Rad, FloatPt, 9, Rect, Decimal(Real), Equation, Auto
Basic keyboard: Deg, FloatPt, 9, Rect, Mixed, Equation, Auto

☆ Format ([2nd F] [FORMAT]): Advanced keyboard: OFF, OFF, ON, OFF, RectCoord
Basic keyboard: OFF, OFF, ON, OFF

Stat Plot ([STAT PLOT] [E]): 2. PlotOFF
Shade ([2nd F] [DRAW] [G]): 2. INITIAL
Zoom ([ZOOM] [A]): 5. Default
Period ([2nd F] [FINANCE] [C]): 1. PmtEnd (Advanced keyboard only)

Note: ☆ returns to the default setting in the following operation.
([2nd F] [OPTION] [E] [1] [ENTER])

4. Using the keys

Press [2nd F] to use secondary functions (in yellow).
To select "X^{-1}": [2nd F] [X^2] → Displayed as follows: [2nd F] [X^{-1}]

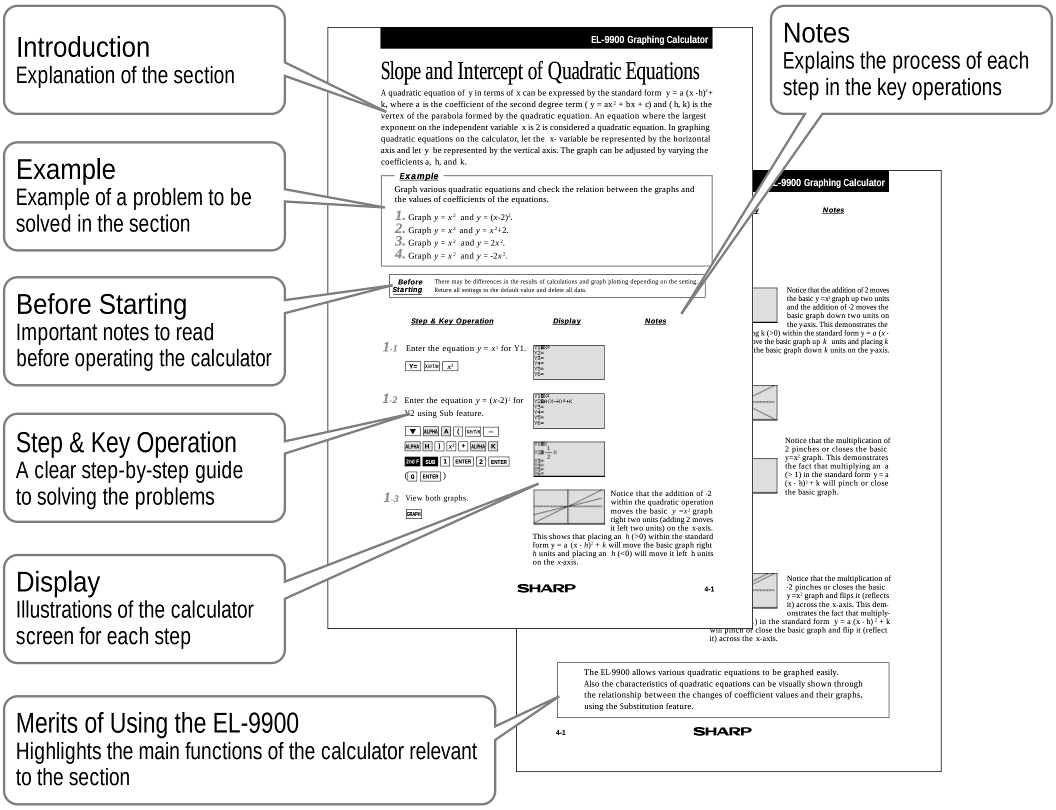
Press [ALPHA] to use the alphabet keys (in violet).
To select F: [ALPHA] [X^2] → Displayed as follows: [ALPHA] [F]

5. Notes

- Some features are provided only on the Advanced keyboard and not on the Basic keyboard. (Solver, Matrix, Tool etc.)
- As this handbook is only an example of how to use the EL-9900, please refer to the manual for further details.

Using this Handbook

This handbook was produced for practical application of the SHARP EL-9900 Graphing Calculator based on exercise examples received from teachers actively engaged in teaching. It can be used with minimal preparation in a variety of situations such as classroom presentations, and also as a self-study reference book.



We would like to express our deepest gratitude to all the teachers whose cooperation we received in editing this book. We aim to produce a handbook which is more replete and useful to everyone, so any comments or ideas on exercises will be welcomed.

(Use the attached blank sheet to create and contribute your own mathematical problems.)

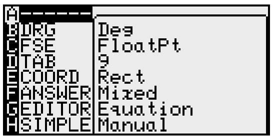
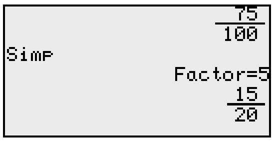
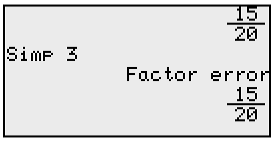
Fractions and Decimals

To convert a decimal into a fraction, form the numerator by multiplying the decimal by 10^n, where n is the number of digits after the decimal point. The denominator is simply 10^n. Then, reduce the fraction to its lowest terms.

Example

Convert 0.75 into a fraction.

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data. We recommend using the Basic keyboard to calculate fractions.

	Step & Key Operation	Display	Notes
1	Choose the manual mode for reducing fractions. [2nd F] [SET UP] [H] [2]		
2	Convert 0.75 into a fraction. [CL] [0] [•] [7] [5] [→b/c] [ENTER]		
3	Reduce the fraction. [Simp] [ENTER]		The fraction can be reduced by a factor of 5.
4	Enter 3 to further reduce the fraction. [Simp] [3] [ENTER]		The fraction cannot be reduced by a factor of 3, even though the numerator can be. (15 = 3 x 5)
5	Enter 5 to reduce the fraction. [Simp] [5] [ENTER]		0.75 = 3/4

The EL-9900 can easily convert a decimal into a fraction. It also helps students learn the steps involved in reducing fractions.

Pie Charts and Proportions

Pie charts enable a quick and clear overview of how portions of data relate to the whole.

Example

A questionnaire asking students about their favourite colour elicited the following results:

Red: 20 students
Blue: 12 students
Green: 25 students
Pink: 10 students
Yellow: 6 students

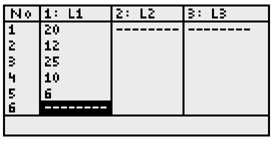
1. Make a pie chart based on this data.
2. Find the percentage for each colour.

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Step & Key Operation	Display	Notes

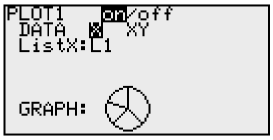
1-1 Enter the data.

STAT A ENTER 2 0 ENTER 1 2 ENTER 2 5 ENTER 1 0 ENTER 6 ENTER



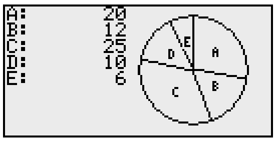
1-2 Choose the setting for making a pie chart.

STAT PLOT A ENTER ENTER ▼ ▼ ▼ STAT PLOT F 1



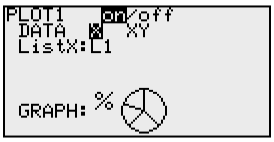
1-3 Make a pie chart.

GRAPH



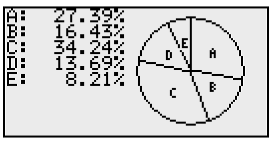
2-1 Choose the setting for displaying by percentages.

STAT PLOT A ENTER ▼ ▼ ▼ STAT PLOT F 2



2-2 Make another pie chart.

GRAPH



Notes:
Red: 27.39%
Blue: 16.43%
Green: 34.24%
Pink: 13.69%
Yellow: 8.21%

Pie charts can be made easily with the EL-9900.

Slope and Intercept of Linear Equations

A linear equation of y in terms of x can be expressed by the slope-intercept form $y = mx+b$, where m is the slope and b is the y- intercept. We call this equation a linear equation since its graph is a straight line. Equations where the exponents on the x and y are 1 (implied) are considered linear equations. In graphing linear equations on the calculator, we will let the x variable be represented by the horizontal axis and let y be represented by the vertical axis.

Example

Draw graphs of two equations by changing the slope or the y- intercept.

1. Graph the equations $y = x$ and $y = 2x$.
2. Graph the equations $y = x$ and $y = \frac{1}{2}x$.
3. Graph the equations $y = x$ and $y = -x$.
4. Graph the equations $y = x$ and $y = x + 2$.

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

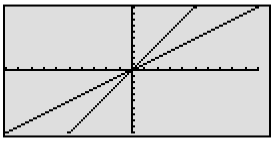
Step & Key Operation	Display	Notes

1-1 Enter the equation $y = x$ for Y1 and $y = 2x$ for Y2.

[Y=] [X/θ/T/n] [ENTER] [2] [X/θ/T/n]

1-2 View both graphs.

[GRAPH]

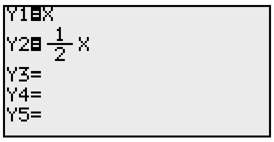


The equation Y1 = x is displayed first, followed by the equation Y2 = $2x$. Notice how Y2 becomes steeper or climbs faster. Increase the size of the slope (m>1) to make the line steeper.

2-1 Enter the equation $y = \frac{1}{2}x$ for Y2.

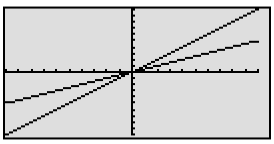
[Y=] [▼] [CL]

[1] [a/b] [2] [▶] [X/θ/T/n]



2-2 View both graphs.

[GRAPH]



Notice how Y2 becomes less steep or climbs slower. Decrease the size of the slope (0<m<1) to make the line less steep.

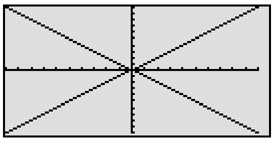
Step & Key Operation	Display	Notes

3-1 Enter the equation $y = -x$ for Y2.

[Y=] [▼] [CL] [(-)] [X/θ/T/n]

3-2 View both graphs.

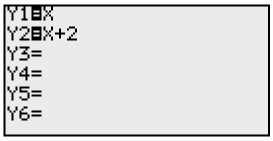
[GRAPH]



Notice how Y2 decreases (going down from left to right) instead of increasing (going up from left to right). Negative slopes ($m<0$) make the line decrease or go down from left to right.

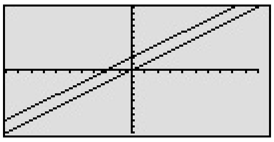
4-1 Enter the equation $y = x + 2$ for Y2.

[Y=] [▼] [CL] [X/θ/T/n] [+] [2]



4-2 View both graphs.

[GRAPH]



Adding 2 will shift the $y = x$ graph upwards.

Making a graph is easy, and quick comparison of several graphs will help students understand the characteristics of linear equations.

SHARP

Parallel and Perpendicular Lines

Parallel and perpendicular lines can be drawn by changing the slope of the linear equation and the y intercept. A linear equation of y in terms of x can be expressed by the slope-intercept form $y = mx + b$, where m is the slope and b is the y-intercept.

Parallel lines have an equal slope with different y-intercepts. Perpendicular lines have slopes that are negative reciprocals of each other ($m = -\frac{1}{m}$). These characteristics can be verified by graphing these lines.

Example

Graph parallel lines and perpendicular lines.

1. Graph the equations $y = 3x + 1$ and $y = 3x + 2$.

2. Graph the equations $y = 3x - 1$ and $y = -\frac{1}{3}x + 1$.

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

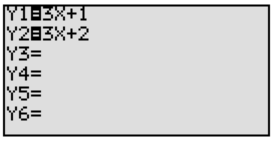
Set the zoom to the decimal window: [ZOOM] [C] ([ENTER] [ALPHA] [▼]) [7]

Step & Key Operation	Display	Notes

1-1 Enter the equations $y = 3x + 1$ for Y1 and $y = 3x + 2$ for Y2.

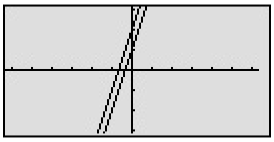
[Y=] [3] [X/θ/T/n] [+] [1] [ENTER]

[3] [X/θ/T/n] [+] [2]



1-2 View the graphs.

[GRAPH]



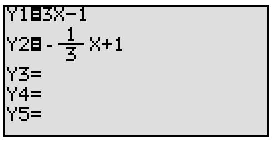
These lines have an equal slope but different y-intercepts. They are called parallel, and will not intersect.

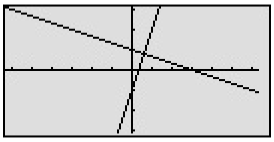
2-1 Enter the equations $y = 3x - 1$ for Y1 and $y = -\frac{1}{3}x + 1$ for Y2.

[Y=] [CL] [3] [X/θ/T/n] [—] [1] [ENTER]

[CL] [(-)] [1] [a/b] [3] [▶] [X/θ/T/n]

[+] [1]



Step & Key Operation	***Display***	***Notes***
***2**-2* View the graphs. GRAPH		These lines have slopes that are negative reciprocals of each other ($m = -\frac{1}{m}$). They are called perpendicular. Note that these intersecting lines form four equal angles.

The Graphing Calculator can be used to draw parallel or perpendicular lines while learning the slope or *y*-intercept of linear equations.

Slope and Intercept of Quadratic Equations

A quadratic equation of y in terms of x can be expressed by the standard form $y = a(x - h)^2 + k$, where a is the coefficient of the second degree term ($y = ax^2 + bx + c$) and (h, k) is the vertex of the parabola formed by the quadratic equation. An equation where the largest exponent on the independent variable x is 2 is considered a quadratic equation. In graphing quadratic equations on the calculator, let the x-variable be represented by the horizontal axis and let y be represented by the vertical axis. The graph can be adjusted by varying the coefficients a, h, and k.

Example

Graph various quadratic equations and check the relation between the graphs and the values of coefficients of the equations.

1. Graph $y = x^2$ and $y = (x - 2)^2$.
2. Graph $y = x^2$ and $y = x^2 + 2$.
3. Graph $y = x^2$ and $y = 2x^2$.
4. Graph $y = x^2$ and $y = -2x^2$.

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Step & Key Operation | **Display** | **Notes**

1-1 Enter the equation $y = x^2$ for Y1.

[Y=] [X/θ/T/n] [X^2]

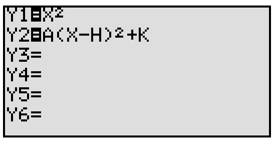
1-2 Enter the equation $y = (x - 2)^2$ for Y2 using Sub feature.

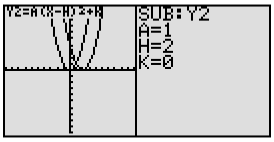
[▼] [ALPHA] [A] [(] [X/θ/T/n] [—]

[ALPHA] [H] [)] [X^2] [+] [ALPHA] [K]

[2nd F] [SUB] [1] [ENTER] [2] [ENTER]

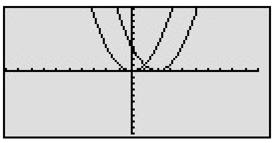
([0] [ENTER])





1-3 View both graphs.

[GRAPH]

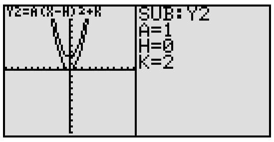


Notice that the addition of -2 within the quadratic operation moves the basic $y = x^2$ graph right two units (adding 2 moves it left two units) on the x-axis. This shows that placing an h (>0) within the standard form $y = a(x - h)^2 + k$ will move the basic graph right h units and placing an h (<0) will move it left h units on the x-axis.

Step & Key Operation	***Display***	***Notes***

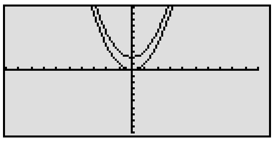
2-1 Change the equation in Y2 to $y = x^2+2$.

Y= ▼ 2nd F SUB ▼ 0
ENTER 2 ENTER



2-2 View both graphs.

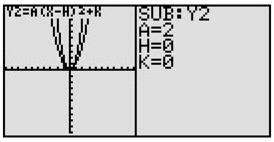
GRAPH



Notice that the addition of 2 moves the basic $y = x^2$ graph up two units and the addition of -2 moves the basic graph down two units on the *y*-axis. This demonstrates the fact that adding k (>0) within the standard form $y = a(x - h)^2 + k$ will move the basic graph up k units and placing k (<0) will move the basic graph down k units on the *y*-axis.

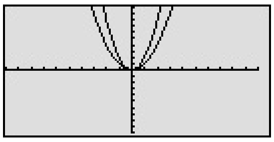
3-1 Change the equation in Y2 to $y = 2x^2$.

Y= ▼ 2nd F SUB 2 ENTER
▼ 0 ENTER



3-2 View both graphs.

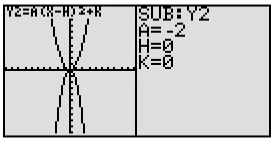
GRAPH



Notice that the multiplication of 2 pinches or closes the basic $y = x^2$ graph. This demonstrates the fact that multiplying an a (> 1) in the standard form $y = a(x - h)^2 + k$ will pinch or close the basic graph.

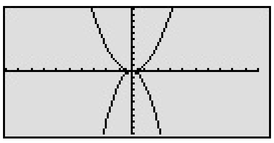
4-1 Change the equation in Y2 to $y = -2x^2$.

Y= ▼ 2nd F SUB (-) 2
ENTER



4-2 View both graphs.

GRAPH



Notice that the multiplication of -2 pinches or closes the basic $y =x^2$ graph and flips it (reflects it) across the *x*-axis. This demonstrates the fact that multiplying an a (<-1) in the standard form $y = a(x - h)^2 + k$ will pinch or close the basic graph and flip it (reflect it) across the *x*-axis.

The EL-9900 allows various quadratic equations to be graphed easily. Also the characteristics of quadratic equations can be visually shown through the relationship between the changes of coefficient values and their graphs, using the Substitution feature.

Solving a Literal Equation Using the Equation Method (Amortization)

The Solver mode is used to solve one unknown variable by inputting known variables, by three methods: Equation, Newton's, and Graphic. The Equation method is used when an exact solution can be found by simple substitution.

Example

Solve an amortization formula. The solution from various values for known variables can be easily found by giving values to the known variables using the Equation method in the Solver mode.

The formula : $P = L\left[\dfrac{1-\left(1+\frac{I}{12}\right)^{-N}}{\frac{I}{12}}\right]^{-1}$

P= monthly payment
L= loan amount
I= interest rate
N=number of months

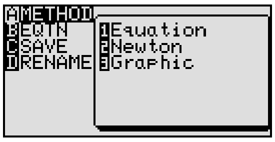
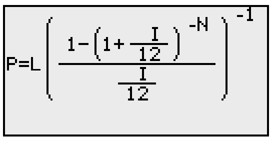
1. Find the monthly payment on a $15,000 car loan, made at 9% interest over four years (48 months) using the Equation method.

2. Save the formula as "AMORT".

3. Find amount of loan possible at 7% interest over 60 months with a $300 payment, using the saved formula.

Before Starting

There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.
As the Solver feature is only available on the Advanced keyboard, this section does not apply to the Basic keyboard.

Step & Key Operation	Display	Notes
1-1 Access the Solver feature. 2nd F SOLVER		This screen will appear a few seconds after "SOLVER" is displayed.
1-2 Select the Equation method for solving. 2nd F SOLVER A 1		
1-3 Enter the amortization formula. 2nd F ALPHA P = L ALPHA (a/b 1 — (1 + ALPHA I a/b 1 2 ▶) a^b (-) ALPHA N ▶ ▶ ALPHA I a/b 1 2 ▶ ▶) a^b (-) 1		

Step & Key Operation	Display	Notes

1-4 Enter the values L=15,000, I=0.09, N=48.

ENTER ▼ 1 5 0 0 0
ENTER • 0 9 ENTER 4
8 ENTER

```
Solver:Equation
 P=0
 L=15000
 I=0.09
 N=48
```

1-5 Solve for the payment(P).

▲ ▲ ▲ 2nd F EXE
(CL)

```
Equation solver
 P=373.2756356
```

The monthly payment (P) is $373.28.

2-1 Save this formula.

2nd F SOLVER C ENTER

```
AMETHOD
BEQTN
CSAVE
DRENAME
        Press[ENTER]
```

2-2 Give the formula the name AMORT.

A M O R T ENTER

```
Equation title
 [AMORT    ]
```

3-1 Recall the amortization formula.

2nd F SOLVER B
0 1

```
AMETHOD
BEQTN   01AMORT
CSAVE
DRENAME
```

3-2 Enter the values: P = 300, I = 0.01, N = 60

ENTER 3 0 0 ENTER 0 ENTER
• 0 1 ENTER 6 0 ENTER

```
Solver:Equation
 P=300
 L=0
 I=0.01
 N=60
```

3-3 Solve for the loan (L).

▲ ▲ 2nd F EXE

```
Equation solver
 L=17550.27685
```

The amount of loan (L) is $17550.28.

With the Equation Editor, the EL-9900 displays equations, even complicated ones, as they appear in the textbook in easy to understand format. Also it is easy to find the solution for unknown variables by recalling a stored equation and giving values to the known variables in the Solver mode when using the Advanced keyboard.

Solving a Literal Equation Using the Graphic Method (Volume of a Cylinder)

The Solver mode is used to solve one unknown variable by inputting known variables. There are three methods: Equation, Newton's, and Graphic. The Equation method is used when an exact solution can be found by simple substitution. Newton's method implements an iterative approach to find the solution once a starting point is given. When a starting point is unavailable or multiple solutions are expected, use the Graphic method. This method plots the left and right sides of the equation and then locates the intersection(s).

Example

Use the Graphic method to find the radius of a cylinder giving the range of the unknown variable.

The formula : $V = \pi r^2 h$ (V = volume r = radius h = height)

1. Find the radius of a cylinder with a volume of $30in^3$ and a height of 10in, using the Graphic method.

2. Save the formula as "V CYL".

3. Find the radius of a cylinder with a volume of $200in^3$ and a height of 15in, using the saved formula.

Before Starting

There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.
As the Solver feature is only available on the Advanced keyboard, this section does not apply to the Basic keyboard.

Step & Key Operation | **Display** | **Notes**

1-1 Access the Solver feature.

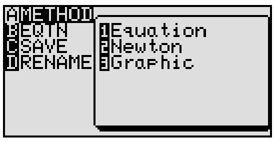
[2nd F] [SOLVER]

This screen will appear a few seconds after "SOLVER" is displayed.

1-2 Select the Graphic method for solving.

[2nd F] [SOLVER] [A]

[3]



1-3 Enter the formula $V = \pi r^2 h$.

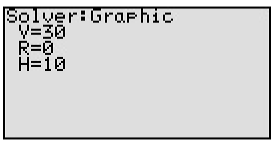
[ALPHA] [V] [ALPHA] [=] [2nd F] [π] [ALPHA]

[R] [X^2] [ALPHA] [H]

1-4 Enter the values: V = 30, H = 10. Solve for the radius (R).

[ENTER] [3] [0] [ENTER] [▼] [1]

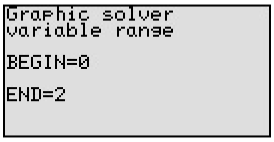
[0] [ENTER] [▲] [2nd F] [EXE]



Step & Key Operation	***Display***	***Notes***

1-5 Set the variable range from 0 to 2.

[0] [ENTER] [2] [ENTER]



The graphic solver will prompt with a variable range for solving.

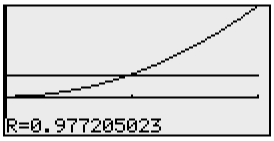
$$r^2 = \frac{30}{10\pi} = \frac{3}{\pi} < 3$$

$r = 1 \rightarrow r^2 = 1^2 = 1 < 3$
$r = 2 \rightarrow r^2 = 2^2 = 4 > 3$

Use the larger of the values to be safe.

1-6 Solve.

[2nd F] [EXE] ([CL])



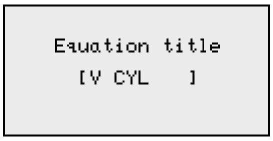
The solver feature will graph the left side of the equation (volume, $y = 30$), then the right side of the equation ($y = 10r^2$), and finally will calculate the intersection of the two graphs to find the solution.
The radius is 0.98 in.

---

2 Save this formula.
Give the formula the name “V CYL”.

[2nd F] [SOLVER] [C] [ENTER]

[V] [SPACE] [C] [Y] [L] [ENTER]

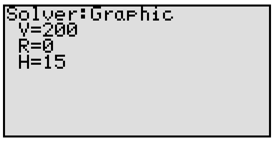


3-1 Recall the formula.
Enter the values: V = 200, H = 15.

[2nd F] [SOLVER] [B] [0] [1]

[ENTER] [2] [0] [0] [ENTER] [0] [ENTER]

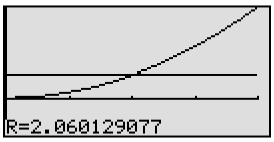
[1] [5] [ENTER]



3-2 Solve the radius setting the variable range from 0 to 4.

[▲] [2nd F] [EXE] [0] [ENTER] [4]

[ENTER] [2nd F] [EXE]



$$r^2 = \frac{200}{15\pi} = \frac{14}{\pi} < 14$$

$r = 3 \rightarrow r^2 = 3^2 = 9 < 14$
$r = 4 \rightarrow r^2 = 4^2 = 16 > 14$

Use 4, the larger of the values, to be safe.
The answer is : $r = 2.06$

One very useful feature of the calculator is its ability to store and recall equations. The solution from various values for known variables can be easily obtained by recalling an equation which has been stored and giving values to the known variables. The Graphic method gives a visual solution by drawing a graph.

Solving a Literal Equation Using Newton's Method (Area of a Trapezoid)

The Solver mode is used to solve one unknown variable by inputting known variables. There are three methods: Equation, Newton's, and Graphic. The Newton's method can be used for more complicated equations. This method implements an iterative approach to find the solution once a starting point is given.

Example

Find the height of a trapezoid from the formula for calculating the area of a trapezoid using Newton's method.

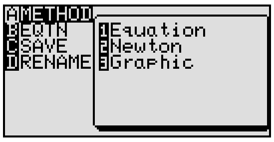
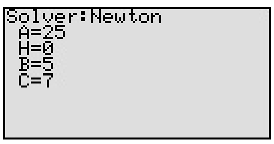
The formula : $A=\frac{1}{2}h(b+c)$ (A = area h = height b = top face c = bottom face)

1. Find the height of a trapezoid with an area of $25in^2$ and bases of length 5in and 7in using Newton's method. (Set the starting point to 1.)

2. Save the formula as "A TRAP".

3. Find the height of a trapezoid with an area of $50in^2$ with bases of 8in and 10in using the saved formula. (Set the starting point to 1.)

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.
As the Solver feature is only available on the Advanced keyboard, this section does not apply to the Basic keyboard.

Step & Key Operation	Display	Notes
1-1 Access the Solver feature. [2nd F] [SOLVER]		This screen will appear a few seconds after "SOLVER" is displayed.
1-2 Select Newton's method for solving. [2nd F] [SOLVER] [A] [2]		
1-3 Enter the formula A = $\frac{1}{2}h(b+c)$. [ALPHA] [A] [ALPHA] [=] [1] [a/b] [2] [▶] [ALPHA] [H] [(] [ALPHA] [B] [+] [ALPHA] [C] [)]		
1-4 Enter the values: A = 25, B = 5, C = 7 [ENTER] [2] [5] [ENTER] [▼] [5] [ENTER] [7] [ENTER]		

Step & Key Operation	Display	Notes
1-5 Solve for the height and enter a starting point of 1. ▲ ▲ 2nd F EXE 1 ENTER	Newton solver START=1 STEP=0.001	Newton's method will prompt with a guess or a starting point.
1-6 Solve. 2nd F EXE (CL)	Newton solver H=4.166666667 RIGHT=25 LEFT =25 L-R =-0.000000002	The answer is : *h* = 4.17
2 Save this formula. Give the formula the name “A TRAP”. 2nd F SOLVER C ENTER A SPACE T R A P ENTER	Equation title [A TRAP]	
3-1 Recall the formula for calculating the area of a trapezoid. 2nd F SOLVER B 0 1	$A=\frac{1}{2}H(B+C)$	
3-2 Enter the values: A = 50, B = 8, C = 10. ENTER 5 0 ENTER ▼ 8 ENTER 1 0 ENTER	Solver:Newton A=50 H=4.166666667 B=8 C=10	
3-3 Solve. ▲ ▲ 2nd F EXE 1 ENTER 2nd F EXE	Newton solver H=5.555555556 RIGHT=50 LEFT =50 L-R =0	The answer is : *h* = 5.56

One very useful feature of the calculator is its ability to store and recall equations. The solution from various values for known variables can be easily obtained by recalling an equation which has been stored and giving values to the known variables in the Solver mode. If a starting point is known, Newton's method is useful for quick solution of a complicated equation.

Graphing Polynomials and Tracing to Find the Roots

A polynomial $y = f(x)$ is an expression of the sums of several terms that contain different powers of the same originals. The roots are found at the intersection of the x-axis and the graph, i. e. when $y = 0$.

Example

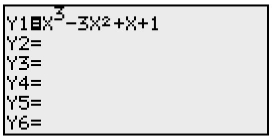
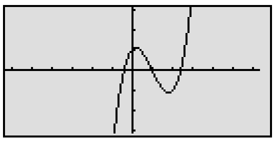
Draw a graph of a polynomial and approximate the roots by using the Zoom-in and Trace features.

1. Graph the polynomial $y = x^3 - 3x^2 + x + 1$.
2. Approximate the left-hand root.
3. Approximate the middle root.
4. Approximate the right-hand root.

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Set the zoom to the decimal window: ZOOM A (ENTER ALPHA ▼) 7

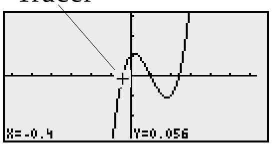
Setting the zoom factors to 5 : ZOOM B ENTER 5 ENTER 5 ENTER 2nd F QUIT

Step & Key Operation	Display	Notes
1-1 Enter the polynomial $y = x^3 - 3x^2 + x + 1$. Y= X/θ/T/n a^b 3 ▶ — 3 X/θ/T/n x^2 + X/θ/T/n + 1		
1-2 View the graph. GRAPH		

Step & Key Operation	***Display***	***Notes***

2*-1* Move the tracer near the left-hand root.

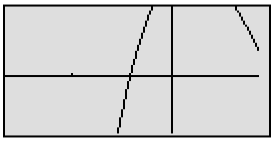
[TRACE] [◀] (repeatedly)



Note that the tracer is flashing on the curve and the *x* and *y* coordinates are shown at the bottom of the screen.

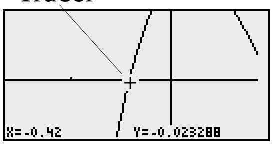
2*-2* Zoom in on the left-hand root.

[ZOOM] [A] [3]



2*-3* Move the tracer to approximate the root.

[TRACE] [◀] or [▶] (repeatedly)

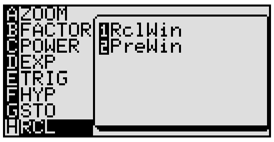


The root is : $x \fallingdotseq -0.42$

3*-1* Return to the previous decimal viewing window.

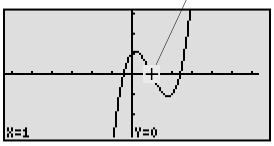
[ZOOM] [H]

[2]



3*-2* Move the tracer to approximate the middle root.

[TRACE] [▶] (repeatedly)



The root is exactly $x = 1$. (Zooming is not needed to find a better approximate.)

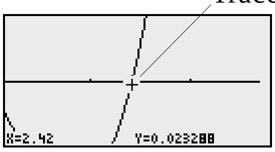
---

4 Move the tracer near the right-hand root.
Zoom in and move the tracer to find a better approximate.

[▶] (repeatedly)

[ZOOM] [A] [3]

[TRACE] [▶] or [◀] (repeatedly)



The root is : $x \fallingdotseq 2.42$

The calculator allows the roots to be found (or approximated) visually by graphing a polynomial and using the Zoom-in and Trace features.

Graphing Polynomials and Jumping to Find the Roots

A polynomial $y = f(x)$ is an expression of the sums of several terms that contain different powers of the same originals. The roots are found at the intersection of the x-axis and the graph, i. e. when $y = 0$.

Example

Draw a graph of a polynomial and find the roots by using the Calculate feature.

1. Graph the polynomial $y = x^4 + x^3 - 5x^2 - 3x + 1$.

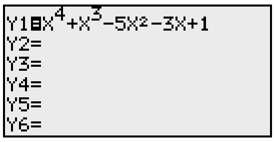
2. Find the four roots one by one.

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.
Setting the zoom factors to 5 : [ZOOM] [A] [ENTER] [A] [ENTER] [A] [ENTER] [2nd F] [QUIT]

Step & Key Operation	Display	Notes

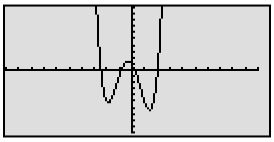
1-1 Enter the polynomial $y = x^4 + x^3 - 5x^2 - 3x + 1$

[Y=] [X/θ/T/n] [a^b] [4] [▶] [+] [X/θ/T/n] [a^b] [3] [▶] [—] [5] [X/θ/T/n] [X^2] [—] [3] [X/θ/T/n] [+] [1]



1-2 View the graph.

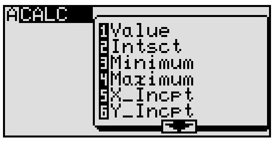
[GRAPH]

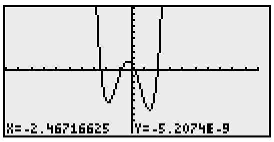


2-1 Find the first root.

[2nd F] [CALC]

[5]

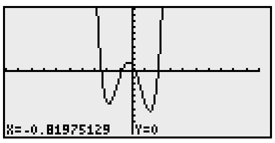




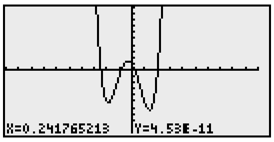
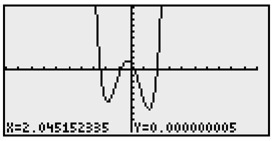
$x \fallingdotseq -2.47$
Y is almost but not exactly zero. Notice that the root found here is an approximate value.

2-2 Find the next root.

[2nd F] [CALC] [5]



$x \fallingdotseq -0.82$

Step & Key Operation	Display	Notes
2-3 Find the next root. 2nd F CALC 5		$x \fallingdotseq 0.24$
2-4 Find the next root. 2nd F CALC 5		$x \fallingdotseq 2.05$

The calculator allows jumping to find the roots by graphing a polynomial and using the Calculate feature, without tracing the graph.

Solving a System of Equations by Graphing or Tool Feature

A system of equations is made up of two or more equations. The calculator provides the Calculate feature and Tool feature to solve a system of equations. The Calculate feature finds the solution by calculating the intersections of the graphs of equations and is useful for solving a system when there are two variables, while the Tool feature can solve a linear system with up to six variables and six equations.

Example

Solve a system of equations using the Calculate or Tool feature. First, use the Calculate feature. Enter the equations, draw the graph, and find the intersections. Then, use the Tool feature to solve a system of equations.

1. Solve the system using the Calculate feature.

$$\begin{cases} y = x^2 - 1 \\ y = 2x \end{cases}$$

2. Solve the system using the Tool feature.

$$\begin{cases} 5x + y = 1 \\ -3x + y = -5 \end{cases}$$

Before Starting

There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Set viewing window to "-5 < X < 5", "-10 < Y < 10".

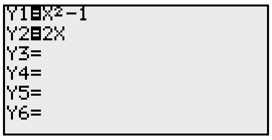
[WINDOW] [(-)] [5] [ENTER] [5] [ENTER]

As the Tool feature is only available on the Advanced keyboard, example 2 does not apply to the Basic keyboard.

Step & Key Operation	Display	Notes

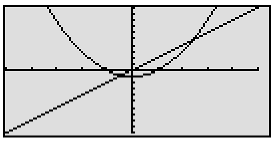
1-1 Enter the system of equations $y = x^2 - 1$ for Y1 and $y = 2x$ for Y2.

[Y=] [X/θ/T/n] [X^2] [—] [1] [ENTER] [2] [X/θ/T/n]



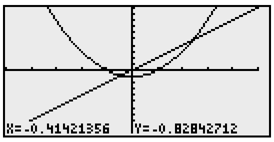
1-2 View the graphs.

[GRAPH]



1-3 Find the left-hand intersection using the Calculate feature.

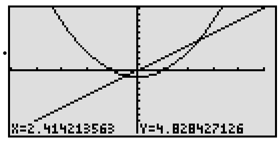
[2nd F] [CALC] [2]



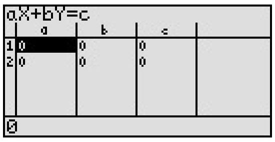
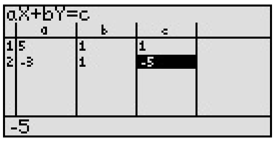
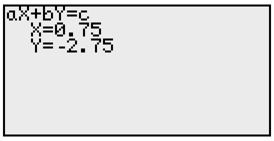
Note that the x and y coordinates are shown at the bottom of the screen. The answer is : x = -0.41 y = -0.83

1-4 Find the right-hand intersection by accessing the Calculate feature again.

[2nd F] [CALC] [2]



The answer is : x = 2.41 y = 4.83

Step & Key Operation	Display	Notes
2-1 Access the Tool menu. Select the number of variables. 2nd F TOOL B 2		Using the system function, it is possible to solve simultaneous linear equations. Systems up to six variables and six equations can be solved.
2-2 Enter the system of equations. 5 ENTER 1 ENTER 1 ENTER (-) 3 ENTER 1 ENTER (-) 5 ENTER		
2-3 Solve the system. 2nd F EXE		$x = 0.75$ $y = -2.75$

A system of equations can be solved easily by using the Calculate feature or Tool feature.

SHARP

Entering and Multiplying Matrices

A matrix is a rectangular array of elements in rows and columns that is treated as a single element. A matrix is often used for expressing multiple linear equations with multiple variables.

Example

Enter two matrices and execute multiplication of the two.

1. Enter a 3x3 matrix A
2. Enter a 3x3 matrix B
3. Multiply the matrices A and B

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

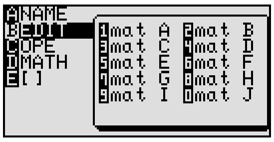
Before Starting
There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.
As the Matrix feature is only available on the Advanced keyboard, this section does not apply to the Basic keyboard.

Step & Key Operation	Display	Notes

1-1 Access the matrix menu.

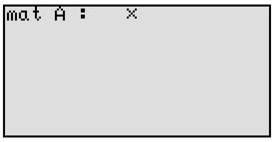
2nd F MATRIX B

1



1-2 Set the dimension of the matrix at three rows by three columns.

3 ENTER 3 ENTER

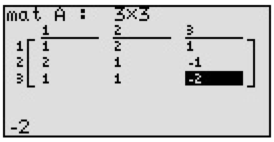


1-3 Enter the elements of the first row, the elements of the second row, and the elements of the third row.

1 ENTER 2 ENTER 1 ENTER

2 ENTER 1 ENTER (-) 1 ENTER

1 ENTER 1 ENTER (-) 2 ENTER



Step & Key Operation	Display	Notes

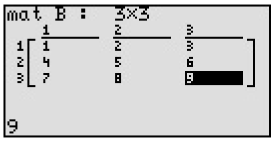
2 Enter a 3x3 matrix B.

2nd F MATRIX B 2 3 ENTER 3 ENTER

1 ENTER 2 ENTER 3 ENTER

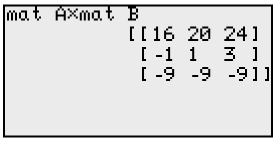
4 ENTER 5 ENTER 6 ENTER

7 ENTER 8 ENTER 9 ENTER



***3**-1* Multiply the matrices A and B together at the home screen.

[home] 2nd F MATRIX A 1 × 2nd F MATRIX A 2 ENTER



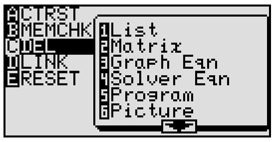
Matrix multiplication can be performed if the number of columns of the first matrix is equal to the number of rows of the second matrix. The sum of these multiplications (1•1 + 2•4 + 1•7) is placed in the 1,1 (first row, first column) position of the resulting matrix. This process is repeated until each row of A has been multiplied by each column of B.

***3**-2* Delete the input matrices for future use.

2nd F OPTION C

2 ENTER ENTER

2nd F QUIT



Matrix multiplication can be performed easily by the calculator.

SHARP

Solving a System of Linear Equations Using Matrices

Each system of three linear equations consists of three variables. Equations in more than three variables cannot be graphed on the graphing calculator. The solution of the system of equations can be found numerically using the Matrix feature or the System solver in the Tool feature.

A system of linear equations can be expressed as AX = B (A, X and B are matrices). The solution matrix X is found by multiplying $A^{-1}B$. Note that the multiplication is "order sensitive" and the correct answer will be obtained by multiplying BA^{-1}. An inverse matrix A^{-1} is a matrix that when multiplied by A results in the identity matrix I ($A^{-1} \times A = I$). The identity matrix I is defined to be a square matrix (n×n) where each position on the diagonal is 1 and all others are 0.

Example

Use matrix multiplication to solve a system of linear equations.

1. Enter the 3×3 identity matrix in matrix A.
2. Find the inverse matrix of the matrix B.
3. Solve the equation system.

$$\begin{cases} x + 2y + z = 8 \\ 2x + y - z = 1 \\ x + y - 2z = -3 \end{cases}$$

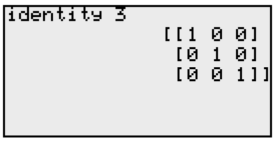
$$B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{bmatrix}$$

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.
As the Matrix feature is only available on the Advanced keyboard, this section does not apply to the Basic keyboard.

Step & Key Operation	Display	Notes

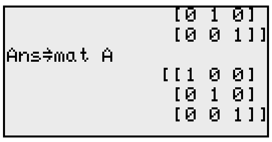
1-1 Set up 3×3 identity matrix at the home screen.

2nd F MATRIX C 0 5 3 ENTER



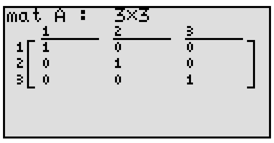
1-2 Save the identity matrix in matrix A.

STO 2nd F MATRIX A 1 ENTER



1-3 Confirm that the identity matrix is stored in matrix A.

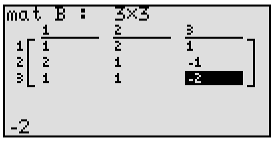
2nd F MATRIX B 1



Step & Key Operation	***Display***	***Notes***

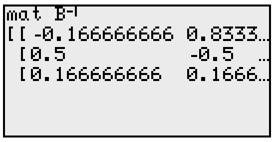
2-1 Enter a 3x3 matrix B.

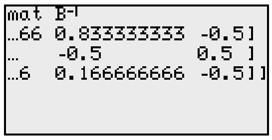
[2nd F] [MATRIX] [B] [2] [3] [ENTER] [3] [ENTER]
[1] [ENTER] [2] [ENTER] [1] [ENTER]
[2] [ENTER] [1] [ENTER] [(-)] [1] [ENTER]
[1] [ENTER] [1] [ENTER] [(-)] [2] [ENTER]



2-2 Exit the matrix editor and find the inverse of the square matrix B.

[2nd F] [QUIT] [CL]
[2nd F] [MATRIX] [A] [2] [2nd F] [X^{-1}] [ENTER]
[▶] (repeatedly)



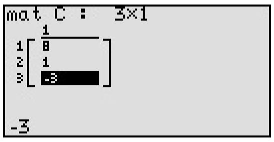


Some square matrices have no inverse and will generate error statements when calculating the inverse.

$$B^{-1}=\begin{bmatrix} -0.17 & 0.83 & -0.5 \\ 0.5 & -0.5 & 0.5 \\ 0.17 & 0.17 & -0.5 \end{bmatrix}$$

3-1 Enter the constants on the right side of the equal sign into matrix C (3x1).

[2nd F] [MATRIX] [B] [3] [3] [ENTER] [1] [ENTER]
[8] [ENTER] [1] [ENTER] [(-)] [3] [ENTER]



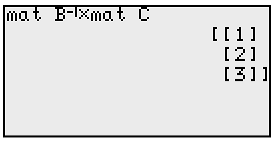
The system of equations can be expressed as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ -3 \end{bmatrix}$$

Let each matrix B, X, C :
BX = C
$B^{-1}BX = B^{-1}C$ (multiply both sides by B^{-1})
$I = B^{-1}$ ($B^{-1}B = I$, identity matrix)
$X = B^{-1}C$

3-2 Calculate $B^{-1}C$.

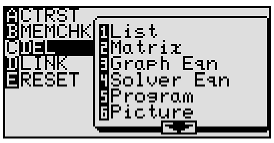
[⊞] [CL] [2nd F] [MATRIX] [A] [2]
[2nd F] [X^{-1}] [X] [2nd F] [MATRIX] [A] [3] [ENTER]



The 1 is the x coordinate, the 2 the y coordinate, and the 3 the z coordinate of the solution point.
$(x, y, z)=(1, 2, 3)$

3-3 Delete the input matrices for future use.

[2nd F] [OPTION] [C]
[2] [ENTER]
[2nd F] [QUIT]



The calculator can execute calculation of inverse matrix and matrix multiplication. A system of linear equations can be solved easily using the Matrix feature.

Solving Inequalities

To solve an inequality, expressed by the form of $f(x) \leq 0$, $f(x) \geq 0$, or form of $f(x) \leq g(x)$, $f(x) \geq g(x)$, means to find all values that make the inequality true.

There are two methods of finding these values for one-variable inequalities, using graphical techniques. The first method involves rewriting the inequality so that the right-hand side of the inequality is 0 and the left-hand side is a function of x. For example, to find the solution to $f(x) < 0$, determine where the graph of $f(x)$ is below the x-axis. The second method involves graphing each side of the inequality as an individual function. For example, to find the solution to $f(x) < g(x)$, determine where the graph of $f(x)$ is below the graph of $g(x)$.

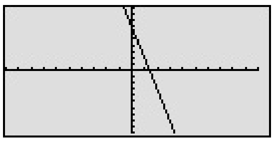
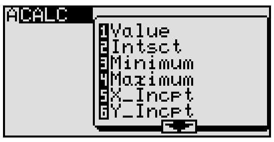
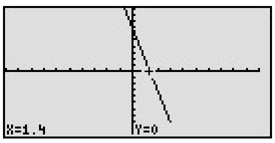
Example

Solve an inequality in two methods.

1. Solve $3(4 - 2x) \geq 5 - x$, by rewriting the right-hand side of the inequality as 0.

2. Solve $3(4 - 2x) \geq 5 - x$, by shading the solution region that makes the inequality true.

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

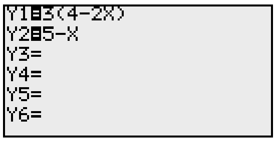
Step & Key Operation	Display	Notes
1-1 Rewrite the equation $3(4 - 2x) \geq 5 - x$ so that the right-hand side becomes 0, and enter $y = 3(4 - 2x) - 5 + x$ for Y1. [Y=] [3] [(] [4] [—] [2] [X/θ/T/n] [)] [—] [5] [+] [X/θ/T/n]		$3(4 - 2x) \geq 5 - x$ → $3(4 - 2x) - 5 + x \geq 0$
1-2 View the graph. [GRAPH]		
1-3 Find the location of the x-intercept and solve the inequality. [2nd F] [CALC] [5]	 	The x-intercept is located at the point (1.4, 0). Since the graph is above the x-axis to the left of the x-intercept, the solution to the inequality $3(4 - 2x) - 5 + x \geq 0$ is all values of x such that $x \leq 1.4$.

Step & Key Operation	***Display***	***Notes***

2-1 Enter y = 3(4 - 2x) for Y1 and y = 5 - x for Y2.

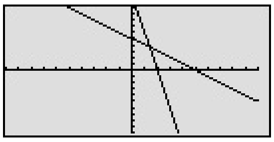
[Y=] [▶] (7 times) [DEL] (4 times)

[ENTER] [5] [—] [X/θ/T/n]



2-2 View the graph.

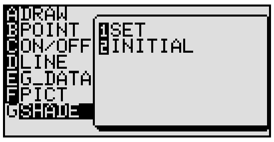
[GRAPH]



2-3 Access the Set Shade screen.

[2nd F] [DRAW] [G]

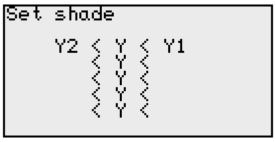
[1]



2-4 Set up the shading.

[2nd F] [VARS] [A] [ENTER] [A] [2] [▶]

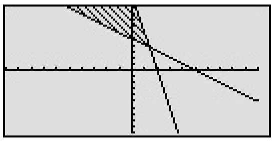
[2nd F] [VARS] [ENTER] [1]



Since the inequality being solved is Y1 ≥ Y2, the solution is where the graph of Y1 is "on the top" and Y2 is "on the bottom."

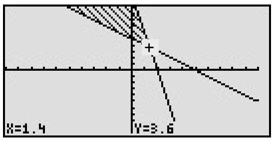
2-5 View the shaded region.

[GRAPH]



2-6 Find where the graphs intersect and solve the inequality.

[2nd F] [CALC] [2]



The point of intersection is (1.4, 3.6). Since the shaded region is to the left of x = 1.4, the solution to the inequality 3(4 - 2x) ≥ 5 - x is all values of x such that x ≤ 1.4.

Graphical solution methods not only offer instructive visualization of the solution process, but they can be applied to inequalities that are often difficult to solve algebraically. The EL-9900 allows the solution region to be indicated visually using the Shade feature. Also, the points of intersection can be obtained easily.

Solving Double Inequalities

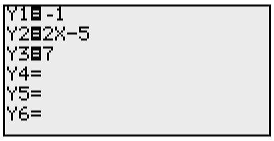
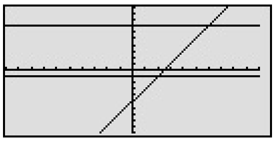
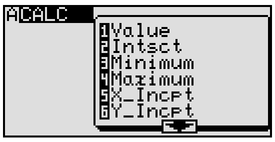
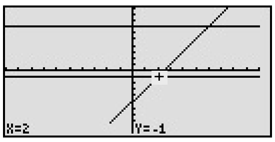
The solution to a system of two inequalities in one variable consists of all values of the variable that make each inequality in the system true. A system $f(x) \geq a$, $f(x) \leq b$, where the same expression appears on both inequalities, is commonly referred to as a “double” inequality and is often written in the form $a \leq f(x) \leq b$. Be certain that both inequality signs are pointing in the same direction and that the double inequality is only used to indicate an expression in x “trapped” in between two values. Also a must be less than or equal to b in the inequality $a \leq f(x) \leq b$ *or* $b \geq f(x) \geq a$.

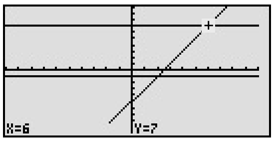
Example

Solve a double inequality, using graphical techniques.

$2x - 5 \geq -1$
$2x - 5 \leq 7$

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Step & Key Operation	***Display***	***Notes***
1 Enter $y = -1$ for Y1, $y = 2x - 5$ for Y2, and $y = 7$ for Y3. [Y=] [(-)] [1] [ENTER] [2] [X/θ/T/n] [—] [5] [ENTER] [7]		The “double” inequality given can also be written to $-1 \leq 2x - 5 \leq 7$.
2 View the lines. [GRAPH]		
3 Find the point of intersection. [2nd F] [CALC] [2]	 	$y = 2x - 5$ and $y = -1$ intersect at (2, -1).

Step & Key Operation	Display	Notes
4 Move the tracer and find another intersection. ▲ 2nd F CALC 2		$y = 2x - 5$ and $y = 7$ intersect at (6,7).
5 Solve the inequalities.		The solution to the "double" inequality $-1 \le 2x - 5 \le 7$ consists of all values of x in between, and including, 2 and 6 (i.e., $x \ge 2$ and $x \le 6$). The solution is $2 \le x \le 6$.

Graphical solution methods not only offer instructive visualization of the solution process, but they can be applied to inequalities that are often difficult to solve algebraically. The EL-9900 allows the solution region to be indicated visually using the Shade feature. Also, the points of intersection can be obtained easily.

SHARP

System of Two-Variable Inequalities

The solution region of a system of two-variable inequalities consists of all points (a, b) such that when $x = a$ and $y = b$, all inequalities in the system are true. To solve two-variable inequalities, the inequalities must be manipulated to isolate the y variable and enter the other side of the inequality as a function. The calculator will only accept functions of the form y = ___. (where y is defined explicitly in terms of x).

Example

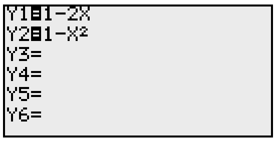
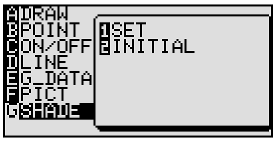
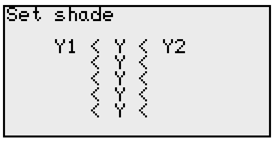
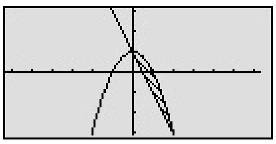
Solve a system of two-variable inequalities by shading the solution region.

$2x + y \geq 1$

$x^2 + y \leq 1$

Before Starting

There may be differences in the results of calculations and graph plotting depending on the setting.
Return all settings to the default value and delete all data.
Set the zoom to the decimal window: [ZOOM] [A] ([ENTER] [2nd F] [▼]) [7]

Step & Key Operation	Display	Notes
1 Rewrite each inequality in the system so that the left-hand side is y :		$2x + y \geq 1 \rightarrow y \geq 1 - 2x$ $x^2 + y \leq 1 \rightarrow y \leq 1 - x^2$
2 Enter $y = 1 - 2x$ for Y1 and $y = 1 - x^2$ for Y2. [Y=] [1] [—] [2] [X/θ/T/n] [ENTER] [1] [—] [X/θ/T/n] [X^2]		
3 Access the set shade screen [2nd F] [DRAW] [G] [1]		
4 Shade the points of y -value so that Y1 $\leq y \leq$ Y2. [2nd F] [VARS] [A] [ENTER] [A] [1] [▶] [2nd F] [VARS] [ENTER] [2]		
5 Graph the system and find the intersections. [GRAPH] [2nd F] [CALC] [2] [2nd F] [CALC] [2]		The intersections are (0, 1) and (2, -3)
6 Solve the system.		The solution is $0 \leq x \leq 2$.

Graphical solution methods not only offer instructive visualization of the solution process, but they can be applied to inequalities that are often difficult to solve algebraically. The EL-9900 allows the solution region to be indicated visually using the Shade feature. Also, the points of intersection can be obtained easily.

Graphing Solution Region of Inequalities

The solution region of an inequality consists of all points (a, b) such that when $x = a$, and $y = b$, all inequalities are true.

Example

Check to see if given points are in the solution region of a system of inequalities.

1. Graph the solution region of a system of inequalities:

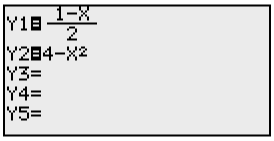
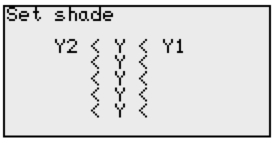
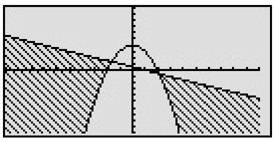
$x + 2y \le 1$

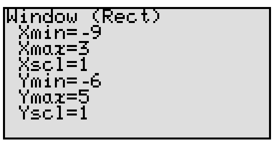
$x^2 + y \ge 4$

2. Which of the following points are within the solution region?

(-1.6, 1.8), (-2, -5), (2.8, -1.4), (-8,4)

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

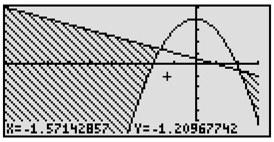
Step & Key Operation	Display	Notes
1-1 Rewrite the inequalities so that the left-hand side is *y*.		$x + 2y \le 1 \rightarrow y \le \frac{1-x}{2}$ $x^2+y \ge 4 \rightarrow y \ge 4 - x^2$
1-2 Enter $y = \frac{1-x}{2}$ for Y1 and $y = 4 - x^2$ for Y2. Y= a/b 1 — X/θ/T/n ▼ 2 ENTER 4 — X/θ/T/n x^2		
1-3 Set the shade and view the solution region. 2nd F DRAW G 1 2nd F VARS A ENTER A 2 ▶ 2nd F VARS ENTER 1 GRAPH	 	Y2 ≤ *y* ≤ Y1

Step & Key Operation	Display	Notes
2-1 Set the display area (window) to : $-9 < x < 3$, $-6 < y < 5$. WINDOW (-) 9 ENTER 3 ENTER ENTER (-) 6 ENTER 5 ENTER		

Step & Key Operation | ***Display*** | ***Notes***

2-2 Use the cursor to check the position of each point. (Zoom in as necessary).

[GRAPH] [▶] or [◀] or [▲] or [▼]

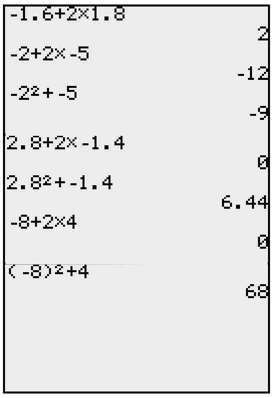


Points in the solution region are (2.8, -1.4) and (-8, 4). Points outside the solution region are (-1.6, 1.8) and (-2, -5).

2-3 Substitute points and confirm whether they are in the solution region.

[⊞] [(-)] [1] [•] [6] [+] [2] [×] [1] [•] [8] •••

(Continuing key operations omitted.)



- (-1.6, 1.8): -1.6 + 2 × 1.8 = 2
  → This does not materialize.
- (-2, -5): -2 + 2 × (-5) = -12
  $(-2)^2 + (-5) = -1$
  → This does not materialize.
- (2.8, -1.4): 2.8 + 2 × (-1.4) = 0
  $(2.8)^2 + (-1.4) = 6.44$
  → This materializes.
- (-8, 4): -8 + 2 × 4 = 0
  $(-8)^2 + 4 = 68$
  → This materializes.

Graphical solution methods not only offer instructive visualization of the solution process, but they can be applied to inequalities that are often very difficult to solve algebraically. The EL-9900 allows the solution region to be indicated visually using the Shading feature. Also, the free-moving tracer or Zoom-in feature will allow the details to be checked visually.

Slope and Intercept of Absolute Value Functions

The absolute value of a real number x is defined by the following:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x \leq 0 \end{cases}$$

If n is a positive number, there are two solutions to the equation $|f(x)| = n$ because there are exactly two numbers with the absolute value equal to n: n and $-n$. The existence of two distinct solutions is clear when the equation is solved graphically.
An absolute value function can be presented as $y = a|x - h| + k$. The graph moves as the changes of slope a, x-intercept h, and y-intercept k.

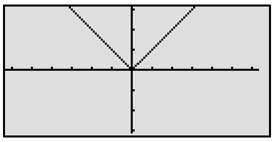
Example

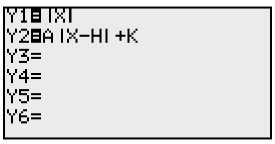
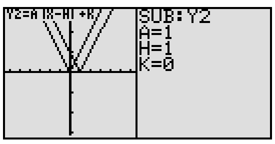
Consider various absolute value functions and check the relation between the graphs and the values of coefficients.

1. Graph $y = |x|$

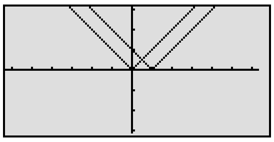
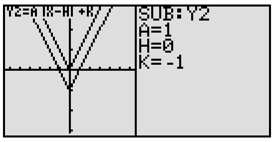
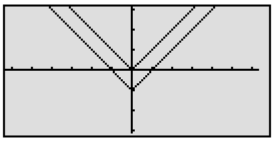
2. Graph $y = |x -1|$ and $y = |x|-1$ using Rapid Graph feature.

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.
Set the zoom to the decimal window: [ZOOM] [A] ([ENTER] [2nd F] [▼]) [7]

Step & Key Operation	Display	Notes
1-1 Enter the function $y = \|x\|$ for Y1. [Y=] [MATH] [B] [1] [X/θ/T/n]		
1-2 View the graph. [GRAPH]		Notice that the domain of $f(x) = \|x\|$ is the set of all real numbers and the range is the set of non-negative real numbers. Notice also that the slope of the graph is 1 in the range of X > 0 and -1 in the range of X ≤ 0.

Step & Key Operation	Display	Notes
2-1 Enter the standard form of an absolute value function for Y2 using the Rapid Graph feature. [Y=] [▼] [ALPHA] [A] [MATH] [B] [1] [X/θ/T/n] [—] [ALPHA] [H] [▶] [+] [ALPHA] [K]		
2-2 Substitute the coefficients to graph $y = \|x - 1\|$. [2nd F] [SUB] [1] [ENTER] [1] [ENTER] [0] [ENTER]		

SHARP

Step & Key Operation	***Display***	***Notes***
2-3 View the graph. [GRAPH]		Notice that placing an *h* (>0) within the standard form *y* = *a* | *x* - *h* | + *k* will move the graph right *h* units on the *x*-axis.
2-4 Change the coefficients to graph *y* = | *x* | -1. [Y=] [▼] [2nd F] [SUB] [ENTER] [1] [ENTER] [0] [ENTER] [(-)] [1] [ENTER]		
2-5 View the graph. [GRAPH]		Notice that adding a *k* (>0) within the standard form y=*a* | *x*-*h* | +*k* will move the graph up *k* units on the *y*-axis.

The EL-9900 shows absolute values with | |, just as written on paper, by using the Equation editor. Use of the calculator allows various absolute value functions to be graphed quickly and shows their characteristics in an easy-to-understand manner.

Solving Absolute Value Equations

The absolute value of a real number *x* is defined by the following:

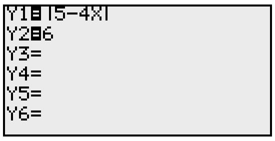
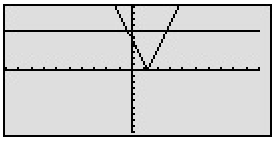
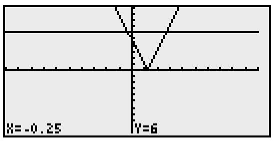
$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x \leq 0 \end{cases}$$

If *n* is a positive number, there are two solutions to the equation $|f(x)| = n$ because there are exactly two numbers with the absolute value equal to *n*: *n* and -*n*. The existence of two distinct solutions is clear when the equation is solved graphically.

Example

Solve an absolute value equation $|5 - 4x| = 6$

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Step & Key Operation	Display	Notes
1 Enter $y = |5 - 4x|$ for Y1. Enter $y = 6$ for Y2. [Y=] [MATH] [B] [1] [5] [—] [4] [X/θ/T/n] [ENTER] [6]		
2 View the graph. [GRAPH]		There are two points of intersection of the absolute value graph and the horizontal line $y = 6$.
3 Find the points of intersection of the two graphs and solve. [2nd F] [CALC] [2] [2nd F] [CALC] [2]		The solution to the equation $|5 - 4x| = 6$ consists of the two values -0.25 and 2.75. Note that although it is not as intuitively obvious, the solution could also be obtained by finding the *x*-intercepts of the function $y = |5x - 4| - 6$.

The EL-9900 shows absolute values with | |, just as written on paper, by using the Equation editor. The graphing feature of the calculator shows the solution of the absolute value function visually.

SHARP

Solving Absolute Value Inequalities

To solve an inequality means to find all values that make the inequality true. Absolute value inequalities are of the form $|f(x)| < k$, $|f(x)| \le k$, $|f(x)| > k$, *or* $|f(x)| \ge k$. The graphical solution to an absolute value inequality is found using the same methods as for normal inequalities. The first method involves rewriting the inequality so that the right-hand side of the inequality is 0 and the left-hand side is a function of x. The second method involves graphing each side of the inequality as an individual function.

Example

Solve absolute value inequalities in two methods.

1. Solve $|20 - \frac{6x}{5}| < 8$ by rewriting the inequality so that the right-hand side of the inequality is zero.

2. Solve $|3.5x + 4| > 10$ by shading the solution region.

Before Starting

There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Set viewing window to "-5< x <50," and "-10< y <10".

WINDOW (-) 5 ENTER 5 0 ENTER

Step & Key Operation | **Display** | **Notes**

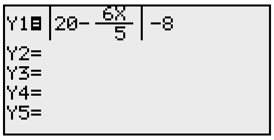
1-1 Rewrite the equation.

Notes: $|20 - \frac{6x}{5}| < 8$

$\rightarrow |20 - \frac{6x}{5}| - 8 < 0.$

1-2 Enter $y = |20 - \frac{6x}{5}| - 8$ for Y1.

Y= MATH B 1 2 0 — a/b 6 X/θ/T/n ▶ 5 ▶ ▶ — 8



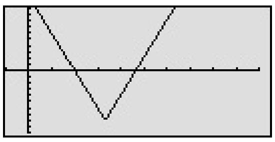
1-3 View the graph, and find the x-intercepts.

GRAPH

2nd F CALC 5 → $x = 10,\ y = 0$

2nd F CALC 5 → $x = 23.33333334$

$y = 0.00000006$ (∗ Note)



Notes: The intersections with the x-axis are (10, 0) and (23.3, 0) (∗ Note: The value of y in the x-intercepts may not appear exactly as 0 as shown in the example, due to an error caused by approximate calculation.)

1-4 Solve the inequality.

Notes: Since the graph is below the x-axis for x in between the two x-intercepts, the solution is $10 < x < 23.3$.

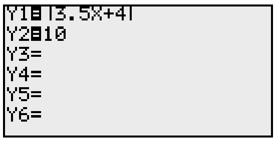
Step & Key Operation	***Display***	***Notes***

2-1 Enter the function
$y = |3.5x + 4|$ for Y1.
Enter y = 10 for Y2.

[Y=] [CL] [MATH] [B] [1]

[3] [•] [5] [X/θ/T/n] [+] [4] [ENTER]

[1] [0]

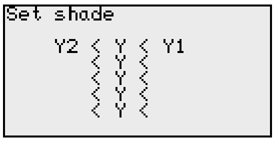


2-2 Set up shading.

[2nd F] [DRAW] [G] [1]

[2nd F] [VARS] [A] [ENTER] [A] [2] [▶]

[2nd F] [VARS] [ENTER] [1]



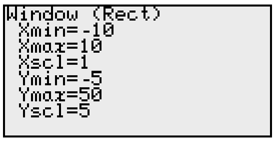
Since the inequality you are solving is Y1 > Y2, the solution is where the graph of Y2 is "on the bottom" and Y1 in "on the top."

2-3 Set viewing window to "$-10 < x < 10$" and "$-5 < y < 50$", and view the graph.

[WINDOW] [(-)] [1] [0] [ENTER] [1] [0]

[ENTER] [ENTER] [(-)] [5] [ENTER] [5] [0]

[ENTER] [5] [ENTER]

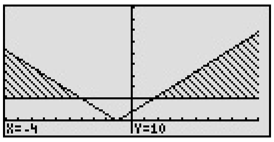


2-4 Find the points of intersection.
Solve the inequality.

[2nd F] [CALC] [2] → $x = -4$, $y = 10$

[2nd F] [CALC] [2] → $x = 1.714285714$

$y = 9.999999999$ (∗ Note)



The intersections are (-4, 10) and (1.7, 10.0). The solution is all values of x such that $x < -4$ or $x > 1.7$.
(∗ Note: The value of y in the intersection of the two graphs may not appear exactly as 10 as shown in the example, due to an error caused by approximate calculation.)

The EL-9900 shows absolute values with | |, just as written on paper, by using the Equation editor. Graphical solution methods not only offer instructive visualization of the solution process, but they can be applied to inequalities that are often difficult to solve algebraically. The Shade feature is useful to solve the inequality visually and the points of intersection can be obtained easily.

Evaluating Absolute Value Functions

The absolute value of a real number x is defined by the following:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x \leq 0 \end{cases}$$

Note that the effect of taking the absolute value of a number is to strip away the minus sign if the number is negative and to leave the number unchanged if it is nonnegative. Thus, $|x| \geq 0$ for all values of x.

Example

Evaluate various absolute value functions.

1. Evaluate $|-2(5-1)|$

2. Is $|-2+7| = |-2| + |7|$?
Evaluate each side of the equation to check your answer.
Is $|x + y| = |x| + |y|$ for all real numbers x and y?
If not, when will $|x + y| = |x| + |y|$?

3. Is $\left|\frac{6-9}{1+3}\right| = \frac{|6-9|}{|1+3|}$?
Evaluate each side of the equation to check your answer. Investigate with more examples, and decide if you think $|x / y| = |x| / |y|$

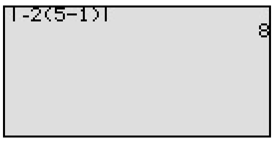
Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Step & Key Operation	Display	Notes

1-1 Access the home or computation screen.

1-2 Enter $y = |-2(5-1)|$ and evaluate.

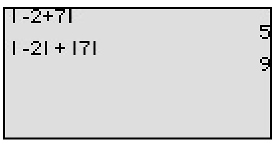
[MATH] [B] [1] [(-)] [2] [(] [5] [—] [1] [)] [ENTER]



The solution is ±8.

2-1 Evaluate $|-2 + 7|$. Evaluate $|-2|+|7|$.

[CL]
[MATH] [1] [(-)] [2] [+] [7] [ENTER]
[MATH] [1] [(-)] [2] [▶] [+] [MATH] [1] [7] [ENTER]



$|-2 + 7| = 5$, $|-2| + |7| = 9$
$\rightarrow |-2 + 7| \neq |-2| + |7|$.

Step & Key Operation	Display	Notes

2-2 Is $|x + y| = |x| + |y|$? Think about this problem according to the cases when x or y are positive or negative.

If $x \geq 0$ and $y \geq 0$ [e.g.; $(x, y) = (2,7)$]

$|x + y| = |2 + 7| = 9$
$|x| + |y| = |2| + |7| = 9$
$\rightarrow |x + y| = |x| + |y|$.

If $x \leq 0$ and $y \geq 0$ [e.g.; $(x, y) = (-2, 7)$]

$|x + y| = |-2 + 7| = 5$
$|x| + |y| = |-2| + |7| = 9$
$\rightarrow |x + y| \neq |x| + |y|$.

If $x \geq 0$ and $y \leq 0$ [e.g.; $(x, y) = (2, -7)$]

$|x + y| = |2-7| = 5$
$|x| + |y| = |2| + |-7| = 9$
$\rightarrow |x + y| \neq |x| + |y|$.

If $x \leq 0$ and $y \leq 0$ [e.g.; $(x, y) = (-2, -7)$]

$|x + y| = |-2-7| = 9$
$|x| + |y| = |-2| + |-7| = 9$
$\rightarrow |x + y| = |x| + |y|$.

Therefore $|x + y| = |x| + |y|$ when $x \geq 0$ and $y \geq 0$, and when $x \leq 0$ and $y \leq 0$.

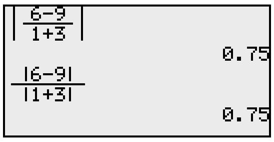
3-1 Evaluate $\left|\frac{6-9}{1+3}\right|$. Evaluate $\frac{|6-9|}{|1+3|}$.

[CL] [MATH] [1] [a/b] [6] [—] [9]

[▶] [1] [+] [3] [ENTER]

[MATH] [1] [6] [—] [9] [▶] [a/b]

[MATH] [1] [1] [+] [3] [ENTER]



$\left|\frac{6-9}{1+3}\right| = 0.75$, $\frac{|6-9|}{|1+3|} = 0.75$

$\rightarrow \left|\frac{6-9}{1+3}\right| = \frac{|6-9|}{|1+3|}$

3-2 Is $|x / y| = |x| / |y|$? Think about this problem according to the cases when x or y are positive or negative.

If $x \geq 0$ and $y \geq 0$ [e.g.; $(x, y) = (2,7)$]

$|x / y| = |2/7| = 2/7$
$|x| / |y| = |2| / |7| = 2/7$
$\rightarrow |x / y| = |x| / |y|$

If $x \leq 0$ and $y \geq 0$ [e.g.; $(x, y) = (-2, 7)$]

$|x / y| = |(-2)/7| = 2/7$
$|x| / |y| = |-2| / |7| = 2/7$
$\rightarrow |x / y| = |x| / |y|$

If $x \geq 0$ and $y \leq 0$ [e.g.; $(x, y) = (2, -7)$]

$|x / y| = |2/(-7)| = 2/7$
$|x| / |y| = |2| / |-7| = 2/7$
$\rightarrow |x / y| = |x| / |y|$

If $x \leq 0$ and $y \leq 0$ [e.g.; $(x, y) = (-2, -7)$]

$|x / y| = |(-2)/-7| = 2/7$
$|x| / |y| = |-2| / |-7| = 2/7$
$\rightarrow |x / y| = |x| / |y|$

The statement is true for all $y \neq 0$.

The EL-9900 shows absolute values with | |, just as written on paper, by using the Equation editor. The nature of arithmetic of the absolute value can be learned through arithmetical operations of absolute value functions.

Graphing Rational Functions

A rational function $f(x)$ is defined as the quotient $\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are two polynomial functions such that $q(x) \neq 0$. The domain of any rational function consists of all values of x such that the denominator $q(x)$ is not zero.

A rational function consists of branches separated by vertical asymptotes, and the values of x that make the denominator $q(x) = 0$ but do not make the numerator $p(x) = 0$ are where the vertical asymptotes occur. It also has horizontal asymptotes, lines of the form $y = k$ (k, a constant) such that the function gets arbitrarily close to, but does not cross, the horizontal asymptote when $|x|$ is large.

The x intercepts of a rational function $f(x)$, if there are any, occur at the x-values that make the numerator $p(x)$, but not the denominator $q(x)$, zero. The y-intercept occurs at $f(0)$.

Example

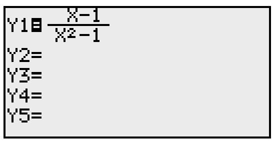
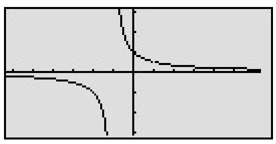
Graph the rational function and check several points as indicated below.

1. Graph $f(x) = \frac{x-1}{x^2-1}$.
2. Find the domain of $f(x)$, and the vertical asymptote of $f(x)$.
3. Find the x- and y-intercepts of $f(x)$.
4. Estimate the horizontal asymptote of $f(x)$.

Before Starting

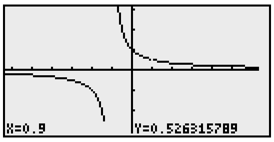
There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Set the zoom to the decimal window: [ZOOM] [A] ([ENTER] [ALPHA] [▼]) [7]

Step & Key Operation	Display	Notes
1-1 Enter $y = \frac{x-1}{x^2-1}$ for Y1. [Y=] [a/b] [X/θ/T/n] [—] [1] [▼] [X/θ/T/n] [x^2] [—] [1]		
1-2 View the graph. [GRAPH]		The function consists of two branches separated by the vertical asymptote.

Step & Key Operation | ***Display*** | ***Notes***

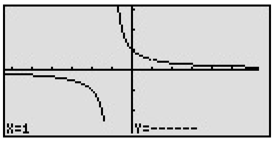
2 Find the domain and the vertical asymptote of $f(x)$, tracing the graph to find the hole at $x = 1$.



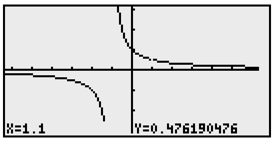
Since $f(x)$ can be written as $\frac{x - 1}{(x + 1)(x - 1)}$, the domain consists of all real numbers x such that $x \neq 1$ and $x \neq -1$. There is no vertical asymptote where $x = 1$ since this value of x also makes the numerator zero. Next to the coordinates $x = 0.9$, $y = 0.52$, see that the calculator does not display a value for y at $x = 1$ since 1 is not in the domain of this rational function.

TRACE ▶ (repeatedly)

▶



▶



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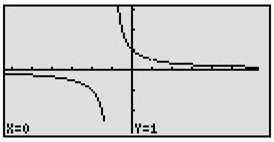
3 Find the x- and y-intercepts of $f(x)$.

2nd F CALC 6



The y-intercept is at (0 ,1). Notice that there are no x-intercepts for the graph of $f(x)$.

---

4 Estimate the horizontal asymptote of $f(x)$.

The line $y = 0$ is very likely a horizontal asymptote of $f(x)$.

The graphing feature of the EL-9900 can create the branches of a rational function separated by a vertical asymptote. The calculator allows the points of intersection to be obtained easily.

Solving Rational Function Inequalities

A rational function $f(x)$ is defined as the quotient $\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are two polynomial functions such that $q(x) \neq 0$. The solutions to a rational function inequality can be obtained graphically using the same method as for normal inequalities. You can find the solutions by graphing each side of the inequalities as an individual function.

Example

Solve a rational inequality.

Solve $\left|\frac{x}{1-x^2}\right| \leq 2$ by graphing each side of the inequality as an individual function.

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

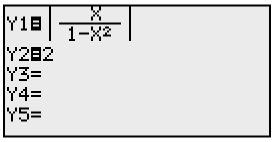
Set the zoom to the decimal window: ZOOM A (ENTER ALPHA ▼) 7

Step & Key Operation | **Display** | **Notes**

1 Enter $y = \left|\frac{x}{1-x^2}\right|$ for Y1. Enter $y = 2$ for Y2.

Y= MATH B 1 a/b X/θ/T/n ▼

1 — X/θ/T/n X^2 ENTER 2

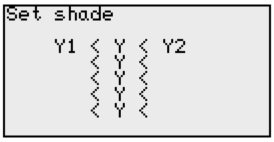


2 Set up the shading.

2nd F DRAW G 1

2nd F VARS A ENTER A 1 ▶

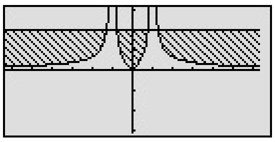
2nd F VARS ENTER 2



Since Y1 is the value "on the bottom" (the smaller of the two) and Y2 is the function "on the top" (the larger of the two), Y1 < Y < Y2.

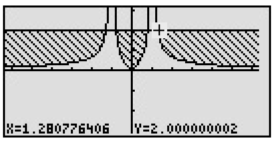
3 View the graph.

GRAPH



4 Find the intersections, and solve the inequality.

2nd F CALC 2 Do this four times



The intersections are when $x = -1.3$, -0.8, 0.8, and 1.3. The solution is all values of x such that $x \leq -1.3$ or $-0.8 \leq x \leq 0.8$ or $x \geq 1.3$.

The EL-9900 allows the solution region of inequalities to be indicated visually using the Shade feature. Also, the points of intersections can be obtained easily.

Graphing Parabolas

The graphs of quadratic equations ($y = ax^2 + bx + c$) are called parabolas. Sometimes the quadratic equation takes on the form of $x = ay^2 + by + c$.
There is a problem entering this equation in the calculator graphing list for two reasons:
a) it is not a function, and only functions can be entered in the Y= list locations,
b) the functions entered in the Y= list must be in terms of x, not y.
There are, however, two methods you can use to draw the graph of a parabola.

Method 1: Consider the "top" and "bottom" halves of the parabola as two different parts of the graph because each individually is a function. Solve the equation of the parabola for y and enter the two parts (that individually are functions) in two locations of the Y= list.

Method 2: Choose the parametric graphing mode of the calculator and enter the parametric equations of the parabola. It is not necessary to algebraically solve the equation for y. Parametric representations are equation pairs $x = F(t)$, $y = F(t)$ that have x and y each expressed in terms of a third parameter, t.

Example

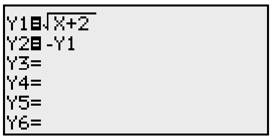
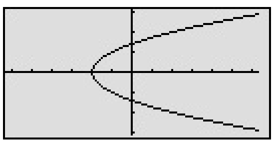
Graph a parabola using two methods.

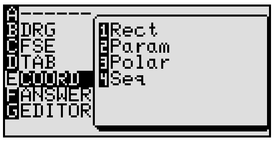
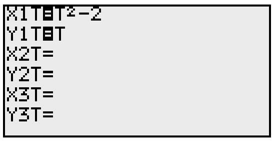
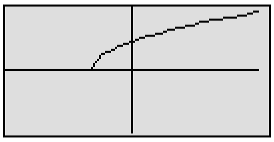
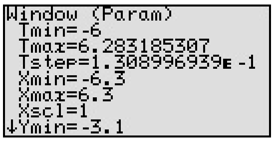
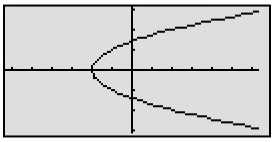
1. Graph the parabola $x = y^2 -2$ in rectangular mode.

2. Graph the parabola $x = y^2 -2$ in parametric mode.

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Set the zoom to the decimal window: [ZOOM] [A] ([ENTER] [ALPHA] [▼]) [7]

Step & Key Operation	Display	Notes
1-1 Solve the equation for y.		$x = y^2 -2$ $x + 2 = y^2$ $y = \pm\sqrt{x + 2}$
1-2 Enter $y = \sqrt{x+2}$ for Y1 and enter y = -Y1 for Y2. [Y=] [2nd F] [√] [X/θ/T/n] [+] [2] [ENTER] [(-)] [2nd F] [VARS] [A] [ENTER] [1]		
1-3 View the graph. [GRAPH]		The graph of the equation $y = \sqrt{x+ 2}$ is the "top half" of the parabola and the graph of the equation $y = -\sqrt{x + 2}$ gives the "bottom half."

Step & Key Operation	***Display***	***Notes***
2-1 Change to parametric mode. [2nd F] [SET UP] [E] [2]		
2-2 Rewrite $x = y^2 -2$ in parametric form. Enter X1T = T^2 -2 and Y1T = T. [Y=] [X/θ/T/n] [x^2] [—] [2] [ENTER] [X/θ/T/n]		Let y = T and substitute in $x = y^2 - 2$, to obtain $x = T^2 - 2$.
2-3 View the graph. Consider why only half of the parabola is drawn. (To understand this, use Trace feature.) [GRAPH] ([TRACE] [▶])		The graph starts at T =0 and increases. Since the window setting is T ≥ 0, the region T < 0 is not drawn in the graph.
2-4 Set Tmin to -6. [WINDOW] [(-)] [6] [ENTER]		
2-5 View the complete parabola. [GRAPH]		

The calculator provides two methods for graphing parabolas, both of which are easy to perform.

Graphing Circles

The standard equation of a circle of radius r that is centered at a point (h, k) is $(x - h)^2 + (y - k)^2 = r^2$. In order to put an equation in standard form so that you can graph in rectangular mode, it is necessary to solve the equation for y. You therefore need to use the process of completing the square.

Example

Graph the circles in rectangular mode. Solve the equation for y to put it in the standard form.

1. Graph $x^2 + y^2 = 4$.

2. Graph $x^2 - 2x + y^2 + 4y = 2$.

Before Starting There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

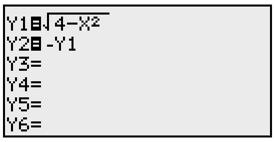
Set the zoom to the decimal window: ZOOM A (ENTER ALPHA ▼) 7

Step & Key Operation | ***Display*** | ***Notes***

1-1 Solve the equation for y. Enter $y = \sqrt{4 - x^2}$ for Y1 (the top half). Enter $y = -\sqrt{4 - x^2}$ for Y2.

Y= 2nd F √ 4 — X/θ/T/n X^2

ENTER (-) 2nd F VARS A ENTER 1

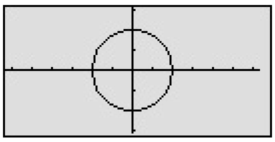


$y^2 = 4 - x^2$

$y = \pm\sqrt{4 - x^2}$

1-2 View the graph.

GRAPH



This is a circle of radius r, centered at the origin.

2-1 Solve the equation for y, completing the square.

$x^2 - 2x + y^2 + 4y = 2$	Place all variable terms on the left and the constant term on the right-hand side of the equation.
$x^2-2x+y^2+4y+4=2+4$	Complete the square on the y-term.
$x^2 - 2x + (y+2)^2 = 6$	Express the terms in y as a perfect square.
$(y+2)^2 = 6 - x^2 + 2x$	Leave only the term involving y on the left hand side.
$y+2 = \pm\sqrt{6-x^2+2x}$	Take the square root of both sides.
$y = \pm\sqrt{6-x^2+2x} - 2$	Solve for y.

Step & Key Operation | **Display** | **Notes**

2-2 Enter $y = \sqrt{6 - x^2 + 2x}$ for Y1, y = Y1 - 2 for Y2, and y = -Y1 -2 for Y3.

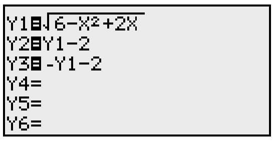
[Y=] [CL] [2nd F] [√] [6] [—] [X/θ/T/n]

[X^2] [+] [2] [X/θ/T/n] [ENTER] [CL]

[2nd F] [VARS] [A] [ENTER] [1] [—]

[2] [ENTER]

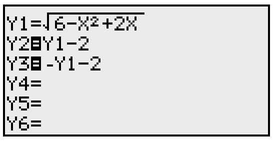
[(-)] [2nd F] [VARS] [ENTER] [1] [—] [2]



Notice that if you enter $y = \sqrt{6 - x^2 + 2x}$ - 2 for Y1 and y = - Y1 for Y2, you will not get the graph of a circle because the "±" does not go with the "-2".

2-3 "Turn off" Y1 so that it will not graph.

[▲] [▲] [◀] [ENTER]

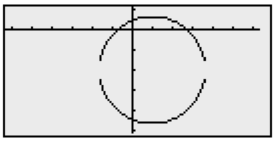


Notice that "=" for Y1 is no longer darkened. You now have the top portion and the bottom portion of the circle in Y2 and Y3.

2-4 Adjust the screen so that the whole graph is shown. Shift 2 units downwards.

[WINDOW] [▼] (3 times) [—] [2] [ENTER]

[—] [2] [ENTER] [GRAPH]



-1.3 < Y < 3.1
↓
-5.1 < Y < 1.1

Graphing circles can be performed easily on the calculator display.

Graphing Ellipses

The standard equation for an ellipse whose center is at the point (h, k) with major and minor axes of length a and b is $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$.

There is a problem entering this equation in the calculator graphing list for two reasons:

a) it is not a function, and only functions can be entered in the Y = list locations.

b) the functions entered in the Y = list locations must be in terms of x, not y.

To draw a graph of an ellipse, consider the “top” and “bottom” halves of the ellipse as two different parts of the graph because each individual is a function. Solve the equation of the ellipse for y and enter the two parts in two locations of the Y = list.

Example

Graph an ellipse in rectangular mode. Solve the equation for y to put it in the standard form.

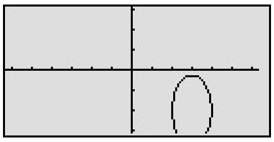
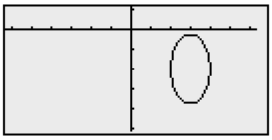
Graph the ellipse $3(x - 3)^2 + (y + 2)^2 = 3$

Before Starting

There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Set the zoom to the decimal window: [ZOOM] [A] ([ENTER] [ALPHA] [▼]) [7]

Step & Key Operation	Display	Notes
1 Solve the equation for y, completing the square. Enter $Y1 = \sqrt{3 - 3(x - 3)^2}$ $Y2 = Y1 - 2$ $Y3 = -Y1 - 2$ [Y=] [2nd F] [√] [3] [—] [3] [(] [X/θ/T/n] [—] [3] [)] [X^2] [ENTER] [2nd F] [VARS] [A] [ENTER] [1] [—] [2] [ENTER] [(-)] [2nd F] [VARS] [ENTER] [1] [—] [2]	Y1=√3−3(X−3)² Y2=Y1−2 Y3=-Y1−2 Y4= Y5= Y6=	$3(x - 3)^2 + (y + 2)^2 = 3$ $(y + 2)^2 = 3 - 3(x - 3)^2$ $y + 2 = \pm\sqrt{3 - 3(x - 3)^2}$ $y = \pm\sqrt{3 - 3(x - 3)^2} - 2$
2 Turn off Y1 so that it will not graph. [▲] [▲] [◀] [ENTER]	Y1=√3−3(X−3)² Y2=Y1−2 Y3=-Y1−2 Y4= Y5= Y6=	

Step & Key Operation	Display	Notes
3 View the graph. GRAPH		
4 Adjust the screen so that the whole graph is shown. Shift 2 units downwards. WINDOW ▼ (3 times) — 2 ENTER — 2 ENTER GRAPH		-3.1 < Y < 3.1 ↓ -5.1 < Y < 1.1

Graphing an ellipse can be performed easily on the calculator display.

Graphing Hyperbolas

The standard equation for a hyperbola can take one of two forms:

$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$ with vertices at $(h \pm a, k)$ or

$\frac{(x - k)^2}{b^2} - \frac{(y - h)^2}{a^2} = 1$ with vertices at $(h, k \pm b)$.

There is a problem entering this equation in the calculator graphing list for two reasons:

a) it is not a function, and only functions can be entered in the Y= list locations.

b) the functions entered in the Y= list locations must be in terms of *x*, not *y*.

To draw a graph of a hyperbola, consider the “top” and “bottom” halves of the hyperbola as two different parts of the graph because each individual is a function. Solve the equation of the hyperbola for *y* and enter the two parts in two locations of the Y= list.

Example

Graph a hyperbola in rectangular mode. Solve the equation for *y* to put it in the standard form.

Graph the hyperbola $x^2 + 2x - y^2 - 6y + 3 = 0$

Before Starting

There may be differences in the results of calculations and graph plotting depending on the setting. Return all settings to the default value and delete all data.

Set the zoom to the decimal window: [ZOOM] [A] ([ENTER] [ALPHA] [▼]) [7]

Step & Key Operation | **Display** | **Notes**

1 Solve the equation for *y* completing the square.

Enter

$Y1 = \sqrt{x^2 + 2x + 12}$

Y2 = Y1 -3

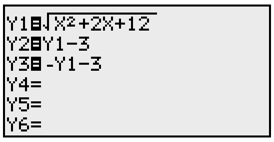
Y3 = -Y1 -3

[Y=] [2nd F] [√] [X/θ/T/n] [X^2] [+] [2]

[X/θ/T/n] [+] [1] [2] [ENTER]

[2nd F] [VARS] [A] [ENTER] [1] [—] [3] [ENTER]

[(-)] [2nd F] [VARS] [A] [ENTER] [1] [—] [3]



Notes:

$x^2 + 2x - y^2 - 6y = -3$

$x^2 + 2x - (y^2 + 6y + 9) = -3 - 9$

$x^2 + 2x - (y + 3)^2 = -12$

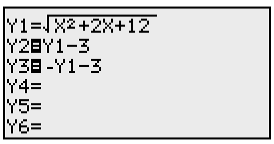
$(y + 3)^2 = x^2 + 2x + 12$

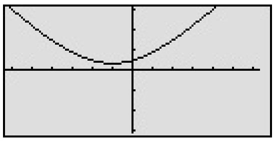
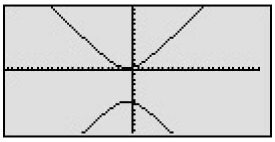
$y + 3 = \pm\sqrt{x^2 + 2x + 12}$

$y = \pm\sqrt{x^2 + 2x + 12} - 3$

2 Turn off Y1 so that it will not graph.

[▲] [▲] [◀] [ENTER]

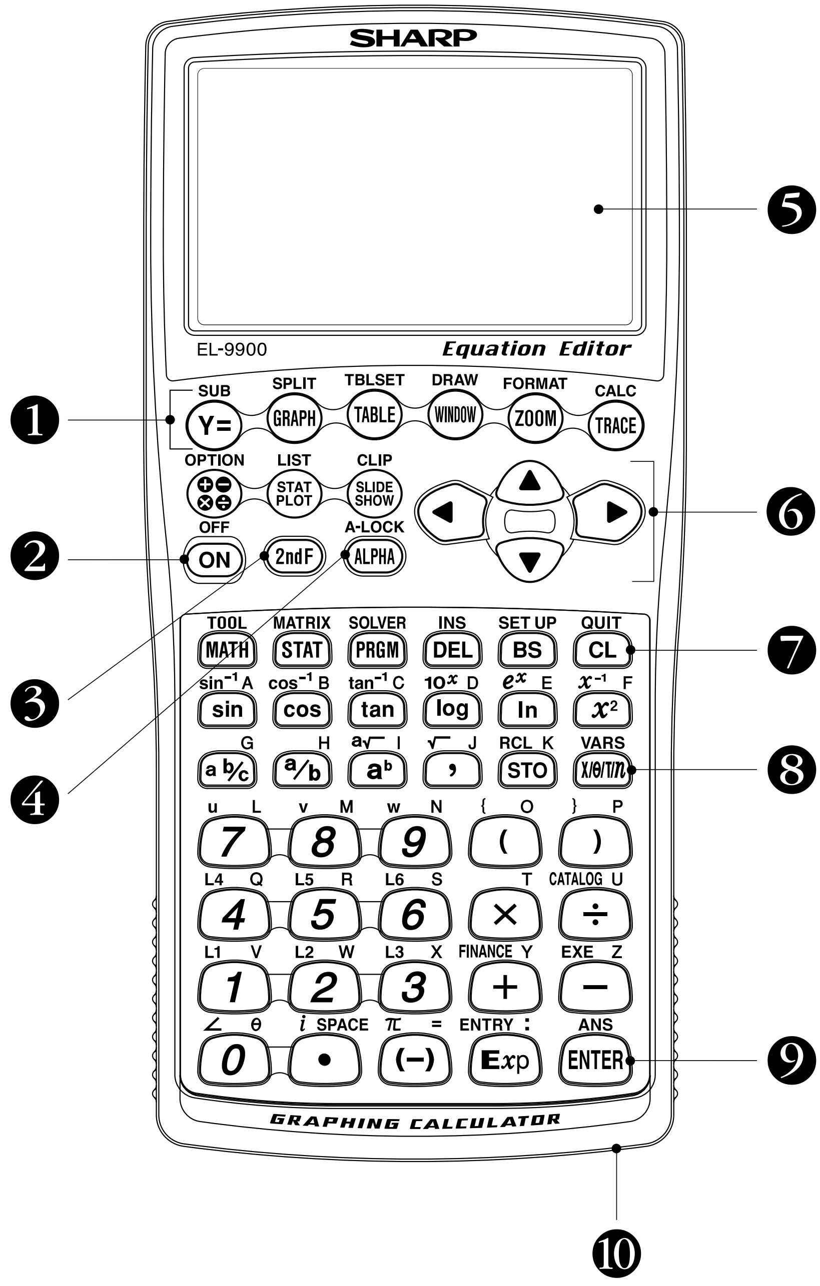


Step & Key Operation		***Display***	***Notes***
3	View the graph. GRAPH		
4	Zoom out the screen. ZOOM A 4		

Graphing hyperbolas can be performed easily on the calculator display.

Key pad for the SHARP EL-9900 Calculator

Advanced Keyboard



❶ Graphing keys

❷ Power supply ON/OFF key

❸ Secondary function specification key

❹ Alphabet specification key

❺ Display screen

❻ Cursor movement keys

❼ Clear/Quit key

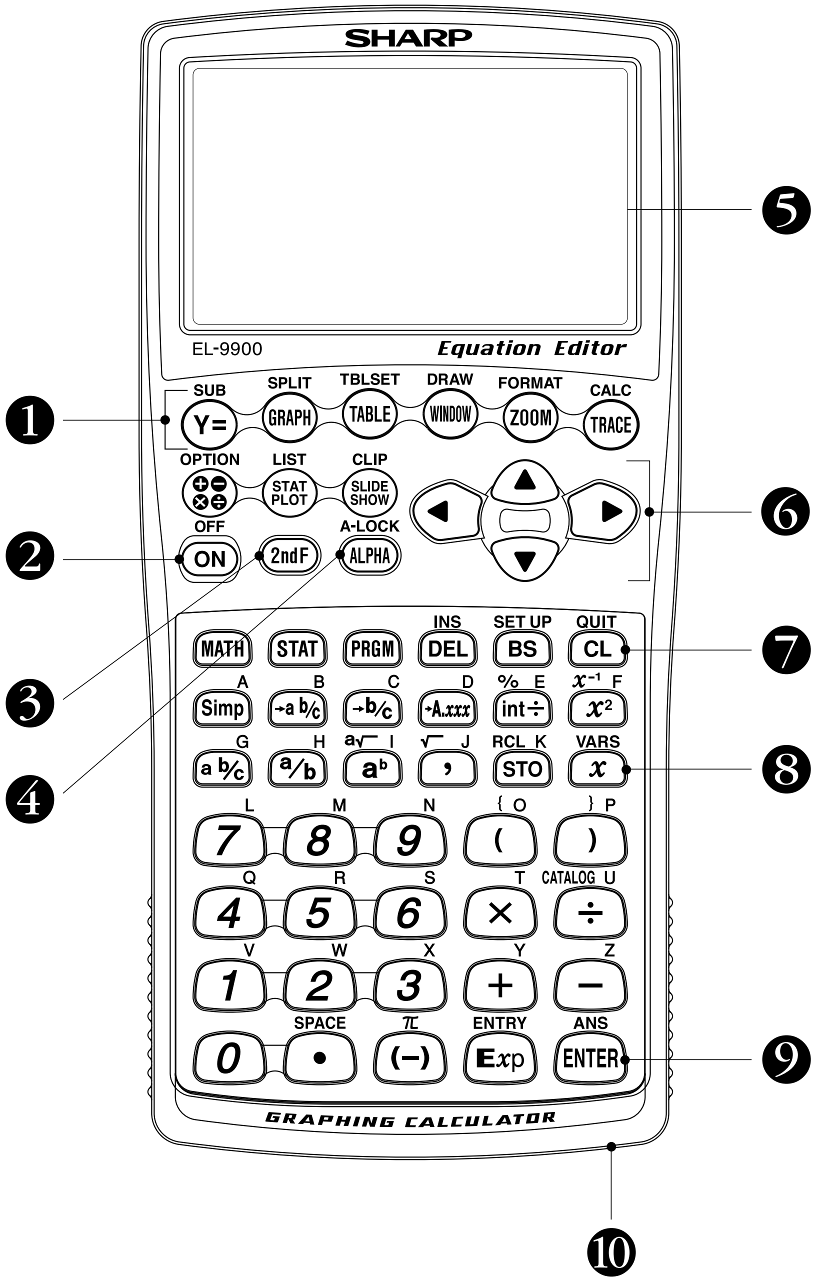
❽ Variable enter key

❾ Calculation execute key

❿ Communication port for peripheral devices

Key pad for the SHARP EL-9900 Calculator

Basic Keyboard



❶ Graphing keys

❷ Power supply ON/OFF key

❸ Secondary function specification key

❹ Alphabet specification key

❺ Display screen

❻ Cursor movement keys

❼ Clear/Quit key

❽ Variable enter key

❾ Calculation execute key

❿ Communication port for peripheral devices

SHARP

Use this form to send us your contribution

Dear Sir/Madam

We would like to take this opportunity to invite you to create a mathematical problem which can be solved with the SHARP graphing calculator EL-9900. For this purpose, we would be grateful if you would complete the form below and return it to us by fax or mail.

If your contribution is chosen, your name will be included in the next edition of The EL-9900 Graphing Calculator Handbook. We regret that we are unable to return contributions.

We thank you for your cooperation in this project.

Name: (☐ Mr. ☐ Ms.) ____________________

School/College/Univ.: ____________________

Address: ____________________

____________________ Post Code: __________ Country: __________

Phone: ____________________ Fax: ____________________

E-mail: ____________________

SUBJECT : Write a title or the subject you are writing about.

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INTRODUCTION : Write an explanation about the subject.

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EXAMPLE : Write example problems.

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. . . .

BEFORE STARTING : Write any conditions to be set up before solving the problems.

STEP

NOTES

SHARP

SHARP CORPORATION Osaka, Japan
Fax:

SHARP

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