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# USER MANUAL

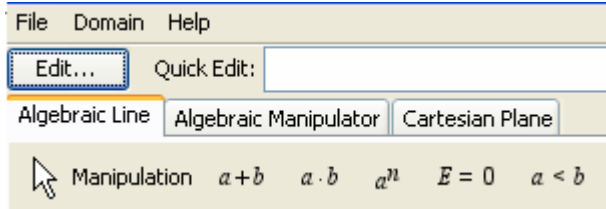
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# ALNUSET

## THE MAIN MENU OF ALNUSET



### **File**

	Use the File popup menu to: <ul style="list-style-type: none"><li>- Open a saved file</li><li>- Save a file</li><li>- Save file in a specific folder</li><li>- Quit ALNUSET</li></ul>
--	---

Note: file drop opportunity is available.

### **Domain**

Use the Domain popup menu in the ALNUSET menu bar to choose the numerical set you want to operate on.

	The choice of numerical set modifies how data are displayed, on three components of Alnuset: for example, if the domain of natural integers is selected on the Algebraic Line, only positive numbers are displayed. Note: Full Range domain is the extension of Rational Numbers to rational powers
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### **Help**

Function not yet available

# ALGEBRAIC EDITORS

Two Algebraic Editors are available in Alnuset:

- Linear Editor
- Two-dimensional Editor

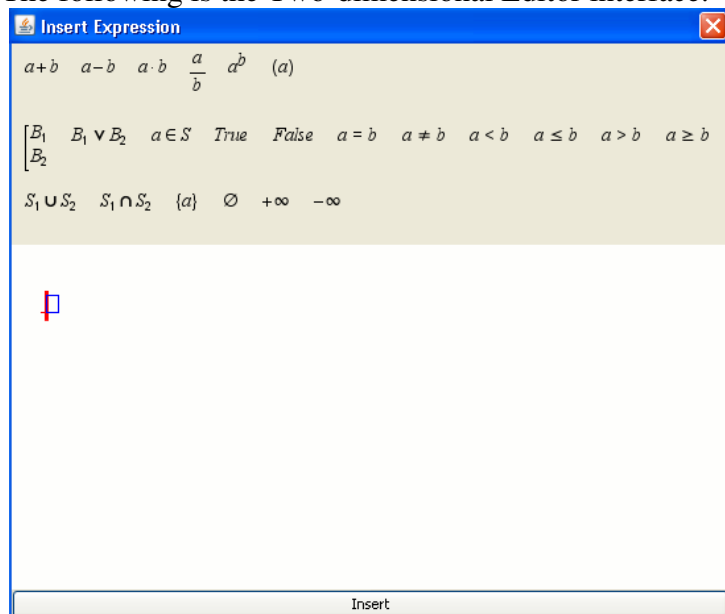
## 1. Linear Editor

In this space it is possible to edit expressions in linear form using keyboard only.

Quick Edit:  $((3*x+1)^2)/(x+1)$

## 2. Two-dimensional Editor

The following is the Two-dimensional Editor interface.





The interface is composed by:

- the list of commands that are available for the editing;
- the editing space.

### Interaction

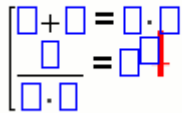
Two modalities of editing are available:

1. using keyboard only
  2. using commands of the interface.
- These two modalities can be used in integrated way.
  - These two modalities of editing are available compared to the position of the red cursor:  
when red cursor is on the left of the square  the user can edit by means of the keyboard;

when red cursor is on the right of the square  the user can select a command on the interface.

- Use the arrow keys to move the cursor on the left and on the right of the square
- Use the mouse to move the cursor on the expression
- It is possible to select a part of the edited expression in order to delete it or to apply on it a command of the interface
- Press the “Insert” button or the key “return” to use the edited expression.

Note that it is possible to construct an empty structure to be filled successively both using the key board and the commands of the interface.



This is the list of keys that the user can use when the red cursor is on the left of the square:

0-9, numerical constants

a-z, variables

Left Right Home End

move cursor without modifications

Backspace Delete

erase char to the left or right

Esc

clear the expression

+ - \* / ^

insert numerical operator

()

parentheses

[ ], { }

subsets

T F

boolean constants

U I

union, intersection of sets

& |

boolean and, or

O

empty set

M P

negative, positive infinity

> < = !

equations, comparisons

#

in set operator

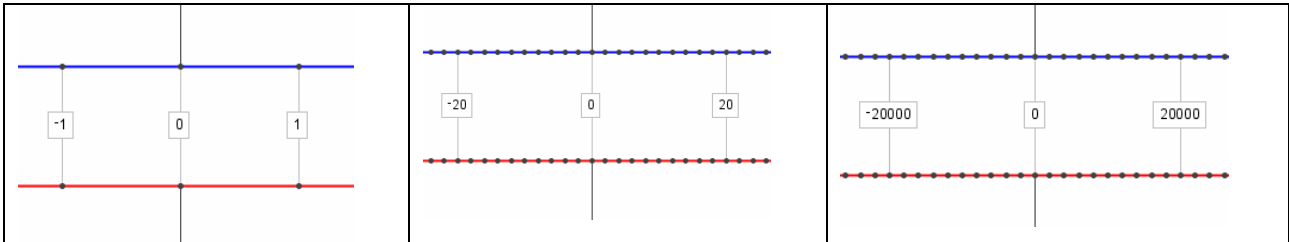
# ALGEBRAIC LINE COMPONENT

## 1. General characteristics of Algebraic Line

### Zoom

Two different ways to modify the unit measure on the lines are available:

- drag any point corresponding to any integer number both on the blue and on red line;
- use the mouse wheel.

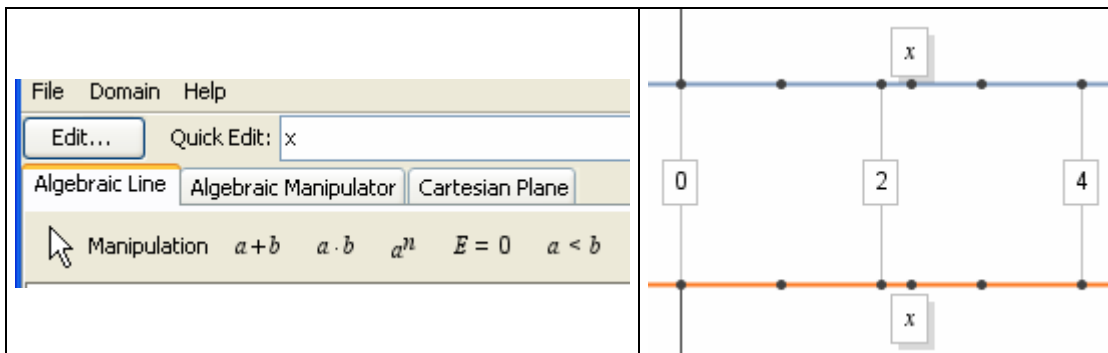


### Scroll

Click the right mouse button and drag the mouse along the X dimension of the window to scroll the Algebraic Line

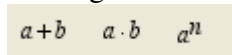
### Insertion of a mobile point on the line corresponding to an algebraic variable

Edit a letter in the Linear Editor of Alnuset to insert a mobile point on the line that will be labeled with such a letter.



### Geometrical editor

Three geometrical models for the algebraic operations are available through the following icons:



## Addition/Subtraction $a + b$

	<p>Click on the label <math>\square + \square</math> or <math>\square - \square</math> (or on their corresponding green points) to pass from addition to subtraction and vice-versa.</p>	
--	--	--

	<p>Drag the two green points corresponding to the label <math>\square</math> and the label <math>\square</math> onto two any point of the line you want to add or subtract. Click on the label of the result to accept it.</p>	
--	--	--

## Multiplication/Division $a \cdot b$

	<p>Click on the label <math>\square \cdot \square</math> or the label <math>\frac{\square}{\square}</math> (or on their corresponding green points) to pass from multiplication to division and vice-versa.</p>	
--	---	--



	<p>Drag the two green points corresponding to the label <input type="text"/> and the label <input type="text"/> onto any point of the line you want to multiply or divide.</p> <p>Click on the label of the result to accept it.</p>	
--	--	--

**Integer Power/Rational Power**

$a^n$

	<p>Use the button <input type="text"/> or the button <input type="text"/> to increase or decrease the grade of the power.</p> <p>Click on the first label or any other labels (or on the corresponding points), to pass from integer power to rational power and vice-versa.</p>	
--	--	--

	<p>Drag the green point corresponding to the label <input type="text"/> onto any point of the line you want to compute the power.</p> <p>Click on the label of the result to accept it.</p>	
--	---	--

## 2. Visualize the constructed expressions

The diagram shows an Algebraic Line with two horizontal axes. The top axis is blue and the bottom axis is orange. Both axes have tick marks at 0, 2, 4, and 6. On the blue axis, there are three expressions:  $x$  at position 2,  $x+1$  at position 3, and  $\frac{(3 \cdot x + 1)}{(x - 1)}$  at position 6. On the orange axis, there are three expressions:  $x$  at position 2,  $x+1$  at position 3, and  $\frac{(3 \cdot x + 1)}{(x - 1)}$  at position 6. Below the axes is a red box labeled "Expressions" containing the following text:

$x$   
 $\frac{(3 \cdot x + 1)}{(x - 1)}$   
 $x + 1$

To the right of the Algebraic Line is a context menu with the following options:

- Show
- Start Tracking
- Show Construction
- Delete
- Send to Manipulator
- Show/Hide Graph ▶
- Copy

The expressions constructed on the Algebraic Line will be displayed also in the specific space Expressions

Clicking the right mouse button onto an expression (both on the line and on the Expressions space) different visualization functions are made available for that expression:

- Show
- Start Tracking
- Show Construction
- Delete
- Send to Manipulator
- Show/Hide Graph ▶
- Copy

These functions will be described successively

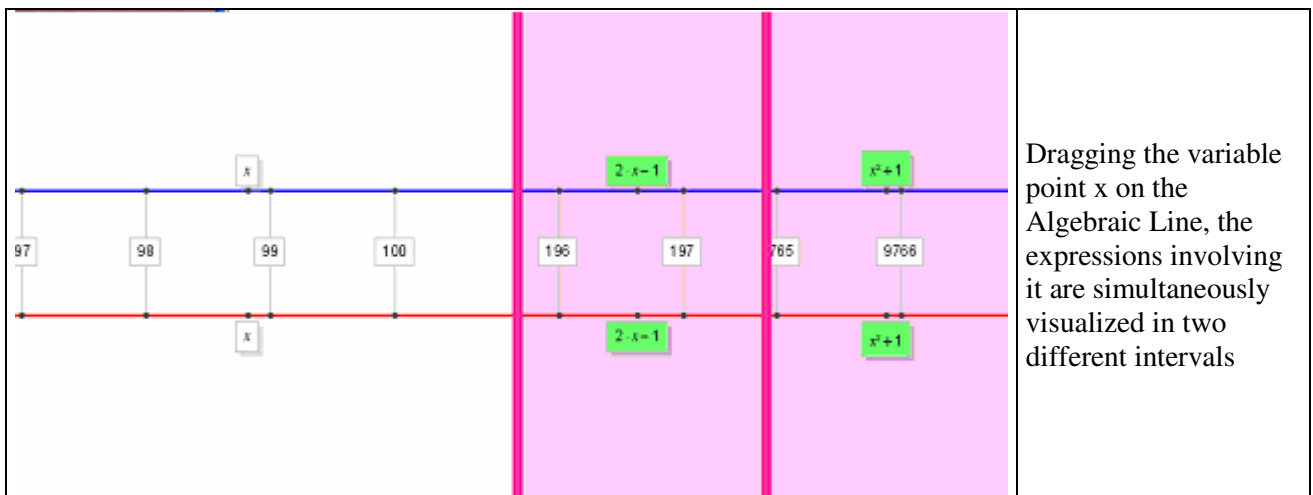
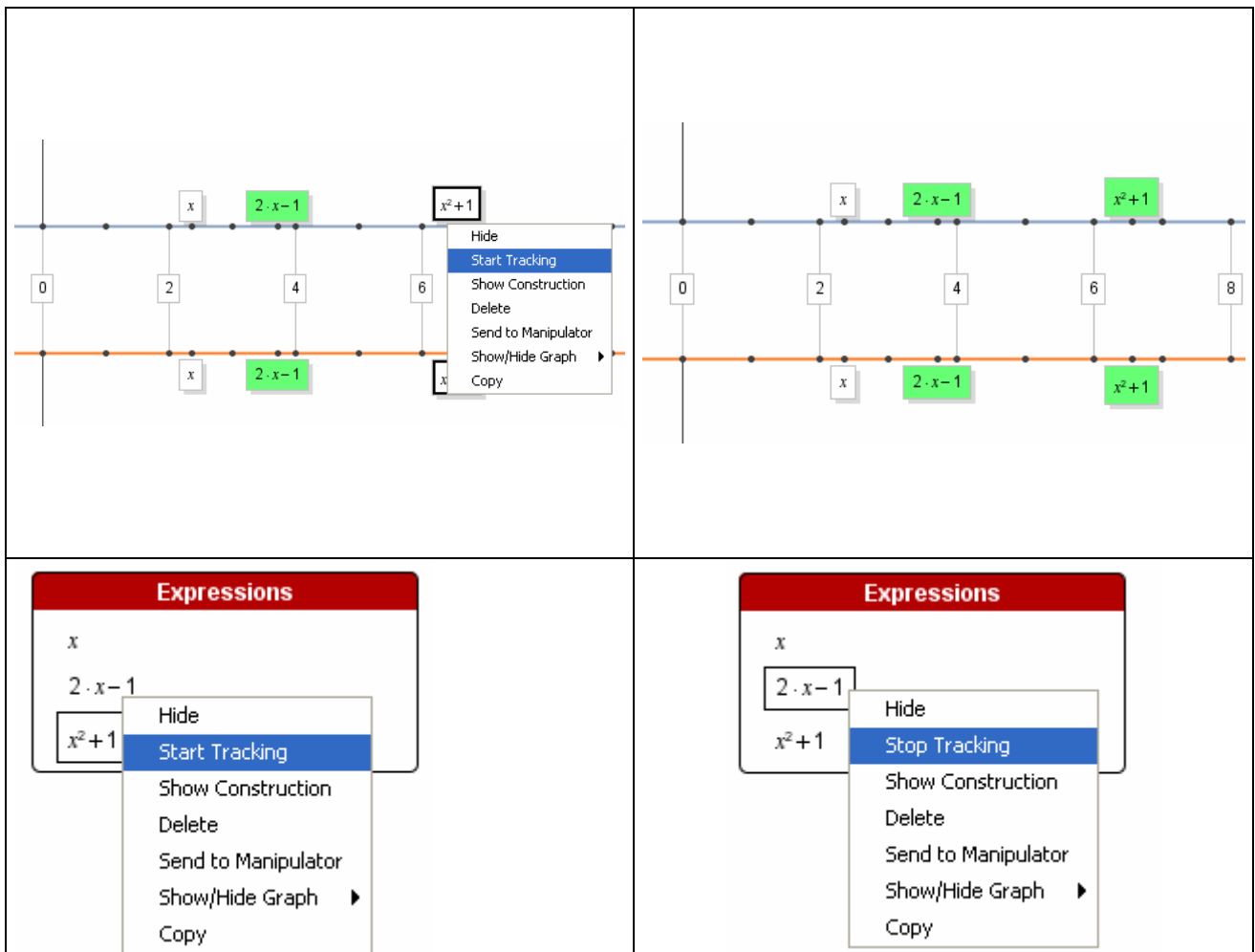
### ***Drag mobile points corresponding to algebraic variables***

Mobile points can be dragged on the Algebraic Line in accordance with the restraints imposed by the chosen numerical domain. Dragging a mobile point corresponding to an algebraic variable, all the expressions involving that variable move accordingly.

### ***Start/Stop Tracking***

Use the Show Tracking function to display an expression which comes out of the screen window during the drag of the variable point from which it depends on.

The expressions on which the Tracking function is applied are highlighted in green.



**Hide/Show**

This function makes possible to show or hide a point constructed on the Algebraic Line and its label.

**Show/Hide construction**

This function makes possible to show or hide (default: hide) the geometrical construction of a point.

## Delete

This function allows the user to delete a point and its corresponding label.

## Send to manipulator

This function allows the user to send the selected expression to the Manipulator component to be transformed.

## Show Graph

This function allows the user to send the selected expressions on the Cartesian Plane component selecting the independent variable and to show automatically the corresponding graph.

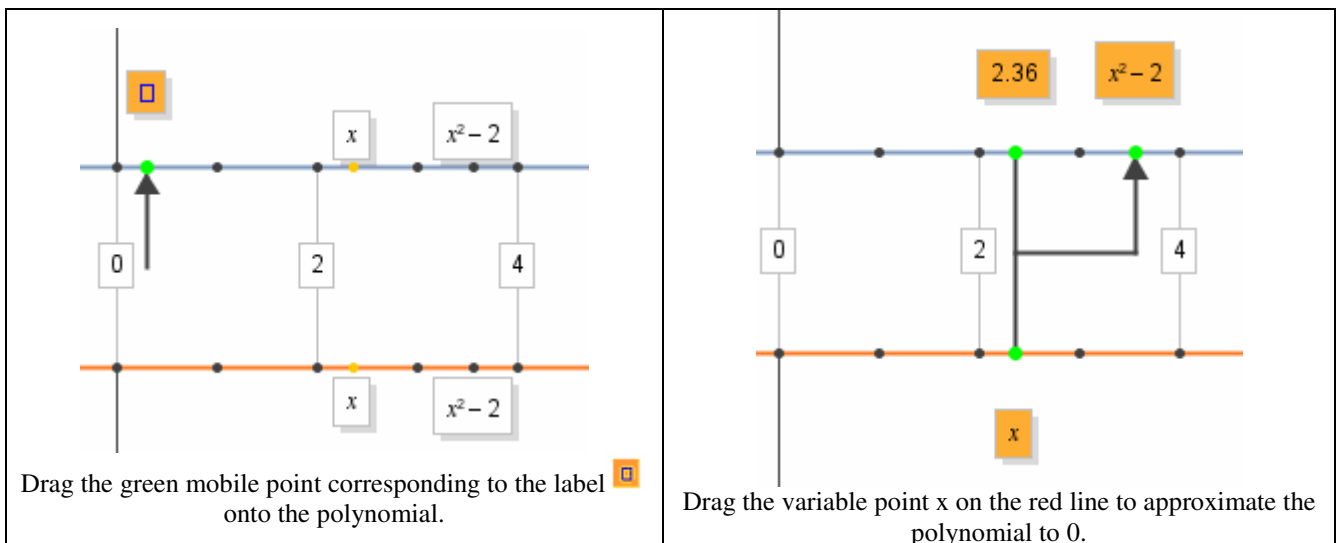
## Copy

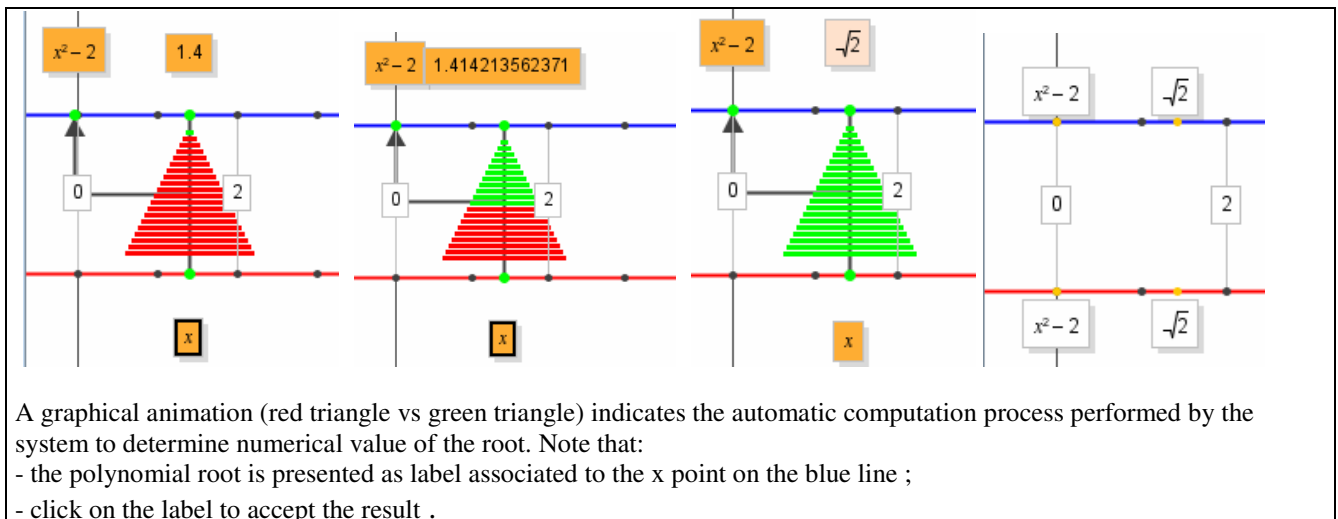
Not yet available. This function allows the user to translate an expression of Alnuset in MathML language in order to integrate its use within other applications

## Polynomial Roots

The following command allows the user to find real roots of a polynomial with integer coefficients.

$$E = 0$$





Once the result has been accepted the polynomial roots is reported in the Roots window under the corresponding polynomial.

Roots
Roots of $Z^2 - 2$
$\sqrt{2} = 1.4142135623730951$
$-\sqrt{2} = -1.4142135623730951$

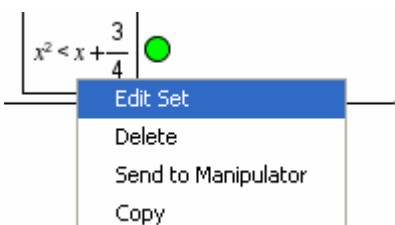
### ***Truth set of algebraic proposition***

To explore and to define the truth set of algebraic propositions, namely the truth set of equations and inequations, the user has to edit the proposition by means of the bidimensional editor.

The proposition defined by the user is automatically reported in the specific space named Sets.

Sets
$x^2 < x + \frac{3}{4}$ ●

In order to define the truth set of the proposition, the user has to click on that proposition with the right mouse button and select "Edit set".



Drag the variable point on the points already represent on the lines that you consider important points to define the truth set of the proposition.

Click once on the button in order to select the point and to include it to the truth set (a green point appears on the line), or twice to select the point without include it to the truth set (a red point appears on the line).

In case of inequalities the user can define on the new red line the interval where the defined proposition is “true”. This definition is completely mouse based.

The actions performed by the user on the line (selecting of points, definition of intervals) are automatically expressed in formal language by the system and reported in the Sets space containing the proposition.

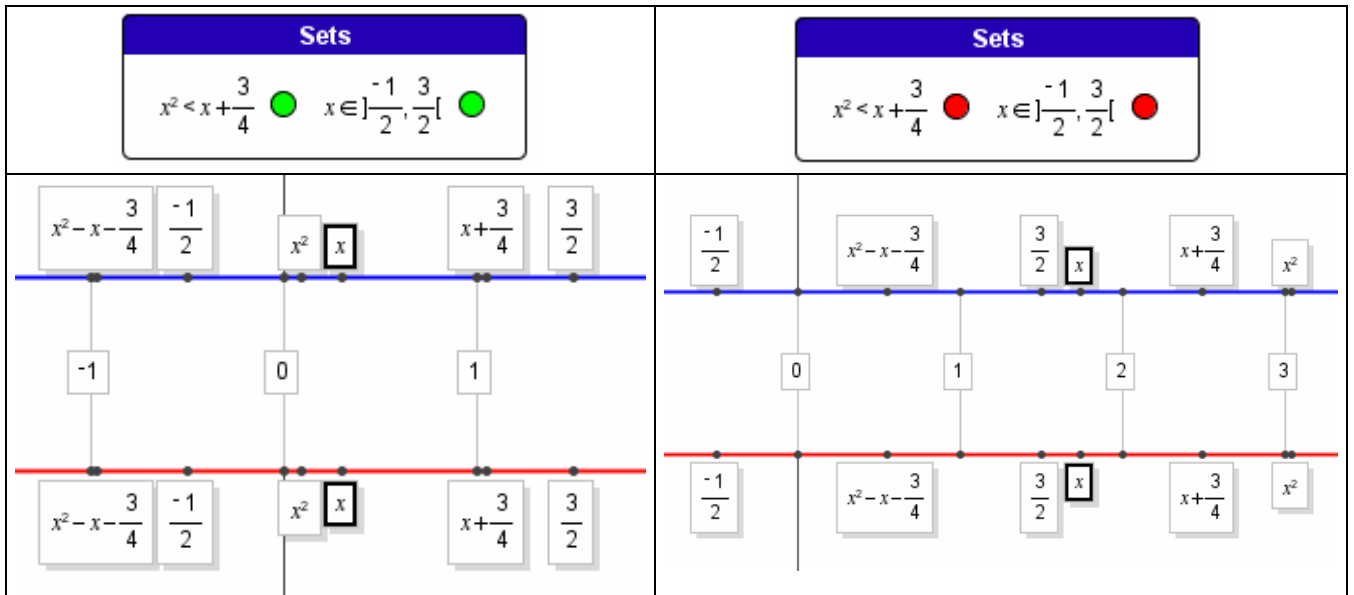
Sets	
$x^2 < x + \frac{3}{4}$	$x \in ]-\frac{1}{2}, \frac{3}{2}[$

The system offers the following feedbacks.

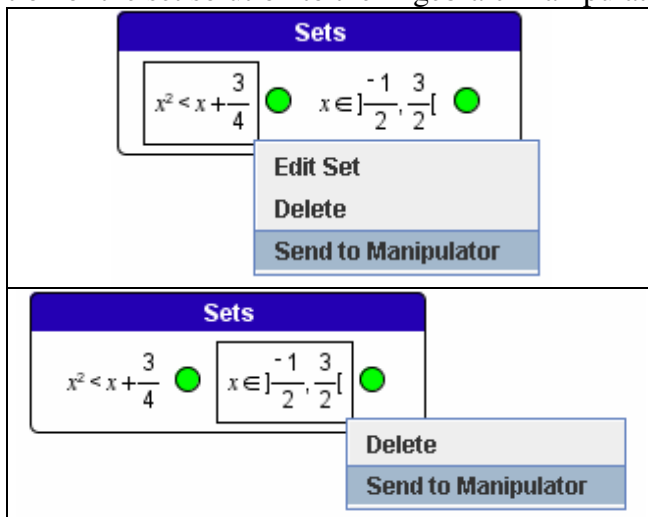
Dragging the  $x$  point on the line, a colored ball appears in the Sets space near the proposition and another one near the set solution.

If  $x$  is dragged on points where proposition is true, then the mark becomes green. On the contrary, if  $x$  is dragged on points where proposition is false, then the associated ball become red.

The ball associated to the set solution becomes green when  $x$  is dragged on points that are contained in the edited set. The concordance of the color between the two balls when the  $x$  point is dragged on any values of the algebraic line, is index of the fact that the edited set is the truth set of the proposition.



Clicking the right mouse button on the proposition or on the set solution it is possible to send the proposition or the set solution to the Algebraic Manipulator component.



# ALGEBRAIC MANIPLULATOR COMPONENT

## 1. General characteristics of Algebraic Manipulator

The interface makes available two kinds of commands:

- Basic commands for manipulation of algebraic expressions (see the following figure)
- Commands constructed by the user (see User Rules commands)

The figure below illustrates an example of use of the interface of Algebraic Manipulator component.

The screenshot shows the Algebraic Manipulator software interface. At the top, there is a menu bar with 'File', 'Domain', and 'Help'. Below the menu bar is a toolbar with 'Edit...' and 'Quick Edit:'. The main interface is divided into several sections:

- Algebraic Line:** Contains 'Algebraic Manipulator' and 'Cartesian Plane' tabs.
- User Rules:** Contains 'Show', 'Import...', 'Export...', and 'Clear' buttons.
- Rules List:** A large table of mathematical rules categorized into:
  - Addition:**
    - $A+B \Leftrightarrow B+A$
    - $A+(B+C) \Leftrightarrow (A+B)+C$
    - $A \Leftrightarrow A+0$
    - $A+(-A) \Leftrightarrow 0$
    - $A-B \Leftrightarrow A+(-B)$
    - $a_1+a_2+\dots \Rightarrow x$
    - $n \Rightarrow a+b$
  - Multiplication:**
    - $A \cdot B \Leftrightarrow B \cdot A$
    - $A \cdot (B \cdot C) \Leftrightarrow (A \cdot B) \cdot C$
    - $A \Leftrightarrow A \cdot 1$
    - $A \cdot 0 \Leftrightarrow 0$
    - $-A \Leftrightarrow -1 \cdot A$
    - $1 \Leftrightarrow -1 \cdot -1$
    - $A \cdot \frac{1}{A} \Leftrightarrow 1$
    - $\frac{A}{B} \Leftrightarrow A \cdot \frac{1}{B}$
    - $\frac{1}{A_1 \cdot A_2 \cdot \dots} \Leftrightarrow \frac{1}{A_1} \cdot \frac{1}{A_2} \cdot \dots$
    - $a_1 \cdot a_2 \cdot \dots \Rightarrow x$
    - $n \Rightarrow p_1 \cdot p_2 \cdot \dots$
  - Distribute / Factor:**
    - $A \cdot (B_1+B_2+\dots) \Leftrightarrow A \cdot B_1+A \cdot B_2+\dots$
  - Powers:**
    - $A^n \Leftrightarrow A \cdot A \cdot \dots$
    - $A^{n_1+n_2+\dots} \Leftrightarrow A^{n_1} \cdot A^{n_2} \cdot \dots$
    - $(A_1 \cdot A_2 \cdot \dots)^n \Leftrightarrow A_1^n \cdot A_2^n \cdot \dots$
    - $A^{\frac{p}{q}} \Leftrightarrow A^{\frac{1}{q}} \cdot A^{\frac{1}{q}} \cdot \dots$
    - $A^{\frac{1}{2}} \Leftrightarrow \sqrt{A}$
  - Computation:**
    - $A \Rightarrow (A)$
    - Remove extra ()
    - Simplify numerical expression
    - Expand
    - Collect
    - Eliminate variable
  - Solving:**
    - $A \leq B \Leftrightarrow B \leq A$
    - $A \leq B \Rightarrow A-B \leq 0$
    - $A \leq B+T \Rightarrow A-T \leq B$
    - $A+T \leq B \Rightarrow A \leq B-T$
    - $T \cdot A \leq B \Rightarrow A \leq \frac{B}{T}$
    - $A^{\frac{p}{q}} \leq B \Rightarrow A^p \leq B^q$
    - $A^2 \leq B \Rightarrow A \leq \sqrt{B}$
    - $T \cdot A \leq 0 \Rightarrow T \leq 0 \vee A \leq 0$
    - $\frac{A}{B} \leq 0 \Rightarrow A \leq 0 \vee B \leq 0$
  - Logic and Set:**
    - Simplify boolean expression
    - Simplify set
    - $L \Leftrightarrow x \in S$
    - $x \in S_1 \vee x \in S_2 \vee \dots \Rightarrow x \in S_1 \cup S_2 \cup \dots$
    - $\begin{cases} x \in S_1 \\ x \in S_2 \end{cases} \Rightarrow x \in S_1 \cap S_2 \cap \dots$
  - Insert from Algebraic Line:**
    - Factor roots
    - Insert solution set
    - Instantiate variable
- Result:** A large box on the right displays the result of a calculation:  $\frac{(2 \cdot x - 1)^2 + (x + 1)}{(x - 2)}$ .



## 2. Interaction

### Selection of a part of expression

Operators, brackets and literal or numerical elements define the parts of the expression and their hierarchical organization. When the mouser pointer is positioned over any part of the expression, the system dynamically displays all the elements in the hierarchical structure of that part of expression. Hierarchical selection of a part of the expression corresponds to what mouse points and the part of the expression selected is highlighted in yellow.

$\frac{((2 \cdot x - 1)^2 + (x + 1))}{(x - 2)}$ <p>mouse points to the operator “-“ of <math>2x-1</math></p>	$\frac{((2 \cdot x - 1)^2 + (x + 1))}{(x - 2)}$ <p>mouse points to the brackets of the following part of the expression <math>(2x-1)^2</math></p>
$\frac{((2 \cdot x - 1)^2 + (x + 1))}{(x - 2)}$ <p>mouse points to the operator “+“ of <math>(2x-1)^2 + (x+1)</math></p>	$\frac{((2 \cdot x - 1)^2 + (x + 1))}{(x - 2)}$ <p>mouse points to the line fraction of the expression</p>
<p>If operators have the same hierarchic level, for instance the operator “+” in the following expression: <math>7+5+3+4</math></p> <p>the selection of a part of the expression , for instance <math>5+3+4</math>, is performed by dragging mouse from the first to the last element of that part of the expression you want to select</p>	
$7+5+3+4$ <p>the selection of a part of the expression , for instance <math>5+3+4</math>, is performed by dragging mouse from the first to the last element of that part of the expression you want to select</p> $7+5+3+4$	

### Commands for manipulation

In the following table the description and the action of each command for the algebraic manipulation is listed.

Note that the list makes reference to the full range domain.

Legend:

A, B, C, ...= structural elements of an algebraic expression;

a, b, c, ...= integer positive numbers.

Commands	Description - action
<b>ADDITION</b>	
$A+B \Leftrightarrow B+A$	Commutative property of addition.
$A+(B+C) \Leftrightarrow (A+B)+C$	Associative property of the addition.
$A \Leftrightarrow A+0$	Neutral element of the addition.
$A+-A \Leftrightarrow 0$	Sum of two opposite elements.
$A-B \Leftrightarrow A+-B$	Relation between binary minus and unary minus.
$a_1 + a_2 + \dots \Rightarrow x$	Addition of positive integer numbers.
$n \Rightarrow a+b$	Decomposition of a positive integer number into an addition of two positive integer numbers. Note that input for “a” is request

MULTIPLICATION	
$A*B \Leftrightarrow B*A$	Commutative property of multiplication.
$A*(B*C) \Leftrightarrow (A*B)*C$	Associative property of the multiplication.
$A \Leftrightarrow A*1$	Neutral element of the multiplication.
$A*0 \Leftrightarrow 0$	Zero-product property Note that selecting 0, input for A is request
$-A \Leftrightarrow -1*A$	Relation between an expression and its opposite.
$1 \Leftrightarrow -1 * -1$	Rule of signs for multiplication
$A \cdot \frac{1}{A} \Leftrightarrow 1$	Product of two reciprocal expressions. Note that selecting 1, input for A is request
$\frac{A}{B} \Leftrightarrow A \cdot \frac{1}{B}$	Fundamental property of fraction.
$\frac{1}{A_1 \cdot A_2 \cdot \dots} \Leftrightarrow \frac{1}{A_1} \cdot \frac{1}{A_2} \cdot \dots$	Multiplication of fractions with unitary numerator.
$a_1 * a_2 * \dots \Rightarrow x$	Multiplication of positive integer numbers
$n \Rightarrow p_1 * p_2 * \dots$	Factorization of an integer positive number in its prime factors.
DISTRIBUTE/FACTOR	
$A* (B_1 + B_2 + \dots) \Leftrightarrow A* B_1 + A* B_2 \dots$	Distributive property of multiplication over addition
POWERS	
$A^n \Leftrightarrow A * A * \dots$	Transform the integer power “n” of an expression into n products of its base, and vice-versa.
$A_1^{n_1} * A_2^{n_2} * \dots \Leftrightarrow A_1^{n_1} * A_2^{n_2} * \dots$	Rule of transformation involving product of powers with the same base
$(A_1 * A_2 * \dots)^n \Leftrightarrow A_1^n * A_2^n * \dots$	Rule of transformation involving the product of powers with the same exponent.
$\frac{p}{A^q} \Leftrightarrow A^{\frac{1}{q}} * A^{\frac{1}{q}} * \dots$	Rule of transformation involving rational power
$\frac{1}{A^2} \Leftrightarrow \sqrt{A}$	Rule of transformation of a rational power into a radical and vice-versa.
COMPUTATION	
$A \Rightarrow (A)$	Insertion of the selected expression A into brackets
Remove extra ()	Remove redundant brackets from the selected expression.

Simplify numerical expression	Compute the result (integer or rational) of a numerical expression
Expand	Perform the selected polynomial computation and simplify the result
Collect	Order the selected polynomial according to the variable assigned as input
Eliminate variable	Eliminate a variable within a system
<b>SOLVING</b>	
Legend:	
$\frac{>}{<}$ : indicates whatever operator of comparison between two algebraic expressions. $A \frac{>}{<} B$ : indicates a proposition obtained through the comparison of two expressions;	
$A \frac{>}{<} B \Leftrightarrow B \frac{>}{<} A$	Transform a proposition into its symmetrical.
$A \frac{>}{<} B \Rightarrow A - B \frac{>}{<} 0$	Transform the comparison between two expressions into a comparison between their difference and zero
$A \frac{>}{<} B + T \Rightarrow A - T \frac{>}{<} B$	Move a structural element from the right side to the left side of the proposition
$A + T \frac{>}{<} B \Rightarrow A \frac{>}{<} B - T$	Move a structural element from the left side to the right side of the proposition
$T * A \frac{>}{<} B \Rightarrow A \frac{>}{<} \frac{B}{T}$	Fundamental solving rule for any proposition
$A^{\frac{p}{q}} \frac{>}{<} B \Rightarrow A^p \frac{>}{<} B^q$	Solving rule for any irrational proposition
$A^2 \frac{>}{<} B \Rightarrow A \frac{>}{<} \sqrt{B}$	Solving rule for a pure second degree proposition
$T * A \frac{>}{<} 0 \Rightarrow T \frac{>}{<} 0 \vee A \frac{>}{<} 0$	Solving rule for propositions where the product of two factors is compared with zero
$A/B \frac{>}{<} 0 \Rightarrow A \frac{>}{<} 0 \vee B \frac{>}{<} 0$	Solving rule for propositions where the division of two expressions is compared with zero
<b>LOGIC AND SET</b>	
Simplify Boolean expression	
Simplify set	
$L \Leftrightarrow x \in S$	Transform a solution expressed in propositional form into a set form

$x \in S_1 \vee x \in S_2 \vee \dots \Rightarrow x \in S_1 \cup x \in S_2 \cup \dots$	Transform the logic “or” among two or more sets into their union
$x \in S_1 \Rightarrow x \in S_1 \cap x \in S_2 \cap \dots$ $x \in S_2$ ...	Transform the logic “and” among two or more sets into their intersection
<b>Insert from Algebraic Line</b>	
Factor roots	Factorize the selected polynomial (available only if the roots of the selected polynomial are previously found in Algebraic Line component)
Insert solution set	Insert the true set of the selected proposition previously edited in the Algebraic Line (available only if the true set is correct)
Instantiate variable	Assign to the selected variable the value assumed on the Algebraic Line

## Application of the rule

The part of the expression selected for the manipulation come under the system's control. When a part of the symbolic representation is selected for manipulation, only those commands that can be applied to that part are available.

The figure below illustrates the manipulation of an expression. It can be noted that, in accordance with the current selection, only some commands are available.

The screenshot shows a software interface for algebraic manipulation. At the top, there are tabs for 'Algebraic Line', 'Algebraic Manipulator', and 'Cartesian Plane'. Below these are 'User Rules' and buttons for 'Show', 'Import...', 'Export...', and 'Clear'. The main area is divided into several panels:

- Addition:**
  - $A+B \Leftrightarrow B+A$
  - $A+(B+C) \Leftrightarrow (A+B)+C$
  - $A \Leftrightarrow A+0$
  - $A+-A \Leftrightarrow 0$
  - $A-B \Leftrightarrow A+-B$
  - $a_1+a_2+\dots = x$
  - $n = a+b$
- Multiplication:**
  - $A \cdot B \Leftrightarrow B \cdot A$
  - $A \cdot (B \cdot C) \Leftrightarrow (A \cdot B) \cdot C$
  - $A \Leftrightarrow A \cdot 1$
  - $A \cdot 0 \Leftrightarrow 0$
  - $-A \Leftrightarrow -1 \cdot A$
  - $1 \Leftrightarrow -1 \cdot -1$
  - $A \cdot \frac{1}{A} \Leftrightarrow 1$
  - $\frac{A}{B} \Leftrightarrow A \cdot \frac{1}{B}$
  - $\frac{1}{A_1 \cdot A_2 \cdot \dots} \Leftrightarrow \frac{1}{A_1} \cdot \frac{1}{A_2} \cdot \dots$
  - $a_1 \cdot a_2 \cdot \dots = x$
  - $n = p_1 \cdot p_2 \cdot \dots$
- Distribute / Factor:**
  - $A \cdot (B_1+B_2+\dots) \Leftrightarrow A \cdot B_1+A \cdot B_2+\dots$
- Powers:**
  - $A^n \Leftrightarrow A \cdot A \cdot \dots$
  - $A^{n_1+n_2+\dots} \Leftrightarrow A^{n_1} \cdot A^{n_2} \cdot \dots$
  - $(A_1 \cdot A_2 \cdot \dots)^n \Leftrightarrow A_1^n \cdot A_2^n \cdot \dots$
  - $A^{\frac{p}{q}} \Leftrightarrow A^{\frac{1}{q}} \cdot A^{\frac{1}{q}} \cdot \dots$
  - $A^{\frac{1}{2}} \Leftrightarrow \sqrt{A}$
- Computation:**
  - $A = (A)$
  - Remove extra ()
  - Simplify numerical expression
  - Expand
  - Collect
  - Eliminate variable
- Solving:**
  - $A \leq B \Leftrightarrow B \leq A$
  - $A \leq B \Rightarrow A-B \leq 0$
  - $A \leq B+T \Rightarrow A-T \leq B$
  - $A+T \leq B \Rightarrow A \leq B-T$
  - $T \cdot A \leq B \Rightarrow A \leq \frac{B}{T}$
  - $A^{\frac{p}{q}} \leq B \Rightarrow A^p \leq B^q$
  - $A^2 \leq B \Rightarrow A \leq \sqrt{B}$
  - $T \cdot A \leq 0 \Rightarrow T \leq 0 \vee A \leq 0$
  - $\frac{A}{B} \leq 0 \Rightarrow A \leq 0 \vee B \leq 0$
- Logic and Set:**
  - Simplify boolean expression
  - Simplify set
  - $L \Leftrightarrow x \in S$
  - $x \in S_1 \vee x \in S_2 \vee \dots \Rightarrow x \in S_1 \cup S_2 \cup \dots$
  - $\begin{cases} x \in S_1 \Rightarrow x \in S_1 \cap S_2 \cap \dots \\ x \in S_2 \\ \dots \end{cases}$
- Insert from Algebraic Line:**
  - Factor roots
  - Insert solution set
  - Instantiate variable

On the right side, a list of expressions is shown, with some parts highlighted in green and others in yellow:

- $x^2+2 \cdot x+1$
- $x \cdot x+2 \cdot x+1$
- $x \cdot x+(1+1) \cdot x+1$
- $x \cdot x+x \cdot (1+1)+1$
- $x \cdot x+x \cdot 1+x \cdot 1+1$
- $x \cdot (x+1)+x \cdot 1+1$
- $x \cdot (x+1)+1 \cdot x+1$
- $x \cdot (x+1)+1 \cdot x+1 \cdot 1$
- $x \cdot (x+1)+1 \cdot (x+1)$
- $(x+1) \cdot x+1 \cdot (x+1)$
- $(x+1) \cdot x+(x+1) \cdot 1$
- $(x+1) \cdot (x+1)$

Clicking with the right mouse button on an expression, different functions are available for that expression:

- Re-Insert at End
- Delete
- Create User Rule
- Send to Line

These functions will be described in the following.

## RE-Insert at End

This function makes possible to insert the expression selected to the end of the list of manipulations.

$x^2+2 \cdot x+1$	Re-Insert at End	$x^2+2 \cdot x+1$
$x \cdot x+2 \cdot x+1$	Delete	$x \cdot x+2 \cdot x+1$
$x \cdot x+(1+1) \cdot x+1$	Create User Rule	$x \cdot x+(1+1) \cdot x+1$
$x \cdot x+x \cdot (1+1)+1$	Send to Line	$x \cdot x+x \cdot (1+1)+1$
$x \cdot x+x \cdot 1+x \cdot 1+1$		$x \cdot x+x \cdot 1+x \cdot 1+1$
$x \cdot (x+1)+x \cdot 1+1$		$x \cdot (x+1)+x \cdot 1+1$
$x \cdot (x+1)+1 \cdot x+1$		$x \cdot (x+1)+1 \cdot x+1$
$x \cdot (x+1)+1 \cdot x+1 \cdot 1$		$x \cdot (x+1)+1 \cdot x+1 \cdot 1$
$x \cdot (x+1)+1 \cdot (x+1)$		$x \cdot (x+1)+1 \cdot (x+1)$
$(x+1) \cdot x+1 \cdot (x+1)$		$(x+1) \cdot x+1 \cdot (x+1)$
$(x+1) \cdot x+(x+1) \cdot 1$		$(x+1) \cdot x+(x+1) \cdot 1$
$(x+1) \cdot (x+1)$		$(x+1) \cdot (x+1)$
$(x+1)^2$		$(x+1)^2$
		$x^2+2 \cdot x+1$

## Delete

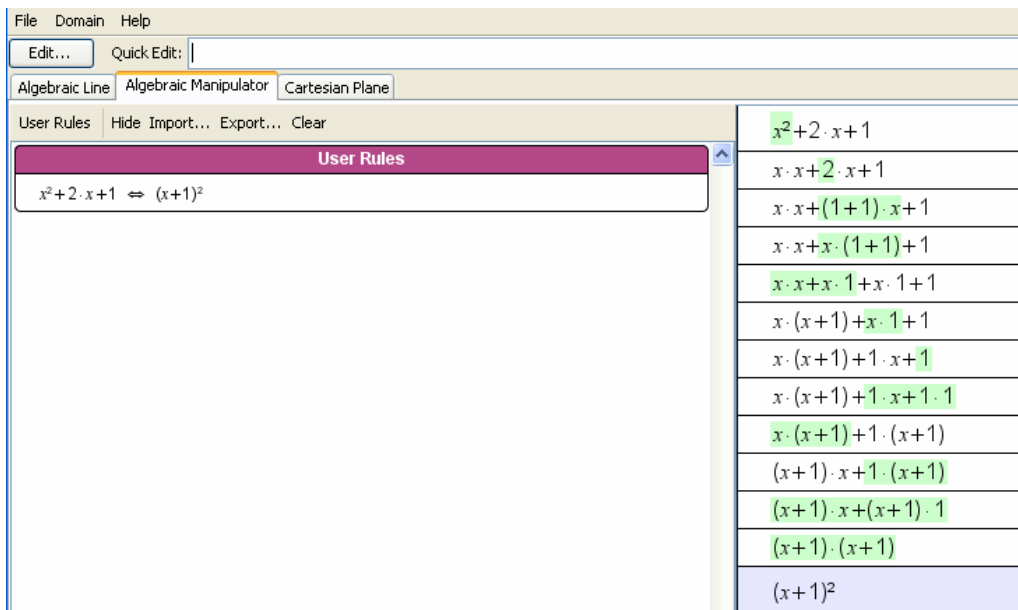
This function allows the user to delete the expression selected.

## Create User Rule

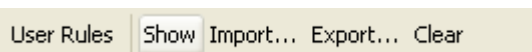
Once the algebraic manipulation has proved that expressions E is equivalent to expression F, the function “Create User Rule” allows the user to create a new symbolic manipulation command which the rewriting rule  $E \Leftrightarrow F$  is associated to.

The new command is immediately added to the list of the user-created commands in the User Rules space.

$x^2+2 \cdot x+1$	Re-Insert at End
$x \cdot x+2 \cdot x+1$	Delete
$x \cdot x+(1+1) \cdot x+1$	Create User Rule
$x \cdot x+x \cdot (1+1)+1$	Send to Line
$x \cdot x+x \cdot 1+x \cdot 1+1$	
$x \cdot (x+1)+x \cdot 1+1$	
$x \cdot (x+1)+1 \cdot x+1$	
$x \cdot (x+1)+1 \cdot x+1 \cdot 1$	
$x \cdot (x+1)+1 \cdot (x+1)$	
$(x+1) \cdot x+1 \cdot (x+1)$	
$(x+1) \cdot x+(x+1) \cdot 1$	
$(x+1) \cdot (x+1)$	
$(x+1)^2$	



The User Rules space is visible clicking on the button “Show”.



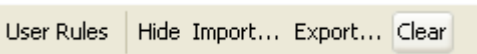
To make available the expression of user rules in a new working session, the user has to save it by means of the “Export” button



“Import” button allows the user to import a last of user rules saved in a previously working session.

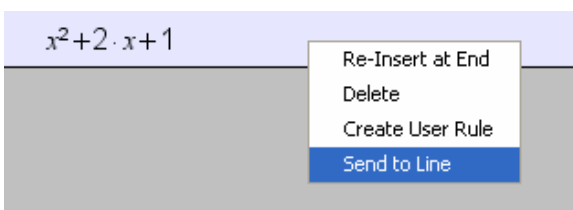


To delete the user rules from the interface the user has to click on the “Clear” button



### Send to Line

This function makes possible to send a whole expression or proposition or selected part of them into the Algebraic Line component.



# CARTESIAN PLAN COMPONENT

## 1. General characteristics of Cartesian plan

The interface of the Cartesian Plan component is composed by an algebraic line and a Cartesian plane. The algebraic line contained in this component is a copy of the line of the Algebraic Line component. To visualize the algebraic line it is necessary to widen the window (default: not visible).

### **Zoom**

To modify the unit measure on the Cartesian plan the user has to use the mouse wheel on the algebraic line. In this way, both the units measure of vertical axis and horizontal axis are modified accordingly.

To modify the unit measure of the vertical axis only the user has to use the mouse wheel on the Cartesian plan.

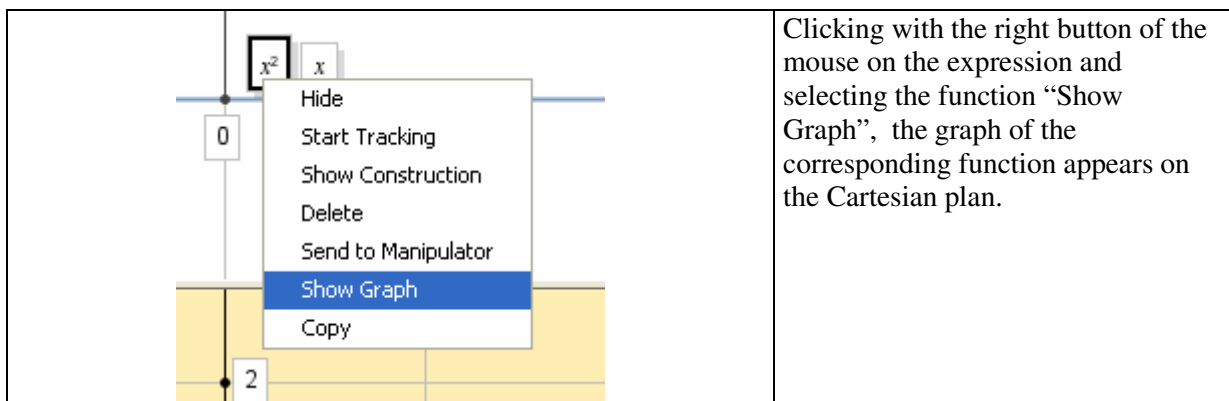
### **Scroll**

Click the right mouse button and drag the mouse to scroll the Cartesian Plan.

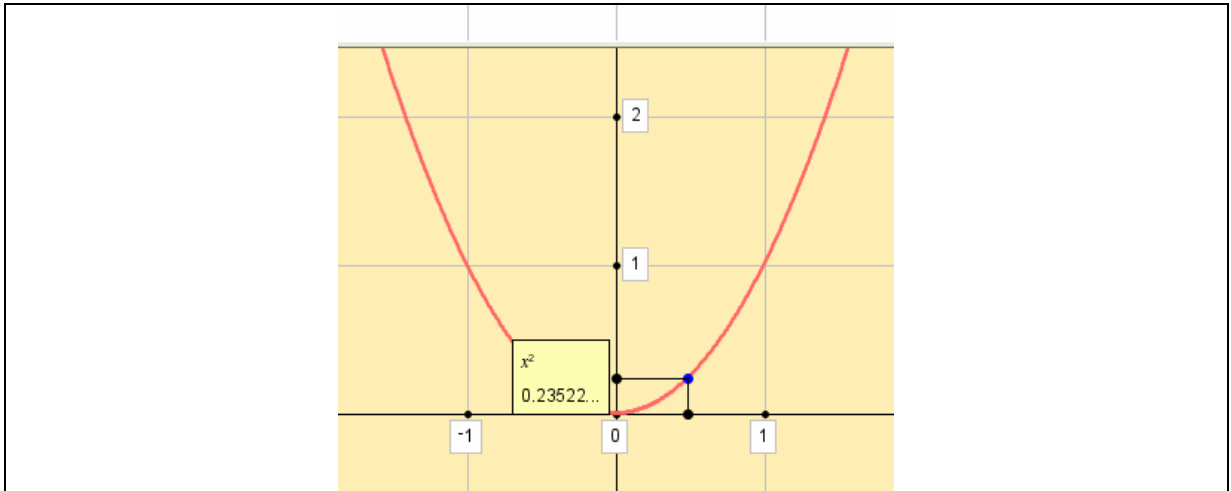
## 2. Interaction

### **Representation of Graphs**

An expression visualized on the algebraic line can be represented as the graph of the corresponding function in the Cartesian plane through the “Show graph” function.





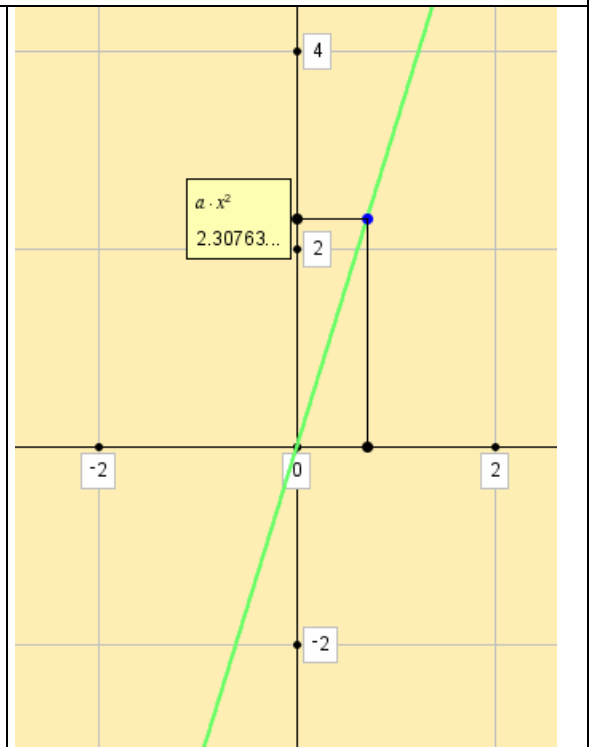
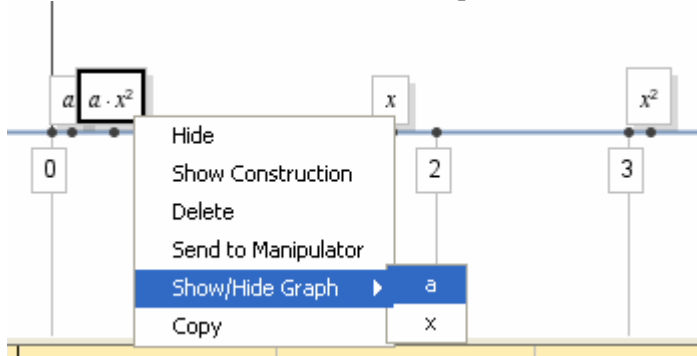


Dragging the mobile point  $x$  on the Algebraic Line, the expression move accordingly on the same line and the point defined by the pairs  $(x, x^2)$  move accordingly on the graph of the Cartesian plane.

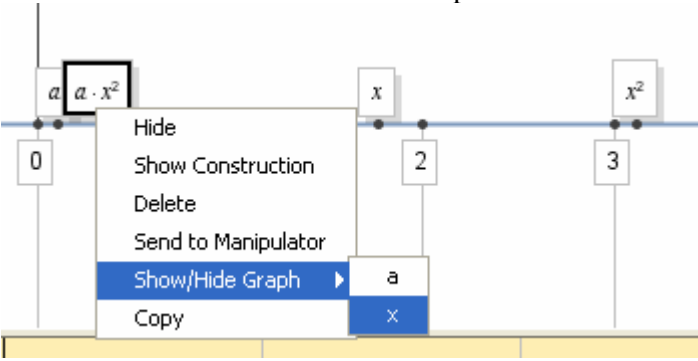
To delete a graph the user can click the right button of the mouse on the graph and select “clear”.

If the expression represented on the Algebraic Line contains more than one letter (i.e.  $a \cdot x^2$ ), the user can choose what letter s/he want to assume as independent variable. The other letters are considered as parameter.

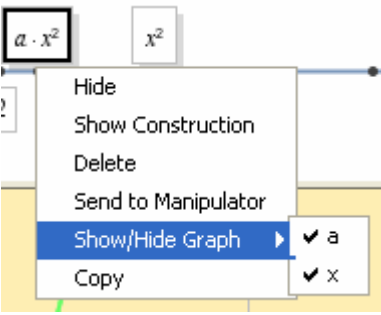
The letter “a” assumes the role of independent variable.



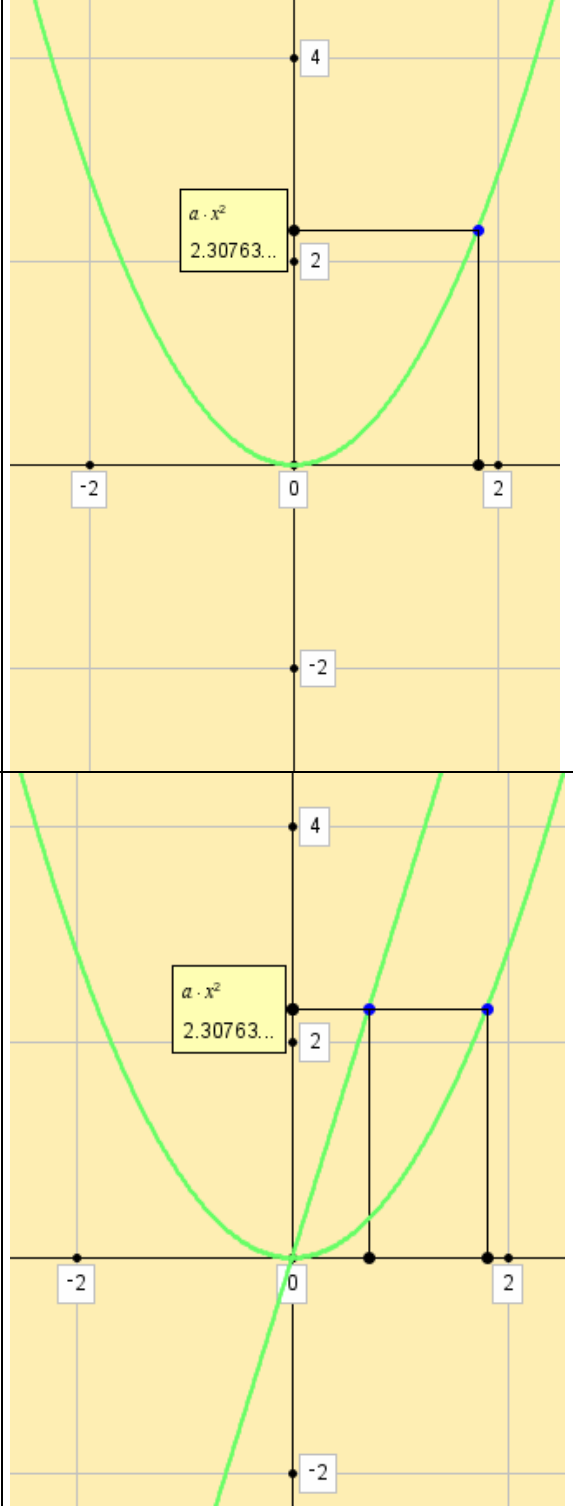
The letter “x” assumes the role of independent variable.



The two previously graphs can be visualized simultaneously on the Cartesian Plane.



Note that dragging the “a” mobile point on the algebraic line, its role as parameter will modify the graph of the parable while its role as independent variable will modify the point on the graph of the line.



### ***Drag mobile point corresponding to algebraic variable***

A mobile point can be dragged on the Algebraic Line in accordance with the restraints imposed by the chosen numerical domain. Dragging a mobile point corresponding to an algebraic variable, both the expression involving that variable in the algebraic line and the point on the curb in the Cartesian plan move accordingly.