FLQ: A Software for Facility Layout with Queueing

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Abstract

We describe a software application, FLQ, that we have developed to implement the layout design methodology described in Benjaafar (2000). For convenience, details of the methodology are reproduced in Appendix 1. FLQ is a windows-based application that combines traditional layout design methods with queueing analysis. FLQ allows designers to examine the impact of layout decisions on various operational performance measures and provides them with the ability to design layouts with one or more of these operational performance measures as criteria. The software is applicable to facilities where a material-handling system consisting of discrete devices provides transportation for material between departments. The software accounts for multiple products with product-specific routing, product-specific processing times, product-specific production requirements, processing departments with multiple servers, material-handling system with multiple devices, both empty and full travel by the material handling devices, product and process-specific holding costs, multiple performance measures and design criteria, and user-specified solution methods.

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1. Introduction

In this paper, we describe a software application, FLQ, that we have developed to implement the layout design methodology described in Benjaafar [1]. For convenience, details of the methodology are reproduced in Appendix 1. FLQUEUE is a windows-based application that combines traditional layout design methods with queueing analysis. FLQ allows designers to examine the impact of layout decisions on various operational performance measures and provides them with the ability to design layouts with one or more of these operational performance measures as criteria. The software is applicable to facilities where a material-handling system consisting of discrete devices provides transportation for material between departments. The software accounts for the following:

- multiple products with product-specific routing,
- product-specific processing times,
- product-specific production requirements,
- processing departments with multiple servers,
- material-handling system with multiple devices,
- both empty and full travel by the material handling devices,
- product and process-specific holding costs,
- multiple performance measures and design criteria, and
- user-specified solution methods.

FLQ has a menu driven interface that allows easy input and display of data. (see Figure 1). It also provides for an efficient way to store, retrieve and manage all layout design data. FLQ offers the user the choice to work on new designs or retrieve and modify existing ones. The software can be used to evaluate the performance of an existing layout or to design a layout from scratch.

2. Software and Hardware Requirements

In order to run and use FLQUEUE, certain minimum requirements of hardware, software, and memory must be met. These requirements are listed below:
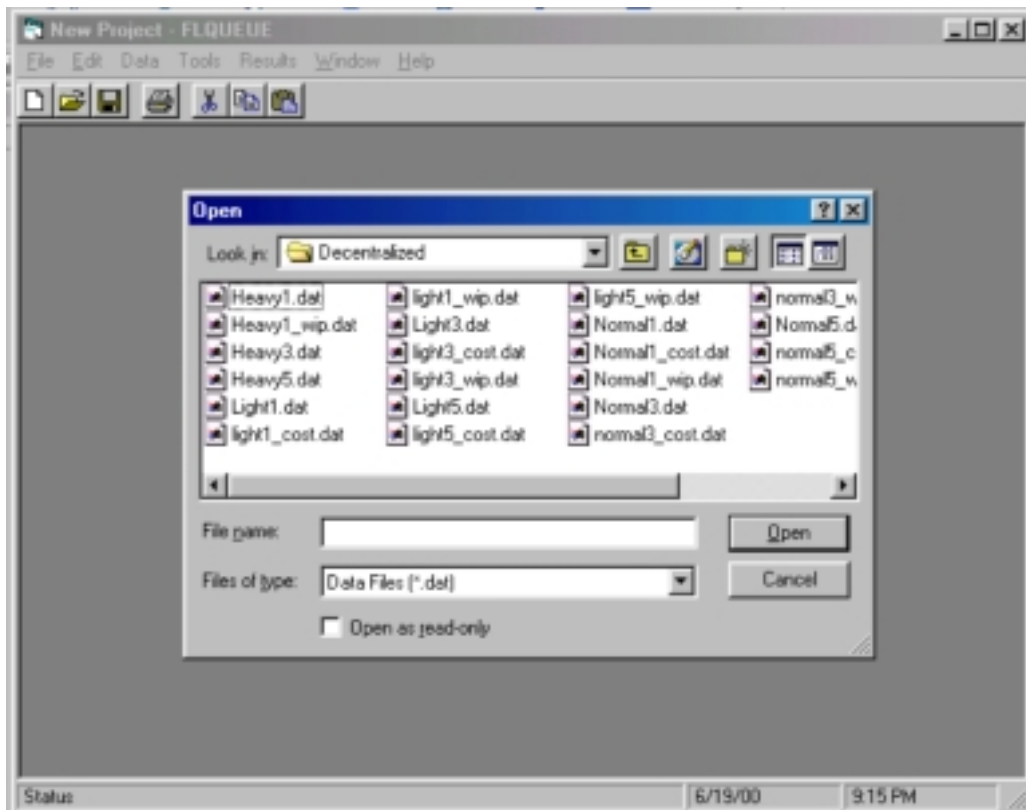
FLQUEUE requires the following hardware:

- IBM PC compatible with Pentium class processor or later model,

- Hard disk drive with a minimum of 4Mb of free space, and

- at least 32Mb of RAM memory.

Software

FLQUEUE requires MS windows 95/98/NT/2000 or later operating systems.

Figure 1 - FLQUEUE User Interface

3. Installing FLQUEUE

The FLQUEUE package includes high-density 1.44Mb floppy disks or a CD-ROM disc. The following steps describe how to install FLQUEUE:

- insert the disk labeled "FLQUEUE disk 1" in the floppy drive, or insert the FLQUEUE CD-ROM.
- double-click the icon named "setup.exe", and

setup.exe

- follow the installer program's instructions.

4. Data Management

The numerous data involved in plant layout design can be classified into the following categories:

- department information,
- product information,
- facility information,
- material-handling system information, and
- layout configuration information.

FLQUEUE manages these categories separately so that small changes in one category do not affect the others. This allows the testing of several layout configurations without having to input all the information each time. Users enter this information through menus, each one dedicated to a specific category. These menus can be accessed through the data menu as shown in Figure 1.
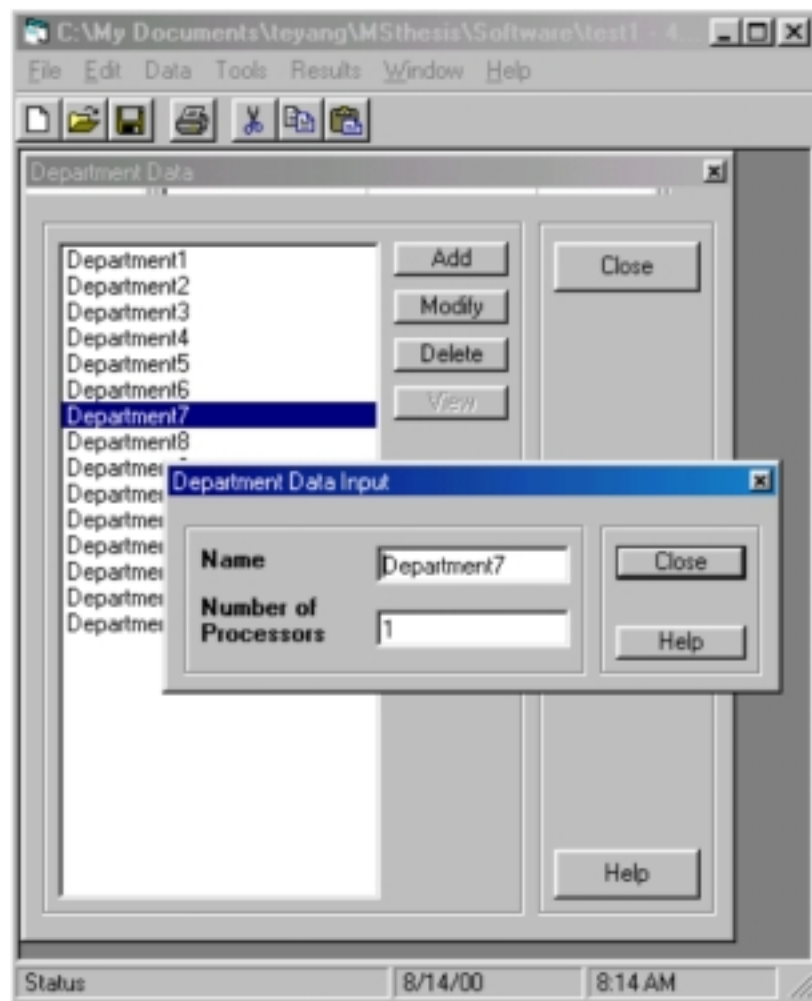
4.1 Department Data

Departments are the processing departments in the factory. Each department is composed of one or more processors. FLQUEUE requires the following information:

- department name and,
- number of processors.

The processing time is not a department characteristic. It is product and operation-specific. Therefore, it will be defined in the product routing menu. Figure 2 shows an example of the department data menu. The department menu shows the list of departments entered. This menu

allows adding, modifying or removing a department by choosing the corresponding command button. Once a button has been pressed, the input menu appears waiting for the information to be entered.

Figure 2 - Department Data Menu

4.2 Product Data

FLQUEUE requires the following information for each product:
- a product name,
- the demand (mean and squared coefficient of variation),
- a target lead-time,

- a penalty cost if the target lead-time is not met and,
- product routing.

The demand is expressed in term of units per unit of time. The target lead-time defines the desired flow time for this product. If the target lead-time is not met, there is a user-specified penalty cost. The part routing consists of several operations and each of them is characterized by the following information:

- name of the department to be used for this operation,
- processing time of this operation (mean and squared coefficient of variation) and,
- holding costs of the product after this operation.

Figure 3 shows an example of this menu. The product menu shows the list of products to be made in the facility. It allows adding, modifying or removing a product by pressing on the corresponding command button. If the "Add" and "Modify" buttons are chosen, the product data input window pops up. The user can then specify product characteristics. The user can also specify the routing for the product by choosing the command buttons in the routing sub-menu. This will show the route data input window, where the user can specify routing information.

4..3 Facility Data

The user specifies the number of locations and the distances between them. Therefore, the facility information needed is:

- number of locations, and
- distances between locations.

FLQUEUE provides the option for the user to use a rectangular shape for the facility. By selecting "Specify Dimensions" as shown in Figure 4, the user can choose the width and length of the facility.
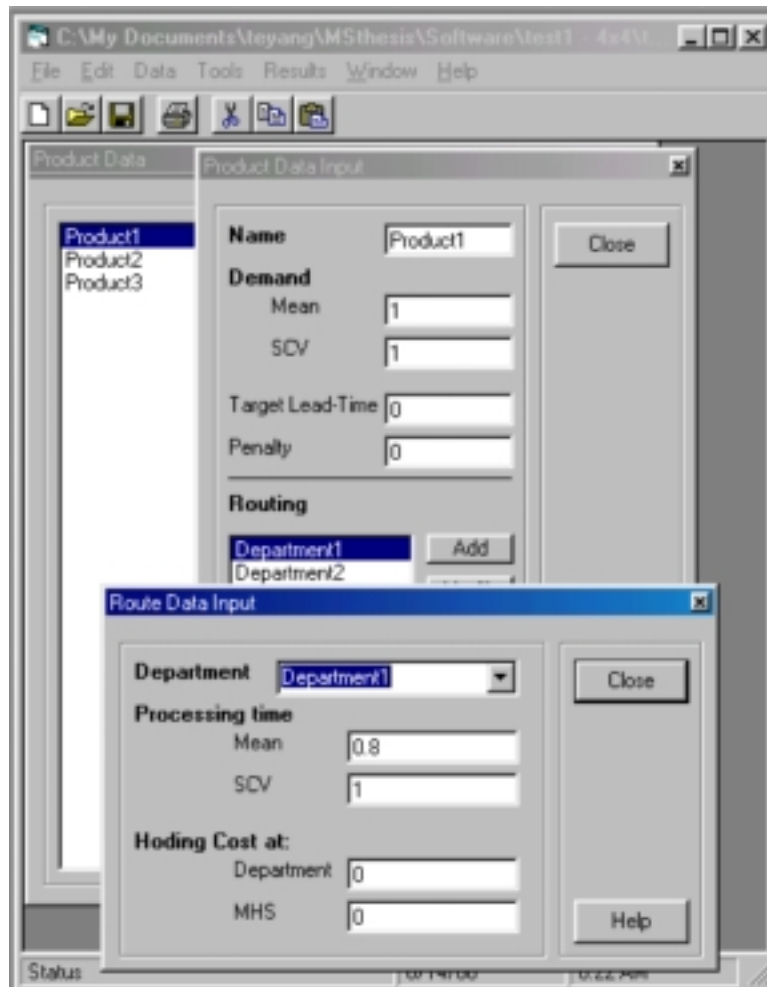
Figure 3 - Product Data Menu

Figure 4 - Facility Menu

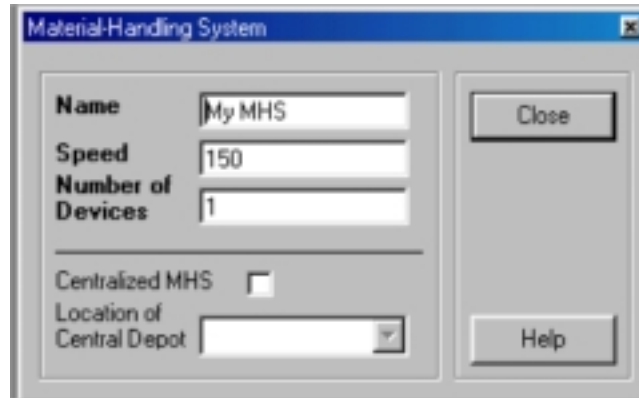## 4.4 Material-Handling System Data

The material-handling system is characterized by the following information:

- name,
- the type of material-handling (centralized or decentralized),
- number of devices available and,
- speed of devices.

The material-handling system can be either decentralized or centralized. In a decentralized system, the devices stay at the location of their last delivery. In the centralized system, the devices return to a central depot after delivery. By default, FLQUEUE assumes a decentralized material-handling system. An optional check box allows the user to specify a centralized material-handling system. In that case, the central depot needs to be defined, as one of the departments defined in the department menu. Figure 5 shows an example of this menu:
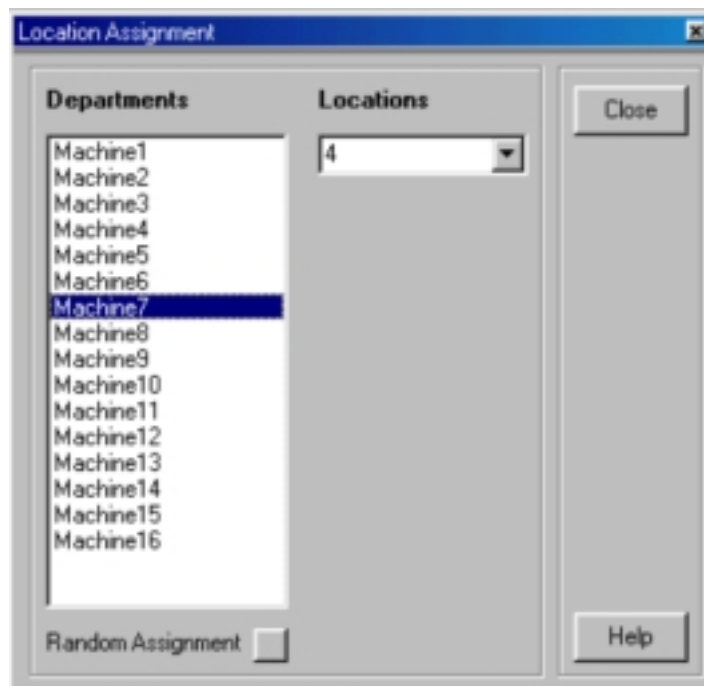
Figure 5 - Material-Handling System Menu

4..5 Layout Configuration Data

A layout configuration corresponds to an assignment of processing departments to locations. We need to specify the location in the facility where a department will be placed. Figure 6 illustrates how these assignments are specified:


Figure 6 - Location Assignment Menu

The left panel shows the list of departments defined using the department menu. For a selected department, we can use the location list on the right to select the location where it will be placed or we can let FLQUEUE generate randomly an assignment by pressing the button provided.

5. Performance Evaluation

FLQUEUE provides an easy way to obtain various operational performance measures of using the data menu. This tool can be accessed through the "tools" menu, then using "Run" command as shown in Figure 7 below.
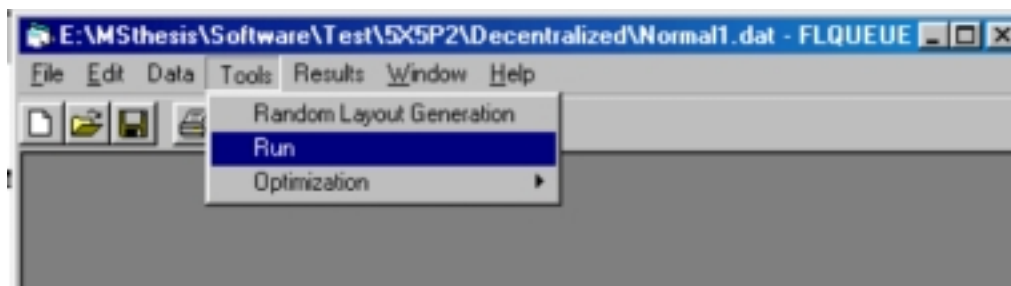

Figure 7 Operational Performance Evaluation

The results can be viewed using the "Results" menu. This menu contains several sub-menus, which group results by category. The categories include:

- results for departments,
- results for products,
- results for the material-handling system,
- results for the entire system,
- matrix of product flow,
- matrix of travel time between machines and,
- matrix of empty travel time between machines.

This menu is shown in Figure 8.
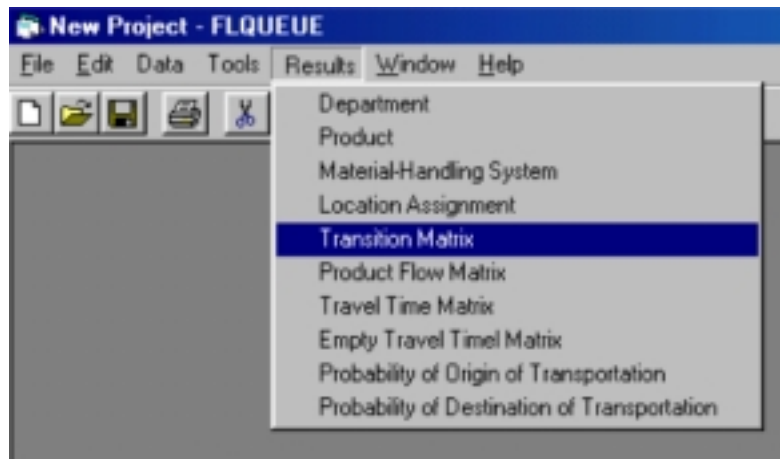
Figure 8 - Results Menu

5.1 Output for Departments

This sub-menu provides department-related performance. Two windows constitute this sub-menu, one providing the list of departments and the other providing department-related performance. The following information is provided:

- expected processing time at the department (mean and squared coefficient of variation),
- part arrival rate to this department,
- departure rate of parts leaving the system at this department,
- average utilization,
- expected waiting time in queue,
- expected flow time through this department,
- expected work-in-process at this department, and
- squared coefficient of variation of departure time from this department.

Figure 9 Output for Departments

An advantage of FLQUEUE is that we can use this information to identify the bottleneck in the system and see what factors could improve its performance. The usual factors are variability and department capacity among others. Figure 9 shows an example of this sub menu.

5. 2 Output for Products

This sub-menu displays two windows. The first one provides the list of products and the second one provides the performance measures for a selected product. Product-related operational performances are:

- expected flow time through the system,

- expected work-in-process of this product in the system,

- expected holding cost for this product,

- tardiness compared to the target lead-time and,

- operational performance measures at each operation of the product routing.

The operational performance measures at each operation of the product routing and material-handling system include the following:

- expected processing time of a product at this operation,

- holding cost per unit at each department,

- expected flow time through each department,

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- expected work-in-process in front of each department,

- expected total holding cost at each department,

- holding cost per unit at the material-handling system,

- expected travel time at the material-handling system,

- expected work-in-process at the material-handling system and,

- expected total holding cost at the material-handling system.

Figure 10 gives an illustration of this menu. Presenting the results for each individual product allows us to identify the effect of each product on the performance of the entire system. For example, using the above information we can check if a product costs too much in term of holding cost or contributes the most WIP.

5.3 Output for Material-Handling System

This sub-menu provides operational performance measures for the material-handling system. These measures include:

- expected travel time (mean and squared coefficient of variation),

- expected travel time experienced by the products being moved,

- part arrival rate,

- average utilization,

- expected waiting time in the material-handling system queue,

- expected flow time through the material-handling system,

- expected work-in-process at the material-handling system,

- squared coefficient of variation of the inter-departure time from the material-handling system.

Figure 11 shows an example of the menu.

Figure 10 - Output for Products

Figure 11 - Output for the Material-Handling System

5.4 Output for the Entire System

This sub-menu provides a summary of system operational performance measures, which include:

- material-handling system utilization,
- expected total work-in-process,
- average flow time and,
- average tardiness.

The average flow time displayed here is the average over all products' expected flow times. The average tardiness is the average over all products' tardiness. Figure 12 shows an example of this menu.

Figure 12 - Output for the Entire System

5.5 Other Outputs

The six other sub-menus provide information that can be used for checking the accuracy of the data input. This information concerns the rate of product flow, matrix of transitions, origin of transportation requests, destination of transportation requests, travel time and empty travel time between machines. Figure 13 shows these six menus:
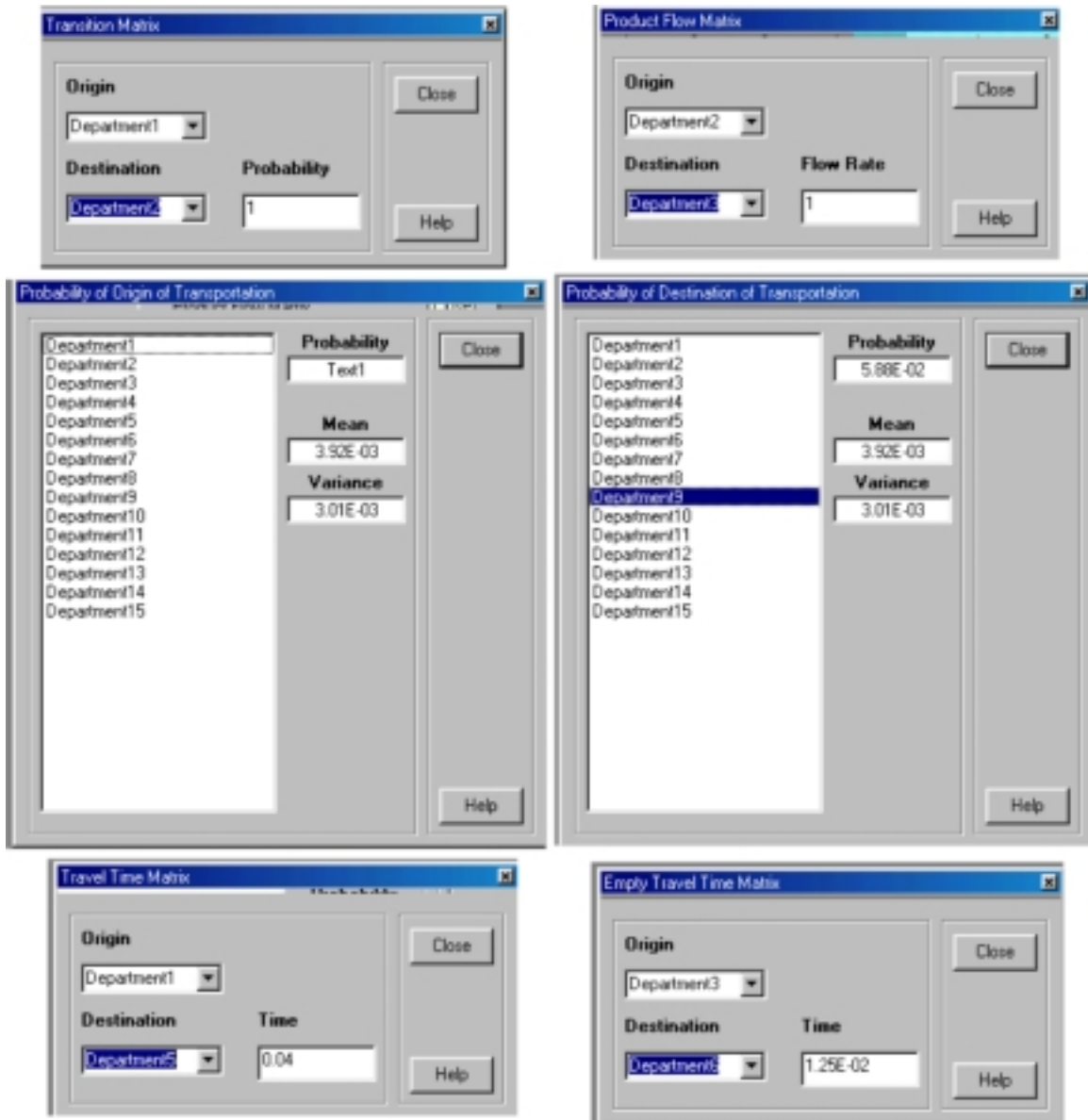
Figure 13 - Other Outputs

6. Optimization Tools

FLQUEUE offers several tools for optimizing layout designs. These tools can be accessed from the "tools" menu as shown in Figure 7. They include the following algorithms:

- layout design optimization using full enumeration,
- layout design optimization using a 2-opt exchange algorithm, and

- layout design optimization using a simulated annealing algorithm.

FLQUEUE allows layout optimization using various criteria of performance. They include the following:

- expected work-in-process,
- expected holding cost,
- expected tardiness,
- material-handling system utilization,
- material-handling system utilization due to empty travel and,
- material-handling system utilization due to full travel

The last criterion is identical to the criterion used in the traditional QAP formulation.

6.1 Full Enumeration Algorithm

This algorithm looks at all possible layouts. The best solution is saved and given at the end of the process. Since the algorithm considers all combinations, we are guaranteed an optimal solution. However this can be computing intensive for large systems. Therefore, it is only suitable for small problems. For larger problems, heuristic algorithms are more suitable. Figure 14 shows an example of the selection menu for this algorithm.
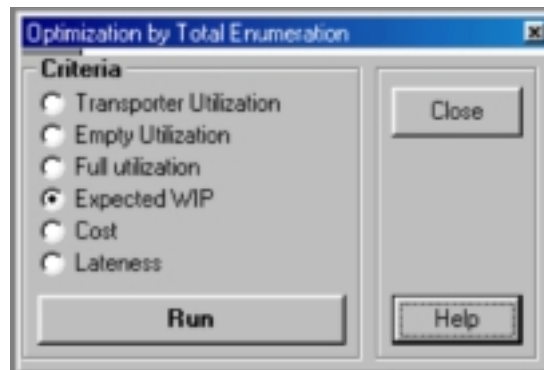
Figure 14 - Full Enumeration Optimization Menu

6.2 The 2-opt Exchange Algorithm

The 2-opt (or pairwise) exchange algorithm is a well-known algorithm. It considers two departments at a time for exchange as explained in the appendix. Figure 15 gives an illustration of the selection menu for this algorithm.
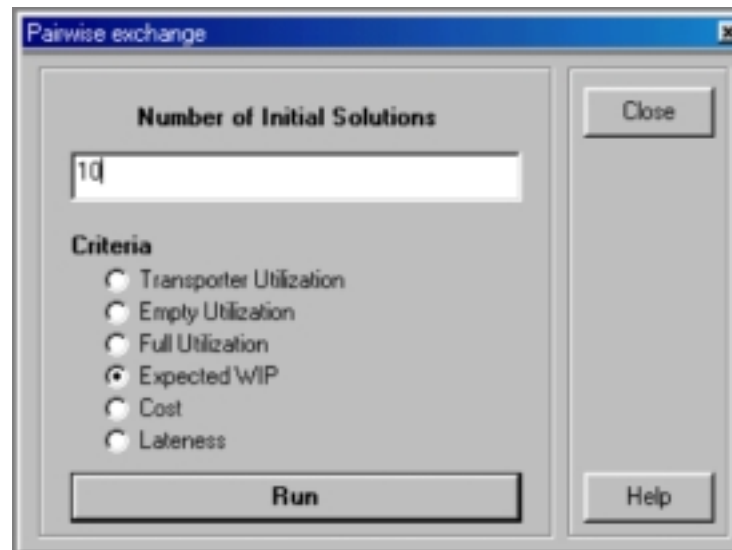
Figure 15 - Pairwise Exchange Algorithm Menu

Since the algorithm considers pairwise exchange and accepts only improved solutions, the optimal solution obtained at the end of the optimization process is affected by the initial solution. To maximize the probability of finding a good solution, the algorithm uses several randomly generated starting solutions. In this menu, the user is asked to provide the number of initial solutions to be generated by the algorithm.

6.3 The Simulated Annealing Algorithm

The simulated annealing algorithm provides a mean of introducing non-improving solutions in the optimization process. By doing so, we prevent the algorithm from getting stuck in a local optimum. The algorithm is controlled by the following parameters:
- initial temperature,
- final temperature,

- coefficient of reduction, and

- number of iterations at each temperature level.

The initial and final temperatures define the starting and stopping condition of the algorithm. The temperature controls how likely a no-improving solution will be accepted. The coefficient of reduction characterizes how fast the temperature will drop. The number of iterations defines the number of solutions that will be tested at each temperature. Figure 16 shows an example of the simulated annealing menu.
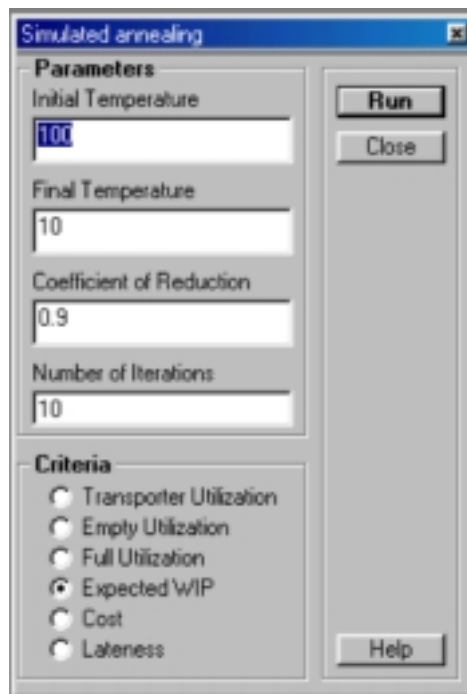
Figure 16 - Simulated Annealing Algorithm Menu

7. File Management

The data that is entered can be saved for later use. This option also allows saving different scenarios for comparison and analysis. Since it is easy to change, multiple scenarios can be generated without difficulty. Saving and restoring files can be accessed through the file menu as shown in Figure 1.

8. How to Create a New Layout Design Project

This section provides a step by step description for creating a new, including:

- manual data input or random layout generation,
- performance evaluation and layout optimization, and
- output analysis.

8.1 Data Input

From the file menu, choose **new** to create a new project. Then from the data menu, choose the appropriate sub-menu in order to enter information regarding:

- department data,
- product data,
- facility data,
- material-handling system data, and
- layout configuration information.

For the department data, enter successively each department by providing the corresponding specifications. The departments entered in this step will be used for product routing.

For the product data, enter each product with their specifications and routing. The part routing uses departments' information. Therefore, it is necessary to enter department data first.

For the facility data, the only information needed is the number of locations and the distances between them. An optional feature allows the user to input the dimensions. This feature allows, if used, FLQUEUE to generate automatically the distances using rectilinear distance between locations. This eliminates the need for inputting, one by one, distances between locations, which can be tedious if the number of locations is large. Nevertheless, the user still has the possibility to make changes to the distances by editing them one by one.

For the material-handling system, the user is required to choose the type, speed and number of devices.

For the layout configuration information, the user needs to have previously entered information regarding the departments and the locations. In this step, we need to assign each

department to a location and one location only. The objective of this step is to provide a layout solution that can be used in the performance evaluation menu and optimization menu.
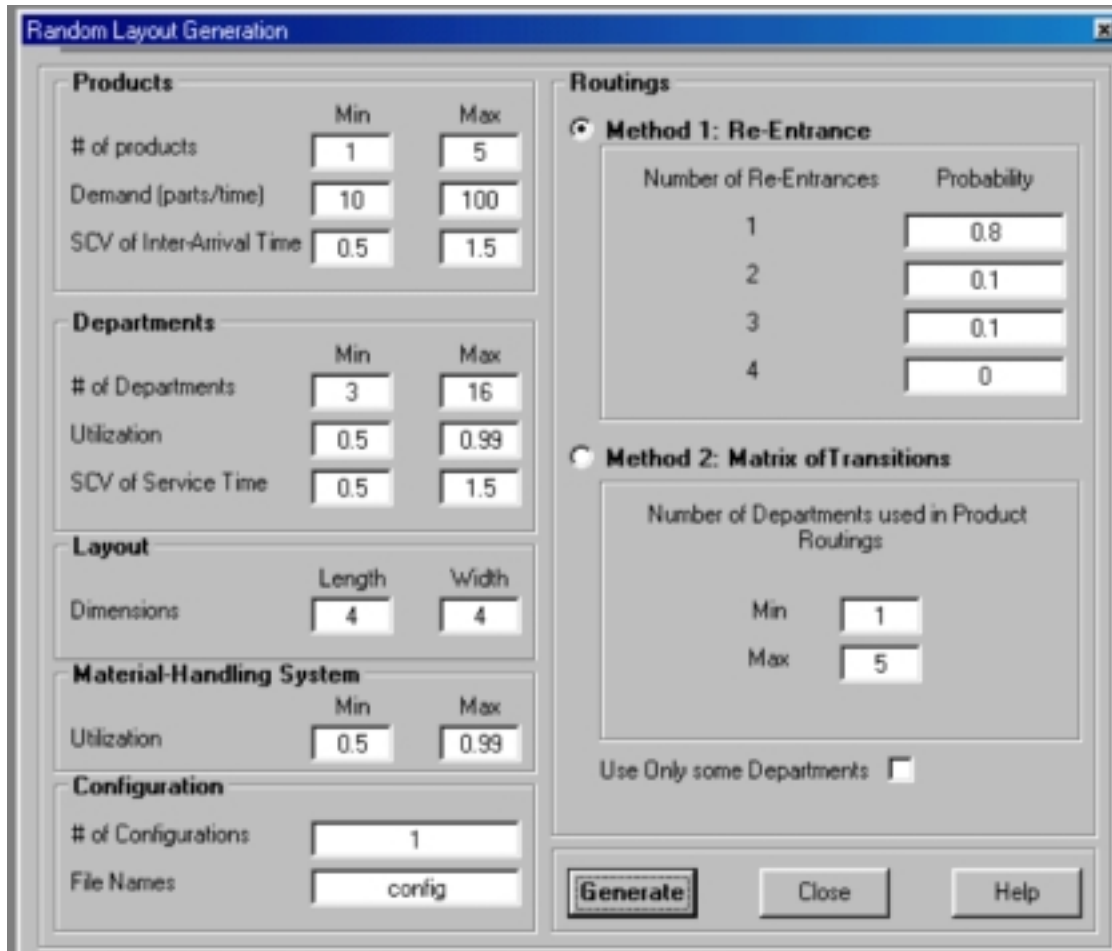
8.2 Random Layout Generation

The software provides a tool for randomly generating data for problem instances. This is useful for quickly generating data sets for numerical analysis. However, the use of this tool is optional and not required for running the software to solve a specific layout problem. The menu for this feature can be found under TOOLS then RANDOM LAYOUT GENERATION as illustrated in Figure 17. Figure 18 provides an illustration of the menu, which presents the different parameters that the user can specify. FLQUEUE generates the layout problem instances in accordance to these parameters.
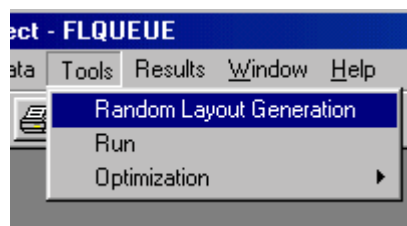
Figure 17 - Random Layout Generation Tool

FLQUEUE selects randomly the number of products and the number of processing departments between the intervals specified. Then for each product, demand is generated randomly from a uniform distribution ranging from these user-specified intervals. The number of times a product visits a department to which it has been assigned is either 1, 2, 3 or 4 with certain probabilities. These probabilities can be specified using one of two methods. The first method allows the user the probabilities with which a products visits the same a certain number of times. The second method uses a random transition matrix to generate the sequence. These two methods assume that all machines are equally likely to be chosen. However, if the *Use Only Some Departments* is checked, a certain number of departments will have a higher probability to be chosen in the sequence. These departments are chosen arbitrarily by the software. The software always ensure that the product does not immediately revisit the same department. The utilization of each

Figure 18 - Random Layout Generation Menu

department is determined by sampling randomly from a uniform distribution with minimum and maximum values specified by the user. From the utilization and the demands of all products that visit a department, we determine the department average processing time. For each department, the squared coefficient of variation is generated randomly from a specified range. FLQUEUE then generates an initial layout. The utilization of the transporter is randomly generated from a pre-specified range. Using the initial layout, it then calculates the corresponding transporter speed. FLQUEUE offers the possibility to generate multiple examples simultaneously. The user can specify the number of examples in the configurations box. The configurations generated are saved using a file name composed of the specified name to which an index is added. The files are saved under the current directory path.

8.3 Performance Evaluation and Layout Optimization

Completion of the data input step provides a layout solution for which we can obtain various measures of operational performance. This is available by selecting the tools menu and clicking on run. We can optimize the initial layout by selecting the optimization menu and choosing the algorithm of our choice.

9. Technical Appendix

In this appendix, we provide a technical description of the mathematical models and assumptions used by FLQUEUE. Additional discussion and examples can be found in Benjaafar [1].

9.1. Notation

The following notation is used throughout the thesis. Additional notation will be given as needed.

- λ_t: total arrival rate to the material-handling system.
- $C_{a_t}^2$: squared coefficient of variation of job inter-arrival time to the material-handling system.
- $E(S_t)$: expected transportation time.
- $C_{s_t}^2$: squared coefficient of variation of transportation time.
- m_t: number of material-handling devices.
- v: speed of the material-handling devices.
- ρ_t: utilization of the material-handling system.
- ρ_t^f: utilization of the material-handling system due to travel when full.
- ρ_t^e: utilization of the material-handling system due to travel when empty.
- $C_{d_t}^2$: squared coefficient of variation of inter-departure rate from the material-handling system.
- M: total number of processing departments.
- λ_i: total arrival rate to department i.

- $C_{a_i}^2$: squared coefficient of variation of job inter-arrival time at department i.

- $E(S_i)$: expected service time at department i.

- $C_{s_i}^2$: squared coefficient of variation of service time to department i.

- m_i: number of processors at department i.

- ρ_i: utilization of department i.

- $C_{d_i}^2$: squared coefficient of variation of inter-departure time from department i.

- N: total number of product types.

- D_j: demand for product type j.

- C_j^2: squared coefficient of variation of external inter-arrival time of product type j.

- $R_{jki} = \begin{cases} 1 & \text{if product } j \text{ visits department } i \text{ for its } k^{th} \text{ operation} \\ 0 & \text{otherwise.} \end{cases}$

- $E(S_{jk})$: expected processing time for operation k of product j.

- Cs_{jk}^2: squared coefficient of variation of processing time for operation k of product j.

- n_j: total number of operations for product j.

- λ_{ij}: rate of flow from department i to department j.

- λ_{ti}: rate of flow from the material-handling system to processing department i.

- $C_{d_{t,i}}^2$: squared coefficient of variation of job inter-arrival time from the material-handling system to processing department i.

- $\lambda_{out,i}$: rate of external arrivals to department i.

- $C_{a_{out,j}}^2$: squared coefficient of variation of external job inter-arrival time at department j.

- λ_{it}: rate of flow from department i to the material-handling system.

- $C_{d_{it}}^2$: squared coefficient of variation of job inter-arrival time to the material-handling system from department i.

- $\lambda_{i,out}$: rate of flow from department i to outside the system.

- $C_{d_{j,out}}^2$: squared coefficient of variation of inter-departure time from department i to outside the system.

- $X=\{x_{ik}\}$, where $x_{ik} = \begin{cases} 1 & \text{if department } i \text{ is assigned to location } k \\ 0 & \text{otherwise.} \end{cases}$

- d_{kl}: distance between locations k and l.

- t_{ij}: travel time between departments i and j.

9.2. Model Description

We consider a plant with N products. Product demands are independently distributed random variables characterized by an average demand D_i and a squared coefficient of variation C_i^2 for $i=1, 2,.., N$. The squared coefficient of variation denotes the ratio of the squared mean over the variance. Each product follows its own routing. Products can be released to the plant at any of the M departments and exit the plant through any department. Depending on its routing, a product can visit the same department several times and with a different processing time during each visit. The rate of flow from department i to department j, denoted by λ_{ij}, is determined from the part routing.

The plant consists of M processing departments, with each department consisting of one or more processors with ample storage for work-in-process (WIP). Jobs in the queue are processed according to a First-Come-First-Served (FCFS) policy. The processing time for product i at operation k of its routing is a random variables with mean $E(S_{ik})$ and squared coefficient of variation $C_{s_{ik}}^2$. We refer to $E(S_i)$ and $C_{s_i}^2$ as the resulting average processing time and resulting squared coefficient of variation for department i respectively.

The material-handling system consists of multiple discrete devices such as automated guided vehicles (AGV), forklift trucks or overhead cranes. Material transfer requests are serviced in a First-Come-First-Served (FCFS) fashion. We will assume that all devices move at speed v and vehicle congestion is assumed to be insignificant. Hence the travel time between any pair of locations k and l, is assumed to be deterministic and is given by d_{kl}/v, where d_{kl} is the shortest distance traveled by a device between locations k and l.

Each processing department can be assigned to only one location in the factory. Hence a layout configuration corresponds to a unique assignment of departments to locations. We use the vector notation $X=\{x_{ik}\}$ to represent a layout configuration, where x_{ik} takes a value of 1 if department i is assigned to location k and a value of 0 otherwise.

The facility is divided into L areas, which will serve as locations for processing departments. Each location is able to receive at most one processing department. We define the distance matrix $\{d_{kl}\}$, which represents the shortest distance traveled by a material-handling device from location k to l.

In order to analyze the system, we will treat the processing departments and the material-handling system as being stochastically independent nodes and model each node as a *GI/G/m* queue, with the material-handling system playing the role of a central server queue (see figure 1).

Exact analysis of a network of *GI/G/m* queues is difficult. Therefore, we use well-known decomposition and approximation techniques, where each department, as well as the material-handling system, is treated as being stochastically independent, with the arrival process to and the departure process from each department and the material-handling system being approximated by renewal processes. We will evaluate the performance of the network by considering the first two moments, mean and squared coefficient of variation, of the inter-arrival time and processing time distributions.

9.3 Flow Analysis

The rate of flow from department i to department j, λ_{ij}, can be obtained from the product routings as follows:

$$\lambda_{ij} = \sum_{k=1}^{N} \sum_{l=1}^{n_k-1} D_k R_{kli} R_{k,l+1,j} \quad i = 1,..,M \quad j = 1,..,M . \tag{1}$$

Since all products have to be transported, the arrival rate to the material-handling system, λ_t, is given by:
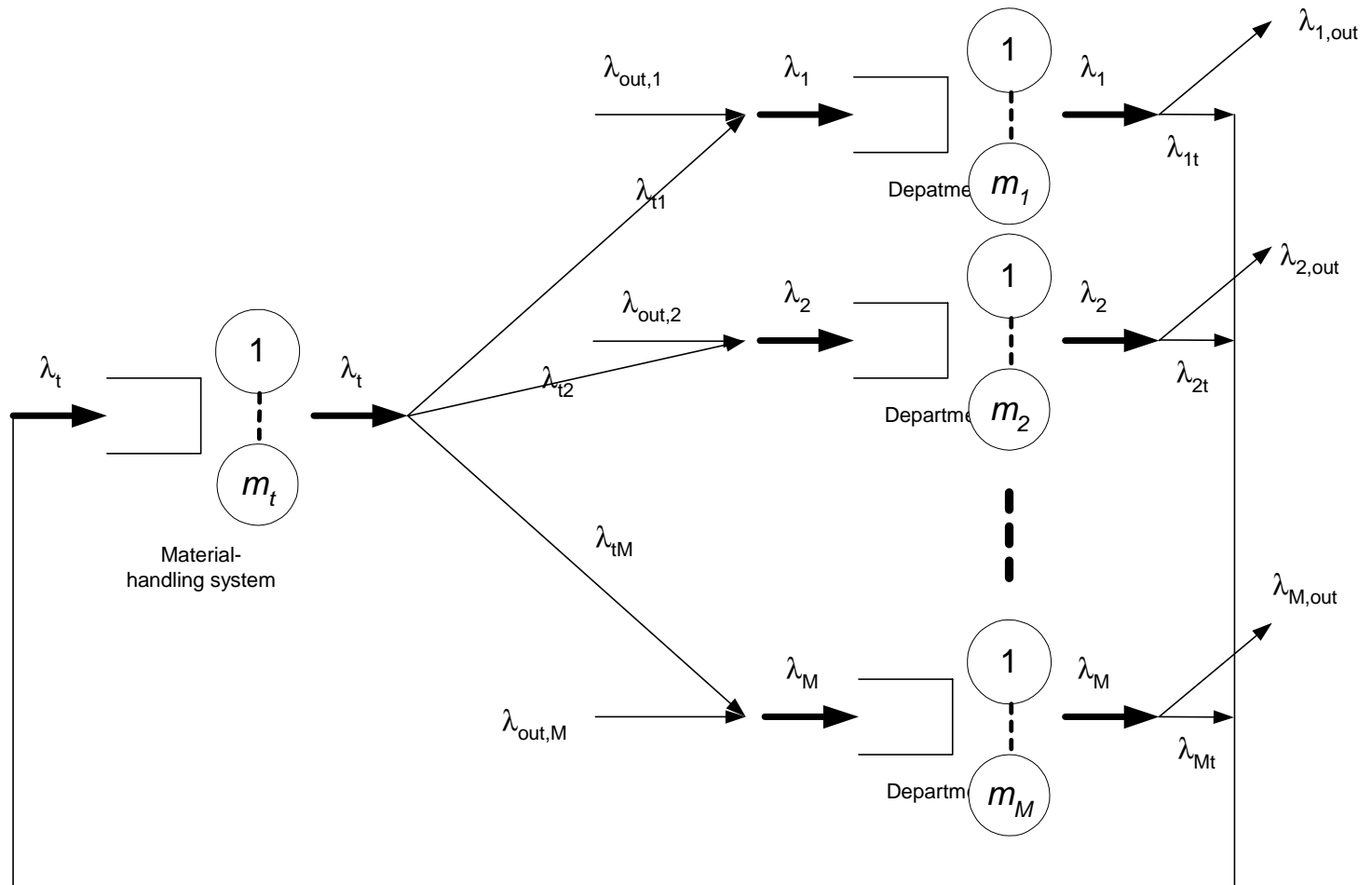
$$\lambda_t = \sum_{i=1}^{M} \sum_{j=1}^{M} \lambda_{ij} , \tag{2}$$

which can be also rewritten as:

$$\lambda_t = \sum_{i=1}^{M} \lambda_{it} , \tag{3}$$

where:

$$\lambda_{it} = \sum_{j=1}^{M} \lambda_{ij} \tag{4}$$

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Figure 1 The Queueing Network Model

and stands for the rate of flow from department i to the material-handling system. Similarly, the arrival rate to department i, λ_i, can be obtained as:

$$\lambda_i = \lambda_{out,i} + \sum_{j=1}^{M} \lambda_{ji} = \lambda_{out,i} + \lambda_{ti} \quad i=1,..,M \ , \tag{5}$$

where λ_{ti} represents the rate of flow from the material-handling system to department i and $\lambda_{out,i}$ is the external rate of flow that enters the system at department i. The two rates can be calculated as follows:

$$\lambda_{out,i} = \sum_{j=1}^{N} D_j R_{j1i} \quad i=1,..,M \tag{6}$$

and

$$\lambda_{ti} = \sum_{j=1}^{M} \lambda_{ji} \quad i=1,..,M \ . \tag{7}$$

The total external arrival rate to the system as a whole is given by:

$$\lambda_{out} = \sum_{i=1}^{M} \lambda_{out,i} \quad i=1,..,M \tag{8}$$

The expected service time at department i is given by:

$$E(S_i) = \frac{\sum\limits_{j=1}^{N}\sum\limits_{k=1}^{n_j} D_j R_{jki} E(S_{jk})}{\sum\limits_{j=1}^{N}\sum\limits_{k=1}^{n_j} D_j R_{jki}} \quad i=1,..,M \ , \tag{9}$$

and the corresponding squared coefficient of variation of service time at department i, $C_{s_i}^{2}$, is obtained by:

$$C_{s_i}^{2} = \frac{1}{\lambda_i E(s_i)^2} \sum_{j=1}^{N}\sum_{k=1}^{n_j} D_j R_{jki} E(s_{jk})^2 \left(1+C_{s_{jk}}^{2}\right) - 1 \quad i=1,..,M \ . \tag{10}$$

In the following discussion, we assume that the mean and squared coefficient of variation of travel time ($E(S_t)$ and $C_{S_t}^{2}$) are known. In section 3.5, we describe how these two parameters can be calculated for different material-handling system and layout configurations.

The squared coefficient of variation of inter-arrival time to the material-handling system, $C_{a_t}{}^2$, can be approximated as follows [3][4]:

$$C_{a_t}{}^2 = \sum_{i=1}^{M} \left(\frac{\lambda_{it}}{\lambda_t} \right) C_{d_{it}}{}^2 ,$$ (11)

where $C_{d_{it}}{}^2$ is the square coefficient of the inter-arrival time from department i to the material-handling system and is given by:

$$C_{d_{it}}{}^2 = \frac{\lambda_{it}}{\lambda_i} C_{d_i}{}^2 + 1 - \frac{\lambda_{it}}{\lambda_i} \quad i = 1,..,M .$$ (12)

The squared coefficients of variation of inter-arrival time, $C_{a_i}{}^2$, and external inter-arrival time, $C_{a_{out,i}}{}^2$, to department i can be obtained as follows:

$$C_{a_i}{}^2 = \frac{\lambda_{out,i} C_{a_{out,i}}{}^2 + \lambda_{ti} C_{a_{ti}}{}^2}{\lambda_i} \quad i = 1,..,M ,$$ (13)

where

$$C_{a_{out,i}}{}^2 = \sum_{j=1}^{N} \left(\frac{D_j R_{j1i}}{\sum_{l=1}^{N} D_l R_{l1i}} \right) C_j{}^2 \quad i = 1,..,M .$$ (14)

The squared coefficients of variation of inter-departure time at department i and at the material-handling system are approximated as follows [3][4]:

$$C_{d_t}{}^2 = 1 + (1 - \rho_t{}^2)(C_{a_t}{}^2 - 1) + \frac{\rho_t{}^2}{\sqrt{m_t}}(C_{s_t}{}^2 - 1),$$ (15)

and

$$C_{d_i}{}^2 = 1 + (1 - \rho_i{}^2)(C_{a_i}{}^2 - 1) + \frac{\rho_i{}^2}{\sqrt{m_i}}(C_{s_i}{}^2 - 1) \quad i = 1,..,M .$$ (16)

The squared coefficient of variation of inter-departure time from the material-handling system to department i, $C_{a_{ti}}{}^2$, can be approximated as follows [3][4]:

$$C_{a_{ti}}{}^2 = p_i C_{d_t}{}^2 + 1 - p_i \quad i = 1,..,M ,$$ (17)

where p_i is given by:

$$p_i = \frac{\lambda_{ti}}{\lambda_t} \quad i = 1,..,M \ . \tag{18}$$

Using equations (13), (14), (15), (16), (17) and (18), we can now solve for the squared coefficients of inter-arrival time to the material-handling system, Ca_t^2, and to each department i, Ca_i^2, as follows:

$$Ca_t^2 = \frac{A + B + C + D + E}{1 - F} , \tag{19}$$

where

$$\begin{cases} A = \sum_{i=1}^{M} \left(1 - \frac{\lambda_{it}}{\lambda_i}\right)\frac{\lambda_{it}}{\lambda_t} \\[2mm] B = \sum_{i=1}^{M} \left(\frac{\lambda_{it}^2}{\lambda_t \lambda_i}\right)\left(1 + \frac{1}{\sqrt{m_i}}(Cs_i^2 - 1)\right)\rho_i^2 \\[2mm] C = \sum_{i=1}^{M} \left(\frac{\lambda_{it}^2 \lambda_{out,i}}{\lambda_t \lambda_i^2}\right)(1 - \rho_i^2)Ca_{out,i}^2 \\[2mm] D = \sum_{i=1}^{M} \left(\frac{\lambda_{it}^2 \lambda_{ti}}{\lambda_t \lambda_i^2}\right)(1 - \rho_i^2)(1 - p_i) \\[2mm] E = \sum_{i=1}^{M} \left(\frac{\lambda_{it}^2 \lambda_{ti}}{\lambda_t \lambda_i^2}\right)\left(1 + \frac{1}{\sqrt{m_t}}(C_T^2 - 1)\right)(1 - \rho_i^2)\rho_t^2 p_i \\[2mm] F = \sum_{i=1}^{M} \left(\frac{\lambda_{it}^2 \lambda_{ti}}{\lambda_t \lambda_i^2}\right)(1 - \rho_i^2)(1 - \rho_t^2)p_i \end{cases}, \tag{20}$$

and

$$Ca_i^2 = G + H + ICa_t^2 \quad i = 1,..,M , \tag{21}$$

where

$$\begin{cases} G = \frac{\lambda_{out,i}Ca_{out,i}^2}{\lambda_i} + \frac{\lambda_{ti}}{\lambda_i}(1 - p_i) \\[2mm] H = \frac{\lambda_{ti}}{\lambda_i} p_i \rho_t^2 \left(1 + \frac{C_T^2 - 1}{\sqrt{m_t}}\right) \\[2mm] I = \frac{\lambda_{ti}}{\lambda_i} p_i (1 - \rho_t^2) \end{cases}. \tag{22}$$

9.4 Performance Evaluation

The expected flow times at each department i and at the material-handling system can, respectively, be approximated as follows [3][4]:

$$E(F_i) = \left(\frac{C_{a_i}^2 + C_{s_i}^2}{2} \right) \left(\frac{(m_i \rho_i)^{m_i} P_{0i} \rho_i}{m_i!(1 - \rho_i)^2} \right) \frac{1}{\lambda_i} + E(S_i) \quad i = 1,..,M \tag{23}$$

and

$$E(F_t) = \left(\frac{C_{a_t}^2 + C_{s_t}^2}{2} \right) \left(\frac{(m_t \rho_t)^{m_t} P_{0t} \rho_t}{m_t!(1 - \rho_t)^2} \right) \frac{1}{\lambda_t} + E(S_t), \tag{24}$$

where:

$$P_{0i} = \frac{1}{\frac{(m_i \rho_i)^{m_i}}{m_i!(1 - \rho_i)} + \sum_{n=0}^{m_i-1} \frac{(m_i \rho_i)^n}{n!}} \quad i = 1,..,M \tag{25}$$

$$P_{0t} = \frac{1}{\frac{(m_t \rho_t)^{m_t}}{m_t!(1 - \rho_t)} + \sum_{n=0}^{m_t-1} \frac{(m_t \rho_t)^n}{n!}}. \tag{26}$$

Using Little's law, expected work-in-process at the processing departments and material-handling system are then, respectively, given by:

$$E(WIP_i) = \lambda_i E(F_i) \quad i = 1,..,M \tag{27}$$

and

$$E(WIP_t) = \lambda_t E(F_t). \tag{28}$$

We can now estimate total expected work-in-process in the system as:

$$E(WIP) = E(WIP_t) + \sum_{i=1}^{M} E(WIP_i), \tag{29}$$

and by applying Little's law, we can obtain expected flow time:

$$E(F) = \frac{E(WIP)}{\lambda_{out}}, \tag{30}$$

where λ_{out} is given by equation (9).

The average utilization at each department i and at the material-handling system are, respectively, given by:

$$\rho_i = \frac{\lambda_i E(S_i)}{m_i} \quad i = 1,.., M \tag{31}$$

and

$$\rho_t = \frac{\lambda_t E(S_t)}{m_t}, \tag{32}$$

which we shall assume to be less than one, i.e. $\rho_i < 1$ and $\rho_t < 1$.

Similarly, we can estimate operational performance measures for each product and at each stage of its completion. The utilization of a processing department i due to operation k of product j is given by:

$$\rho_{jki} = D_j \sum_{i=1}^{M} R_{jki} \frac{E(S_{jk})}{m_i} \quad j = 1,.., N \quad k = 1,.., n_j \tag{33}$$

The expected flow time of product j during its kth operation at department i is given by:

$$E(F_{jki}) = \left(E(W_i) + E(S_{jk})\right) R_{jki} \quad j = 1,.., N \quad k = 1,.., n_j, \tag{34}$$

where, $E(W_i)$ is given by:

$$E(W_i) = \left(\frac{C_{a_i}^2 + C_{s_i}^2}{2}\right) \left(\frac{(m_i \rho_i)^{m_i} P_{0i} \rho_i}{m_i!(1 - \rho_i)^2}\right) \frac{1}{\lambda_i} \quad i = 1,.., M, \tag{35}$$

and the expected work-in-process of product j during operation k at department i is:

$$E(WIP_{jki}) = D_j E(F_{jki}) \quad j = 1,.., N \quad k = 1,.., n_j. \tag{36}$$

Since each product has to be transported after each operation, we can also estimate product and operation specific performance measures for the material-handling system. The average utilization of the material-handling system due to product j after its kth operation is:

$$\rho_{jkt} = \frac{D_j}{m_t} \sum_{u=1}^{M} \sum_{v=1}^{M} R_{jku} R_{j,k+1,v} \left(t_{uv}^e + t_{uv}^f\right) \quad j = 1,.., N \quad k = 1,.., n_j - 1, \tag{37}$$

where t_{uv}^e and t_{uv}^f are defined in section 3.5. Note that we do not include operation m_j since the product exits the plant without using the material-handling system. The expected flow time through the material-handling system of product j after its kth operation is given by:

$$E(F_{jkt}) = E(W_t) + \sum_{u=1}^{M} \sum_{v=1}^{M} R_{jku} R_{jkv} \left(t_{uv}^e + t_{uv}^f\right) \quad j = 1,.., N \quad k = 1,.., n_j - 1, \tag{38}$$

where:

$$E(W_t) = \left(\frac{C_{a_t}^2 + C_{s_t}^2}{2} \right) \left(\frac{(m_t \rho_t)^{m_t} P_{0t} \rho_t}{m_t!(1-\rho_t)^2} \right) \frac{1}{\lambda_t}. \quad (39)$$

From which we can obtain the corresponding work-in-process as:

$$E(WIP_{jkt}) = D_j * E(F_{jkt}) \quad j = 1,.., N \quad k = 1,.., n_j - 1 \quad (40)$$

The expected flow time and the expected work-in-process for each product can now be calculated as:

$$E(F_j) = \sum_{k=1}^{n_j-1} \left(\sum_{i=1}^{M} E(F_{jki}) + E(F_{jkt}) \right) + \sum_{i=1}^{M} E(F_{jn_j i}) \quad j = 1,.., N \quad (41)$$

and

$$E(WIP_j) = \sum_{k=1}^{n_j-1} \left(\sum_{i=1}^{M} E(WIP_{jki}) + E(WIP_{jkt}) \right) + \sum_{i=1}^{M} E(WIP_{jn_j i}) \quad j = 1,.., N. \quad (42)$$

The benefit of using product-specific performance measures is the ability to examine the impact of a layout configuration on each product. This is important for systems where different products have different priorities. We are also able to differentiate work-in-process at different stages of the manufacturing process and assign different holding costs for different stages and different products. In many applications, the value of a product appreciates significantly as more operations are completed. In these cases, it is important to optimize the performance at the later stages of the manufacturing process. This can be easily accommodated by assigning, for example, individual holding costs for each product at each operation. Our measures can then be rewritten as follows:

$$E(F_j)^c = \sum_{k=1}^{n_j-1} \left(\sum_{i=1}^{M} h_{jki} E(F_{jki}) + h_{jkt} E(F_{jkt}) \right) + \sum_{i=1}^{M} h_{jn_j i} E(F_{jn_j i}) \quad j = 1,.., N \quad (43)$$

and

$$E(WIP_j)^c = \sum_{k=1}^{n_j-1} \left(\sum_{i=1}^{M} h_{jki} E(WIP_{jki}) + h_{jkt} E(WIP_{jkt}) \right) + \sum_{i=1}^{M} h_{jn_j i} E(WIP_{jn_j i}) \quad j = 1,.., N, \quad (44)$$

where:

- h_{jki} is the holding cost for product j before operation k at department i.
- h_{jkt} is the holding cost for product j after operation k at the material-handling system.

9.5 Distribution of Transportation Times

In the previous section, we assumed that the distribution of travel time is known. In this section, we characterize this distribution for two cases:

- the decentralized material-handling system case, and
- the centralized material-handling system case.

9.5.1 The Decentralized Material-Handling System Case

The material-handling system is composed of a set of discrete devices. Requests for transportation are processed on a First-Come-First-First (FIFO) basis. When more than one vehicle is available, a vehicle is chosen randomly among all available vehicles to serve the current request. The vehicle moves from its current position to the department where the request originated and then transports the job to the next department on its routing. In the absence of any requests, the vehicles remain at the location of their last delivery. Therefore, with each request, there is an empty trip from the vehicle's current position to the origin of the request followed by a full trip from the origin of the request to the destination of the request.

We use the methodology described in Benjaafar [1] to estimate expected service time at the material-handling system. Given a layout configuration $X=\{x_{ik}\}$, the time to perform a trip from department i to department j is given by:

$$t_{ij}(x) = \sum_{k=1}^{L}\sum_{l=1}^{L} x_{ik} x_{jl} \frac{d_{kl}}{v}. \tag{45}$$

The probability of a full trip from department i to department j is given by:

$$p_{ij} = \frac{\lambda_{ij}}{\lambda_t}, \tag{46}$$

with λ_{ij} and λ_t are defined, respectively, by (1) and (2). Therefore, the mean and variance of travel time due to full travel is:

$$E(S_t^{f}) = \sum_{i=1}^{M}\sum_{j=1}^{M} p_{ij} t_{ij}(X) = \sum_{i=1}^{M}\sum_{j=1}^{M} \frac{\lambda_{ij}}{\lambda_t} t_{ij}(X) \tag{47}$$

and

$$Var(S_t^{f}) = E((S_t^{f})^2) - E(S_t^{f})^2, \tag{48}$$

where,

$$E((S_t^f)^2) = \sum_{i=1}^{M} \sum_{j=1}^{M} p_{ij}(t_{ij}(X))^2 = \sum_{i=1}^{M} \sum_{j=1}^{M} \frac{\lambda_{ij}}{\lambda_t}(t_{ij}(X))^2 .\tag{49}$$

The probability of having a transfer request from department j is:

$$p_j' = \sum_{k=1}^{M} \frac{\lambda_{jk}}{\lambda_t} ,\tag{50}$$

and the probability that the destination of a material-transfer request is department i is:

$$p_i = \sum_{k=1}^{M} \frac{\lambda_{ki}}{\lambda_t} .\tag{51}$$

This probability can also be interpreted as the probability of the material-handling devices being located at department i. Let π_i be the probability of an empty trip originating in i, n_i denotes the number of transporters that are idle at department i and n_s the total number of idle transporters in the system. If a transporter is selected randomly among those that are idle to carry out the current material handling, then the probability that an empty trip originates from i given n_i idle transporters at department i and a total n_s idle transporters in the system ($n_i=1,..,n_s$ and $n_s=1,..,m_t$) is given by:

$$\pi_{i|n_i,n_s} = \frac{n_i}{n_s}\binom{n_s}{n_i}p_i^{n_i}(1-p_i)^{n_s-n_i} .\tag{52}$$

The probability of an empty trip originating in department i can now be obtained as:

$$\pi_i = \sum_{n_s=1}^{n} \sum_{n_i}^{n_s} \pi_{i|n_i,n_s} p(n_s) .\tag{53}$$

Noting that

$$\pi_i = \sum_{n_s=1}^{n} \frac{p(n_s)}{n_s} \sum_{n_i}^{n_s} n_i \binom{n_s}{n_i}p_i^{n_i}(1-p_i)^{n_s-n_i}\tag{54}$$

and since

$$\sum_{n_i}^{n_s} n_i \binom{n_s}{n_i}p_i^{n_i}(1-p_i)^{n_s-n_i} = n_s p_i ,\tag{55}$$

we have

$$\pi_i = p_i\tag{56}$$

We can now calculate the mean and variance of travel time due to empty travel as follows:

$$E(S_t^e) = \sum_{i=1}^{M} \sum_{j=1}^{M} p_i p_j' t_{ij}(X) \tag{57}$$

and

$$Var(S_t^e) = E((S_t^e)^2) - E(S_t^e)^2, \tag{58}$$

where,

$$E((S_t^e)^2) = \sum_{i=1}^{M} \sum_{j=1}^{M} p_i p_j' (t_{ij}(X))^2. \tag{59}$$

Hence, the expected service time at the material-handling system is:

$$E(S_t) = E(S_t)^f + E(S_t)^e, \tag{60}$$

which can be rewritten as follows:

$$E(S_t) = \sum_{i=1}^{M} \sum_{j=1}^{M} \frac{\lambda_{ij}}{\lambda_t} t_{ij}(X) + \sum_{i=1}^{M} \sum_{j=1}^{M} \left(\sum_{k=1}^{M} \frac{\lambda_{ki}}{\lambda_t} \right) \left(\sum_{l=1}^{M} \frac{\lambda_{jl}}{\lambda_t} \right) t_{ij}(X). \tag{61}$$

The variance of the expected travel time is given by:

$$Var(S_t) = Var(S_t^f) + Var(S_t^e). \tag{62}$$

We can obtain the utilization of the material-handling system as follows:

$$\rho_t = \frac{\lambda_t E(S_t)}{m_t} = \frac{1}{m_t} \left(\sum_{i=1}^{M} \sum_{j=1}^{M} \lambda_{ij} t_{ij}(X) + \frac{1}{\lambda_t} \sum_{i=1}^{M} \sum_{j=1}^{M} \left(\sum_{k=1}^{M} \lambda_{ki} \right) \left(\sum_{l=1}^{M} \lambda_{jl} \right) t_{ij}(X) \right), \tag{63}$$

or equivalently:

$$\rho_t = \rho_t^f + \rho_t^e, \tag{64}$$

where

$$\rho_t^f = \sum_{i=1}^{M} \sum_{j=1}^{M} \frac{\lambda_{ij} t_{ij}(X)}{m_t} \tag{65}$$

corresponds to the utilization of the material-handling system due to full travel, and

$$\rho_t^e = \frac{1}{m_t \lambda_t} \sum_{i=1}^{M} \sum_{j=1}^{M} \left(\sum_{k=1}^{M} \lambda_{ki} \right) \left(\sum_{l=1}^{M} \lambda_{jl} \right) t_{ij}(X) \tag{66}$$

is the utilization of the material-handling system due to empty travel.

We also define t_{ij}^e as the average empty travel time made by a material handling device when processing a transportation request from department i to j, and similarly t_{ij}^f as the average full travel time. These two parameters are given by:

$$t_{ij}^{\ e} = \sum_{u=1}^{M} p_u t_{ui} \tag{67}$$

and,

$$t_{ij}^{\ f} = t_{ij} . \tag{68}$$

9.5.2 The Centralized Material-Handling System Case

In this case, we assume that there is a central depot for the material-handling system. This means that vehicles always return to the central depot after making a delivery. Therefore, with each request, there is an empty trip from the depot to the origin of the request followed by a full trip from the origin of request to the destination of request. After delivery, the transporter returns empty to the depot.

So given a layout configuration $X=\{x_{ik}\}$, the first and the second moments of transporter travel time are given respectively by the following:

$$E(S_t) = \sum_{i=1}^{M} \sum_{j=1}^{M} p_{ij} \left(t_{depot,i}(X) + t_{ij}(X) + t_{j,depot}(X) \right), \tag{69}$$

and

$$E\left(S_t^{\ 2}\right) = \sum_{i=1}^{M} \sum_{j=1}^{M} p_{ij} \left(t_{depot,i}(X) + t_{ij}(X) + t_{j,depot}(X) \right)^2 . \tag{70}$$

The expected travel time can be reformulated as follows:

$$E(S_t) = \sum_{i=1}^{M} p_i t_{depot,i}(X) + \sum_{i=1}^{M} \sum_{j=1}^{M} p_{ij} t_{ij}(X) + \sum_{j=1}^{M} p_j' t_{j,depot}(X), \tag{71}$$

where p_{ij}, p_j' and p_i are given respectively by equations (46), (50) and (51). From which we can derive the expected travel time due to full travel as:

$$E(S_t^{\ f}) = \sum_{i=1}^{M} \sum_{j=1}^{M} p_{ij} t_{ij}(X), \tag{72}$$

and the expected travel time due to empty travel as:

$$E(S_t^{\ e}) = \sum_{i=1}^{M} p_i t_{depot,i}(X) + \sum_{j=1}^{M} p_j' t_{j,depot}(X). \tag{73}$$

Note that from the point of view of the material being transferred, the empty-trips made by the material-handling devices after delivery have no impact on its travel time. However, this additional empty travel affects waiting time at the material-handling system queue since

$\rho_t=\lambda_t E(S_t)/m_t$. In order to account for this difference, expected flow time through the material-handling system is calculated as follows:

$$E(F_t) = \left(\frac{C_{a_t}^2 + C_{s_t}^2}{2} \right) \left(\frac{(m_t \rho_t)^{m_t} P_{0t} \rho_t}{m_t!(1-\rho_t)^2} \right) \frac{1}{\lambda_t} + \sum_{i=1}^{M} \sum_{j=1}^{M} p_{ij} \left(t_{depot,i}(X) + t_{ij}(X) \right). \quad (74)$$

The two parameters, t_{ij}^e and t_{ij}^f, are given by:

$$t_{ij}^{e} = t_{depot,i} + t_{j,depot} \quad (75)$$

and,

$$t_{ij}^{f} = t_{ij}. \quad (76)$$

9.6 Layout Design Formulation

The previous performance measures can now be used to reformulate the layout design problem so that operational performance measures other than the simple material handling cost measure of the quadratic assignment problem (QAP) formulation are used.

Let us first restate the QAP formulation in terms of our notation. Given a layout configuration $X=\{x_{ij}\}$, the QAP can be formulated as follows:

$$\text{Minimize} \qquad \sum_{i}^{M} \sum_{j}^{M} \sum_{k}^{L} \sum_{l}^{L} x_{ik} x_{jk} \lambda_{ij} d_{kl} \quad (77a)$$

Subject to:

$$\sum_{k=1}^{L} x_{ik} = 1 \quad i = 1,..,M \quad (77b)$$

$$\sum_{i}^{M} x_{ik} = 1 \quad k = 1,..,L \quad (77c)$$

$$x_{ik} = 0,1 \quad i = 1,..,M \quad k = 1,..,L \quad (77d)$$

The objective function (73a) minimizes material handling cost by minimizing average distance traveled by an arbitrary unit load of material. Since our model deals with discrete material-handling devices, this QAP formulation also minimizes the average distance traveled by the devices when they are full – i.e., while carrying a load. Constraints (73b) and (73c) ensure, respectively, that each department is assigned one location and each location is assigned at most one department. Since the QAP formulation ignores empty travel, a QAP-optimal layout is not

guaranteed to be feasible (See chapter 5). An alternative to minimizing ρ_t^f is to minimize the total utilization of material-handling system $r_t = \rho_t^f + \rho_t^e$. This ensures that empty travel is taken into account. This will also ensure that a layout design is feasible (this is guaranteed whenever there is at least one solution for which $\rho_t < 1$).

Alternatively, other performance measures, such as expected flow time or expected work-in-process in the system, can be used as optimization criteria. A layout design formulation that minimizes expected work-in-process, is given by:

$$\text{Minimize} \qquad E(WIP) = E(WIP_t) + \sum_{i=1}^{M} E(WIP_i) \qquad (78a)$$

Subject to:

$$\sum_{k=1}^{L} x_{ik} = 1 \quad i = 1,..,M \qquad (78b)$$

$$\sum_{i}^{M} x_{ik} = 1 \quad k = 1,..,L \qquad (78c)$$

$$x_{ik} = 0,1 \quad i = 1,..,M \quad k = 1,..,L \qquad (78d)$$

$$\rho_t < 1 \qquad (78e)$$

This formulation has the same constraints as the QAP formulation. We include an additional constraint, (74e), to ensure that the layout is feasible.

We could also minimize a weighted average of work-in-process to account for differences in holding costs between products at different stages of their manufacturing processes. Our objective function would then be rewritten as follows:

$$\text{Minimize} \qquad E(WIP)^c = \sum_{j=1}^{N} E(WIP_j)^c \,, \qquad (79)$$

where $E(WIP_j)^c$ is given by equation (44). Instead of WIP, we could minimize expected flow time for one or more products. In cases where different products have different priority, a weighted average flow time could be used instead. Alternatively, we could minimize average flow time subject to a flow time constraint on individual product (e.g., a target lead-time).

9.7 Solution Approach

Most facility design and planning problems are computationally hard. The quadratic assignment problem used to formulate the plant layout problem is known is NP-complete [2]. Since the objective function of our models is a non-linear transformations of the QAP objective function, it is also NP-complete. Thus no known algorithm can solve these problems optimally in polynomial time. Therefore, for large problems, we must resort to heuristic methods. Several heuristics have been proposed for solving the QAP problem (see for example Pardalos [2]). In our case, we used both a modified 2-opt exchange heuristic and a simulated annealing algorithm. Implementation of either approach is straightforward.

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