Gaigen User Manual
Version 0.99

Daniël Fontijne
University of Amsterdam

November 24, 2003

Contents

Chapter 1

Introduction

Gaigen is a program that can generate implementations of geometric algebras. It generates C++, C and assembly source code that implements a geometric algebra requested by the user. This is an unconventional approach. The choice to create such a program / package was made because we wanted performance similar to optimized hand-written code, while maintaning full generality; for (scientific) research and experimentation, many geometric algebras with different dimensionality, signatures and other properties may be required. Instead of coding each algebra by hand, Gaigen provides the possibility to generate the code for exactly the geometric algebra the user requires. This code may be less efficient than fully optimized hand-written code, but is likely to be much more efficient than one library that tries to support all possible algebras at once.

One such a library, CLU [6], uses C++ templates and *style* classes, which specify the properties of the geometric algebra. While the CLU approach has its advantages, (e.g. special code can easily be added to the style classes that represent an algebra, and the size of the code implementation may be smaller, especially when more than 1 algebra is used), initial experiments show that the performance of code generated by Gaigen is about an order of magnitude more efficient in terms of computation time. Another disadvantage of CLU is that for each algebra you have to write a new style file (four style files are provided in the package), you have to write your own style file. In Gaigen you simply specify what you want in the user interface, hit the **generate** button and you can use your new algebra.

The Gaigen package consists of

- this manual, which describes how to use Gaigen and what its internals look like,
- an installation manual,
- a paper describing and discussing the design of Gaigen, and reporting its performance relative to other packages and itself (using different settings),
- the Gaigen executable for win32 (most flavours of Windows), Sun Solaris and Linux and its source code,

- pre-generated algebras for euclidean (**e3ga**), projective (**p3ga**) and conformal model for 3d geometry (**c3ga**),
- tutorials showing how to use the pre-generated algebras and how to generate and use your own algebras, and
- a quick reference page for the high level C++ interface.

This user manual is divided into six chapters, of which most users, not interested in modifying or changing Gaigen, will only have to read the first two to get started. These chapters are chapter 2 that describes the Gaigen user interface, and chapter 3 which describes the high level C++ programming interface of the source code Gaigen generates. The source code generated by Gaigen is exposed to the user as a C++ class with member functions and overloaded operators. No fancy C++ features (except for operator overloading) are used to keep everything as basic and simple as possible. Gaigen does not depend on other software packages, except for its user interface, which uses the FLTK library [7].

Chapter 4 describes the intermediate C++ layer that lies between the low level C or assembly code and the high level interface. The low level C or assembly code implements the actual computation of products of the geometric algebra and is described in chapter 5. This chapter also describes how one could implement his or her own version of this code, optimized for a specific (processor) architecture. Finally chapter 6 describes the various file formats used by Gaigen.

Chapter 2

The User Interface

It may seem a bit weird to download a programming package and the first thing to do with it is start its user interface, but that is exactly how Gaigen works (unless you use one of pre-generated sample implementions that come with Gaigen). The user interface, that pops up after starting the *gaigenui* executable, is used to specify the properties (e.g. the dimension of the algebra, signature of the basis vectors, optimizations) of the geometric algebra you would like to use. After selecting those properties you hit the generate button, and source code for your algebra is generated. Then you can exit the Gaigen program and are ready to use your algebra. You don't have to use the Gaigen user interface again, unless you would want to change properties of your algebra (or perhaps generate source code for an entirely new one), or if you would like to change the performance optimizations of your algebra.

Changing the optimizations of the algebra can increase the performance of your program drastically. Gaigen can include profiling code in your algebra that tracks what products / multivector combinations you use and how often you use them. With this code included, you can run your program and, at any time, request a dump of that information, and use it to enable specific optimizations, either by hand. (see section 2.3).

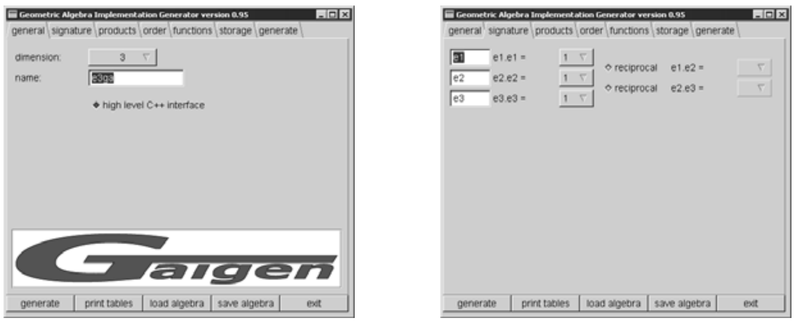
The user interface consists of a number of tabs at the top of the window, a number of buttons at the bottom and a large field displaying the contents of the currently active tab in the center.

Clicking on a tab will raise its contents. The contents of each tab allow you to change a specific set of properties of the algebra that will be generated by clicking on the **generate** button in the lower left of the window. The contents of each tab and the properties they control is discussed in the following sections.

2.1 General

The contents of the **general** tab, shown in figure 2.1a, control the dimension, the name of the class and source files, whether the high level C++ interface will be included, in what directory the source files will be written, and the what type of low level computational code will be generated.

The dimension is set to 0 by default and can be changed by using the dropdown box. Only dimensions 0 to 8 are allowed, because the approach used



dimension: 3
name: e3ga
◆ high level C++ interface

e1 e1.e1 = 1
e2 e2.e2 = 1
e3 e3.e3 = 1
◇ reciprocal e1.e2 =
◇ reciprocal e2.e3 =

a b

Figure 2.1: The general and signature tabs with settings for the e3ga algebra.

by Gaigen to implement the products becomes infeasible for higher dimension (see [1]). Generating a 0 dimensional algebra will basically give you a scalar algebra.

If you want to use the high level C++ interface described in section 3, make sure the checkbox **high level C++ interface** is ticked. Gaigen will include two files (gaigenhl.h and gaigenhl.cpp) in the source code for your algebra.

The name of your algebra can be entered at the **name** textfield. This name is used to generate the output filenames and as the name of the class which is generated. Also, the name is used internally, with an *i* appended to it, as the name of the class which sits between the low level code and the high level C++ interface. In the discussion and figures below, we assume that you call your algebra **e3ga** [1].

When you click the **generate** button at the lower right, the source code and other files will be generated in your *algebras* directory (which you set in your configuration file; see the installation manual). The files you have to include into your programming project are *e3ga.cpp*, *e3ga.h*, and, depending which checkboxes you activated in the **generate** tab, either *e3ga_optc.c*, *e3ga_optc2.c* or *e3ga_optlapack.c*. You need exactly one of these optimized files, or you'll run into link errors. If you use the *e3ga_optlapack.c* file, be sure to include the LA-Pack library in your project.

Also, you can click the **print tables** button, which will cause *e3ga.txt* and *e3ga.tex* to be generated. These text and TEX-files will contain the multiplication tables for the products of your algebra. These may be useful for educational purposes. To learn more about these tables, see [2] or [4].

To save the specification of your algebra to a file, such that you can reuse it later, click the **save algebra** button. Gaigen will then ask you for a location to store the specification. This stores every property of your algebra you can control through the user interface in the file. The file format used is plain text

[1]If you take a look inside *gaigenhl.cpp* and *gaigenhl.h* you will see that the classnames **GAIM_CLASSNAME** and **CLASSNAME** are used in these files. **CLASSNAME** is **#define**d as the name of your class (e.g. **e3ga**) and **GAIM_CLASSNAME** is **#define**d as the internal name of your class (e.g. **e3gai**). **GAIM** stands for *Geometric Algebra IMplementation* and is used in Gaigen source code as a prefix to macros that have to do with the intermediate C++ layer (section 4) and the selections made in the *gaigenui* program.

and is described in section 6. To load the specification of an algebra, click the **load algebra** button. A nice place to save your specifications is in the *algebras* directory (which is the default location where the **save algebra** dialog starts).

2.2 Signature

Clicking the signature tab will raise its contents, which depend on the dimension of the algebra. Since the signature tab is used to modify the signature of the basis vectors, its contents are empty if you selected '0' as the dimension of your algebra in the **general** tab. In figure 2.1b you can see the signature tab for a 3 dimension geometric algebra.

First of all, the signature tab can be used to change the names of the basis vectors to something appropriate for your algebra using the textfields on the left. E.g., if you want to generate an algebra to work with the conformal model of euclidean space, you could give a special name to the basis vector representing the point at the origin and the point at infinity. Or you could call the basis vectors of a 3d euclidean geometric algebra *x*, *y* and *z*, or *red*, *green* and *blue*, if you like.

The main purpose of the signature tab is to set the signature of the basis vectors. The signature of a basis vector is the value it squares to; e.g. we can define $\mathbf{e}_1 \odot \mathbf{e}_1 = \mathbf{1}$, $\mathbf{e}_1 \cdot \mathbf{e}_1 = -\mathbf{1}$ or even $\mathbf{e}_1 \cdot \mathbf{e}_1 = \mathbf{0}$ (a null vector). The signature of all basis vectors is $\mathbf{1}$ by default, but this can be changed to $-\mathbf{1}$ and to $\mathbf{0}$ using the dropdown box provided for each basis vector. It is also possible to create pairs of reciprocal null basis vectors. A pair of reciprocal null vectors is a pair of vectors which square to $\mathbf{0}$ with itself, but to $-\mathbf{1}$ or $\mathbf{1}$ with the other. A pair of null vectors with these properties can be created by checking the **reciprocal** checkbox between two basis vectors. An example of the use of reciprocal null vectors are $\mathbf{e}_0$ and $\mathbf{e}_\infty$ in the conformal model. Gaigen can support these directly. The dropdown box to the right of the checkbox can then be used to select the value the pair of vectors squares to. Because a **reciprocal** checkbox is only provided between neighboring basis vectors, so only a limited set of reciprocal null vectors can be created like this. This is not just a user interface limitation, but a limitation in the way the code that Gaigen generates works. It is not a limitation of geometric algebra in general, but we believe an algebra specification can always be modified slightly to archive the same result with 'neightboring' reciprocal null vectors.

2.3 Products

Currently, Gaigen supports seven basic products between multivectors. Besides these basic products, some special products are available, such as the outermorphism, meet and join, and the delta product. But only the seven basic products are controlled from the **products** tab, which is shown in figure 2.2a.

The **products** tab itself contains another set of tabs, one for each product. The names of these tabs are abbreviated versions of the products:

abbreviation	full name
gp	Geometric Product
hip	Hestenes Inner Product
mhip	Modified Hestenes Inner Product
lcont	Left Contraction
rcont	Right Contraction
op	Outer Product
scp	Scalar Product

The abbreviations are also used inside the generated source code as names of the functions which compute the products.

The **products** tab is used to select which of these products you want to include in your algebra. A common selection would be geometric, outer and scalar product and either the left contraction or one of the Hestenes inner products. To include a product in your algebra, click on the tab of that product, and check the checkbox left of the full name of the product. The color of the text in the tab will change from black to red to reflect the inclusion of that product.

Below the checkbox is a large field that is used to control the optimizations implemented for each product. You are not required to add any optimizations to generate a basic algebra, but you can significantly increase the performance of your application by doing so, sometimes by an order of magnitude. But, by making the wrong optimizations you could in theory decrease the performance of your application. So it is important that you understand how the optimizations work if you care about performance.

The optimizations are based on the assumption that it is likely that your application will use certain products between certain multivectors much more often than others. Suppose you use lots of rotations of 3D vectors in your program, like:

$$\mathbf{w} = (\mathbf{R}\mathbf{v})\mathbf{R}^{-1}. \tag{2.1}$$

Then the performance of your program could be increased if Gaigen would generate an optimized function for the geometric product between an even multivector and a vector ($\mathbf{R}\mathbf{v}$), and an optimized function for the geometric product between an odd multivector and an even multivector ($(\mathbf{R}\mathbf{v})\mathbf{R}^{-1}$).

Because Gaigen tracks the *grade usage* (which grade parts of a multivector are equal to **0** (*empty*) and which are not) of all multivectors you use in your application, it is in theory capable to generate and invoke an optimized function for every combination of grade usages and products. Of course, generating an optimized function for every possible combination is not feasible for high dimensional algebras, because the amount of code generated would get too large. That's why you can select a specific set of combinations from the **products** tab.

You turn on optimization for a specific combination of grade usage and product by first swiching to that product (click its tab). Then you use the two sets of little checkboxes (labeled $[0...d]$ where d is the dimension you selected for your algebra in the **general** tab) to specify the grade usage combination you want to optimize for, and add it to the set of optimizations by clicking **add**.

So suppose you want to optimize for the example above ($\mathbf{w} = (\mathbf{R}\mathbf{v})\mathbf{R}^{-1}$). You would check **0** and **2** in the left set of check boxes (a general 3d rotor has a non-empty grade 0 and 2), **1** in the right set (a 3d vector has a non-empty grade 1) , and then click **add**. This would optimize the product $\mathbf{R}\mathbf{v}$. The combination will appear in the list below the two sets of checkboxes. Then you would check **1** and **3** in the left set (the product $\mathbf{Rv}$ will have and non-empty grade 1 and 3), and you check **0** and **2** in the right set (for the inverted rotor). Adding this combination will optimize the product $(\mathbf{Rv})\mathbf{R}^{-1}$. To remove any combination already added, you simply click the **remove** button for that combination.

2.3.1 Automatic optimizations using profiles, and optimization options

Of course, once your application gets more complicated, you won't be able to tell easily what combinations of products and grade usages you use most often. That's why Gaigen can include profiling code in your application. You can enable profiling by ticking the **enable profiling** checkbox in the **optimizations** tab in the **products** tab. This tab is new in Gaigen version 0.95. The profiling code (usage will be explained in the next section) will count how often you use each combination and on request print or save a list of most used combinations. You can then use this list to manually optimize your algebra, but more conveniently, you can let Gaigen handle the optimizations for you.

First of all, you can use the **remove all optimizations** button to remove all product optimizations from the algebra specification. This is recommended before adding automatic optimizations to make sure no old optimizations are left behind.

Then, you can use the **automatically add optimizations from profile** button to add optimizations automatically. When you push the button, you are prompted to select a *.gap* (Geometric Algebra Profile) file. These files are written by the **saveProfile** function (see section 3.16). Gaigen will then automatically add optimizations for all product / multivector combinations that are used more than 2.0% of the time. You can use the **optimize threshold in usage percentage** slider to change this value of 2.0%. I.e., 0.0% would add optimizations for every product / multivector combination that is used in your application.

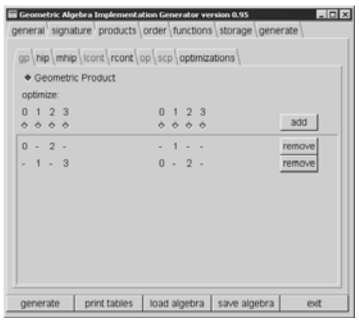
If you tick the **inline products** checkbox, all generated products will be prefixed with an **inline** statement. This might improve performance a few percent.

You can use the **Dispatch method** radio buttons to select what dispatch method to use. As shown in [1], the **ifelse** method is usually fastest, followed by **switch**.

2.4 Order

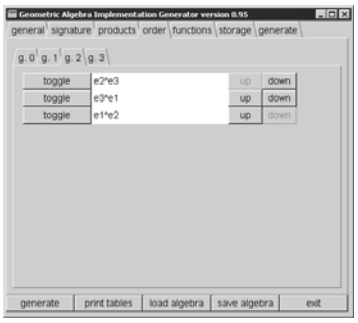
Using the **order** tab (figure 2.2b) you can modify the order in which the coordinates referring to basis blades are stored, and you can change the orientation of the basis blades. This part of the user interface is a bit primitive, though functional.

You are concerned mostly with coordinates when you enter them into a multivector object, e.g. by using the **set** function, when you retrieve them from a multivector object, e.g. by using the **coordinates** function, or when you inspect them, e.g. by using the **print** function.



```
optimize:
0 1 2 3          0 1 2 3
0 - 2 -          - 1 - -
- 1 - 3          0 - 2 -
```

a



```
toggle  e2*e3  up  down
toggle  e3*e1  up  down
toggle  e1*e2  up  down
```

b

Figure 2.2: The products and order tabs with settings for the e3ga algebra.

In Gaigen all multivectors are stored as (compressed) arrays of coordinates. A multivector

$$\mathbf{A} = 1 + 2\mathbf{e}_1 + 3\mathbf{e}_1 + 4\mathbf{e}_1 + 5\mathbf{e}_1 \wedge \mathbf{e}_2 + 6\mathbf{e}_1 \wedge \mathbf{e}_3 + 7\mathbf{e}_2 \wedge \mathbf{e}_3 + 8\mathbf{e}_1 \wedge \mathbf{e}_2 \wedge \mathbf{e}_3 \quad (2.2)$$

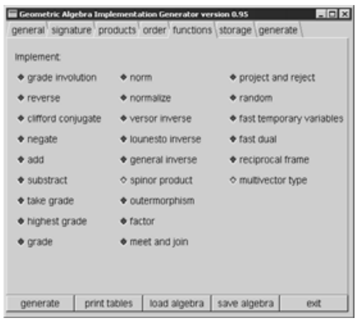
would, by default, be stored as an array $\{1, 2, 3, 4, 5, 6, 7, 8\}$. But suppose you don't like the orientation of the basis blade $\mathbf{e}_1 \wedge \mathbf{e}_3$ and prefer $\mathbf{e}_3 \wedge \mathbf{e}_1$ instead. You could do this by clicking the **g.2** tab (g.2 stands for grade 2), and clicking the **toggle** button for $\mathbf{e}_1 \wedge \mathbf{e}_3$, which will then change into $\mathbf{e}_3 \wedge \mathbf{e}_1$. The multivector $\mathbf{A}$ with the same value as above would then have to be stored as $\{1, 2, 3, 4, 5, -6, 7, 8\}$. Toggle buttons are active only for basis blades with a grade higher than 1.

Now suppose you want to change the order in which the coordinates are stored in the array, e.g. because the order in which you store your coordinate data is different. You would use the **up** and **down** buttons to change the order of basis blades. Note that you can only modify the order of basis blades within a grade. All coordinates for one grade are always packed together in the coordinate array.

As an example, if you change the order and orientation of the grade 2 basis blades of a 3d algebra to $[\mathbf{e}_2 \wedge \mathbf{e}_3, \mathbf{e}_3 \wedge \mathbf{e}_1, \mathbf{e}_1 \wedge \mathbf{e}_2]$, then the coordinates of the multivector $\mathbf{A}$ would be stored as $\{1, 2, 3, 4, 7, -6, 5, 8\}$. This order and orientation is used by the pregenerated **e3ga** algebra and is the one shown in figure 2.2b.

2.5 Functions

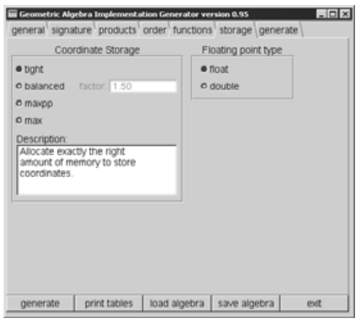
The **functions** tab (figure 2.3a) simply contains a lot of checkboxes which can be used to include certain functions into the algebra code. Some functions are interdependant on each other, some only work for specific dimensions and most functions require that a product (usually the geometric product, outer product, scalar product or left contraction) is included in the algebra. While you are still unexperienced with Gaigen (and while Gaigen isn't finished yet) it might be best to always include these 4 products in your algebra, because



Implement:

[✓ grade involution] [✓ norm] [✓ project and reject]
[✓ reverse] [✓ normalize] [✓ random]
[✓ clifford conjugate] [✓ versor inverse] [✓ fast temporary variables]
[✓ negate] [✓ lounesto inverse] [✓ fast dual]
[✓ add] [✓ general inverse] [✓ reciprocal frame]
[✓ substract] spinor product multivector type
[✓ take grade] [✓ outermorphism]
[✓ highest grade] [✓ factor]
[✓ grade] [✓ meet and join]

a



Coordinate Storage: [✓ tight] balanced (factor: 1.50) maxpp max

Description: Allocate exactly the right amount of memory to store coordinates.

Floating point type: [✓ float] double

b

Figure 2.3: The functions and memory tabs with settings for the e3ga algebra.

otherwise you might run into compilation errors[2]. If you try to compile an algebra and the compiler complains about a certain function or product which does not exist, include it from the **functions** or **products** tabs.

Because the names of most functions speak for themselves, what follows is a list of a functions with peculiarities or names that do not make 100 percent clear what the function does. How to use the functions is explained in section 3.

- **take grade**. This function takes one grade part from a multivector and copies it to a new multivector.
- **highest grade**. This function takes the highest grade part that is non-zero from a multivector and copies it to a new multivector. It is used to compute the delta product.
- **grade of a blade**. When given a blade (or a homogenous multivector), this function returns its grade. If a non-homogenous multivector which is is passed, the function returns an error.
- **norm**. We have implemented several (currently two) functions to compute the norm of multivectors. However, every application seems to have its own idea of what the of a multivector norm is, so these functions have not entirely 'stabilized' yet.
- **normalize**. Normalizes a multivector by dividing it by its magnitude (norm). Uses the outer product and the norm.
- **versor inverse**. Computes the inverse of a multivector, assuming it is a versor (a versor is a multivector that can be written as the geometric product of vectors). If the multivector is not a versor, something other than the inverse is returned. The versor inverse is very efficient and should always be preferred over the general inverse function if possible. Requires the reverse, scalar product and the outer product.

[2]This should never happen, but we still have to build a good dependency system that checks that all required products and functions are present.

- **lounesto inverse**. This function can compute the inverse of any invertible multivector in a *3 dimensional* algebra. Requires the clifford conjugate and take grade functions and the geometric product. See [4] and [1] for more information on how it works.

- **general inverse**. This function can compute the inverse of any invertible multivector in an algebra of *any* dimension. It computes the inverse by explicitly inverting the geometric product matrix expansion. It is slow compared to the versor inverse or the lounesto inverse. See Gaigen tutorial 1 [3] for an example of relative efficiency and [1] for details. Does not require any other functions or products.

- **outermorphism**. Use this function to create an outermorphism operator. If you have a linear function, you can construct an outermorphism operator of it. Applying the outermorphism operator to multivector has several advantages (efficiency, precision, floating point noise) over simply applying the original function to multivector. Requires take grade and negate function and the outer product.

- **spinor product**. Including the spinor product function will add special code for constructing outermorphism operators from spinors. This function will be removed from Gaigen in the future, since it was superceded by the outer morphism. Requires take grade and negate function and the geometric product and outer product.

- **factor blade/versor**. This function factors blades and versors into arbirary vector factors. It is used to compute the meet and join. Requires norm, project, versor inverse, and take grade function, and the outer product, geometric product and left contraction.

- **meet and join**. This function computes the meet and join of blades. Requires factor blade / versor.

- **project and reject**. These functions project and reject blades onto / from blades and versors. Requires the versor inverse function, left contraction and for projection onto versors the geometric product.

- **random blade/versor**. This function generates random blades and versors.

- **fast temporary variables**. This function is only of interest if you use the high level C++ interface, which is what most people will do.When you write an C++ expression such as

  ```
  a = (b + c) * d;
  ```

  temporary variables are used to store the intermediate results (i.e. **(b + c)** and **((b + c) * d)**). A method exists to allocate these variables very quickly (compared to the default method used by the C++ compiler). The downside of this method is that it allocates the temporary variables from an array with a fixed (e.g. 64 variables) size. When it comes to the end of the array, it cycles back to first entry, whose contents will be overwritten with the new intermediate results. If your program still had a reference to this variable (as the (intermediate) result of a previous computation, this refenence will be useles. Using the reference as if it still contained the old value will cause your program to malfunction (but no crash). Checking the **fast temporary variables** checkbox will make your algebra about twice as fast, but you will have to make sure you don't run out of temporary variables. The main rule of thumb is *never* to pass references to temporary variables to functions. Instead of writing something like

```
e3ga b, c, d;
someFunction((b + c) * d);
```

write something like

```
e3ga a, b, c, d;
a = (b + c) * d;
someFunction(a);
```

This will make sure you don't pass references to temporary variables to other functions and prevent most problems. The only other ways you can get into trouble with fast temporary variables is by explicitly keeping references to temporary variables like this:

```
e3ga &a = b + c;
```

or by writing expressiong which are so long that they use all (64) temporary variables at once. The number of temporary variable is controlled by the line

```
#define MV_MAX_TEMP 64
```

in *gaigenhl.h*. In any case, if you suspect that a malfunction of your program is caused by using fast temporary variables, you can simply turn off the **fast temporary variables** button, regenerate your algebra, recompile your application and see if the malfunction disappears.

- **fast dual**. The fast dual function can compute the dual of a multivector with respect to the pseudoscalar of the algebra very quickly (compared to the default dual function) by simply shuffling and flipping the sign of the coordinates.
- **reciprocal frame**. The reciprocal frame function can compute the reciprocal frame of a set of vectors.
- **multivector type**. This function can compute the type (blade, versor, or general) of a multivector.

2.6 Memory

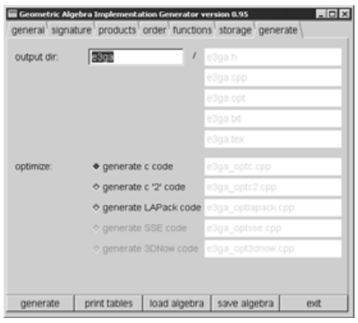
The memory tab, as shown in figure 2.3b, is used to control the memory allocation method and the floating point type used to store the coordinates. The coordinates of multivectors are stored in arrays of floating variables. Gaigen stores only coordinates of grade parts of which it knows that they are not equal to $\mathbf{0}$. To save the amount of memory used to store the coordinates, it can allocate just enough memory for each multivector to store its coordinates. But memory allocation costs computation time. If memory has to be reallocated for each basic product or sum or other operation, performance might suffer.

That is why Gaigen allows you to make a compromise between minimal memory usage and minimal memory allocation computation time. You can choose from four memory allocation schemes, ranging from lowest memory usage to lowest computational time:

- Tight: Exactly the right amount of memory is allocated to store the coordinates of the non-zero grades of a multivector. This implies frequent memory reallocation, which is done via a simple and efficient memory heap.
- Balanced: To prevent abundant memory reallocation, the balanced allocation scheme does not reallocate when less memory is required to store the coordinates, up to a certain *waste factor* which the user can specify. Suppose a variable holds a 3D rotor (4 coordinates), and is assigned a vector (3 coordinates); the memory waste would be 33.3% if 4 coordinates memory locations would still be used to store 3 coordinates. The balanced memory allocation algorithm then decides to either release the 4 old memory locations and allocate 3 new memory locations, or to just waste 1 memory location. This depends on the the waste factor. If the number of *used* memory location divided by the number of *required* memory locations will become larger than or equal to the waste factor, the algorithm will decide to reallocate. So the larger the waste factor, the more memory may be wasted, but the less memory reallocations will occur. However, this memory allocation scheme doesn't work well in practice, as shown in [1].
- Maximum: The maximum number of memory locations to store all 2^d coordinates is allocated when a multivector variable is created. Gaigen never has to reallocate memory.
- Maximum parity pure: We call a multivector parity pure if it is either odd or even. If the dimension of the algebra is larger than 0, only half of the 2^d coordinates have to be allocated to store the coordinates of a parity pure multivector variable. The user must guarantee that he will never create multivectors that are not parity pure, or weird things can happen (a crash or incorrect results).

You can switch between the **float** (32 bit precision) and **double** (64 bit precision) types for coordinate storage by toggling the radio buttons on the right side of the tab. If you always use the type **GAIM_FLOAT** (defined as either **float** or **double**) in your application, you can switch back and forth painlessly between **floats** or **doubles**. So instead of writing

```
float c[1.0, 2.0, 3.0];
e3ga a;
a.set(GRADE1, c);
```



output dir: e3ga / e3ga.h
e3ga.cpp
e3ga.opt
e3ga.txt
e3ga.tex

optimize:
◆ generate c code e3ga_optc.cpp
◇ generate c '2' code e3ga_optc2.cpp
◇ generate LAPack code e3ga_optlapack.cpp
◇ generate SSE code e3ga_optsse.cpp
◇ generate 3DNow code e3ga_opt3dnow.cpp

Figure 2.4: The generate tab with settings for the e3ga algebra.

write

```
GAIM_FLOAT c[1.0, 2.0, 3.0];
e3ga a;
a.set(GRADE1, c);
```

When you switch floating point type, the type of the array of coodinates **c** is automatically switched too. If you use multiple algebras in one application, make sure they use the same floating point type or you might run into trouble. This might be improved in the future.

As shown with benchmarks in [1], the combination of *maximum parity pure* memory allocation and the use of *floats* as floating point type leads to the highest performance. For high dimensional algebras (e.g. 5D) and *doubles*, the *tight* memory allocation method might be more efficient however.

2.7 Generate

The **generate** tab, shown in figure 2.4, is used to control several aspects of the code generation.

The **output dir** textfield has the same contents as the **name** field in the **general** tab by default; when you change the **name** field, the **output dir** field is set to the same contents as the **name** field. However, you may find it useful to have several algebras with the same name (e.g. **e3ga**), but with different properties (such as optimizations, products and functions). One instance where this may be useful is when you use the same algebra in different applications. Different applications may benefit from different product optimizations, but instead of using two algebras with the different names (which would become bothersome when you use code from one application in the other), you can use two algebras with the same name, but with (slightly) different properties. You would store the algebras in seperate directories (e.g. *e3ga_app1* and *e3ga_app2*) and compile and link each application with its own algebra which was tailored to its own specific needs.

The checkboxes at the lower part of the **general** tab control what kind of optimized code is generated to implement the computation of the products. Only **C**, **C2** and **LAPack** are available currently. Moreover LAPack is more like a test implementation to compare the performance of the C implementation to a brute force LAPack approach. In the future, SSE and 3DNow support may be added.

The **C** code generator generates slightly more (a few percent) efficient code than the **C2** code generator. However, the code generated by the **C2** generator might be *much* smaller (especially when you add lots of optimizations in the **products** tab, or when you generate a high dimensional algebra). When you have finished an application, you may want to try the other generator to see which one gives the best results.

Chapter 3

Layer 2: high level C++ interface

The most convenient way to use the source code generated by Gaigen is the high level C++ interface. It consists of two classes (one for mulivectors, one for outermorphism operators) which sit on top of the actual code generated by Gaigen. The interface is contained in two files (*gaigenhl.h* and *gaigenhl.cpp*) which you can easily change if you wanted to (for instance, if you don't like the operator definitions).

This chapter discusses not only the features provided by *gaigenhl.h* and *gaigenhl.cpp*, but also some of the more convenient features provide by the lower level C++ interface discussed in chapter 4, because those features are directly accessible. In short we could say that this chapter discusses everything the casual user needs to know about programming geometric algebra using source code generated by Gaigen.

One of the most important features introduced by the high level interface is operator overloading. Operator overloading allows a programmer to give new meaning to a symbol (e.g. =, *, +, - and —) to almost whatever he or she wants. This allows us to write things like

```
a = R * b * !R;
```

to express $\mathbf{a} = \mathbf{R}\mathbf{b}\mathbf{R}^{-1}$. This example immediately shows a problem with operator overloading: picking the right operator symbol for the right operation. The $*$ symbol is used for the scalar product in geometric algebra literature, but is used for general multiplication in C++; thus one could reason it is a good choice for the geometric product. The ! symbol is used in C++ arithmetic as the 'not' or 'binary complement' operator for integers, which might make it a good choice for the inverse (but also for the dual). As you can see, it is not easy to pick the right symbol for each operation. Furthermore there are not enough operator symbols available to give every operation its own symbol, so some will have to do with regular function names. Functions are available for all operations, and the code above could be written as

```
a.copy(gp(gp(R,b), R.inverse()));
```

which gives the same result, but isn't half as readable. We have done our best to make a reasonable operator selection for the operators and also provided the reasoning behind each operator, but there will always be people who disagree with our choice. Already, our selection differs at some points from the selection made in another package, CLU [6].

We now list the functions assigned to all overloaded operators, with a comment as to why we assigned that specific function to that specific operator. We will discuss the use of operators later on, this is only a reference table:

operator	function	example	comment
=	assignment	a = b;	

$*$, $*=$	geometric product	c = a * b;
		a $*=$ b;
			The * symbol is used for multiplication in C++. Thus it is the best for the most fundamental product in geometric algebra.

/, /=	division inverse geometric product	c = a / b;
		a /= b;	

$\wedge$, $\wedge=$	outer product	c = a $\wedge$ b;
		a $\wedge=$ b;
			Best visual match to wegde symbol.

<<, <<=	left contraction	c = a << b;
		a <<= b;
			<< is the binary shifting operator in C++. The left contraction could be thought of as 'shifting' or removing the lhs argument from the right hand side argument. A trick to remember the symbol: the < symbol is used to express the 'smaller than' relation in mathematics. For the left contraction to make sense, the grade of the object on the lhs should be smaller than the grade of the object on the right hand side.

>>, >>=	right contraction	c = a >> b;
		a >>= b;
			Same reasoning as with the left contraction.

%, %=	scalar product	c = a % b;
		a %= b;
			The % symbol is used for modulo division in C++; the scalar product kind of works like a modulo, returning only the scalar part of a geometric product. To remember the symbol, imagine the two circles in the % symbol being two zeros, indicating that only the grade 0 part will be returned.

+, +=	addition	c = a + b;
		a += b;	

−, −=	subtraction, negation	c = a − b;
		a −= b;
		a = −b;	

!	inversion	a = !b;	The ! symbol is used to denote the binary complement of integers in C++. This suggests its use as the inversion operator for multivector, since the inverse of a multivector could be considered its complement.

∼	reversion	a = b∼;
		a = ∼b;
			The superscript ∼ symbol is often used in geometric algebra literature to denote the reverse.

−−	clifford conjugate	a = b−−;
		a = −−b;
			This operator can be used both pre- and postfix. Both do the same, and do *not* change the operand, unlike the −− operator when applied to standard C++ integers of floats.

++	grade involution	a = ++b;	Same comment as the −− operator.

&, &=	meet	c = a & b;
		a &= b;
			The & symbol is used for the binary *and* operation for integers in C++. The meet is like an *and* operation for subspaces.

|, |=	join	c = a | b;
		a |= b;
			The | symbol is used for the binary *or* operation for integers in C++. The join is like an *or* operation for subspaces.

()	grade part selection	c = a(GRADE1);	The () operator selects the specified grade part of a multivector variable.

[]	grade part selection	f0 = a[GRADE1][0];
		f1 = a[GRADE1][1];
		f2 = a[GRADE1][2];
			The [] operator returns a *pointer* to an array of floating point values representing the coordinates for the specified grade part of a multivector variable.

We now discuss how to use the C++ high level interface. Reading this section, and tutorial 1 which demonstrates the **e3ga** algebra (which stands for Euclidian 3 dimensional Geometric Algebra), will give you a good understanding of how to write your own application using geometric algebra as implemented by Gaigen. We will assume the classname is **e3ga**, but in your application it might be any name, since you can set it in the Gaigen user interface (see section 2.1).

3.1 Construction

To create a new multivector variable **a** with the value **0** you would use:

```
e3ga a;
```

If you want a scalar valued multivector variable **b** use:

```
e3ga b(1.0);
```

which creates a variable **b** with the value 1.0.

To explicitly set the coordinates of a homogenous multivector variable (a multivector for which all grade parts are 0 except for one) or a blade, you can use this constructor:

```
e3ga c(GRADE1, 1.0, 2.0, 3.0);
e3ga t(GRADE3, 8.0);
```

This creates a vector valued multivector **c** with the value $1\mathbf{e}_1 + 2\mathbf{e}_2 + 3\mathbf{e}_3$ and a multivector variable **t** with the value $8\mathbf{e}_1 \wedge \mathbf{e}_2 \wedge \mathbf{e}_3$. The **GRADE1** macro tells the constructor that the coordinates specified are for grade 1 of the multivector. The **GRADE3** macro tells the constructor that the coordinate specified is for grade 3 of the multivector. Note that the order of coordinates matters here. It depends on the order you specified in the **order** tab in the user interface when you created the algebra. For instance, if you wanted to create a multivector with the value of $1\mathbf{e}_1 \wedge \mathbf{e}_2 + 2\mathbf{e}_1 \wedge \mathbf{e}_3 + 3\mathbf{e}_2 \wedge \mathbf{e}_3$ you would have to use

```
e3ga c(GRADE2, 3.0, -2.0, 1.0);
```

because the **e3ga** algebra stores its grade 2 coordinates in the following the order and orientation: $[\mathbf{e}_2 \wedge \mathbf{e}_3, \mathbf{e}_3 \wedge \mathbf{e}_1, \mathbf{e}_1 \wedge \mathbf{e}_2]$. You can inspect and change this order and orientation by loading the specification file *e3ga.gas* for the algebra into the Gaigen UI (click on **load algebra**) and clicking on the **order** and **g.2** tabs.

Of course, you can also construct such an object explicitly from basis vector as follows

```
e3ga c = 1.0 * e3ga::e1 ^ e3ga::e2 +
      2.0 * e3ga::e1 ^ e3ga::e3 +
      3.0 * e3ga::e2 ^ e3ga::e3;
```

This removes all doubt, but is less efficient In this code snippet we have used some features we have not discussed yet (such as the **=**, **+** and **ˆ** operators, and the use of **e3ga::e1** to denote the $\mathbf{e}_{e1}$ basis vector). We will treat them later.

Now suppose you want explicitly set the coordinates of multiple grade parts of a new multivector variable. You can do that like this:

```
float coordinates[4] = {1.0, 2.0, 3.0, 4.0};
e3ga d(GRADE0 | GRADE2, coordinates);
```

This creates a multivector variable **d** with the value $1.0 + 2\mathbf{e}_2 \wedge \mathbf{e}_3 + 3\mathbf{e}_3 \wedge \mathbf{e}_1 + 4\mathbf{e}_1 \wedge \mathbf{e}_2$. The macros **GRADE0** and **GRADE2** can be added together using the standard binary 'or' operator, telling the constructor that you will supply coordinates for the grade 0 and grade 2 parts. The coordinates have to be supplied in an array, because supplying a separate constructor for almost every grade combination would grow out of hand. If the supplied array is too short to contain all coordinates, behaviour of the constructor will be unpredictable.

There is one more constructor; the copy constructor is used to create a new multivector variable with the same value as another variable. The following code creates a variable **e** with the same value as **d**:

```
e3ga e(d);
```

The definitions of all of these constuctors are:

```
// null constructor:
e3ga::e3ga();

// copy constructor:
e3ga::e3ga(e3ga &a);

// set to scalar value:
e3ga::e3ga(float scalar);

// construct & set single grade with N coordinates:
e3ga::e3ga(int gradeUsage, float c1, ..., float cN);

// construct & set multiple grades:
e3ga::e3ga(int gradeUsage, const float *coordinates);
```

3.2 Assignment

To set an *existing* multivector variable to the value **0** the function **null** can be used:

```
a.null();
```

To set the coordinates of an existing multivector variable explicitly, you can use one of the **set** functions. They are available in two flavours, just like the constructors above. The first flavour can only be used to set a multivector variable to a homogenous value:

```
a.set(GRADE1, 1.0, 2.0, 3.0);
```

This sets the existing multivector variables **a** to $1\mathbf{e}_1 + 2\mathbf{e}_2 + 3\mathbf{e}_3$.

To set the value to a non-homogenous value, the other flavour of the **set** function can be used:

```
float coordinates[8] = {1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0};
a.set(GRADE0 | GRADE1 | GRADE2 | GRADE3, coordinates);
```

This example would assign the value $1+2\mathbf{e}_1+3\mathbf{e}_2+4\mathbf{e}_3+5\mathbf{e}_2\wedge\mathbf{e}_3+6\mathbf{e}_3\wedge\mathbf{e}_1+7\mathbf{e}_1\wedge\mathbf{e}_2+8\mathbf{e}_1\wedge\mathbf{e}_2\wedge\mathbf{e}_3$ to **a**.

A variation of the **set** function are the named set functions:

```
a.setScalar(1.0);
b.setVector(2.0, 3.0, 4.0);
c.set3Vector(8.0);
float coordinates[3] = {4.0, 5.0, 6.0};
d.set2Vector(coordinates);
```

These can take both single coordinates and coordinates provided in arrays and are available for all grades of the algebra.

A final variation of the **set** functions is the use of the $=$ operator to assign a scalar value to a multivector variable:

```
a = 1.0f;
```

Note that the use of the $=$ is made possible through operator overloading.

To copy the value of one multivector variable to another, use

```
a = b;
```

or

```
a.copy(b);
```

which would copy the value of **b** into **a**.

To set a multivector variable to a random value, the random blade / versor function can be used. The following example sets the multivector variable **a** to a random scalar:

```
a.randomBlade(GRADE0, 1.0f);
```

The distribution of the random value is linear from the range $[-1.0, 1.0]$. The range is controlled by the second argument.

To set a multivector variable to a higher grade value the following would be used:

```
b.randomBlade(GRADE2, 5.0f);
c.randomVersor(GRADE3, 0.5f);
```

This sets **b** to a grade 2 *blade* with a random value, and **c** to a *versor* with the highest non-empty grade part 3. The blades and versors are created by generating the required number of random vectors within the range specified by the second argument and respectively wedging and multiplying these together. Higher grade random blades and versors are thus not taken from a linear distribution.

The definitions of all of these assignment functions are:

```
// set to 0:
void e3ga::null();

// set single grade with N coordinates:
int e3ga::set(int grade, float c1, ..., float cN);

// set multiple grades:
void e3ga::set(int gradeUsage, const float *coordinates);

// set single grade with N seperate coordinates:
int e3ga::setScalar(float c1);
int e3ga::setVector(float c1, ..., float cN1);
int e3ga::set2Vector(float c1, ..., float cN2);
.
.
.
int e3ga::setNVector(float c1);

// set single grade with an array of coordinates:
```

```
int e3ga::setScalar(const float coordinates[1]);
int e3ga::setVector(const float coordinates[N1]);
int e3ga::set2Vector(const float coordinates[N2]);
.
.
.
int e3ga::setNVector(const float coordinates[1]);

// copy and assign multivector or float:
void e3ga::copy(const e3ga &a);
e3ga& e3ga::operator=(const e3ga &a);
e3ga& e3ga::operator=(float f);

// set to random blade or versor:
int e3ga::randomBlade(int grade, float scale);
int e3ga::randomVersor(int grade, float scale);
int e3ga::random(int grade, float scale, int versor);
```

3.3 Operators

As already discussed in at the start of this section, a number of often used operations such as addition, geometric product and outer product have special symbols assigned to them (i.e. $+$, $*$ and $\wedge$. This is done through a C++ feature called operator overloading, which allows the programmer to assign new meaning to a symbol when it is used in combination with a class. It drastically increases the readability of code, e.g. compare

```
a = R * b * R.inverse();
```

with

```
a.copy(gp(gp(R,b), R.inverse()));
```

The former, using operator overloading, is much easier understood, though the C++ compiler will generate the same machine code for both cases.

One could classify the operators into thee types: binary and unary and 'in place'. Binary operators (such as $+$) take two arguments, e.g.:

```
a = b + c;
```

which is equal to

```
a = add(b, c);
```

On the contrary, unary operators (e.g. $\sim$) take one argument, e.g.:

```
a = ~b;
```

which is equal to

```
a = b.reverse();
```

And last of all, the 'in place' operators take two arguments, one of which is also used to store the result. e.g.:

```
a += b;
```

which is equal to

```
a = a + b; // ...which in turn is equal to...
a.copy(add(a, b));
```

3.4 The basic products

We use the name *basic products* for all products which are included using the **products** tab of *gaigenui*. Thus this definition excludes the delta product, the meet and join, multiplying by the inverse and other derived products.

3.4.1 Geometric Product

The function **gp** computes the geometric product of two multivector variables. The ***** symbol is used as the operator for the geometric product. The definitions of the geometric product functions are:

```
e3ga& e3ga::operator*(const e3ga &a) const;
e3ga& e3ga::operator*=(const e3ga &a);
e3ga& gp(const e3ga &a, const e3ga &b);
```

3.4.2 Outer Product

The function **op** computes the outer product of two multivector variables. The **ˆ** symbol is used as the operator for the outer product. The definitions of the outer product functions are:

```
e3ga& e3ga::operator^(const e3ga &a) const;
e3ga& e3ga::operator^=(const e3ga &a);
e3ga& op(const e3ga &a, const e3ga &b);
```

3.4.3 The Scalar Product

The function **scp** computes the scalar product of two multivector variables. The scalar product returns the grade 0 part of the geometric product. The **%** symbol is used as the operator for the scalar product. The definitions of the scalar product functions are:

```
e3ga& e3ga::operator%(const e3ga &a) const;
e3ga& e3ga::operator%=(const e3ga &a);
e3ga& scp(const e3ga &a, const e3ga &b);
```

3.4.4 The Inner Products

The are currently four inner products available in Gaigen, two of which have no operator symbol. The number of inner products arises from the lack of consensus in the geometric algebra research community of which inner product should be preferred. The definitions for the inner products are:

```
// left contraction:
e3ga &e3ga::operator<<(const e3ga &a) const;
e3ga &e3ga::operator<<=(const e3ga &a);
e3ga &lcont(const e3ga &a, const e3ga &b);
// right contraction:
e3ga &e3ga::operator>>(const e3ga &a) const;
e3ga &e3ga::operator>>=(const e3ga &a);
e3ga &rcont(const e3ga &a, const e3ga &b);
// Hestenes inner product:
e3ga &hip(const e3ga &a, const e3ga &b);
// Modified Hestenes inner product:
e3ga &mhip(const e3ga &a, const e3ga &b);
```

3.5 Inversion

If three inversion functions are available (versor, lounesto and general inverse), then which one does Gaigen pick when you write one of the following equivalent line of code?

```
a = b / c;
a = b * !c;
a = b * c.inverse();
```

The answer is that Gaigen will prefer the versor inverse over the lounesto inverse, and the lounesto inverse over the general inverse. So if you have included all three inversion functions in your algebra, Gaigen will automatically use the fastest (versor) inverse. If the versor inverse is not available, the lounesto inverse is used, and finally it resort to the general inverse. This is handled by the following piece of code in *gaigenhl.h*:

```
#ifdef GAIM_FUNCTION_VERSORINVERSE
inline void inverse(const CLASSNAME &a) {versorInverse(a);};
#elif defined(GAIM_FUNCTION_LOUNESTOINVERSE)
inline void inverse(const CLASSNAME &a) {lounestoInverse(a);};
#elif defined(GAIM_FUNCTION_GENERALINVERSE)
inline void inverse(const CLASSNAME &a) {generalInverse(a);};
#endif
```

This automatic selection of the **inverse** function shouldn't problem be a problem, unless you want to *force* the use of specific inversion function. One case where this might occur is if you want to write

```
a = b * c.inverse();
```

where **c** is not a versor. If you have included the versor inverse, Gaigen will apply it even though it will give the incorrect answer. To force Gaigen to use a specific inversion function, you have to explictly name the inversion function you want:

```
a = b * c.versorInverse();
a = b * c.lounestoInverse();
a = b * c.generalInverse();
```

This of course rules out the use of the **inverse()** function and the **/** operator, because they always resort to the default inversion function.

The definitions of the inversion functions are:

```
// compute the inverse using the default algorithm:
e3ga& e3ga::inverse() const;
e3ga& e3ga::operator!() const;

// the overloaded '/' operator in all its variations:
e3ga& e3ga::operator/(const e3ga &a) const;
e3ga& operator/(float a, const e3ga &b);
e3ga& e3ga::operator/(float a) const;
e3ga& e3ga::operator/=(const e3ga &a);
e3ga& e3ga::operator/=(float a);

// the three inverse functions:
e3ga& e3ga::versorInverse() const;
e3ga& e3ga::lounestoInverse() const;
e3ga& e3ga::generalInverse() const;

// the functions which do the same as the operators:
// igp stands for 'inverse geomtric product'
e3ga& igp(const e3ga &a, const e3ga &b);
e3ga& igp(const e3ga &a, float b);
e3ga& igp(float a, const e3ga &b);
```

Remember that every time you use the inverse geometric product, or the **/** operator, you are implictly inverting a multivector. So if you have to divide by the same multivector value many times, it is more efficient to first invert the multivector, store that inverse, and then multiply by the inverse instead of dividing by the original multivector.

3.6 Addition, subtraction, negation

Multivector variable can be added and subtracted using the **add**, **sub**, **+** and **-** functions and operators. The **negate-** operator is also used to compute the negation of a multivector variable.

```
e3ga a, b, c;
a = -b;  // this is equivalent to...
a = b.negate(); // ... this, which is equivalent to...
a = 0.0 - b; // ... this
```

The definitions of these the addition, subtraction and negation functions are:

```
// addition:
e3ga& e3ga::operator+(const e3ga& a) const;
e3ga& e3ga::operator+=(const e3ga& a);
e3ga& add(const e3ga& a, const e3ga& b);
// subtraction:
```

```
e3ga& e3ga::operator-(const e3ga& a) const;
e3ga& e3ga::operator-=(const e3ga& a);
e3ga& sub(const e3ga& a, const e3ga& b);
// negation:
e3ga& e3ga::operator-() const;
e3ga& e3ga::negate() const;
```

Note that the **–** and **++** operators, which are used to decrement and increment integer and floating point variables by one in C++, have nothing to do with addition of subtraction for multivector variables. They are used for the Clifford conjugate and the grade involution in Gaigen (see section 3.7).

3.7 Reverse, Clifford Conjugate and Grade Involution

The reverse, Clifford conjugate and grade involution all toggle the sign of certain grade parts. They each have a unary operator. The **++** and **–** operators can be applied both pre- and post-fix:

```
e3ga a, b;
a = ++b;
a = b++;
a = b.gradeInvolition();
```

all of which are equivalent. They do not alter the operand like the standard **++** and **–** operators for integer and floating point variables do. The definitions of these functions and operators are:

```
// reverse:
e3ga& e3ga::operator˜() const;
e3ga& e3ga::reverse() const;
// clifford conjugate:
e3ga& e3ga::operator--() const;
e3ga& e3ga::operator--(int) const;
e3ga& e3ga::cliffordConjugate() const;
// grade involution:
e3ga& e3ga::operator++() const;
e3ga& e3ga::operator++(int) const;
e3ga& e3ga::gradeInvolution() const;
```

3.8 Grade Part Selection

The **grade** function and the **()** operator can be used to select a certain grade part from a multivector variable. This may be useful when you are only interested in a certain grade part and not in any others, such as in this example:

```
e3ga vector, rotor;
vector = rotor * vector * rotor.inverse();
vector = vector(GRADE1);
```

Here some vector is rotated by some rotor. We know that the vector is a grade 1 blade, and should still be a grade 1 blade after the rotation. However, due to floating point round off errors, a small grade 3 part may arise. This grade 3 part is thrown away by the the third line in the example: the **(GRADE1)** operator call selects only the grade 1 parts of **vector**. This example could have been one line shorter like this:

```
e3ga vector, rotor;
vector = (rotor * vector * rotor.inverse())(GRADE1);
```

The definitions of the **grade** function and the **()** operator are:

```
e3ga& e3ga::grade(int g) const;
e3ga& e3ga::operator()(int g) const;
```

3.9 Meet and Join

The & and | symbols are used as operators for the meet and join of blades or subspaces. These symbols are used as the binary *and* and *or* operations when used on integers. The meet and join compute the *and* and *or* of subspaces, hence the & and | are good operator symbols for them.

The join is on based the delta product [5] and algorithms described in [1]. The delta product is used to compute the grade of the join of the two input blades. Then one blade is factored into a number of vectors, which are then repeatedly wedged to the other blade until a blade best representing the join of the input blades is found. The meet is compute using the join, as described in [1].

The following shows how to use the meet and join:

```
// Assuming b and c are two blade valued multivector variables
// this code computes their meet and join
e3ga m = b | c;
e3ga j = b & c;
// which is equivalent to:
e3ga m = meet(b, c);
e3ga j = join(b, c);
```

The definitions of the meet and join functions are:

```
e3ga &meet(const e3ga &a, const e3ga &b);
e3ga &join(const e3ga &a, const e3ga &b);
e3ga &e3ga::operator&(const e3ga &a) const;
e3ga &e3ga::operator|(const e3ga &a) const;
```

3.10 Factorization

The function **factor** and **factorVersor** can be used to factor blade or versors into vectors. When the vectors are wedged or multiplied together, they rebuild the blade of versor. The **factor** function is used by the **meet** and **join** functions, and a use of the **factorVersor** function might be to retrieve the vectors in the plane of rotation of a rotor. The definitions of the factorization functions are:

```
// Factor a blade ('versor' = 0) or a versor ('versor' = 1)
// The function returns the number of factors and...
// stores them in 'f'.
int e3ga::factor(e3ga f[], int versor = 0) const;
// Factor a versor
// The function returns the number of factors and...
// stores them in 'f'.
int e3ga::factorVersor(e3ga f[]) const;
```

3.11 Exponentiation

The function **exp**, which computes the exponentiation of a multivector, is present in the algebra whenver the geometric product is included. The function computes the taylor series expansion

$$exp(\mathbf{A}) \approx \sum_{i=0}^{n} \frac{\mathbf{A}^i}{i!} \tag{3.1}$$

n is specified by the optional integer argument which defaults to 9. This is identical to the CLU **exp** function.

The definition of **exp** is:

```
e3ga& e3ga::exp(int order = 9) const;
```

3.12 Outermorphism

If you have a function f for which the following is true for any pair of input blades $\mathbf{a}$ and $\mathbf{b}$.

$$f(\mathbf{a} \wedge \mathbf{b}) = f(\mathbf{a}) \wedge f(\mathbf{b}) \tag{3.2}$$

then it is an outermorphism. Examples of outermorphisms are rotations. Another example are all operations that you traditionally can model using a 4×4 when you use homogeneous coordinates (translation, rotation, scaling, skewing and so on). If you have to apply such a function to multivectors many times, you might consider constructing an outermorphism for it. It can be applied to blades of any grade. Once initialized, the outermorphism operator can probably compute the result faster, and will assure that the result is the same grade as the input.

The outermorphism operator is represented by its own class. The of this class is the name of your algebra class (e.g. **e3ga**) with **_om** concatenated to it (e.g. **e3ga_om**). You can initialize the outermorphism operator when you construct it, or later on using one of the **init** functions. The outermorphism is initialized by either passing it the images of all basis vectors under the linear transformation, or by passing it a spinor. If you pass a spinor, the initialization function will assume that the right way to apply the spinor is

```
e3ga spinor, vector, vectorImage;
vectorImage = spinor * vector / spinor;
```

It will compute the vector images of all basis vectors and then pass these to the vector images initilization function.

A typical use of the outermorphism operator would be something like this:

```
const int nb = 1000;
int i;
e3ga rotor, lotsOfVectors[nb], lotsOfVectorImages[nb];
e3ga_om om;

// initialize the outermorphism operator:
om.initSpinor(rotor);

// apply it to the vectors:
for (i = 0; i < nb; i++) {
    lotsOfVectorImages[i] = om * lotsOfVectors[i];
    /*
    //this is the other way to compute the vector images:
    lotsOfVectorImages[i] =
        (rotor * lotsOfVectors[i] / rotor)(GRADE1);
    */
}
```

Of course the outermorphism operators doesn't work on vectors only: you can also use it to transform blades of any grade.

The outermorphism operator internally constructs a matrix representation for the linear operator. The matrix representation for the grade 1 part is exactly the traditional matrix that would be used when one would use linear algebra to do geometry. The 1x1 'matrix' that transforms the pseudoscalar is the determinant of the transformation [1].

The matrix representation leads to another advantage of the outermorphism; because of the matrix form, it can easily be executed using Single Instruction Multiple Data (SIMD) instructions sets, such as supplied by the SSE(2), 3DNow! or AltiVec. This could be exploited by a future opt2X compiler.

The definitions of the outermorphism constructors are:

```
// construct but don't initialize:
e3ga_om::e3ga_om();
// construct & initialize using array of vector images:
e3ga_om::e3ga_om(const e3ga vectorImages[3]);
// construct & initialize using array of pointer to vector images:
e3ga_om::e3ga_om(const e3ga *vectorImages[3]);
// construct & initialize using spinor/rotor/versor:
e3ga_om::e3ga_om(const e3ga &spinor);
```

The initialization functions are:

```
// init using a spinor/rotor/versor to create the vector images:
int e3ga_om::initSpinor(const CLASSNAME &spinor);
// init using images under the outermorphism of the basis vectors:
```

[1] The other grade parts of the matrix represention are not widely know traditionally, but are also very useful

```
int e3ga_om::initVectorImages(const CLASSNAME vectorImages[3]);
int e3ga_om::initVectorImages(const CLASSNAME *vectorImages[3]);
```

To apply the outermorphism to a multivector variable use either of these functions:

```
// apply outermorphism L to multivector A, returns the result:
e3ga& om(const e3ga_om& L, const e3ga& A);
// the '*' operator does the same as the 'om' function:
e3ga &operator*(const e3ga_om& L, const e3ga& A);
```

3.13 Coordinate Output and Access

You may want access to coordinate for several reasons. One reason is that you may want to store your multivectors in files, to retrieve them later. This can be done by storing their coordinates and later restoring them when your application reads the file.

Another reason is inspection of the coordinates, either graphically or by printing them as numbers. Although geometric algebra is coordinate free (and Gaigen is as well, after you have entered the coordinates into multivectors), most people find it useful (at least while still somewhat uncomfortable with geometric algebra) to inspect the coordinates of multivectors.

We will first treat printing the coordinates via Gaigen, and then show how your application can get get access the coordinates.

If you want to print the coordinates to the standard output, you can use the **print** function like this:

```
e3ga a(GRADE1, 1.0, 2.0, 3.0);
a.print("a: ");
```

This example would print:

```
a: 1.00*e1  + 2.00*e2 + 3.00*e3
```

The default way the floating point coordinates are printed is using the **printf** format **¨%2.2f¨**. You can change this format by specifying it as the second argument of **print**, e.g.:

```
e3ga a(GRADE1, 1.0, 2.0, 3.0);
a.print("a: ", "%e");
```

This prints:

```
a: 1.000000e+000*e1  + 2.000000e+000*e2 + 3.000000e+000*e3
```

You can also change it using the **setFPPrecision()** function described below.

If you want to print to something else than the standard output, you can use the **fprint** function to print to any file. For total control of where your output goes, you can print the coordinates to a string using the **string** function. You can then do whatever you want with that string. The definitions of these three functions are:

```
// get a string representation of a multivector:
const char *e3ga::string(const char *prec = NULL) const;

// print a string representation of a multivector to...
// the standard output:
void e3ga::print(const char *text = NULL,
    const char *prec = NULL) const;

// print a string representation of a multivector to...
// a file:
void e3ga::fprint(FILE *F, const char *text = NULL,
    const char *prec = NULL) const;
```

Starting with Gaigen 0.99, two more function are available that control the printing:

```
// set the default precision for all future prints/strings
static int e3ga::setFPPrecision(const char *prec);

//set the string delimiters for all future prints/strings
static int e3ga::setStringDelimiters(char start, char end);
```

The first function takes as argument an ASCIIZ string (e.g., "%e") that describes how to format floating point coordinates. The second function takes as arguments to characters (e.g., '[' and ']') that will be used as start and end of every string.

Two functions and one operator provide direct (read only) access to the coordinates of a multivector: **scalar**, **coordinates** and **[]**. The **scalar** function returns the scalar coordinate of a multivector and is used like this:

```
e3ga a(1.0);
float sc = a.scalar();
```

The **coordinates** function and the **[]** operator can retrieve a pointer to the coordinates of any grade part of a multivector. You specify which grade part you'll get the the coordinates using the integer argument. This argument can be any of the **GRADE0** ... **GRADEN** macros, but you can not combine then to get the coordinates of multiply grades in one call. This example demonstratres the use of the **coordinates** function and the **[]** operator:

```
e3ga a(GRADE1, 1.0, 2.0, 3.0);
// get a pointer to the grade 1 coordinates of a:
float *g1c = a.coordinates(GRADE1);
// get the scalar coordnate of a:
float sc = a.coordinates(GRADE0)[0];
// get each of the grade 2 coordinates of a:
float *g2c = a.coordinates(GRADE2);
float g2c_e1_e2 = g2c[2];
float g2c_e2_e3 = g2c[0];
float g2c_e3_e1 = g2c[1];
```

In the last four lines of the example above, you can how the each of the three coordinates of the grade 2 part of **a** is retrieved. This piece of code is very dependent on the orientation and order of the grade 2 basis blades. If you change the orientation and order of these blades (using the **order** tab, see section 2.4), this code would function incorrectly.

A better way to retrieve individual coordinates is using the **E3GA_X** macros defined in **e3ga.h**. For each coordinate, a macro is generated which specifies the index of the coordinate in the coordinates array *relative* to the first coordinate of that grade part. For the **e3ga** algebra, these macros look like this:

```
#define E3GA_S 0
#define E3GA_I 0
#define E3GA_E1 0
#define E3GA_E2 1
#define E3GA_E3 2
#define E3GA_E2_E3 0
#define E3GA_E3_E1 1
#define E3GA_E1_E2 2
```

If you change the order of the coordinates , these macros automaticly change as well when you regenerate the code. It's easy and readable to use the macros like this to retrieve the individual coordinates:

```
// get each of the grade 2 coordinates of a:
float *g2c = a.coordinates(GRADE2);
float g2c_e1_e2 = g2c[E3GA_E1_E2];
float g2c_e2_e3 = g2c[E3GA_E2_E3];
float g2c_e3_e1 = g2c[E3GA_E3_E1];
```

The only instance where you could run into trouble with accessing coordinates like this is when you flip the orientation of the basis blades. For instance this could change

```
#define E3GA_E3_E1 1
```

into

```
#define E3GA_E1_E3 1
```

and then your application won't compile anymore if you still use **E3GA_E3_E1**. But this is better than compiling successfully and then failing at runtime because your application retrieve the wrong coordinates.

```
// get the scalar coordinate of a multivector:
float scalar() const;

// get the GRADEX coordinates of a multivector:
const float *e3ga::operator[](int grade) const;
const float *e3ga::coordinates(int grade) const;
```

You can retrieve the largest coordinate of a multivector using **largestCoordinate()**:

```
// Get the fabs() value of the (absolutely) largest coordinate
// of a multivector:
float e3ga::largestCoordinate() const;
```

For example:

```
e3ga a(GRADE1, 1.0, 2.0, -3.0);
GAIM_FLOAT lca = a.largestCoordinate(); // lca = 3
```

3.14 Coordinate string parsing

Gaigen can parse its own string output and a bit more. The function **parseString()** takes as input a string describing a multivector (as formatted for instance by the **string()** function), parses it, and sets the value of the multivector accordingly:

```
int e3ga::parseString(const char *str, const c3ga_ben *ben = NULL);
```

Usage example:

```
e3ga a;
int nb;
nb = a.parseString("1.0*e1+2.0*e1^e2^e3");
```

The function returns the number of characters read from **str**, or -1 on failure.

The optional second argument of **parseString()** describes optional extra Basis Elements Names and the start and end delimited of the string. These are contained in a small C++ class:

```
class e3ga_ben {
public:
    e3ga_ben();
    e3ga_ben(char startDelimiter, char endDelimiter);
    ~e3ga_ben();

    // add a basis element name and value to this 'ben'
    int addName(const char *name, const e3gai &mv);
    // remove a basis element name from this 'ben'
    int removeName(const char *name);

    // look up a basis element name and its value from this 'ben'
    int lookupName(const char *name, e3gai &mv) const;

    // remove all basis element names
    int removeAll();

    // set the start and end delimiters (e.g. '[' and ']')
    // this can also be done using the
    int setDelimiters(char startDelimiter, char endDelimiter);

    // set 'this' to defaults (no extra names, no delimiters)
    int setDefaults();
};
```

Usage example:

```
e3ga a;
e3ga_ben eb('[', ']');
int nb;
eb.addName("I", e3ga::e1 ^ e3ga::e2 ^ e3ga::e3);
nb = a.parseString("[1.0*I]", &eb);
```

A future version of Gaigen may contain a better multivector parser that uses ANTLR, much like the parser that is already present in GAViewer.

3.15 Miscellaneous functions

3.15.1 normal

The **normal** function returns a normalized version of the multivector it is applied to, e.g.:

```
e3ga a, b;
//...
b = a.normal(); // set b to the normalized version of a
```

The normal function takes and optional integer argument, which specifies the norm to be used. If the argument is 1, the 'Euclidean' norm is take: the sum of the square of all coordinates. If the argument is 2, the norm is computed as

```
GAIM_FLOAT norm = a % a.reverse();
```

3.15.2 dual

The **dual** function returns the dual with respect to the pseudoscalar **I** of the algebra:

```
e3ga a, b;
//...
b = a.dual(); // set b to the dual of a
```

the dual is computed as $\mathbf{b} = \mathbf{a}\mathbf{I}^{-1}$. If the **fastDual** function is included in the algebra, then that function is used by default. It works by swapping and negating the coordinates which is faster than explicitly computing the dual.

3.15.3 mvType

mvType() returns the type of a multivector: **GA_BLADE**, **GA_VERSOR** or **GA_MULTIVECTOR**. If the multivector is a blade, it also returns the grade of the blade. It's declaration is:

```
int mvType(int *grade, double epsilon) const;
```

The pointer to **grade** can be **NULL** if you don't care about the grade. The value in **grade** is one of the constants **GRADE0**, **GRADE1** ... **GRADEN**. The value of epsilon defaults to 1e-14 for doubles, 1e-7 for floats. A usage example:

```
e3ga a(GRADE1, 1.0, 2.0, -3.0);
int grade;
int type = a.mvType(&grade); //type = GA_BLADE, grade = GRADE1 (=2)
```

3.15.4 layer 1 functions

There are also a number of functions from layer 1 that are useful and can be accessed directly from the high level C++ interface:

- **null**. The **null** sets its argument to 0
- **compress** compresses its argument in place. This means to remove any grade part that is equal to 0. You can supply an option floating point argument ϵ to **compress** to tell it what is still considered to be 0:

  ```
  e3ga a;
  //...
  a.compress(1e-10); // compress all grade parts < 1e-10
  ```

- **reciprocalFrame**. The static reciprocal frame function computes the reciprocal frame of a set of basis vectors. Its definition is

  ```
  static int reciprocalFrame(c3gai f[], const c3gai e[],
  int nbVectors);
  ```

  This sets the set of '**nbVectors** vectors **f** to the reciprocal frame of **e**.

3.16 Profiling

To use the profiler, the **profile** checkbox in the **function** must be ticked. Gaigen will then include profiling code when the algebra is generated. Only two *static* functions accessible by your application are added: **resetProfile** and **printProfile**. Static means that the functions are accessible without reference to an instantiation of the class. The following:

```
e3ga::printProfile();
```

will invoke the static **printProfile** function in the class **e3ga**. Thus, when you use a static function of a class, you don't have to have access to a variable of that class.

Actually the three profiling functions are always present in the algebra code, whether the checkbox is ticked or not. But when it is not ticked, the generated function does nothing. If the functions would not be included at all in the source code, you would have to remove or add your calls to the profiler each time you turn the profiling option off or on. The profiling code slows down the performance of your application, so its best to turn it off when you don't use it anyway.

resetProfile resets the count of all product/multivector combinations. You should call this function at the start of your application.

printProfile prints out all product/multivector combinations to the standard output. It has an optional argument, which defaults to 2.0, which specifies up to what usage percentage the product/multivector combinations are shown. **printProfile** computes how often each product/multivector combination is used relatively, and if a product/multivector combination falls below the usage percentage threshold, it is not shown. So to print the usage of *all* product/multivector combinations, use

```
e3ga::printProfile(0.0);
```

where the **0.0** tells the function to show *all* product / multivector combinations. The default argument,

```
e3ga::printProfile();
```

which is equivalent to

```
e3ga::printProfile(2.0);
```

will print the product / multivector combinations with a usage above 2.0%.

saveProfile saves the profile to *.gap* file which can be read back by the Gaigen user interface to automatically add optimizations (section 2.3). Use the function as follows:

```
e3ga::saveProfile("profile_name.gap");
```

The definitions of **resetProfile**, **printProfile** and **saveProfile** are:

```
static int e3ga::resetProfile();
static int e3ga::printProfile(float threshold = 2.0);
static int e3ga::saveProfile(const char *filename = NULL);
```

3.17 Basis Vectors and (Inverse) Pseudoscalar

The internal class (see section 4) contains the multivector variables with the values of the basis vectors and (inverse) pseudoscalar as *static* [2] members. These are also available from the high level C++ interface.

Using the basis vectors and (inverse) pseudoscalar you can write things like:

```
e3ga a, b, one;
a = e3ga::e1 ^ e3ga::e2;
b = a << e3ga::Ii;
one = e3ga::Ii * e3ga::I;
```

The definitions of the basis vectors and (inverse) pseudoscalar are:

```
static e3gai::e3gai e1;
static e3gai::e3gai e2;
static e3gai::e3gai e3;
static e3gai::e3gai I; // pseudoscalar
static e3gai::e3gai Ii; // inverse pseudoscalar
static e3gai::e3gai *bv[3]; // array of pointers to e1, e2, e3
```

bv is an array of pointers to the basis vectors. The basis vectors (**e1**, **e2** and **e3** in the example of the **e3ga** algebra) can have any name. In the 5d **c3ga** algebra for instance, the basis vectors are called **e1**, **e2**, **e3**, **e0**, **einf**; a 3d 'color algebra' could name the basis vectors **red**, **green** and **blue**. To change the name of the basis vectors, use the **signature** tab (section 2.2).

[2]Static in this context means that there is only one global copy of such a variable; it is not included in every instantiation of a multivector variable.

3.18 Use of Internal Class and Floating Point Variables in Functions

Besides writing statements like:

```
e3ga a, b, c;
c = a * b;
```

you of course also want to write

```
e3ga a, b, c;
c = 2.0 * a * b;
```

and

```
e3ga a, b, c;
c = e3ga::e1 ^ e3ga::e2;
```

even though **2.0**, **e3ga::e1** and **e3ga::e2** are not variables of type **e3ga** (**2.0** is of type **float** or **double**; **e3ga::e1** and **e3ga::e2** are of the internal class type (**e3gai**).

That's why *gaigenhl.h* and *gaigenhl.cpp* contain a combination of automatic *type casting* operator and extra *inline* defintions for some and operators functions such that you can write the statements given above.

The definition of the casting operator looks like this:

```
inline e3gai::operator e3ga &() const {return *((e3ga*)this);}
```

The operator casts variables of type **e3gai** to **e3ga** [3].

Examples of extra functions for using floating point variables and internal class variable in combination with multivectors are the following:

```
e3ga& gp(float a, const e3ga &b);
e3ga& gp(const e3gai &a, float b);
```

These function computes the geometric product of a **float** and an **e3ga** variable, and of an **e3gai** and a **float**. Such extra definitions are present in *gaigenhl.h* and *gaigenhl.cpp* for all functions where appropriate.

3.19 Temporary Variables

One may wonder where the temporary multivector variables come from in code like

```
a = (b + c) + d; // both lines are equivalent
a = add(add(b, c), d); // both line do the same thing
```

since the program should first compute **b + c**, store the answer to that somewhere, and then compute **(b + c) + d**, possibly store that somewhere, and then assign the result to **a**. Where the temporary variables come from, depends on whether you selected the **fast temporary variable**.

If you turned the fast temporary variables off, the temporary variables are handled in the default C++ way. All functions and operators like **add** and **operator+** return plain multivector variables like this:

[3]Note that because of this casting operator, you can not add non-static member variables or virtual functions to *gaigenhl.h* and *gaigenhl.cpp*. Doing so anyway might mess up the simple but naive casting operator.

```
e3ga e3ga::operator+(const e3ga &a);
e3ga add(const e3ga &a, const e3ga &b);
```

Note the absence of the **&** symbol in the return type. This means that an actual variable is returned, instead of just a reference. These variables are allocated from the stack and then initialized using the standard constructor (the code to do so is generated by the C++ compiler). One can imagine that allocating and initializing a multivector variable for each intermediate result can lower the performance of your application.

On the other hand, if you turned on the fast temporary variables, functions and operators returning multivector variables will be defined like this:

```
e3ga& e3ga::operator+(const e3ga &a);
e3ga& add(const e3ga &a, const e3ga &b);
```

The functions and operators now return *references* to multivector variables (note the **&** symbol in the return type). The actual temporary variables are allocated by Gaigen source code from an array of multivectors variables with a fixed size. This is more efficient than letting the C++ compiler allocate them from the stack, but it has a drawback. Since the array of temporary variables has a fixed size[4], at some point old temporary variables will have to be recycled. If these old temporary variables are still in use somewhere in you program, their value will be overwritten. This will lead to bugs in you program which are very hard to find, especially of you don't know what the symptoms of the problem are. The section on temporary variables in tutorial 1 [3] demonstrates the problem.

The rule of thumb is to never keep a reference to a temporary multivector variable when you have enabled fast temporary variables in the **functions** tab. The two main ways to keep a reference to a temporary variable are passing them on to another function like this:

```
    someFunction(a + b);
```

and explictly keeping a reference:

```
    e3ga &c = a + b;
```

Explictly keeping a reference can be easily avoided, and pass a temporary variable to a function can be prevented like this:

```
    c = a + b;
    someFunction(c);
```

If you have a look at *gaigenhl.h* and *gaigenhl.cpp*, you will see that functions and operators return the type **GAIM_RETURN_TYPE**. This is actually a macro **#define**d as either **e3ga** or **e3ga&**, depending on whether the fast temporary variables are enabled or not. The allocation of temporary variables is managed by the **GAIM_RETURN_VAR(variableName)** macro. Both these macros are defined in *gaigenhl.h*.

[4]this size is defined by **MV_MAX_TEMP**, in *gaigenhl.h*.

3.20 Multitheading

Multitheading support has not yet been fully implemented in Gaigen. The main functions which are not mt-safe are part of the tight and balanced memory allocation algorithms, the fast temporary variable allocation, and the **string** function. Use of the **string** (and **print** and **fprint** functions can be avoided if you really want to use multithreading, and the max or maxpp memory allocation algorithms can be used. Multitheading support might be more complete in the future.

3.21 Drawing

There are no multivector drawing functions available in Gaigen. We have implemented GAViewer that allows you to view multivectors interactively with minimal intrusion in your application. Your application only has to be able to write multivectors and commands to a file, which can be conviniently done using the print functions of Gaigen. This allows you to view results of computions and to debug your application without adding an (OpenGL) UI to it.

Chapter 4

Layer 1: Internal C++ class

This chapters describes the internal C++ class. The source code for this class is generated by Gaigen. Much of this generated source code is copied and pasted (with adjustments) from *gaspectemplates.txt*, a file which is described in section 6.1. Most people will not want to use the internal C++ class to actually program applications, since it's a rather crude interface. This chapter is intended for people who want to create their own high level C++ interface (to replace *gaigenhl.cpp*), who want to squeeze the last bit of performance out of their application, or those who have to change or maintain the internal C++ class.

Many functions of the internal C++ class return **void**, unlike functions of the high level C++ interface, which return the result. Instead, most internal C++ class functions store the result in the calling class, e.g.:

```
c.add(a, b);
```

adds **a** and **b** and stores the result in **c**.

4.1 C++ Class

The storage parts of the C++ class declaration looks something like this:

```
class e3gai {
.
.
.
// Depending on the memory allocation algorithm...
// Either a pointer, or an array of floats
    float *c; // pointer
    float c[8]; // array of floats
// bitfield for grade part and memory usage administration
    int usage;

// information about the class:
    static const int dim;
    static const int nbCoor;
```

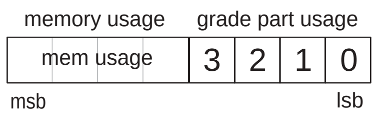


Figure 4.1: Storage of grade part and memory usage in the usage field.

```
// the basis vectors and (inverse) pseudoscalar
    static e3gai e1;
    static e3gai e2;
    static e3gai e3;
    static e3gai *bv[3];
    static e3gai I;
    static e3gai Ii;
.
.
.
}
```

First of all, the class contains either a pointer to an array of floating point variables, or an array of floating point variables. In either case, it is called **c** for coordintes. The type used depends on the memory allocation algorithm selected in the **storage** tab. The difference is, that an array of floating point variables *inside* the class always uses a certain amount of memory, while a pointer can be used to point to any amount of memory. So with the array of floating point variables *inside* the class, the **e3gai** class will still use 8 or 4 floating point variables to store a single coordinate (e.g. a scalar), while the pointer can point to a much lower amount of memory (this is done by the tight and balanded memory allocation algorithms). For coordinate access, there is no difference between the pointer and the array.

The coordinates are always stored in 'compressed' form in the **c** array. Even if 8 floating point variable are allocated anyway, only 4 of them are used to store (for example) a rotor.

The **usage** integer variable is to store the grade part usage (as a bitfield) and the memory usage (as an integer). These two n-bit[1] words are stored as illustrated in figure 4.1. If a grade part is in use by the multivector, the bit in the grade usage field will be set to 1, otherwise it will be set to 0. The memory usage field stores the number of floating point variables *allocated* for this multivector. It is used by the balanced and tight memory allocation algorithms to determine from which heap they allocated the coordinate array[2].

The other storage variables of the C++ class are *static*, which means that only one copy of them is used in your application. They include **dim**. the dimension of the algebra, **nbCoor**, the maximum number of coordinates used by a multivector variable, and the basis vectors and (inverse)pseuodscalar. The basis vectors are available individually, named as they in the **signature** tab, and from an array of length 'd' (the dimension of the algebra). The pseudoscalar

[1]'n' is the dimension of the algebra plus 1.

[2]One heap is available for each array length, from 1 up to 2^d.

and inverse pseudoscalar are always called **I** and **Ii**. All these static multivector variables are initialized to their proper values when your application starts.

4.2 Constructors, Assignment Functions

The constructors and assignment functions are identical to those used for the high level C++ interface except that the operator symbol '=' is not available. The **random**, **randomBlade** and **randomVersor** *are* available.

4.3 The Products

The product functions like **gp**, **lcont** and **op** all have the same syntax:

```
void e3gai::gp(const e3gai &a, const e3gai &b);
void e3gai::op(const e3gai &a, const e3gai &b);
void e3gai::lcont(const e3gai &a, const e3gai &b);
void e3gai::rcont(const e3gai &a, const e3gai &b);
void e3gai::hip(const e3gai &a, const e3gai &b);
void e3gai::mhip(const e3gai &a, const e3gai &b);
void e3gai::scp(const e3gai &a, const e3gai &b);
```

They are used differently than those from the high level C++ interface; the following line

```
    c.gp(a, b);
```

computes the geometric product of **a** and **b** and stores it in **c**. No temporary variables are used.

Versions of the outer product function are available in which one argument can be a floating point value:

```
void e3gai::op(const e3gai &a, float scalar);
void e3gai::op(float scalar, const e3gai &a);
```

All product functions first check to see if an optimized function exists to compute the product of the arguments. If so, they set the grade part and memory usage of the result, and call the optimized function. Otherwise they expand the arguments to arrays of pointers to arrays of floating point variables (like the one shown in figure 5.1), call the general implementation of the function, and compress the result into the multivector variable.

4.3.1 Euclidean Metric Products

Only when the meet and join or factoring functions are included, in algebras with a non-euclidean signature, special euclidean implementations of some metric products (geometric product, left contraction and scalar product) are available. They are used to implemented a LIFT (see [5]). Their definitions are

```
void e3gai::gp_em(const e3gai &a, const e3gai &b);
void e3gai::lcont_em(const e3gai &a, const e3gai &b);
void e3gai::scp_em(const e3gai &a, const e3gai &b);
```

4.4 Other Functions

To add or subtract two multivectors, use

```
void e3gai::add(const e3gai &a, const e3gai &b);
void e3gai::sub(const e3gai &a, const e3gai &b);
```

To add a multivector and a floating point variable, these functions are available:

```
void e3gai::add(float scalar, const e3gai &b);
void e3gai::add(const e3gai &b, float scalar);
void e3gai::sub(float scalar, const e3gai &b);
void e3gai::sub(const e3gai &b, float scalar);
```

These functions compute the reverse, clifford conjugate and grade involution of a multivector and store it in the calling multivector:

```
void e3gai::reverse(const e3gai &a);
void e3gai::cliffordConjugate(const e3gai &a);
void e3gai::gradeInvolution(const e3gai &a);
```

The three flavours of the inverse can be used with these functions:

```
// compute inverse, for versors only
int e3gai::versorInverse(const e3gai &a);

// compute inverse, for general 3d multivectors
int e3gai::lounestoInverse(const e3gai &a);

// compute inverse, for general multivectors;
// uses gaussian elimination
// (slower and less stable than versorInverse)
int e3gai::generalInverse(const e3gai &a);
```

These four functions can be used to extract grade parts, or to get information about the grade of a multivector:

```
void e3gai::takeGrade(const e3gai &a, int grade);
int e3gai::highestGrade(const e3gai &a);
int e3gai::grade() const;
int e3gai::maxGrade() const;
```

takeGrade extracts a single grade part from a multivector. **highestGrade** extracts the highest non-zero grade part from a multivector, and returns its grade index. **grade** returns the grade of a homogenous multivector and -1 if the is not homogenous. **maxGrade** returns the the index of the highest non-zero grade.

The norm of a multivector, according to one definition, can be computed using the **norm_a** function:

```
float e3gai::norm_a() const;
```

The function is called **norm_a** (for now) because other norm definitions will also be implemented. To normalize a multivector use:

```
void e3gai::normalize(const e3gai &a, int norm);
```

The **norm** argument is used to specify which definition of the norm you want to use; currently the only valid value is **1**, which uses the **norm_a** norm.

To project a multivector onto a blade or versor, or to reject a multivector from a blade, one of the following functions can be used:

```
int e3gai::project(const e3gai &blade, const e3gai &a);
int e3gai::projectOntoVersor(const e3gai &versor, const e3gai &a);
int e3gai::reject(const e3gai &blade, const e3gai &a);
```

To compute the meet and join of two blades, use

```
int e3gai::join(const e3gai &a, const e3gai &b, int algorithm = 1);
int e3gai::meet(const e3gai &a, const e3gai &b, int algorithm = 1);
```

The **algorithm** argument (either **1** or **2**) is used to specify which of the two available join algorithms to use. The meet is computed with respect to the join.

These are three support functions used by the meet and join:

```
int e3gai::deltaProduct(const e3gai &a, const e3gai &b);
int e3gai::factor(e3gai factors[], int versor = 0) const;
int e3gai::factorVersor(e3gai factors[]) const;
```

The coordinates function returns a pointer to the coordinates of a certain grade part. You should not try to modify the floating point variables this pointer points to.

```
const float *e3gai::coordinates(int grade) const;
```

To compute the reciprocal frame of a set of **nbVectors** vectors **e**, use **reciprocalFrame**. The result goes into **f**.

```
static int e3gai::reciprocalFrame(e3gai f[], const e3gai e[], int nbVectors);
```

The outermorphism has the same construction and initialization functions as the high level C++ interface outermorphism, except that there is no 'init with spinor' initializer.

The print and string functions are exactly those described in the high level C++ interface.

The **fastDual** function can be used to compute the dual with respect to the entire space.

```
int e3gai::fastDual(const e3gai &a);
```

4.5 Internal Functions

This section described several interesting internal functions.

The **setUsage** function sets the grade part and memory usage of a multivector variable. It makes sure enough memory has been allocated to store the coordinates. It's declaration is:

```
void e3gai::setUsage(int u);
```

These are several functions to expand the coordinates of a multivector.

```
// Expand a multivector into a matrix, according to table:
void e3gai::expand(float matrix[], const int table[]) const;

// Expand a multivector into an array of pointers
// to arrays of coordinates for each grade:
void e3gai::expand(const float *pa[4]) const;

// Expand 2 multivectors ('a' and 'b') into arrays of pointers
// to arrays of coordinates for each grade:
void e3gai::expand(const e3gai &b, float const *pa[4],
    float const *pb[4]) const;
```

The profiling functionality is implemented by three functions (two of which have already been described in chapter 3). The third, internal, function is **incrementUsage**. This function is called by the product functions when profiling is enable. It counts (in the array **usageCount**) how many times each combination of product and multivectors has been used. This information is printed when you call **printProfile**. The **productNames** are used for printing purposes.

```
static int e3gai::resetProfile();
static int e3gai::printProfile(float threshold = 2.0);
int e3gai::incrementUsage(int product, int gua, int gub);
static int e3gai::usageCount[5][16][16];
static const char *e3gai::productNames[5];
```

Two algorithms are available to compute the join:

```
int e3gai::joinAlg1(const e3gai &a, const e3gai &b,
    int ga, int gb, int gj);
int e3gai::joinAlg2(const e3gai &a, const e3gai &b,
    int ga, int gb, int gj);
```

Algorithm 2 builds up candidates for the join by wedging vectors together, algorithm 1 projects blades of the right grade onto on of the multivectors **a** or **b**, to find a good candidate for the join. The best candidate (with the largest norm) is returned as the join. Both algorithms take as arguments two multivectors (**a** and **b**), the grade of each multivector (**ga**, **gb**) and the required grade of the join (**gj**).

The inline functions **gradeUsage** and **memUsage** return the grade part and memory usage of a multivector. This information is extracted from the **usage** variable in the multivector.

```
inline int gradeUsage() const;
inline int memUsage() const;
```

4.6 Global Internal Variables

There are a number of interesting variables outside the C++ which are assumed to be useful to the internal class only; that's why they are not included in the class itself. To name a few:

```
const char *e3gai_basisElementNames[8];
int e3gai_gradeSize[4];
int e3gai_mvSize[16];
```

e3gai_basisElementNames is an array of strings which holds the names of all the basis vectors (e.g. **e1**, **e2** and **e3**). These names are used for printing the coordinates of a multivector variable. The integer array **e3gai_gradeSize** contains the number of coordinates each grade part holds. **e3gai_mvSize** is also an integer array and holds, for each possible combination of grade usage, the minimum number of coordinates required to store the coordinates. This is used (among others) for the memory allocation algorithms. The grade part usage, which serves as an index into the **e3gai_mvSize** array, is specified in the same binary format as in figure 4.1.

Chapter 5

Layer 0: low level computational functions

Layer 0 is responsible for actually computing the (optimized) products and computing other basic functions (such as addition and reversion). It is defined as an partially static and partially dynamic interface which must be implemented. An opt2X compiler is responsible for generating the layer 0 files (although you can of course them by hand if you want). The compiler takes an *.opt* file (see section 6.2), which contains specifications of the functions that should be implemented, and generates a C++, C or assembly file which adhers to the interface.

The static part of the interface currently consists of the following functions:

```
// copy 'src' to 'dst'; length of arrays is 'length'
void e3gai_copy(float *dst, const float *src, int length);

// compute reverse multivector 'a'
void e3gai_reverse(float *a[]);

// compute clifford conjugate of multivector 'a'
void e3gai_cliffordConjugate(float *a[]);

// compute the grade involution of multivector 'a'
void e3gai_involution(float *a[]);

// negate 'src' to 'dst'; length of array is 'length'
void e3gai_negate(float *dest, const float *src, int length);

// compute the norm (all coordinates squared) of 'a'
// length of array is 'length'
float e3gai_norm_a(float a[], int length);

// add two multivectors with the same grade part usage
// 'c' = 'a' + 'b'
// length of arrays is 'length'
void e3gai_addSameGradeUsage(float *c,
```

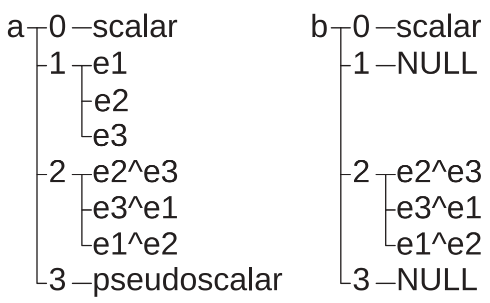


Figure 5.1: Array representation of multivector variables $\mathbf{a}$ and $\mathbf{b}$.

```
    const float *a, const float *b, int length);

// add two multivectors with the different grade part usage
// 'c' = 'a' + 'b'
void e3gai_add(const float *a[4], const float *b[4], float *c[4]);

// subtraction function, simular to addition functions
void e3gai_subSameGradeUsage(float *c,
    const float *a, const float *b, int length);
void e3gai_sub(const float *a[4], const float *b[4], float *c[4]);
```

All these functions must always be implemented by the code generated by the opt2X compiler. As you can see, all functions except **e3gai_copy**, **e3gai_norm_a**, **e3gai_negate**, **e3gai_addSameGradeUsage** and **e3gai_subSameGradeUsage** take arrays of pointers to arrays of floating point variables as arguments. These arrays contain the coordinates of each grade part. A pointer to a coordinate array can also be **NULL** when a grade part is not in occupied by the multivector. This is illustrated in figure 5.1. It shows the array representation of multivectors $\mathbf{a}$ and $\mathbf{b}$. $\mathbf{a}$ occupies all grade parts, so all 4 pointers actually point to coordinates. $\mathbf{b}$ on the other hand is a rotor and only occupies grade part 0 and 2. The pointers for grade part 1 and 3 are **NULL**.

The *.opt* file contains the specifications of the dynamic part of the interface. It specifies the name of the algebra (with an *i* appended to it, since this is an internal part of the algebra implementation), the dimension, the floating point type and the (optimized) product functions which should be implemented. The *.opt* file format is described in section 6.2.

The general product functions, which can compute the product of any type of multivector with any other type of multivector, always look something like this:

```
void e3gai_general_gp(const float *a[], const float *b[], float *c)
```

This function should compute a product of multivector $\mathbf{a}$ and $\mathbf{b}$, and store it in $\mathbf{c}$. The coordinates of the multvectors are given in the format explain above and illustrated in figure 5.1.

The optimized product functions, which can compute only a product of two specific types of multivectors, always look something like this:

```
void e3gai_opt_0A_gp_05(const float *a, const float *b, float *c);
```

The functions receive the coordinates of their arguments **a**, **b** and **c** as arrays of floating point variables. The opt2X compiler knows what it should do with each coordinate because of the description of the function in the *.opt* file.

An opt2X compiler is free to implement all these function anyway it wants. The opt2c compiler for instance, take the most straightforward approach and almost directly converts the .opt file to C++. The opt2LAPack compiler, an experimental compiler, uses LAPack to compute the general products. The opt2c2 compiler splits up the products into many little functions and lets the product functions call those as required.

Chapter 6

File Formats

6.1 *gaspectemplates.txt* file

Gaigen reads and copies most piece of code that it generates from the *gaspectemplates.txt* file. The syntax of that file is discussed in this section. To read, parse and output the code contained in the file, the **codeTemplateContainer** class in *codetemplate.cpp* and *codetemplate.h* is used.

The *gaspectemplates.txt* file consists of *codeblocks*. Codeblocks are blocks of code which Gaigen copies into the source code it generates. In early versions of Gaigen, all code was simply printed to the sourcefiles using **fprintf**. This became very messy, since all of the code that now resides *gaspectemplates.txt* (over a thousand lines) was contained in *gaspec.cpp*. Instead of that, we put almost all of the code in templates (or codeblocks) in *gaspectemplates.txt*. That makes them easy to read, create and maintain. Only the code which is very difficult to 'copy and paste' (e.g. which is extremely dependent on the dimension of the algebra), is printed directly from *gaspec.cpp*.

Of course, Gaigen does not literally copy code from *gaspectemplates.txt*. Many parts of the generated code depend on the dimension, name, floating point type and many other properties of the required algebra. Thus special keywords are used, which are replaced by the appropriate strings when the sourcefiles are generated. These keyword are set by Gaigen and stored in a lookup table (which also resides in the **codeTemplateContainer** class)[1].

Each codeblock in *gaspectemplates.txt* has the following structure:

```
${CODEBLOCK codeBlockName}
.
.
.
${ENDCODEBLOCK}
```

Each code block starts with a **CODEBLOCK** statement, followed by the name of the code block. That name is used to identify it the **codeTemplateContainer** class. The statement starts with a dollar sign and is surrounded by curly braces.

[1]If a keyword is used in *gaspectemplates.txt* which has not been entered into the lookup table, the **codeTemplate** class prints out the name of the keyword in the generated source file, making it easy to find such mistakes.

Each code block ends with a **ENDCODEBLOCK** statement. Between these two statements, the code resides. *Anything* can be put between the two statements, so not just (parts of) C++ code. Code or text outside a codeblock is ignored and generates a warning when parsed by the **codeTemplateContainer** class.

In the code block, you can use keywords which will be replaced by strings from the lookup table. An example of such a keyword is **$**{**FLOAT**}. When the code is output by the **codeTemplateContainer** class, this keyword is changed to either **float** or **double**, depending on the floating point type specified in the **storage** tab.

These are often used keywords:

- {**CLASSNAME**}: the name of the class as it appears in the high level C++ interface.
- {**INTERNAL_CLASSNAME**}: the name of the class used in the low level C++ glue.
- {**FLOAT**}: the floating point type (**float** or **double**).
- {**FLOATSIZE**}: the size of the floating point type in bytes (e.g. **sizeof(float)**).
- {**FLOATCAST**}: used to cast a type to a floating type (e.g. **(double)**).
- {**FLOAT_EPSILON**}: the value that is consider 'small' by default. $1e-8$ for **float** and $1e-15$ for **double**.
- {**NUMBER_OF_COORDINATES**}: the total number of coordinates (2^d).
- {**DIMENSION**}: the dimension of the algebra (e.g. **3**).
- {**DIMENSION+1**}:the dimension of the algebra plus 1.
- {**1<<(DIMENSION+1)**}: 2 to the power of the dimension plus 1 (2^{d+1}).
- {**MAX_NUMBER_OF_COORDINATES**}: only used when the memory allocation algorithm is **max** or **maxpp**. Is equal to the maximum number of coordinates which can be stored in the coordinate array.
- {**MEMFACTOR**}: the balaned memory allocation algorithms 'waste factor'.
- {**NUMBER_OF_PRODUCTS**}: how many products are included in the algebra (includes euclidean versions of the geometric and scalar product and left contraction.
- {**GP_EM**}: the name of the function which can compute the *euclidean* metric geometric product (used for **meet** and **join**).
- {**LCONT_EM**}: the name of the function which can compute the *euclidean* metric left contraction (used for **meet** and **join**).
- {**SCP_EM**}: the name of the function which can compute the *euclidean* metric scalar product (used for **meet** and **join**).
- {**VERSORINVERSE_EM**}: the name of the function which can compute the *euclidean* metric versor inverse (used for **meet** and **join**).

- {**PROJECT_EM**}: the name of the function which can compute the *euclidean* metric projection (used for **meet** and **join**).
- {**TMPVAR1**}: temporary variable, set right before the code block is output. The value of these temporary variable varies from code block to code block.
- {**TMPVAR2**}: temporary variable.
- {**TMPVAR3**}: temporary variable.

6.2 *.opt* files

.opt files are genereted by Gaigen and contain descriptions of the low level computational functions which must be implemented to compute the products. There are two types of low level computational functions: general and optimized. The general functions must always be implemented for every product that is included in the algebra; they must are able to compute the product of *any* multivector with any other multivector (independent of grade usage). Descriptions of optimized functions however, are generated for every optimization that was entered in the **products** tab. Optimized function can only compute the product of two specific multivectors (e.g. the outer product of a vector and a bivector).

The *.opt* files are read by special opt2X compilers, which compile them into source code (e.g. C, C++ or assembly). The job of the compilers is to do this as efficient as possible. The separation between the Gaigen program and the opt2X compiles was made to make it easy for 3rd party developers to generate low level computational functions for specific platforms they use. The standard opt2X which come with Gaigen (opt2c, opt2c2 and opt2LAPack) are called directly by Gaigen when it generates the source code and the *.opt* file. Which compiler is used can be controlled using the **generate** tab in the user interface. *opt.cpp* and *opt.h* can be used to read and parse *.opt* files[2]

Besides the function descriptions, the *.opt* files must contain three other keywords:

- **dimension**: the dimension of the algebra
- **classname**: the *internal* classname of the algebra (e.g. **e3gai**).
- **floattype**: the floating point type to be used (**float** or **double**).

A general function description starts with a line like this:

```
general e3gai_general_gp gp 0
```

[2]*opt.cpp* and *opt.h* are also able to extract certain extra information from the products, called *product patterns*. These are used by the opt2c2 compiler. Product patterns are the basic building blocks of the products. Only a limited number of product patterns exist for each algebra, and all of them are used by the geometric product. The other products are made of 'selections' of the product patterns of the geometric products. Thus, by implementing the product patterns of the geometric product, *all* products can be implemented by calling the function which computes each required product pattern. This is how the opt2c2 compiler works.

The **general** keyword is used to identify the start of a general function description. The next string is the name of the function to be generated (**e3gai_general_gp** in this case). The next string is the abbreviated name of the product (**gp**). Finally the integer indicates that the products belongs to 'group' 0 or 'group' 1 (this information is used to build the product patterns). Multiple groups are used when euclidean and non-euclidean versions of the metric products are required.

The function which should be generated in response to the example above is (in C syntax):

```
void e3gai_general_gp(const float *a[], const float *b[],
    float *c) {
.
.
.
}
```

a and **b** are arrays of pointers to arrays of floating point values which contain the coordinates of the input multivector variables. E.g. **a[0]** points to the scalar coordinate of the first input multivector, and **b[2]** points to the 3 bivector coordinates of the second input multivector. When a grade part is not used in the multivector variables, the pointer is set to **NULL**. **c** points to a *single* array of floating point variables. This is where the results of the computition go.

The lines following the start of the general function description all look something like this:

```
c[4] = + a[0][0] * b[2][0] - a[1][0] * b[3][0]
```

What this line says is that at **c[4]** the compiler should store **a[0][0] * b[2][0] - a[1][0] * b[3][0]**. This could almost directly be copied and pasted to make up a valid C function, except that one should check for **NULL** pointers in arrays **a** and **b**.

The optimized function descriptions have a slightly different syntax than the general functions. Here is an example of an optimized function description which is to compute the geometric product of a rotor and a vector:

```
optimize e3gai_opt_05_gp_02 gp 0 05 02
c[0] = + a[0] * b[0] + a[3] * b[1] - a[2] * b[2]
c[1] = - a[3] * b[0] + a[0] * b[1] + a[1] * b[2]
c[2] = + a[2] * b[0] - a[1] * b[1] + a[0] * b[2]
c[3] = + a[1] * b[0] + a[2] * b[1] + a[3] * b[2]
```

Again, the first keyword **optimize** is used to identify the start of the optimized function description. This is followed by the name of the function, tha abbriviated name of the product, the group number (**0**), and the grade usage of the input multivectors in hexadecimal format (**05** and **02** in this case). *opt2c* generates this function is response to the functino description above:

```
void e3gai_opt_05_gp_02(const float *a, const float *b,
    float *c) {
    c[0] = + a[0] * b[0] + a[3] * b[1] - a[2] * b[2] ;
    c[1] = - a[3] * b[0] + a[0] * b[1] + a[1] * b[2] ;
    c[2] = + a[2] * b[0] - a[1] * b[1] + a[0] * b[2] ;
    c[3] = + a[1] * b[0] + a[2] * b[1] + a[3] * b[2] ;
}
```

As you can see, compiling optimized functions is even simpler than compiling general functions.

Besides the general and optimized functions, the opt2X compilers also generate a number of low level computational functions which can add, subtract, negate, reverse, etc multivectors. These are to be described elsewhere.

6.3 *.gas* files

.gas files store Gaigen's specifications of geometric algebras. *.gas* stands for Geometric Algebra Specification. The *.gas* file format is very simple. Every line consists of a keyword (e.g. **dimension**) and several arguments. Keywords can appear in any order, with one exception: the **dimension** keyword is always the first keyword of the file. Comments begin was the hash symbol '#' and end at the end of a line. Keywords are case insensitive and if they are unknown to the parser, the entire line is ignored.

If you look in the *gaspec.cpp* source code, functions that have to do with *.gas* files have names like **loadProfile** and **saveProfile**. Initially the word *profile* instead of *specification* was used to refer to the properties of an Gaigen algebra. The word *profile* was not exposed to the user later on because it may cause confusion in relation with the **profile** checkbutton and function.

This is a list of all keywords, arguments and descriptions in alphabetic order.

- **basiselementname** The **basiselementname** keyword has two arguments. The first argument is a number followed by a colon (e.g. **0:**), the second a word which is a valid C++ variable name (e.g. **e1**). The **basiselementname** keyword is used to set the name of the basis elements (or vectors) of the algebra. Basis elements are labeled from 0 to $d-1$, where d is the dimension of the algebra, as specified by the **dimension** keyword.

- **dimension** The single argument of the **dimension** keyword specifies the dimension of the algebra. It should always be the first keyword in the *.gas* file. Only lines with comment or blanks can preceed it.

- **dirname** The string argument to the **dirname** keyword specifies the directory (relative to the algebras directory) where the generated code will be stored. This is equal to the name of the algebra by default, but this be changed in the **generate** tab.

- **floattype** The **floattype** keyword is used to set the type of floating point numbers (**float**s or **double**s) which the generated source code uses. Gaigen is not yet fully able to generate source code which uses **double**s, so the user interface does allow the user to set the the type of floating point numbers yet.

- **function** The **function** keyword has a single argument which is an integer. The value of the integer specifies which functions are included in the algebra. These functions correspond to the checkboxes in the **function** tab, and the **gaFunction** enumeration in *gaspec.h*. The integer should be interpreted as a binary word. Each bit which is on signals the inclusion of specific function in the algebra. Which function corresponds to which bit can be looked up in *gaspec.h*, in the **gaFunction** enumeration.

- **generateoption** Like the **function** keyword, the **generateoption** keyword is followed by an integer which should be interpreted as a binary word. Each bit which is on signals the (de)activation of a specific *generate* option. Which options corresponds to chich bit can be looked up in *gaspec.h*, in the **gaGenerateOption** enumeration.

- **memfactor** The single floating point argument of the **memfactor** keyword is the factor entered in the **storage** tab. It is only used when the memory allocation algorithm is set to **balanced**. It must have a value of 1.0 or higher.

- **memman** The memory allocation (or management) algorithm is specified by the string argument of the **memman** keyword. The string argument can be one of **tight**, **balanced**, **maxpp** or **max**. These values correspond to each of the four choices which can be made in the **storage** tab.

- **name** The name of the algebra follows the **name** keyword.

- **optimize** The **optimize** keyword specifies the optimization of a specific multivector / product combination. The syntax is

  ```
  optimize product (gradeUsage1, gradeUsage2) usage
  ```

  where **product** can be any of the abbreviated product names (e.g. **gp** or **lcont**), and **gradeUsage1** and **gradeUsage2** are grade usage of each of the multivectors involved. The grade usage should be interpreted as a binary word, just like they are in Gaigen internally. E.g. when bit 2 (4 in decimal) is on, then grade 2 is in use. The optional floating point value **usage** specifies how often this product was used according to a profile. It is used by the *ifelse* and *switch* dispatching methods to ensure that the most often used function are found most efficiently.

- **order** The **order** keyword specifies for each index in the uncompressed coordinate array (from 0 to $2^d - 1$), what basis blade coordinate is stored there. The basis blades are specified by which basis vectors are included in them. The basis blades are again given as a binary word, where each bit signals the in- or exclusion of a basis vector. So the line

  ```
  order 7: 6
  ```

  tells us that at uncompressed coordinate array location 7 the coordinate relative to the basis blade $\mathbf{e}_2 \wedge \mathbf{e}_3$ is given. If an optimization is specified for a product which is not included in the algebra, it is still read and stored by *gaigenui*, but it won't be visible in the **products** tab until the product is included.

- **product** The **product** keyword is followed by any of the any of the abbreviated product names (e.g. **gp** or **lcont**) and specifies the inclusion of that product in the algebra.

- **sign** The **sign** keyword is followed by a basis blade (specifed as a binary word) and either **1** or **-1**. It specifies the orientation of the basisblade. When you to toggle the orientation for a basis blade in the **order** tab, you toggle

- **reciprocal** The **reciprocal** keyword is used to specify reciprocal null basis vectors. In the user interface, this is done by checking the **reciprocal** checkbutton between two basis vectors (in the **signature** tab). The **reciprocal** keyword is used like this:

  ```
  reciprocal 3 4 signature: 1
  ```

  The **3** and **4** indicate which basis vectors are involved (they are indices in the range $[0 \ldots d-1]$). The signature can be either **-1** or **1**. The signature **0** is forbidden, since then you could just as well create two ordinary null vectors.

- **signature** The **signature** keyword specifies the signature of a basis vector. The first argument is the index of the basis vector (range $[0 \ldots d-1]$) and the second argument is either -1, 0 or 1.

6.4 *.gap* files

.gap files are written by the **saveProfile** function. There are four possible keywords in a *.gap* file:

- **profiledate** The **profiledate** keyword is followed by a string specifying the date and time when the file was written.

- **name**. The argument to the **name** keyword specifies the name of the algebra.

- **dirname**. The argument to the **dirname** keyword specifies the directory name where the algebra was generated. Both **name** and **dirname** are used to identify the algebra (i.e. to make sure you don't use the wrong argument

- **productusage**. The **productusage** specifies what product of what types of multivectors is used how often. These lines form the bulk of a *.gap* file. The format of the keyword arguments is identical to those of the **optimize** keyword in a *.gap* file:

  ```
  productusage product (gradeUsage1, gradeUsage2) usage
  ```

Bibliography

[0] To do: fix some of these references.

[1] D. Fontijne, T. Bouma, L. Dorst *Gaigen: A Geometric Algebra Implementation Generator.* Available at http://carol.science.uva.nl/~fontijne/gaigen

[2] D. Fontijne. *Gaigen user manual.* Available at http://carol.science.uva.nl/~fontijne/gaigen

[3] D. Fontijne, *Gaigen Tutorial 1* Available at http://carol.science.uva.nl/~fontijne/gaigen

[4] S. Mann, L. Dorst, T Bouma, *The Making of GABLE: A Geometric Algebra Learning Environment in Matlab,* in Geometric Algebra with Applications in Science and Engineering, E Corrochano and G Sobczyk (eds), Birkhauser, 2001, pg 491-511. Also available at ftp://cs-archive.uwaterloo.ca/cs-archive/CS-99-27/ and http://carol.science.uva.nl/~leo/clifford/gable.html, 1999

[5] T. Bouma *About the delta product* Where: ask Tim.... / online

[6] C. Perwass *The CLU and CLUDraw Library* Available at http://www.perwass.de/cbup/clu.html

[7] Bill Spitzak et al. *FLTK: the Fast Light Tool Kit* Available at http://www.fltk.org/