Physics 218

Alternating Current Circuits and Filters
Due Monday, February 16, 2015

1 INTRODUCTION

Many of the ideas we will discuss in the context of mechanical oscillations carry over into the realm of oscillations in electrical circuits. Broadly speaking, the process of analyzing time-varying signals is known as “signal processing” .

In this experiment you will examine the effects of resistors and capacitors on alternating current signals. You will construct high pass, low pass, and bandpass filters. These circuits will have a response that depends on the frequency of the incoming signal.

1.1 Notation Conventions

We will typically use lower case letters, e.g. v, for time-varying quantities, and upper case letters, e.g. V, for constant quantities. Of course, sometimes those “constant” quantities themselves will be slowly varying in time, so the distinction is not absolute. Typically, we will consider an input signal of the form

$$v_{in}(t) = V_{in} \cos \omega t \,,$$

and the corresponding output signal will be of the form

$$v_{out}(t) = V_{out} \cos(\omega t + \phi) \,.$$

The constants V_{in} and V_{out} are known as amplitudes. The ratio $A = V_{out}/V_{in}$ is known as the voltage gain. Even though A is less than 1 in the absence of amplification of some sort, it is still called the voltage “gain”.

2 Digital Oscilloscope

For this experiment, we will use the Tektronix TDS 1012 digital oscilloscope. These oscilloscopes have several built-in tools that will enable us to quickly analyze the response of various circuits. (You may be tempted to use a DMM instead of an oscilloscope to measure V_{in} and V_{out}, but you should beware that DMMs typically only work up to a maximum frequency of about 5 to 10 kHz. Consult the specifications for a particular DMM if in doubt.)

2.1 Voltage Measurements

Plug the Hi-Ω output of the function-generator into the CH 1 input of the oscilloscope. Set the function generator to 1000 Hz sine waves, and put the amplitude about half-way up to the maximum.

Note that on the right-hand side of the screen there is space for a menu. The menu available is selected by pressing the appropriate button on the front panel. For example, you can select either `CH 1 menu` or `CH 2 menu`, `Math menu` (this one includes the Fast Fourier Transform, or FFT option), `Measure`, or `Cursor`. Try pressing each one and looking at the different options available. For the `Measure` button, the `freq` (frequency) and `Pk-Pk` (peak-to-peak voltage) settings are often quite useful. Note that when you press the `Cursor` menu, small lights come on beside the vertical position knobs—these allow you to move two cursors on the screen. These will be useful for measuring phase differences.

Experiment with the various buttons and dials on the front of the oscilloscope until you are comfortable with how to use it to make measurements.

2.2 Calibration

In this experiment, you will use the oscilloscope in dual-trace mode. Channel 1 will be used to measure V_{in} and Channel 2 will be used to measure V_{out}. The gain A will then simply be the amplitude of Ch. 2 divided by the amplitude of Ch. 1.

Before getting started, it is important to check that the two channels are calibrated. Set the function generator to 1500 Hz, and turn the amplitude knob about half-way up. Connect both Ch. 1 and Ch. 2 to to function generator `HIΩ` and `GND` outputs. Adjust the Variable Sensitivity knobs until both channels agree on the input voltage.

3 RC High Pass Filter

In this part, you will study the voltage gain and phase shift of a simple RC circuit as a function of frequency.

Construct the circuit shown in Fig. 1. Use $R = 10,000\,\Omega$ and $C = 0.01\,\mu F$. (These are nominal values only. Measure, record, and use the actual resistance and capacitance values.) Set the function generator to sine waves.

It is useful to think of this as a voltage divider. The amplitude of the output voltage is

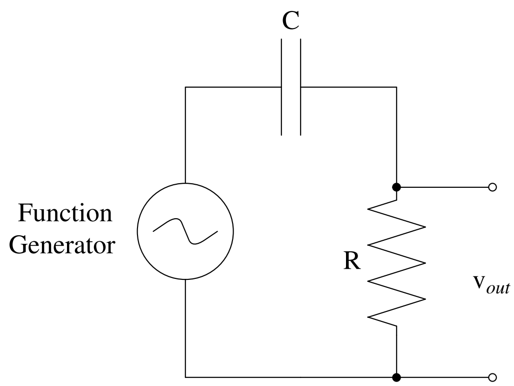

Figure 1: RC High Pass Filter

then $V_{out} = V_{in}\frac{R}{Z}$, where the Z is the total impedance, given by

$$Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}} \; .$$

The voltage gain $A(\omega)$ of the circuit is defined to be V_{out}/Vin, and is given for this circuit by

$$A(\omega) = \frac{V_{out}}{V_{in}} = \frac{R}{Z} = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}} \; , \tag{1}$$

where the last expression follows from simple algebra. At small ω the gain is near 0, but as the frequency increases, the gain rises and approaches 1. Hence this circuit is known as a high-pass filter—high frequency signals are passed through, but low frequency signals are blocked.

The "breakpoint" frequency is defined to be $\omega_b = 1/(RC)$. At this frequency, A has decreased to a value $1/\sqrt{2}$, and the power has decreased to 1/2 of its peak value.

Calculate the expected linear breakpoint frequency $f_b = \omega_b/(2\pi) = 1/(2\pi RC)$ for your circuit.

The phase ϕ of the output voltage also varies with frequency, and is given by

$$\phi(\omega) = \tan^{-1}\left(\frac{1}{\omega RC}\right) \tag{2}$$

Set up the oscilloscope for dual trace mode. Monitor the input voltage on Channel 1, and the output voltage (The voltage across R) on Channel 2. Use the "DC" settings for both channels. (The "AC" setting is effectively just a high-pass filter that subtracts out the average value of the incoming signal.)

3.1 Voltage Gain and Phase Difference

Use the `Measure` feature to measure the voltage gain. Note that since you are taking a ratio V_{out}/Vin, it doesn't matter if you use the "peak" or the "peak-to-peak" values.

To measure the phase difference, use the `Cursor` menu. (Note that if you use `Cursor 1` and `Cursor 2` to mark the horizontal positions of two peaks, the oscilloscope can calculate the time difference Δt for you. [1] The phase is then given by

$$\phi = 2\pi \frac{\Delta t}{T} = 2\pi(\Delta t) f$$

where $T = 1/f$ is the oscillation period.

3.1.1 Data Acquisition

Take the data necessary to measure the voltage gain and phase difference over a wide range of frequencies, going from well below f_b to well above f_b. Record your *raw data* in a clear table. Note that the *raw data* is the data you get directly from the oscilloscope, not the results of calculations you perform on that data.

You don't need a lot of data points. Take a few data points near the ends of the range, and more near the breakpoint frequency. If you *plot your data as you go along*, it should become clear when you have enough data. Don't waste time taking lots of closely-spaced data points.

3.1.2 Analysis

From your raw data, calculate the voltage gain A and phase shift ϕ as a function of frequency. Plot your results. Include the theoretical curve on your graph as well. (This is easy to do in `Excel`, `LoggerPro`, and `Mathematica`. Please feel free to ask for help setting up the graphs and calculations in any of them.)

Discuss your findings. If everything agrees well, there is usually little more to say. If there are significant discrepancies, try to resolve them, perhaps by going back and re-measuring some points, if necessary. No sophisticated error analysis is required for this lab, just a qualitative assessment of how well the theory fits the data.

[1] See the example "Measuring Ring Frequency" on pg. 48 of the User Manual for step-by-step instructions.

4 RC Low Pass Filter

Interchange the position of the resistor and capacitor in Fig. 1. Measure the output voltage across the capacitor.[2]

The theoretical gain for this circuit is

$$A = \frac{V_{out}}{V_{in}} = \frac{X_c}{Z} = \frac{1}{\sqrt{1 + (\omega RC)^2}} \; . \tag{3}$$

The breakpoint frequency is the same as before.

4.1 Data Acquisition

Measure the voltage gain and phase difference over a wide range of frequencies, going from well below f_b to well above f_b.

4.2 Analysis

Again, plot the theoretical predictions along with your measured values for A *vs.* f and ϕ *vs.* f. Discuss your results briefly.

5 Bandpass Filter

A "bandpass" filter is a filter that passes only a narrow range (or "band") of frequencies. Signals with frequencies either higher or lower than that band are blocked. One simple way to build a bandpass filter is to start with a low-pass filter (which blocks high frequencies) and follow it with a high-pass filter (which blocks low frequencies).

Design and build a bandpass filter by combining low-pass and high-pass filters. Start by using your low-pass filter from the previous part. This will tend to pass all signals with frequencies lower than f_{lowpass}. Use the *output* of your low-pass filter as the *input* to a high pass filter. Your high-pass filter should have a breakpoint frequency $f_{highpass}$ slightly *lower*

[2] You may be tempted to simply move the oscilloscope probes to measure the voltage across the capacitor, but this won't work because of the way the oscilloscope probes are grounded. The negative terminals of the oscilloscope probes are connected internally to the oscilloscope ground, which is also connected to the third prong on the power cord. Thus the negative terminals of both probes must always be connected to the same spot in the circuit. In fact, you don't even need to connect the negative terminal of the second probe, though it's good practice to do so.

than f_{lowpass}. That is, the combination of filters will tend to pass through only frequencies f in the range

$$f_{\text{highpass}} < f < f_{\text{lowpass}} \quad . \tag{4}$$

If possible, design the second stage of the filter to have a higher impedance to prevent it from "loading down" the first stage. Include a diagram of your working circuit.

Qualitatively, your design should look something like Fig. 2.

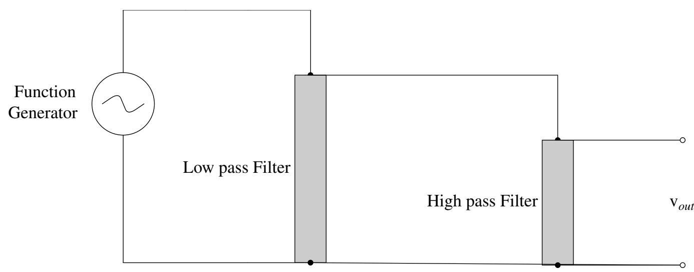

Figure 2: Block diagram of a bandpass filter, consisting of a low-pass filter followed by a high-pass filter.

5.1 Data Acquisition

Measure the voltage gain and phase difference for your circuit as a function of frequency over a vary wide range of frequencies. Again, don't waste time taking too many closely-spaced data points. You are looking for the general overall behavior over a very wide range of frequencies. **If you *plot your data as you go along*, it should become clear when you have enough data.** See the Analysis section below for further details.

5.2 Analysis

Plot $\log(A)$ as a function of $\log(f)$. (The log-log plot is a useful way to show a wide range of data on a single plot.) Include on your graph the theoretical prediction for each stage of the filter operating independently. (You don't need to compute a combined theoretical response.)

Similarly, plot ϕ as a function of $\log(f)$, and include the the theoretical predictions for each stage of the filter operating independently.

Discuss your observations.

With more complex circuits, it is possible to get a much sharper frequency response, but this simple circuit illustrates the basic features of a bandpass filter.