# CHAPTER 1

# Introduction

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# OUTLINE

1.1 Advantages of the Bayesian Approach to Statistics	2
1.1.1 Numerical Tractability	2
1.1.2 Absence of Asymptotics	2
1.1.3 Ease of Error Propagation	2
1.1.4 Formal Framework for Combining Information	3
1.1.5 Intuitive Appeal	3
1.1.6 Coherence and Intellectual Beauty	4
1.2 So Why Then Isn't Everyone a Bayesian?	4
1.3 WinBUGS	4
1.4 Why This Book?	5
1.4.1 This Is Also an R Book	6
1.4.2 Juxtaposition of Classical and Bayesian Analyses	6
1.4.3 The Power of Simulating Data	7
1.5 What This Book Is Not About: Theory of Bayesian Statistics	
and Computation	8
1.6 Further Reading	9
1.7 Summary	11

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WinBUGS (Gilks et al., 1994; Spiegelhalter et al., 2003; Lunn et al., 2009)
is a general-purpose software program to fit statistical models under the
Bayesian approach to statistics. That is, statistical inference is based on
the posterior distribution, which expresses all that is known about the
parameters of a statistical model, given the data and existing knowledge.
In recent years, the Bayesian paradigm has gained tremendous momentum

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#### 1. INTRODUCTION

1 in statistics and its applications, including ecology, so it is natural to wonder

2 about the reasons for this.

# s0010 1.1 ADVANTAGES OF THE BAYESIAN APPROACH TO STATISTICS

p0015 3 Key assets of the Bayesian approach and of the associated computa-4 tional methods include the following:

#### s0015 1.1.1 Numerical Tractability

p0020 5 Many statistical models are currently too complex to be fitted using
6 classical statistical methods, but they can be fitted using Bayesian compu7 tational methods (Link et al., 2002). However, it may be reassuring that, in
8 many cases, Bayesian inference gives answers that numerically closely
9 match those obtained by classical methods.

#### s0020 1.1.2 Absence of Asymptotics

Asymptotically, that is, for a "large" sample, classical inference based on maximum likelihood (ML) is unbiased, i.e., in the long run right on target. However, for finite sample sizes, *i.e., for your data set*, ML may well be biased (Le Cam, 1990). Similarly, standard errors and confidence intervals are valid only for "large" samples. Statisticians never say what "large" exactly means, but you can be assured that typical ecological data sets aren't large. In contrast, Bayesian inference is *exact* for any sample rsize. This issue is not widely understood by ecological practitioners of statistics but may be particularly interesting for ecologists since our data sets are typically small to very small.

## s0025 1.1.3 Ease of Error Propagation

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In classical statistics, computing the uncertainty of functions of random variables such as parameters is not straightforward and involves approximations such as the delta method (Williams et al., 2002). For instance, consider obtaining an estimate for a population growth rate ( $\hat{r}$ ) that is composed of two estimates of population size in subsequent years ( $\hat{N}_1, \hat{N}_2$ ). We have  $\hat{N}_1$  and  $\hat{N}_2$  and we want  $\hat{r}$ : what should we do? Getting the point estimate of  $\hat{r}$  is easy, but what about its standard error? In a Bayesian analysis with Markov chain Monte Carlo, estimating such, and much more complex, derived quantities including their uncertainty is 1.1 ADVANTAGES OF THE BAYESIAN APPROACH TO STATISTICS

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1 trivial once we have a random sample from the posterior distribution of 2 their constituent parts, such as  $\hat{N}_1$  and  $\hat{N}_2$  in our example.

#### s0030 1.1.4 Formal Framework for Combining Information

3 By basing inference on both what we knew before (the prior) and what 4 we see now (the data at hand), and using solely the laws of probability for 5 that combination, Bayesian statistics provides a formal mechanism for 6 introducing external knowledge into an analysis. This may greatly 7 increase the precision of the estimates (McCarthy and Masters, 2005); 8 some parameters may only become estimable through precisely this com-9 bination of information.

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Using existing information also appears a very sensible thing to do: after 11 all, only rarely don't we know anything at all about the likely magnitude of 12 an estimated parameter. For instance, when estimating the annual survival 13 rate in a population of some large bird species such as a condor, we would 14 be rather surprised to find it to be less than, say, 0.9. Values of less than, say, 15 0.5 would appear downright impossible. However, in classical statistics, by not using any existing information, we effectively say that the survival rate 16 17 in that population could be just as well 0.1 as 0.9, or even 0 or 1. This is not 18 really a sensible attitude since every population ecologists knows very well 19 a priori that no condor population would ever survive for very long with a 20 survival rate of 0.1. In classical statistics, we always feign total ignorance 21 about the system under study when we analyze it. However, within some limits, it is also possible to specify ignorance in a

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However, within some limits, it is also possible to specify ignorance in a Bayesian analysis. That is, also under the Bayesian paradigm, we can base our inference on the observed data alone and thereby obtain inferences that are typically very similar numerically to those obtained in a classical analysis.

26 analysis.

# s0035 1.1.5 Intuitive Appeal

p0050 27 The interpretation of probability in the Bayesian paradigm is much more 28 intuitive than in the classical statistical framework; in particular, we directly 29 calculate the probability that a parameter has a certain value rather than the 30 probability of obtaining a certain kind of data set, given some Null hypo-31 thesis. Hence, popular statements such as "I am 99% sure that ..." are only 32 possible in a Bayesian mode of inference, but they are impossible in princi-33 ple under the classical mode of inference. This is because, in the Bayesian 34 approach, a probability statement is made about a parameter, whereas in 35 the classical approach, it is about a data set.

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36 Furthermore, by drawing conclusions based on a combination of what 37 we knew before (the prior, or the "experience" part of learning) and what

#### 1. INTRODUCTION

1 we see now (the likelihood, or the "current observation" part of learning),

2 Bayesian statistics represent a mathematical formalization of the learning

*3 process,* i.e., of how we all deal with and process information in science

4 as well as in our daily life.

# s0040 1.1.6 Coherence and Intellectual Beauty

5 The entire Bayesian theory of statistics is based on just three axioms of 6 probability (Lindley, 1983, 2006). This contrasts with classical statistics 7 that Bayesians are so fond to criticize for being a patchwork of theory 8 and *ad hoc* amendments containing plenty of internal contradictions.

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# 1.2 SO WHY THEN ISN'T EVERYONE A BAYESIAN?

Given all the advantages of the Bayesian approach to statistics just men-tioned, it may come as a surprise that currently almost all ecologists stilluse classical statistics. Why is this?

12 Of course, there is some resistance to the Bayesian philosophy with 13 its perceived subjectivity of prior choice and the challenge of avoiding 14 to, unknowingly, inject information into an analysis via the priors, see 15 Chapter 2. However, arguably, the lack of a much more widespread adop-16 tion of Bayesian methods in ecology has mostly practical reasons.

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First, a Bayesian treatment shines most for complex models, which may 17 18 not even be fit in a frequentist mode of inference (Link et al., 2002). Hence, until very recently, most applications of Bayesian statistics featured rather 19 20 complex statistical models. These are neither the easiest to understand in 21 the first place, nor may they be relevant to the majority of ecologists. 22 Second, typical introductory books on Bayesian statistics are written in 23 what is fairly heavy mathematics to most ecologists. Hence, getting to the 24 entry point of the Bayesian world of statistics has been very difficult for 25 many ecologists. Third, Bayesian philosophy and computational methods 26 are not usually taught at universities. Finally, and perhaps most impor-27 tantly, the practical implementation of a Bayesian analysis has typically 28 involved custom-written code in general-purpose computer languages 29 such as Fortran or C++. Therefore, for someone lacking a solid knowledge 30 in statistics and computing, Bayesian analyses were essentially out of reach.

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#### 1.3 WinBUGS

p0080 31 This last point has radically changed with the advent of WinBUGS
32 (Lunn et al., 2009). Arguably, WinBUGS is the only software that allows
33 an average numerate ecologist to conduct his own Bayesian analyses of

1.4 WHY THIS BOOK?

realistically complex, customized statistical models. By customized I mean 1 that one is not constrained to run only those models that a program lets 2 3 you select by clicking on a button. However, although WinBUGS has been 4 and is increasingly being used in ecology, the paucity of really accessible 5 and attractive introductions to WinBUGS for ecologists is a surprise (but see McCarthy, 2007). I believe that this is the main reason for why Win-6 7 BUGS isn't even more widely used in ecology.

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# 1.4 WHY THIS BOOK?

8 This book aims at filling this gap by gently introducing ecologists to p0085 9 WinBUGS for exactly those methods they use most often, i.e., the linear, 10 generalized linear, linear mixed, and generalized linear mixed model 11 (GLMM). Table 1.1 shows how the three latter model classes are all gen-12 eralizations of the simple Normal linear model (LM) in the top left cell of 13 the body of the table. They extend the Normal model to contain either more than a single random process (represented by the residual in the 14 15 Normal LM) and/or to exponential family distributions other than the Normal, e.g., Poisson and Binomial. Alternatively, starting from the 16 17 GLMM in the bottom right cell, the other three model classes can be 18 viewed as special cases obtained by imposing restrictions on a general 19 GLMM.

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These four model classes form the core of modern applied statistics. 21 However, even though many ecologists will have applied them often 22 using click-and-point programs or even statistics packages with a pro-23 gramming language such as GenStat, R, or SAS, I dare express doubts 24 whether they all really always understand the models they have fitted. 25 Having to specify a model in the elementary way that one has to in Win-26 BUGS will prove to greatly enhance your understanding of these models, 27 whether you fit them by some sort of likelihood analysis (e.g., ML or 28 restricted maximum likelihood [REML]) or in a Bayesian analysis. p0095 29 Apart from the gentle and nonmathematical presentation by examples,

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 TABLE 1.1
 Classification of Some Core Models Used for Applied
 Statistical Analysis

	Single Random Process	Two or More Random Processes
Normal response	Linear model (LM)	Linear mixed model (LMM)
Exponential family response	Generalized linear model (GLM)	Generalized linear mixed model (GLMM)

the unique selling points of this book, which distinguish it from others, are

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#### 1. INTRODUCTION

- 1 the full integration of all WinBUGS analyses into program R, the parallel
- 2 presentation of classical and Bayesian analyses of all models and the use of
- 3 simulated data sets. Next, I briefly expand on each of these points.

# s0060 1.4.1 This Is Also an R Book

One key feature of this book as an introduction to WinBUGS is that we conduct all analyses in WinBUGS fully integrated within program R (R Development Core Team, 2007). R has become the *lingua franca* of mod-7 ern statistical computing and conducting your Bayesian analysis in 8 WinBUGS from within an R session has great practical benefits. Moreover, 9 we also see how to conduct all analyses using common R functions such as 1m(), glm(), and glmer(). This has the added bonus that this book will be useful to you even if you only want to learn to understand and fit the models in Table 1 in a classical statistical setting.

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#### 1.4.2 Juxtaposition of Classical and Bayesian Analyses

13 Another key feature is the juxtaposition of analyses using the classical 14 methods provided for in program R (mostly ML) and the analyses of the 15 same models in a Bayesian mode of inference using WinBUGS. Thus, with the exception of Chapters 20 and 21, we fit every model in both the clas-16 17 sical and the Bayesian mode of inference. I have two reasons for creating 18 parallel examples. First, this should increase your confidence into the 19 "new" (Bayesian) solutions since with vague priors they give numerically 20 very similar answers as the "old" solutions (e.g., ML). Second, the analysis 21 of a single model by both classical and Bayesian methods should help to 22 demystify Bayesian analysis. One sometimes reads statements like "we used a Bayesian model," or "perhaps a Bayesian model should be tried 23 24 on this difficult problem." This is nonsense! Since any model exists inde-25 pendently of the method we choose to analyze it. For instance, the linear regression model is not Bayesian or non-Bayesian; rather, this model may 26 27 be *analyzed* in a Bayesian or in a frequentist mode of inference. Even that 28 class of models which has come to be seen as almost synonymous with 29 Bayesian inference, hierarchical models which specify a hierarchy of sto-30 chastic processes, is not intrinsically Bayesian; rather, hierarchical models can be analyzed by frequentist (de Valpine and Hastings, 2002; Lee et al., 31 32 2006; de Valpine, 2009; Ponciano et al., 2009) or by Bayesian methods 33 (Link and Sauer, 2002; Sauer and Link, 2002; Wikle, 2003; Clark et al., 34 2005). Indeed, many statisticians now use the two modes of inference 35 quite opportunistically (Royle and Dorazio, 2006, 2008). Thus, the juxta-36 position of classical and Bayesian analysis of the same models should 37 make it very clear that a model is one thing and its analysis another 38 and that there really is no such thing as a "Bayesian model."

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1.4 WHY THIS BOOK?

# s0070 1.4.3 The Power of Simulating Data

<ul> <li>p0115 7</li> <li>p0115 7</li> <li>analysis of a model can be compared with what we know they show be in the long-run average.</li> <li>p0120 10</li> <li>When coding an analysis in WinBUGS, especially in more complex</li> </ul>	he Ild
<ul> <li>analysis of a model can be compared with what we know they show</li> <li>be in the long-run average.</li> <li>p0120 10 2. When coding an analysis in WinBUGS, especially in more complex</li> </ul>	uld
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p0120 10 2. When coding an analysis in WinbUGS, especially in more complex	vn
11 cases but over for simpler and it is very easy to make mistakes	v <b>n</b>
11 cases but even for simpler ones, it is very easy to make mistakes. 12 Encuring that an analysis recovers estimates that recomble the know	
12 Ensuring that an analysis recovers estimates that resemble the know	v11
15 Input values used to generate a data set can be an important check in 14 it has been coded correctly.	lat
<b>50195</b> 15 3 It has been said that one of the most difficult but absolutely necessi	177
16 statistical concepts to grasp is that of the sampling variation of an	Тy
17 estimator. For nonstatisticians. I don't see any other way to grasp t	he
18 meaning of sampling variation other than literally experiencing it h	v
19 repeatedly simulating data under the same model analyzing them a	y nd
20 seeing how estimates differ randomly from one sample to the next; t	nis
21 variation is exactly what the standard error of an estimate quantifies.	In
real life, one typically only ever observes a single realization (i.e., d	ata
set) from the stochastic system about which one wants to make an	
24 inference in a statistical analysis. Hence, for ecologists it may be hard	to
25 make the connection with the concept of repeated samples from a	
26 system, when all we have is a single data set (and related to that, t	Э
27 understand the difference between a standard deviation and a standard	rd
28 error).	
p0130 29 4. Simulating data can be used to study the long-run average	
30 characteristics of estimates, given a certain kind of data set, by	
31 repeating the same data generation-data analysis cycle many times.	In
32 this way, the (frequentist) operating characteristics of an estimator	
33 (bias, or "is it on target on average?"; efficiency, or "how far away fro	m
34 the target is the individual estimate on average?") can be studied b	у
35 packaging both the simulation and the analysis into a loop and	
comparing the distribution of the resulting estimates to the known	
57 truth. Further, required sample sizes to obtain a desired level of	
20 All this can be done for exactly the energifications of one's data act	y. ~
All this can be upper for exactly the specifications of one's data set, e	g., nd
40 replicate usia sets can be generated and analyzed with sample size a	uu
42 impression say of the precision of the estimates that one is likely t	0

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8 1. INTRODUCTION 1 obtain. This is also the idea behind posterior predictive checks of 2 goodness-of-fit, where the "natural" lack of fit for a model is studied 3 using ideal data sets and then compared with the lack of fit observed 4 for the actual data set (see Section 8.4.2). 5 5. Simulated data sets can be used to study effects of assumption p0135 6 violations. All models embody a set of assumptions that will be 7 violated to some degree. Whether this has serious consequences for 8 those estimates one is particularly interested in, can be studied using 9 simulation. 10 6. Finally, and perhaps most importantly, I would claim that the ultimate p0140 proof that one has really understood the analysis of a statistical model 11 12 is when one is able to simulate a data set under that very model. 13 Analyzing data is a little like fixing a motorbike but in reverse: it 14 consists of breaking a data set into its parts (e.g., covariate effects and 15 variances), whereas fixing a bike means putting all the parts of a bike 16 into the right place. One way to convince yourself that you really 17 understand how a bike works is to first dismantle and then reassemble 18 it again to a functioning vehicle. Similarly, for data analysis, by first 19 assembling a data set and then breaking it apart into recognizable parts 20 by analyzing it, you can prove to yourself that you really understand 21 the analysis. 22 p0145 In summary, I believe that the value of simulation for analysis and 23 understanding of complex stochastic systems can hardly be overstated. 24 On a personal note, what has helped me most to understand nonnormal 25 GLMs or mixed models, apart from having to specify them in the intuitive 26 BUGS language, was to simulate the associated data sets in program R, 27 which is great for simulating data. 28 Finally, I hope that the slightly artificial flavor of my data sets is more p0150 29 than made up for by their nice ecological setting and the attractive organisms we pretend to be studying. I imagine that many ecologists will by far 30 31 prefer learning about new statistical methods using artificial ecological data 32 sets than using real, but "boring" data sets from the political, social,

- 33 economical, or medical sciences, as one has to do in many excellent intro-
- 34 ductory books.

# s0075 1.5 WHAT THIS BOOK IS NOT ABOUT: THEORY OF BAYESIAN STATISTICS AND COMPUTATION

p0155 35 The theory of Bayesian inference is treated only very cursorily in this
36 book (see Chapter 2). Other authors have done this admirably, and
37 I refer you to them. Texts that should be accessible to ecologists include

#### 1.6 FURTHER READING

Ellison (1996), Wade (2000), Link et al. (2002), Bernardo (2003), Brooks
 (2003), Gelman et al. (2004), Woodworth (2004), McCarthy (2007), Royle
 and Dorazio (2008), King et al. (2009), and Link and Barker (2010).

4 Furthermore, I don't dwell on explaining Markov chain Monte Carlo 5 (MCMC) or Gibbs sampling, the computational methods most frequently used to fit models in the Bayesian framework. Arguably, a deep under-6 7 standing of the details of MCMC is not required for an ecologist to conduct an adequate Bayesian analysis using WinBUGS. After all, very few 8 9 ecologists who nowadays fit a GLM or a mixed model understand the 10 (possibly restricted) likelihood function or the algorithms used to find its maximum. (Or can you explain the Newton-Raphson algorithm? 11 And how about iteratively reweighted least squares?) Rather, by using 12 13 WinBUGS we are going to experience some of the key features of 14 MCMC. This includes the chain's initial transient behavior, the resultant 15 need for visual or numerical assessment of convergence that leads to dis-16 carding of initial ("burn-in") parts of a chain, and the fact that successive 17 iterations are not independent. If you want to read more on Bayesian computation, most of the above references may serve as an entry point to a 18 19 rich literature.

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# **1.6 FURTHER READING**

p0165 20 If you seriously consider going Bayesian for your statistical modeling, 21 you will probably want to purchase more than a single book. McCarthy 22 (2007) is an accessible introduction to WinBUGS for beginners, although it 23 presents WinBUGS only as a standalone application (i.e., not run from R) 24 and the coverage of model classes dealt with is somewhat more limited. 25 Gelman and Hill (2007) is an excellent textbook on linear, generalized, and 26 mixed (generalized) linear models fit in both the classical and the Bayesian 27 mode of inference and using both R and WinBUGS. Thus, its concept is 28 somewhat similar to that of this book, though it does not feature the rig-29 orous juxtaposition of both kinds of analysis. All examples are from the 30 social and political sciences, which will perhaps not particularly interest 31 an ecologist. However, the book contains a wealth of information that should be digestible for the audience of this book, as does Gelman et al. 32 33 (2004). Ntzoufras (2009) is a new and comprehensive introduction to Win-34 BUGS focusing on GLMs. It is very useful, but has a higher mathematical 35 level and uses WinBUGS as a standalone application only. Woodworth (2004) is an entry-level introduction to Bayesian inference and also has 36 37 some WinBUGS code examples.

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Link and Barker (2010) is an excellent textbook on Bayesian inferencespecifically for ecologists and featuring numerous WinBUGS examples.

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#### 1. INTRODUCTION

As an introduction to Bayesianism written mostly in everyday language,
 Lindley, an influential Bayesian thinker, has written a delightful book,
 where he argues, among others, that *probability is the extension of logic to all events, both certain (like classical logic) and uncertain* (Lindley, 2006,
 p. 66). His book is not about practical aspects of Bayesian analysis, but
 very informative, quite amusing and above all, written in an accessible
 way.

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8 In this book, we run WinBUGS from within program R; hence, some 9 knowledge of R is required. Your level of knowledge of R only needs to be minimal and any simple introduction to R would probably suffice to 10 enable you to use this book. I like Dalgaard (2001) as a very accessible 11 12 introduction that focuses mostly on linear models, and at a slightly higher level, featuring mostly GLMs, Crawley (2005) and Aitkin et al. 13 14 (2009). More comprehensive R books will also contain everything 15 required, e.g., Venables and Ripley (2002), Clark (2007), and Bolker 16 (2008).

17 This book barely touches some of the statistical models that one 18 would perhaps particularly expect to see in a statistics book for ecolo-19 gists, namely, Chapters 20 and 21. I say nothing on such core topics in 20 ecological statistics such as the estimation of population density, survival and other vital rates, or community parameters (Buckland et al., 21 22 2001; Borchers et al., 2002; Williams et al., 2002). This is intentional. I 23 hope that my book lays the groundwork for a much better understand-24 ing of statistical modeling using WinBUGS. This will allow you to better 25 tackle more complex and specialized analyses, including those featured 26 in books like Royle and Dorazio (2008), King et al. (2009), and Link and 27 Barker (2010).

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Free documentation for WinBUGS abounds, see http://www.mrc-bsu 28 29 .cam.ac.uk/bugs/winbugs/contents.shtml. The manual comes along with 30 the program; within WinBUGS go Help > User Manual or press F1 and 31 then scroll down. Recently, an open-source version of BUGS has been developed under the name of OpenBugs, see http://mathstat.helsinki 32 33 .fi/openbugs/, and the latest release contains a set of ecological example analyses including those featured in Chapters 20 and 21. WinBUGS can 34 35 be run in combination with other programs such as R, GenStat, Matlab, 36 SAS; see the main WinBUGS Web site. There is even an Excel front-end (see http://www.axrf86.dsl.pipex.com/) that allows you to fit a wide range 37 38 of complex models without even knowing the BUGS language. However, 39 most serious WinBUGS users I know run it from R (see Chapter 5). It turns out that one of the main challenges for the budding WinBUGS program-40 41 mer is to really understand the linear model (see Chapter 6). One particu-42 larly good introduction to the linear model in the context of survival and 43 population estimation is Chapter 6 in Evan Cooch's Gentle introduction to 44 MARK (see http://www.phidot.org/software/mark/docs/book/pdf/chap6.pdf).

#### 1.7 SUMMARY

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# 1.7 SUMMARY

p0190 1 This book attempts the following:

2 1. *demystify Bayesian analyses* by showing their application in the most p0195 3 widely used general-purpose Bayesian software WinBUGS, in a gentle 4 tutorial-like style and in parallel with classical analyses using program 5 R, for a large set of ecological problems that range from very simple to 6 moderately complex; 7 p0200 **2.** enhance your understanding of the *core of modern applied statistics*: 8 linear, generalized linear, linear mixed, and generalized linear mixed 9 models and features common to all of them, such as statistical

- 10 distributions and the design matrix;
- p0205 11 **3.** demonstrate the great value of simulation; and

p0210 12 4. thereby building a solid grounding of the use of WinBUGS (and R) for

relatively simple models, so you can tackle more complex ones, and tohelp *free the modeler in you*.

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