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UNIVERSITY OF VICTORIA

Department of Computer Science

# MATLAB User Manual

DEPARTMENT OF COMPUTER SCIENCE

# MATLAB User Manual

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# Accessing and Quitting MATLAB

## Accessing MATLAB

MATLAB is available on both UNIX and MS Windows platforms in the Department of Computer Science. MATLAB can be accessed from the workstations located in ELW B215, B228 and B203 or from any UNIX workstation that allows access to **SHELL**, where the UNIX version of MATLAB is located. The PCs in ELW B228 and B203 can be used to access MATLAB from a Windows Environment.

### In a UNIX Environment

- **Starting MATLAB on an X-Windows Desktop:** This is the environment available in ELW B215. The basic steps to access MATLAB are as follows.

  1. **Start Up A SSH Client Session to SHELL:** In an X-Terminal, type **ssh shell.csc.uvic.ca** at the prompt, and then enter your password. You should get a UNIX prompt on the remote server after this step. For example:

     **mayne%**

  2. **Determine the Name of Your Computer:** At the prompt, type

     **who am i**

     This command will give you a result back similar to the line below.

     **username termid date (*hostname*)**

  3. **Set Environment Variable for the Output Display:** Simply type the UNIX command below.

**setenv DISPLAY *hostname*:0**

For example, **setenv DISPLAY boxter:0**

4. **Start MATLAB:** Type the command below.

**matlab**

## In a Microsoft Windows Environment

- **Starting MATLAB on a Windows Desktop:** MATLAB for Windows is available in ELW B228 and B203. To start the Windows version of MATLAB on a PC, click on the **Start** button on the taskbar at the bottom of the desktop and then select **Productivity/MATLAB 6.1**.

- **Starting MATLAB Using X-Windows on the NT/2000 Workstations:** This is the environment available in ELW B228 and B203. The basic steps to access the UNIX version of MATLAB on **SHELL** are as follows.

  1. **Start Up An X-Window Session:** Click on the **Start** button, and then select **network/X-Win32.** An icon should appear on the taskbar at the bottom of the desktop.

  2. **Start Up An SSH Client to SHELL:** Click on the **Start** button, and then select **network/SSH Client**. After this step, a SSH Client window should appear. To complete the connection to **shell.csc.uvic.ca**, follow the steps from a to d below.

     a. Click on **Quick Connect** on the menu bar at the top of the SSH widow, or click on **File** on the menu bar and then select **Connect…**.

     b. Type the name for the remote server and your user name in the popup window as illustrated below. Then click on the **Connect** button.

c. When a popup window appears for Host Identification, just click on the button **No**.

d. Type your password when the last popup window appears.

**Note:** After these steps, a command line prompt in the remote server should appear in the SSH Client window. For example:

**mayne%**

3. **Determine the Name of Your Computer:** At the prompt in the SSH Client window, type

   **who am i**

   This command will give you a result back similar to the line below.

   **username termid date (*hostname*)**

4. **Set Environment Variable for the Output Display:** Simply type the UNIX command below.

   **setenv DISPLAY *hostname*:0**

   For example, **setenv DISPLAY boxter:0**

5. **Start MATLAB:** Type the command below.

   **matlab**

## Quitting MATLAB

Select **Exit MATLAB** from **File** on the menu bar to exit MATLAB. Alternatively, type **exit** or **quit** in the Command Window to quit MATLAB. To quit MATLAB on **SHELL** from the NT/2000 workstations in ELW B228 or B203, do as follows.

1. **Quitting MATLAB:** In the MATLAB Command Window, type **exit** or **quit**.

2. **Logout from SHELL:** In the SSH Client window, type **exit**.

3. **Quitting SSH Client Session:** Click on **File** on the menu bar at the top of the SSH widow, and then select **Exit**.

4. **Quitting X-Window Session:** Click on the icon **X-Win32** on the taskbar at the bottom of the desktop, and then select **Close**.

# Some Useful Help Commands

| COMMAND | DESCRIPTION |
|---|---|
| help | List all help topics in the Command Window |
| helpwin | List all help topics in the Help Window |
| help *topic* | Give help on the specified topic |
| quit | Terminate MATLAB |
| version | Version of MATLAB |
| type *filename* | Display the contents of the specified file |
| what | List MATLAB specific files in the current directory |
| more | Control paged output for the Command Window |

Chapter

2

# Basics of MATLAB

## The General Structure of the MATLAB Environment

The Command Window, Graphics Window and Edit Window are the three basic windows available in MATLAB .

### MATLAB Windows

- **Command Window:** This is the primary and default window in which a user interacts with MATLAB. Similar to any other command shells, the prompt >> is displayed and a blinking cursor appears to the right of the prompt. A user can type an individual command on the command line or run a program in this window. For example, to create a vector e with two elements, type

```
>> e = [ 1  0 ]
e =
    1     0
```

- **Graphics Window:** This is a graphics editor as well as an output window for graphs or figures generated from commands entered in the Command Window. To invoke the graphics editor, type **figure** in the Command Window.

- **Edit Window:** This is a program editor where you create and modify your own programs called 'M-files'. To invoke the editor, type **edit** in the Command Window.

## Other Features of the MATLAB Desktop

- **MATLAB Desktop:** In addition to the three basic windows above, MATLAB also has a number of other windows, including Command History, Launch Pad, Workspace, and Directory Browser. These windows, which make up the MATLAB Desktop, are opened automatically after MATLAB gets started. They can be closed or reopened by clicking on the corresponding menu entry from the **View** menu on the desktop.

  - **Command History:** The commands that you have previously entered in the Command Window are listed in the Command History Window. You can view and run the previous commands by selecting and pasting them into the Command Window. Or you can use the up-arrow key ↑ in the Command Window to recall previous commands.

  - **Launch Pad:** Launch Pad provides easy access to all of the MATLAB products installed in your system. To view a list of all the products, select **Launch Pad** from the **View** menu on the desktop. To run a product, double click on the selected product listed in the Launch Pad Window.

  - **Workspace:** The data and variables created in the Command Window are stored in the system memory called the MATLAB Workspace. To view the variables in the current Workspace, type **who** or **whos** in the Command Window. Similarly, to clear the variables, type **clear** or **clear yourvariablename**. The content of the Workspace Window is equivalent of the **whos** command.

  - **Directory Browser:** This directory management system can be used to search, open, view, and edit files. To launch the Directory Browser, select **Current Directory** from the **View** menu on the desktop or type **filebrowser** in the Command Window. Alternatively, you can use the following file management commands.

| COMMAND | DESCRIPTION |
|---|---|
| cd | Changes the current working directory |
| pwd | Shows the current working directory |
| dir | Lists contents of the current directory |
| ls | Lists contents of the current directory |
| mkdir | Creates a directory |

# Data Structures and The Operators

A matrix is the fundamental data structure in MATLAB; scalars and vectors are special cases of a matrix. The entries of a matrix can be either real numbers or complex numbers.

## Scalars

- **Definition:** A scalar is a number that can be either a real or complex number. A scalar is a special case of a 1 × 1 matrix. For example:

```
>> x = 0.75
x =
     0.7500

>> y = 3 + 4i
y =
     3.0000 + 4.0000i
```

## Vectors

- **Definition:** A vector is a special case of a matrix with one row or one column. For example:

```
>> u = [ 1  2 ]
u =
    1    2

>>  v = [ 1,  -1.1,  0 ]
v =
    1.0000    −1.1000    0

>>  w = [ 2;  3.6;  - 1 ]
v =
      2.0000
      3.6000
     −1.0000
```

## Matrices

- **Definition:** An m × n matrix is a two dimensional array of scalars, consisting m rows and n columns. A space or a comma separates consecutive entries in a row, and a semicolon or a carriage return separates consecutive rows. For example:

```
>>  A = [ 1  2;  3  4 ]
A =
    1    2
    3    4

>> B = [  i,  -1,      1 + i
            2,  -2 - i,  3 ]
B =
    0 + 1.0000i    - 1.0000                  1.0000 + 1.0000i
    2.0000         - 2.0000 - 1.0000i    3.0000
```

## The Colon Notation and Subscripting

- **The Colon Notation:** The colon notation is useful for constructing vectors with equally spaced entries. The syntax for using the colon notation to generate a vector is **m:s:n**, which generates entries from m to n with an increment for each step of s. If the required increment is 1, then the syntax becomes **m:n**. Note that **m**, **s** and **n** need not be integers. For example:

```
>> v = 1 : 4
v =
    1    2    3    4

>> w = 12 : -3 : 0
w =
    12    9    6    3    0

>> y = 5 : -2 : 0
y =
    5    3    1

>> x = 0 : 2 : 5
x =
    0    2    4

>> z = 0.2 : 0.3 : 1.2
z =
    0.2000    0.5000    0.8000    1.1000
```

- **Subscripting:** Each of the entries in a matrix A can be accessed by A (i, j), where i ≥ 1 and j ≥ 1. If v is a vector of the row indices of a matrix A and w is a vector of the column indices of A, then A (v, w) is the submatrix of

A from the selected rows and columns. If the row and column indices are consecutive, then A(r : s, p : q) denotes the submatrix from rows r, …, s and from columns p, …, q. A colon ( : ) can be used to select all of the row or column indices. For example:

```
>> A = [ 1 2 3; 4 5 6; 7 8 9 ]
A =
    1   2   3
    4   5   6
    7   8   9

>> A ( 3, 3 )
ans =
    9

>> B = A ( [ 1 3 ], [ 2 3 ] )
B =
    2   3
    8   9

>> C = A ( 1 : 2, 2 : 3 )
C =
    2   3
    5   6

>> D = A ( :, 1 : 2 )
D =
    1   2
    4   5
    7   8
```

## Operators

- **Arithmetic Operators:** The table below lists all of the MATLAB arithmetic operators. Other operators, such as logical and relational operators, are described in the section Flow of Control.

| OPERATOR | DESCRIPTION |
|---|---|
| + | Addition |
| - | Subtraction |
| * | Matrix multiplication |
| .* | Entry-wise multiplication |
| / | Matrix left division |
| ./ | Left entry-wise Division |
| \ | Matrix right division |
| .\ | Right entry-wise division |
| ^ | Matrix exponentiation |
| .^ | Entry-wise exponentiation |
| ' | Matrix transpose |
| .' | Nonconjugated transpose |

- **Examples:**

Suppose that $x = 0.75$, $v = \begin{bmatrix} 1 \\ -1.1 \\ 0 \end{bmatrix}$, $w = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $u = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 7 & 2 \end{bmatrix}$.

```
>> 2 * x / 5
ans =
   0.3000

>> v – w
ans =
         0
   -1.1000
         0

>> x + 10 * v
ans =
    10.7500
   -10.2500
     0.7500
```

```
>> 2 + A/2
ans =
    2.5000    3.0000
    3.5000    4.0000

>> A \ u
ans =
    1
    0

>> A * u
ans =
     7
    15

>> u * u'
ans =
    1    3
    3    9

>> u .* u
ans =
    1
    9

>> u' * u
ans =
    10

>> A./B
ans =
    1.0000    0.6667
    0.4286    2.0000

>> A/B
ans =
    0.6316    0.0526
    1.1579    0.2632
```

```
>> A * B
ans =
    15    7
    31   17

>> A( 1, 2 )^2 * B( 2, 2 )
ans =
    8
```

## Precedence Rules for Operators

- **Operator precedence:** The precedence rules for MATLAB operators are summarized in the table below. They are ordered from the highest (Level 1) to the lowest (Level 9).

| LEVEL | OPERATOR |
|---|---|
| 1 | Parentheses ( ) |
| 2 | Transpose (.'), power (.^), complex conjugate transpose ( ' ), matrix power(^) |
| 3 | Unary plus (+), unary minus (-), logical negation (~) |
| 4 | Multiplication (.*), right division (./), left division (.\), matrix multiplication (*), |
| | matrix right division (/), matrix left division (\) |
| 5 | Addition (+), subtraction (-) |
| 6 | Colon operator (:) |
| 7 | Less than (<), less than or equal to (<=), greater than (>), |
| | greater than or equal to (>=), equal to (==), not equal to (~=) |
| 8 | Logical AND (&) |
| 9 | Logical OR (\|) |

## Character Strings

- **Definition:** A character string, which is enclosed by a pair of single quotes, is an array of characters. The internal representation of each character is a numerical value, and requires 2 bytes for storage. For example,

```
>> myString = ' This is my first string.'
myString =
This is my first string.
```

- **String Functions:** The most common commands that manipulate character strings are summarized in the table below.

| COMMAND | DESCRIPTION |
|---|---|
| char | Create character array |
| blank(n) | A string of n blanks |
| deblank(s) | Strip trailing blanks from the end of a string |
| eval | Execute a string containing an expression |
| findstr (s1,s2) | Find one string within another |
| int2str(n) | Integer to string conversion |
| ischar(s) | True for character arrays |
| isletter (s) | True for alphabetical characters |
| isstring (s) | True for if the argument is a string (version 5) |
| lower | Convert string to lower case |
| mat2str | Convert a matrix into a string |
| num2str | Convert numbers to a string |
| strcmp(s1,s2) | Compare strings |
| strcmpi(s1,s2) | Compare strings ignoring case |
| strncmp(s1,s2,n) | Compare the first n characters of two strings |
| strncmpi(s1,s2,n) | Compare the first n characters of two strings ignoring case |
| strcat | String concatenation |
| strvcat | Vertical concatenation of strings |
| upper(s) | Convert string to upper case |

# Variables

As in other programming languages, you can use variables to store values in the current session or in an M-file. There are two types of variables, local and global.

## Declaration

- **Implicit Declaration:** MATLAB does not require explicit declarations for its variables (with the exception of global variables used in MATLAB functions; see 'Global Variables' below). When MATLAB encounters a new variable name, it automatically creates the variable and allocates the appropriate amount of storage.

- **Length of a Variable:** A variable name begins with a letter, optionally followed by a number of letters, digits, or underscores to a maximum of 31 characters. Variable names are case sensitive.

### Global Variables

- **Explicit Declaration:** A global variable can be declared using the **global** command so that more than one function can share a single copy of the variable. You must declare the variable as **global** at the beginning of every function that requires access to it. Similarly, you must declare it as **global** from the command line to enable your active workspace to access it. Using uppercase characters for a global variable name is recommended.

## Flow of Control

In MATLAB, flow of control depends on the results of evaluating logical expressions using relational and logical operators defined in the tables below. These operators compare corresponding entries of matrices with the same dimensions. The Boolean values true and false are stored and displayed as 1 and 0, respectively.

### Relational and Logical Operators

- **Relational Operators:**

| OPERATOR | DESCRIPTION |
|---|---|
| < | Less then |
| > | Greater then |
| <= | Less than or equal |
| >= | Greater than or equal |
| == | Equal |
| ~= | Not equal |

- **Logical Operators:**

| OPERATOR | DESCRIPTION |
|---|---|
| & | And |
| \| | Or |
| ~ | Not |

- **Examples:**

```
>> ( 2^3 < 9 ) + ( 3^2 >= 9 )
ans =
    2

>> [ 2 3 5 ] > [ 0 3 4 ]
ans =
    1    0    1

>> [ 1 2; 3 4 ] <= [ 1 5; 6 2 ]
ans =
    1    1
    1    0
```

**Note:** To test if two matrices are identical, use **isequal .** For example, if A $= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$, then

```
>> isequal( A,C )
ans =
    0
```

## Control Statements

- **if statement:** The **if** statement executes a group of statements if the evaluated expression is true. The optional **elseif** and **else** provide alternatives for execution of different groups of statements.

  - **Syntax:**

```
if expression
  statements
else
  statements
end

if expression1
  statements
elseif expression2
  statements
...
else
  statements
end
```

  - **Example:**

```
>>if x > 10
   z = 1;
  else if y > 0
   z = 2;
  else
   z = 3;
  end
```

- **switch and case Statements:** The **switch** statement evaluates an expression and then executes a group of statements under the first matching **case** statement. If no matching **case** statement is found, then the statements under the optional **otherwise** statement are executed.

  - **Syntax:**

```
switch expression
case test_expression1
 statements
case test_expression2
 statements
...
otherwise
 statements
end
```

  - **Example:**

```
>>x = input(' Enter a number: ');

>>switch x
    case 0
      y = 0;
    case 1
      y = x + 2;
    otherwise
      y = 10;
  end
```

- **for Statement:** The **for** loop repeatedly executes a group of statements a fixed and predetermined number of times.

  - **Syntax:**

```
for  variable = expression
 statements
end
```

- **Example:**

```
>> for n = 1 : 4
      x( n ) = n/10 * pi;
    end

>> x
x =
    0.3142    0.6283    0.9425    1.2566
```

- **while Loop Statement:** The **while** loop allows a group of statements to be repeatedly executed as long as the evaluated expression is true.

  - **Syntax:**

```
while  expression
  statements
end
```

  - **Example:**

```
>> p =1; u = 1;
while p < 16
   p = p * 2;
   u = [ u, p ];
end

>> u
u =
    1    2    4    8    16
```

- **continue and break Statements:** The **continue** statement causes execution of a **for** or **while** loop to jump immediately to the next iteration of the loop, and it skips any remaining statements in the loop. In contrast to the **continue** statement, the **break** statement terminates the execution of the loop.

  - **Syntax:**

```
while  expression
  statements
 continue
 statements
end
```

```
for  variable = expression
  statements
 continue
 statements
end

while  expression
  statements
 break
 statements
end

for  variable = expression
  statements
 break
 statements
end
```

- **Example:**

```
>> m = 1; n = 0;
>> while n <= 1000
        m = m/3;
        if ( 1 + m ) > 1
           n = n + 1;
           continue
       end
       m = m*3
       break
   end

   m =
      1.7989e-016
```

# M-File Basics

Besides using the interactive computational environment, you can also write programs in the MATLAB language and store them in files. These files are called M-files.

## Creating M-Files

An M-file is just an ordinary text file and hence it can be created using any text editor. As mentioned earlier, MATLAB provides a default M-file editor for all platforms. To open the default editor, select **New** and then **M-File** from the **File** menu, or type **edit** in the Command Window. To save an M-file, from the **File** menu select **Save** for an existing file or **Save as** for a new file. An M-file name must have a '.m' extension after the file name. There are two types of M-files, script files and function files.

- **Scripts:** A script file is a file that contains a sequence of valid MATLAB commands, and has no input or output arguments. For example:

**M-file: myScriptFile.m**

```
% Script M-file myScriptFile.m
% 1. Create a 3 × 3 matrix A
% 2. Compute the coefficients of the characteristic polynomial,
%    det(λI – A)
% 3. Compute the roots of this polynomial (eigenvalues of matrix A)

A = [1 2 3; 4 5 6; 7 8 0]
p = poly (A)
r = roots (p)
```

- **Function Files:** Similar to a script file, a function file is a file that contains one or more functions. The first function in the file is the primary function and the rest are subfunctions. A subfunction can only be called by the primary function and other subfunctions within the same file. The primary function or a subfunction can contain any valid MATLAB statements.

  A function or subfunction starts with a function definition line, which specifies a list of input and/or output arguments. The syntax of the function definition line is defined as follows.

  **function [output variables] = function_name(input variables)**

  The output variables and the input variables are both optional. Note that MATLAB function names are specified in the same way as variable names (that is, they begin with a letter and are up to 31 characters long).

  To save a function file, one must use the primary function name for the M-file. For example, if the primary function name is EigValues, the file name is EigValues.m

  - **Passing Parameters to and Returning Parameters from a Function:** Below are two simple examples to illustrate how a MATLAB function works.

**M-file: EigValues.m**

```
% Script M-file EigValues.m
% This function takes a matrix A as input and returns a list
% of the eigenvalues of A and the coefficients of the
% characteristic polynomial, det(rI - A).

% To call this function, type:
% [eigvalues, coeffs] = EigValues(A);

function [eigvalues,coeffs] = EigValues(A)
p = poly (A);
eigvalues = roots (p);
coeffs = p;
```

```
>> C = [ 1  2  3;  4  5  6;  7  8  0 ]
C =
    1    2    3
    4    5    6
    7    8    0

>> [eig, coef] = EigValues( C )
eig =
    12.1229
    -5.7345
    -0.3884

coef =
    1.0000   -6.0000  -72.0000  -27.0000
```

Note that a function can be called with a different number of input or output arguments by using the built-in functions **nargin** and **nargout.** The example below is the same function as EigValues above except the output of the coefficients is optional.

**M-file: EigValues.m**

```
% Script M-file EigValues.m
% This function takes a matrix A as input, returns a list
% of the eigenvalues of A, and optionally returns the coefficients
% of the characteristic polynomial, det(rI - A).
```

```
% To call this function, type:
% [eigvalues, coeffs] = EigValues(A);

function [eigvalues,coeffs] = EigValues(A)
p = poly (A);
eigvalues = roots (p);

if ( nargout == 2 )
   coeffs = p;
end
```

```
>> eig = EigValues( C )
eig =
    12.1229
    - 5.7345
    - 0.3884
```

- **Subfunctions:** As mentioned earlier, an M-file can contain subfunctions besides the primary function. Any subfunction must appear after the primary function. Subfunctions are local and can be called only by the primary function and other subfunctions in the same M-file.

**M-file: BigTrace.m**

```
function [result] = BigTrace(A,B) % Primary function
% The variable result is set to equal maximum of
% trace(A) and trace(B)
result = max(trace(A),trace(B));

function t = trace(C)          % Subfunction
%  Return the sum of the diagonal elements of the matrix C.

t = sum(diag(C));
```

- **Recursive Functions:** Note that MATLAB supports recursive function calls.

- **Syntax of Comments:** In an M-file, MATLAB treats all text after a percent sign % as a comment statement. Comments can appear anywhere in an M-file, as shown in the examples above.

## Running M-Files

- **From the Command Line:** To invoke an M-file (either a function file or a script file), type the name of the file without the '.m' extension from the command line in the Command Window. For example, you can call the function BigTrace from the command line as follows.

  Suppose A = $[1 \quad 2 \quad 3; \quad 4 \quad 5 \quad 6; \quad 7 \quad 8 \quad 0]$ and B = $[3 \quad 4; \quad 5 \quad 6]$.

  ```
  >> BigTrace( A, B )

  ans =
      9
  ```

- **Within Another M-file:** A function or a script can similarly be called from another M-file.

## Summary of Useful Commands

| COMMAND | DESCRIPTION |
|---|---|
| type *filename* | Display the contents of a specified file |
| edit *filename* | Invoke the default editor |
| path | Display the current MATLAB search path. |
| tic/toc | tic starts a timer and toc returns elapsed time |
| profile | A debugging utility |
| lookfor | Search for the specified keyword in all help entries |
| dbstop | Set breakpoints in an M-file function |
| dbclear | Clear breakpoints in an M-file function |
| dbcont | Resume execution |
| dbquit | Quit debug mode |
| keyboard/return | Invoke and terminate the keyboard mode in an M-file |
| nargin | Number of input function arguments |
| nargout | Number of output function arguments |
| which | Locate functions and files |
| pcode | Create pre-parsed pseudocode file |

# Input/Output and Data Formatting

MATLAB allows user input during runtime, saves a copy of a MATLAB session in a file, and saves data files in a variety of formats. In addition, there are commands to control how data are displayed.

## Input

- **User Input:** User input can be prompted and obtained interactively during runtime. The syntax for the command **input** is given below. The value entered by a user can be any valid MATLAB expression or a character string if the second argument 's' is used.

  ```
  inValues = input(prompt_string)
  inValues = input(prompt_string,'s')
  ```

- **Example:**

  ```
  >> isQuit = input( ' Do you want to exit the current session? Y/N [Y]: ',
  's' );
  >> if ( isQuit == 'Y' | isempty( isQuit ) )
         save;
         quit;
     else
         quit cancel;
     end
  ```

## Output

- **Save and Load Variables:** Before you exit or quit the current workspace, you can use the **save** command to save all the variables and their current values. In a new session, you can use the **load** command to restore them. For example:

  >> save *filename*

  To restore the variables, type

  >> load *filename*

  Note that if **save** and **load** are used without a specified file name, MATLAB uses a default file name, *matlab.mat.*

- **Output to the Screen:** Several output functions are available. Here are a few examples.

When you type a variable name, MATLAB displays the variable name and its value by default. Sometimes it is desirable to display only the value. For example, suppose B = $[3 \quad 4; \quad 5 \quad 6]$. Then to display the value of the matrix B with column labels, type

```
>> disp('    B1    B2 '), disp( B )
    B1  B2
     3   4
     5   6
```

Similarly, if x = 0.756, then to display the formatted value of x, type

```
>> fprintf( '%7.2f\n', x )
    0.76
```

Note that the number 7 in the format string is the field width, the number 2 is the number of decimal digits after the decimal point, and the escape character \n is a new line terminator.

- **Suppress Output from MATLAB Commands:** If you place a semicolon (; ) at the end of a statement line, MATLAB executes the statement but does not display any output. For example:

```
>> u = [ 1  2 ];
>> v = u + 3;
```

- **Keep a Session Log:** The **diary** command can be used to save the entire working session. This command spools all the activities or events in the Command Window to a text file. For example, to save the current session, type

  >> diary *filename*

  To suspend the diary, type

```
>> diary off
```

  If **diary** is used without a specified file name, then MATLAB uses a default file name *diary*.

## Format

**For online help type help format or select MATLAB Help from Help menu.**

MATLAB stores numbers to a relative precision of approximately 16 decimal digits. By default MATLAB displays numbers in the short format (4 decimal places). To print a value in any of the formats given below, enter **format** *type* on the command line. For example:

>> format long

The following table illustrates the additional format types supported by MATLAB.

| FORMAT | EXAMPLES |
|---|---|
| format short | 17.3205 |
| format short e | 1.7321e+001 |
| format short g | 17.321 |
| format long | 17.32050807568877 |
| format long e | 1.732050807568877e+001 |
| format long g | 17.3205080756888 |
| format bank | 17.32 |
| format hex | 4031520cd1372fea |
| format rat | 1351/78 |

## Summary of Useful Commands

| COMMAND | DESCRIPTION |
|---|---|
| save *filename* | Save variables |
| load *filename* | Restore variables |
| clc | Clear the Command Window |
| disp | Display text or array |
| input | Wait for input from the keyboard |
| pause(n) | Halt execution temporarily |
| diary | Save a session to a disk file |
| format | Control the output format |

# Graphics

MATLAB has extensive tools for displaying various data as graphs. It also provides facilities for annotating and printing graphs. In the following section, some examples are presented to illustrate how a graph can be created using these tools. To invoke the graphics editor, type **figure** in the Command Window.

## Basic Plots

**For online help type help graph2d or select MATLAB Help from Help menu.**

The most basic graph is a simple 2-D plot. One form of the syntax for the plot command is **plot (x_values,y_values,'style-option'). x_values** is a vector that contains the points on the x-coordinate, while **y_values** contains the points on the y-coordinate. **style-option** is a parameter that defines the line style, the marker, and the color used in a graph. If this parameter is not entered, MATLAB uses the default style. Style options are summarized in the following table.

| COLOR | | LINE STYLE | | MARKER | |
|---|---|---|---|---|---|
| y | Yellow | - | Solid | o | Circle |
| m | magenta | -- | Dashed | * | Asterisk |
| c | Cyan | : | Dotted | . | Point |
| r | Red | -. | Dash-dot | + | Plus |
| g | Green | none | No line | x | Cross |
| b | Blue | | | s | Square |
| w | White | | | d | Diamond |
| k | Black | | | ^ | Upward triangle |
| | | | | v | Downward triangle |
| | | | | > | Right triangle |
| | | | | < | Left triangle |
| | | | | p | Five-point star |
| | | | | h | Six-point start |

Here is an example of a simple plot. The output is shown in Figure 2.1

```
>> t = 0 : 0.001 : 2 * pi;
>> x = cos( 3 * t );
>> y = sin( 2 * t );
>> plot( x, y )
```

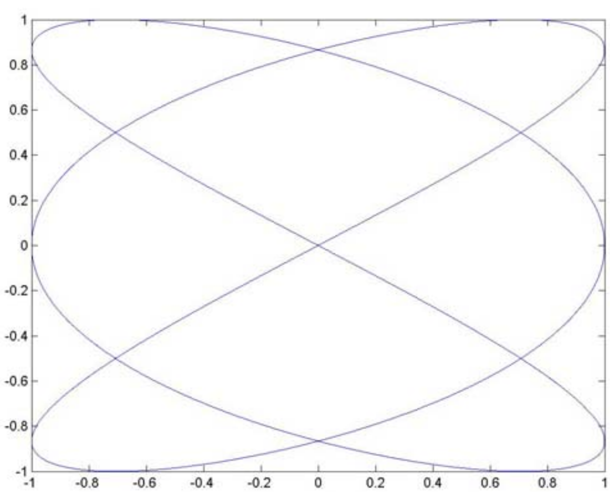



**Figure 2.1: x-y Plot**

## Graph of a Function

MATLAB provides commands **ezplot** and **fplot** for plotting mathematical functions. The syntaxes for the commands are **ezplot ('function', [xmin, xmax, ymin, ymax])** and **fplot ('function', [xmin, xmax, ymin, ymax])**, respectively. Both commands plot a function in a specified range. However, if a line style different from the default is required, then **fplot ( 'function', [xmin, xmax, ymin, ymax], 'style-option')** should be used. For example, let us first create a function in an M-file called **myFunction.m** and then use the command **ezplot ('myFunction',[ 0 2*pi 0 12 ])** to generate the graph on the specified range. The output is shown in Figure 2.2a. To generate the same plot using a dotted line instead of a solid line (default), use **fplot ('myFunction',[ 0 2*pi 0 12 ], ' :xr ')**. Note that ':xr' means a dotted red line with cross markers is used in the plot. The output is shown in Figure 2.2b.

**M-file: myFunction.m**

```
function y = myFunction(x)

y = exp( sqrt(x) .* sin(12 * x) );
```

```
>> ezplot( 'myFunction', [ 0 2*pi 0 12 ] )
```

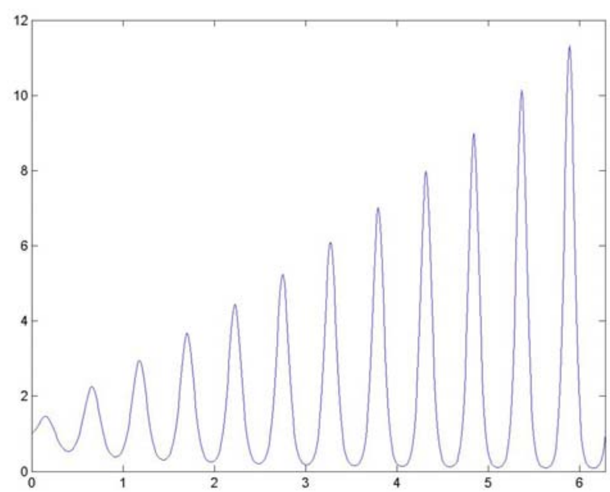



**Figure 2.2a: Plot of a Function Using ezplot**

```
>> fplot( 'myFunction', [ 0 2*pi 0 12 ], ' :xr ' )
```

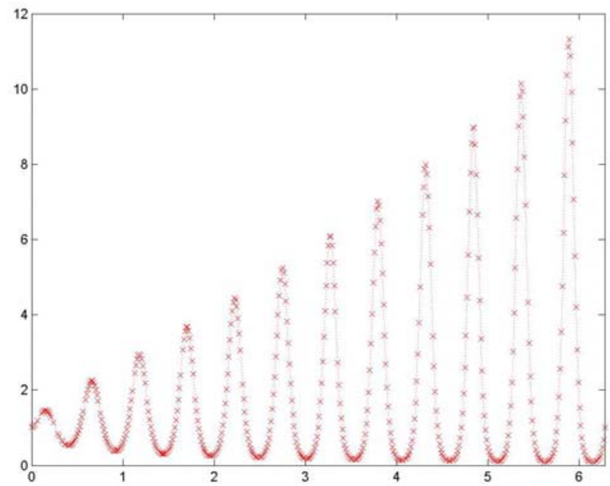



**Figure 2.2b: Plot of a Function Using fplot**

## Define Titles, Labels and Text in a Graph

You can add a title to a graph and add labels to axes. The general syntax for the **title** function is **title('string')**, and the syntax for the **xlabel** and **ylabel** commands are **xlabel('string')** and **ylabel('string').** Moreover, you can also add a text object to a graph. The syntax for the command **text** is **text(x, y, 'string').** For example, we add a title, labels and a text object to Figure 2.1. The output is shown in Figure 2.3.

```
>> plot( x, y )
>> title( 'X-Y Plot' )
>> ylabel( 'cos(2*t)' )
>> xlabel( 'sin(3*t)' )
>> text( -0.2, 0.4, 'A symmetry graph' )
```

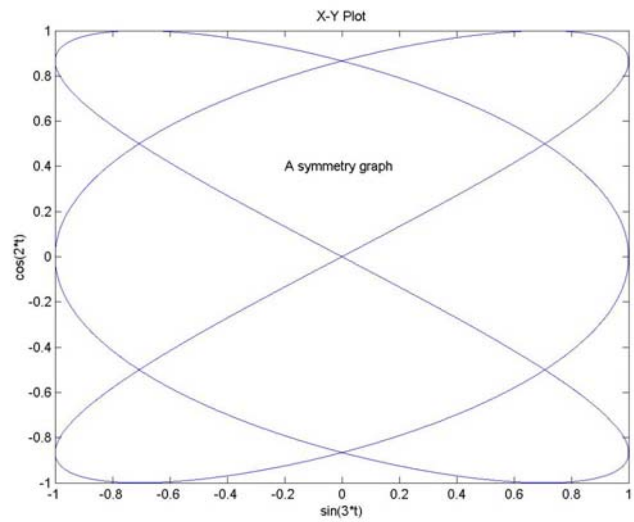



**Figure 2.3: x-y Plot with Title and Labels**

## Commands for Controlling the Axes

After you generate a graph, you can modify or change an axis range with the command **axis**. The most basic syntax is **axis([xmin xmax ymin ymax])**. **xmin** and **xmax** define the smallest and largest end points for x-axis; similarly, **ymin** and **ymax** define the smallest and largest end points for y-axis. For example, we can change the ranges for the axes in Figure 2.2a to [ 0, 4 ] and [ 0, 8 ] as follows. The output is shown in Figure 2.4.

**For online help type help axis or select MATLAB Help from Help menu.**

```
>> ezplot( 'myFunction', [ 0  2*pi  0 12 ] )
>> axis( [  0  4  0  8 ] )
```

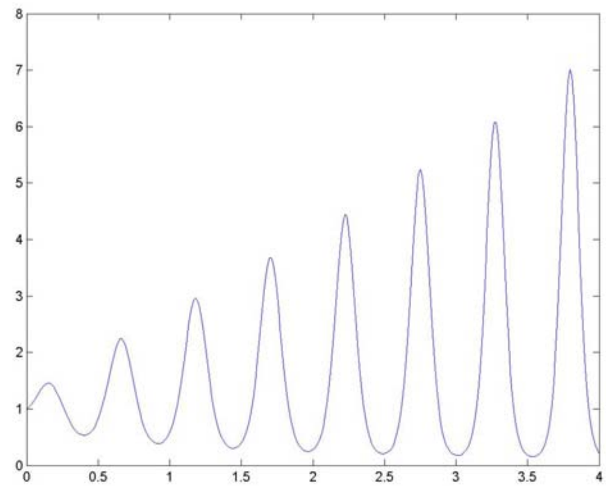



**Figure 2.4: Plot of a Function: myFunction with Modified Axis Ranges**

## Multiple Plots in One Figure

There are three ways to create multiple plots on a single graph.

**For online help type help subplot or select MATLAB Help from Help menu.**

- **Using the Command subplot:** The MATLAB function **subplot** can be used to plot data into different sub-regions within the same graphics window. The command **subplot(m,n,i)** divides the graphics window into an m by n matrix of small sub-regions and generates the next figure in the ith sub-region. The sub-regions are numbered row-wise.

For example, the following statements plot a set of data in four different sub-regions of the graphics window in Figure 2.5. The command **subplot(2,2,1)** is set for **plot(t,z)** to be generated in the first sub-region in first row. Similarly, **subplot(2,2,2**) is set for **plot(t,2*q)** in the second sub-region in the first row, and so on.

```
>> t = 0 : pi/20 : 2*pi;
>> z = cos( 3*t );
>> subplot( 2, 2, 1 )
>> plot( t, z )
>> subplot( 2, 2, 2 )
>> q = exp( -t );
>> plot( t, 2*q )
>> subplot( 2, 2, 3 )
>> fplot( 'myFunction',[ 0  2*pi ] )
```

```
>> subplot( 2, 2, 4 )
>> fplot( '[ sin( x ), cos( 2*x ), 1/( 1+x ) ]',[ 0  5*pi  -1.5  1.5 ] )
```

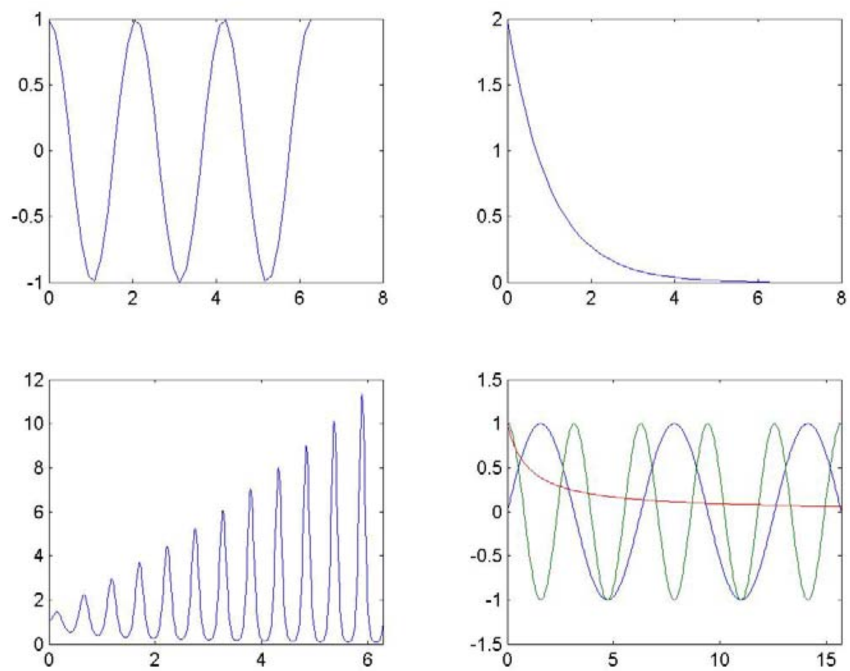


**Figure 2.5: Generating Multiple Plots Using subplot**

- **Using a Matrix:** To create multiple plots in a single graph (as in the last sub-region in Figure 2.5), one can also use a matrix. Each column of the matrix contains the functional values that are to be plotted as one graph. In the following, note that cos(x) is a row vector of functional values, and cos(x)' is a column vector ( ' is the transpose operator). The following example is illustrated in Figure 2.6a.

  ```
  >> x = 0 : 0.01 : 2*pi;
  >> Y = [ cos( x )', cos( 2*x )', cos( 4*x )' ];
  >> plot( x, Y )
  ```

  If different styles are desired for different plots, replace **plot(x,Y)** with, for example, **plot(x,Y(:,1),'--',x,Y(:,2),'-.',x,Y(:,3))**. The result is shown in Figure 2.6b.

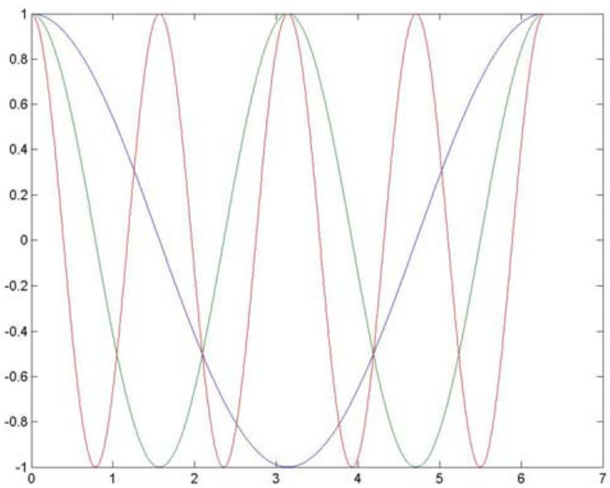



**Figure 2.6a: Generating Multiple Plots Using a Matrix**

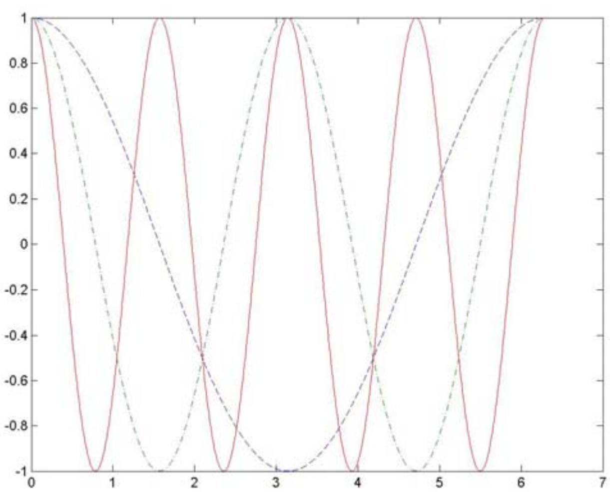



**Figure 2.6b: Multiple Plots with Different Styles**

- **Using the Command hold:** The third way to create multiple plots in the same graphics window is to use the command **hold**. **hold on** freezes the current plot in a graphics window and allows subsequent plots to be generated in the same window. An example is shown below and the output is displayed in Figure 2.7.

  ```
  >> t = [ 0 : 0.01 : 2*pi ];
  >> plot( sin( t ) )
  ```

```
>> hold on
>> plot( cos( t ) )
>> hold off
```

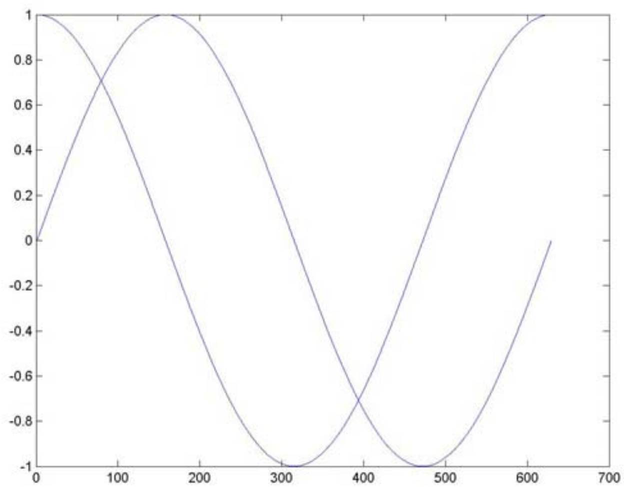



**Figure 2.7: Generating Multiple Plots Using hold**

## Save and Print a Figure

**For online help type help print or type help saveas select MATLAB Help from Help menu.**

To print a hardcopy of a graph, the simplest way is to select **Print** from the **File** menu in a graphics window or alternatively type the **print** command directly in the Command Window. Similarly to save a graph into a file, select **Save** from the **File** menu or enter the **print** command along with a file name in the Command Window. The basic syntax of the **print** command is defined as follows.

**print –d*devicetype* –*options filename***

For example, the following statement will save the current graph into a file named as mygraph.eps.

```
>>print –deps mygraph.eps
```

You can export a graph with a specified format using the command **saveas**. For example, the command below saves the current graph in the jpg format.

```
>> saveas( gcf, 'myGraph.jpg' )
```

## Commands for 2D Plotting Functions

| COMMAND | DESCRIPTION |
|---|---|
| area | Create an area graph |
| bar | Create a bar graph |
| barh | Create a horizontal bar graph |
| compass | Create an arrow graph for complex numbers |
| contour | Make a contour graph |
| hist | Create a histogram graph |
| pie | Create a pie graph |
| scatter | Create a scatter graph |
| stairs | Create a stair step graph |
| polar | Plot polar coordinates |
| plotmatrix | Draw scatter plots |
| plot | Create a 2-D plot |
| fplot | Plot a function between specified ranges ( with line styles) |
| ezplot | Plot a function between specified ranges |
| subplot | Create and control multiple axes |
| grid | Grid lines for two-dimensional plots |
| xlabel/ylabel | Create labels for x-axis and y-axis |
| title | Add titles to current axes |
| legend | Create a legend on a graph |
| hold | Hold current graph in a figure |
| text | Create text objects in current axes |
| fill | Filled two-dimensional polygons |
| line | Create a line object |
| axis | Set ranges for x-axis and y-axis |

Chapter

3

# Basic Functions for Linear Algebra and Numerical Analysis

## Linear Algebra

MATLAB has an extensive set of functions for computations in linear algebra, such as functions for computing the inverse and the determinant of a matrix. In the following section, several fundamental concepts from linear algebra are defined and some examples are given to illustrate how to use these MATLAB functions to solve linear algebra problems.

### Vector and Matrix Norms

Norms, which are scalars, are measures of the size of vectors and matrices.

- **P-norm of a Vector:** The p-norm of a vector x is

$$\|x\|_p \equiv \left( \sum_{i=1}^{n} |x_i|^p \right)^{1/p} \text{ for } 1 \le p < \infty .$$

The sum norm ( $p = 1$ ) and the Euclidean norm ( $p = 2$ ) are particular cases of p-norms. In MATLAB, a norm is calculated by using the function **norm(x, p)**. Suppose x = $\begin{bmatrix} 2 & 4 & 6 & 8 \end{bmatrix}$.

```
>> [ norm( x, 1 ) norm( x, 2 ) ]
ans =
   20.0000   10.9545
```

Note that norm( x, 2 ) can be computed by norm( x ).

When $p \to \infty$, the p-norm becomes the max norm, defined as

$$\|x\|_\infty \equiv \max\{|x_1|,\ldots,|x_n|\}$$

For example, to compute the max norm of the vector x used in the previous example, type

```
>> norm( x, inf )
ans =
    8
```

- **P-norm of a Matrix:** The p-norm of a matrix A is

$$\|A\|_p \equiv \max_{x\neq 0} \frac{\|Ax\|_p}{\|x\|_p}$$

The maximum column sum matrix norm ( $p = 1$ ), the maximum row sum matrix norm ( $p = \infty$ ), and the spectral norm ( $p = 2$ ) are particular cases of the p-norms and they can be computed, respectively, by

$$\|A\|_1 \equiv \max_{1\le j\le n} \sum_{i=1}^{n} |a_{ij}| \ ,$$

$$\|A\|_\infty \equiv \max_{1\le i\le n} \sum_{j=1}^{n} |a_{ij}| \ , \text{and}$$

$$\|A\|_2 \equiv \max\{\sqrt{\lambda} : \lambda \text{ is an eigenvalue of } A^*A\} .$$

Suppose matrix A = $[1 \quad 2; \quad 3 \quad 4]$. To use the function **norm(A, p)** to compute these matrix norms, type

```
>> [ norm(A, 1) norm(A, inf) norm(A, 2) ]
ans =
    6.0000    7.0000    5.4650
```

Note that norm(A, 2) can be computed by norm(A).

## Inverses

- **Inverse of a Square Matrix:** The inverse of a square matrix A is the matrix $A^{-1}$ such that $\mathbf{A^{-1}A = AA^{-1} = I}$, where I is the identity matrix. The function **inv(A)** is used to compute the inverse of a square matrix A. For example:

```
>>  A = [ 1  2;  3  4 ]
A =
    1    2
    3    4

>> B = inv( A )
B =
    - 2.0000     1.0000
      1.5000   - 0.5000

>> I = B * A
I =
    1.0000           0
    0.0000    1.0000
```

- **Pseudo-inverse:** If a matrix A is a square and singular matrix, or a rectangular matrix, it does not have an inverse. However, A has a unique pseudo-inverse which can be computed using **pinv(A)**. For example:

```
>> A = [ 1  2  3;  5  7  9 ]
A =
    1    2    3
    5    7    9

>> B = pinv( A )
B =
    - 1.3889     0.4444
    - 0.2222     0.1111
      0.9444   - 0.2222

>> B * A
ans =
      0.8333    0.3333   - 0.1667
      0.3333    0.3333     0.3333
    - 0.1667    0.3333     0.8333
```

```
>> A * B
ans =
     1.0000    0.0000
    -0.0000    1.0000

>> A * B * A
ans =
    1.0000    2.0000    3.0000
    5.0000    7.0000    9.0000

>> B * A * B
ans =
    -1.3889     0.4444
    -0.2222     0.1111
     0.9444    -0.2222
```

## Transposes

The transpose of a matrix A is obtained by interchanging the rows and columns of A, and is computed by A' in MATLAB.

- **Transpose of a Real Matrix:** For example:

```
>>  A = [ 1  2;  3  4 ]
A =
    1    2
    3    4

>> A'
ans =
    1    3
    2    4
```

- **Conjugate Transpose of a Complex Matrix:** In addition to interchanging the rows and columns, the conjugate transpose of a complex matrix A also replaces each entry by its complex conjugate. For example:

```
>>  A = [ 1 + i,  2 - i; -3,  -2i ]
A =
    1.0000 + 1.0000i    2.0000 - 1.0000i
             -3.0000            0 - 2.0000i
```

```
>> A'
ans =
    1.0000 - 1.0000i        - 3.0000
    2.0000 + 1.0000i    0 + 2.0000i
```

## Determinants

- **Determinant of a Square Matrix:** The determinant of a square matrix A is calculated using the triangular factors obtained from Gaussian elimination and can be computed by **det(A)**. Suppose A = $[1 \quad 2; \quad 3 \quad 4]$.

```
>> det( A )
ans =
    -2
```

## Rank

- **Rank of a Matrix:** The rank of a matrix A is the largest number of columns (or rows) of A that constitutes a linearly independent set. To compute the rank of a matrix, use the function **rank(A)** in MATLAB. For example, for the matrix A = $[1 \quad 2; \quad 3 \quad 4]$:

```
>> rank( A )
ans =
    2
```

## Factorizations

- **LU Factorization of a Matrix:** For every square matrix A, there exist a lower triangular matrix L, an upper triangular matrix U, and a permutation matrix P such that **PA = LU**. To compute the LU factors for a matrix by Gaussian elimination with partial pivoting, use the function **lu(A)** in MATLAB. For example, suppose A = $[1 \quad 3; \quad 2 \quad 4]$.

```
>> [L, U, P] = lu( A )
L =
    1.0000        0
    0.5000   1.0000

U =
    2   4
    0   1
```

```
P =
    0   1
    1   0
```

- **Cholesky Factorization of a Matrix:** The Cholesky factorization is a special case of LU factorization. Suppose A is a real symmetric matrix. If A is positive definite (that is, x'Ax > 0 for all nonzero column vectors x), then A can be factored as **A = R'R**, where R is an upper triangular matrix. The function **chol(A)** in MATLAB is used to compute the Cholesky factor for a matrix A. For example:

```
>> A = [ 1  1  1;  1  2  3;  1  3  6 ]
A =
    1   1   1
    1   2   3
    1   3   6

>> R = chol( A )
R =
    1   1   1
    0   1   2
    0   0   1

>> R' * R
ans =
    1   1   1
    1   2   3
    1   3   6
```

- **QR Factorization of a Matrix:** Any matrix A can be factored as a product QR, where Q is orthogonal (or unitary) and R is upper triangular. This decomposition is used, for example, to compute the eigenvalues of a matrix and to solve least-squares problems. To compute the QR factorization of a matrix A, use the function **qr(A)**.

Suppose A = [1  0  1;  2  2  3;  0  1  3].

```
>> [Q, R] = qr( A )
Q =
   -0.4472    0.5963    0.6667
   -0.8944   -0.2981   -0.3333
         0   -0.7454    0.6667

R =
   -2.2361   -1.7889   -3.1305
         0   -1.3416   -2.5342
         0         0    1.6667
```

## Eigenvalues

- **Eigenvalues and Eigenvectors of a Matrix:** If A is a square matrix and if a scalar λ and a nonzero vector x satisfy the equation **Ax = λx**, then λ is called an eigenvalue and x is called an eigenvector of the matrix A. The function **eig(A)** allows you to compute eigenvalues and eigenvectors of a matrix. Suppose $A = \begin{bmatrix} 5 & -3 & 2; & -3 & 8 & 4; & 1 & 3 & -9 \end{bmatrix}$.

```
>> [V, D] = eig( A )
V =
   -0.4690    0.8714   -0.1763
    0.8760    0.4583   -0.2421
    0.1129    0.1751    0.9541

D =
   10.1218         0         0
         0    3.8243         0
         0         0   -9.9461
```

To verify that 10.1218 is an eigenvalue of A with corresponding eigenvector $\begin{bmatrix} -0.4690 \\ 0.8760 \\ 0.1129 \end{bmatrix}$, enter the following.

```
>> A * V( :, 1 )
ans =
   -4.7472
    8.8666
    1.1429
```

```
>>D( 1, 1 ) * V( :, 1 )
ans =
   - 4.7472
     8.8666
     1.1429
```

## Singular Value Decomposition

- **Singular Values of a Matrix:** If A is an m × n matrix with rank r, then there exist real numbers $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r > 0$, an orthonormal basis $v_1, \ldots, v_m$, and an orthonormal basis $u_1, \ldots, u_n$ such that

$$
\begin{array}{llll}
Av_i = \sigma_i u_i & i = 1,\ldots,r & A'u_i = \sigma_i v_i & i = 1,\ldots,r \\
Av_i = 0 & i = r+1,\ldots,m & A'u_i = 0 & i = r+1,\ldots,n
\end{array}
$$

In MATLAB, the function **svd(A)** computes the singular value decomposition for a matrix A.. Suppose A is the 2 × 3 matrix $\begin{bmatrix} 1 & 2 & 0; & 2 & 0 & 2 \end{bmatrix}$.

```
>> [U D V] = svd( A )
U =
   0.4472    0.8944
   0.8944  - 0.4472

D =
   3   0   0
   0   2   0

V =
   0.7454  - 0.0000  - 0.6667
   0.2981    0.8944    0.3333
   0.5963  - 0.4472    0.6667
```

The function returns two orthogonal matrices U and V, and a matrix D that contains the singular values of A in its diagonal entries. To verify **A = UDV'**, enter the following statement.

```
>> U * D * V'
ans =
   1.0000    2.0000  - 0.0000
   2.0000  - 0.0000    2.0000
```

## Sparse Matrices

A sparse matrix is a matrix that contains a relatively large number of zero entries. MATLAB provides a set of functions that stores only the nonzero entries of a sparse matrix and eliminates arithmetic operations on the zero entries.

- **Storage Information:** To find out the information about sparse and full versions of the same matrix, use the command **whos**. Suppose A = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$ and B = sparse (A).

  >> whos

  | Name | Size | Bytes | Class |
  |---|---|---|---|
  | A | 4x4 | 128 | double array |
  | B | 4x4 | 80 | sparse array |

- **Create a Sparse Matrix:** For a sparse matrix, MATLAB uses three arrays to store only the nonzero entries, their row indices, and their column indices. To create a sparse matrix, use the command **sparse( i, j, s, m, n )**, where i and j are the vectors that contain row and column indices for the nonzero entries of the matrix; s is a vector that contains a list of nonzero values whose indices are defined by the corresponding i, j pairs; m is the row dimension of the sparse matrix; and similarly n is the column dimension. To create a sparse matrix B of the above matrix A, for example, enter the following.

```
>> i = [ 1  2  3  3  4 ];
>> j = [ 1  4  2  3  3 ];
>> s =[ 1  2  1  5  3 ];
>> B = sparse( i, j, s, 4, 4 )
B =
   (1,1)      1
   (3,2)      1
   (3,3)      5
   (4,3)      3
   (2,4)      2
```

  Note that the resulting matrix above is the same as the one obtained using **sparse(A)**.

- **View Sparse Matrices:** There are a few useful commands to compute additional information about a sparse matrix. For example, to find the number of nonzero entries in the above sparse matrix B, use the command **nnz(B)**.

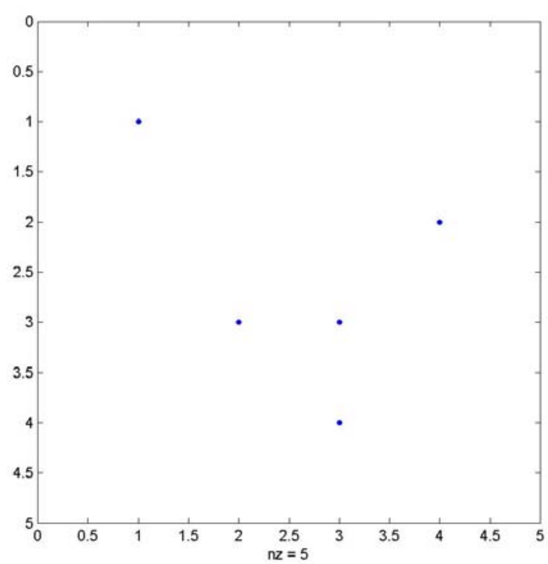
```
>> nnz( B )
ans =
    5
```

To obtain the list of nonzero entries of B, use the command **nonzeros(B)**.

```
>> nonzeros( B )
ans =
    1
    1
    5
    3
    2
```

To view the distribution of the nonzero entries of B, use the command **spy(B)**.

```
>> spy( B )
```



- **Sparse Matrix Functions:** There are numerous sparse matrix functions in MATLAB, e.g., for the solution of simultaneous linear equations, and for factorizations such as LU, QR, and Cholesky. The table below lists a set of commonly used sparse matrix functions.

| COMMAND | DESCRIPTION |
|---|---|
| isspars | True if matrix is sparse |
| find | Find indices and values of nonzero entries |
| spalloc | Allocate space for sparse matrix |
| sprank | Structure rank |
| speys | Sparse identity matrix |
| svds | A few singular values |
| eigs | A few eigenvalues |
| cholinc | Incomplete Cholesky factorization |
| luinc | Incomplete LU factorization |
| bicg | Biconjugate gradient iterative linear equation solution |
| bicstab | Biconjugate gradient stabilized iterative linear equation solution |
| cgs | Conjugate gradient squared iterative linear equation solution |
| gmres | Generalized minimum residual iterative linear equation solution |
| minres | Minimum residual iterative linear equation solution |
| pcg | Preconditioned conjugate gradient iterative linear equation solution |
| qmr | Quasi-minimal residual iterative linear equation solution |

## Iterative Methods

Two classes of methods can be used to solve systems of simultaneous linear equations, direct methods and iterative methods. Direct methods are more efficient for small linear systems; however, they may be very costly in terms of storage and computational time for large sparse linear systems. If convergent, iterative methods compute an approximate solution to a linear system and this may be much more efficient than using a direct method. In this section, we describe how to solve a linear system using MATLAB functions based on iterative methods.

- **Description:** The functions in MATLAB are intended to solve $Ax = b$ or $\min | b - Ax |$. A linear system is usually replaced by an equivalent system $M^{-1}Ax = M^{-1}b$, where M is a preconditioner that is chosen to make computation of the solution more efficient. The goal is to find a simple matrix M so that $M^{-1}Ax$ is near to the identity matrix. The table below lists a set of MATLAB functions corresponding to iterative methods.

| COMMAND | DESCRIPTION |
|---|---|
| bicg | Biconjugate gradient |
| bicgstab | Biconjugate gradient stabilized |
| cgs | Conjugate gradient squared |
| gmres | Generalized minimum residual |
| lsqr | Conjugate Gradients on the normal equations |
| minres | Minimum residual |
| pcg | Preconditioned conjugate gradient |
| qmr | Quasiminimal residual |
| symmlq | Symmetric LQ |

The basic syntax of the functions above is

**function_name (A, b, restart, tol, maxit, M)**

where **function_name** is the name of a function in the table above; **restart** defines the number of inner iterations such that the method restarts after every **restart** inner iterations; **tol** specifies the error tolerance of the method; **maxit** specifies the maximum number of outer iterations; and **M** is the preconditioner.

- **Example:** Suppose A is a 139 × 139 five-point discrete negative Laplacian, and b is a 139 × 1 column vector. Solve the linear system **Ax = b** using the generalized minimum residual method **gmres**. Note that A is a symmetric positive definite sparse matrix.

```
>> A =delsq( numgrid( 'C', 15 ) );
>> b = ones( 1, 139 )';
```

Perform the incomplete Cholesky factorization and use the factor R' of the matrix A as the preconditioner M. Note that $M^{-1}Ax = ( R' )^{-1}Ax = ( R' )^{-1}b$, and $( R' )^{-1}A$ is better conditioned than A.

```
>> R = cholinc( A, '0' );
>> condest( A )
ans =
   86.2192
>> condest( inv( R' ) * A )
ans =
   31.8511
```

Complete the computation of the solution x by typing the following command.

```
>> x = gmres( A, b, 12, 1e-5, 3, R' );
```

To verify the solution, type

```
>> y = A * x;
```

The entries of the vector y should all be equal to 1.

# Polynomial Roots and Interpolation

MATLAB provides a number of functions for manipulating polynomials, such as for root finding and curve fitting. In the following section, some examples are given to illustrate their use.

## Polynomials

- **Representation of a Polynomial:** MATLAB stores the coefficients of a polynomial in a row vector, ordered by descending powers. For example, the coefficients of the polynomial $p(x) = x^2 - 1$ can be entered in MATLAB as follows.

```
>> p = [ 1  0  -1 ]
p =
    1    0   -1
```

- **Find the Roots of a Polynomial Equation:** The roots of a polynomial equation $p(x) = 0$ are the real or complex values $\hat{x}$ for which $p(\hat{x}) = 0$. The roots can be computed using the command **roots(p)**. In the case of the above polynomial, the roots are calculated as follows.

```
>> r = roots( p )
r =
    -1
     1
```

- **Characteristic Polynomial of a Matrix:** The characteristic polynomial of an n × n matrix A is defined as **det(rI - A)**, where r is a variable and I is the n × n identity matrix . The characteristic polynomial is calculated using the command **poly(A)**. Suppose A = [1 2 3; 4 5 6; 7 8 0].

```
>> p = poly ( A )
p =
     1.0000   - 6.0000   - 72.0000   - 27.0000
```

The roots of this characteristic polynomial are the eigenvalues of A.

```
>> roots( p )
ans =
     12.1229
     - 5.7345
     - 0.3884
```

- **Polynomial Evaluation:** To evaluate a polynomial at a specified point, use the command **polyval(p, x)**. This function returns the value of the given polynomial p at the point x. Suppose $p(x) = x^2 - 1$.

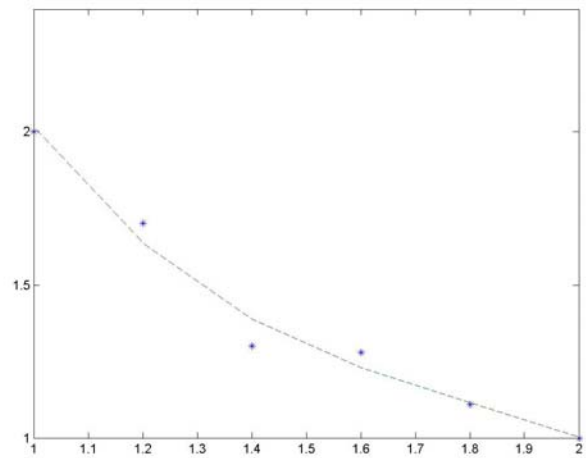
```
>> p = [ 1  0  -1 ];

>> polyval( p, 2 )
ans =
    3
```

- **Data Fitting:** The function **polyfit(x, y, n)** determines the polynomial $p(x)$ of degree of n that best fits the given data $(x_i, y_i)$, $1 \le i \le$ n, in the least squares sense. That is, $p(x) \approx y_i$ for $i$ = 1, 2,…, n. For example:

```
>> x =1 : 0.2 : 2
x =
    1.0000   1.2000   1.4000   1.6000   1.8000   2.0000

>> y =  [ 2  1.7  1.3  1.28  1.11  1 ]
y =
     2.0000   1.7000   1.3000   1.2800   1.1100   1.0000

>> p = polyfit( x, y, 3 )
p =
     - 0.9144   4.9494   - 9.4616   7.4432
```

To obtain a plot of the best least squares polynomial approximation of degree 3 to the data points, enter the following.

```
>> plot( x, y, '*', x, polyval(p, x), '--' )
```




- **Other Useful Functions:** The table below lists some useful polynomial functions.

| COMMAND | DESCRIPTION |
|---|---|
| conv | Polynomial multiplication |
| deconv | Polynomial division |
| polyder | Polynomial derivative |
| polyvalm | Matrix polynomial evaluation |
| residue | Partial fraction expansion |

## Polynomial Interpolation

- **One Dimensional Interpolation:** A polynomial $p(x)$ in one variable $x$ interpolates a given set of values $(x_i, y_i)$, $1 \leq i \leq$ n, if $p(x_i) = y_i$ for $i$ = 1, 2,…, n.

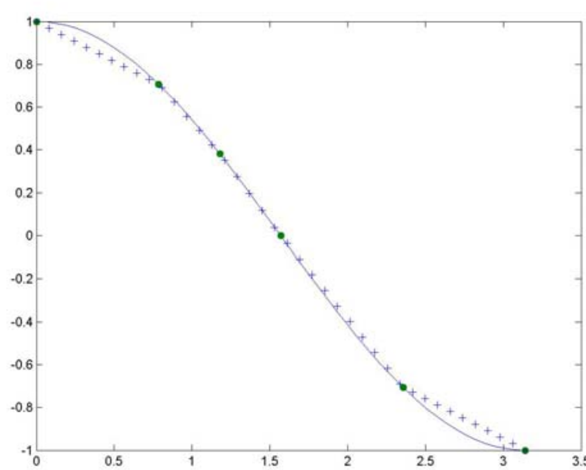
  - **Methods:** There are six methods available for one-dimensional interpolation. The default interpolation method is **linear**.

| METHOD | DESCRIPTION |
|---|---|
| linear | Linear interpolation |
| spline | Cubic spline interpolation |
| nearest | Nearest neighbor interpolation |
| pchip | Piecewise cubic Hermite interpolation |
| cubic | Piecewise cubic Hermite interpolation |
| v5cubic | Cubic interpolation used in MATLAB 5 |

- **Use:** The command **interp1( x, y, xx, method )** computes an interpolating polynomial p(x) of the type specified by the parameter method (see above) for the data specified in the vectors x and y, and returns the interpolated values p(xx). For example:

```
>> x = [ 0   pi/4   3*pi/8   pi/2   3*pi/4   pi ];
>> y = cos( x );
>> xx = linspace( 0, pi, 40 )';
>> yy = interp1( x, y, xx, 'linear' );
>> z = linspace( 0, pi, 50 )';
>> plot( z, cos( z ), '-', x, y, '.', 'MarkerSize', 20 )
>> hold on
>> plot( xx, yy, '+' )  % plot interpolated data
>> hold off
```

The result is shown in the figure below.



- **Two Dimensional Interpolation:** A polynomial $p(x, y)$ in two variables $x$ and $y$ interpolates a given set of values $(x_i, y_j, z_{ij})$, $1 \le i \le$ n and $1 \le j \le$ m, if $p(x_i, y_j) = z_{ij}$ for $i$ = 1, 2,…, n and $j$ = 1, 2,…, m.

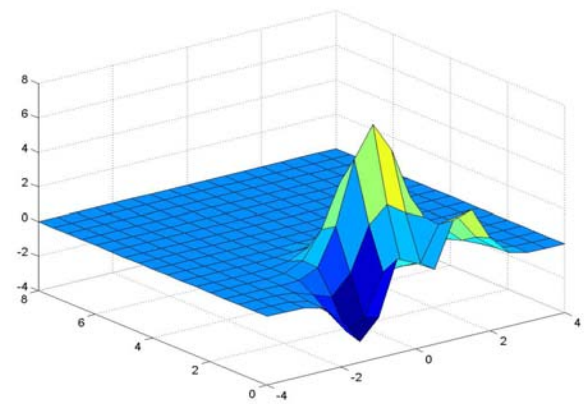
  - **Methods:** Four methods are available for two-dimensional interpolation. The default interpolation method is **linear**.

| METHOD | DESCRIPTION |
|---|---|
| linear | Bilinear interpolation |
| spline | Cubic spline interpolation |
| nearest | Nearest neighbor interpolation |
| cubic | Bicubic interpolation |

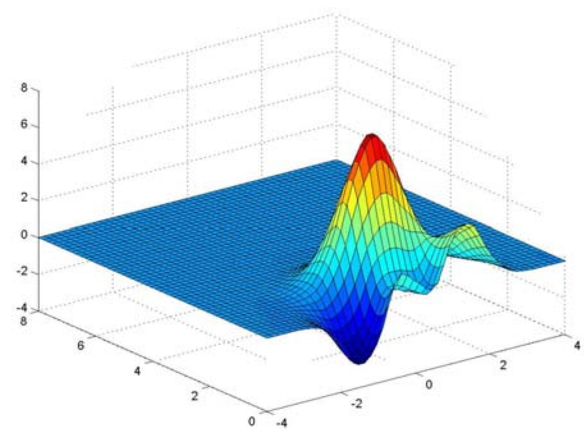
  - **Functions:** Similar to the command **interp1,** the command **interp2 ( X, Y, Z, XX, YY, method )** performs two-dimensional interpolation of the type specified by the parameter method (see above). The matrices X, Y and Z specify the given data to be interpolated; for example, $z_{ij} = f(x_{ij}, y_{ij})$. The arrays XX and YY specify the points at which the interpolating polynomial is evaluated. For example:

```
>> x = -4 : 0.5 : 4;
>> y = 0 : 0.5 : 8;
>> [X, Y] = meshgrid( x, y );
>> Z = peaks( X, Y );
>> xx = linspace( -4, 4, 50 );
>> yy = linspace( 0, 8, 50 );
>> [XX, YY] = meshgrid( xx, yy );
>> ZZ = interp2( X, Y, Z, XX, YX, 'bicubic' );
```

  Enter the command **surf( X, Y, Z )** to generate a plot of the original data.




Enter the command **surf( XX, YY, ZZ )** to generate a plot of the interpolated data.

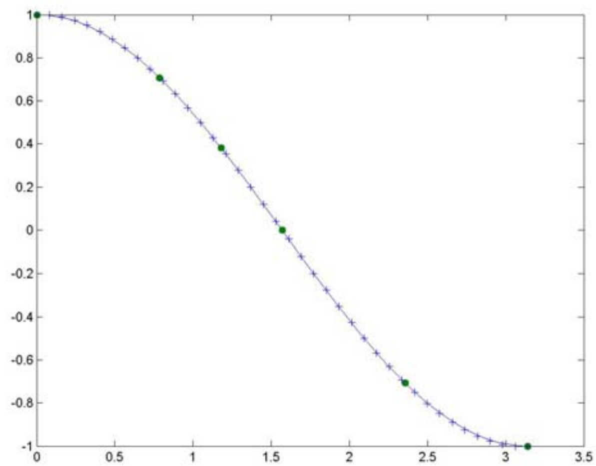



- **Spline Function:** The **spline** function can be used to do cubic spline interpolation. This function has two forms, **yy = spline( x, y, xx )** and **pp = spline( x, y )**. Given vectors x and y, the function computes the cubic spline interpolating polynomial S that interpolates the given data specified by the vectors x and y, and then it returns the values S(xx) in the vector yy. Alternatively, the **spline** function returns a data structure **pp** that contains the piecewise polynomial form of the cubic spline interpolant. This data structure is called the pp-form and can be used by other functions such as **ppval**. For example:

```
>> x = [ 0  pi/4  3*pi/8  pi/2  3*pi/4  pi ];
>> y = cos( x );
>> xx = linspace( 0, pi, 40 )';
>> yy = spline( x, y, xx );
```

```
>> z = linspace( 0, pi, 50 )';
>> plot( z, cos( z ), '-', x, y, '.', 'MarkerSize', 20 )
>> hold on
>> plot( xx, yy, '+' )
>> axis( [ 0  3.5  -1  1 ] )
>> hold off
```

The resulting plot is shown below. This is the same example as the one in the one-dimensional interpolation, except that the **spline** function is used instead.




If there is more than one set of interpolated values, the pp-form of the **spline** function can be used in combination with the function **ppval(pp, xx)**. For example:

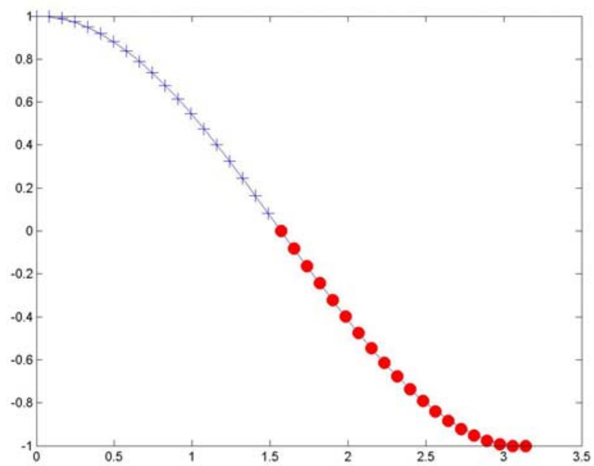
```
>> x = [ 0   pi/4   3*pi/8   pi/2   3*pi/4   pi ];
>> y = cos( x );
>> pp = spline( x, y );
>> xx1 = linspace( 0, pi/2, 20 )';
>> yy1 = ppval( pp, xx1 );
>> z = linspace( 0, pi, 50 )';
>> plot( z, cos( z ), '-' )
>> hold on
>> plot( xx1, yy1, '+', 'MarkerSize', 10 )
```

The statements above compute interpolated values yy1 on the interval [ 0, pi/2 ]. Similarly,

```
>> xx2 = linspace( pi/2, pi, 20 )';
>> yy2 = ppval( pp, xx2 );
>> plot( xx2, yy2, 'o',  'MarkerSize', 10, 'MarkerEdgeColor', 'r',
```

'MarkerFaceColor', 'r' )
>> axis( [ 0  3.5  -1  1 ] )
>> hold off

compute interpolated values yy2 on the interval [ pi/2,  pi ]. Note that there is no need to recompute the same set of cubic spline coefficients a second time; the previously computed pp-form can be used. The resulting plot is shown below.




To get details of the piecewise polynomial or the pp-form, use the function **unmkpp(pp)**.  For example, to print the knots and the coefficients of the computed spline function above, type the commands below.

```
>> [breaks, coefs] = unmkpp(pp)
breaks =
    0    0.7854    1.1781    1.5708    2.3562    3.1416

coefs =
    0.1159   -0.6123    0.0365    1.0000
    0.1159   -0.3392   -0.7108    0.7071
    0.1858   -0.2026   -0.9236    0.3827
    0.1356    0.0163   -0.9968    0.0000
    0.1356    0.3357   -0.7203   -0.7071
```

Note that the values of the breaks are the entries of the vector x above, and each row of the matrix coefs contains the coefficients of one of the cubic polynomials of the spline function.

# Quadrature

MATLAB provides a set of functions for evaluating definite integrals. In the following section, examples are given to illustrate the basic usage of these functions.

## Integrating Functions of One Variable

- **Description:** The numerical approximation of the definite integral $\int_a^b f(x)dx$ is called quadrature. The basic syntax of the MATLAB quadrature functions is

  **q = quad( fun, a, b )**

  where **fun** is the function to be integrated; **a** and **b** specify the interval of integration.

- **Example1:** Consider the function below.

$$F(x) = \int_a^b \frac{dx}{2x^2 + 3}$$

1. Write a MATLAB function for the function to be integrated. Note that the MATLAB function should allow the argument x to be a vector; that is, the ./ and .* operators are required in this function.

**M-file: myIntegral.m**

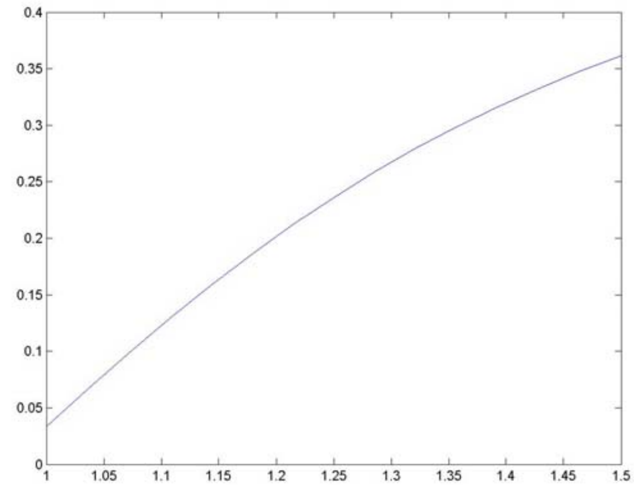
```
function y = myIntegral(x)

% Example for Quadrature

y = 1 ./ ( 2 .* x.^2 + 3 );
```

2. Run the following script to solve the given problem.

```
>> n = 15;
>> for m = 1 : n
        b = m * 0.1;
        Int(m) = quad( @myIntegral, 0, b );
   end
```

3. View the result.

```
>> x = linspace( 1, 1.5, 15 );
>> plot( x, Int )
```




- **Example2:** Another quadrature function in MATLAB is **trapz(x, y)**. This function computes a numerical approximation of the definite integral $\int_a^b f(x)dx$ by applying the trapezoidal rule. Consider the example below.

  Suppose $F(x) = \int_1^2 \frac{dx}{x}$. An approximation of this integral using **trapz** can be obtained as follows.

```
>> format long
>> x = linspace ( 1, 2, 50 );
>> y = 1 ./ x;
>> area = trapz( x,y )
area =
    0.69317321002551
```

  Note that the exact solution is ln 2 = 0.69314718055995….

- **Summary of Quadrature Functions:** The table below lists the quadrature functions in MATLAB.

| SOLVERS | DESCRIPTION | METHOD |
|---|---|---|
| quad | Adaptive Simpson quadrature | Simpson quadrature |
| quad1 | Adaptive Lobatto quadrature | Lobatto quadrature |
| dblquad | Evaluate double integral | Double integral |
| trapz | Trapezoidal numerical integration | Trapezoidal Rule |

# Ordinary Differential Equations

MATLAB provides software for solving both initial value and boundary value problems. In the following section, examples are given to illustrate how to solve initial value problems (a single differential equation and a system of differential equations).

## Initial Value Problems

- **Description:** These problems have the form

$$y' = f(t, y) \text{ subject to } y(t_0) = y_0,$$

where $t$ is a scalar variable (the independent variable), $y = y(t)$, and the initial condition is $y(t_0) = y_0$. The functions $y(t)$ and $f(t, y)$, and the constant $y_0$, can be vectors with more than one component.

- **Example1:** Solve the initial value problem

$$y' = y - t^2 + 1, \quad 0 \le t \le 2, \qquad y(0) = 0.5.$$

First create the function myODE below and save the function in the file myODE.m.

**M-file: myODE.m**

```
function dy = myODE(t,y)

% Initial Value Problem
% y' = y – t * t + 1,   0 ≤ t ≤ 2,           y( 0 ) = 0.5,
```

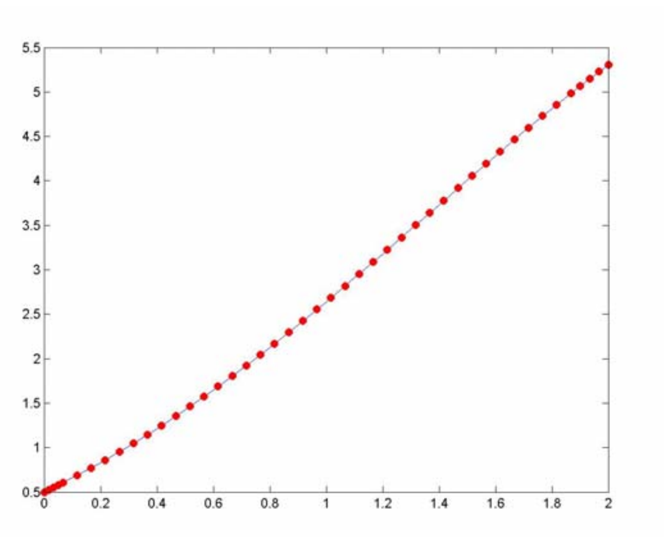
```
dy = y - t * t + 1;
```

To run the function myODE, enter the following.

```
>> tspan = [ 0  2 ];
>> yzero = 0.5;
>> [t, y] = ode45( @myODE, tspan, yzero );
```

The exact solution to the problem is $y(t) = (t+1)^2 - 0.5e^t$. The following script generates a plot, which compares the (approximate) computed solution with the exact solution.

```
>> w = [ 0 : .1 : 2 ];
>> f = ( w + 1 ) .^ 2 - 0.5 .* exp( w );
>> % Plot the exact solution
>> plot( w, f, '-' )
>> hold on
>> % Plot the computed solution
>> plot( t, y, 'o', 'MarkerEdgeColor', 'r', 'MarkerFaceColor', 'r' )
```




- **Example2:** Solve the second order initial value problem

$$y''-2y'+2y = e^{2t}\sin t, \quad 0 \le x \le 1, \quad y(0) = -0.4, \quad y'(0) = -0.6.$$

1. Rewrite the problem as a first order system.

Set $y_1 = y$ and $y_2 = y'$, and rewrite the second order equation as a system of two first order equations:

$$y'_1 = y_2, \quad y'_2 = e^{2t}\sin t + 2y_2 - 2y_1, \quad y_1(0) = -0.4,$$
$$y_2(0) = -0.6.$$

2. Write a function that evaluates the differential equations as follows.

**M-file: myODE2.m**

```
function dy = myODE2(t,y)

% Initial Value Problem for a Second-order Equation
% y1' = y2,    y2' = exp(2t)sin t + 2y2 - 2y1
%  y1(0) = -0.4, y2(0) = -0.6

dy = [y(2); exp(2 * t) * sin(t) + 2 * y(2) - 2 * y(1)];
```

3. Run the following script to solve the given problem.

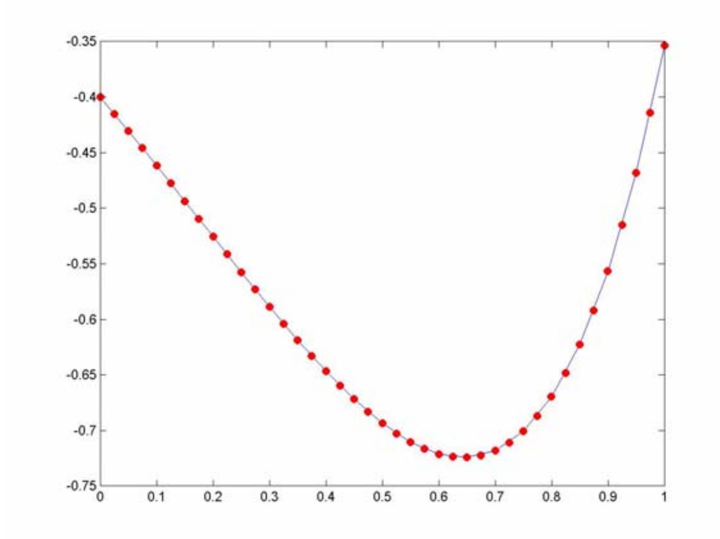
```
>> tspan = [ 0  1 ];
>> yzero =[ -0.4; -0.6 ];
>> [t, y] = ode45( @myODE2, tspan, yzero );
```

4. View the computed solutions.

The exact solution to the problem is $y(t) = 0.2e^{2t}(\sin t - 2\cos t)$ and $y'(t) = 0.2e^{2t}(4\sin t - 3\cos t)$. The following script generates a plot, which compares the (approximate) computed solution with the exact solution.

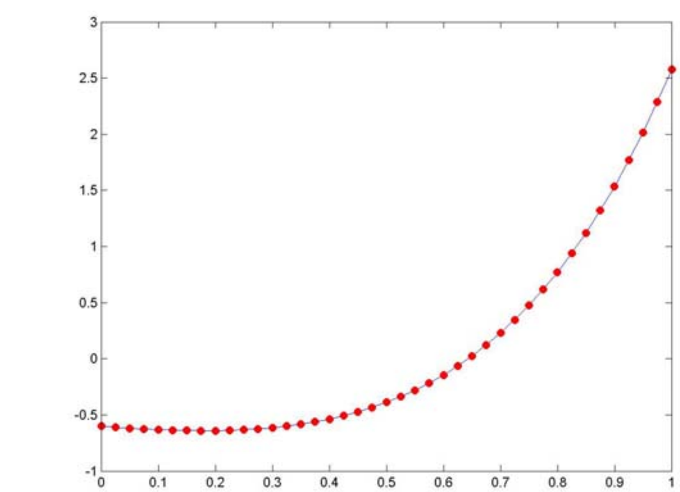
To plot the actual values and the computed values of $y$, type

```
>> w = [ 0 : 0.05 :1 ];
>> f = 0.2 .* exp( 2 .*  w ) .* ( sin( w ) - 2 .* cos( w ) );
>> % Plot the exact solution
>> plot( w, f, '-' )
>> hold on
>> % Plot the computed solution
>> plot( t, y(:, 1), 'o', 'MarkerEdgeColor', 'r', 'MarkerFaceColor', 'r' )
```




To plot the actual values and the computed values of $y'$, type

```
>> w = [ 0 : 0.05 :1 ];
>> f = 0.2 .* exp( 2 .*  w ) .* ( 4 .* sin( w ) - 3 .* cos( w ) );
>> % Plot the exact solution
>> plot( w, f, '-' )
>> hold on
>> % Plot the computed solution
>> plot( t, y(:, 2), 'o', 'MarkerEdgeColor', 'r', 'MarkerFaceColor', 'r' )
```



- **ODE Function Summary:** The table below lists the MATLAB initial value problem solvers.

| SOLVERS | DESCRIPTION | METHOD |
|---|---|---|
| ode45 | Nonstiff differential equations | Runge-Kutta |
| ode23 | Nonstiff differential equations | Runge-Kutta |
| ode113 | Nonstiff differential equations | Adams |
| ode15s | Stiff differential equations and DAEs | NDFs (BDFs) |
| ode23s | Stiff differential equations | Rosenbrock |
| ode23t | Moderately stiff differential equations and DAEs | Trapezoidal rule |
| ode23tb | Stiff differential equations | TR-BDF2 |

# Partial Differential Equations

Version 6 of MATLAB provides a solver for solving certain classes of parabolic and elliptic partial differential equations. In the following section, examples are given to illustrate how to use the solver.

## Parabolic and Elliptic Equations

- **Description:** The class of parabolic and elliptic partial differential equations that MATLAB can solve is of the form

$$c\left(x,t,u,\frac{\partial u}{\partial x}\right)\frac{\partial u}{\partial t} = x^{-m}\frac{\partial}{\partial x}\left(x^m f\left(x,t,u,\frac{\partial u}{\partial x}\right)\right) + s\left(x,t,u,\frac{\partial u}{\partial x}\right),$$

where $t_0 \le t \le t_f$, $a \le x \le b$, and $m =$ 0, 1 or 2.

The vector-valued function $u$ is a function of a space variable $x$ and a time variable $t$. At the initial time $t = t_0$, the solution must satisfy initial conditions of the form $u(x,t_0) = u_0(x)$. In addition, at the boundaries $x = a$ and $x = b$, the solution must satisfy a boundary condition of the form

$$p(x,t,u) + q(x,t)f\left(x,t,u,\frac{\partial u}{\partial x}\right) = 0.$$

MATLAB provides a PDE solver **pdepe**. The basic syntax of this solver is:

**sol = pdepe(m, pdefun, icfun, bcfun, xmesh, tspan)**

where **m** corresponds to $m$ in the form of PDE; **pdefun** computes the terms $c$, $f$, and $s$ in the form of PDE; **icfun** evaluates the initial conditions; **bcfun** evaluates the terms $p$ and $q$ in the form of the boundary condition; **xmesh** is a vector specifying the points between $a$ and $b$; and **tspan** is a vector specifying the points between $t_0$ and $t_f$.

- **Example:** The example below illustrates the steps to solve a given parabolic partial differential equation problem.

Consider the heat equation

$$\frac{\partial u}{\partial t}(x,t) - \frac{\partial^2 u}{\partial x^2}(x,t) = 0, \qquad 0 < x < 1, \qquad t > 0,$$

with boundary conditions

$$u(0,t) = u(1,t) = 0, \quad t > 0,$$

and initial conditions

$$u(x,0) = \sin(\pi x), \qquad 0 \le x \le 1.$$

1. Rewrite the PDE in the required form.

$$\frac{\partial u}{\partial x} = x^0 \frac{\partial}{\partial x}\left( x^0 \frac{\partial u}{\partial x}\right) + 0$$

Given the form above, $m = 0$ and

$$c\left( x,t,u,\frac{\partial u}{\partial x}\right) = 1$$

$$f\left( x,t,u,\frac{\partial u}{\partial x}\right) = \frac{\partial u}{\partial x}$$

$$s\left( x,t,u,\frac{\partial u}{\partial x}\right) = 0$$

2. Write a function that evaluates the terms $c$, $f$, and $s$ in the differential equation as follows.

**M-file: pdeHeat.m**

```
function [c,f,s] = pdeHeat(x, t, u, DuDx)

% Set required c, f, s for the equation

c = 1;
f = DuDx;
s = 0;
```

3. Write the function that represents the initial conditions as follows.

**M-file: pdeHeatic.m**

```
function u0 = pdeHeatic (x)

% inital condition u(x,0) = sin(pi*x)

u0 = sin(pi*x);
```

4. Rewrite the boundary conditions in the required form.

$$u(0,t) + 0 \cdot \frac{\partial u}{\partial x}(0,t) = 0 \qquad \text{at } x = 0$$

$$u(1,t) + 0 \cdot \frac{\partial u}{\partial x}(1,t) = 0 \qquad \text{at } x = 1$$

5. Write the function that represents the boundary conditions.

**M-file: pdeHeatbc.m**

```
function [pl,ql,pr,qr] = pdeHeatbc(xl,ul,xr,ur,t)

% Set boundary conditions u(0,t) = u(1,t) = 0

pl = ul;
ql = 0;
```

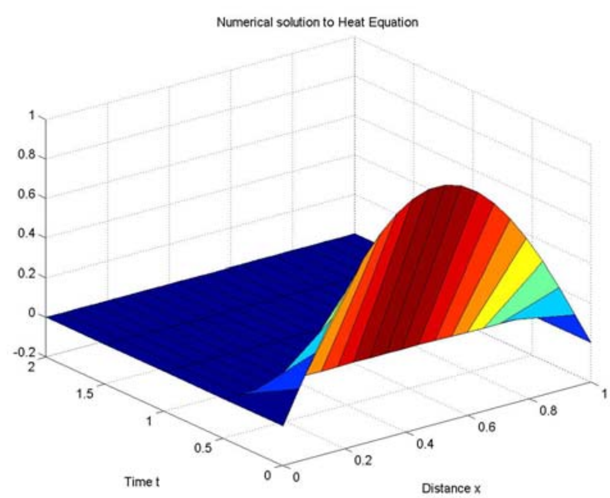
```
pr = ur;
qr = 0;
```

6. Run the following script to solve the given problem.

```
>> m = 0;
>> x = linspace( 0, 1, 20 );
>> t = linspace( 0, 2, 5 );
>> sol = pdepe( m, @pdeHeat, @pdeHeatic, @pdeHeatbc,x,t );
```

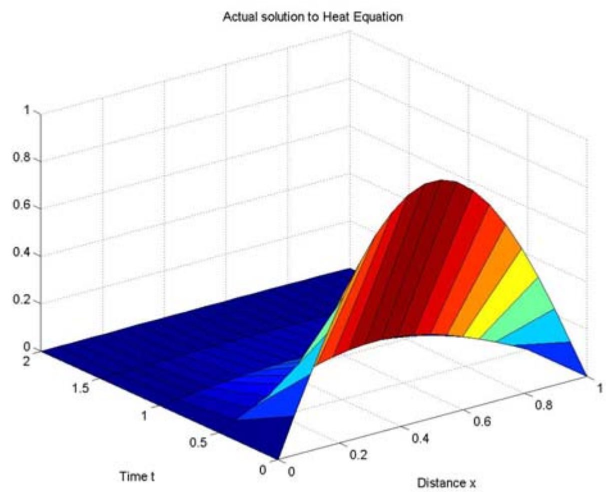
The following script generates a plot of the numerical solution.

```
>> u = sol( :, :, 1 );
>> surf( x, t, u )
>> title( 'Numerical solution to Heat Equation' )
>> xlabel( 'Distance x' )
>> ylabel( 'Time t' )
```




The actual solution to this problem is $u(x,t) = e^{-\pi^2 t}\sin(\pi x)$. Run the following script to generate a plot of this solution and compare the two solutions.

```
>> ureal = exp( -pi .* t )' * sin( pi .* x );
>> surf( x, t, ureal )
>> title( 'Actual solution to Heat Equation' )
>> xlabel( 'Distance x' )
>> ylabel( 'Time t' )
```




# Other Useful Functions

There are a few other useful algebraic functions and functions for numerical analysis, which are described as follows.

## Functions for Nonlinear Algebraic Equations

- **Find a Zero of a Function of One Variable:** The function **fzero** finds a zero of a function of one variable near a point $x_0$ or within a given range. For example:

  ```
  >> fzero ( @sin, [ 2  4 ])
  ans =
      3.1416
  ```

## Functions for Data Analysis

- **Minimize a Function of One Variable:** The function **fminbnd** finds a local minimizer of a function of one variable within a given range. A local minimizer $\hat{x}$ of $f(x)$ is a value of $x$ such that $f(\hat{x})$ is minimum in an interval around $\hat{x}$. For example:

  ```
  >> fminbnd ( 'sin', -pi, pi )
  ans =
      -1.5708
  ```

  Note that the minimum value returned is -pi/2.

- **Maximum and Minimum Entries of an Array:** The functions **max** and **min** return the largest and the smallest entries, respectively, along a specified dimension of an array. For example:

```
>> w = [ 1  3  -3.56  4.1 ];
>> min ( w )
ans =
   -3.5600

>> max ( w )
ans =
   4.1000
```

- **Sum and Cumulative Sum:** The function **sum** computes the sum of the entries of an array along a specified dimension. Similarly, the function **cumsum** computes the cumulative sum of the entries of an array. For example:

```
>> A = [ 1  2  3;  5  7  9;  0  4  1 ]
A =
    1    2    3
    5    7    9
    0    4    1

>> sum( A )
ans =
    6   13   13

>> cumsum( A )
ans =
    1    2    3
    6    9   12
    6   13   13
```

- **Product and Cumulative Product:** The function **prod** computes the product of the entries of an array along a specified dimension. Similarly, the function **cumprod** computes the cumulative product. For example:

```
>> A = [ 1  2  3;  5  7  9;  0  4  1 ]
A =
    1   2   3
    5   7   9
    0   4   1

>> prod( A )
ans =
    0   56   27

>> cumprod( A )
ans =
    1    2    3
    5   14   27
    0   56   27
```

- **Sort Elements:** The function **sort** sorts elements in ascending order. For example:

```
>> w = [ -1  6  -4  0 ];
>> sort( w )
ans =
   -4   -1   0   6
```

- **Differences:** The function **diff** computes differences between adjacent entries of an array along a specified dimension. For example:

```
>> C = [ 2  5  8;  1  6  10;  3  6  5 ]
C =
    2   5    8
    1   6   10
    3   6    5
>> diff( C )
ans =
   -1   1    2
    2   0   -5
```

A

# References

[1] Desmond J. Higham and Nicholas J. Higham. *MATLAB Guide*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2000. ISBN 0-89871-469-9.

[2] *Getting Started with MATLAB Version 6*. The Math Works, Inc., Natick, MA, USA, 2000.

[3] Duane Hanselman and Bruce Littlefield. *Mastering MATLAB 6: a comprehensive tutorial and reference*. Prentice Hall, Upper Saddle River, New Jersey, USA, 2001. ISBN 0-13-019468-9

[4] Rudra Pratap. *Getting Started with MATLAB 5: A Quick Introduction for Scientists and Engineers*. Oxford University Press, Inc., New York, New York, USA, 1999. ISBN 0-19-512947-4.

[5] Richard L. Burden and J. Douglas Faires. *Numerical Analysis*. PWS Publishing Company, USA, 1993. ISBN 0-534-93219-3.

[6] *Using MATLAB Version 6*. The Math Works, Inc., Natick, MA, USA, 2000.

[7] Roger A. Horn. *Matrix Analysis*. Cambridge University Press, Cambridge, United Kingdom, 1985. ISBN 0-521-38632-2.

[8] David S. Watkins. *Fundamentals of Matrix Computations*. Jon Wiley & Sons, Inc, New York, New York, USA, 1991. ISBN 0-471-61414-9.