CHAPTER

5

TEXAS INSTRUMENTS TI-92

5.1 Getting Started with the TI-92

In this book, the key with the green diamond symbol inside a green border will be indicated by ◆, the key with the white arrow pointing up inside a white border (the *shift* key*)* will be indicated by ⇧, and the key with the white arrow (the *backspace* key) pointing to the left will be indicated by ⇦. Although the cursor pad allows for movements in eight directions, we will mainly use the four directions of up, down, right, and left. These directions will be indicated by 🡅, 🡇, ➔, and 🡄, respectively

There are eight blue keys on the left side of the calculator labeled F1 through F8. These *function keys* have different effects depending on the screen that is currently showing. The effect or menu of the function keys corresponding to a screen are shown across the top of the display.

5.1.1 Basics: Press the ON key to begin using your TI-92. If you need to adjust the display contrast, first press ◆, then press – (the minus key) to lighten or + (the plus key) to darken. To lighten or darken the screen more, press ◆ then + or – again. When you have finished with the calculator, turn it off to conserve battery power by pressing 2nd and then OFF. Note that the TI-92 has three ENTER keys and two 2nd keys which can be used interchangeably.

Check your TI-92's settings by pressing MODE. If necessary, use the cursor pad to move the blinking cursor to a setting you want to change You can also use F1 to go to page 1 or F2 to go to page 2 of the MODE menu. To change a setting, use 🡇 to get to the setting that you want to change, then press ➔ to see the options available. Use 🡅 or 🡇 to highlight the setting that you want and press ENTER to select the setting. To start with, select the options shown in Figures 5.1 and 5.2: function graphs, main folder, floating decimals with 10 digits displayed, radian measure, normal exponential format, real numbers, rectangular vectors, pretty print, full screen display, Home screen showing, and approximate calculation mode. Note that some of the lines on page 2 of the MODE menu are not readable. These lines pertain to options that are not set as above. Details on alternative options will be given later in this guide. For now, leave the MODE menu by pressing ◆ HOME or 2nd QUIT. Some of the current settings are shown on the status line of the Home screen.

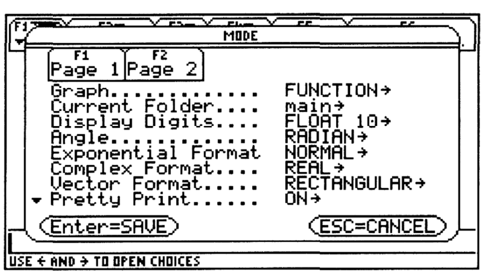


Figure 5.1: MODE menu, page 1

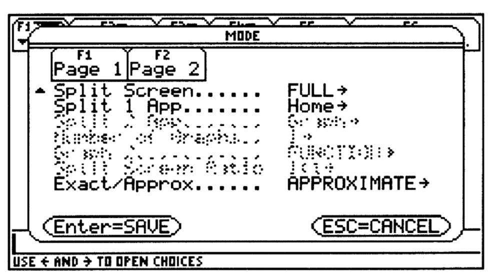


Figure 5.2: MODE menu, page 2

Technology Tip: There are many different ways to get to the most commonly used screens on your TI-92. One method is by using the APPS menu (Figure 5.3) which is accessed by pressing the blue APPS key on the right side of the calculator. Thus, to get to the Home screen you can press 2nd QUIT, ◆ HOME, or APPS ENTER.

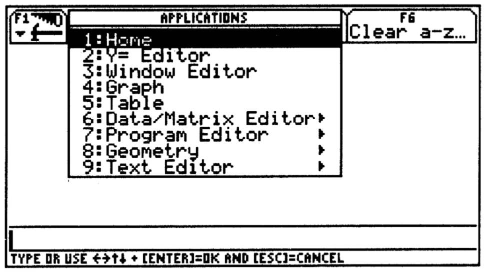


Figure 5.3: APPS menu

5.1.2 Editing: One advantage of the TI-92 is that you can use the cursor pad to scroll in order to see a long calculation. For example, type this sum (Figure 5.4):

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20

Then press ENTER to see the answer. The sum is too long for both the entry line and the history area. The direction(s) in which the line extends off the screen is indicated by an ellipsis at the end of the entry line and arrows (🡐 or 🡒) in the history area. You can scroll through the entire calculation by using the cursor pad (🡑 or ,🡓) to put the cursor on the appropriate line and then using 🡒 or 🡐 to move the cursor to the part of the calculation that you wish to see.

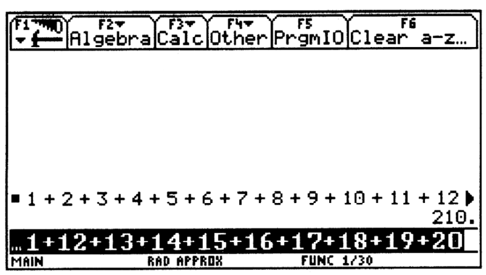


Figure 5.4: Home screen

Often we do not notice a mistake until we see how unreasonable an answer is. The TI-92 permits you to redisplay an entire calculation, edit it easily, then execute the *corrected* calculation.

Suppose you had typed 12 + 34 + 56 as in Figure *5.5* but had *not yet* pressed ENTER, when you realize that 34 should have been 74. Simply press the 🡐 direction on the cursor pad as many times as necessary to move the blinking cursor line until it is to the immediate right of the 3, press ⇦ to delete the 3, and then type 7. On the other hand, if 34 should have been 384, move the cursor until it is between the 3 and the 4 and then type 8. If the 34 should have been 3 only, move the cursor to right of the 4, and press ⇦ to delete the 4.

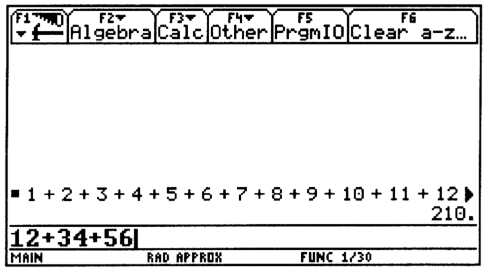


Figure 5.5: Editing a calculation

Technology Tip: The TI-92 has two different inputing modes: *insert* and *overtype.* The default mode is the insert mode, in which the cursor is a blinking vertical line and new text will be inserted at the cursor's position and other characters are pushed to the right. In the overtype mode, the cursor is a blinking square and the characters that you type replace the existing characters. To change from one mode to another, press 2nd INS. The TI-92 remains in whatever the last input mode was, even after being turned off.

Even if you had pressed ENTER, you may still edit the previous expression. Immediately after you press ENTER your entry remains on the entry line. Pressing the 🡐 direction on the cursor pad moves the cursor to the beginning of the line, while pressing the ➔ direction on the cursor pad puts the cursor at the end of the line. Now the expression can be edited as above. To edit a previous expression that is no longer on the entry line, press 2nd and then ENTRY to recall the prior expression. Now you can change it. In fact, the TI-92 retains as many entries as the current history area holds in a "last entry" storage area, including entries that have scrolled off of the screen. Press 2nd ENTRY repeatedly until the previous line you want is on the entry line. (The number of entries that the history area can hold may be changed, see your user's manual for more information.)

To clear the entry line, press CLEAR while the cursor is on that line. To clear previous entry/answer pairs from the history area, use the cursor pad to either the entry or the answer and press CLEAR (both the entry and the answer will be deleted from the display). To clear the entire history area, press F1[*Tools*] 8 *[Clear Home],* although this will not clear the entry line.

Technology Tip: When you need to evaluate a formula for different values of a variable, use the editing feature to simplify the process. For example, suppose you want to find the balance in an investment account if there is now $5000 in the account and interest is compounded annually at the rate of 8.5%. The formula for the balance is $P\left(1+\frac{r}{n}\right)^{nt}$, where P = principal, r = rate of interest (expressed as a decimal), n = number of times interest is compounded each year, and t = number of years. In our example, this becomes $5000(1 + .085)^t$. Here are the keystrokes for finding the balance after t = 3, 5, and 10 years.

Years	*Keystrokes*	*Balance*
3	5000 (1 + .085) ∧ 3 ENTER	$6,386.45
5	➔ ⇦ 5 ENTER	$7,518.28
10	➔ ⇦ 10 ENTER	$11,304.92

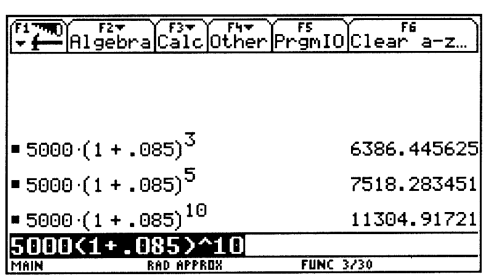


Figure 5.6: Editing expressions

Then, to find the balance from the same initial investment but after 5 years when the annual interest rate is 7.5%, press the following keys to change the last calculation above: ➔ ⇦ ⇦ 5 🡐 🡐 🡐 🡐 ⇦ 7 ENTER. You could also use the CLEAR key to erase everything to the right of the current location of the cursor. Then, changing the calculation from 10 years at the annual interest rate of 8.5% to 5 years at the annual interest rate of 7.5% is then done by pressing ➔ 🡐 🡐 CLEAR 5 🡐 🡐 🡐 🡐 ⇦ 7 ENTER.

5.1.3 Key Functions: Most keys on the TI-92 offer access to more than one function, just as the keys on a computer keyboard can produce more than one letter ("g" and "G") or even quite different characters ("5" and "%"). The primary function of a key is indicated on the key itself, and you access that function by a simple press on the key.

To access the *second* function indicated in yellow or to the *left* above a key, first press 2nd ("2nd" appears on the status line) and *then* press the key. For example to calculate $\sqrt{25}$, press 2nd √ 25) ENTER.

Technology Tip: The TI-92 automatically places a left parenthesis, (, after many functions and operators (including LN, 2nd e^x, SIN, COS, TAN, and 2nd √). If a right parenthesis is not entered, the TI-92 will respond with an error message indicating that the right parenthesis is missing.

When you want to use a function printed in green or to the *right* above a key, first press ◆ ("◆"appears on the status line) and then press the key. For example, if you are in EXACT calculation mode and want to find the approximate value of $\sqrt{45}$ press 2nd √ 45) ◆ ≈. The QWERTY keyboard on the TI-92 is similar to a typewriter and can produce both upper and lower case letters. To switch from one case to another, press 2nd CAPS. For a single upper case letter, use the ⇧ key. There are also additional symbols available from the keyboard by using the 2nd and ◆ keys. Some of the most commonly used symbols are marked on the keyboard, but most are not. See your TI-92 user's manual for more information.

5.1.4 Order of Operations: The TI-92 performs calculations according to the standard algebraic rules. Working outwards from inner parentheses, calculations are performed from left to right. Powers and roots are evaluated first, followed by multiplications and divisions, and then additions and subtractions.

Enter these expressions to practice using your TI-92.

Expression	*Keystrokes*	*Display*
$7-5\cdot 3$	7 – 5 × 3 ENTER	–8
$(7-5)\cdot 3$	(7 – 5) × 3 ENTER	6
$120-10^2$	120 – 10 ∧ 2 ENTER	20
$(120-10)^2$	(120 – 10) ∧ 2 ENTER	12100
$\frac{24}{2^3}$	24 ÷ 2 ∧ 3 ENTER	3
$\left(\frac{24}{2}\right)^3$	(24 ÷ 2) ∧ 3 ENTER	1728
$(7--5)\cdot -3$	(7 – (-) 5 × (-) 3 ENTER	–36

5.1.5 Algebraic Expressions and Memory: Your calculator can evaluate expressions such as $\frac{N(N+1)}{2}$ *after* you have entered a value for *N*. Suppose you want *N* = 200. Press 200 STO▸ N ENTER to store the value 200 in memory location *N*. Whenever you use *N* in an expression, the calculator will substitute the value 200 until you make a change by storing *another* number in *N*. Next enter the expression $\frac{N(N+1)}{2}$ by typing N x (N + 1) ÷ 2

ENTER. For $N = 200$, you will find that $\frac{N(N+1)}{2} = 20100$. Note that there is no distinction made between upper and lower case letters in this case.

The contents of any memory location may be revealed by typing just its letter name and then ENTER. And the TI-92 retains memorized values even when it is turned off, so long as its batteries are good.

5.1.6 Repeated Operations with ANS: As many entry/answer pairs as the history area shows are stored in memory. The last result displayed can be entered on the entry line by pressing 2nd ANS, while the last entry computed is entered on the entry line by pressing 2nd ENTRY. This makes it easy to use the answer from one computation in another computation. For example, press 30 + 15 ENTER so that 45 is the last result displayed. Then press 2nd ANS ÷ 9 ENTER and get 5 because 45 ÷ 9 = 5.

The answer locations are indexed by ans(#), and the entry locations are indexed by entry(#), where # indicates the number of the entry/answer. The pairs are numbered with the most recent computation as 1. Hence the number of a pair changes with each successive computation that is entered. The number of an entry or answer can be found by using the cursor pad (↑) to scroll up to the entry or answer. The number, which is the same for both the entry and the answer, is shown on the bottom of the screen.

To use an earlier answer or entry in a computation, to calculate, say 15 times answer 3 plus 75, press 1 5 x A N S (3) + 7 5 ENTER, using the keyboard to type the letters A, N, and S.

With a function like division, you press the ÷ *after* you enter an argument. For such functions, whenever you would start a new calculation with the previous answer followed by pressing the function key, you may press just the function key. So instead of 2nd ANS ÷ 9 in the previous example, you could have pressed simply ÷ 9 to achieve the same result. This technique also works for these functions: + – × ∧ 2nd x^{-1}.

Here is a situation where this is especially useful. Suppose a person makes $5.85 per hour and you are asked to calculate earnings for a day, a week, and a year. Execute the given keystrokes to find the person's incomes during these periods (results are shown in Figure 5.7).

Pay Period	*Keystrokes*	*Earnings*
8-hour day	5.85 × 8 ENTER	$46.80
5-day week	× 5 ENTER	$234
52-week year	× 52 ENTER	$12,168

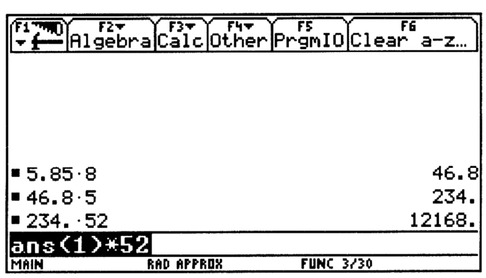


Figure 5.7: ANS variable

5.1.7 The MATH Menu: Operators and functions associated with a scientific calculator are available either immediately from the keys of the TI-92 or by the 2nd keys. You have direct access to common arithmetic operations (2nd $\sqrt{\ }$, 2nd x^{-1}, ∧), trigonometric functions (SIN, COS, TAN), and their inverses (2nd SIN^{-1}, 2nd COS^{-1}, 2nd TAN^{-1}), exponential and logarithmic functions (LN, 2nd e^x), and a famous constant (2nd π).

A significant difference between the TI-92 graphing calculators and most scientific calculators is that TI-92 requires the argument of a function *after* the function, as you would see in a formula written in your textbook. For example, on the TI-92 you calculate $\sqrt{16}$ by pressing the keys 2nd √ 16) in that order.

Here are keystrokes for basic mathematical operations. Try them for practice on your TI-92.

Expression	*Keystrokes*	*Display*
$\sqrt{3^2+4^2}$	2nd √ (3 ∧ 2 + 4 ∧ 2) ENTER	5
$2\frac{1}{3}$	2 + 3 2nd x^{-1} ENTER	2.333333333
ln 200	LN 200) ENTER	5.298317367
$2.34 \cdot 10^5$	2.34 × 10 ∧ 5 ENTER	234000

Technology Tip: Note that if you had set the calculation mode to either AUTO or EXACT (the last line of page 2 of the MODE menu), the TI-92 would display $\frac{7}{3}$ for $2\frac{1}{3}$ and 2 ln(5) + 3 ln(2) for ln 200. Thus, you can use either fractions and exact numbers or decimal approximations. The AUTO mode will give exact rational results whenever all of the numbers entered are rational, and decimal approximations for other results.

Additional mathematical operations and functions are available from the MATH menu. Press 2nd MATH to see the various sub-menus. Press 1*[Number]* or just ENTER to see the options available under the Number sub-menu. You will learn in your mathematics textbook how to apply many of them. As an example, calculate the remainder of 437 when divided by 49 by pressing 2nd MATH 1*[Number]* then *either* A*[remain(] or* ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ENTER; finally press 437 , 49) ENTER to see 45. To leave the MATH menu (or any other menu) and take no other action, press 2nd QUIT or just ESC.

Note that you can select a function or a sub-menu from the current menu by pressing either ↓ until the desired item is highlighted and then ENTER, or by pressing the number or letter corresponding to the function or sub-menu. It is easier to press the letter A than to press ↓ nine times to get the remain(function.

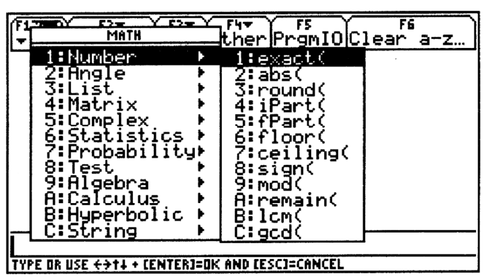


Figure 5.8: MATH menu and Number sub-menu

The *factorial* of a non-negative integer is the *product* of *all* the integers from 1 up to the given integer. The symbol for factorial is the exclamation point. So 4! (pronounced *four factorial) is* $1 \cdot 2 \cdot 3 \cdot 4 = 24$. You will learn more about applications of factorials in your textbook, but for now use the TI-92 to calculate 4! Press these keystrokes: 4 2nd MATH 7*[Probability]* 1*[!]* ENTER.

On the TI-92 it is possible to do calculations with complex numbers. To enter the imaginary number *i*, press 2nd *i*. For example, to divide 2 + 3*i* by 4 – 2*i*, press (2 + 3 2nd *i*) ÷ (4 – 2 2nd *i*) ENTER. The result is 0.1 + 0.8*i* (Figure 5.9).

To find the complex conjugate of 4 + 5*i* press 2nd MATH 5*[Complex]* ENTER 4 + 5 2nd *i*) ENTER (Figure 5.9).

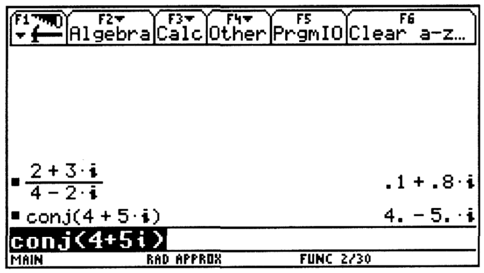


Figure 5.9: Complex number calculations

The TI-92 can also solve for the real and complex roots of an equation. This is done by using the cSolve(function which is not on any of the keys, but can be found in the CATALOG. From the Home screen, pressing 2nd CATALOG gives an alphabetical list of all functions and operations available on the TI-92. You can scroll through the CATALOG page-by-page by pressing 2nd ↓, or if you know what letter the function starts with, pressing the letter moves the cursor to the beginning of the listings for that letter.

The format of cSolve(is cSolve(*expression, variable*). For example, to find the zeros of $f(x) = x^3 - 4x^2 + 14x - 20$, from the Home screen press 2nd CATALOG and move the cursor down to cSolve(, then press ENTER. The display will return to the Home screen, with cSolve(on the entry line. To complete the computation, press X ∧ 3 – 4 X ∧ 2 + 14 X – 20 = 0 , X) ENTER. The TI-92 will display the real and complex roots of the equation, as shown in Figure 5.10.

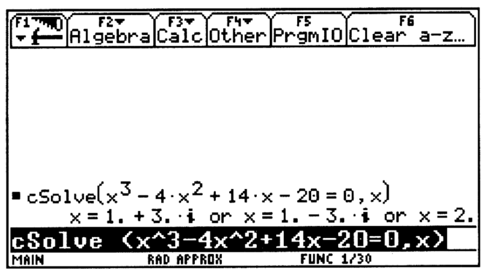


Figure 5.10: cSolve function

All functions and commands found in the CATALOG, can also be used by merely typing the command using the keyboard. Hence, in the Home screen, you could also press C S 0 L V E (X ∧ 3 – 4 X ∧ 2 + 14 X – 20 = 0 , X) ENTER to find the zeros of $f(x) = x^3 - 4x^2 + 14x - 20$.

5.2 Functions and Graphs

5.2.1 Evaluating Functions: Suppose you receive a monthly salary of $1975 plus a commission of 10% of sales. Let x = your sales in dollars; then your wages W in dollars are given by the equation $W = 1975 + .10x$. If your January sales were $2230 and your February sales were $1865, what was your income during those months?

Here's one method to use your TI-92 to perform this task. Press the ◆ Y= key (above the letter W) or APPS *2[Y= Editor]* to display the function editing screen (Figure 5.11). You may enter as many as ninety-nine different functions for the TI-92 to use at one time. If there is already a function y1 press ↑ or ↓ as many times as necessary to move the cursor to y1 and then press CLEAR to delete whatever was there. Then enter the expression 1975 +. 10x by pressing these keys: 1975 + .1 0 X ENTER. Now press ◆ HOME.

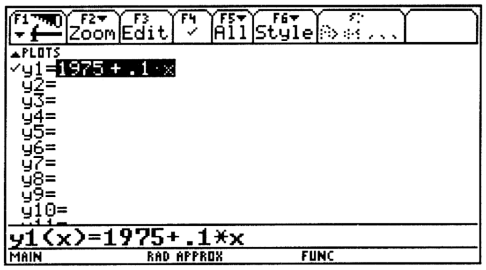


Figure 5.11: Y= screen

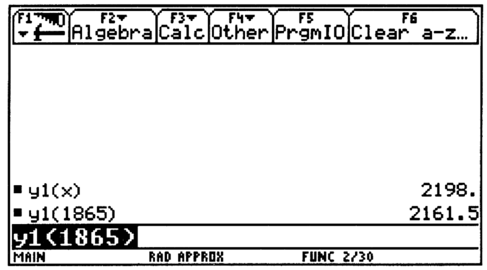


Figure 5.12: Evaluating a function

Assign the value 2230 to the variable x by these keystrokes: 2230 STO▸ X ENTER. Then press the following keystrokes to evaluate y1 and find January's wages: Y 1 (X) ENTER, completes the calculation. It is not necessary to repeat all these steps to find the February wages. Simply press ➔ to begin editing the previous entry, change X to 1865, and press ENTER (see Figure 5.12).

You may also have the TI-92 make a table of values for the function. Press ◆ TblSet to set up the table (Figure 5.13). Move the blinking cursor down to the fourth line beside Independent:, then press ➔ and *2[ASK]* ENTER. This configuration permits you to input values for x one at a time. Now press ◆ TABLE or APPS *5[Table],* enter 2230 in the x column, and press ENTER (see Figure 5.14). Press ➔ to move to the next line and continue to enter additional values for x. The TI-92 automatically completes the table with the corresponding values of y1. Press 2nd QUIT to leave the TABLE screen.

Technology Tip: The TI-92 requires multiplication to be expressed between variables, so *xxx* does not mean x^3, rather it is a new variable named *xxx*. Thus, you must use either ×'s between the x's or ∧ for powers of x. Of course, expressed multiplication is not required between a constant and a variable. See your TI-92 manual for more information about the allowed usage of implied multiplication.

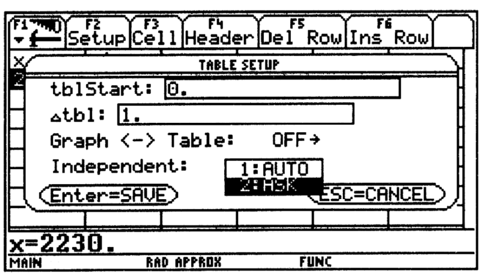


Figure 5.13: Table Setup screen

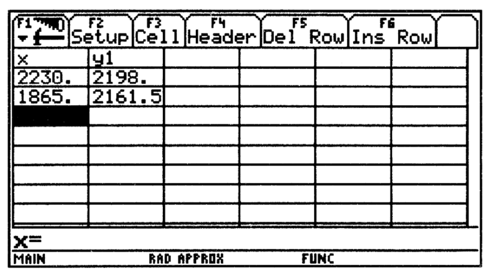


Figure 5.14: Table of values

5.2.2 Functions in a Graph Window: Once you have entered a function in the Y= screen of the TI-92, just press ◆ GRAPH to see its graph. The ability to draw a graph contributes substantially to our ability to solve problems.

For example, here is how to graph $y = -x^3 + 4x$. First press ◆ Y= and delete anything that may be there by moving with the arrow keys to y1 or to any of the other lines and pressing CLEAR wherever necessary. Then, with the cursor on the (now cleared) top line (y1), press (-) X ∧ 3 + 4 X ENTER to enter the function (as in Figure 5.15). Now press ◆ GRAPH and the TI-92 changes to a window with the graph of $y = -x^3 + 4x$. While the TI-92 is calculating coordinates for a plot, it displays a the word BUSY on the status line.

Technology Tip: If you would like to see a function in the Y= menu and its graph in a graph window, both at the same time, press MODE to open the MODE menu and press F2 to go to the second page. The cursor will be next to Split Screen. Select either TOP-BOTTOM or LEFT-RIGHT by pressing ➔ and 2 or 3, respectively. Now the 2 lines below the Split 1 App line have become readable, since these options apply only when the calculator is in the split screen mode. The Split 1 App will automatically be the screen you were on prior to pressing MODE. You can choose what you want the top or left-hand screen to show by moving down to the Split 1 App line, pressing ➔ and the number of the application you want in that window. The Split 2 App determines what is shown in the bottom or right-hand window. Press ENTER to confirm your choices and your TI-92's screen will now be divided either horizontally or vertically (as you choose). Figure 5.15 shows the graph and the Y= screen with the settings shown in Figure 5.16. The split screen is also useful when you need to do some calculations as you trace along a graph. In split screen mode, one side of the screen will be more heavily outlined. This is the *active screen*, i.e., the screen that you can currently modify. You can change which side is active by using 2nd to access the symbol above the APPS key. For now, restore the TI-92 to Full screen.

Technology Tip: Note that if you set one part of your screen to contain a table and the other to contain a graph, the table will not necessarily correspond to the graph unless you use ◆ TblSet to generate a new table based on the functions(s) being graphed(as in Section 5.2.1).

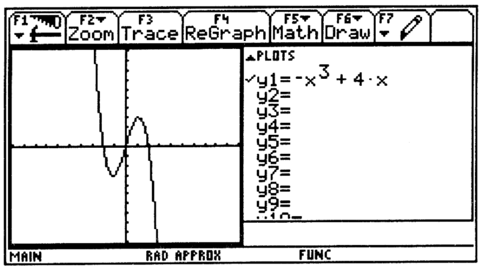


Figure 5.15: Split screen: LEFT-RIGHT

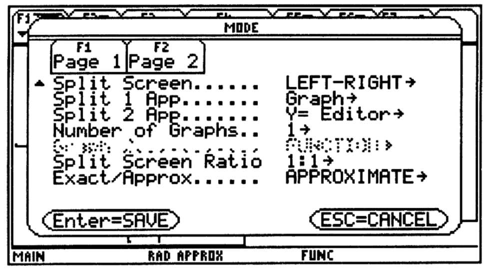


Figure 5.16: MODE settings for Figure 5.15

Your graph window may look like the one in Figure 5.17 or it may be different. Since the graph of $y = -x^3 + 4x$ extends infinitely far left and right and also infinitely far up and down, the TI-92 can display only a piece of the actual graph. This displayed rectangular part is called a *viewing rectangle.* You can easily change the viewing rectangle to enhance your investigation of a graph.

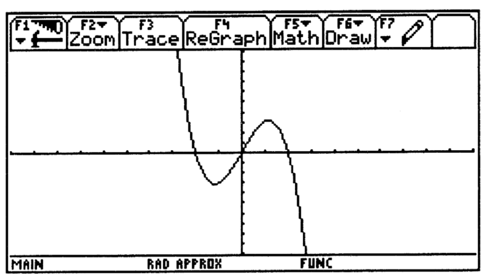


Figure 5.17: Graph of $y = -x^3 + 4x$

The viewing rectangle in Figure 5.17 shows the part of the graph that extends horizontally from –10 to 10 and vertically from –10 to 10. Press ◆ WINDOW to see information about your viewing rectangle. Figure 5.18 shows the WINDOW screen that corresponds to the viewing rectangle in Figure 5.17. This is the *standard* viewing rectangle for the TI-92.

The variables xmin and xmax are the minimum and maximum *x*-values of the viewing rectangle; ymin and ymax are the minimum and maximum *y*-values.

xscl and yscl set the spacing between tick marks on the axes.

xres sets pixel resolution (1 through 10) for function graphs.

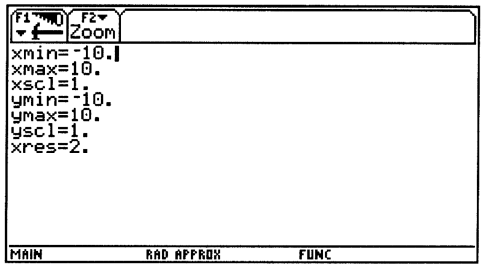


Figure 5.18: Standard WINDOW

Technology Tip: Small xres values improve graph resolution, but may cause the TI-92 to draw graphs more slowly.

Use 🡅 and 🡇 to move up and down from one line to another in this list; pressing the ENTER key will move down the list. Enter a new value to over-write a previous value and then press ENTER. Remember that a minimum *must* be less than the corresponding maximum or the TI-92 will issue an error message. Also, remember to use the (-) key, not – (which is subtraction), when you want to enter a negative value. Figures 5.17–18, 5.19–20, and 5.21–22 show different WINDOW screens and the corresponding viewing rectangle for each one.

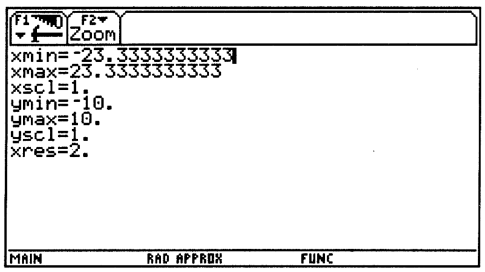


Figure 5.19: Square window

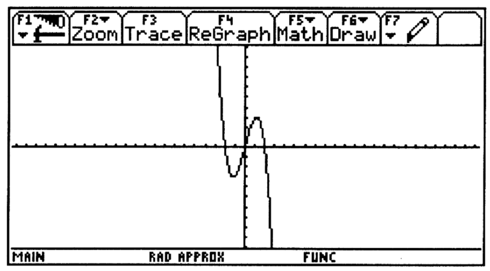


Figure 5.20: Graph of $y = -x^3 + 4x$

To initialize the viewing rectangle quickly to the *standard* viewing rectangle (Figure 5.18), press *F2[Zoom]* 6*[ZoomStd]*. To set the viewing rectangle quickly to a square (Figure 5.19), press *F2[Zoom]* 5*[ZoomSqr]*. More information about square windows is presented later in Section 5.2.4.

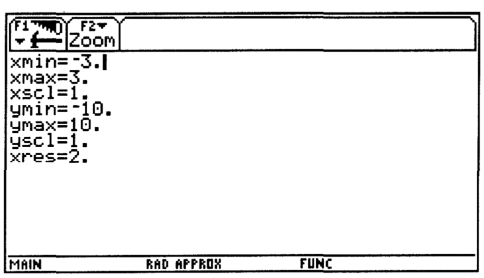


Figure 5.21: Custom window

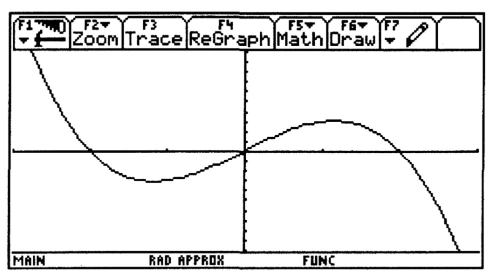


Figure 5.22: Graph of $y = -x^3 + 4x$

Sometimes you may wish to display grid points corresponding to tick marks on the axes. This and other graph format options may be changed while you are viewing the graph by pressing F1 to get the ToolBar menu (Figure 5.23) and then pressing 9*[Format]* to display the Format menu (Figure 5.24) or by pressing ◆ F as indicated on the ToolBar menu in Figure 5.23. Use the cursor pad to move the blinking cursor to Grid; press ➔ 2*[On]* ENTER to redraw the graph. Figure 5.25 shows the same graph as in Figure 5.22 but with the grid turned on.

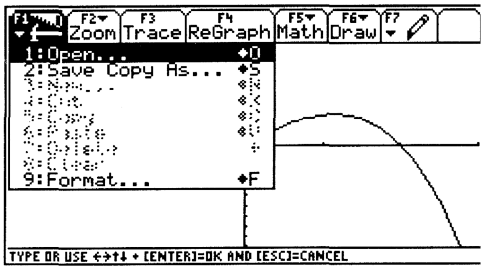


Figure 5.23: ToolBar menu

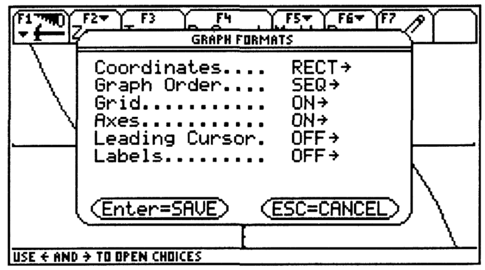


Figure 5.24: Format menu

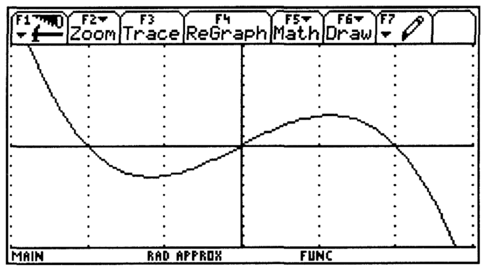


Figure 5.25: Grid turned on for $y = -x^3 + 4x$

In general, you'll want the grid turned *off,* so do that now by pressing ◆ F and turning the Grid option to OFF, then pressing ENTER.

5.2.3 Graphing Step and Piecewise-Defined Functions: The greatest integer function, written [[*x*]] , gives the greatest *integer* less than or equal to a number *x*. On the TI-92, the greatest integer function is called floor(and is located under the Number sub-menu of the MATH menu (Figure 5.8). From the Home screen, calculate [[6.78]] = 6 by pressing 2nd MATH➔*6[floor(]* 6.78) ENTER.

To graph *y* = [[*x*]], go into the Y= menu, move beside y1 and press CLEAR 2nd MATH ➔ 6*[floor(]* X) ENTER ◆ GRAPH. Figure 5.26 shows this graph in a viewing rectangle from –5 to 5 in both directions.

The true graph of the greatest integer function is a step graph, like the one in Figure 5.27. For the graph of *y* = [[*x*]], a segment should *not* be drawn between every pair of successive points. You can change this graph from a Line to a Dot graph on the TI-92 by going to the Y= screen, moving up until this function is selected (highlighted) and then pressing F6. This opens the Graph Style menu. Move the cursor down to the second line and press ENTER or press 2; to have the selected graph plotted in Dot style. Now press ◆ GRAPH to see the result.

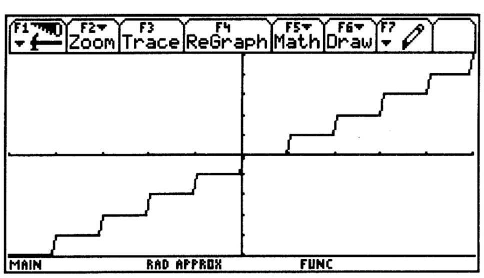


Figure 5.26: Connected graph of *y* = [[*x*]]

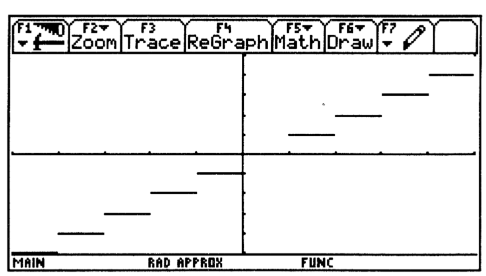


Figure 5.27: Dot graph of *y* = [[*x*]]

Technology Tip: When graphing functions in the Dot style, it improves the appearance of the graph to set xres to 1. Figure 5.27 was graphed with xres = 1. Also, the default graph style is Line, so you have to set the style to Dot each time you wish to graph a function in Dot mode.

The TI-92 can graph piecewise-defined functions by using the "when" function. The "when" function is not on any of the keys but can be found in the CATALOG or typed from the keyboard. The format of the when(function is when(*condition, trueResult, falseResult, unknownResult*) where the *falseResult* and *unknownResult* are optional arguments.

For example, to graph the function $f(x) = \begin{cases} x^2+2, & x<0 \\ x-1, & x \geq 0 \end{cases}$, you want to graph $x^2 + 2$ when the condition $x < 0$ is true and graph x – 1 when the condition is false. First, clear any existing functions in the Y= screen. Then move to the y1 line and press W H E N (X 2nd < 0, X ∧ 2 + 1 , X – 1) ENTER (Figure 5.28). Then press ◆ GRAPH to display the graph. Figure 5.29 shows this graph in a viewing rectangle from –5 to 5 in both directions. This was done in Dot style, since the TI-92 will (incorrectly) connect the two sides of the graph at $x = 0$ if the function is graphed in Line style.

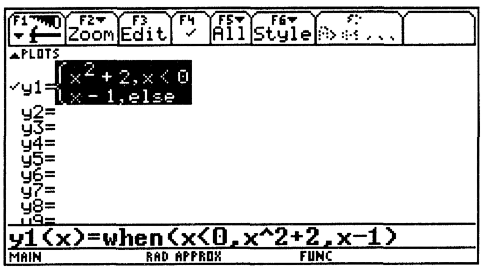


Figure 5.28: Piecewise-defined function

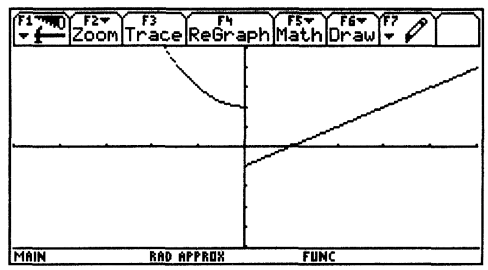


Figure 5.29: Piecewise-defined graph

Other *test* functions, such as ≤, ≥ and ≠ as well as logic operators can be found on the Test sub-menu of the 2nd MATH menu.

5.2.4 Graphing a Circle: Here is a useful technique for graphs that are not functions but can be "split" into a top part and a bottom part, or into multiple parts. Suppose you wish to graph the circle of radius 6 whose equation is $x^2 + y^2 = 36$. First solve for y and get an equation for the top semicircle, $y = \sqrt{36 - x^2}$, and for the bottom semicircle, $y = -\sqrt{36 - x^2}$. Then graph the two semicircles simultaneously.

Use the following keystrokes to draw this circle's graph. First clear any existing functions on the Y= screen. Enter $\sqrt{36 - x^2}$ as y1 and $-\sqrt{36 - x^2}$ as y2 (see Figure 5.30) by pressing 2nd √ 36 – X ∧ 2) ENTER (-) 2nd √ 36 – X ∧ 2) ENTER. Then press ◆ GRAPH to draw them both (Figure 5.31).

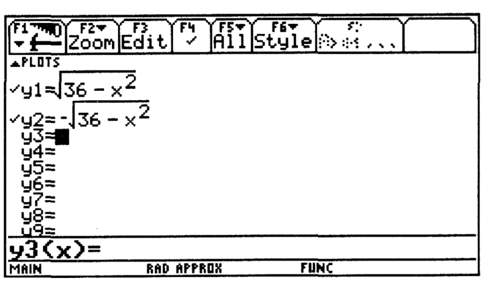


Figure 5.30: Two semicircles

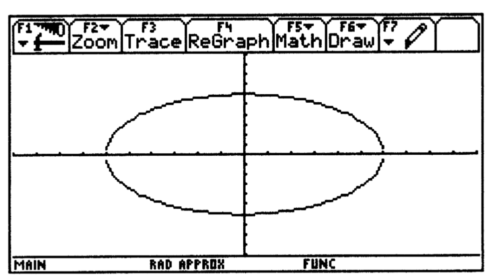


Figure 5.31: Circle's graph – standard WINDOW

Instead of entering $-\sqrt{36 - x^2}$ as y2, you could have entered –y1 as y2 and saved some keystrokes. On the TI-92, try this by going into the Y= screen and pressing 🡅 to move the cursor up to y2. Then press CLEAR (-) Y 1 (X) ENTER (Figure 5.32). The graph should be as before.

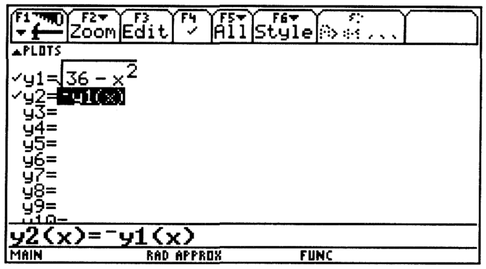


Figure 5.32: Using y1 in y2

If your range were set to a viewing rectangle extending from –10 to 10 in both directions, your graph would look like Figure 5.31. Now this does *not* look a circle, because the units along the axes are not the same. You need what is called a "square" viewing rectangle. Press F2[Zoom] 5*[ZoomSqr]* and see a graph that appears more circular.

Technology Tip: Another way to get a square graph is to change the range variables so that the value of ymax – ymin is approximately $\frac{3}{7}$ times xmax – xmin. For example, see the WINDOW in Figure 5.33 to get the corresponding graph in Figure 5.34. This method works because the dimensions of the TI-92's display are such that the ratio of vertical to horizontal is approximately $\frac{3}{7}$.

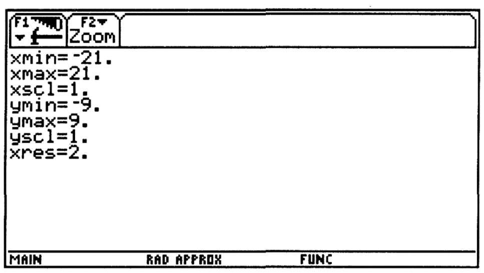


Figure 5.33: $\frac{\text{vertical}}{\text{horizontal}} = \frac{18}{42} = \frac{3}{7}$

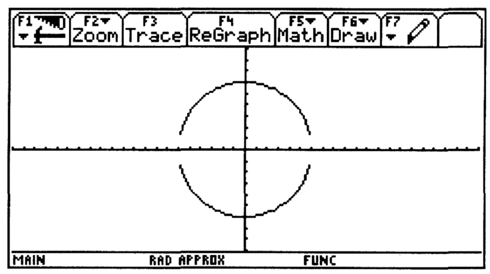


Figure 5.34: A "square" circle

The two semicircles in Figure 5.34 do not meet because of an idiosyncrasy in the way the TI-92 plots a graph.

5.2.5 TRACE: Graph the function $y = -x^3 + 4x$ from Section 5.2.2 using the standard viewing rectangle. (Remember to clear any other functions in the Y= screen.) Press any of the cursor directions ↑ ↓ → ← and see the cursor move from the center of the viewing rectangle. The coordinates of the cursor's location are displayed at the bottom of the screen, as in Figure 5.35, in floating decimal format. This cursor is called a *free-moving cursor* because it can move from dot to dot *anywhere* in the graph window.

Remove the free-moving cursor and its coordinates from the window by pressing ◆ GRAPH, CLEAR, ESC or ENTER. Press the cursor pad again and the free-moving cursor will reappear at the same point you left it.

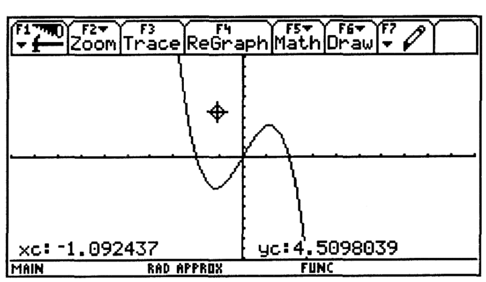


Figure 5.35: Free-moving cursor

Press F3*[TRACE]* to enable the left 🡐 and right 🡒 directions to move the cursor along the function. The cursor is no longer free-moving, but is now constrained to the function. The coordinates that are displayed belong to points on the function's graph, so the *y*-coordinate is the calculated value of the function at the corresponding *x*-coordinate.

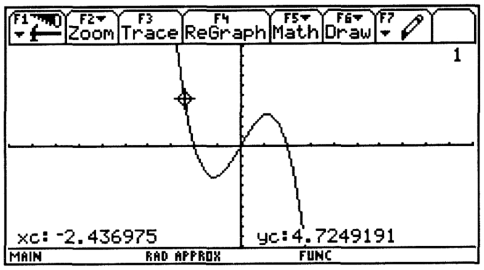


Figure 5.36: TRACE

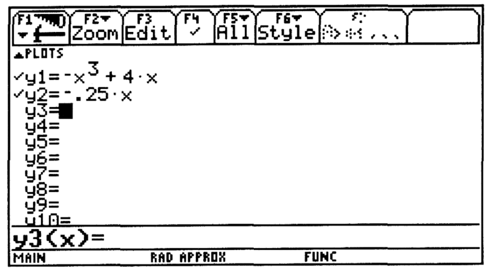


Figure 5.37: Two functions

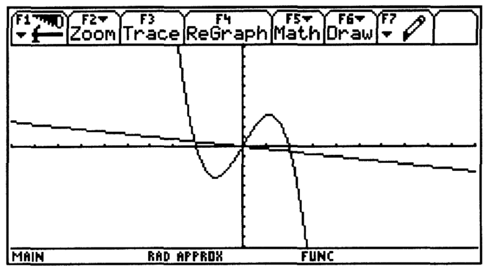


Figure 5.38: $y = -x^3 + 4x$ and $y = -.25x$

Now plot a second function, $y = -.25x$, along with $y = -x^3 + 4x$. Press ◆ Y= and enter $-.25x$ for y2, then press ◆ GRAPH to see both functions (Figure 5.38).

Notice that in Figure 5.37 there are check marks ✓ to the left of *both* y1 and y2. This means that *both* functions will be graphed. In the Y= screen, move the cursor onto y1 and press F4*[✓]*. The check mark left of y1 should disappear (Figure 5.39). Now press ◆ GRAPH and see that only y2 is plotted (Figure 5.40).

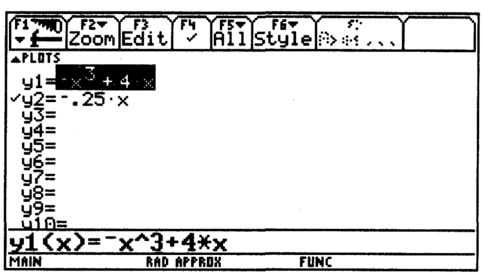


Figure 5.39: only y2 active

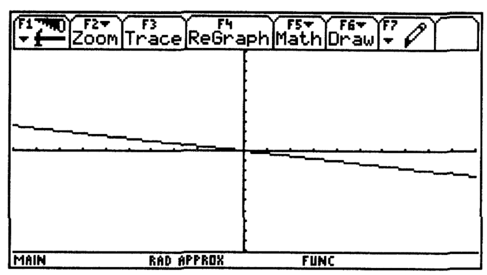


Figure 5.40: Graph of $y = -.25x$

Many different functions can be stored in the Y= list and any combination of them may be graphed simultaneously. You can make a function active or inactive for graphing by pressing F4 when the function is highlighted to add a check mark (activate) or remove the check mark (deactivate). Now go back to the Y= screen and do what is needed in order to graph y1 but not y2.

Now activate both functions so that both graphs are plotted. Press *F3[TRACE]* and the cursor appears first on the graph of $y = -x^3 + 4x$ because it is higher up on the Y= list. You know that the cursor is on this function, y1, because of the numeral 1 that is displayed in the upper right corner of the screen. Press the up 🡑 or down 🡓 direction to move the cursor vertically to the graph of $y = -.25x$. Now the numeral 2 is shown in the upper right corner of the screen. Next press the left and right arrow keys to trace along the graph of $y = -.25x$. When more than one function is plotted, you can move the trace cursor vertically from one graph to another with the ↑ and, ↓ directions.

Technology Tip: *By* the way, trace the graph of $y = -.25x$ and press and hold either the ← or → direction. The cursor becomes larger and pulses as it moves along the graph. Eventually you will reach the left or right edge of the window. Keep pressing the direction and the TI-92 will allow you to continue the trace by panning the viewing rectangle. Check the WINDOW screen to see that the xmin and xmax are automatically updated.

The TI-92 has a display of 239 horizontal columns of pixels and 103 vertical rows, so when you trace a curve across a graph window, you are actually moving from xmin to xmax in 238 equal jumps, each called Δx. You would calculate the size of each jump to be $\Delta x = \frac{\text{xmax} - \text{xmin}}{238}$. Sometimes you may want the jumps to be friendly numbers like 0.1 or 0.25 so that, when you trace along the curve, the x-coordinates will be incremented by such a convenient amount. Just set your viewing rectangle for a particular increment Δx by making xmax = xmin + 238 · Δx. For example, if you want xmin = –5 and $\Delta x = 0.3$, set xmax = –5 + 238 · 0.3 = 66.4. Likewise, set ymax = ymin + 102 Δy if you want the vertical increment to be some special Δy.

To center your window around a particular point, say (h, k), and also have a certain Δx, set xmin = h – 119 · Δx and make xmax = h + 119 · Δx. Likewise, make ymin = k – 51 · Δy and make ymax = k + 51 · Δy. For example, to center a window around the origin (0, 0), with both horizontal and vertical increments of 0.25, set the range so that xmin = 0 – 119 · 0.25 = –29.75, xmax = 0 + 119 · 0.25 = 29.75, ymin = 0 – 51 · 0.25 = –12.75 and ymax = 0 + 51 · 0.25 = 12.75.

See the benefit by first plotting $y = x^2 + 2x + 1$ in a standard graphing window. Trace near its y-intercept, which is (0, 1), and move towards its x-intercept, which is (–1, 0). Then press *F2[Zoom]* 4*[ZoomDec]* and trace again near the intercepts.

5.2.6 Zoom: Plot again the two graphs, for $y = -x^3 + 4x$ and $y = -.25x$. There appears to be an intersection near x = 2. The TI-92 provides several ways to enlarge the view around this point. You can change the viewing rectangle directly by pressing ◆ WINDOW and editing the values of xmin, xmax, ymin, and ymax. Figure 5.42 shows a new viewing rectangle for the range displayed in Figure 5.41. The cursor has been moved near the point of intersection; move your cursor closer to get the best approximation possible for the coordinates of the intersection.

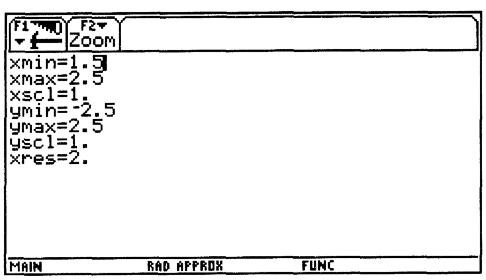


Figure 5.41: New WINDOW

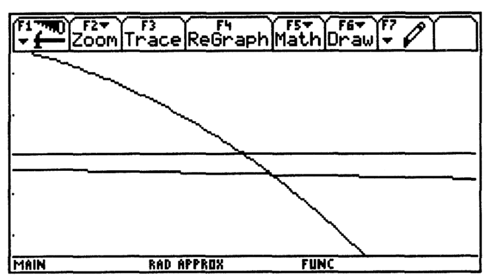


Figure 5.42: Closer view

A more efficient method for enlarging the view is to draw a new viewing rectangle with the cursor. Start again with a graph of the two functions $y = -x^3 + 4x$ and $y = -.25x$ in a standard viewing rectangle. (Press F2*[Zoom]* 6*[ZoomStd]* for the standard viewing window.)

Now imagine a small rectangular box around the intersection point, near $x = 2$. Press F2*[Zoom]* 1*[ZoomBox]* (Figure 5.43) to draw a box to define this new viewing rectangle. Use the arrow keys to move the cursor, whose coordinates are displayed at the bottom of the window, to one corner of the new viewing rectangle you imagine.

Press ENTER to fix the corner where you moved the cursor; it changes shape and becomes a blinking square (Figure 5.44). Use the arrow keys again to move the cursor to the diagonally opposite corner of the new rectangle

(Figure 5.45). (Note that you can use the diagonal directions on the cursor pad for this.) If this box looks all right to you, press ENTER. The rectangular area you have enclosed will now enlarge to fill the graph window (Figure 5.46).

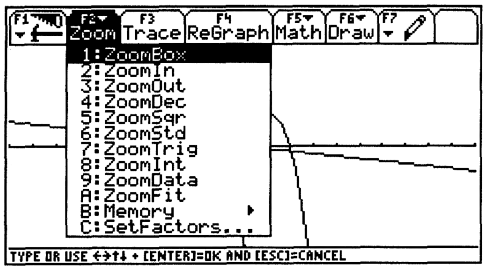


Figure 5.43: F2*[Zoom]* menu

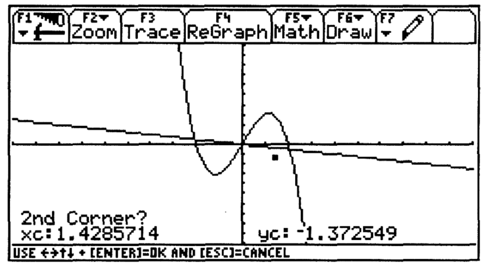


Figure 5.44: One corner selected

You may cancel the zoom any time *before* you press this last ENTER. Press F2*[Zoom]* once more and start over. Press ESC or ◆ GRAPH to cancel the zoom, or press 2nd QUIT to cancel the zoom and return to the Home screen.

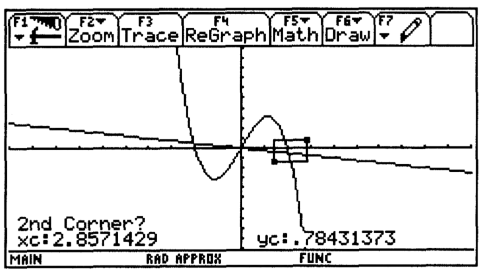


Figure 5.45: Box drawn

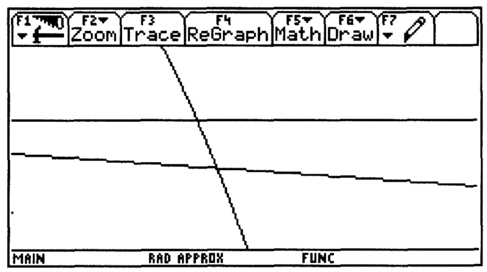


Figure 5.46: New viewing rectangle

You can also quickly magnify a graph around the cursor's location. Return once more to the standard window for the graph of the two functions $y = -x^3 + 4x$ and $y = -.25x$. Press F2*[Zoom]* 2*[ZoomIn]* and then use the cursor pad to move the cursor as close as you can to the point of intersection near $x = 2$ (see Figure 5.47). Then press ENTER and the calculator draws a magnified graph, centered at the cursor's position (Figure 5.48). The range variables are changed to reflect this new viewing rectangle. Look in the WINDOW menu to verify this.

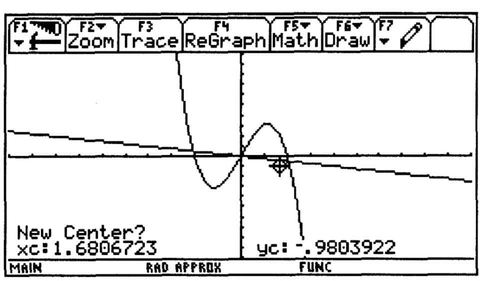


Figure 5.47: Before a zoom in

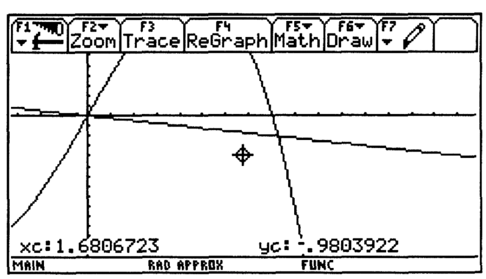


Figure 5.48: After a zoom in

As you see in the F2*[Zoom]* menu (Figure 5.43), the TI-92 can zoom in (press F2*[Zoom]* 2) or zoom out (press F2*[Zoom]* 3). Zoom out to see a larger view of the graph, centered at the cursor position. You can change the horizontal and vertical scale of the magnification by pressing F2*[Zoom] C[SetFactors]* (see Figure 5.49) and editing xFact and yFact, the horizontal and vertical magnification factors. (The zFact is only used when dealing with 3-dimensional graphs.)

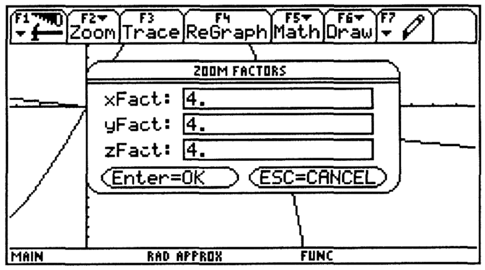


Figure 5.49: ZOOM FACTORS menu

Technology Tip: An advantage of zooming in from square viewing window is that subsequent windows will also be square. Likewise, if you zoom in from a friendly viewing rectangle, the zoomed windows will also be friendly.

The default zoom factor is 4 in both direction. It is not necessary for xFact and yFact to be equal. Sometimes, you may prefer to zoom in one direction only, so the other factor should be set to 1. Press ESC to leave the ZOOM FACTORS menu and go back to the graph. (Pressing 2nd QUIT will take you back to the Home screen.)

Technology Tip: The TI-92 remembers the window it displayed before a zoom. So if you should zoom in too much and lose the curve, press F2*[Zoom]* B*[Memory]* 1*[ZoomPrev]* to go back to the window before. If you want to execute a series of zooms but then return to a particular window, press F2*[Zoom]* B*[Memory]* 2*[ZoomSto]* to store the current window's dimensions. Later, press F2*[Zoom]* B*[Memory]* 3*[ZoomRcl]* to recall the stored window.

5.2.7 Relative Minimums and Maximums: Graph $y = -x^3 + 4x$ once again in the standard viewing rectangle. This function appears to have a relative minimum near $x = -1$ and a relative maximum near $x = 1$. You may zoom and trace to approximate these extreme values.

First trace along the curve near the local minimum. Notice by how much the *x*-values and *y*-values change as you move from point to point Trace along the curve until the *y*-coordinate is as *small* as you can get it, so that you are as close as possible to the local minimum, and zoom in (press F2*[Zoom]* 2*[ZoomIn]* ENTER or use a zoom box). Now trace again along the curve and, as you move from point to point, see that the coordinates change by smaller amounts than before. Keep zooming and tracing until you find the coordinates of the local minimum point as accurately as you need them, approximately (–1.15, –3.08).

Follow a similar procedure to find the local maximum. Trace along the curve until the *y*-coordinate is as *great* as you can get it, so that you are as close as possible to the local maximum, and zoom in. The local maximum point on the graph of $y = -x^3 + 4x$ is approximately (1.15, 3.08).

The TI-92 can automatically find the maximum and minimum points. While viewing the graph, press F5*[Math]* to display the Math menu (Figure 5.50). Choose 3*[Minimum]* to calculate the minimum value of the function and 4*[Maximum]* for the maximum. You will be prompted to trace the cursor along the graph first to a point *left* of the minimum/maximum (press ENTER to set this *lower bound).* Note the arrow near the top of the display marking the lower bound (as in Figure 5.51).

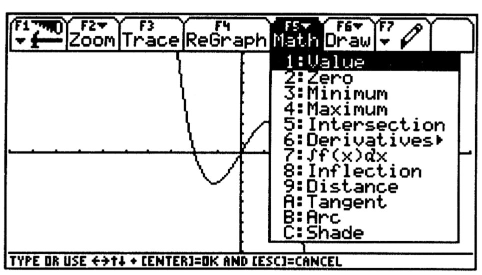


Figure 5.50: Math menu

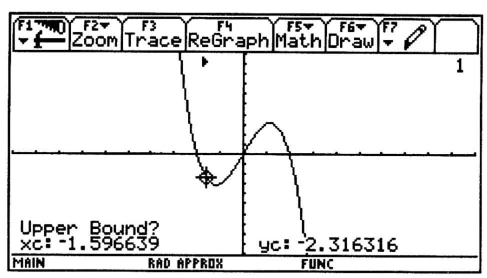


Figure 5.51: Finding a minimum

Now move to a point *right* of the minimum/maximum and set *a upper bound* by pressing ENTER. The coordinates of the relative minimum/maximum point will be displayed (see Figure 5.52). Good choices for the left bound and right bound can help the TI-92 work more efficiently and quickly.

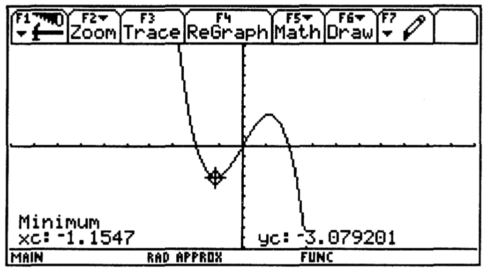


Figure 5.52: Relative minimum on $y = -x^3 + 4x$

Note that if you have more than one graph on the screen, the upper right corner of the TI-83 screen will show the number of the function whose minimum/maximum is being calculated.

5.3 Solving Equations and Inequalities

5.3.1 Intercepts and Intersections: Tracing and zooming are also used to locate an x-intercept of a graph, where a curve crosses the x-axis. For example, the graph of $y = x^3 - 8x$ crosses the x-axis three times (Figure 5.53). After tracing over to the x-intercept point that is farthest to the left, zoom in (Figure 5.54). Continue this process until you have located all three intercepts with as much accuracy as you need. The three x-intercepts of $y = x^3 - 8x$ are approximately –2.828, 0, and 2.828.

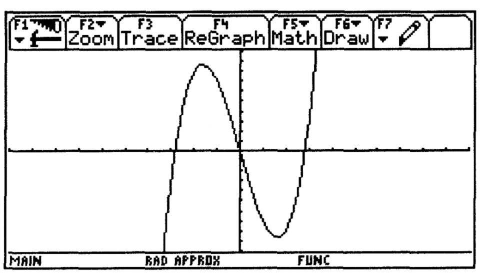


Figure 5.53: Graph of $y = x^3 - 8x$

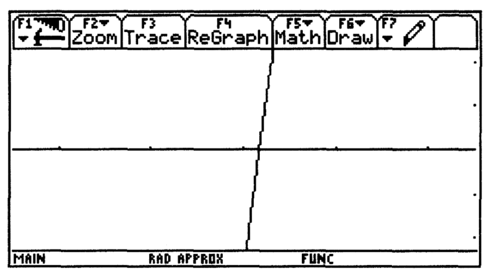


Figure 5.54: Near an x-intercept of $y = x^3 - 8x$

Technology Tip: As you zoom in, you may also wish to change the spacing between tick marks on the x-axis so that the viewing rectangle shows scale marks near the intercept point. Then the accuracy of your approximation will be such that the error is less than the distance between two tick marks. Change the x-scale on the TI-92 from the WINDOW menu. Move the cursor down to xscl and enter an appropriate value.

The x-intercept of a function's graph is a *zero* of the function, so while viewing the graph, press F5*[Math]* (Figure 5.50) and choose 2*[Zero]* to find a zero of this function. Set a lower bound and upper bound as described in Section 5.2.7. The TI-92 shows the coordinates of the point and indicates that it is a zero (Figure 5.55)

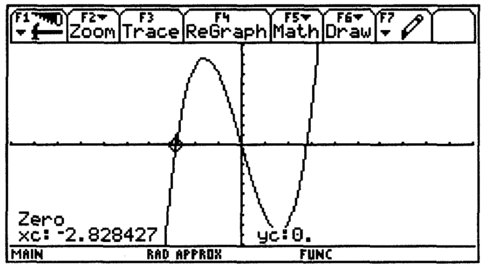


Figure 5.55: A zero of $y = x^3 - 8x$

TRACE and ZOOM are especially important for locating the intersection points of two graphs, say the graphs of $y = -x^3 + 4x$ and $y = -.25x$. Trace along one of the graphs until you arrive close to an intersection point. Then press ↑ or ↓ to jump to the other graph. Notice that the x-coordinate does not change, but the y-coordinate is likely to be different (Figures 5.56 and 5.57).

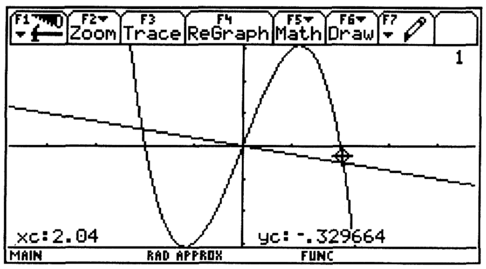


Figure 5.56: Trace on $y = -x^3 + 4x$

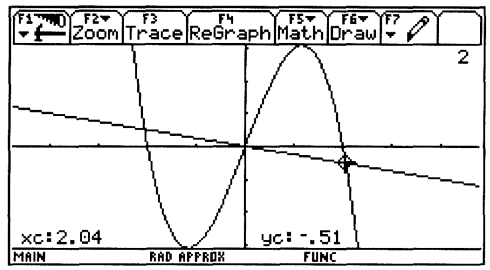


Figure 5.57: Trace on $y = -.25x$

When the two y-coordinates are as close as they can get, you have come as close as you now can to the point of intersection. So zoom in around the intersection point, then trace again until the two y-coordinates are as close as possible. Continue this process until you have located the point of intersection with as much accuracy as necessary.

You can also find the point of intersection of two graphs by pressing F5*[Math]* 5*[Intersection]*. Trace with the cursor first along one graph near the intersection and press ENTER; then trace with the cursor along the other graph and press ENTER. Marks + are placed on the graphs at these points. Then set lower and upper bounds for the x-coordinate of the intersection point and press ENTER again. Coordinates of the intersection will be displayed at the bottom of the window (Figure 5.58).

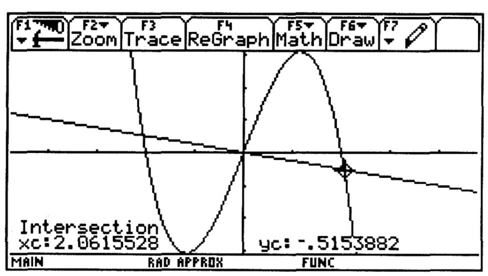


Figure 5.58: An intersection of $y = -x^3 + 4x$ and $y = -.25x$

5.3.2 Solving Equations by Graphing: Suppose you need to solve the equation $24x^3 - 36x + 17 = 0$. First graph $y = 24x^3 - 36x + 17$ in a window large enough to exhibit *all* its x-intercepts, corresponding to all the equation's zeros (roots). Then use trace and zoom, or the TI-92's zero finder, to locate each one. In fact, this equation has just one solution, approximately $x = -1.414$.

Remember that when an equation has more than one x-intercept, it may be necessary to change the viewing rectangle a few times to locate all of them.

The TI-92 has a solve(function. To use this function, you must be in the Home screen. To use the solve(function, press S O L V E (24 X ∧ 3 – 35 X + 17 = 0 , X) ENTER. The TI-92 displays the value of the zero (Figure 5.59). Note that any letter could have been used for the variable. This is the reason that you must indicate to the TI-92 that the variable being used is X.

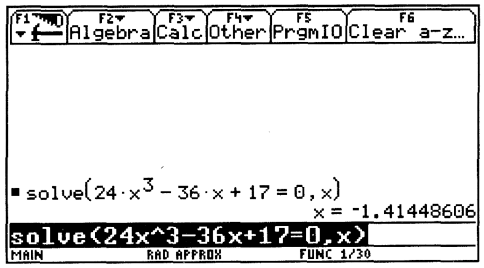


Figure 5.59: solve(function

Technology Tip: To solve an equation like $24x^3 + 17 = 36x$, you may first transform it into standard form, $24x^3 - 36x + 17 = 0$, and proceed as above. However, the solve(function does not require that the function be in standard form. You may also graph the *two* functions $y = 24x^3 + 17$ and $y = 36x$, then zoom and trace to locate their point of intersection.

5.3.3 Solving Systems by Graphing: The solutions to a system of equations correspond to the points of intersection of their graphs (Figure 5.60). For example, to solve the system $y = x^3 + 3x^2 - 2x - 1$ and $y = x^2 - 3x - 4$, first graph them together. Then use zoom and trace or the intersection option in the F5*[Math]* menu, to locate their point of intersection, approximately (–2.17, 7.25).

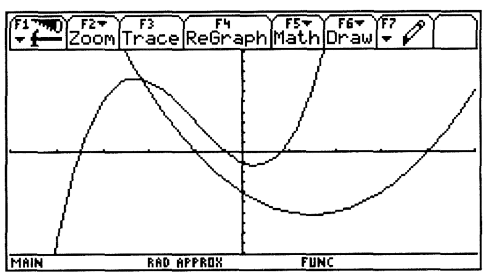


Figure 5.60: Graph of $y = x^3 + 3x^2 - 2x - 1$ and $y = x^2 - 3x - 4$

If you do not use the Intersection option**,** you must judge whether the two current y-coordinates are sufficiently close for $x = -2.17$ or whether you should continue to zoom and trace to improve the approximation.

The solutions of the system of two equations $y = x^3 + 3x^2 - 2x - 1$ and $y = x^2 - 3x - 4$ correspond to the solutions of the single equation $x^3 + 3x^2 - 2x - 1 = x^2 - 3x - 4$, which simplifies to $x^3 + 2x^2 + x + 3 = 0$. So you may also graph $y = x^3 + 2x^2 + x + 3$ and find its x-intercepts to solve the system or use the solve(function.

5.3.4 Solving Inequalities by Graphing: Consider the inequality $1 - \frac{3x}{2} \ge x - 4$. To solve it with your TI-92, graph the two functions $y = 1 - \frac{3x}{2}$ and $y = x - 4$ (Figure 5.61). First locate their point of intersection, at $x = 2$. The inequality is true when the graph of $y = 1 - \frac{3x}{2}$ lies *above* the graph of $y = x - 4$, and that occurs when $x < 2$. So the solution is the half-line $x \le 2$, or $(-\infty, 2]$.

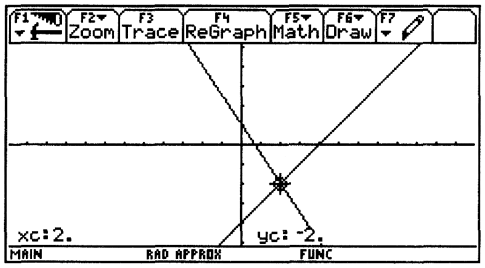


Figure 5.61: Solving $1-\frac{3x}{2} \geq x-4$

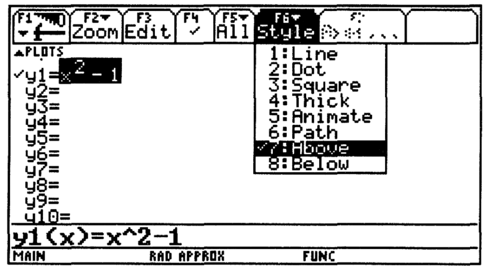


Figure 5.62: Shade Above style

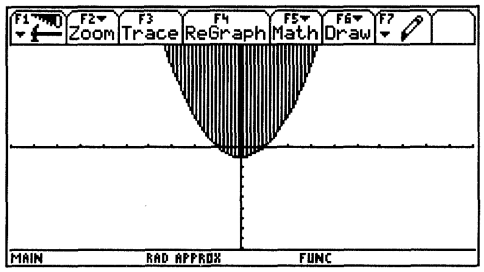


Figure 5.63: Graph of $y \geq x^2 - 1$

The TI-92 is capable of shading the region above or below a graph, or between two graphs. For example, to graph $y \geq x^2 - 1$, first enter the function $y = x^2 - 1$ as y1. Then, highlight y1 and press F6*[Style] 7[Above]* (see Figure 5.62). These keystrokes instruct the TI-92 to shade the region above $y = x^2 - 1$. Press ◆ GRAPH to see the graph. The region above the graph will be shaded using the first shading option of vertical lines, as in Figure 5.63.

Now use shading to solve the previous inequality, $1-\frac{3x}{2} \geq x-4$. The solution is the region which is *below* the graph of $1-\frac{3x}{2}$ and *above* $x - 4$. First graph both equations. Then, from the graph screen, press F5*[Math] C[Shade].* The TI-92 will prompt for the function that you want to have the shading *above.* Use 🡅 or 🡇 to move the cursor to the graph of x – 4, then press ENTER. The TI-92 will then prompt for the function that you want to have the shading *below,* so use 🡅 or 🡇 to move the cursor to the graph of $1-\frac{3x}{2}$ and press ENTER. The TI-92 will then prompt for the *lower bound* then the *upper bound,* which are the left and right edges, respectively, of the extent of the shading. If you do not enter a lower or upper bound, the values of xmin and xmax will be used. So, in this case, press ENTER twice to set the lower and upper bounds. The shaded area extends left from $x = -2$, hence the solution to $1-\frac{3x}{2} \geq x-4$ is the half-line $x \leq 2$, or $(-\infty, 2]$.

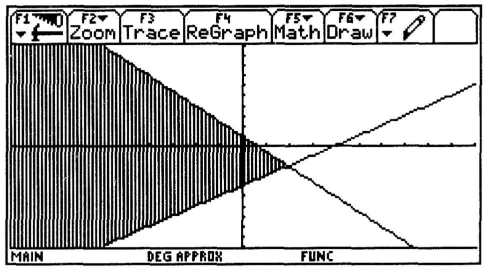


Figure 5.64: Graph of $1-\frac{3x}{2} \geq x-4$

5.4 Trigonometry

5.4.1 Degrees and Radians: The trigonometric functions can be applied to angles measured either in radians or degrees, but you should take care that the TI-92 is configured for whichever measure you need. Press MODE to see the current settings. Press ↓ three times and move down to the fourth line of the first page of the mode menu where angle measure is selected. Then press → to display the options. Use ↑ or ↓ to move from one option to the other. Either press the number corresponding to the measure or, when the measure is highlighted, press ENTER to select it. Then press ENTER to confirm your selection and leave the MODE menu.

It's a good idea to check the angle measure setting before executing a calculation that depends on a particular measure. You may change a mode setting at any time and not interfere with pending calculations. From the Home screen, try the following keystrokes to see this in action.

Expression	*Keystrokes*	*Display*
sin 45°	MODE ↓↓↓→↓ENTER ENTER SIN 45) ENTER	.7071067812
sin π°	SIN 2nd π) ENTER	.0548036651
sin π	MODE ↓↓↓→↑ENTER ENTER SIN 2nd π) ENTER	0
sin 45	SIN 45) ENTER	.8509035245
$\sin \frac{\pi}{6}$	SIN 2nd π ÷ 6) ENTER	.5

The first line of keystrokes sets the TI-92 in degree mode and calculates the sine of 45 *degrees.* While the calculator is still in degree mode, the second line of keystrokes calculates the sine of π *degrees,* approximately 3.1415°. The third line changes to radian mode just before calculating the sine of π *radians.* The fourth line calculates the sine of 45 *radians* (the calculator remains in radian mode).

The TI-92 makes it possible to mix degrees and radians in a calculation. Execute these keystrokes to calculate tan 45°+ sin $\frac{\pi}{6}$ as shown in Figure 5.65: TAN 45 2nd MATH 2*[Angle]* 1) + SIN (2nd π ÷ 6) 2nd MATH 2*[Angle]* 2) ENTER. Do you get 1.5 whether your calculator is set *either* in degree mode *or* in radian mode?

The degree sign can also be entered by pressing 2nd D, which saves keystrokes. There is no corresponding key for the radian symbol.

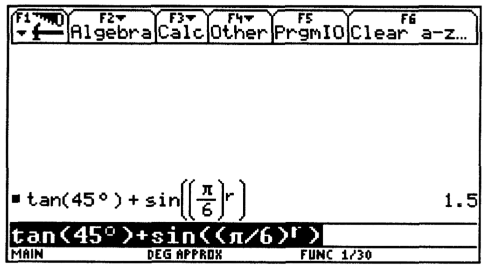


Figure 5.65: Angle measure

Technology Tip: The automatic left parenthesis that the TI-92 places after functions such as sine, cosine, and tangent (as noted in Section 5.1.3) *can* affect the outcome of calculations. In the previous example, the degree sign must be *inside* of the parentheses so that when the TI-92 is in radian mode, it calculates the tangent of 45 degrees, rather than converting the tangent of 45 radians into an equivalent number of degrees. Also, the parentheses around the fraction $\frac{\pi}{6}$ are required so that when the TI-92 is in radian mode, it converts $\frac{\pi}{6}$ into radians, rather than converting merely the 6 to radians. Experiment with the placement of parentheses to see how they affect the result of the computation.

5.4.2 Graphs of Trigonometric Functions: When you graph a trigonometric function, you need to pay careful attention to the choice of graph window. For example, graph $y = \frac{\sin 30x}{30}$ in the standard viewing rectangle. Trace along the curve to see where it is. Zoom in to a better window, or use the period and amplitude to establish better WINDOW values.

Technology Tip:. Since $\pi \approx 3.1$, when in radian mode, set xmin = 0 and xmax = 6.2 to cover the interval from 0 to 2π.

Next graph $y = \tan x$ in the standard window first, then press F2*[Zoom]* 7*[Zoom Trig]* to change to a special window for trigonometric functions in which the horizontal increment is $\frac{\pi}{24}$ or 7.5° and the vertical range is from –4 to 4. The TI-92 plots consecutive points and then connects them with a segment, so the graph is not exactly what you should expect. You may wish to change the plot style from Line to Dot (see Section 5.2.3) when you plot the tangent function.

5.5 Scatter Plots

5.5.1 Entering Data: The table shows the total prize money (in millions of dollars) awarded at the Indianapolis 500 race from 1981 to 1989. *(Source:* Indianapolis Motor Speedway Hall of Fame.)

Year	*1981*	*1982*	*1983*	*1984*	*1985*	*1986*	*1987*	*1988*	*1989*
Prize ($million)	$1.61	$2.07	$2.41	$2.80	$3.27	$4.00	$4.49	$5.03	$5.72

We'll now use the TI-92 to construct a scatter plot that represents these points and find a linear model that approximates the given data.

The TI-92 holds data in *lists.* You can create as many list names as your TI-92 memory has space to store. Before entering data, clear the data in the lists that you want to use. To delete a list press 2nd VAR-LINK. This will display the list of folders showing the variables defined in each folder. Highlight the name of the list that you wish to delete and press *F1 [Manage]* 1 *[Delete]* ENTER. The TI-92 will ask you to confirm the deletion by pressing ENTER once more.

Now press *APPS* 6*[Data/Matrix Editor] 3[New]* ↓ ↓ P R I Z E ENTER to open a new variable called PRIZE (Figure 5.66). Press ENTER to then begin entering the variable values, with the years going in column c1. Instead of entering the full year 198x, enter only x. Here are the keystrokes for the first three years: 1 ENTER 2 ENTER 3 ENTER and so on, then press → to move to the next list. Use the cursor pad to move up to the first row and press 1.61 ENTER 2.07 ENTER 2.41 and so on (see Figure 5.67).

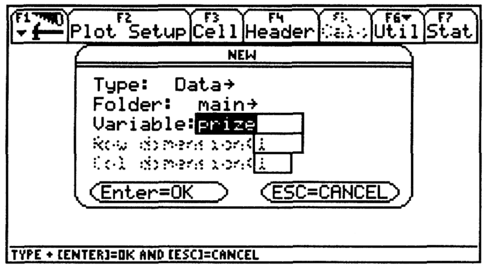


Figure 5.66: Entering a new variable

You may edit statistical data in almost the same way you edit expressions in the Home screen. ⇦ will delete the *entire cell,* not just the character or value to the left of the cursor. Thus, move the cursor to any value you wish to change, then type the correction. To insert or delete a data point, move the cursor over the data point (cell) you wish to add or delete. To insert a cell, move to the cell *below* the place where you want to insert the new cell and press *F6[Util]* 1 *[Insert]* 1 *[cell]* and a new empty cell is open.

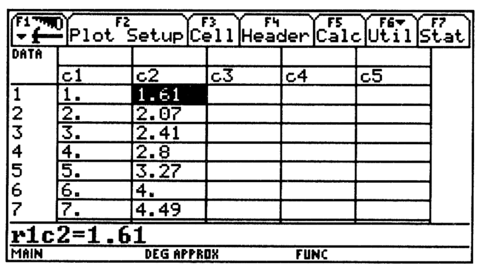


Figure 5.67: Entering data points

5.5.2 Plotting Data: First check the MODE screen (Figure 5.1) to make sure that you are in FUNCTION graphing mode. With the data points showing, press F2*[Plot Setup]* to display the Plot Setup screen. If no other plots have been entered, Plot 1 is highlighted by default. Press F1 *[Define]* to select the options for the plot. Use ↑, ↓, and ENTER to select the Plot Type as Scatter and the Mark as a Box. Use the keyboard to set the independent variable, *x*, to c1 and the dependent variable, *y*, to c2 as shown in Figure 5.68, then press ENTER to save the options and press ◆ GRAPH to graph the data points. (Make sure that you have cleared or turned off any functions in the Y= screen, or those functions will be graphed simultaneously.) Figure 5.69 shows this plot in a window from 0 to 10 horizontally and vertically. You may now press F3*[Trace]* to move from data point to data point.

To draw the scatter plot in a window adjusted automatically to include all the data you entered, press *F2[Zoom]* 9 *[ZoomData]*.

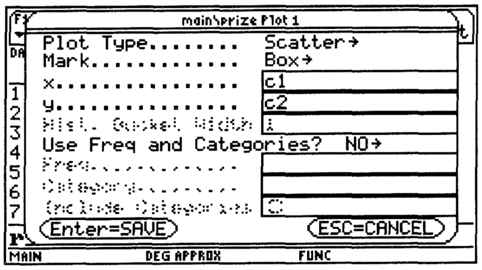


Figure 5.68: Plot1 menu

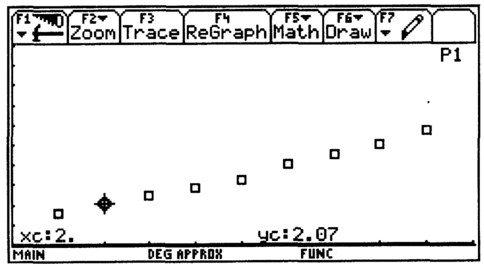
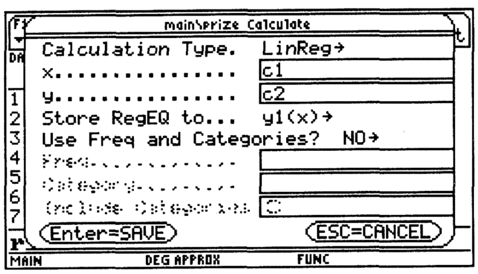


Figure 5.69: Scatter plot

When you no longer want to see the scatter plot, press APPS 6*[Data/Matrix Editor]* 1 *[Current]* F2*[Plot Setup],* highlight Plot 1 and use F4[✓] to deselect plot 1. The TI-92 still retains all the data you entered.

5.5.3 Regression Line: The TI-92 calculates slope and *y*-intercept for the line that best fits all the data. After the data points have been entered, while still in the Data/Matrix Editor, press F5*[Calc].* For the Calculation Type, choose 5*[LinReg]* and set the *x* variable to c1 and the *y* variable to c2. In order to have the TI-92 graph the regression equation, set Store RegEQ to as y1(x) as shown in Figure 5.70. Press ENTER and the TI-92 will calculate a linear regression model with the slope named a and the *y*-intercept named b (Figure 5.71). The *correlation coefficient* measures the goodness of fit of the linear regression with the data. The closer the absolute value of the correlation coefficient is to 1, the better the fit; the closer the absolute value of the correlation coefficient is to 0, the worse the fit. The TI-92 displays both the correlation coefficient and the coefficient of determination (R^2).



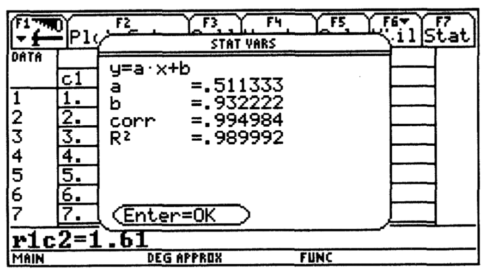


Figure 5.70: Linear regression: Calculate dialog box

Figure 5.71: Linear regression model

Press ENTER to accept the regression equation and close the STAT VARS screen. To see both the data points and the regression line (Figure 5.72), go to the Plot Setup screen and select Plot1, then press ◆ GRAPH to display the graph.

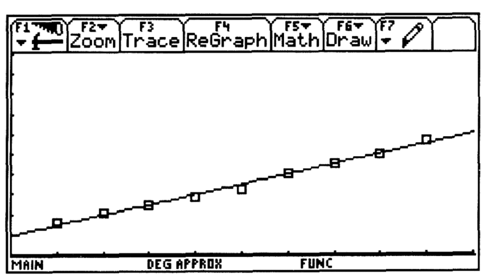


Figure 5.72: Linear regression line

5.5.4 Exponential Growth Model: The table shows the world population (in millions) from 1980 to 1992.

Year	*1980*	*1985*	*1986*	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*
Population (millions)	4453	4850	4936	5024	5112	5202	5294	5384	5478

Clear the previous data by going to the current variable in the Data/Matrix Editor and pressing F1 *[Tools]* 8*[Clear Editor]* ENTER. Follow the procedure described above to enter the data in order to find an exponential model that approximates the given data. Use 0 for 1980, 5 for 1985, and so on.

The TI-92 will not compute the exponential growth model $y = ae^{cx}$. The exponential regression that the TI-92 will compute is of the form $y = ab^x$. To get this exponential growth model press F5*[Calc]* and set the Calculation Type to 4*[ExpReg],* the x variable to c1*,* and the y variable to c2. Then press ENTER to find the values of a and b (Figure 5.73). In this case, the exponential growth model is $y = 4451(1.017454^x)$. To convert this to the form $y = ae^{cx}$, the required equation is $c = \ln b$, and the exponential growth model in this case is $y = 4451e^{x \ln 1.017454}$ or $y = 4451e^{0.017303t}$.

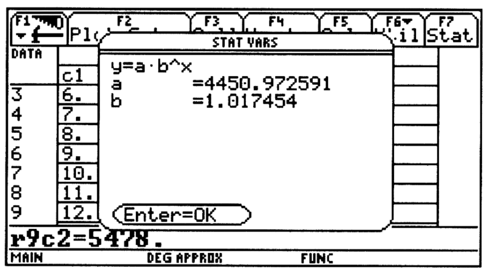


Figure 5.73: Exponential growth model

If you wish to plot and graph the data, follow the method for linear regression. Set an appropriate range for the data and then press ◆ GRAPH. The data will now be plotted in the range. To graph the regression equation also, store the regression equation to a y plot that is free. As in the linear regression model, press ◆ Y= and inactivate or clear any other existing functions, then press ◆ GRAPH to graph the exponential growth model. Note that the exponential regression model does not need to be converted to the form $y = ae^{cx}$ before graphing.

Remember to clear or deselect the plot before viewing graphs of other functions.

5.6 Matrices

5.6.1 Making a Matrix: The TI-92 can work with as many different matrices as the memory will hold. Here's how to create this 3×4 matrix $\begin{bmatrix} 1 & -4 & 3 & 5 \\ -1 & 3 & -1 & -3 \\ 2 & 0 & -4 & 6 \end{bmatrix}$ in your calculator.

From the Home screen, press APPS 6*[Data/Matrix Editor]* 3*[New]*. Set the Type to Matrix, the Variable to a (this is the 'name' of the matrix), the Row Dimension to 3 and the Col Dimension to 4 (Figure 5.74). Press ENTER to accept these values.

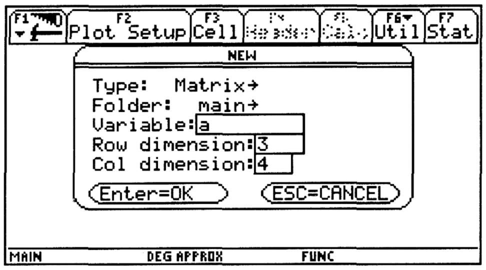


Figure 5.74: Data/Matrix menu

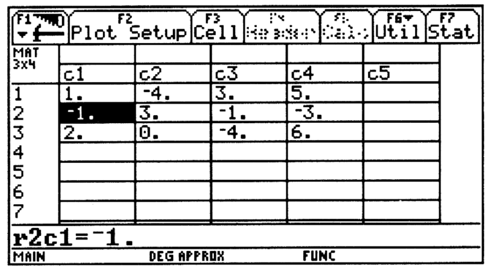


Figure 5.75: Editing a matrix

The display will show the matrix by showing a grid with zeros in the rows and columns specified in the definition of the matrix.

Use the cursor pad or press ENTER repeatedly to move the cursor to a matrix element you want to change. If you press ENTER, you will move right across a row and then back to the first column of the next row. The lower left of the screen shows the cursor's current location within the matrix. The element in the second row and first column in Figure 5.75 is highlighted, so the lower left of the window is r2c1 = –1. showing that element's current value. Enter all the elements of matrix a; pressing ENTER after inputting each value.

When you are finished, leave the editing screen by pressing 2nd QUIT or ◆ HOME to return to the Home screen.

5.6.2 Matrix Math: From the Home screen, you can perform many calculations with matrices. To see matrix a, press A ENTER (Figure 5.76).

Perform the scalar multiplication 2 a pressing 2A ENTER. The resulting matrix is displayed on the screen. To create matrix b as 2a press 2 A STO▸ B ENTER (Figure 5.77), or if you do this immediately after calculating 2a, press only STO▸ B ENTER. The calculator will display the matrix.

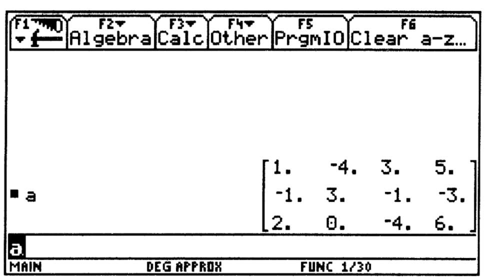


Figure 5.76: Matrix a

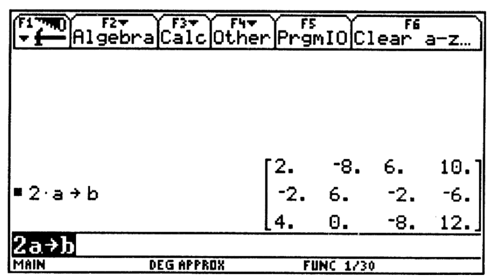


Figure 5.77: Matrix b

To add two matrices, say a and b, create b (with the same dimensions as a) and then press A + B ENTER. Again, if you want to store the answer as a specific matrix, say m, then press STO▸ M. Subtraction is performed in similar manner.

Now create a matrix called c with dimensions of 2×3 and enter the matrix $\begin{bmatrix} 2 & 0 & 3 \\ 1 & -5 & -1 \end{bmatrix}$ as c. For matrix multiplication of c by a, press C × A ENTER. If you tried to multiply a by c, your TI-92 would notify you of an error because the dimensions of the two matrices do not permit multiplication in this way.

The *transpose* of a matrix is another matrix with the rows and columns interchanged. The symbol for the transpose of a is a^T. To calculate a^T, press A 2nd MATH 4*[Matrix]* 1*[^{T}]* ENTER.

5.6.3 Row Operations: Here are the keystrokes necessary to perform elementary row operations on a matrix. Your textbook provides a more careful explanation of the elementary row operations and their uses.

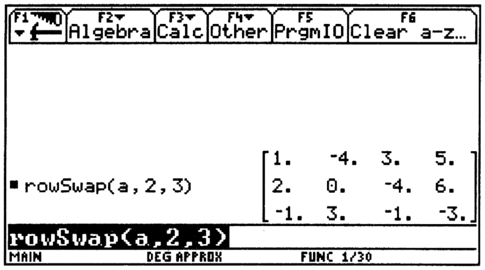


Figure 5.78: Swap rows 2 and 3

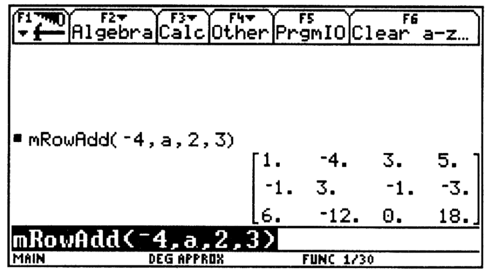


Figure 5.79: Add –4 times row 2 to row 3

To interchange the second and third rows of the matrix a that was defined above, press 2nd MATH *4[Matrix]* D*[Row ops]* 1 *[rowSwap(]* A , 2 , 3) ENTER (see Figure 5.78). The format of this command is rowSwap*(matrix1, rIndex1, rIndex2).*

To add row 2 and row 3 and *store* the results in row 3, press 2nd MATH 4*[Matrix]* D*[Row ops]* 2*[rowAdd(]* A, 2, 3) ENTER. The format of this command is rowAdd*(matrix1, rIndex1, rIndex2).*

To multiply row 2 by –4 and *store* the results in row 2, thereby replacing row 2 with new values, press 2nd MATH 4*[Matrix]* D*[Row ops]* 3*[mRow(]* (-) 4 , A , 2) ENTER. The format of this command is mRow*(expression, matrix1, index).*

To multiply row 2 by –4 and *add* the results to row 3, thereby replacing row 3 with new values, press 2nd MATH 4*[Matrix]* D*[Row ops]* 4*[mRowAdd(]* (-) 4, A, 2, 3) ENTER (see Figure 5.79). The format of this command is m RowAdd*(expression, matrix1, Index1, Index2).*

Note that your TI-92 does *not* store a matrix obtained as the result of any row operation. So, when you need to perform several row operations in succession, it is a good idea to store the result of each one in a temporary place.

For example, use row operations to solve this system of linear equations: $\begin{cases} x-2y+3z=9 \\ -x+3y=-4 \\ 2x-5y+5z=17 \end{cases}$.

First enter this *augmented matrix* as a in your TI-92: $\begin{bmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{bmatrix}$. Then return to the Home screen and store this matrix as e (press A STO➧ E ENTER), so you may keep the original in case you need to recall it.

Here are the row operations and their associated keystrokes. At each step, the result is stored in e and replaces the previous matrix e. The last two steps of the row operations are shown in Figure 5.80.

Row Operations	*Keystrokes*
add row 1 to row 2	2nd MATH 4 D 2 E, 1, 2) STO▸ E ENTER
add –2 times row 1 to row 3	2nd MATH 4 D 4 (-) 2, E, 1 , 3) STO▸ E ENTER
add row 2 to row 3	2nd MATH 4 D 2 E, 2, 3) STO▸ E ENTER
multiply row 3 by $\frac{1}{2}$	2nd MATH 4 D 3 1 ÷ 2, E, 3) STO▸ E ENTER

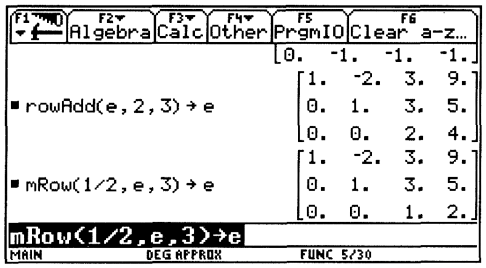


Figure 5.80: Final matrix after row operations

Thus $z = 2$, so $y = -1$, and $x = 1$.

Technology Tip: The TI-92 can produce a row-echelon form and the reduced row-echelon form of a matrix. The row-echelon form of matrix a is obtained by pressing 2nd MATH 4*[Matrix]* 3*[ref(]* A) ENTER and the reduced row-echelon form is obtained by pressing 2nd MATH 4*[Matrix]* 4*[rref(]* A) ENTER. Note that the row-echelon form of a matrix is not unique, so your calculator may not get exactly the same matrix as you do by using row operations. However, the matrix that the TI-92 produces will result in the same solution to the system.

5.6.4 Determinants and Inverses: Enter this 3×3 square matrix as a: $\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}$. Since this consists of the first three columns of the matrix a that was previously used, you can go to the matrix, move the cursor into the fourth column and press *F6[Util]* 2*[Delete]* 3*[column]*. This will delete the column that the cursor is in. To calculate its determinant $\begin{vmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{vmatrix}$, go to the Home screen and press 2nd MATH 4*[Matrix]* 2*[det(]* A) ENTER. You should find that the determinant is 2 as shown in Figure 5.81.

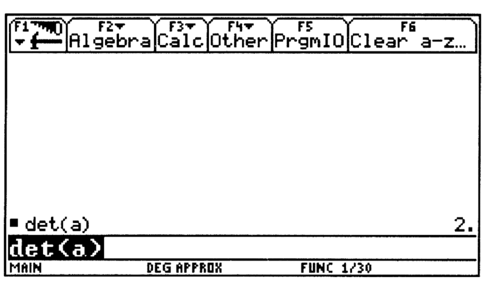


Figure 5.81: Determinant of a

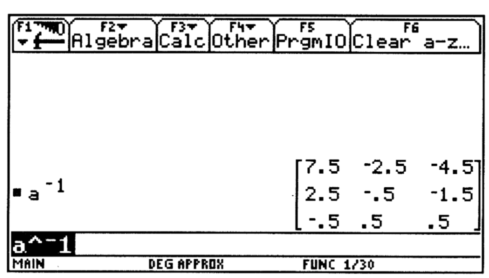


Figure 5.82: Inverse of a

Since the determinant of the matrix is not zero, it has an inverse matrix. Press A 2nd x^{-1} ENTER to calculate the inverse. The result is shown in Figure 5.82.

Now let's solve a system of linear equations by matrix inversion. Once again consider $\begin{cases} x-2y+3z=9 \\ -x+3y=-4 \\ 2x-5y+5z=17 \end{cases}$. The coefficient matrix for this system is the matrix $\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}$ which was entered as matrix a in the previous example. Now enter the matrix $\begin{bmatrix} 9 \\ -4 \\ 17 \end{bmatrix}$ as b. Since b was used before, when we stored 2a as b, press APPS 6*[Data/Matrix Editor]* 2*[Open]* ➔ 2*[Matrix]* 🡻 🡻 ➔ and use 🡻 to move the cursor to b, then press ENTER twice to go to the matrix previously saved as b, which can be edited. Return to the Home screen (◆ HOME) and press A 2nd x^{-1} × B ENTER to get the answer as shown in Figure 5.83.

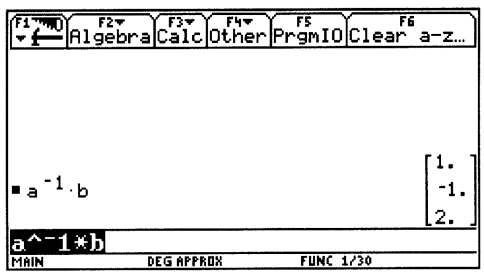


Figure 5.83: Solution matrix

The solution is still $x = 1$, $y = -1$, and $z = 2$.

5.7 Sequences

5.7.1 Iteration with the ANS key: The ANS key enables you to perform *iteration,* the process of evaluating a function repeatedly. As an example, calculate $\frac{n-1}{3}$ for $n = 27$. Then calculate $\frac{n-1}{3}$ for n = the answer to the previous calculation. Continue to use each answer as n in the *next* calculation. Here are keystrokes to accomplish this iteration on the TI-92 calculator. (See the results in Figure 5.84.) Notice that when you use ANS in place of n in a formula, it is sufficient to press ENTER to continue an iteration.

Iteration	*Keystrokes*	*Display*
1	27 ENTER	27
2	(2nd ANS – 1) ÷ 3 ENTER	8.666666667
3	ENTER	2.555555556
4	ENTER	.5185185185

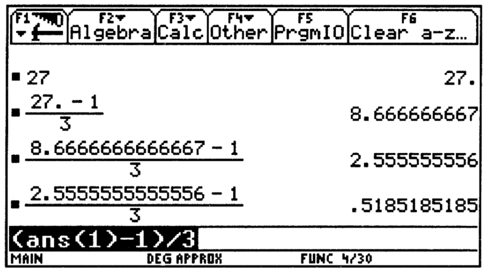
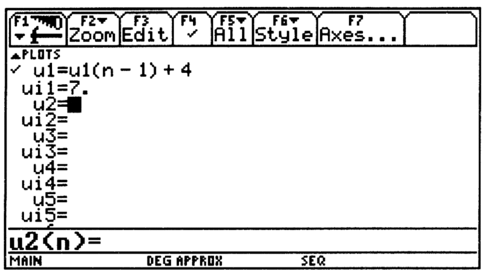


Figure 5.84: Iteration

Press ENTER several more times and see what happens with this iteration. You may wish to try it again with a different starting value.

5.7.2 Arithmetic and Geometric Sequences: Use iteration with the ANS variable to determine the *n*-th term of a sequence. For example, find the 18th term of an *arithmetic* sequence whose first term is 7 and whose common difference is 4. Enter the first term 7, then start the progression with the recursion formula, 2nd ANS + 4 ENTER. This yields the 2nd term, so press ENTER sixteen more times to find the 18th term. For a *geometric* sequence whose common ratio is 4, start the progression with 2nd ANS x 4 ENTER.



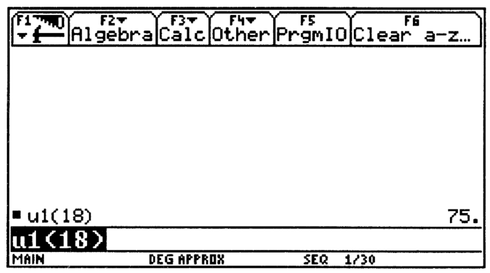


Figure 5.85: Sequential Y= menu

Figure 5.86: Sequence mode

You can also define the sequence recursively with the TI-92 by selecting Sequence in the Graph type on the first page of the MODE menu (see Figure 5.1). Once again, let's find the 18th term of an *arithmetic* sequence whose first term is 7 and whose common difference is 4. Press MODE ➔ 4*[Sequence]* ENTER. Then press ◆ Y= to edit any of the TI-92's sequences, $u1$ through $u99$. Make $u1(n) = u1(n-1) + 4$ and $u1(1) = 7$ by pressing U 1 (N – 1) + 4 ENTER 7 ENTER (Figure 5.85). Press 2nd QUIT to return to the Home screen. To find the 18th term of this sequence, calculate $u1(18)$ by pressing U 1 (18) ENTER (Figure 5.86).

Of course, you could also use the *explicit* formula for the *n*-th term of an arithmetic sequence $t_n = a + (n-1)d$. First enter values for the variables *a, d,* and *n,* then evaluate the formula by pressing A + (N – 1) D ENTER. For a geometric sequence whose *n*-th term is given by $t_n = a \cdot r^{n-1}$, enter values for the variables *a, d,* and *r*, then evaluate the formula by pressing A R ∧ (N – 1) ENTER.

To use the explicit formula in Seq MODE, make $u_1(n) = 7 + (n-1) \cdot 4$ by pressing ◆ Y= then using 🡅 to move up to the u1(n) line and pressing CLEAR 7 + (N – 1) × 4 ENTER 2nd QUIT. Once more, calculate $u1(18)$ by pressing U 1 (1 8) ENTER.

5.7.3 Finding Sums of Sequences: You can find the sum of a sequence by combining the features sum(and seq(feature on the LIST sub-menu of the MATH menu. The format of the sum(command is sum(*list*). The format of the seq(command is seq(*expression, variable, low, high, step*) where the *step* argument is optional and the default is for integer values from *low* to *high.* For example, suppose you want to find the sum $\sum_{n=1}^{12} 4(0.3)^n$. Press 2nd MATH 3*[LIST]* 6*[sum(]* 2nd MATH 3*[LIST]* 1*[seq(]* 4 (. 3) ∧ K , K , 1, 12)) ENTER (Figure 5.87). The seq(

command generates a list, which the sum(command then sums. Note that any letter can be used for the variable in the sum, i.e., the K could just have easily been an A or an N.

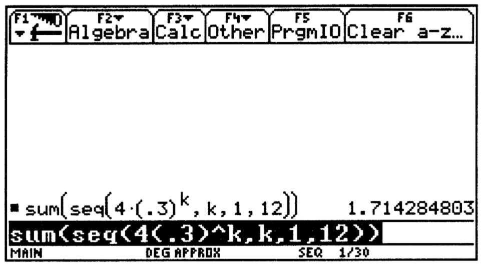


Figure 5.87: $\sum_{n=1}^{12} 4(0.3)^n$

Now calculate the sum starting at $n = 0$ by using ➔, 🡐, and ⇦ to edit the range. You should obtain a sum of approximately 5.712848.

5.8 Parametric and Polar Graphs

5.8.1 Graphing Parametric Equations: The TI-92 plots parametric equations as easily as it plots functions. Up to ninety nine pairs of parametric equations can be plotted. In the first page of the MODE menu (Figure 5.1) change the Graph setting to PARAMETRIC. Be sure, if the independent parameter is an angle measure, that the angle measure in the MODE menu has been set to whichever you need, RADIAN or DEGREE.

You can now enter the parametric functions. For example, here are the keystrokes needed to graph the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$. First check that angle measure is in radians. Then press ◆ Y= (COS T)) ∧ 3 ENTER (SIN T)) ∧ 3 ENTER (Figure 5.88).

Press ◆ WINDOW to set the graphing window and to initialize the values of t. In the standard window, the values of t go from 0 to 2π in steps of $\frac{\pi}{24} \approx 0.1309$, with the view from –10 to 10 in both directions. In order to provide a better viewing rectangle press ENTER three times and set the rectangle to go from –2 to 2 horizontally and vertically (Figure 5.89). Now press ◆ GRAPH to draw the graph (Figure 5.90).

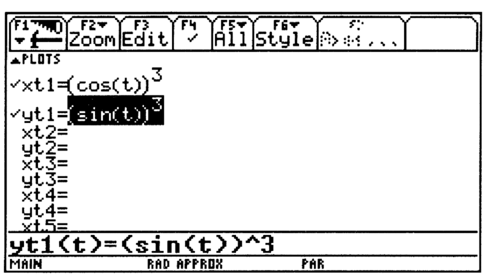


Figure 5.88: $x = \cos^3 t$ and $y = \sin^3 t$

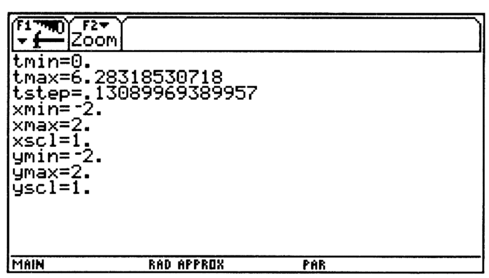


Figure 5.89: Parametric WINDOW menu

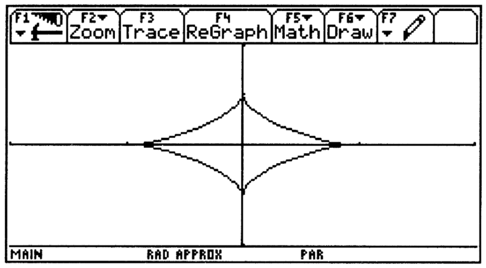


Figure 5.90: Parametric graph of $x = \cos^3 t$ and $y = \sin^3 t$

You may Zoom and Trace along parametric graphs just as you did with function graphs. However, unlike with function graphs, the cursor will not move to values outside of the t range, so 🡐 will not work when t = 0, and 🡒 will not work when t = 2π. As you trace along this graph, notice that the cursor moves in the *counterclockwise* direction as t increases.

5.8.2 Rectangular-Polar Coordinate Conversion: The Angle sub-menu of the MATH menu provides a function for converting between rectangular and polar coordinate systems. These functions use the current angle measure setting, so it is a good idea to check the default angle measure before any conversion. Of course, you may override the current angle measure setting, as explained in Section 5.4.1. For the following examples, the TI-92 is set to radian measure.

Given the rectangular coordinates $(x, y) = (4, -3)$, convert to polar coordinates (r, θ) in the Home screen by pressing 2nd MATH 2*[Angle]* 5*[R ►Pr(]* 4 , (-) 3) ENTER. The value of r is displayed; now press 2nd MATH 2*[Angle]* 6*[R ►Pθ(]* 4, (-) 3) ENTER to display the value of θ (Figure 5.91). The polar coordinates are approximately (5, –0.6435).

Suppose $(r, \theta) = (3, \pi)$. Convert to rectangular coordinates (x, y) by pressing 2nd MATH 2*[Angle]* 3*[P ► Rx(]* 3 , 2nd π) ENTER. The x-coordinate is displayed. Press 2nd MATH 2*[Angle]* 4*[P►Ry(]* 3 , 2nd π) ENTER to display the y-coordinate (Figure 5.92). The rectangular coordinates are (–3, 0).

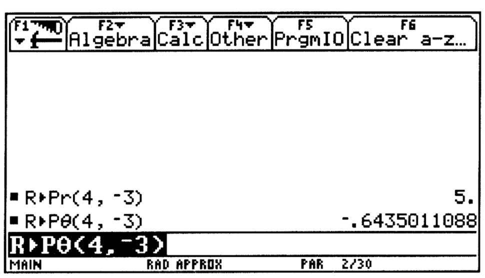


Figure 5.91: Rectangular to polar coordinates

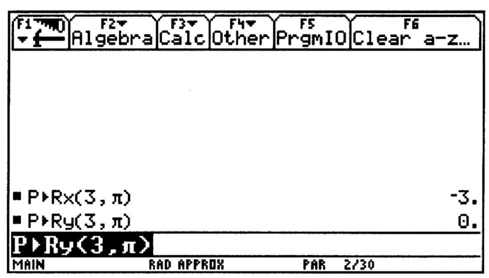


Figure 5.92: Polar to rectangular coordinates

5.8.3 Graphing Polar Equations: The TI-92 graphs polar functions in the form $r = f(\theta)$. In the Graph line of the MODE menu, select POLAR for polar graphs. You may now graph up to ninety nine polar functions at a time. Be sure that the angle measure has been set to whichever you need, RADIAN or DEGREE. Here we will use radian measure.

For example, to graph $r = 4\sin\theta$, press ◆ Y= for the polar graph editing screen. Then enter the expression 4 sin θ by pressing 4 SIN θ) ENTER. The θ key is on the lower right of the keyboard, near the ENTER key.

Choose a good viewing rectangle and an appropriate interval and increment for θ. In Figure 5.93, the viewing rectangle is roughly "square" and extends from –14 to 14 horizontally and from –6 to 6 vertically. (Refer back to the Technology Tip in Section 5.2.4.)

Figure 5.93 shows *rectangular* coordinates of the cursor's location on the graph. You may sometimes wish to trace along the curve and see *polar* coordinates of the cursor's location. The first line of the Graph Format menu (Figure 5.24) has options for displaying the cursor's position in rectangular (RECT) or polar (POLAR) form.

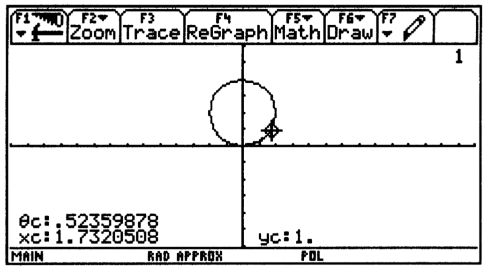


Figure 5.93: Polar graph of $r = 4 \sin \theta$

5.9 Probability

5.9.1 Random Numbers: The command rand(generates numbers. You will find this command in the Probability sub-menu of the MATH menu in the Home screen. Press 2nd MATH 7*[Probability]* 4*[rand()* *)* ENTER to generate a random number between 0 and 1. Press ENTER to generate another number; keep pressing ENTER to generate more of them.

If you need a random number between, say, 0 and 10, then press 10 2nd MATH 7*[Probability]* 4*[rand()* *)* ENTER. To get a random number between 5 and 15, press 5 + 10 2nd MATH 7*[Probability]* 4*[rand()* *)*ENTER.

If you need the random number to be an *integer* between 1 and 10 (inclusive), press 2nd MATH 7*[Probability]* 4*[rand()* 10) ENTER. For a random negative integer between –1 and –10 (inclusive), press 2nd MATH 7*[Probability]* 4*[rand()* (-) 10) ENTER

5.9.2 Permutations and Combinations: To calculate the number of permutations of 12 objects taken 7 at a time, ${}_{12}P_7$, press 2nd MATH 7*[Probability]* 2*[nPr()* 12 , 7*)* ENTER (Figure 5.94). Thus ${}_{12}P_7 = 3{,}991{,}680$.

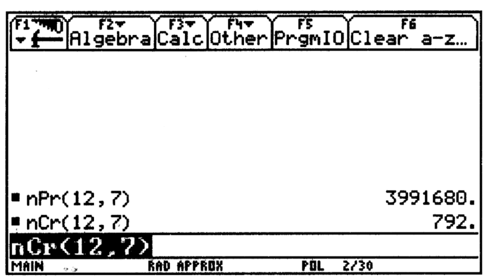


Figure 5.94: ${}_{12}P_7$ and ${}_{12}C_7$

For the number of combinations of 12 objects taken 7 at a time, ${}_{12}C_7$, press 2nd MATH 7*[Probability]* 3*[nCr]* 12 , 7) ENTER (Figure 5.94). Thus ${}_{12}C_7 = 792$.

5.9.3 Probability of Winning: A state lottery is configured so that each player chooses six different numbers from 1 to 40. If these six numbers match the six numbers drawn by the State Lottery Commission, the player wins the top prize. There are ${}_{40}C_6$ ways for the six numbers to be drawn. If you purchase a single lottery ticket, your probability of winning is 1 in ${}_{40}C_6$. Press 1 ÷ 2nd MATH 7*[Probability]* 3*[nCr]* 40 , 6) ENTER to calculate your chances, but don't be disappointed.

5.10 Programming

5.10.1 Entering a Program: The TI-92 is a programmable calculator that can store sequences of commands for later replay. Here's an example to show you how to enter a useful program that solves quadratic equations by the quadratic formula.

Press APPS 7*[Program Editor]* to access the programming menu. The TI-92 has space for many programs, each named by a name you give it. To create a new program now, start by pressing APPS 7*[Program Editor]* 3*[New]*.

Set the Type to Program and the Folder to main (unless you have another folder in which you want to have the program). Enter a descriptive title for the program in the Variable line. Name this program Quadrat and press ENTER twice to go to the program editor. The program name and the beginning and ending commands of the program are automatically displayed with the cursor on the first line after Prgm, the begin program command.

In the program, each line begins with a colon : supplied automatically by the calculator. Any command you could enter directly in the TI-92's Home screen can be entered as a line in a program. There are also special programming commands.

Input the program Quadrat by pressing the keystrokes given in the listing below. You may interrupt program input at any stage by pressing 2nd QUIT. To return later for more editing, press APPS 7*[Program Editor]* 2*[Open]*, move the cursor down to the Variable list, highlight this program's name, and press ENTER twice.

Each time you press ENTER while writing a program, the TI-92 *automatically* inserts the : character at the beginning of the next line.

The instruction manual for your TI-92 gives detailed information about programming. Refer to it to learn more about programming and how to use other features of your calculator.

Note that this program makes use of the TI-92's ability to compute complex numbers. Make sure that Complex Format on the MODE screen (Figure 5.1) is set to RECTANGULAR.

Enter the program Quadrat by pressing the given keystrokes. A space entered by using the spacebar on the keyboard is indicated by ␣ .

Program Line	*Keystrokes*
: Input "Enter a", a	⇧ I N P U T ␣ 2nd "⇧ E N T E R ␣ A 2nd ", A ENTER

displays the words ENTER A on the TI-92 screen and waits for you to input a value that will be assigned to the variable A

: Input "Enter b", b	⇧ I N P U T ␣ 2nd "⇧ E N T E R ␣ B 2nd ", B ENTER
: Input "Enter c", c	⇧ I N P U T ␣ 2nd "⇧ E N T E R ␣ C 2nd ", C ENTER
: $b^2 - 4*a*c \rightarrow d$	B ∧ 2 – 4 × A × C STO▸ D ENTER

calculates the discriminant and stores its value as d

: (–b + √ (d))/(2a) → m	((-) B + 2nd √ D)) ÷ (2 A) STO▸ M ENTER

calculates one root and stores it as m

: (–b – √ (d))/(2a) → n	((-) B – 2nd √ D)) ÷ (2 A) STO▸ N ENTER

calculates the other root and stores it as n

: If d < 0 Then	⇧ I F ␣ D 2nd < 0 ␣ ⇧ T H E N ENTER

tests to see if the discriminant is negative;

: Goto Label1	⇧ GOTO ␣ ⇧ L A B E L 1 ENTER

if the discriminant is negative, jumps to the line Label l below

: EndIf	⇧ E N D ⇧ I F ENTER

if the discriminant is not negative, continues on to the next line

: If d = 0 Then	⇧ I F ␣ D = 0 ␣ T H E N ENTER

tests to see if the discriminant is zero;

: Goto Label2	⇧ GOTO ␣ ⇧ L A B E L 2 ENTER

if the discriminant is zero, jumps to the line Label 2 below

: EndIf	⇧ E N D ⇧ I F ENTER

if the discriminant is not zero, continues on to the next line

: Disp “Two real roots”, m, n	⇧ D I S P ␣ 2nd “⇧ T W O ␣ R E A L ␣ R O O T S 2nd ” , M , N ENTER

displays the message “Two real roots” and both roots

: Stop	⇧ S T O P ENTER

stops program execution

: Lbl Label1	⇧ L B L ␣ ⇧ L A B E L 1 ENTER

jumping point for the Goto command above

: Disp “Complex roots”, m, n ⇧ D I S P ␣ 2nd “⇧ C O M P L E X ␣ R O O T S 2nd “ , M , N ENTER

displays the message “Complex roots” and both roots

: Stop ⇧ S T O P ENTER

: Lbl Label2 ⇧ L B L ␣ ⇧ L A B E L 2 ENTER

: Disp “Double root”, m ⇧ D I S P ␣ 2nd “⇧ D O U B L E ␣ R O O T 2nd “, M ENTER

displays the message “Double root” and the solution (root)

When you have finished, press 2nd QUIT to leave the program editor and move on.

If you want to remove a program from memory, press 2nd VAR-LINK, use the cursor pad to highlight the name of the program you want to delete, then press F1*[Manage]* 1*[Delete]* ENTER and then ENTER again to confirm the deletion from the calculator's memory.

Technology Tip: The program uses the variables a, b, c, d, m, and n. Note that any previous values for these variables, including matrices, will be replaced by the values used by the program. The TI-92 does not distinguish between A and a in these uses. Note that you will have to clear the variables (using 2nd VAR-LINK) in order to use these names again in the current folder. From the Home screen, F6 will clear all 1-character variables. Another way to deal with this is to create a new folder. From the Home screen, press F4*[Other]* B*[NewFold]* and type the name of the new folder. The work you do from that point on will be in the new folder, as indicated by the folder name in the lower left corner of the Status line. You can change folders from the MODE menu or, from the Home screen, by typing setFold(*foldername*), where *foldername* is the existing folder that you wish to be in.

5.10.2 Executing a Program: To execute the program you have entered, go to the Home screen and type the name of the program, including the parentheses and then press ENTER to execute it. If you have forgotten its name, press 2nd VAR-LINK to list all the variables that exist. The programs will have PRGM after the name. You can execute the program from this screen by highlighting the name and then pressing ENTER. The screen will return to the Home screen and you will have to enter the closing parenthesis) and press ENTER to execute the program.

The program has been written to prompt you for values of the coefficients *a, b,* and c in a quadratic equation $ax^2 + bx + c = 0$. Input a value, then press ENTER to continue the program.

If you need to interrupt a program during execution, press ON.

After the program has run, the TI-92 will display the appropriate message and the root(s). The TI-92 will be on the Program I/O screen *not* the Home screen. The F5 key toggles between the Home screen and the Program I/O screen or you can use 2nd QUIT, ◆ HOME to go to the Home screen, or the APPS menu to go any screen.

The instruction manual for your TI-92 gives detailed information about programming. Refer to it to learn more about programming and how to use other features of your calculator.

5.11 Differentiation

5.11.1 Limits: Suppose you need to find this limit: $\lim_{x\to 0}\frac{\sin 4x}{x}$. Plot the graph of $f(x) = \frac{\sin 4x}{x}$ in a convenient viewing rectangle that contains the point where the function appears to intersect the line $x = 0$ (because you want the limit as $x \to 0$). Your graph should lend support to the conclusion that $\lim_{x\to 0}\frac{\sin 4x}{x} = 4$. (Figure 5.95)

To test whether the conclusion that $\lim_{x\to\infty}\frac{2x-1}{x+1} = 2$ is reasonable, evaluate the function $f(x) = \frac{2x-1}{x+1}$ for several large positive values of x (since you want the limit as $x \to \infty$). For example, evaluate $f(100)$, $f(1000)$, and $f(10{,}000)$. Another way to test the reasonableness of this result is to examine the graph of $f(x) = \frac{2x-1}{x+1}$ in a viewing rectangle that extends over large values of x. See, as in Figure 5.96 (where the viewing rectangle extends horizontally from 0 to 90), whether the graph is asymptotic to the horizontal line $y = 2$. Enter $\frac{2x-1}{x+1}$ for y1 and 2 for y2.

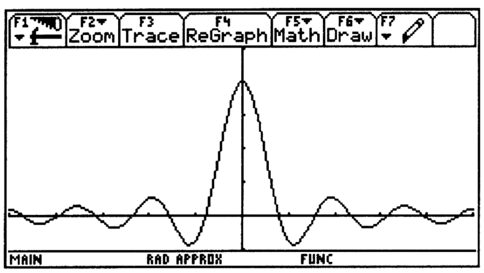


Figure 5.95: Checking $\lim_{x\to 0}\frac{\sin 4x}{x} = 4$

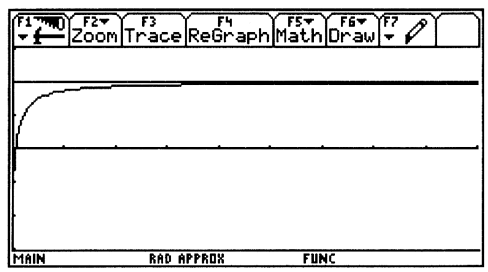


Figure 5.96: Checking $\lim_{x\to\infty}\frac{2x-1}{x+1} = 2$

5.11.2 Numerical Derivatives: The derivative of a function f at x can be defined as the limit of the slopes of secant lines, so $f'(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x-\Delta x)}{2\Delta x}$. And for small values of Δx, the expression $\frac{f(x+\Delta x)-f(x-\Delta x)}{2\Delta x}$ gives a good approximation to the limit.

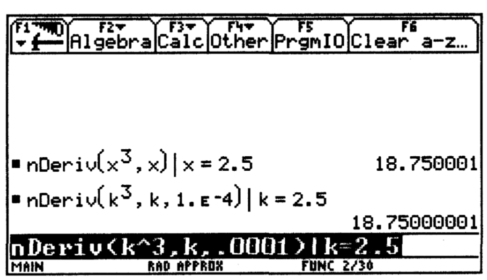


Figure 5.97: Using nDeriv(

The TI-92 has a function , nDeriv(, which is available in the Calculus sub-menu of the MATH menu, that will calculate the *symmetric difference,* $\frac{f(x+\Delta x)-f(x-\Delta x)}{2\Delta x}$. So, to find a numerical approximation to $f'(2.5)$ when $f(x) = x^3$ and with $\Delta x = 0.001$, go to the Home screen and press 2nd MATH A*[Calculus]* A*[nDeriv(]* X ∧ 3 , X) 2nd | X = 2.5 ENTER as shown in Figure 5.97. The format of this command is nDeriv*(expression, variable, Δx)*, where the optional argument Δx controls the accuracy of the approximation. The added expression, 2nd | X = 2.5 give the value of x at which the derivative is evaluated. The | is found on the keyboard above the K, so press 2nd K to enter it. If no value for Δx is provided, the TI-92 automatically uses $\Delta x = 0.001$. If no value for x is given, the TI-92 will give the symmetric difference as a function of x. The same derivative is also approximated in Figure 5.97 using $\Delta x = 0.0001$. For most purposes, $\Delta x = 0.001$ gives a very good approximation to the derivative. Note that in Figure 5.97 any letter can be used for the variable.

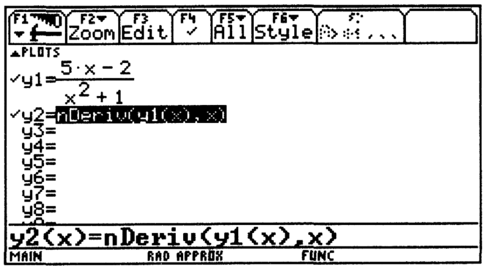


Figure 5.98: Entering $f(x)$ and $f'(x)$

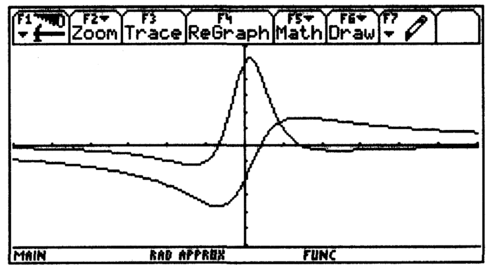


Figure 5.99: Graphs of $f(x)$ and $f'(x)$

Technology Tip: It is sometimes helpful to plot both a function and its derivative together. In Figure 5.99, the function $f(x) = \dfrac{5x-2}{x^2+1}$ and its numerical derivative (actually, an approximation to the derivative given by the symmetric difference) are graphed on viewing window that extends from –6 to 6 vertically and horizontally. You can duplicate this graph by first entering $\dfrac{5x-2}{x^2+1}$ for y1 and then entering its numerical derivative for y2 by pressing 2nd *MATH* A*[Calculus]* A*[nDeriv(]* Y 1 (X) , X*)* ENTER (Figure 5.98).

Graphing the derivative will be quite slow. Making the xres value larger on the WINDOW screen will speed up the plotting of the graph.

Technology Tip: To approximate the *second* derivative $f''(x)$ of a function $y = f(x)$ or to plot the second derivative, first enter the expression for y1 and its derivative for y2 as above. Then enter the second derivative for y3 by pressing 2nd *MATH* A*[Calculus]* A*[nDeriv(]* Y 2 (X) , X) ENTER.

You may also approximate a derivative while you are examining the graph of a function. When you are in a graph window, press *F5[Math]* 6*[Derivatives]* 1*[dy/dx],* then use the cursor pad to trace along the curve to a point where you want the derivative or enter a value and press ENTER. For example, with the TI-92 in Function graphing mode, graph the function $f(x) = \dfrac{5x-2}{x^2+1}$ in the standard viewing rectangle. Then press *F5[Math]* 6*[Derivatives]* 1*[dy/dx].* The coordinates of the point in the center of the range will appear. To find the numerical derivative at $x = -2.3$, press –2.3 ENTER. Figure 5.100 shows the derivative at that point to be about –0.7746922.

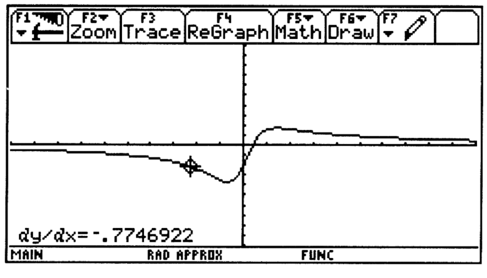


Figure 5.100: Derivative of $f(x) = \dfrac{5x-2}{x^2+1}$ at $x = -2.3$

If more than one function is graphed you can use ↑ and ↓ to scroll between the functions.

Note that different options are available from pressing *F5[Math]* 6*[Derivatives]* depending on whether the function(s) being graphed are in FUNCTION, PARAMETER, or POLAR mode.

5.11.3 Newton's Method: With the TI-92, you may iterate using Newton's method to find the zeros of a function. Recall that Newton's Method determines each successive approximation by the formula $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$.

As an example of the technique, consider $f(x) = 2x^3 + x^2 - x + 1$. Enter this function as y1 and graph it in the standard viewing window. A look at its graph suggests that it has a zero near $x = -1$, so start the iteration by going to the Home screen and storing –1 as x. Then press these keystrokes: X – Y 1 (X) ÷ 2nd MATH A*(Calculus]* *A[nDeriv(]* Y 1 (X) , X) STO▶ X ENTER ENTER (Figure 5.101) to calculate the first two iterations of Newton's method. Press ENTER repeatedly until two successive approximations differ by less than some predetermined value, say 0.0001. Note that each time you press ENTER, the TI-92 will use the *current* value of *x,* and that value is changing as you continue the iteration.

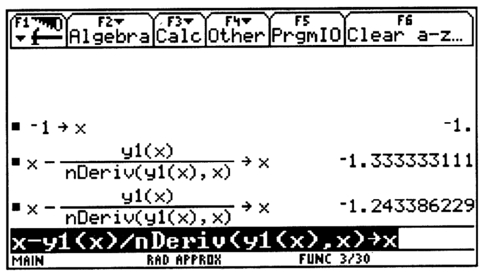


Figure 5.101: Newton's method

Technology Tip: Newton's Method is sensitive to your initial value for *x*, so look carefully at the function's graph to make a good first estimate. Also, remember that the method sometimes fails to converge!

You may want to write a short program for Newton's Method. See your calculator's manual for further information.

5.12 Integration

5.12.1 Approximating Definite Integrals: The TI-92 has a function, nInt(,which is available in the Calculus sub-menu of the MATH menu, that will approximate a definite integral. For example, to find a numerical approximation to $\int_0^1 \cos x^2 \, dx$ go to the Home screen and press 2nd MATH A*[Calculus]* B*[inInt(]* COS X ∧ 2) ,X , 0 , 1) ENTER (Figure 5.102). The format of this command is nInt(*expression, variable, lower limit, upper limit).* The algorithm that the TI-92 uses to calculate the numerical integral is adaptive, and has an accuracy goal of six significant digits. If it seems that this goal has not been achieved, the calculator will display the warning "Questionable accuracy."

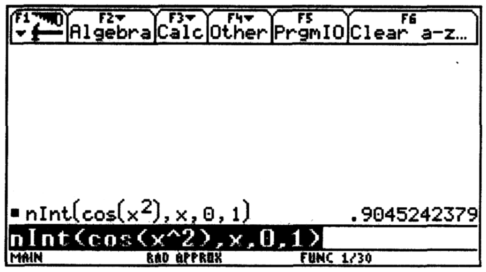


Figure 5.102: Using nInt(

5.12.2 Areas: You may approximate the area under the graph of a function $y = f(x)$ between $x = A$ and $x = B$ with your TI-92. To do this you use the F5*[Math]* menu when you have a graph displayed. For example, here are the keystrokes for finding the area under the graph of the function $y = \cos x^2$ between $x = 0$ and $x = 1$. The area is represented by the definite integral $\int_0^1 \cos\ x^2\,dx$. First clear any existing graphs and then press COS X ∧ 2) ENTER followed by ◆ GRAPH to draw the graph. The range in Figure 5.103 extends from –5 to 5 horizontally and from –2 to 2 vertically. Now press F5*[Math]* 7*[∫f(x)dx]*. The TI-92 will prompt you for the lower and upper limits which are entered by pressing 0 ENTER 1 ENTER. The region between the graph and the x-axis from the lower limit to the upper limit is shaded and the approximate value of the integral is displayed (Figure 5.104).

Technology Tip: If the function takes on negative values between the lower and upper limits, the value that the TI-92 displays it the value of the integral, *not* the area of the shaded region.

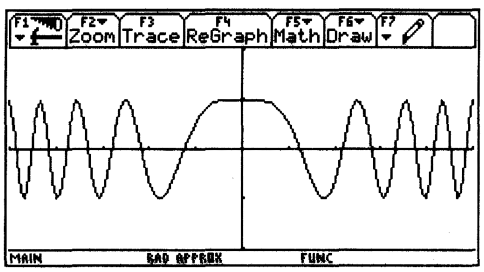


Figure 5.103: Graph of $y = \cos x^2$

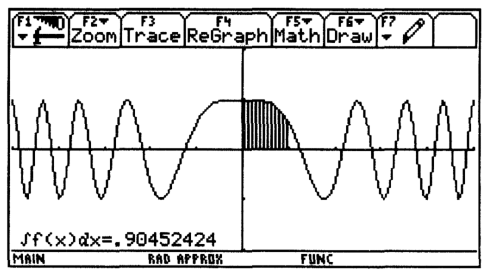


Figure 5.104: Graph and area

Technology Tip: Suppose that you want to find the area between two functions, $y = f(x)$ and $y = g(x)$ from $x = A$ and $x = B$. If $f(x) \geq g(x)$ for $A \leq x \leq B$, then enter the expression $f(x) - g(x)$ and use the method above to find the required area.