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User Guide for Program CARE-2

Anne Chao and Hisn-Chou Yang

Institute of Statistics, National Tsing Hua University, Hsin-Chu, Taiwan

Table of Contents:

1. Introduction

Program CARE-2 calculates population size estimates for various closed capture-recapture models. The program consists of two parts: one part, written in C Language, deals with models without covariates and the other part, written in GAUSS language, deals with models with covariates.

In this manual, we outline the downloading and setup procedures (Section 2), data input formats (Section 3). Operation procedures, models and estimators featured in CARE-2 are described in Section 4 (for models without covariates) and Section 5 (for models with covariates). Examples are provided and sample outputs are shown. Results for each example are also discussed to help the user interpret the numerical output.

Before using CARE-2, the user is suggested to read two introductory chapters in a Handbook of Capture-Recapture (Chao and Huggins, 2003) where some backgrounds and historical development are provided.

You are welcome to use CARE-2 for your own research and applications as long as you will not distribute CARE-2 in any commercial form. If you publish your work based on the results from CARE-2, please use the following reference for citing CARE-2.

Chao, A. and Yang, H.-C. (2003) Program CARE-2 (for Capture-Recapture Part. 2). Program and User's Guide published at http://chao.stat.nthu.edu.tw.

The maximum input size in CARE-2 is 2000 individuals and 80 occasions. If your data exceed these sizes, please send a mail to us indicating your size; we will send you a modified program that fits your data input.

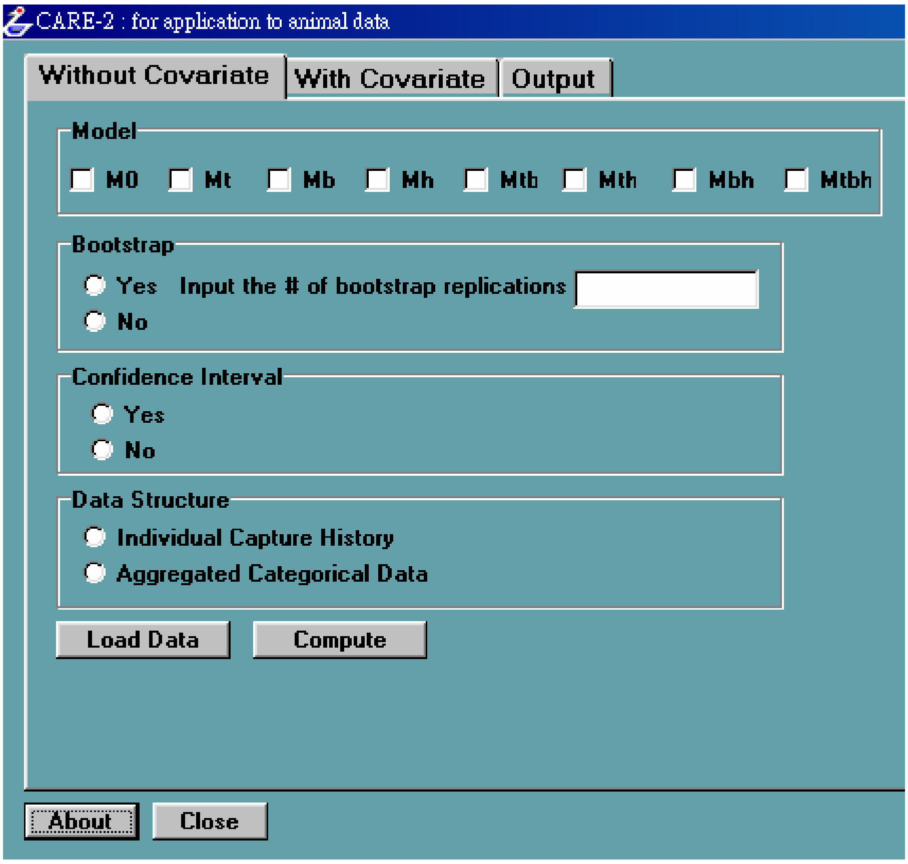
2. Download and Setup

Program CARE-2 can be downloaded from Anne Chao's website at http://chao.stat.nthu.edu.tw/softwareCE.html. First doubly click the downloaded file "care-2.exe" to unzip all files to a specified folder. Then doubly click the executable file "setup.exe" to install the program. The source files along with six illustrative data sets will be stored automatically in the specified folder in your computer.

Analysis without covariates

After the setup, doubly click the executable file "CARE-2.exe" to start the program with the interface shown in Figure 1.

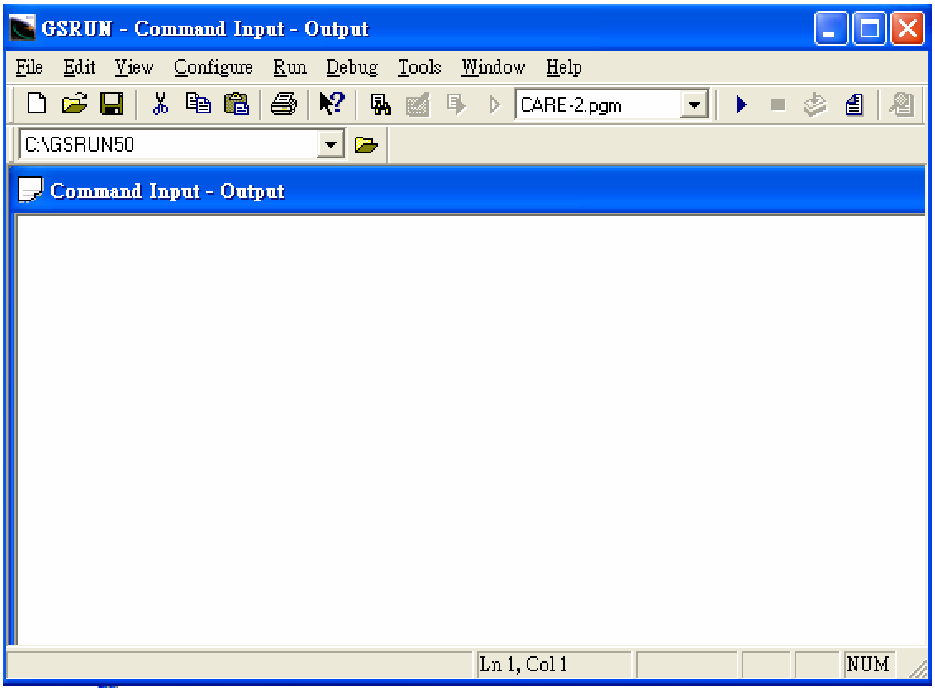
Figure 1. The interface of CARE-2 for analysis without covariates.



Analysis with covariates

The covariate analysis is not embedded in the interface of Figure 1. A working environment of Gauss is provided by the following procedure: first doubly click the "GRTM.exe" to unzip all files of the Gauss Run-Time Module (GRTM) in the previously specified folder. Then doubly click the executable file "setup.exe" to install the Gauss Run-Time Module, which is GUASS free-ware for non-commercial redistribution. (The GRTM allows licensee to redistribute licensee's compiled GAUSS programs free of charge to other users who do not have GAUSS so long as licensee's GAUSS program is distributed free of charge.) Then doubly click the icon "GSRUN50" on the desktop of your computer to initialize the Gauss Run-Time Module and then the interface is shown below.

Figure 2. The interface of CARE-2 for analysis with covariates.



3. Data Input Format

Data must be read from an ascii file. There are two types of data input formats:

(1) Individual Capture History:

Data are arranged in a matrix, called "individual capture history" matrix, with the rows representing the capture histories of each captured individual and the columns representing the captures on each occasion. The capture history of each captured individual is expressed as a series of 0's (non-captures) and 1's (captures) possibly followed by some individual covariates. The maximum size for capture history matrix input in CARE-2 is 2000 individuals and 80 occasions.

(2) Aggregated Categorical Data:

In some studies with many captured individuals, the individual capture history matrix becomes very large. It is more convenient to represent the raw data in a categorical data by a tally of the frequencies of each capture history.

The two types of data input will be illustrated by examples in the following sections.

4. Analysis Without Covariates

Models/Estimators Featured

The models considered in CARE-2 are originally proposed in Otis et al. (1978) and White et al. (1982) and are tabulated in Table 1. Assume that there are N animals in the study area and capture-recapture experiments are conducted over t occasions. The purpose is to estimate the unknown parameter N. Under each model, there are many available estimators in the literature. The estimators featured in CARE-2 and their abbreviations in output (see later sample output for four examples) are shown in Table 2. All the estimators are shown in the Appendix.

Table 1. Models without covariates in CARE-2.

P_{ij} denotes the capture probability of the ith animal on the jth occasion

p_i: heterogeneity effect of the ith individual, i =1, 2, …, N;

e_j: time or occasional effect of the jth occasion, j = 1, 2, …, t;

ϕ: behavioral response effect.

Model	Assumption	Restriction in model $\mathbf{M}_{tbh}$
$\mathbf{M}_{tbh}$	$P_{ij} = \begin{cases} p_i e_j & \text{until first capture} \\ \phi p_i e_j & \text{for any recapture} \end{cases}$	
$\mathbf{M}_{bh}$	$P_{ij} = \begin{cases} p_i & \text{until first capture} \\ \phi p_i & \text{for any recapture} \end{cases}$ (Generalized removal model)	set e_j = 1
$\mathbf{M}_{tb}$	$P_{ij} = \begin{cases} e_j & \text{until first capture} \\ \phi e_j & \text{for any recapture} \end{cases}$	set p_i = 1
$\mathbf{M}_{th}$	$P_{ij} = p_i e_j$	set ϕ = 1
$\mathbf{M}_{h}$	$P_{ij} = p_i$	set e_j = 1, ϕ = 1
$\mathbf{M}_{b}$	$P_{ij} = \begin{cases} p & \text{until first capture} \\ \phi p & \text{for any recapture} \end{cases}$ (Removal model)	set $p_i = p$, e_j = 1
$\mathbf{M}_{t}$	$P_{ij} = e_j$	set p_i = 1, ϕ = 1
$\mathbf{M}_{0}$	$P_{ij} = p$	set $p_i = p$, e_j = 1, ϕ = 1

Table 2. Estimators and their abbreviations in program CARE-2.

Model	Estimators/Approaches	Estimators in Software CARE-2
$\mathbf{M}_0$	Unconditional MLE (UMLE) Conditional MLE (CMLE) Estimating equations (EE)	Otis et al. (1978) Darroch (1958) Yip (1991)
$\mathbf{M}_t$	Unconditional MLE (UMLE) Conditional MLE (CMLE) Estimating equations (EE)	Otis et al. (1978) Darroch (1958) Yip (1991)
$\mathbf{M}_b$	Unconditional MLE (UMLE) Conditional MLE (CMLE) Estimating equations (EE)	Otis et al. (1978) Zippin (1956) Lloyd (1994)
$\mathbf{M}_{tb}$	Unconditional MLE (UMLE) Conditional MLE (CMLE) Estimating equations (EE)	Chao et al. (2000) Chao et al. (2000) Lloyd (1994); Chao et al. (2000)
$\mathbf{M}_h$	Jackknife (JK1, JK2, IntJK) Sample coverage (SC1 & SC2) Estimating equations (EE)	Burnham and Overton (1978) Lee and Chao (1994) Chao et al. (2001)
$\mathbf{M}_{th}$	Sample coverage (SC1 & SC2) Estimating equations (EE)	Lee and Chao (1994) Chao et al. (2001)
$\mathbf{M}_{bh}$	Jackknife (JK) Sample coverage (SC) Estimating equations (EE)	Pollock and Otto (1983) Lee and Chao (1994) Chao et al. (2001)
$\mathbf{M}_{tbh}$	Estimating equations (EE)	Chao et al. (2001)

Program CARE-2 calculates two standard error estimates. One is the asymptotic s.e. (Asy_s.e. in output) which is obtained by inverting a Fisher information matrix (for models without heterogeneity) or by a delta method (for heterogeneous models). For the estimating equation (EE) approach, the asymptotic s.e. is not obtainable for models $\mathbf{M}_h$, $\mathbf{M}_{th}$, $\mathbf{M}_{bh}$ and $\mathbf{M}_{tbh}$ because of complexity. The other method is bootstrap s.e. (Boot_s.e. in output), which is always obtainable for all estimators.

For interval estimation, CARE-2 provides two 95% confidence intervals based on a log-transformation method (Chao, 1987) and percentile method (Efron and Tibshirani, 1993) respectively. Both intervals are constructed from the bootstrap s.e. We remark that the bootstrap standard error (Boot_s.e.) and confidence intervals may vary from trial to trial because the bootstrap replication data vary with trials.

Running Procedures

(1) Doubly click the executable file CARE-2.exe, it prompts you the interface window as shown in Figure 1.

(2) Click “Without Covariate” from the top menu of CARE-2. There are four items to be specified before executing CARE-2 as shown in Figure1. They are Model, Bootstrap, Confidence Interval and Data Structure as explained in the following four steps.

(3) Model Selection: select suitable model(s) for your data. You can check all model boxes to include eight models for comparisons. The model description is listed in Table 1.

(4) Bootstrap Selection: select whether you like to do the bootstrap for obtaining standard error estimates and confidence intervals or not. If yes, then select the number of replications (1000 is suggested).

(5) Confidence Interval Selection: select whether you like to have a 95% confidence interval or not. If your selection is “yes”, you must also check “yes” in step (4) for the bootstrap selection and specify the number of replications.

(6) Data Structure Selection: select the format of your data set. Two types of data formats are described in Section 3.

(7) Click “Load Data” to input the filename of your data file (e.g. c:\program files\CARE-2\data\example1.dat).

(8) Click “Compute” to get the results. (Wait a while for executing the program. The execution time depends on the size of data and the number of bootstrap replications.)

(9) Click “Output” from the top menu to view the results. You can click “Save Output” to save all the output results to a designated file; click “Print” to print the output from your printer; or click “Clear” to remove all results and to proceed another run.

Examples

Four examples are used to demonstrate the use of CARE-2 for analyzing animal capture-recapture data without covariates. All data sets used in this guide are distributed with CARE-2 and stored by default in the directory c:\program files\CARE-2\data. The output will be shown and briefly described. The four examples used in this section are:

Example 1: Deer mice data in a format of individual capture history (data in file: example1.dat). Refer to Chao and Huggins (2003) for detailed analysis.

Example 2: Mouse data in a format of individual capture history (data in file: example2.dat). Refer to Chao et al. (2001) for detailed analysis.

Example 3: Same data set as in Example 2, but in a format of aggregated categorical form (data in file: example3.dat).

Example 4: Cottontail rabbit data in a format of individual capture history form (data in file: example4.dat). Refer to Chao et al. (1992) for detailed analysis.

Example 1: Deer mice data (individual capture history data)

These data were collected by V. Reid and are distributed with program CAPTURE (Otis et al., 1978; White et al., 1982; Rexstad and Burnham, 1991). The data arose from a live-trapping experiment that was conducted for six consecutive nights with a total of 38 mice captured over these six capture occasions. In data file example1.dat, a matrix of 38 x 6 is recorded. Analyses of these data include Otis et al. (1978, p. 32), Huggins (1991) and Chao and Huggins (2003).

Using the procedure as described in the above and selecting all models in step (3), the following output is shown in the Output window after execution. The output contains three parts: (1) basic data information; (2) summary statistics; and (3) results of estimation.

Table 3. The output of deer mice data analysis.

```
(1) Basic Data Information:
        ---------------------------------------------
                    Data filename : c:\program files\CARE-2\data\example1.dat
          Total # of distinct animals : 38
          Number of capture occasions : 6
               Bootstrap replications : 1000
        ---------------------------------------------

(2) Summary Statistics:

      i   |   u[i]   m[i]   n[i]   M[i]   ft[i]   f1[i]
  --------+---------------------------------------------
      1   |    15      0     15      0      9      15
      2   |     8     12     20     15      6      11
      3   |     6     10     16     23      7      14
      4   |     3     16     19     29      6      11
      5   |     3     22     25     32      6       8
      6   |     3     22     25     35      4       9
      7   |                          38
```

ft[i]: # of individuals that were captured exactly i times on occasions 1, 2, ..., t.
f1[i]: # of individuals that were captured exactly once on occasions 1, 2, ..., i.

(3) Estimation Results:

Model	Estimate	Boot_s.e.	Asy_s.e.	Phi	CV	95% CI (log-transf.)	95% CI (percentile)
M0(CMLE)	38.5	0.36	0.72			(38.12, 39.81)	(38.13, 39.55)
M0(UMLE)	38.0	0.24	0.67			(38.00, 38.00)	(38.00, 38.83)
M0(EE)	38.0	0.36	0.68			(38.00, 38.00)	(38.00, 39.22)
Mt(CMLE)	38.4	0.31	0.66			(38.11, 39.51)	(38.08, 39.27)
Mt(UMLE)	38.0	0.14	0.62			(38.00, 38.00)	(38.00, 38.53)
Mt(EE)	38.0	0.21	0.62			(38.00, 38.00)	(38.00, 38.73)
Mb(CMLE)	42.3	7.30	3.75	1.92		(38.43, 80.28)	(38.77, 57.41)
Mb(UMLE)	40.8	6.91	3.05	1.79		(38.18, 81.43)	(38.00, 51.98)
Mb(EE)	41.9	5.29	3.58	1.89		(38.53, 66.84)	(38.00, 53.28)
Mh(SC1)	43.5	3.81	3.72		0.50	(39.64, 56.78)	(39.65, 50.94)
Mh(SC2)	42.4	3.52	3.40		0.48	(39.09, 55.48)	(38.73, 49.29)
Mh(JK1)	45.5	3.58	3.71			(41.09, 56.22)	(41.33, 49.67)
Mh(JK2)	48.3	5.78	5.68			(41.69, 66.72)	(39.73, 57.83)
Mh(IntJK)	45.5	8.35	3.71			(39.29, 81.58)	(41.33, 70.24)
Mh(EE)	40.2	2.14	----		0.50	(38.44, 48.89)	(38.00, 43.76)
Mtb(CMLE)	48.0	12.78	11.98	2.95		(39.46,106.76)	(38.78, 85.55)
Mtb(UMLE)	43.6	11.12	6.90	2.34		(38.47,104.74)	(38.07, 80.31)
Mtb(EE)	47.1	8.51	10.78	2.82		(39.91, 81.09)	(38.00, 68.25)
Mth(SC1)	43.6	3.97	3.77		0.51	(39.62, 57.57)	(39.70, 51.76)
Mth(SC2)	42.5	3.41	3.45		0.49	(39.18, 54.85)	(38.90, 48.89)
Mth(EE)	40.3	2.20	----		0.51	(38.48, 49.14)	(38.00, 44.26)
Mbh(SC)	50.5	23.43	----		0.60	(39.13,176.57)	(38.89,125.72)
Mbh(JK)	53.0	9.43	----			(42.84, 84.47)	(38.00, 73.00)
Mbh(EE)	43.5	4.44	----	1.68	0.40	(39.36, 60.04)	(38.00, 51.33)
Mtbh(EE)	44.2	4.58	----	1.89	0.36	(39.72, 60.60)	(38.10, 53.58)

The first part of the output shows basic information including the data filename, (c:\program files\CARE-2\data\example1.dat for this example), the number of distinct animals caught in the experiment (38 in this case), the number of trapping occasions (6 in this case) and the number of bootstrap replications (1000 in this case).

The summary statistics are listed in the second part of the output. We use these data to introduce some notation. The numbers of captures for the six occasions are (n_1, n_2, ..., n_6) = (15, 20, 16, 19, 25, 25). Out of the n_j animals, there are u_j first-captures and m_j recaptures, so that $u_j + m_j = n_j$, with (u_1, u_2, ..., u_6) = (15, 8, 6, 3, 3, 3) and (m_1, m_2, ..., m_6) = (0, 12, 10, 16, 22, 22). The statistic M_j denotes the number of marked animals just before the jth occasion. Thus $M_j = u_1 + u_2 + \ldots + u_{j-1}$ and (M_1, M_2, ..., M_7) = (0, 15, 23, 29, 32, 35, 38) for these data. That is, the number of marked individuals in the population progressively increased from M_1 = 0 to M_7 = 38. Here M_{t+1} denotes the total number of distinct animals caught in the experiment. The frequency counts for the six occasions are (f_{16}, f_{26}, ..., f_{66}) = (9, 6, 7, 6, 6, 4), where f_{jk} denotes the number of animals captured exactly j times on occasions 1, 2, …, k. Since singleton information is usually important, we also list (f_{11}, f_{12}, …, f_{16}) = (15, 11, 14, 11, 8, 9).

The third part shows estimation results. For these data, Otis et al. (1978, p. 32) indicated that the most suitable model for these data was model $\mathbf{M}_b$. Based on the usual unconditional MLE approach, Mb(UMLE) in Table 3, the estimated population size in model $\mathbf{M}_b$ is 41 with bootstrap s.e. of 6.9 and asymptotic s.e. of 3.1. The 95% confidence intervals are (38.2, 81.4) and (38.0, 52.0) for log-transformation and percentile methods respectively based on the bootstrap procedure. The proportion constant between the re-capture probability and first-recapture probability (ϕ in Table 1 or Phi in Table 3) is estimated to be 1.79, suggesting animals became trap-happy after their first capture.

Chao and Huggins (2003) suggested considering further general models $\mathbf{M}_{bh}$ and $\mathbf{M}_{tbh}$ by use of estimating equation (EE) approach. The two models produce close estimates, Mbh(EE) and Mtbh(EE) in Table 3. So it is reasonable to adopt the most general model $\mathbf{M}_{tbh}$ and conclude that the population size is about 44 (standard error 4.6). The data based on model $\mathbf{M}_{tbh}$ show strong trap-happy behavior (Phi = 1.89 in Table 3), a low degree of heterogeneity (the CV estimate is 0.36, where CV denotes the coefficient of variation of {p_1, p_2, …, p_N}, and slight time-varying effects as the relative time effects are estimated to be $(\bar{p}e_1, \bar{p}e_2, ..., \bar{p}e_6)$ = (0.34, 0.32, 0.26, 0.26, 0.33, 0.33), where $\bar{p}$ denotes the average of p_i's. (Time effects are not shown in the output. Refer to Chao et al. 2001 for calculation formula.)

The 95% confidence interval using a log-transformation under model $\mathbf{M}_{tbh}$ is 40 to 61. This interval is unavoidably wider than that for model $\mathbf{M}_b$ because more parameters are involved. Usually, a simpler model has smaller variance but larger bias whereas a general model has lower bias but larger variance. For interval estimation, a simpler model produces narrow confidence interval with possibly poor coverage probability whereas a more general model produces wide interval with more satisfactory coverage probability. A trade-off clearly occurs with this example.

Example 2: Mouse data (individual capture history)

The mouse data were originally collected by S. Hoffman and described and analyzed in Otis et al. (1978, p. 93). Trapping was conducted on five days and 110 distinct mice were caught. We specifically select this example because a detailed analysis is given in Chao et al. (2001).

For this data set, since Otis et al. (1978) concluded that for these data behavior is the strongest factor affecting capture probabilities, we select three models with behavioral response (models $\mathbf{M}_b$, $\mathbf{M}_{tb}$ and $\mathbf{M}_{tbh}$) in step (3) of the procedures presented earlier. The results are the following:

Table 4. The output of mouse data analysis.

(1) Basic Data Information:

```
Data filename   : c:\program files\CARE-2\data\example2.dat
Total distinct animals   : 110
Number of capture occasions : 5
Bootstrap replications   : 1000
```

(2) Summary Statistics:

i	u[i]	m[i]	n[i]	M[i]	ft[i]	f1[i]
1	37	0	37	0	34	37
2	31	23	54	37	20	45
3	9	49	58	68	28	27
4	21	44	65	77	15	38
5	12	57	69	98	13	34
6				110		

ft[i]: # of individuals that were captured exactly i times on occasions 1, 2, ..., t.
f1[i]: # of individuals that were captured exactly once on occasions 1, 2, ..., i.

(3) Estimation Results:

Model	Est.	Boot_s.e.	Asy_s.e.	Phi	CV	95%CI(log-transf.)	95%CI(percentile)
Mb(CMLE)	145.5	25.40	18.02	2.51		(120.09,235.16)	(124.23,214.34)
Mb(UMLE)	142.2	22.68	16.42	2.42		(119.28,221.72)	(122.92,206.70)
Mb(EE)	139.9	21.84	15.37	2.36		(118.32,217.71)	(120.80,195.35)
Mtb(CMLE)	173.7	46.20	55.69	3.63		(127.83,337.77)	(123.85,293.48)
Mtb(UMLE)	161.1	42.71	41.72	3.19		(122.25,322.82)	(121.45,285.52)
Mtb(EE)	152.0	28.68	32.87	2.87		(122.46,251.21)	(118.99,224.05)
Mtbh(EE)	123.2	11.75	----	1.03	0.52	(112.95,169.00)	(113.51,156.30)

As in Example 1, estimation results for the selected models follow the basic data information and summary statistics. The model selection procedure in Otis et al. (1978, pp. 92-96) shows that the most likely model is model $\mathbf{M}_{tbh}$ and model $\mathbf{M}_b$ is the next most likely model. In the following discussion, we interpret the results for these two models based on the above output.

The unconditional MLE for model $\mathbf{M}_b$, Mb(UMLE) in Table 4, yields an estimate of 142.2 with an asymptotic s.e. of 16.42 and a bootstrap s.e. of 22.68. A 95% confidence interval constructed by a log-transformation is in the range of (119, 222); the bootstrap percentile method gives an interval range of (123, 207). The ratio of recapture and first-capture probabilities, ϕ, is estimated to be 2.42 (Phi = 2.42 in the output), which shows a trap-happy situation. The conditional MLE estimate is 145.5 and the estimate based on an optimal estimating equation is 139.9. Their associated variance and confidence intervals are shown in the above output.

If model $\mathbf{M}_{tbh}$ is assumed, an estimating equation approach (Chao et al., 2001) yields an estimate of 123 with an estimated bootstrap s.e. of 11.75. A 95% confidence interval associated with this estimate under model $\mathbf{M}_{tbh}$ is (113, 169) or (114, 156) based on two methods.

Example 3: Mouse data (aggregated categorical data)

In Example 2, we used the mouse data with individual capture history. Example3.dat files the data in a format of aggregated categorical data. The user can view Example3.dat for the required format for CARE-2. All running procedures are similar to those in Examples 1 and 2 except that aggregated categorical data is selected in step (6). The output is exactly the same as that in Example 2 except for the bootstrap s.e. and confidence intervals.

Example 4: Cottontail rabbit data (individual capture history)

Edwards and Eberhardt (1967) conducted an 18 trapping-occasion capture-recapture experiment on a confined population of known size. In their study, 135 wild cottontail rabbits were penned in a 4-acre rabbit-proof enclosure. Out of 142 captures, there were 76 distinct rabbits. An advantage of this data set is the true population size is known. The basic data information and the summary statistics are shown in Table 5.

Otis et al. (1978, pp. 84-87) found that for these data there was significant time variation and heterogeneity but little behavioral response. Hence we select all models with time and/or heterogeneity (models $\mathbf{M}_t$, $\mathbf{M}_h$ and $\mathbf{M}_{th}$) along with the most general model $\mathbf{M}_{tbh}$. This data was analyzed in the literature (e.g. Burnham and Overton, 1978; Chao et al., 1992). This data set with individual capture history is filed in "example4.dat". The output for models $\mathbf{M}_t$, $\mathbf{M}_h$ and $\mathbf{M}_{th}$ is given in Table 5.

Table 5. The output of cottontail rabbit data analysis.

```
(1) Basic Data Information:
         -----------------------------------------------------------
                      Data filename   : c:\program files\CARE-2\data\example4.dat
               Total distinct animals   : 76
            Number of capture occasions : 18
               Bootstrap replications : 1000
         -----------------------------------------------------------

(2) Summary Statistics:
```

i	u[i]	m[i]	n[i]	M[i]	ft[i]	f1[i]
1	9	0	9	0	43	9
2	6	2	8	9	16	13
3	3	6	9	15	8	12
4	11	3	14	18	6	22
5	4	4	8	29	0	24
6	1	4	5	33	2	23
7	10	8	18	34	1	29
8	7	4	11	44	0	35
9	1	3	4	51	0	35
10	1	2	3	52	0	35
11	9	7	16	53	0	43
12	0	5	5	62	0	41
13	1	1	2	62	0	41
14	5	2	7	63	0	46
15	6	3	9	68	0	50
16	0	0	0	74	0	50
17	0	4	4	74	0	47
18	2	8	10	74	0	43
19				76		

```
ft[i]: # of individuals that were captured exactly i times on occasions 1, 2, ..., t.
f1[i]: # of individuals that were captured exactly once on occasions 1, 2, ..., i.

(3) Estimation Results
```

Model	Est.	Boot_s.e.	Asy_s.e.	Phi	CV	95%CI(log-transf.)	95%CI(percentile)
Mt(CMLE)	96.0	8.13	6.70			(85.27,119.04)	(86.63,112.19)
Mt(UMLE)	95.1	8.36	6.58			(84.39,119.37)	(85.98,110.34)
Mt(EE)	95.0	8.81	6.57			(83.97,121.10)	(85.46,112.73)
Mh(SC1)	137.0	21.50	21.44		0.67	(107.20,195.31)	(106.43,182.11)
Mh(SC2)	132.8	22.05	20.62		0.65	(103.26,194.39)	(103.47,181.51)
Mh(JK1)	116.6	8.54	8.89			(103.01,137.07)	(107.17,125.11)
Mh(JK2)	141.4	14.25	14.87			(118.92,175.79)	(120.76,162.13)
Mh(IntJK)	142.3	38.07	15.18			(99.27,264.74)	(107.17,252.17)
Mh(EE)	125.3	16.41	----		0.67	(102.15,169.10)	(100.39,154.64)
Mth(SC1)	138.9	24.35	22.05		0.70	(106.23,206.84)	(108.82,194.47)
Mth(SC2)	134.6	22.56	21.22		0.68	(104.29,197.46)	(105.93,183.40)
Mth(EE)	-***-	------	----		------		
Mtbh(EE)	-***-	------	----	------	------		

*** iterative steps do not converge

Edwards and Eberhardt (1967) reported that the usual estimators based on equal-catchability considerably underestimated the true number 135. It is readily seen from the output that all estimates based on model $\mathbf{M}_t$, Mt(CMLE), Mt(UMLE) and Mt(EE) in the output, are about 95 or 96. Burnham and Overton (1978) suggested modeling these data by model $\mathbf{M}_h$ and adopted an interpolated jackknife estimator. In the output, the first-order, Mh(JK1), and the second-order jackknife, Mh(JK2), are also shown; the interpolated jackknife, Mh(IntJK) yields an estimate of 142 with an asymptotic s.e. of 15.18. The confidence interval proposed by Burnham and Overton (1978) was (112, 172) based on the asymptotic s.e. This interval is different from ours in Table 5 because we use a bootstrap s.e. The asymptotic s.e. is also tabulated so that user can compute relevant intervals.

If model $\mathbf{M}_{th}$ is assumed, the coefficient of variation (CV) of the capture probabilities for all estimation methods is estimated to be about 0.70 as shown in the output. This relatively large value of the CV gives strong evidence of heterogeneity because the CV = 0 corresponds to no heterogeneity. The two estimators using the sample coverage methods, Mth(SC1) and Mth(SC2), proposed by Chao et al. (1992) and Lee and Chao (1994) are respectively 138.9 (s.e. 24.35) and 134.6 (s.e. 22.56). The latter gives a 95% confidence interval (104, 197) using a log-transformation and (106, 183) using a percentile method. The estimating equation approach does not yield an estimate due to insufficient capture and recapture information, which causes failure of convergence in the numerical iterations. If we adopt the most general model $\mathbf{M}_{tbh}$, similar difficulty arises. Therefore, capture and recapture information is not sufficient for fitting a complicated model with three sources of variations. We caution that in some cases, estimates can still be obtained in the case of insufficient information, but the standard error generally becomes so large that the model is useless.

5. Analysis With Covariates

Models/Estimators Featured

In program CARE-2, we distinguish covariates as two types: individual covariates and occasional covariates as in Huggins (1989, 1991). Individual covariates include individual’s characteristics (age, sex, body weight or wing length) and occasional covariates could be environmental variables (temperature on each occasion) or known catch-effort expended in trapping method (e.g., number of traps on each capture occasion). Occasional covariates should be stored in another file as will be shown in Example 6 below.

Suppose for each animal, there are s individual covariates. Let the individual covariates for the ith animal be denoted as $\boldsymbol{W}_i' = (W_{i1}, W_{i2}, ..., W_{is})$ and $\boldsymbol{\beta}' = (\beta_1, \beta_2, ..., \beta_s)$ denotes the effects of these covariates. It is necessary to assume that the individual covariates are constant across the t capture occasions in the experiment, as they cannot be measured on an occasion if the individual is not captured. If heterogeneity is fully explained by individuals' covariates, then the heterogeneity effect can be expressed conveniently as $\boldsymbol{\beta}' \boldsymbol{W}_i = \beta_1 W_{i1} + \beta_2 W_{i2} + ... + \beta_s W_{is}$.

Assume that there are b occasional covariates: $\{R_{11}, R_{12}, \ldots, R_{1t}\}$, $\{R_{21}, R_{22}, \ldots, R_{2t}\}$, …, $\{R_{b1}, R_{b2}, \ldots, R_{bt}\}$. For example, $\{R_{11}, R_{12}, \ldots, R_{1t}\}$ may represent the temperature on each occasion, and $\{R_{b1}, R_{b2}, \ldots, R_{bt}\}$ may represent the capture effort on each occasion. Let $\boldsymbol{r}' = (r_1, r_2, ..., r_b)$ denote the effects of the occasional covariates. Define $\boldsymbol{R}_j' = \{R_{1j}, R_{2j}, \ldots, R_{bj}\}$, then the occasional effect for the jth occasion can be expressed as $\boldsymbol{r}' \boldsymbol{R}_j = r_1 R_{1j} + r_2 R_{2j} + ... + r_b R_{bj}$.

Define $Y_{ij} = 1$ if the ith animal has been captured at least once before the jth occasion, and $Y_{ij} = 0$ otherwise. The general logistic model incorporating covariates considered in CARE-2 is

$$\text{logit}(P_{ij}) = a + c_j + v\, Y_{ij} + \boldsymbol{\beta}' \boldsymbol{W}_i + \boldsymbol{r}' \boldsymbol{R}_j\,,$$

where a denotes the baseline intercept, $\{c_1, c_2, \ldots, c_{t-1}\}$ represents the unknown occasional or time effect and $c_t \equiv 0$ is used for the reference group. These time effects may or may not be included in the model. You can specify whether these effects are needed for each data analysis. Table 6 summarizes all sub-models.

The interpretation of the coefficient of any β is based on the fact that when $\beta > 0$, the larger the covariate is, the larger the capture probability is, while if $\beta < 0$ then the larger the covariate is, the smaller the capture probability is. Similar interpretation pertains to the coefficient of any r for occasional covariate. The parameter v represents the effect of a recapture, which implies that $v > 0$ corresponds to a case of trap-happy and $v < 0$ corresponds to a case of trap-shy.

The parameters in the logistic models are estimated by a conditional ML method based on the captured individuals (Huggins, 1989, 1991). The default of maximum number of iterations in CARE-2 is 500. Model selection can be performed using Akaike information criterion (AIC) which is defined as -2log$L$+2m, where L denotes the likelihood computed at the conditional MLE and m denotes the number of parameters in the model. A model is selected if AIC is the smallest among all models considered. The population size is estimated by the Horvitz-Thompson estimator, which is $\hat{N}_{HT} = \sum_{i=1}^{M_{t+1}} \{1 - \prod_{j=1}^{t} (1 - \hat{P}_{ij})\}^{-1}$, where $\hat{P}_{ij}$ is the estimated capture probability evaluated at the conditional MLE. The variance of the resulting estimator can be estimated by an asymptotic variance formula derived in Huggins (1989, 1991). Below two examples are used for CARE-2 to illustrate the estimation and model selection.

Table 6. Models with covariates in CARE-2. (The effect c_j is optional.)

Model	Assumption	Restriction in model $\mathbf{M}^*_{\mathrm{tbh}}$
$\mathbf{M}^*_{\mathrm{tbh}}$	$\mathrm{logit}(P_{ij}) = a + c_j + v\,Y_{ij} + \boldsymbol{\beta}'\boldsymbol{W}_i + \boldsymbol{r}'\boldsymbol{R}_j$	
$\mathbf{M}^*_{\mathrm{bh}}$	$\mathrm{logit}(P_{ij}) = a + v\,Y_{ij} + \boldsymbol{\beta}'\boldsymbol{W}_i$	set $c_j = 0$, $\boldsymbol{r} = \mathbf{0}$
$\mathbf{M}^*_{\mathrm{tb}}$	$\mathrm{logit}(P_{ij}) = a + c_j + v\,Y_{ij} + \boldsymbol{r}'\boldsymbol{R}_j$	set $\boldsymbol{\beta} = \mathbf{0}$
$\mathbf{M}^*_{\mathrm{th}}$	$\mathrm{logit}(P_{ij}) = a + c_j + \boldsymbol{\beta}'\boldsymbol{W}_i + \boldsymbol{r}'\boldsymbol{R}_j$	set $v = 0$
$\mathbf{M}^*_{\mathrm{h}}$	$\mathrm{logit}(P_{ij}) = a + \boldsymbol{\beta}'\boldsymbol{W}_i$	set $c_j = 0$, $\boldsymbol{r} = \mathbf{0}$, $v = 0$
$\mathbf{M}^*_{\mathrm{b}}$	$\mathrm{logit}(P_{ij}) = a + v\,Y_{ij}$	set $\boldsymbol{\beta} = \mathbf{0}$, $c_j = 0$, $\boldsymbol{r} = \mathbf{0}$
$\mathbf{M}^*_{\mathrm{t}}$	$\mathrm{logit}(P_{ij}) = a + c_j + \boldsymbol{r}'\boldsymbol{R}_j$	set $\boldsymbol{\beta} = \mathbf{0}$, $v = 0$
$\mathbf{M}^*_{0}$	$\mathrm{logit}(P_{ij}) = a$	set $\boldsymbol{\beta} = \mathbf{0}$, $c_j = 0$, $\boldsymbol{r} = \mathbf{0}$, $v = 0$

Running Procedures by Examples

In the following, we provide two examples to demonstrate the procedure of CARE-2 for covariate analysis. They are:

Example 5: Same capture data as in Example 1, but three individual covariates are included (data in file: example5.dat). Refer to Huggins (1991) and Chao and Huggins (2003) for detailed analysis.

Example 6: Rodent data with two individual covariates and one occasional covariate (capture data and individual covariates are in file: exampl61.dat; occasional data are in file: exampl62.dat). Refer to Huggins (1989) for detailed analysis.

Example 5: Deer mice data (with three individual covariates)

For the data set discussed in Example 1, there were actually three covariates: gender (male or female), age (young, semi-adult or adult) and weight, collected for each individual in the deer mouse data. Only three semi-adult mice were caught, so they were re-classified as adults. The user can view example5.dat for the complete data. Part of the complete data is shown in Table 7.

Table 7. Individual capture history of deer mice with three covariates: Gender (0: male, 1: female); Age (y: young, a: adult); and Weight (in grams).

Occasion 1	Occasion 2	Occasion 3	Occasion 4	Occasion 5	Occasion 6	Gender	Age	Weight
1	1	1	1	1	1	0	y	12
1	0	0	1	1	1	1	y	15
1	1	0	0	1	1	0	y	15
•				•				•
•				•				•
•				•				•
0	0	0	0	0	1	0	a	16
0	0	0	0	0	1	1	a	19

There are three individual covariates and there is no occasional covariate. Since every covariate can be treated as either categorical or continuous, the user has to specify the numbers of each. For example, there are two categorical (gender and age) and one continuous (weight) for individual covariates of this data. In the data format, the order of data entry should be: capture history, categorical covariates followed by the continuous covariates. Occasional covariates are stored in a separate file with the same order of categorical variables first and then continuous variables.

We describe the procedures for analyzing deer mice data with covariates. The following procedure must be executed in a GAUSS environment.

(1) Provoke GAUSS environment either by doubly clicking GSRUN50 on your desktop as described in Download and Setup or by clicking the executable file GSRUN.exe stored in the directory GSRUN50.

(2) Click “File” on the top menu of GAUSS and subsequently click “Run Program” and select the program CARE-2.gcg which is stored in a pre-specified working directory (The default is c:\program files\CARE-2\). It prompts you subsequently the following input steps:

(3) “Please input the number of distinct individuals:” In this example, we input **38**.

(4) “Please input the number of sampling occasions:” Input **6**.

(5) “Please input the number of categorical individual covariates:” Input **2**.

(6) “Please input the number of continuous individual covariates:” Input **1**.

(7) “Please input the filename containing the capture history and individual covariates (continuous type covariates must follow by the categorical type covariates):” Input **c:\program files\CARE-2\data\example5.dat**.

(8) “Please input the number of categorical occasional covariates:” Input **0**.

(9) “Please input the number of continuous occasional covariates:” Input **0**.

(10) “Do you want to include the unknown time effects (y or n)?” (This means that whether the effects $\{c_1, c_2, \ldots, c_{t-1}\}$ are needed in the logistic model). We input **y**.

(11) “Please input the filename to save the output:” Input for example **c:\program files\CARE-2\output.out**. Please wait a moment and the results will be shown in the GAUSS window. Moreover, the output is also saved in c:\program files\CARE-2\output.out. The standard output for CARE-2 with this example with the above input is shown in Table 8.

Remark: If you have abundant data, it may take a long time to get your output due to complicated iterative estimation in GAUSS program operating on a large array or high-dimensional matrix.

Table 8. The output of covariate analysis for deer mice data.

```
############################################################
### CARE-2 for capture-recapture analysis with covariates ###
### Authors: Anne Chao and Hsin-Chou Yang              ###
### Version: 1.5 (April 2006)                          ###
############################################################

 ==========================
 === Summary Statistics ===
 ==========================
------------------------------------------------------
   Total number of distinct animals :      38
          Number of capture samples :       6
------------------------------------------------------

    i   |     u[i]   m[i]   n[i]   M[i]   ft[i]   f1[i]
--------|--------------------------------------------
    1   |      15      0     15      0      9     15
    2   |       8     12     20     15      6     11
    3   |       6     10     16     23      7     14
    4   |       3     16     19     29      6     11
    5   |       3     22     25     32      6      8
    6   |       3     22     25     35      4      9
    7   |                            38
--------|--------------------------------------------

 ==========================================
 === The Fit & Estimation of all models ===
 ==========================================

 Model        Estimate    S.E.    MIN(-LL)    AIC         95% CI       Status
-----------------------------------------------------------------------------------
 M*0           38.47      0.72     157.27    316.54    (38.06,  42.04)  Converge
 M*t           38.40      0.66     152.42    316.84    (38.04,  41.80)  Converge
 M*b           42.25      3.76     150.43    304.87    (38.96,  56.86)  Converge
 M*h           39.85      1.72     144.87    297.75    (38.39,  46.67)  Converge
 M*tb          46.48     12.65     148.18    310.36    (39.02, 108.74)  Converge
 M*th          39.66      1.61     139.55    297.10    (38.34,  46.20)  Converge
 M*bh          47.15      7.17     139.54    289.09    (40.35,  73.52)  Converge
 M*tbh         47.13     10.08     137.33    294.66    (39.59,  90.50)  Converge
------------------------------------------------------------------------------------

=========================
=== Model Description ===
=========================
The general logistic model M*tbh is

      logit(P_ij)=a + c_j + v * Y_ij + beta * W_i + r * R_j

where

i        : refers to the ith individual;
j        : refers to the jth sample or jth capture occasion;
a        : baseline intercept;
c_j      : the unknown time or occasional effect of the jth capture occasion
           (set c_t=0, where t: the number of capture occasions;
v        : (behavioral response) the effect w.r.t. the past capture history indicator Y_ij;
beta     : the effect of individual covariates W_i;
r        : the effect of occasional covariate R_j;

 ===========================================
 === The MLEs of Regression Coefficients ===
 ===========================================
*** Model M*0 ***
         a
```

```
MLE     0.08
S.E.    0.13

*** Model M*t ***
          a      c_1      c_2      c_3      c_4      c_5
MLE     0.62    -1.07    -0.54    -0.96    -0.64     0.00
S.E.    0.24     0.42     0.42     0.42     0.42     0.17

*** Model M*b ***
         a        v
MLE    -0.76     1.22
S.E.    0.34     0.38

*** Model M*h ***
          a   beta1(1)  beta2(1)  beta3
MLE    -1.95     0.81    -1.90     0.16
S.E.    0.71     0.31     0.57     0.06

*** Model M*tb ***
         a        v       c_1      c_2      c_3      c_4      c_5
MLE    -1.16     1.72     0.42     0.31    -0.45    -0.37     0.12
S.E.    1.09     0.98     0.80     0.57     0.49     0.45     0.42

*** Model M*th ***
          a    beta1(1)  beta2(1)  beta3    c_1      c_2      c_3      c_4     c_ 5
MLE    -1.43     0.84    -1.98     0.16    -1.18    -0.59    -1.06    -0.70    0.00
S.E.    0.74     0.32     0.58     0.06     0.44     0.43     0.44     0.43    0.19

*** Model M*bh ***
          a        v    beta1(1)  beta2(1)  beta3
MLE    -2.91     1.18     0.92    -1.88     0.16
S.E.    0.87     0.40     0.35     0.63     0.06

*** Model M*tbh ***
          a        v    beta1(1)  beta2(1)  beta3    c_1      c_2      c_3     c_4      c_5
MLE    -2.76     1.21     0.94    -1.92     0.16    -0.11     0.02    -0.71   -0.50     0.08
S.E.    1.30     0.74     0.36     0.64     0.06     0.87     0.80     0.60    0.56     0.57
```

The first part of the output shows all summary statistics. The second part shows the fitting and estimation results for the logistic model and all sub-models, followed by model description. For each model, the corresponding estimated population size (number under the heading Estimate in Table 8), its s.e. (under the heading S.E.), negative value of the minimum log-likelihood under the heading MIN(-LL), the Akaike information criterion (AIC) and 95% confidence interval (Chao, 1987) are calculated. From the values of AIC, we select model $\mathbf{M}^*_{bh}$ because AIC of this model is the smallest among all models. There are slight differences between our estimates and those in Huggins (1991) because different numerical algorithms are used.

The last part of the output shows all fitted parameter estimates. Under model $\mathbf{M}^*_{bh}$, the fitted intercept is -2.91, the behavioral response effect is 1.18 for re-capture (the first capture effect is set to be 0, so recaptures have higher probabilities). Then there are several coefficients corresponding to the three individual's covariates according to the order of data entry. Generally, one coefficient is associated with a continuous covariate. For a categorical covariate, there are k-1 coefficients associated with a covariate with k categories. When groups are in a numerical order or in an alphabetical order according to the data entry. The category with the largest numerical value or the last alphabetical order is always set to be 0 as the reference group. Suppose there are k categories for the first covariate, then in the output we have k-1 coefficients: beta1(1), beta1(2), …, beta1(k-1), where beta$n$(j) denotes the effect of the jth group relative to the reference group for the nth covariate.

From Table 7, male is coded as 0 and female is coded as 1 in data entry, thus group "1" (the larger numerical value) is set to be the reference group. Therefore, in Table 8, the coefficient, beta1(1) = 0.92, is the effect for male; the female is set to be 0, so males have larger probabilities. Also, young is coded as "y" and adult is coded as "a" in data entry, thus in an alphabetical order the group "y" is used for reference group. The second coefficient, beta2(1) = -1.88, is the effect for adult; the young effect is set to be 0, so young have larger capture probabilities. The last coefficient in the output, beta3 = 0.16 is the effect for a unit change of body weight. This implies the heavier the weight, the larger the capture probability. Then from the summary of model fitting the estimated population size under the selected model $\mathbf{M}^{*}_{bh}$ is 47.2 (s.e. 7.17) with a 95% confidence interval of (40.4, 73.5).

Example 6: Rodents data (two individual covariates and one occasional covariate).

The data of salt marsh rodents were originally collected by Coulombe and analyzed by Otis et al. (1978, pp. 62-67) and Huggins (1989). The experiment was carried out in the morning and night daily for five days. Two individual covariates are recorded: gender (male and female) and age (young, semi-adult and adult). The summary statistics for capture history are shown in Table 9 below. Otis et al. (1978) concluded there is no behavior response effect but time variations and individual heterogeneity are strong. No suitable estimators were available at the time, and thus they suggested the use of the number of the distinct animals caught in the experiment.

There are two types of covariates, individual covariates and occasional covariates in this example. The individual capture history and individual covariates (gender and age) are stored in c:\program files\CARE-2\data\exampl61.dat. The experiment time (morning or night) is treated as an occasional covariate. The data format for filing an occasional covariate is shown in c:\program files\CARE-2\data\exampl62.dat, where “1” denotes for morning and “2” denotes night.

There are two rodents with missing covariates, hence we exclude these two records in the following analysis. It leads to somewhat different results from those in Huggins (1989). The running steps (1) to (3) are similar to those for Example 5, so we begin with step (4).

(4) “Please input the number of distinct individuals:”. In this example, we input **171**.

(5) “Please input the number of sampling occasions:”. Input **10**.

(6) “Please input the number of categorical individual covariates:”. Input **2**.

(7) “Please input the number of continuous individual covariates:”. Input **0**.

(8) “Please input the filename containing the capture history and individual covariates (continuous type covariates must follow by the categorical type covariates):” Input **c:\program files\CARE-2\data\exampl61.dat**.

(9) “Please input the number of categorical occasional covariates:”. Input **1**.

(10) “Please input the number of continuous occasional covariates:”. Input **0**.

(11) “Please input the filename containing the occasional covariates (continuous type covariates must follow by the categorical type covariates):”. Input **c:\program files\CARE-2\data\exampl62.dat**.

(11) “Do you want to include the unknown time effects (y or n)?”. We input **n**.

(12) “Please input the filename to save the output:”. Input for example **c:\program files\CARE-2\output.out**. Please wait a moment and the results will be shown in the GAUSS window. Moreover, the output is also saved in c:\program files\CARE-2\output.out. The standard output is shown in Table 9.

Table 9. The output of covariate analysis for rodent data.

```
############################################################
### CARE-2 for capture-recapture analysis with covariates ###
### Authors: Anne Chao and Hsin-Chou Yang                ###
### Version: 1.5 (April 2006)                            ###
############################################################
```

=========================
=== Summary Statistics ===
=========================

Total number of distinct animals : 171
Number of capture samples : 10

i	u[i]	m[i]	n[i]	M[i]	ft[i]	f1[i]
1	68	0	68	0	2	68
2	33	27	60	68	62	74
3	26	36	62	101	40	74
4	12	40	52	127	31	65
5	15	58	73	139	16	54
6	3	38	41	154	13	45
7	12	64	76	157	5	41
8	0	35	35	169	1	26
9	2	74	76	169	0	9
10	0	38	38	171	1	2
11				171		

===
=== The Fit & Estimation of all models ===
===

Model	Estimate	S.E.	MIN(-LL)	AIC	95% CI	Status
M*0	173.99	1.83	1093.07	2188.14	(171.99, 180.02)	Converge
M*t	173.79	1.76	1071.43	2146.86	(171.90, 179.68)	Converge
M*b	172.99	1.60	1092.39	2188.78	(171.50, 178.96)	Converge
M*h	175.38	2.33	1080.36	2168.72	(172.65, 182.64)	Converge
M*tb	173.74	1.74	1071.43	2148.86	(171.87, 179.57)	Converge
M*th	175.14	2.26	1058.44	2126.89	(172.52, 182.26)	Converge
M*bh	173.86	2.05	1079.44	2168.87	(171.81, 181.09)	Converge
M*tbh	174.86	2.21	1058.42	2128.84	(172.36, 181.95)	Converge

=========================
=== Model Description ===
=========================
The general logistic model M*tbh is

logit(P_ij)=a + c_j + v * Y_ij + beta * W_i + r * R_j

where

i : refers to the ith individual;
j : refers to the jth sample or jth capture occasion;
a : baseline intercept;
c_j : the unknown time or occasional effects of the jth capture occasion
(set c_t=0, where t: the number of capture occasions;
v : (behavioral response) the effect w.r.t. the past capture history indicator Y_ij;
beta : the effect of individual covariates W_i;
r : the effect of occasional covariate R_j;

==
=== The MLEs of Regression Coefficients ===
==
*** Model M*0 ***

	a
MLE	-0.69
S.E.	0.05

```
*** Model M*t ***
          a      r1(1)
MLE     0.31    -0.68
S.E.    0.16     0.10

*** Model M*b ***
          a        v
MLE    -0.58    -0.15
S.E.    0.10     0.11

*** Model M*h ***
          a    beta1(1)  beta2(1)  beta2(2)
MLE    -0.38    -0.28     -0.02     -0.46
S.E.    0.08     0.11      0.13      0.11

*** Model M*tb ***
          a        v       r1(1)
MLE     0.31    -0.01     -0.67
S.E.    0.16     0.00      0.10

*** Model M*th ***
          a    beta1(1)  beta2(1)  beta2(2)  r1(1)
MLE     0.63    -0.28     -0.02     -0.47    -0.68
S.E.    0.17     0.11      0.14      0.12     0.11

*** Model M*bh ***
         a        v    beta1(1)  beta2(1)  beta2(2)
MLE    -0.24    -0.18     -0.28     -0.02     -0.46
S.E.    0.12     0.13      0.11      0.14      0.11

*** Model M*tbh ***
          a         v    beta1(1)  beta2(1)  beta2(2)  r1(1)
MLE     0.65     -0.03     -0.28     -0.02     -0.47    -0.68
S.E.    0.16      0.05      0.11      0.15      0.12     0.11
```

From the results of AIC listed in Table 9, model M_{th} is selected. The conclusion is consistent with that in Otis et al. (1978, pp. 62-64). For gender (data entry is 1 for male and 2 for female), the female is served as the reference group. The negative regression coefficient beta1(1) = -0.28 demonstrates that the females have larger capture probabilities than the males. For age (data entry is 1 for young, 2 for semi-adult and 3 for adult), thus the adult group with the largest numerical value is regarded as a reference group. The regression coefficient beta2(1) = -0.02 is not significant, hence there is no significantly difference of capture probabilities between the young and adult. However, the regression coefficient beta2(2) = -0.47 is significantly different from 0, which implies that adults have higher capture probabilities than the semi-adult. For the occasional covariate (data entry is 1 for morning and 2 for night), the coefficient r1(1) = -0.68 denotes the effect of morning time. Thus the capture probabilities are higher in the night. The population size estimate under model M_{th} is 175.1 with an estimated s.e. of 2.3 and a 95% confidence interval of (172.5, 182.3). These results here are slightly different from those obtained in Huggins (1989) due to the different ways of treating missing covariates.

Reference

Burnham, K.P. and Overton, W.S. (1978). Estimation of the size of a closed population when capture probabilities vary among animals. *Biometrika* **65**, 625-33.

Chao, A. (1987). Estimating the population size for capture-recapture data with unequal catchability. *Biometrics* **43**, 783-91.

Chao, A., Chu, W. and Hsu, C.-H. (2000). Capture-recapture when time and behavioral response affect capture probabilities. *Biometrics* **56**, 427-33.

Chao, A. and Huggins, R.M. (2003). Closed population models. To appear as a chapter in *The Handbook of Capture-Recapture Methods*. Edited by Manly, B., McDonald, T. and Amstrup, S. Princeton University Press.

Chao, A., Lee, S.-M. and Jeng, S.-L. (1992). Estimating population size for capture-recapture data when capture probabilities vary by time and individual animal. *Biometrics* **48**, 201-16.

Chao, A., Yip, P.S.F., Lee, S.-M. and Chu, W. (2001). Population size estimation based on estimating functions for closed capture-recapture models. *Journal of Statistical Planning and Inference* **92**, 213-32.

Darroch, J.N. (1958). The multiple-recapture census I. Estimation of a closed population. *Biometrika* **45**, 343-59.

Edwards, W.R. and Eberhardt, L.L. (1967). Estimating cottontail abundance from live-trapping data. *Journal of Wildlife Management* **31**, 87-96.

Efron, B. and Tibshirani, R.J. (1993). *An Introduction to the Bootstrap*. Chapman and Hall: New York.

Huggins, R.M. (1989). On the statistical analysis of capture experiments. *Biometrika* **76**, 133-40.

Huggins, R.M. (1991). Some practical aspects of a conditional likelihood approach to capture experiments. *Biometrics* **47**, 725-32.

Lee, S.-M. and Chao, A. (1994). Estimating population size via sample coverage for closed capture-recapture models. *Biometrics* **50**, 88-97.

Lloyd, C.J. (1994). Efficiency of martingale methods in recapture studies. *Biometrika* **81**, 305-15.

Otis, D.L., Burnham, K.P., White, G.C. and Anderson, D.R. (1978). Statistical inference from capture data on closed animal populations. *Wildlife Monographs* **62**, 1-135.

Pollock, K.H. and Otto, M.C. (1983). Robust estimation of population size in closed animal populations from capture-recapture experiments. *Biometrics* **39**, 1035-49.

Rexstad, E. and Burnham, K.P. (1991). *User's Guide for Interactive Program CAPTURE*. Colorado Cooperative Fish and Wildlife Research Unit, Fort Collins.

White, G.C., Anderson, D.R., Burnham, K.P. and Otis, D.L. (1982). *Capture-Recapture and Removal Methods for Sampling Closed Populations.* Los Alamos National Lab, LA-8787-NERP, Los Alamos, New Mexico, USA.

Yip, P.S.F. (1991). A martingale estimating equation for a capture-recapture experiment in discrete time. *Biometrics* **47**, 1081-88.

Zippin, C. (1956). An evaluation of the removal method of estimating animal populations. *Biometrics* **12**, 163-89.

Appendix

In this Appendix, we give formulas for the estimators featured in CARE-2 under various models. Refer to Tables 1 and 2 for definitions and references.

1. Model M_0 (Otis et al., 1978; Darroch, 1958; Yip, 1991):

- Unconditional MLE: M0(UMLE) Back to Table2

  Equation for N: $\dfrac{\partial \log L}{\partial N} = \sum_{j=1}^{M_{t+1}} (N - j + 1)^{-1} + t \log(1-p) = 0$,

  Equation for p: $\dfrac{\partial \log L}{\partial p} = \dfrac{n_{\bullet}}{p} - \dfrac{Nt - n_{\bullet}}{1-p} = 0$, where $n_{\bullet} = \sum_{j=1}^{t} n_j$.

- Conditional MLE: M0(CMLE) Back to Table2

  Equation for N: $1 - \dfrac{M_{t+1}}{N} = (1-p)^t$,

  Equation for p: $\dfrac{\partial \log L}{\partial p} = \dfrac{n_{\bullet}}{p} - \dfrac{Nt - n_{\bullet}}{1-p} = 0$, where $n_{\bullet} = \sum_{j=1}^{t} n_j$.

- Estimating Equation: M0(EE) Back to Table2

  Equation for N: $\sum_{j=1}^{t} [(N - M_j)(1-p)]^{-1} [u_j - (N - M_j)p] = 0$,

  Equation for p: $n_{\bullet} - Np = 0$, where $n_{\bullet} = \sum_{j=1}^{t} n_j$.

2. Model M_t (Otis et al., 1978; Darroch, 1958; Yip, 1991):

- Unconditional MLE: Mt(UMLE) Back to Table2

  Equation for N: $\dfrac{\partial \log L}{\partial N} = \sum_{j=1}^{M_{t+1}} (N - j + 1)^{-1} + \sum_{j=1}^{t} \log(1 - e_j) = 0$,

  Equation for e_j: $\dfrac{\partial \log L}{\partial e_j} = \dfrac{n_j}{e_j} - \dfrac{N - n_j}{1 - e_j} = 0$, $j = 1, 2, \ldots, t$.

- Conditional MLE: Mt(CMLE) Back to Table2

  Equation for N: $1 - \dfrac{M_{t+1}}{N} = \prod_{j=1}^{t} (1 - e_j)$,

  Equation for e_j: $e_j = \dfrac{n_j}{N}, j = 1, 2, \ldots, t$.

- Estimating Equation: Mt(EE) Back to Table2

Equation for N: $\sum_{j=1}^{t}[(N-M_j)(1-e_j)]^{-1}[u_j-(N-M_j)e_j]=0$,

Equation for e_j: $n_j-Ne_j=0,\quad j=1,2,\ldots,t.$

3. Model M_b (Otis et al., 1978; Zippin, 1956; Lloyd, 1994):

- Unconditional MLE: Mb(UMLE) Back to Table2

Equation for N: $\dfrac{\partial \log L}{\partial N}=\sum_{j=1}^{M_{t+1}}(N-j+1)^{-1}+\sum_{j=1}^{t}\log(1-p)=0$,

Equation for ϕ: $\dfrac{\partial \log L}{\partial \phi}=\dfrac{m_\bullet}{\phi}-\dfrac{(M_\bullet-m_\bullet)\,p}{1-\phi p}=0$,

Equation for p: $\dfrac{\partial \log L}{\partial p}=\dfrac{n_\bullet}{p}-\dfrac{Nt-M_{j+1}-M_\bullet}{1-p}-\dfrac{(M_\bullet-m_\bullet)\,\phi}{1-\phi p}=0$, where

$n_\bullet=\sum_{j=1}^{t}n_j, m_\bullet=\sum_{j=1}^{t}m_j$ and $M_\bullet=\sum_{j=1}^{t}M_j$.

- Conditional MLE: Mb(CMLE) Back to Table2

Equation for N: $N=\dfrac{M_{t+1}}{1-(1-p)^t}$,

Equation for ϕ: $\dfrac{\partial \log L}{\partial \phi}=\dfrac{m_\bullet}{\phi}-\dfrac{(M_\bullet-m_\bullet)\,p}{1-\phi p}=0$,

Equation for p: $\dfrac{\partial \log L}{\partial p}=\dfrac{n_\bullet}{p}-\dfrac{Nt-M_{j+1}-M_\bullet}{1-p}-\dfrac{(M_\bullet-m_\bullet)\,\phi}{1-\phi p}=0$, where

$n_\bullet=\sum_{j=1}^{t}n_j, m_\bullet=\sum_{j=1}^{t}m_j$ and $M_\bullet=\sum_{j=1}^{t}M_j$.

- Estimating Equation: Mb(EE) Back to Table2

Equation for N: $\sum_{j=1}^{t}[(N-M_j)(1-p)]^{-1}[u_j-(N-M_j)p]=0$,

Equation for ϕ: $\sum_{j=1}^{t}[\phi\, p(1-\phi\, p)]^{-1}[m_j-M_j\phi\, p]=0$,

Equation for p: $\sum_{j=1}^{t}[p(1-p)]^{-1}[u_j-(N-M_j)p]=0$.

4. Model M_{tb} (Chao et al., 2000; Lloyd, 1994):

- Unconditional MLE: Mtb(UMLE) Back to Table2

Equation for N: $$\frac{\partial \log L}{\partial N} = \sum_{j=1}^{M_{t+1}} (N - j + 1)^{-1} + \sum_{j=1}^{t} \log(1 - e_j) = 0\,,$$

Equation for ϕ: $$\frac{\partial \log L}{\partial \phi} = \frac{m_\bullet}{\phi} - \sum_{j=2}^{t} \frac{(M_j - m_j)\, e_j}{1 - \phi e_j} = 0, \quad \text{where} \quad m_\bullet = \sum_{j=1}^{t} m_j\,.$$

Equation for e_j: $$\frac{\partial \log L}{\partial e_j} = \frac{n_j}{e_j} - \frac{N - M_{j+1}}{1 - e_j} - \frac{(M_j - m_j)\,\phi}{1 - \phi e_j} = 0, \quad j = 1, 2, \ldots, t\,.$$

- Conditional MLE: Mtb(CMLE) Back to Table2

Equation for N: $$1 - \frac{M_{t+1}}{N} = \prod_{j=1}^{t} (1 - e_j)\,,$$

Equation for ϕ: $$\frac{\partial \log L}{\partial \phi} = \frac{m_\bullet}{\phi} - \sum_{j=2}^{t} \frac{(M_j - m_j)\, e_j}{1 - \phi e_j} = 0, \quad \text{where} \quad m_\bullet = \sum_{j=1}^{t} m_j\,.$$

Equation for e_j: $$\frac{\partial \log L}{\partial e_j} = \frac{n_j}{e_j} - \frac{N - M_{j+1}}{1 - e_j} - \frac{(M_j - m_j)\,\phi}{1 - \phi e_j} = 0, \quad j = 1, 2, \ldots, t\,.$$

- Estimating Equation: Mtb (EE) Back to Table2

Equation for N: $$\sum_{j=1}^{t} \frac{u_j - (N - M_j) e_j}{(N - M_j)(1 - e_j)} = 0\,,$$

Equation for ϕ: $$\sum_{j=1}^{t} \frac{m_j - M_j \phi e_j}{(1 - \phi e_j)} = 0\,,$$

Equation for e_j: $$\frac{u_j - (N - M_j) e_j}{(1 - e_j)} + \frac{m_j - M_j \phi\, e_j}{(1 - \phi\, e_j)} = 0, \quad j = 1, 2, \ldots, t.$$

5. Model M_h (Burnham and Overton, 1978; Lee and Chao, 1994; Chao et al., 2001):

- The First-order Jackknife: Mh (JK1) Back to Table2

$$\hat{N}_{J1} = M_{t+1} + (\frac{t-1}{t})f_{1t}.$$

- The Second-order Jackknife: Mh(JK2) Back to Table2

$$\hat{N}_{J2} = M_{t+1} + (\frac{2t-3}{t})f_{1t} - \frac{(t-2)^2}{t(t-1)}f_{2t}.$$

- Interpolated Jackknife: Mh(IntJK) Back to Table2

$$\hat{N}_J = \hat{N}_{J1},\ g=1\ ,$$
$$\hat{N}_J = c\hat{N}_{J,g} + (1-c)\hat{N}_{J,g-1},\ 1<g<5,$$
$$\hat{N}_J = \hat{N}_{J5},\ g \ge 5\ ,$$

$\hat{N}_{Jl} = \sum_{j=1}^{t} a_{lj} f_{jt}$, $c = \frac{(0.05-P_{g-1})}{(P_g - P_{g-1})}$, $g = \min\{l : P_l > \alpha\}$, P_l is the P-value and α is the significant level. Coefficients a_{lj} can be referred to Burnham & Overton (1978).

- Sample Coverage1: Mh(SC1) Back to Table2

$$\hat{N}_{sc1} = \frac{M_{t+1}}{\hat{C}_1} + \frac{f_{1t}}{\hat{C}_1}\hat{\gamma}_1^2,\quad \text{where}\quad \hat{C}_1 = 1 - f_{1t} \Big/ \sum_{j=1}^{t} jf_{jt}\ ,$$

$$\hat{\gamma}_1^2 = \max\left\{\frac{\hat{N}_{0,1}t\sum_{j=2}^{t} j(j-1)f_{jt}}{(t-1)(\sum_{j=1}^{t} jf_{jt})^2} - 1, 0\right\} \quad \text{and} \quad \hat{N}_{0,1} = M_{t+1} \big/ \hat{C}_1 .$$

- Sample Coverage2: Mh(SC2) Back to Table2

$$\hat{N}_{SC2} = \frac{M_{t+1}}{\hat{C}_2} + \frac{f_{1t}}{\hat{C}_2}\hat{\gamma}_2^2,\quad \text{where}\quad \hat{C}_2 = 1 - [f_{1t} - 2f_{2t}/(t-1)] \Big/ \sum_{j=1}^{t} jf_{jt}\ ,$$

$$\hat{\gamma}_2^2 = \max\left\{\frac{\hat{N}_{0,2}t\sum_{j=2}^{t} j(j-1)f_{jt}}{(t-1)(\sum_{j=1}^{t} jf_{jt})^2} - 1, 0\right\} \quad \text{and} \quad \hat{N}_{0,2} = M_{t+1} \big/ \hat{C}_2 .$$

- Estimating Equation: Mh(EE) Back to Table2

Equation for N: $\displaystyle\sum_{j=1}^{t} \frac{u_j - (N - \hat{M}_j^*)\overline{p}}{(1-\hat{C}_{j-1})} = 0$,

Equation for $\overline{p}$: $\overline{p} = \sum_{j=1}^{t} n_j / (tN)$.

$\hat{C}_{j-1} = 1 - f_{1j} / \sum_{k=1}^{j} n_k$, $\hat{M}_j^* = M_j + f_{1,j-1}\,\hat{\gamma}_h^2$,

$$\hat{\gamma}_h^2 = \max\left\{\frac{\hat{N}_0\ t\sum_{j=1}^{t} j(j-1)f_{jt}}{(t-1)\ (\sum_{j=1}^{t} jf_{jt})^2} - 1,\quad 0\right\},\ \text{where}\quad \hat{N}_0 = M_{t+1} / [1 - f_{1t} / (\sum_{j=1}^{t} jf_{jt})].$$

6. Model M_{th} (Lee and Chao, 1994; Chao et al., 2001):

- Sample COverage1: Mth(SC1) Back to Table2

$$\hat{N}_{sc1} = \frac{M_{t+1}}{\hat{C}_1} + \frac{f_{1t}}{\hat{C}_1}\hat{\gamma}_1^2 \text{ , where}$$

$$\hat{C}_1 = 1 - f_{1t} \Big/ \sum_{j=1}^{t} j f_{jt} \text{ ,}$$

$$\hat{\gamma}_1^2 = \max\left\{ \frac{\hat{N}_{0,1}\sum_{j=2}^{t} j(j-1) f_{jt}}{\sum\sum_{j<k} n_j n_k} - 1, 0 \right\},$$

$$\hat{N}_{0,1} = M_{t+1} \Big/ \hat{C}_1 \text{ .}$$

- Sample Coverage2: Mth(SC2) Back to Table2

$$\hat{N}_{SC2} = \frac{M_{t+1}}{\hat{C}_2} + \frac{f_{1t}}{\hat{C}_2}\hat{\gamma}_2^2 \text{ , where}$$

$$\hat{C}_2 = 1 - [f_{1t} - 2f_{2t}/(t-1)] \Big/ \sum_{j=1}^{t} j f_{jt} \text{ ,}$$

$$\hat{\gamma}_2^2 = \max\left\{ \frac{\hat{N}_{0,2}\sum_{j=2}^{t} j(j-1) f_{jt}}{2\sum\sum_{j<k} n_j n_k} - 1, 0 \right\} \text{ ,}$$

$$\hat{N}_{0,2} = M_{t+1} \Big/ \hat{C}_2 \text{ .}$$

- Estimating Equation: Mth(EE) Back to Table2

Equation for N: $\displaystyle\sum_{j=1}^{t} \frac{u_j - (N - \hat{M}_j^*)\alpha_j}{(1-\hat{C}_{j-1})[1-(1+\hat{\gamma}_{th}^2)\alpha_j]} = 0$,

Equation for α_j : $\alpha_j = n_j / N, j = 1, 2, \ldots, t,$

where $\alpha_j = \bar{p} e_j, j = 1, 2, \ldots, t,$

$$\hat{C}_{j-1} = 1 - u_j / n_j \text{ , } \hat{M}_j^* = M_j + f_{1,j-1}\hat{\gamma}_{th}^2 \text{ , } \hat{\gamma}_{th}^2 = \max\left\{ \frac{\hat{N}_0 \sum_{j=1}^{t} j(j-1) f_{jt}}{(2\sum_{j<k} n_j n_k)} - 1, \quad 0 \right\},$$

and $\hat{N}_0 = M_{t+1} / [1 - f_{1t} / (\sum_{j=1}^{t} j f_{jt})]$.

7. Model M_{bh} (Pollock and Otto, 1983; Lee and Chao, 1994; Chao et al., 2001):

- Jackknife: Mbh (JK) Back to Table2

$$\hat{N}_{JN} = M_t + t \cdot u_t .$$

- Sample Coverage: Mbh (SC) Back to Table2

$$\hat{N}_{SC} = \frac{M_{j+1}}{\hat{C}_j} + \frac{j \cdot u_j}{\hat{C}_j}\hat{\gamma}_j^2 \quad \text{, where}$$

$$j = \max\{k : u_{k+1}/e_{k+1} < u_1/e_1, k = 1, \cdots, t-1\} ,$$

$$\hat{C}_j = 1 - u_{j+1}/u_1 ,$$

$$\hat{\gamma}_j^2 = \max\left\{\frac{\hat{N}_{0,j}(u_1 - u_2)}{u_1^2} - 1, 0\right\},$$

$$\hat{N}_{0,j} = M_{j+1}/\hat{C}_j .$$

- Estimating Equation: Mbh (EE) Back to Table2

Equation for N: $\displaystyle \sum_{j=1}^{t} \frac{u_j - (N - \hat{M}_j^*)\bar{p}}{(1 - \hat{C}_{j-1})} = 0,$

Equation for ϕ: $\displaystyle \sum_{j=1}^{t} (m_j - \hat{M}_j^* \phi\, \bar{p}) = 0,$

Equation for $\bar{p}$: $\displaystyle \sum_{j=1}^{t} \left\{ \frac{u_j - (N - \hat{M}_j^*)\bar{p}}{[1 - (1 + \hat{\gamma}_{bh}^2)\, \bar{p}\,]} + \frac{m_j - \hat{M}_j^* \phi\, \bar{p}}{[1 - (1 + \hat{\gamma}_{bh}^2)\phi\, \bar{p}]} \right\} = 0,$

$$\hat{C}_{j-1} = \hat{C}_{j-1}(\phi) = 1 - u_j / (u_j + m_j / \phi), \quad \hat{M}_j^* = M_j + (j-1) u_{j-1}\, \hat{\gamma}_{bh}^2 ,$$

$$\hat{\gamma}_{bh}^2 = \hat{\gamma}_{bh}^2(\phi) = \max\left\{ \frac{\hat{N}_h\; t \sum_{j=1}^{t} [j(j-1) f_{jt} + 2(\phi - 1)(j-1) f_{jt}]}{(t-1)[\sum_{j=1}^{t} (m_j + \phi\, u_j)]^2} - 1, \quad 0 \right\}, \text{ where}$$

$\hat{N}_h$ is a simple estimator valid under model **M**$_h$, that is

$$\hat{N}_h = [M_{t+1} + f_{1t}\, \hat{\gamma}_h^2] / [1 - f_{1t} / (\textstyle\sum_{j=1}^{t} j f_{jt})] .$$

8. Model M_{tbh} (Chao et al., 2001):

- Estimating Equation: Mtbh(EE) Back to Table2

Equation for N: $\sum_{j=1}^{t} \frac{\hat{M}_j^*(\phi u_j + m_j) - Nm_j}{(1-\hat{C}_{j-1})[1+(\phi-1)\hat{C}_{j-1} - \phi(1+\hat{\gamma}_{tbh}^2)\hat{\alpha}_j]} = 0,$

Equation for ϕ: $\sum_{j=1}^{t} \frac{\hat{M}_j^*(\phi u_j + m_j) - Nm_j}{[1+(\phi-1)\hat{C}_{j-1} - \phi(1+\hat{\gamma}_{tbh}^2)\hat{\alpha}_j]} = 0.$

$\hat{\alpha}_j = \hat{\alpha}_j(\phi, N) = \frac{A_j - [A_j^2 - 4N\phi n_j(1+\hat{\gamma}_{tbh}^2)]^{1/2}}{2N\phi(1+\hat{\gamma}_{tbh}^2)}$, where

$A_j = A_j(\phi, N) = N + \phi n_j(1+\hat{\gamma}_{tbh}^2) + (\phi-1)[N\hat{C}_{j-1} - (1+\hat{\gamma}_{tbh}^2)m_j].$

$\hat{C}_{j-1} = \hat{C}_{j-1}(\phi) = 1 - u_j/(u_j + m_j/\phi)$, $\hat{M}_j^* = M_j + [\sum_{k=1}^{j-1}\hat{\rho}_{k,j-1}]\ u_{j-1}\hat{\gamma}_{tbh}^2$,

$\hat{\gamma}_{tbh}^2 = \hat{\gamma}_{tbh}^2(\phi) = \max\left\{\frac{\hat{N}_{bh}\sum_{j=1}^{t}[j(j-1)f_{jt} + 2(\phi-1)(j-1)f_{jt}]}{[\sum_{j=1}^{t}(m_j+\phi u_j)]^2 - \sum_{j=1}^{t}(m_j+\phi u_j)^2} - 1,\quad 0\right\}$, where

$\hat{N}_{bh}$ is a simple estimator valid under model $\mathbf{M}_{bh}$. Here, $\rho_{k,j-1} = e_k/e_{j-1}$ denotes the unknown relative time effect of sample k. A convenient estimator of $\rho_{k,j-1} = e_k/e_{j-1}$ is a function of ϕ and can be presented as

$\hat{\rho}_{k,j-1} = \hat{\rho}_{k,j-1}(\phi) = (u_k + m_k/\phi)/(u_{j-1} + m_{j-1}/\phi)$.