Causation to Association I: Unconditional Association

1000: Introduction

1100: Overview

Science endeavors to construct theories that explain and predict the observable world. Toward this end, scientists formulate hypotheses, make predictions from these hypotheses, test their predictions on observable evidence, modify their hypotheses according to the result, etc. Theories and hypotheses come in many forms, as does observable evidence. In our setting, the focus is on qualitative hypotheses of a limited sort: causal graphs. The evidence we consider is also of a limited and qualitative sort: association and independence among variables. Association doesn't have to be a qualitative concept; a substantial part of statistics deals with kinds and degrees of association. For our purposes here, however, we are only concerned with whether two variables are associated or not associated (independent), thus we stay at a qualitative level. Our focus in this module is to connect qualitative causal theories with qualitative associational evidence.

< A link to exercises in the interactive version of this module. >

This module focuses on how causal graphs explain and predict patterns of unconditional association and independence among a set of variables. In the sections that follow, we will break down the task into many sub-tasks. For example, we will cover direct cause, indirect cause, common causes, and common effects. When you finish the module, you should be able to take any causal graph, and write down the set of independence relations predicted to hold in statistical samples involving the variables in the graph. For any two variables **X** and **Y**, you should be able to ascertain whether a causal graph involving **X** and **Y** predicts that **X** and **Y** are associated or independent. In the next module, we extend the ideas to conditional independence.

Consider, for example, a simple diagram that connects a causal theory about **SMOKING**, **YELLOW FINGERS**, and **LUNG CANCER** to the associations that are predicted to exist if this theory is correct. After this module, you should be able to write down the associations and independencies among **SMOKING**, **YELLOW FINGERS**, and **LUNG CANCER** without consulting the right-hand side of the figure, and after the next module you should be able to write down the conditional associations and independencies.

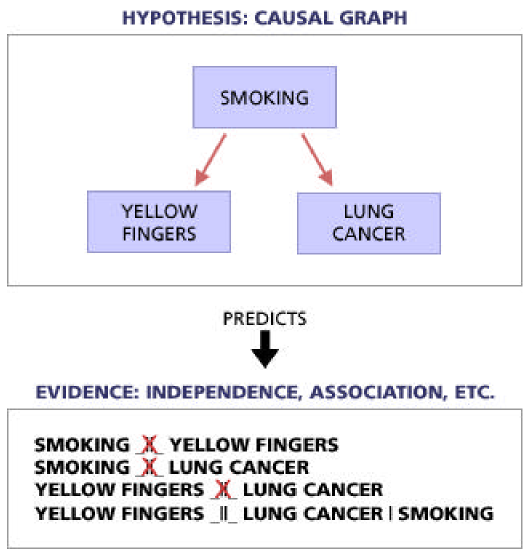

FIGURE 1100-1

1200: The Causality Lab

The Causality Lab is programmed to make the right inference from a particular causal graph to the independence relations that follow from that graph. You can construct a causal graph in the Causality Lab, and then ask it to compute which pairs of variables are predicted to be associated, which pairs independent, and which pairs are conditionally associated or conditionally independent given values for another set of variables.

To learn how to get the Causality Lab to make predictions, read section 4400 of the Causality Lab User Manual. Do that now.

< A link to exercises in the interactive version of this module. >

At any time in the module, you can return to this page, launch the Causality Lab, and explore by constructing any causal graph you like among {**HAPPINESS**, **EDUCATION**, **INCOME**}, or any subset of these variables, and ask for predictions.

JAVA: Link to the Causality Lab for experimenting

1300: Prediction vs Discovery

In this module and the next we move from theory to evidence. After these modules, we consider moving the other way: from evidence to theory. The difference is this: given a causal graph, we can make a unique prediction about association and independencies. Given a set of association and independencies, however, there are **many** causal graphs that would make the same prediction. Associational evidence **underdetermines** causal theories. The situation is not as bad as you might think, however, and therein lies the appeal of the topic. Stay tuned.

2000: Causation and Association

2100: Direct Causation and Association

Consider a causal theory involving only two variables X and Y. If X is a direct cause of Y, then the direct causal relation **produces** association between X and Y. Saying that a direct cause **produces** an association between X and Y is not the same as **predicting** that X and Y are associated in a sample, because the association could come about in other ways, and because in some special cases (to be explained in section 2300) one variable can cause another but because of other causal relationships between the two variables the theory might still predict that the variables are independent. For now, let's focus on how a direct causal relation produces association between X and Y.

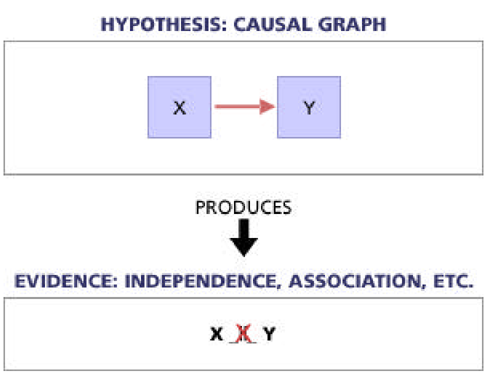

FIGURE 2100-1

To be concrete, consider an example in which everyone begins an experiment feeling sick with a fever. Let X be the treatment variable: TAKES ANTIBIOTIC, with values = [Yes, No], and Y be the variable 1-DAY RECOVERY, with values = [Yes, No]. The causal graph we hypothesize to hold among these two variables is:

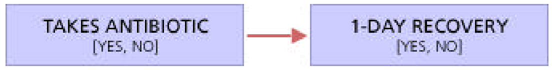

FIGURE 2100-2

By claiming that a direct cause produces association, we are claiming that this graph produces an association between taking antibiotics and recovering from an illness in one day.

< A link to exercises in the interactive version of this module. >

How, in detail, does a direct cause produce an association? Lets examine the Antibiotic and Recovery case in detail. Even though everyone in our experiment begins with a fever, not everyone who takes antibiotics will recover in one day. Some people who take the antibiotic will recover in a day and some won't. Some people who don't take the antiobiotic will recover, and some won't. So the system is indeterministic. Suppose this causal system is like the cell phone system you examined in the module on determinism and indeterminism. Suppose the system involving just the two variables: **TAKES ANTIBIOTIC** and **1-DAY RECOVERY** is psuedo-indeterministic, but underlying it is a deterministic system involving a hidden variable. Suppose this graph tells the whole story:

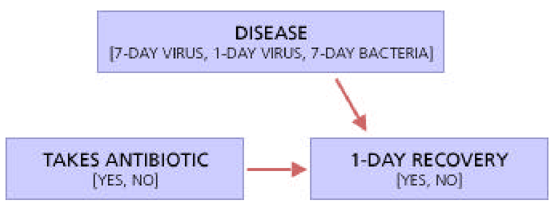

FIGURE 2100-3

Here is the response structure for this system:

TABLE 2100-1: RESPONSE STRUCTURE FOR ANTIBIOTIC, DISEASE, 1-DAY RECOVERY SYSTEM

Causal Assignment	Causal Factor: TAKES ANTIBIOTIC	Causal Factor: DISEASE	Effect: 1-DAY RECOVERY
1	Yes	7-Day Bacteria	Yes
2	No	7-Day Virus	No
3	Yes	1-Day virus	Yes
4	No	7-Day Bacteria	No
5	Yes	7-Day Virus	No
6	No	1-Day Virus	Yes

An antibiotic has no effect on viral infections, so in causal assignments 2, 3, 5, and 6, whether the antibiotic was taken makes no difference to the outcome.

< A link to exercises in the interactive version of this module. >

Below is a simulation of this causal system. To run a single trial in an experiment click the OK button in the "single trial" row and complete the steps that follow. To run multiple trial click on the OK button in the "multiple trials" row and complete the steps that follow. The results of each trial will be displayed as histograms.

< A simulation in the interactive version of this module. >

Run at least 200 trials before you answer these questions.

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Suppose that we were dealing with a mutated bacteria that was resistant to our antibiotic, and taking the antiobiotic had no effect:

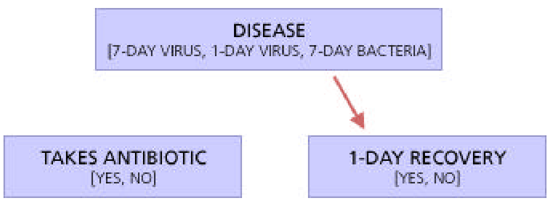

FIGURE 2100-4

Below is a simulation of the mutated bacteria system. To run a single trial in an experiment click the OK button in the "single trial" row and complete the steps that follow. To run multiple trial click on the OK button in the "multiple trials" row and complete the steps that follow. The results of each trial will be displayed as histograms. Run this simulation for at least 200 trials.

< A simulation in the interactive version of this module. >

< A link to exercises in the interactive version of this module. >

So a direct cause $X \rightarrow Y$ produces association between X and Y, and a direct cause $Y \rightarrow X$ also produces association between Y and X. The picture below summarizes the situation for direct causation.

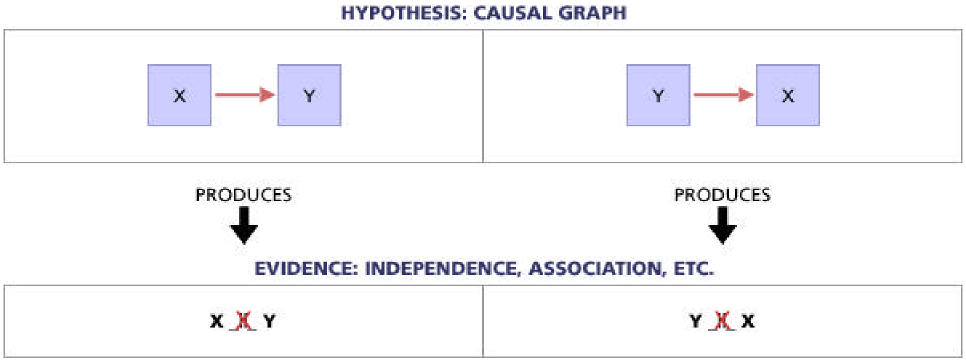

FIGURE 2100-5

Non-independence, like independence, is symmetric, so $X \not\perp\!\!\!\perp Y$ implies $Y \not\perp\!\!\!\perp X$.

2200: Indirect Causes and Association

2210: Simple Causal Paths

Consider 3-year-olds and chicken pox:

TABLE 2210-1: VARIABLES FOR CHICKEN POX AND 3-YEAR-OLDS

Variables	Values
EXPOSED	[Yes, No]
INFECTED	[Yes, No]
SYMPTOMS	[Yes, No]

where **EXPOSED** = Yes means that the 3-year-old has been in close proximity to another person with chicken pox in the past week, **INFECTED** = Yes means that the child has the virus in his or her bloodstream, and **SYMPTOMS** = Yes means that he or she has the typical chicken pox rash.

The causal graph among these variables is clear:

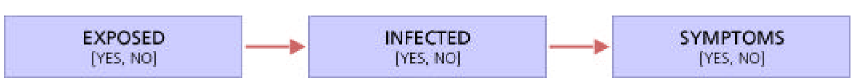

FIGURE 2210-1

EXPOSED is a direct cause of **INFECTED**, which is a direct cause of **SYMPTOMS**.

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From the previous section, we have the following:

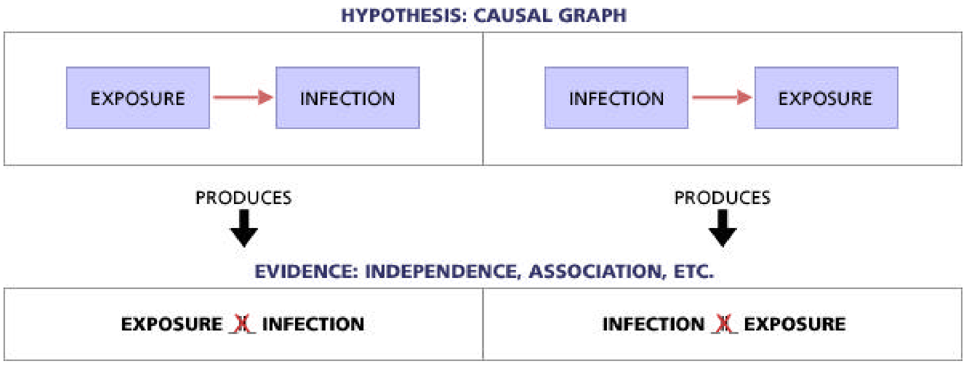

FIGURE 2210-2

The question is: does the indirect causal relation also produce association?

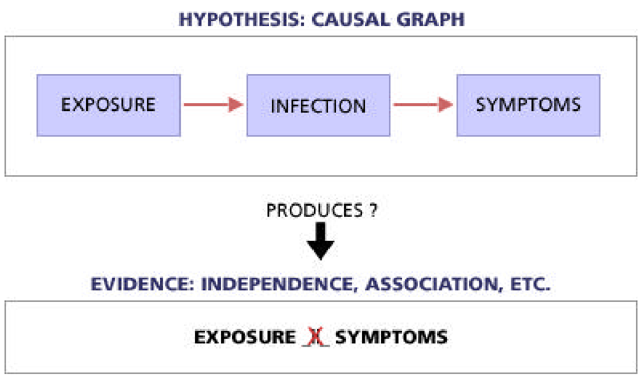

FIGURE 2210-3

In this simple case, the answer is clearly yes. In most any sample of 3-year-olds, the frequency of symptomatic 3-year-olds is clearly lower than the frequency of symptomatic 3-year-olds given exposure. In general the answer is yes as well, but there are rare cases in which indirect causation does not produce association. We will discuss these cases later, but for now, assume that: indirect causation produces association.

How, in detail, does a causal chain X → Y → Z produce an association between X and Z? Let's examine the chicken pox case in same way we did the antibiotics case. Let's assume that the chicken pox system is indeterministic. Not every child who comes in close proximity with another infected child becomes infected with the virus, and not every child who has the virus in his or her bloodstream becomes symptomatic. Like we did with the antibiotics and the cell phone examples, lets suppose that the system is psuedo-indeterministic because of hidden variables. For purposes of illustration, suppose this is the complete, fully deterministic causal system:

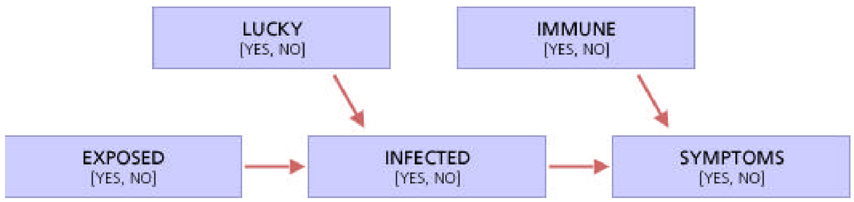

FIGURE 2210-4

Only children who are exposed and unlucky become infected, and only children who are infected and are not immune to chicken pox become symptomatic.

< A link to exercises in the interactive version of this module. >

Below is a simulation of this system. To run a single trial in an experiment click the OK button in the "single trial" row and complete the steps that follow. To run multiple trial click on the OK button in the "multiple trials" row and complete the steps that follow. The results of each trial will be displayed as histograms. Run this simulation for at least 200 trials.

< A simulation in the interactive version of this module. >

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2220: Multiple Causal Paths

In general, one variable X is an indirect cause of another Y if there is a chain of direct causes of any length leading from X to Y. We will refer to indirect causal chains as either causal paths, causal chains, or directed paths, and we define these notions precisely in section 5000.

There can be more than one causal path leading from one variable to another in a causal graph.

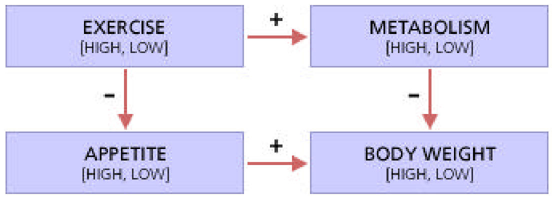

FIGURE 2220-1

For example, in the causal graph above, there are two causal paths from **EXERCISE** to **BODY WEIGHT**:

+ **EXERCISE** → **METABOLISM** → **BODY WEIGHT**
+ **EXERCISE** → **APPETITE** → **BODY WEIGHT**

If we suppose that increasing **EXERCISE** tends to increase **METABOLISM** (thus the "+" on the edge from **EXERCISE** a loss of **BODY WEIGHT** (thus the "-" on the edge from **METABOLISM** to **BODY WEIGHT**), then for a fixed level of **APPETITE**, the frequency of low **BODY WEIGHT** will be less than the frequency of low **BODY WEIGHT** given high amounts of **EXERCISE**. Because high levels of **EXERCISE** go with low levels of **BODY WEIGHT**, we say that path 1 produces negative association between **EXERCISE** and **BODY WEIGHT**.

Similarly, if we assume that **EXERCISE** tends to decrease **APPETITE** (thus the "-" on the edge from **EXERCISE** to **APPETITE**), and that decreased **APPETITE** tends to cause a loss of **BODY WEIGHT** (thus the "+" on the edge from **APPETITE** to **BODY WEIGHT**), then for a fixed **METABOLISM**, the frequency of low **BODY WEIGHT** is less than the frequency of low **BODY WEIGHT** given high **EXERCISE**. So path 2 also produces negative association between **EXERCISE** and **BODY WEIGHT**.

In general, each causal path from **X** to **Y** produces association between **X** and **Y**.

< A link to exercises in the interactive version of this module. >

2230: Offsetting Causal Paths

In the example involving exercise and weight loss above, both causal paths produced negative association between **EXERCISE** and **BODY WEIGHT**. Roughly, the contributions of the individual paths add to create the total association, so if the graph pictured were assumed to be correct and complete, and the signs (+, -) of the causal influences are correct, then it predicts that we would observe a negative association between **EXERCISE** and **BODY WEIGHT**. Suppose, however, that our view of the effect on **APPETITE** of **EXERCISE** was different. Suppose we believed that an increase in **EXERCISE** increased **APPETITE**, instead of decreased it as we assumed above. Then we would have to switch the sign on the edge from **EXERCISE** to **APPETITE** from "-" to "+", as so:

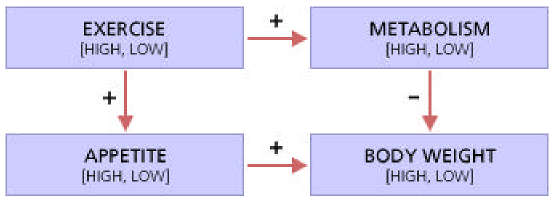

FIGURE 2230-1

Now path 1: **EXERCISE** → **APPETITE** → **BODY WEIGHT** would produce positive association between **EXERCISE** and **BODY WEIGHT**, but path 2 would still produce negative association.

< A link to exercises in the interactive version of this module. >

The total association between **EXERCISE** and **BODY WEIGHT** predicted by this model is not determined by the information given. Since one causal path produces negative association, and the other produces positive association, the overall association depends on the strength of each. It is even possible that both paths are of exactly the same strength and offset each other, thus the overall association between **EXERCISE** and **BODY WEIGHT** predicted by this model might be 0.

If the two paths offset each other exactly, then the model will predict that **EXERCISE** and **BODY WEIGHT** are independent! This is one reason why we said before that producing an association between **X** and **Y** and predicting that **X** and **Y** are associated are different ideas.

In general, whenever there are multiple causal paths connecting two variables in a causal graph, we will assume that the total association from all the paths that produce positive association **do not exactly** offset all the paths that produce negative association, thus producing a total association of exactly zero.

This is one part of an assumption we call **faithfulness**, which we will make more precise later.

< A link to exercises in the interactive version of this module. >

2300: Common Causes

2310: The Idea

Consider the variables:

TABLE 2310-1: VARIABLES FOR BALDNESS

Variables	Values
BALD SON	[Yes, No]
BALD BROTHER	[Yes, No]
BALDNESS GENE (from mother)	[Yes, No]

The causal graph relating these variables is:

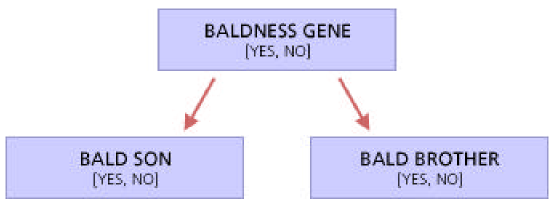

FIGURE 2310-1

From the last section, we know that the variables **BALDNESS GENE** and **BALD SON** are associated because **BALDNESS GENE** is a cause of **BALD SON**, and that the variables **BALDNESS GENE** and **BALD BROTHER** are associated for the same reason. Being a bald son and having a bald brother are also associated, in virtue of being effects of a common cause: having a mother with a baldness gene.

How does this work in detail? Consider a psuedo-indeterministic system of the sort we used to show how direct and indirect causes produce association. A male child gets half of his 46 chromosomes from his father, and half from his mother. The two sets of chromosomes from the parents merge to form 23 pairs -- where each pair has one chromosome from the father and one from the mother. For a male child, one of these pairs is the "XY" pair. The X chromosome in this pair comes from the mother, and the Y from the father. If the X chromosome in the son's pair has the gene for baldness, then the son will be bald. The mother begins with two X chromosomes (say X1 for bald and X2 for not-bald), and then by random luck gives X1 or X2 to the son (The egg that is fertilized may have X1 or X2. Which egg is fertilized is a matter of luck.).

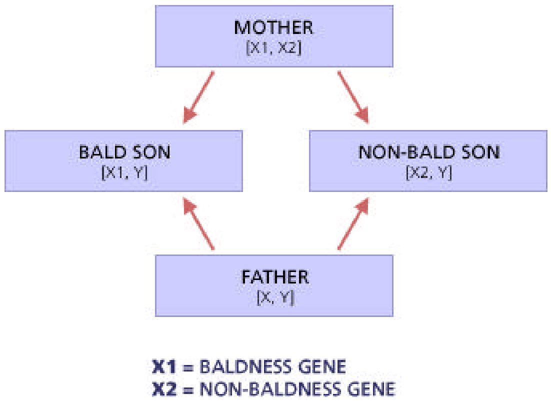

FIGURE 2310-2

So if we consider it just random luck whether a mother carrying the baldness gene gives the gene to her son, then the causal graph for baldness above fills out into a deterministic graph as so:

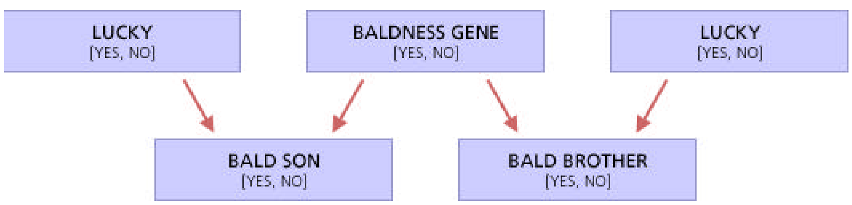

FIGURE 2310-3

The response structures for **BALD** and **BALD BROTHER** are as follows:

TABLE 2310-2: RESPONSE STRUCTURE FOR BALDNESS GENE, LUCKY AND BALD SON

Causal Assignment	Variable 1: BALDNESS GENE	Variable 2: LUCKY	Effect: BALD SON
1	Yes	Yes	No
2	Yes	No	Yes
3	No	Yes	No
4	No	No	No

TABLE 2310-3: RESPONSE STRUCTURE FOR MOTHER, LUCK AND BALD BROTHER

Causal Assignment	Variable 1: BALDNESS GENE	Variable 2: LUCKY	Effect: BALD BROTHER
1	Yes	Yes	No
2	Yes	No	Yes
3	No	Yes	No
4	No	No	No

The following simulation embodies this system. Once again, explore what happens when you manipulate the common cause (**BALDNESS GENE**). To run a single trial in an experiment click the OK button in the "single trial" row and complete the steps that follow. To run multiple trial click on the OK button in the "multiple trials" row and complete the steps that follow. The results of each trial will be displayed as histograms. Produce at least 200 trials, and study the histograms that capture the association between **BALD** and **BALD BROTHER**.

< A simulation in the interactive version of this module. >

< A link to exercises in the interactive version of this module. >

In general, if a variable **C** is a direct cause of **X** and of **Y**, then **C** is a direct common cause of **X** and **Y**. And, in general, direct common causes produce association.

2320: Indirect Common Causes

Like direct and indirect cause -- one variable can be a common causes of a pair without being a direct common cause. For example, consider the graph below.

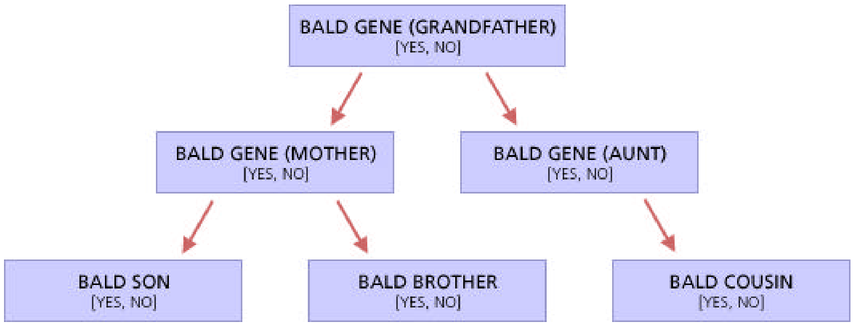

FIGURE 2320-1

The variable **GRANDFATHER** is a common cause of the variable **BALD** and the variable **BALD COUSIN**. It is not a direct common cause, but a common cause nevertheless.

In general, **C** is a common cause of **X** and **Y** if:

+ there is a path from **C** to **X** (**C** is a cause of **X**),
+ there is a path from **C** to **Y** (**C** is a cause of **Y**), and
+ no variable besides **C** is on both of these paths.

Definition: ???

In general, common causes produce association between their effects.

2330: Multiple Common Causes

As there can be multiple causal paths between two variables, there can be multiple common causes. For example, consider the common causes of your own level of athletic achievement and your sibling's level of athletic achievement. below.

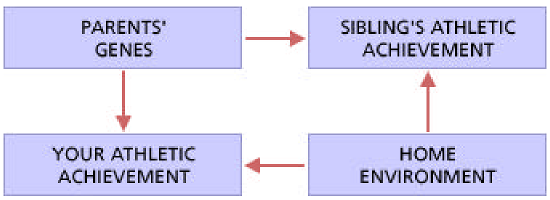

FIGURE 2330-1

There are at least two common causes of these variables, the genetic contribution that you and your sibling share from your parents -- and the environment that you and your sibling both grew up in. Both common causes produce association between the level of athletic achievement among siblings.

Definition: ???

In general, each common cause produces association between its effects.

< A link to exercises in the interactive version of this module. >

2400: Causal Connection

2410: The Idea

We can summarize the preceding sections as follows:

+ Each causal path between X and Y produces association between X and Y, and
+ Each common cause of X and Y produces association between X and Y.

We unify these ideas with the idea of a causal connection. Two variables **X** and **Y** are causally connected if:

+ **X** is a cause of **Y**, or
+ **Y** is a cause of **X**, or
+ there is a third variable **Z** that is a (direct or indirect) cause of **X** and of **Y**.

< A link to exercises in the interactive version of this module. >

2420: Multiple Causal Connections

As you would suspect, there can be many causal connections between a pair of variables. As you might also guess, each causal connection between a pair of variables produces association. In the graph below, for example, a child's exposure to lead and his or her IQ score are causally conncted in two ways.

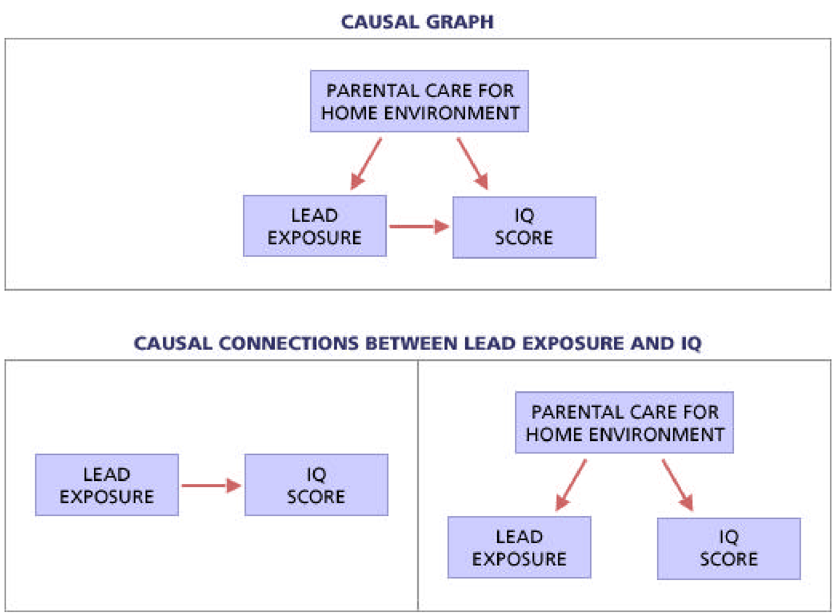

FIGURE 2420-1

Since **LEAD EXPOSURE** is a direct cause of **IQ SCORE**, it constitutes one causal connection, and since **PARENTAL CARE** for the **HOME ENVIRONMENT** is a common cause of **LEAD EXPOSURE** and **IQ SCORE**, it constitutes another. Except for a technical qualification that will be explained later in section 5200, each pathway from one variable to another, and each pair of pathways from a common cause to the two variables, counts as a different causal connection.

< A link to exercises in the interactive version of this module. >

To summarize, each causal connection between a pair of variables produces association between those variables. We assume that when there is at least one form of causal connection between a pair of variables, then they are predicted to have some overall association.

2500: Common Effects

Common causes produce association, but do common effects?

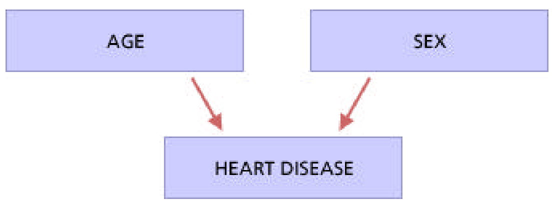

FIGURE 2500-1

For example, in the graph above, the variables **AGE** and **SEX** have a common effect: **HEART DISEASE**. Does having a common effect produce association between **AGE** and **SEX**?

We know that the causal connection between **AGE** and **HEART DISEASE** produces association, as does the causal connection between **SEX** and **HEART DISEASE** (males get it more often). Thus:

AGE $\not\perp\!\!\!\perp$ **HEART DISEASE**, and **HEART DISEASE** $\not\perp\!\!\!\perp$ **SEX**

Because **AGE** and **HEART DISEASE** are associated, and **HEART DISEASE** and **SEX** are associated, does that mean **AGE** and **SEX** are associated? Is association always transitive?

The answer is no to all of these questions. Association is not necessarily transitive, **AGE** and **SEX** are not associated because they have a common effect, and in general, common effects do **not** produce association.

Consider the following example using the Setbuilder. Suppose we create a fantasy world in which being blond causes you to smoke, and being female causes you to smoke as well, but there is no causal connection between **HAIR COLOR** and **SEX**.

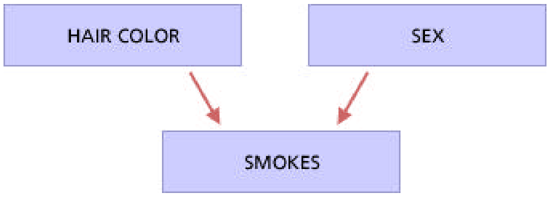

FIGURE 2500-2

If we are correct, then according to this causal graph **HAIR COLOR** and **SMOKING** are associated, **SEX** and **SMOKING** are associated, but **HAIR COLOR** and **SEX** are not associated.

< A link to exercises in the interactive version of this module. >

3000: Summary

The connection between causal graphs and **unconditional** association turns out to be very simple. Two variables X and Y are predicted to be associated just in case they are causally connected in the graph. X and Y are causally connected in the graph if either

+ X is a cause of Y, or
+ Y is a cause of X, or
+ there is a common cause of X and Y.

Variables can be causally connected in several different ways, and each causal connection produces association. Although it is possible, we assume that when multiple causal connections exist between X and Y, the overall association between X and Y is not zero. That is, if X and Y are causally connected, we predict they are associated, and if X and Y are not causally connected, we predict that they are independent.

Section 4000 (Formalities) gives the formal definitions of the concepts we've discussed in this module. It is optional, and provides the technical explanations of directed paths and common causes.

4000: Formalities

4100: Causal Chains

We define a causal path more precisely via an idea from graph theory called a **directed path**.

An edge between A and another variable B is **out of** A just in case A is the cause and B the effect (A → B).

Similarly, an edge between A and another variable B is **into** B just in case A is the cause and B the effect (A → B).

In a causal graph, a sequence of directed edges U is a **directed path** from X to Y just in case:

+ U begins with an edge out of X,
+ U ends with an edge into Y, and
+ every two adjacent edges on U are head to tail, i.e., ZI → ZJ and ZJ → ZK
+ no vertex is a cause of more than one effect on U.

The first two clauses are obvious. The third clause in the definition ensures that all the arrows in the sequence point the same way. In the figure below, for example, the top graph contains a directed path from X to Y that satisfies this clause, but the path in the bottom graph from X to Y is not a directed path because it violates this clause: the arrows connected to Z2 collide, they don't point in the same direction.

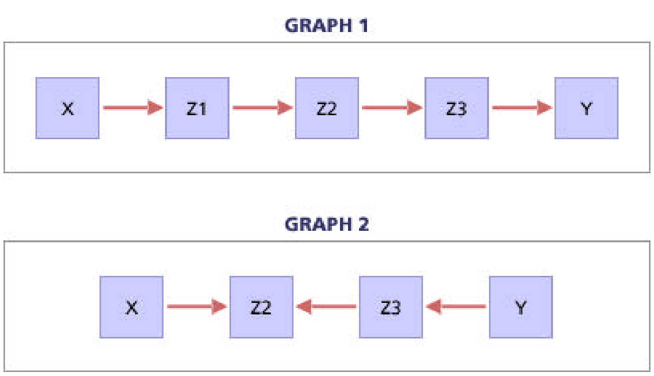

FIGURE 4100-1

Directed paths have a length equal to the number of edges on the path, they have endpoints, and they are described by writing the sequence of edges on the directed path in order from one endpoint to the other. Here is a sampling of directed paths contained in both graphs above, with the length given for each:

TABLE 4100-1: SAMPLING OF DIRECTED PATHS

Graph	Endpoints	Directed Path	Length
1	X, Z2	X → Z1 → Z2	2
1	X, Y	X → Z1 → Z2 → Z3 → Y	4
1	Z3, Y	Z3 → Y	1
2	Y, Z3	Y → Z3	1
2	Y, Z2	Y → Z3 → Z2	2

The second clause in the definition of a directed path is meant to prevent loops. In the figure below, for example, the second candidate path violates this clause because Z2 is a cause of both Z3 and Y.

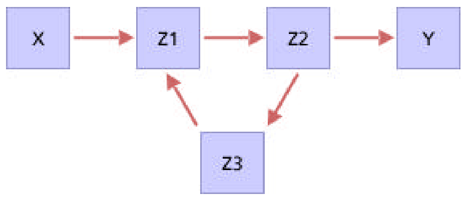

FIGURE 4100-2

TABLE 4100-2: CANDIDATE PATHS

Endpoints	Candidate	Directed Path
X, Y	X → Z1 → Z2 → Y	Yes
X, Y	X → Z1 → Z2 → Z3 → Z1 → Z2 → Y	No

< A link to exercises in the interactive version of this module. >

4200: Common Causes

C is a common cause of X and Y just in case:

- there is a directed path P1 from C to X, and
- there is a directed path P2 from C to Y, and
- C is the only variable on both P1 and P2.

The first two clauses of this definition are obvious, and the third clause is meant to prevent cases like Z1 in the following causal graph:

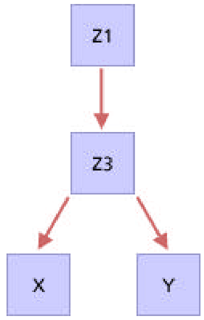

FIGURE 4200-1

Z3 satisfies the definition of common cause, but Z1 does not. In the case of Z3, paths P1 and P2 are:

- P1: Z3 → X
- P2: Z3 → Y

In the case of Z1, the paths P1 and P2 are:

- P1: Z1 → Z3 → X
- P2: Z1 → Z3 → Y

Both causal connections contain X ← Z3 → Y. It would therefore not be appropriate to say that there are two distinct sources of association between X and Y in this graph. Thus we exclude the second case with clause 3 in the definition above.
