ASP modulo CSP: The clingcon system

Max Ostrowski and Torsten Schaub
Institut für Informatik, Universität Potsdam

submitted 1 January 2003; revised 1 January 2003; accepted 1 January 2003

Abstract

We present the hybrid ASP solver *clingcon*, combining the simple modeling language and the high performance Boolean solving capacities of Answer Set Programming (ASP) with techniques for using non-Boolean constraints from the area of Constraint Programming (CP). The new *clingcon* system features an extended syntax supporting global constraints and optimize statements for constraint variables. The major technical innovation improves the interaction between ASP and CP solver through elaborated learning techniques based on *irreducible inconsistent sets*. A broad empirical evaluation shows that these techniques yield a performance improvement of an order of magnitude. To appear in Theory and Practice of Logic Programming.

1 Introduction

clingcon is a hybrid solver for Answer Set Programming (ASP; (Baral 2003)), combining the simple modeling language and the high performance Boolean solving capacities of ASP with techniques for using non-Boolean constraints from the area of Constraint Programming (CP). Although *clingcon*'s solving components follow the approach of modern Satisfiability Modulo Theories (SMT; (Biere et al. 2009, Chapter 26)) solvers when combining the ASP solver *clasp* with the CP solver *gecode* (gecode), *clingcon* furthermore adheres to the tradition of ASP in supporting a corresponding modeling language by appeal to the ASP grounder *gringo*. Although in the current implementation the theory solver is instantiated with the CP solver *gecode*, the principal design of *clingcon* along with the corresponding interfaces are conceived in a generic way, aiming at arbitrary theory solvers.

The underlying formal framework, defining syntax and semantics of constraint logic programs, and the principal algorithms, were presented in (Gebser et al. 2009). This initial *clingcon* system 0.1.0 was based on *clingo* 2.0.2 and *gecode* 2.2.0. Unlike this, the new version of *clingcon* is based on *clingo* 3.0.4 and *gecode* 3.7.1. Apart from major refactoring, it features an extended syntax supporting global constraints and optimize statements for constraint variables. Also, it allows for more fine-grained configurations of constraint-based lookahead, optimization, and propagation delays. However, the major technical innovation improves the interaction between ASP and CP solver through elaborated learning techniques. We introduce filtering methods for conflicts and reasons based on *irreducible inconsistent sets*. A broad empirical evaluation shows that these techniques yield a performance improvement of an order of magnitude.

2 The *clingcon* approach

The input language of *clingcon* extends the one of *gringo* (Gebser et al.) by CP-specific operators marked with a preceding `$` symbol (cf. Section 4). After grounding, a propositional program is then composed of regular and constraint atoms, denoted by $\mathcal{A}$ and $\mathcal{C}$, respectively. The set of constraint atoms induces an ordinary constraint satisfaction problem (CSP) (V, D, C), where V is a set of variables with common domain D, and C is a set of constraints. This CSP is to be addressed by the corresponding CP solver. As detailed in (Gebser et al. 2009), the semantics of such constraint logic programs is defined by appeal to a two-step reduction. For this purpose, we consider a regular Boolean assignment over $\mathcal{A} \cup \mathcal{C}$ (in other words, an interpretation) and an assignment of V to D (for interpreting the variables V in the underlying CSP). In the first step, the constraint logic program is reduced to a regular logic program by evaluating its constraint atoms. To this end, the constraints in C associated with the program's constraint atoms $\mathcal{C}$ are evaluated w.r.t. the assignment of V to D. In the second step, the common Gelfond-Lifschitz reduct (Gelfond and Lifschitz 1991) is performed to determine whether the Boolean assignment is an answer set of the obtained regular logic program. If this is the case, the two assignments constitute a (hybrid) constraint answer set of the original constraint logic program.

In what follows, we rely upon the following terminology. We use signed literals of form $\mathbf{T}$a and $\mathbf{F}$a to express that an atom a is assigned $\mathbf{T}$ or $\mathbf{F}$, respectively. That is, $\mathbf{T}$a and $\mathbf{F}$a stand for the Boolean assignments $a \mapsto \mathbf{T}$ and $a \mapsto \mathbf{F}$, respectively. We denote the complement of such a literal ℓ by $\overline{\ell}$. That is, $\overline{\mathbf{T}a} = \mathbf{F}a$ and $\overline{\mathbf{F}a} = \mathbf{T}a$. We represent a Boolean assignment simply by a set of signed literals. Sometimes we restrict such an assignment A to its regular or constraint atoms by writing $A|_{\mathcal{A}}$ or $A|_{\mathcal{C}}$, respectively. For instance, given the regular atom '`person(adam)`' and the constraint atom '`work(adam) $> 4`', we may form the Boolean assignment $\{\mathbf{T}\texttt{person(adam)}, \mathbf{F}\texttt{work(adam) \$> 4}\}$.

We identify constraint atoms in $\mathcal{C}$ with constraints in (V, D, C) via a function $\gamma : \mathcal{C} \to C$. Provided that each constraint $c \in C$ has a complement $\overline{c} \in C$, like $\overline{\text{`}x = y\text{'}} = \text{`}x \neq y\text{'}$ or $\overline{\text{`}x < y\text{'}} = \text{`}x \geq y\text{'}$ and vice versa, we extend γ to signed constraint atoms over $\mathcal{C}$:

$$\gamma(\ell) = \begin{cases} c & \text{if } \ell = \mathbf{T}c \\ \overline{c} & \text{if } \ell = \mathbf{F}c \end{cases}$$

For instance, we get $\gamma(\mathbf{F}\texttt{work(adam) \$> 4}) = \mathit{work}(\mathit{adam}) \leq 4$, where `work(adam)` $\in V$ is a constraint variable and $(\mathit{work}(\mathit{adam}) \leq 4) \in C$ is a constraint. An assignment satisfying the last constraint is $\{\mathit{work}(\mathit{adam}) \mapsto 3\}$.

Following (Gebser et al. 2007), we represent Boolean constraints issuing from a logic program under ASP semantics in terms of *nogoods* (Dechter 2003). This allows us to view inferences in ASP as unit propagation on nogoods. A *nogood* is a set $\{\sigma_1, \ldots, \sigma_m\}$ of signed literals, expressing that any assignment containing $\sigma_1, \ldots, \sigma_m$ is unintended. Accordingly, a total assignment A is a *solution* for a set Δ of nogoods if $\delta \not\subseteq A$ for all $\delta \in \Delta$. Whenever $\delta \subseteq A$, the nogood δ is said to be *conflicting* with A. For instance, given atoms a, b, the total assignment $\{\mathbf{T}a, \mathbf{F}b\}$ is a solution for the set of nogoods containing $\{\mathbf{T}a, \mathbf{T}b\}$ and $\{\mathbf{F}a, \mathbf{F}b\}$. Likewise, $\{\mathbf{F}a, \mathbf{T}b\}$ is another solution. Importantly, nogoods provide us with reasons explaining why entries must (not) belong to a solution, and look-

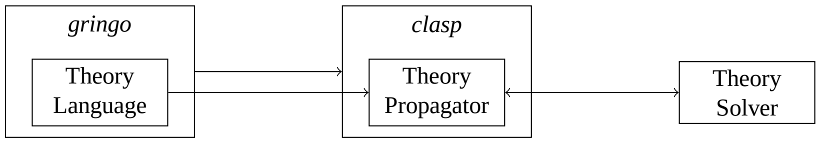


Fig. 1: Architecture of *clingcon*

back techniques can be used to analyze and recombine inherent reasons for conflicts. We refer to (Gebser et al. 2007) on how logic programs are translated into nogoods within ASP.

3 The *clingcon* Architecture

Although *clingcon*'s solving components follow the approach of modern SMT solvers when combining the ASP solver *clasp* with the CP solver *gecode*, *clingcon* furthermore adheres to the tradition of ASP in supporting a corresponding modeling language based on the ASP grounder *gringo*. The resulting tripartite architecture of *clingcon* is depicted in Figure 1. Although in the current implementation the theory solver is instantiated with the CP solver *gecode*, the principal design of *clingcon* along with the corresponding interfaces are conceived in a generic way, aiming at arbitrary theory solvers.

Following the workflow in Figure 1, the first extension concerns the input language of *gringo* with theory-specific language constructs. Just as with regular atoms, the grounding capabilities of *gringo* can be used for dealing with constraint atoms containing first-order variables. As regards the current *clingcon* system, the language extensions allow for expressing constraints over integer variables. As we detail in Section 4, this involves arithmetic constraints as well as global constraints and optimization statements. These constraints are treated as atoms and passed to *clasp* via the standard *gringo*-*clasp* interface, also used in *clingo*, the monolithic combination of *gringo* and *clasp*. Information about these constraints is furthermore directly shared with the theory propagator and in turn the theory solver, viz. *gecode*. In the new version of *clingcon*, the theory propagator is implemented as a post propagator, as furnished by *clasp*.[1] Theory propagation is done in the theory solver until a fixpoint is reached. In doing so, decided constraint atoms are transferred to the theory solver, and conversely constraints whose truth values are determined by the theory solver are sent back to *clasp* using a corresponding nogood. Note that theory propagation is not only invoked when propagating partial assignments but also whenever a total Boolean assignment is found. Whenever the theory solver detects a conflict, the theory propagator is in charge of conflict analysis. Apart from reverting the state of the theory solver upon backjumping, this involves the crucial task of determining a conflict nogood (which is usually not provided by theory solvers, as in the case of *gecode*). This is elaborated upon in Section 5. Similarly, the theory propagator is in charge of enumerating constraint variable assignments, whenever needed. Finally, we note that the theory propagator is informed whenever constraint atoms are decided. This allows for updating watches

[1] Post propagators provide an abstraction easing *clasp*'s extensibility with more elaborate propagation mechanisms. To this end, *clasp* maintains a list of post propagators that are consecutively processed after unit propagation. Also, lookahead and unfounded-set checking are implemented in *clasp* as post propagators.

Listing 1: Example of a constraint logic program

```
1  $domain(0..10).
2  person(adam;smith;lea;john).
3  1{team(A,B) : person(B) : B != A}1 :- person(A), A == adam.
4  {friday}.

6  work(A) $+ work(B) $> 6 :- team(A,B).
7  work(B) $- work(adam) $== 1 :- friday, team(adam,B).
8  :- team(adam,lea), not work(lea) $== work(adam).
9  work(B) $== 0 :- person(B), not team(adam,B), B != adam.

11 $count[work(A) $== 8 : person(A)] $== fulltime.

13 $maximize{work(A) : person(A)}.
```

for constraint literals, even before propagation is launched. This is implemented by *clasp*'s call-back routines, another new feature of *clasp* supporting theory solving.

4 The *clingcon* Language

We explain the syntax of our constraint logic programs via the example in Listing 1. Suppose Adam wants to do house renovation with the help of three friends. We encode the problem as follows. In Line 1 we restrict all constraint variables to the domain $[0, 10]$ as nobody wants to work more than 10 hours a day. In Line 3 we choose teams. They agreed that each team has to work more than six hours a day (Line 6). Within this line we show the syntax of linear constraints. They can be used in the head or body of a rule[2]. We use the $ sign in front of every relation and function symbol referring to the underlying CSP. In this new *clingcon* version this also applies to arithmetic operators to better separate them from *gringo* operators. Many standard arithmetical operators are supported, like plus($+$), times($*$) and absolute(abs). We use the grounding capabilities of *gringo* to create the constraint variables. Grounding Line 6 yields:

```
work(adam) $+ work(smith) $> 6 :- team(adam,smith).
work(adam) $+ work(lea)   $> 6 :- team(adam,lea).
work(adam) $+ work(john)  $> 6 :- team(adam,john).
```

We created three ground rules containing three different constraints, using four different constraint variables. Note that the constraint variables have not been defined beforehand. All variables occurring in a constraint are automatically constraint variables. On Fridays, Adam has to pick up his daughter from sports and therefore works one hour less than his partner (Line 7). Furthermore Lea and Adam are a couple and decided to have an equal work load if they are in the same team (Line 8). Finally, Line 9 prevents persons from working if they are not teammates.

With this little constraint logic program, we want to show how for example quantities can be easily expressed. Constraints and constraint variables fit naturally into the logic program. Not representing quantities explicitly with propositional variables eases the modelling of problems and also decreases the size of the ground logic program.

Global Constraints. In Line 11 we use a global constraint. This is a new feature of *clingcon*. Global constraints capture relations between a non-fixed number of variables. As

[2] Constraint atoms in the head are shifted to the negative body.

with aggregates in *gringo*, we can use conditional literals (Syrjänen) to represent these sets of variables. In the example we can see a *count* constraint. Grounding this yields:

```
$count[work(adam) $== 8, work(smith) $== 8,
       work(lea)  $== 8, work(john)  $== 8] $== fulltime.
```

It constrains the number of variables in $\{$`work(adam)`, `work(smith)`, `work(lea)`, `work(john)`$\}$ that are equal to 8, to be equal to `fulltime`. Constraint variable `fulltime` counts how many persons are working full time. Global constraints do have a similar syntax to propositional aggregates. Also their semantics is similar to *count* aggregates in ASP (cf. (Schulte et al. 2012)). But global constraints only constrain the values of constraint variables, not propositional ones.

Clingcon also supports the global constraint $distinct$, where

```
$distinct{work(A) : person(A)}.
```

means that all persons should have a different workload. That is, all values assigned to constraint variables in $\{$`work(adam)`, `work(smith)`, `work(lea)`, `work(john)`$\}$ have to be distinct from each other. This constraint could also be expressed using a quadratic number of inequalities. Using a single dedicated constraint is usually much more efficient in terms of memory consumption and runtime.

As global constraints are usually not supported in a negated form in a CP solver, we have the syntactic restriction that all global constraints must become facts during grounding and therefore may only occur in the head of rules. Further global constraints can easily be integrated into this generic framework.

A valid solution to our constraint logic program in Listing 1 contains the regular literal **T**`team(adam,smith)`, but also constraint literals like **T**`work(lea)$==0`, **T**`work(john)$==0` and **T**`work(adam)$+work(smith)$>6`. The solution also contains assignments to constraint variables, like `work(adam)`$\mapsto$`8`, `work(smith)`$\mapsto$`3`, `work(lea)`$\mapsto$`0`, `work(john)`$\mapsto$`0` and `fulltime`$\mapsto$`1`.

Optimization. In Line 13 we give a maximize statement over constraint variables. This is also a new feature of *clingcon*. We maximize the sum over a set of variables and/or expressions. In this case, we try to maximize `work(adam) $+ work(smith) $+ work(lea) $+ work(john)`. For optimization statements over constraint variables, we also rely on the syntax of *gringo's* propositional optimization statements. We support minimization/maximization and multi-level optimization. To distinguish propositional and constraint statements, we precede the latter with a `$` sign. One optimal solution to the problem contains the propositional literal **T**`team(adam,john)`, and the constraint literals **T**`work(lea)$==0`, **T**`work(smith)$==0`, and **T**`work(adam)$+work(john)$>6`. To maximize work load, the constraint variables are assigned `work(adam)`$\mapsto$`10`, `work(smith)`$\mapsto$`0`, `work(lea)`$\mapsto$`0`, `work(john)`$\mapsto$`10` and `fulltime`$\mapsto$`0`.

To find a constraint optimal solution, we have to combine the enumeration techniques of *clasp* with the ones from the CP solver. Therefore, when we first encounter a full propositional assignment, we search for an optimal (w.r.t. to the optimize statement) assignment of the constraint variables using the search engine of the CP solver. Let us explain this with the following constraint logic program.

```
$domain(1..100).
```

```
a :- x $* x $< 25.
$minimize{x}.
```

Assume *clasp* has computed the full assignment $\{\mathbf{F}\texttt{x \$* x \$< 25}, \mathbf{F}a\}$. Afterwards, we search for the constraint optimal solution to the constraint variable `x` which yields $\{\texttt{x} \mapsto 5\}$. Given this optimal assignment, a constraint can be added to the CP solver that all further solutions shall be below/above this optimum (`x$<5`). This constraint will now restrict all further solutions to be "better". We enumerate further solutions, using the enumeration techniques of *clasp*. So the next assignment is $\{\mathbf{T}\texttt{x \$* x \$< 25}, \mathbf{T}a\}$ and the CP solver finds the optimal constraint variable assignment $\{\texttt{x} \mapsto 1\}$. Each new solution restricts the set of further solutions, so our constraint is changed to (`x$<1`) which then allows no further solutions to be found.

5 Conflict Filtering in *clingcon*

The development of Conflict Driven Clause Learning (CDCL) algorithms was a major breakthrough in the area of SAT. Also, CDCL is crucial in SMT solving (cf. (Nieuwenhuis et al. 2006)). A prerequisite to combine a CDCL-based SAT solver with a theory solver is the possibility to generate good conflicts and reasons originating in the underlying theory. Therefore, modern SMT solvers use their own specialized theory propagators that can produce such witnesses. *Clingcon* instead uses a black-box approach regarding theory solving. In fact, off-the-shelf CP solvers, like *gecode*, do usually not provide any reason for their underlying inference. As a consequence, conflict and reason information was so far only crudely approximated in *clingcon*. We address this shortcoming by developing mechanisms for extracting minimal reasons and conflicts from any CP solver using monotone propagators. We assume that the reader has basic knowledge on CDCL-based ASP solving, and direct the interested reader to (Gebser et al. 2007).

Whenever the CP solver finds out that the set of constraints is inconsistent under the current assignment A, a conflicting nogood N must be generated, which can then be used by the ASP solver in its conflict analysis. The *simple* version of generating the conflicting nogood N, is just to take the entire assignment of constraint literals. In this way, all yet decided constraint atoms constitute $N = \{\ell \mid \ell \in A|_{\mathcal{C}}\}$. In this case, the corresponding list of inconsistent constraints is

$$I = [\ \gamma(\ell) \mid \ell \in A|_{\mathcal{C}}\ ]. \tag{1}$$

In order to reduce this list of inconsistent constraints and to find the real cause of the conflict, we apply an *Irreducible Inconsistent Set* (IIS) algorithm. The term IIS was coined in (van Loon 1981) for describing inconsistent sets of constraints having consistent subsets only. We use the concept of an IIS to find the minimal cause of a conflict. With this technique, it is actually possible to drastically reduce such exhaustive sets of inconsistent constraints as in (1) and to create a much smaller conflict nogood. It is now possible to apply an IIS algorithm to every conflicting set of constraints in order to provide *clasp* with smaller nogoods. This enhances the information content of the learnt nogood and hopefully speeds up the search process by better pruning the search space. *Clingcon* now features several alternatives to reduce such conflicts. To this end, we build upon the approach of (Chinneck and Dravinieks 1991) who propose different algorithms for computing IISs,

Algorithm 1: DELETION_FILTERING

```
Input  : An inconsistent list of constraints $I = [c_1, \ldots, c_n]$.
Output: An irreducible inconsistent list of constraints.

1 $i \leftarrow 1$
2 while $i \leq |I|$ do
3     if $I \setminus c_i$ is inconsistent then
4         $I \leftarrow I \setminus c_i$
5     else
6         $i \leftarrow i + 1$
7 return $I$
```

among them the so-called *Deletion Filtering* algorithm. In what follows, we first present the original idea of *Deletion Filtering* and afterwards propose several refinements that can then be used to reduce inconsistent lists of constraints in the context of ASP modulo CSP.

Deletion Filtering. Given an inconsistent list of constraints $I = [c_1, \ldots, c_n]$ as in (1) the *Deletion Filtering* Algorithm 1 reduces it to an irreducible list. We test for each $c_i \in I$ whether $I \setminus c_i$ is inconsistent or not. If it is inconsistent we can restart the whole algorithm with the list $I \setminus c_i$ continuing with the next i.

The result of this simple approach is a minimal inconsistent list, as we can see in the following example. Suppose we branch on `Tteam(adam,lea)`. Unit propagation implies the literals `Twork(lea)$==work(adam)`, `Twork(john)$==0`, `Twork(smith)$==0`, and `Twork(adam)$+work(lea)$>6`. At this point we cannot do any constraint propagation[3] and make another choice, `Tfriday`, and some unit propagation, resulting in `Twork(lea)$-work(adam)$==1`. As unit propagation is at fixpoint, the CP solver checks the constraints in the partial assignment $A|_{\mathcal{C}}$ for consistency. As it is inconsistent, a *simple* conflicting nogood would be $N = \{\ell \mid \ell \in A|_{\mathcal{C}}\}$. To minimize this nogood, we now apply *Deletion Filtering* to the list I as defined in (1):

$$\begin{aligned} I &= [\gamma(\ell) \mid \ell \in A|_{\mathcal{C}}] \\ &= [work(lea) = work(adam), work(john) = 0, work(smith) = 0] \\ &\circ\ [work(adam) + work(lea) > 6, work(lea) - work(adam) = 1] \end{aligned}$$

For $i = 1$, we test $[work(john) = 0, work(smith) = 0, work(adam) + work(lea) > 6, work(lea) - work(adam) = 1]$, but it does not lead to inconsistency (Line 3). Next list to test is $[work(lea) = work(adam), work(smith) = 0, work(adam) + work(lea) > 6, work(lea) - work(adam) = 1]$ which restricts the domains of `work(adam)` and `work(lea)` to $\emptyset$. As this is inconsistent, we remove $work(john) = 0$ from I and go on, also removing $work(smith) = 0$ and $work(adam) + work(lea) > 6$. We end up with the irreducible list $I = [work(lea) = work(adam), work(lea) - work(adam) = 1]$, and can now build a much smaller conflicting nogood $N = \{\gamma^{-1}(c) \mid c \in I\} =$

[3] w.l.o.g. we assume arc consistency (Mohr and Henderson 1986)

Algorithm 2: FORWARD_FILTERING

Input : An inconsistent list of constraints $I = [c_1, \dots, c_n]$.
Output: An irreducible inconsistent list of constraints I'.

```
1 $I' \leftarrow []$
2 while $I'$ is consistent do
3     $T \leftarrow I'$
4     $i \leftarrow 1$
5     while $T$ is consistent do
6         $T \leftarrow T \circ c_i$
7         $i \leftarrow i + 1$
8     $I' \leftarrow I' \circ c_i$
9 return $I'$
```

{`Twork(lea)$==work(adam)`, `Twork(lea)$-work(adam)$==1`} as this really describes the cause of the inconsistency.

In most CP solvers, propagation is done in a constraint space. This space contains the constraints and the variables of the problem. After doing propagation, the domains of the variables are restricted. Normally in CP solvers like *gecode* this effect cannot be undone. As long as we add further constraints to the constraint space this is no problem, as another constraint restricts the domain of the variables even more. If we want to remove a constraint from a constraint space we have to create a new space containing only the constraints we want to apply. Then we have to redo all the propagation. This is why we identified Line 4 in Algorithm 1 as an efficiency bottleneck. To address this problem, we propose some derivatives of the algorithm.

Forward Filtering. Algorithm 2 is designed to avoid resetting the search space of the CP solver. It incrementally adds constraints to a testing list T, starting from the first assigned constraint to the last one (lines 5 and 6). Remember that incrementally adding constraints is easy for a CP solver as it can only further restrict the domains. If our test list T becomes inconsistent we add the currently tested constraint to the result I' (lines 5 and 8). If this result is inconsistent (Line 2), we have found a minimal list of inconsistent constraints. Otherwise, we start again, this time adding all yet found constraints I' to our testing list T (Line 1). Now we have to create a new constraint space. But by incrementally increasing the testing list, we already reduced the number of potential candidates that contribute to the IIS, as we never have to check a constraint beyond the last added constraint. We illustrate this again on our example. We start Algorithm 2 with $T = I' = []$ and

$$I = [work(lea) = work(adam), work(john) = 0, work(smith) = 0]$$
$$\circ\ [work(adam) + work(lea) > 6, work(lea) - work(adam) = 1]$$

in Line 3. We add $work(lea) = work(adam)$ to T, as this constraint alone is consistent, we loop and add constraints until $T = I$. As this list is inconsistent, we add the last constraint $work(lea) - work(adam) = 1$ to I' in Line 8. We can do so, as we know that the last constraint is indispensable for the inconsistency. As I' is consistent we restart the whole procedure, but this time setting $T = I' = [work(lea) - work(adam) = 1]$

Algorithm 3: RANGE_FILTERING

```
Input  : An inconsistent list of constraints $I = [c_1, \dots, c_n]$.
Output: A (possibly smaller) inconsistent list of constraints $I'$.
1 $I' \leftarrow []$
2 $i \leftarrow n$
3 while $I'$ is consistent do
4     $I' \leftarrow I' \circ c_i$
5     $i \leftarrow i - 1$
6 return $I'$
```

in Line 3. Please note that, even if I would contain further constraints, we would never have to check a constraint behind $work(lea) - work(adam) = 1$. Our testing list already contained an inconsistent set of constraints, consequently we can restrict ourself to this subset. Now we start the loop again, adding $work(lea) = work(adam)$ to T. On their own, these two constraints are inconsistent, as there exists no valid pair of values for the variables. So we add $work(lea) = work(adam)$ to I', resulting in $I' = [work(lea) - work(adam) = 1, work(lea) = work(adam)]$. This is then our reduced list of constraints and the same IIS as we got with the *Deletion Filtering* method (as it is the only IIS of the example). But this time we only needed one reset of the constraint space (Line 3) instead of five.

Backward Filtering. The basic idea of this algorithm is the same as in Algorithm 2. But this time, we reverse the order of the inconsistent constraint list. Therefore, we first test the last assigned constraint and iterate to the first. In this way we want to accommodate the fact, that one of the literals that was decided on the current decision level has to be included in the conflicting nogood. Otherwise we would have recognized the conflict before.

Range Filtering. This algorithm does not aim at computing an irreducible list of constraints, but tries to approximate a smaller one to find a nice tradeoff between reduction of size and runtime of the algorithm. Therefore, as shown in Algorithm 3, we move through the reversed list of constraints I and add constraints to the result I' until it becomes inconsistent. In our example we cannot reduce the inconsistent list anymore, as the first and the last constraint is needed in the IIS.

Connected Components Filtering. Algorithm 4 tries to make use of the structure of the constraints. Therefore, it does not go forward or backward through the list of constraints but follows their used constraint variables. We start with initializing our result I' and the test list T and set our set of observed constraint variables ω to the variables inside the last assigned constraint of our list I (lines 2 to 4). Then we start our main loop, remembering how many variables we have seen so far (Line 6). We go over our reversed list of constraints I (Line 8). If we find a constraint that contains some of the already inspected variables (Line 9), we add it to our testing list T and extend the set of already seen variables ω. We then continue iterating until our testing list T becomes inconsistent (Line 13). In this case we add the last tested constraint to our result list I' (Line 14). If this list is already inconsistent (Line 5), we return a minimal list of constraints (Line 19). If not, we restart the loop. But this time the set of already seen variables is restricted to the set of variables in the

Algorithm 4: CONNECTED_COMPONENT_FILTERING

```
Input  : An inconsistent list of constraints $I = [c_1, \dots, c_n]$.
Output: An irreducible inconsistent list of constraints $I'$.
1  if $size(I) = 0$ then return $\emptyset$
2  $I' \leftarrow []$
3  $T \leftarrow []$
4  $\omega \leftarrow vars(c_n)$
5  while $I'$ is consistent do
6      $count \leftarrow |\omega|$
7      $i \leftarrow size(I)$
8      while $T$ is consistent and $i \geq 0$ do
9          if $\omega \cap vars(c_i) \neq \emptyset$ then
10             $T \leftarrow T \circ c_i$
11             $\omega \leftarrow \omega \cup vars(c_i)$
12         $i \leftarrow i - 1$
13     if $T$ is inconsistent then
14         $I' \leftarrow I' \circ c_i$
15         $\omega \leftarrow \{y \mid y \in vars(x)$ where $x$ in $I'\}$
16         $I \leftarrow remove(T, c_i)$
17         $T \leftarrow I'$
18     if $count = |\omega|$ then $\omega \leftarrow \{y \mid y \in vars(x)$ where $x$ in $I\}$
19 return $I'$
```

constraints in I'. Furthermore, we only iterate over the constraints from the test list (Line 16), as this possibly shorter list is also an inconsistent list of constraints. If at one point we have not found any non tested constraint that has common variables with the tested ones (this can be the case if the last constraint of our input list I is not contained in the minimal list of constraints), we simply add all variables to ω in Line 18 so that we do not miss any constraint in the next iteration. With this algorithm, we want to take account of the internal structure of the constraints. For our example, this means we start with the last constraint $work(lea) - work(adam) = 1$ and completely ignore $work(john) = 0$ and $work(smith) = 0$ as their variables do not occur anywhere in the other constraints. We end up with the same IIS as with *Forward Filtering* without checking all constraints that do not have common variables with the constraints from the IIS.

Connected Components Range Filtering. This algorithm is a combination of the *Connected Components Filtering* and the *Range Filtering* algorithms. That is why it does not compute an irreducible list of constraints. We move through the list I like in Algorithm 4 and once our test list T becomes inconsistent we simply return it. This shall combine the advantages of using the structure of the constraints in the *Connected Components Filtering* and the simplicity of the *Range Filtering*. We ignore $work(john) = 0$ and $work(smith) = 0$ and end up with $I' = \{work(lea) - work(adam) = 1, work(adam) + work(lea) > 6, work(lea) = work(adam)\}$.

6 Reason Filtering in *clingcon*

Up to now we only considered reducing an inconsistent list of constraints to reduce the size of a conflicting nogood. But we can do even more. If the CP solver propagates the literal l, a *simple* reason nogood is $N = \{\ell \mid \ell \in A|_{\mathcal{C}}\} \cup \{\bar{l}\}$. If we have for example $A|_{\mathcal{C}} = \{\mathbf{T}$`work(john)$==0`$,\mathbf{T}$`work(lea)-work(adam)$==1`$\}$, the CP solver propagates the literal $\mathbf{F}$`work(lea)$==work(adam)`. To use the proposed algorithms to reduce a reason nogood we first have to create an inconsistent list of constraints. As $J = [\gamma(\ell) \mid \ell \in A|_{\mathcal{C}}]$ implies $\gamma(l)$, this inconsistent list is $I = J \circ [\overline{\gamma(l)}] = [work(john) = 0, work(lea) - work(adam) = 1, work(lea) = work(adam)]$. So we can now use these various filtering methods also to reduce reasons generated by the CP solver. In this case the reduced reason is $\{\mathbf{T}$`work(lea)-work(adam)$==1`$, \mathbf{T}$`work(lea)$==work(adam)`$\}$. Smaller reasons reduce the size of conflicts even more, as they are constructed using unit resolution.

These two new features of *clingcon* are available via the command line parameters: `--csp-reduce-conflict=X` and `--csp-reduce-reason=X` where `X` $=$ $\{$`simple,forward,backward,range,cc,ccrange`$\}$.

7 The *clingcon* System

The new filtering methods enhance the learning capabilities of *clingcon*. However, the new version also features *Initial Lookahead*, *Optimization* and *Propagation Delay*. We will now present these in more detail.

Initial Lookahead. As shown in (Yu and Malik 2006), initial lookahead on constraints can be very helpful in the context of SMT. It makes implicit knowledge (stored in the propagators of the theory solver) explicitly available to the propositional solver. Our *Initial Lookahead*, which can be enabled using the option `--csp-initial-lookahead=<true/false>`, is restricted to constraint literals. As a preprocessing step, all of them are separately set to true and constraint propagation is done. In this way, binary relations between constraints become explicitly available to the ASP solver. For example, $\mathbf{T}$`work(smith)$==0` implies $\mathbf{F}$`work(smith)-work(adam)$==1` whereas $\mathbf{T}$`work(lea)$==work(adam)` implies $\mathbf{F}$`work(lea)-work(adam)$==1`. These are then directly translated into a nogood. Or more formal: all constraints c implied by a constraint literal $\mathbf{T}\ell$ w.r.t. the theory are added to *clasp* as the respective binary nogood $\{\mathbf{T}\ell, \overline{\gamma^{-1}(c)}\}$.

Optimization. As shown in Section 4, *clingcon* now supports optimization statements over constraint variables. The option `--csp-opt-val` expects a comma-separated list of values, similar to the *clasp* 1.3 option `--opt-val`. With this option a value for every constraint optimization statement can be given. The solver will then start the search using these values. This is especially useful in combination with the option `--csp-opt-all`, that is used to compute all models that are less or equal to the last found bound. It forms the logical equivalent to the *clasp* 1.3 option `--opt-all`. To compute all constraint-optimal solutions, one first computes one optimal solution. Afterwards, given the optimum value, the same encoding can be used with the options `--csp-opt-val` and `--csp-opt-all` to compute all optimal solutions.

Propagation Delay. With *Propagation Delay* we have a new experimental feature that balances the interplay between the ASP and the CSP part. Constraint propagation can be expensive, especially in combination with the filtering techniques from Section 5. It might be beneficial to give more attention to the ASP solver. This can be done by skipping constraint propagations. Whenever we can propagate a constraint atom or encounter a conflict with the CP solver, filtering methods can be applied. If we therefore skip constraint propagation and only do it every n'th time, *clasp* has the chance to find more conflicts. If we learn less from the CSP side, we learn more from the ASP side. The option `--csp-prop-delay=n` where $n \in \mathbb{N}_0^+$ can be used to set the propagation delay:

- $n = 1$ does constraint propagation every time, similar to the old *clingcon*,
- $n > 1$ does constraint propagation only every n'th time and
- $n = 0$ does constraint propagation only on a full propositional assignment.

Whenever we do constraint propagation we have to catch up on the missed propagation.

8 Experiments

We collected various benchmarks from different categories to evaluate the effects of our new features on a broad range of problems. All of them can be expressed using a mixed representation of Boolean and non-Boolean variables. We restrict ourself to classes where the ASP and the CSP part interact tightly to solve the problem, as we focus on the learning capabilities between the two systems. For encodings where we do not have an ASP or a CSP part, we will not see any effect of our new features. We now present our benchmark classes with a short description of the used encodings. All encodings and instances can be found online at (clingcon).

Benchmarks. Given a set of squares, the *Packing* problem is to pack all squares into a rectangular area with fixed dimension. This problem is directly taken from the 2011 ASP Competition (ASP 2011). The position of the corners of the squares can be represented using integer variables. and are guessed by the CP solver. Within ASP we only have to check whether two squares overlap.

Incremental Scheduling is a problem variant of the well-known job shop scheduling problem which requires rescheduling and ordering of jobs. Furthermore, all jobs have a deadline, and, if a job finishes after its deadline, the difference is taken as “tardiness” of the job. This tardiness multiplied by the importance of the job results in a penalty. The task is to find a solution where the sum of all penalties is below a given maximum. The problem is also taken from the 2011 ASP Competition. We use constraint variables to denote the starting times of the jobs and also to compute the tardiness and penalties.

Taken from the 2011 ASP Competition, the *Weighted Tree* problem is inspired by cost-based “join-order” optimization of SQL queries in databases. The problem is to find a full binary tree with n leaves such that its leaves are pairs (weight, cardinality) of integers, the right child of an inner node is a leaf, where its color is either green, red, or blue, and there are $1, \dots, n-1$ inner nodes such that node $n-1$ is a root node and inner node $i-1$ is the left child of inner node i for $i = 2, \dots, n-1$. The weights of inner nodes are computed recursively based on their colors, and the weights and cardinalities of their children. We use constraint variables to represent the cardinality and the weight of the nodes. We furthermore extended the problem to an optimization problem that tries to find the "cheapest" tree by minimizing the sum of the leaf weights according to the structure of the tree. For this we use the new optimization statement over constraint variables. In our benchmarks, an instance is solved if we have found and proven the optimal solution.

Given an $n \times n$ board, placing numbers in the range $\{1, \ldots, n\}$ such that there are no two equal numbers in the same row/column is called *Quasi Group* problem. For our benchmarks, we let $n = 20$. We assign random numbers to $0 - 80$ percent of the fields.

To increase the spectrum of benchmarks, we conceived a new collection of benchmarks which make use of the CP solver to do the *Unfounded Set Check (USC)* for some normal logic programs. Therefore, we reify (Gebser et al. 2011; Gebser et al.) logic programs (in our case we take *Labyrinth* – the problem of guiding an avatar through a dynamically changing labyrinth to certain fields (ASP 2011), *HashiwoKakero* – a logic puzzle game and *HamiltonianCycle*). Using this reified program we can reason about the structure of the program. In particular, we can add an encoding that does the unfounded set check using level-mapping as proposed in (Niemelä 2008). We assign a level to every atom in a strongly connected component and use the CP solver to find a valid mapping. Using this translation, we can solve any non-tight logic program using the CP solver for the unfounded set check.

Settings. We run our benchmarks single-threaded on a cluster with 24×8 cores with 2.27GHz each. We restricted each run to use 4GB RAM. In all our benchmarks we used the standard configuration of *clingcon*, unless stated otherwise. We now evaluate the new features of *clingcon*.

Global Constraints. We want to check whether the use of global constraints speeds up the computation. Therefore we have chosen the *Quasi Group* problem, as it can be easily expressed using the global constraint *distinct*. We compare two different encodings for *Quasi Group*. The first one uses one *distinct* constraint for every row and every column. The second one uses a cubic number of inequality constraints. We tested 78 randomly generated instances of size 20×20. While the first encoding using the global constraints results in an average runtime of 220 seconds and 18 timeouts over all instances, the decomposed version was much slower. It used 391 seconds on average and had 27 timeouts. *Clingcon* confirms, that global constraints are handled more efficiently than their explicit decomposition.

Initial Lookahead. In Section 7, we presented *Initial Lookahead* over constraints as a new feature of *clingcon*. We now want to study in which cases this technique can be useful in terms of runtime. We run all our benchmarks once with and without *Initial Lookahead*. In Table 1, the first column shows the problem class and its number of instances. The second and the third column show the average runtime in seconds that is used with and without *Initial Lookahead (I.L.)*. Timeouts are shown in parenthesis. The last two columns show the average runtime of the lookahead algorithm and the number of nogoods that have been produced on average per instance. As we can see for the problems *Packing*, *Quasi Group*, and *Weighted Tree*, direct relations between constraints can be detected and the overall runtime can therefore be reduced. But this technique does not work on all benchmark classes. For *Incremental Scheduling* relations between constraints are found but the additional nogoods seem to deteriorate the performance of the solver. In the case of the *Unfounded Set Check*, nearly no relations have been found, so no difference in runtime can be detected.

Conflict and Reason Filtering. We want to analyze how much the different conflict and reason filtering methods presented in Section 5 differ in size of conflicts and average run-

instances (#number)	time (timeouts)	time with *I.L*	time of *I.L.*	nogoods from *I.L*
Packing(50)	888(49)	882(49)	5	7970
Inc. Sched.(50)	30(01)	40(02)	0	73
Quasi Group(78)	390(28)	355(24)	9	105367
Weighted Tree(30)	484(07)	312(04)	0	1520
USC(132)	721(104)	719(103)	3	1

Table 1: Initial Lookahead (*I.L.*)

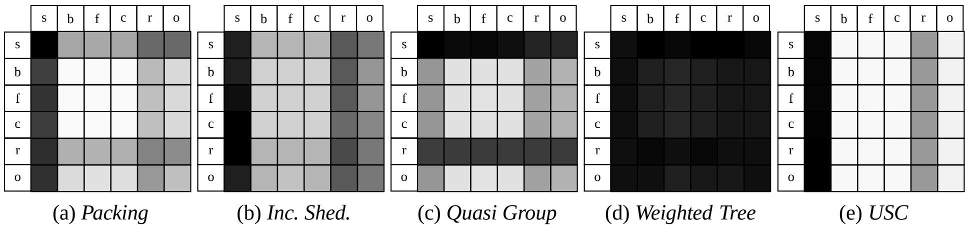
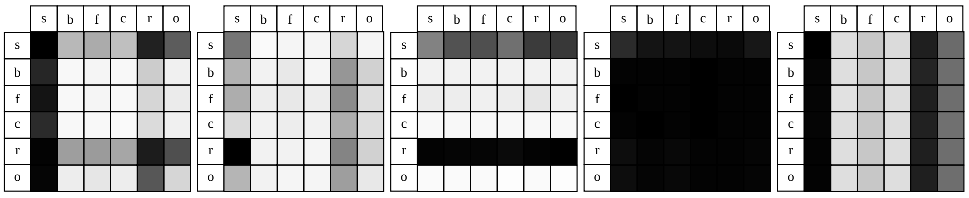


Fig. 2: Average conflict size

time. As conflicts and reasons are strongly interacting in the CDCL framework, we test the combination of all our proposed algorithms. We denote the filtering algorithms with the following shortcuts: s(*Simple*), b(*Backward Filtering*), f(*Forward Filtering*), c(*Connected Component Filtering*), r(*Range Filtering*) and o(*Connected Component Range Filtering*). We name the filtering algorithm for reasons first, separated by a slash from the algorithm used to filter conflicts. To denote the configuration using *Range Filtering* for reasons and *Forward Filtering* for conflicts, we simply write r/f. The original configuration, which can be seen as the "old" *clingcon* is therefore denoted s/s. We start by showing the impact on average conflict size of all configurations using a heat map in Figure 2. It shows the reduction of the conflict size in percentage relative to the worst configuration. The rows represent the used algorithms for reason filtering, the columns represent the algorithms for filtering conflicts. So the worst configuration is represented by a totally black square and a configuration that reduces the average conflict size by half is gray. A completely white field would mean that the conflict size has been reduced to zero. As we can see in Figure 2, the average conflict size is reduced by all combinations of filtering algorithms. Furthermore, we see that the first row and column, respectively, is usually darker than the others, which indicates that filtering either only conflicts or only reasons is not enough. Also we see that for the *Unfounded Set Check (USC)* the filtering of reasons does not have any effect. This is due the encoding of the problem. As nearly no propagation takes place, no reasons are computed at all. The shades on the *Range Filtering* rows/columns (denoted by r) clearly show that the *Range Filtering* produces larger conflicts. But this is improved by incorporating structure to the filtering algorithm using *Connected Component Range Filtering*. Next, we want to see if the reduction of the average conflict size also pays off in terms of runtime. Therefore Figure 3 shows the heat map for average runtime. A black square denotes the slowest configuration, while a gray one is twice as fast. As we can clearly see, the reduction of runtime coincides with the reduction of conflict size in most cases. Furthermore, we



(a) *Packing* (b) *Inc. Sched.* (c) *Quasi Group* (d) *Weighted Tree* (e) *USC*

Fig. 3: Average time in seconds

Instances (#number)	time s/s	time o/b	acs s/s	acs o/b
Packing(50)	888(49)	63(0)	293	40
Inc. Sched.(50)	30(01)	3(0)	15	5
Quasi Group(78)	390(28)	12(0)	480	56
Weighted Tree(30)	484(07)	574(18)	31	31
USC(132)	721(104)	92(1)	454	13

Table 2: Average time in s(timeouts), average conflict size (acs)

can see a clear speedup for all benchmark classes except *Weighted Tree* using the filtering algorithms. Table 2 compares the *Simple* version s/s without using any filtering algorithms, with the configuration o/b (reducing reasons using *Connected Component Range Filtering* and reducing conflicts using *Backward Filtering*), as it has the lowest number of timeouts. We can see a speedup of around one order of magnitude on all benchmarks except *Weighted Tree*. The same picture is given for the reduction of conflict size. So whenever it is possible to reduce the average conflict size, this also pays off in terms of runtime.

Propagation Delay. As the filtering of conflicts and reasons takes a lot of time (e.g configuration o/b uses 43% of the runtime for filtering), we want to reduce the calls to the filtering algorithms. Therefore, with *Propagation Delay*, we can do less propagation with the CP solver and it will produce less conflicts and reasons, hopefully reducing the number of calls. We therefore take the yet best configuration o/b and compare different propagation delays $n \in \{1, 10, 0\}$ (normal, every ten steps, only on model). Table 3 shows the average number of calls to a filtering algorithm for configuration o/b with different delays. We see a reduction of the number of calls on all benchmarks except on *Incremental Scheduling* where it doubled. This is clearly due to a loss of information that is necessary for the search. If the CP solver has less influence on the search, the ASP part gets more control. But the missing knowledge from the CSP part has to be compensated by pure search in

n	1	10	0
Packing	31534	14897	8463
Inc. Sched.	3505	3240	6660
Quasi Group	4245	1535	1726
Weighted Tree	6868k	1168k	1042k
USC	2007	2118	1768

Table 3: Calls to filtering algorithms o/b

n	1	10	0
Packing	63	75	571
Inc. Sched.	3	6	11
Quasi Group	12	9	19
Weighted Tree	574	559	546
USC	92	91	82

Table 4: Times of configuration o/b

the ASP part. Therefore, Table 4 shows that some benchmarks like *Weighted Tree* and *Unfounded Set Check* can relinquish some propagation power gaining additional speedup. On others like *Packing* this propagation is urgently needed to drive the search and cannot be compensated. This feature has to be investigated further to gain benefits for practical usage.

9 Related Work

In the quite young field of ASP modulo CSP a lot of research has been done in the last years. The approaches can be separated roughly into two classes: integration and translation. The integrated approaches like *CASP* (Baselice et al. 2005) and ADSolver/ACSolver (Mellarkod and Gelfond 2008; Mellarkod et al. 2008) are similar to the *clingcon* system. However, no learning is used in the approaches, as the constraint solver just checks the assignment of constraints. Later, (Balduccini 2009) showed how to use ASP as a specification language, where each answer set represents a CSP. In this approach no coupling between the systems was possible and therefore learning facilities were not used. Afterwards GASP (Dal Palù et al. 2009) presented a bottom up approach where the logic program was grounded on the fly. With Dingo (Janhunen et al. 2011) ASP was translated to difference logic using level mapping for the unfounded set check. An SMT solver is used to solve the translated problem. Nowadays, more and more translational approaches arise in the area of SMT. *Sugar* (Tamura et al. 2008) is a very successful solver which translates the various supported theories to SAT. Also, in (Drescher and Walsh 2010) it was shown how to translate constraints into ASP during solving. These translational approaches have the strongest coupling and therefore the highest learning capabilities. The reason generation and conflict handling is directly done by the underlying SAT solver and therefore very efficient. On the other hand, they do have problems to find a compact representation of the constraints without losing propagation strength. *Clingcon* therefore tries to catch up, improving the learning facilities and still preserving the advantages of integrated approaches like compact representation of constraint propagators. Furthermore, *clingcon* is a ASP modulo Theory solver that aims at taking advantage of arbitrary theories in the long run, eg description logics. Such a variety can only be supported by a black box approach. Similar results regarding the filtering methods have been introduced in (Junker 2001) but have not been applied to an SMT framework.

10 Discussion

We extended and improve *clingcon* in various ways. At first, the input language was expanded to support *Global Constraints* and *Optimization Statements* over constraint variables. As the input language is a big advantage over pure SMT systems, complex hybrid problems can now easily be expressed as constraint logic programs. We have shown that *Initial Lookahead* can give advantages in terms of speedup on some problems. We developed *Filtering* methods for conflicts and reasons that can be applied to any theory solver. This enables the ASP solver to learn about the structure of the theory, even if the theory solver does not give any information about it (black box systems). We furthermore show that while applying these filtering methods, that knowledge is discovered that is valuable for the overall search process and can therefore speed up the search by orders of magnitude. Unfortunately, a direct comparison with existing SMT solvers is inapplicable in view of different input formats. However, we want to conduct an indirect comparison by translating ASPmCSP problems to SMT following the line of (Janhunen et al. 2011), using a level mapping for the unfounded set check. This allows us to compare our approach to SMT solvers that do not use a black-box CP solver but do propagation either with dedicated algorithms (eg. (Bofill et al. 2008)) or a translation of CP to SAT (cf. (Tamura et al. 2008)). Such solvers have complete control over reason and conflict generation and can therefore use extra knowledge to create better conflicts. With *Propagation Delay* we developed a method to control the impact of the interaction among both systems to the search. To balance the *Propagation Delay* dynamically during search will be topic of additional research. Our work leaves the burden of choosing the right reason and conflict generation strategy to the user. Although for our benchmark set the optimal filtering configuration was o/b, this may vary on other benchmark classes. This can be counterbalanced by an adaption of the *claspfolio* (Gebser et al. 2011) system to *clingcon* in order to automatically derive an optimal configuration of *clingcon* from the features of problem instances. In the future we still want to focus on learning capacities and increase the coupling of the two systems, such that the CP solver also benefits from the elaborated learning techniques.

Acknowledgments This work was partially funded by the German Science Foundation (DFG) under grants SCHA 550/8-2, SCHA 550/10-1 AOBJ: 593494.

References

ASP Competition. 2011. `https://www.mat.unical.it/aspcomp2011/`.

BALDUCCINI, M. 2009. Representing constraint satisfaction problems in answer set programming. In *Workshop ASPOCP'09*.

BARAL, C. 2003. *Knowledge Representation, Reasoning and Declarative Problem Solving*. Cambridge University Press.

BASELICE, S., BONATTI, P., AND GELFOND, M. 2005. Towards an integration of answer set and constraint solving. In *Proceedings of ICLP'05*, Springer, 52–66.

BIERE, A., HEULE, M., VAN MAAREN, H., AND WALSH, T., 2009. *Handbook of Satisfiability*. Frontiers in Artificial Intelligence and Applications. IOS Press.

BOFILL, M., NIEUWENHUIS, R., OLIVERAS, A., RODRÍGUEZ-CARBONELL, E., AND RUBIO, A. 2008. The barcelogic SMT solver. In *Computer Aided Verification*, Springer, 294–298.

CHINNECK, J. AND DRAVINIEKS, E. 1991. Locating minimal infeasible constraints sets in linear programs. In *ORSA Journal On Computing*. 3, 157–168.

clingcon. `http://www.cs.uni-potsdam.de/clingcon`.

DAL PALÙ, A., DOVIER, A., PONTELLI, E., AND ROSSI, G. 2009. Answer set programming with constraints using lazy grounding. See Hill and Warren (2009), 115–129.

DECHTER, R. 2003. *Constraint Processing*. Morgan Kaufmann Publishers.

DRESCHER, C. AND WALSH, T. 2010. A translational approach to constraint answer set solving. In *TPLP*. 10(4-6). Cambridge University Press, 465–480.

GEBSER, M., KAMINSKI, R., KAUFMANN, B., OSTROWSKI, M., SCHAUB, T., AND THIELE, S. A user's guide to `gringo`, `clasp`, `clingo`, and `iclingo`. Available at `http://potassco.sourceforge.net`.

GEBSER, M., KAMINSKI, R., KAUFMANN, B., SCHAUB, T., SCHNEIDER, M., AND ZILLER, S. 2011. A portfolio solver for answer set programming: Preliminary report. In *Proceedings of LPNMR'11*, Springer, 352–357.

GEBSER, M., KAMINSKI, R., AND SCHAUB, T. 2011. Complex optimization in answer set programming: Extended version. Unpublished draft. Available at (metasp).

GEBSER, M., KAUFMANN, B., NEUMANN, A., AND SCHAUB, T. 2007. Conflict-driven answer set solving. In *Proceedings of IJCAI'07*, AAAI Press/The MIT Press, 386–392.

GEBSER, M., OSTROWSKI, M., AND SCHAUB, T. 2009. Constraint answer set solving. See Hill and Warren (2009), 235–249.

gecode. `http://www.gecode.org`.

GELFOND, M. AND LIFSCHITZ, V. 1991. Classical negation in logic programs and disjunctive databases. *New Generation Computing 9*, 365–385.

HILL, P. AND WARREN, D., Eds. 2009. *Proceedings of ICLP'09*. Springer.

JANHUNEN, T., LIU, G., AND NIEMELÄ, I. 2011. Tight integration of non-ground answer set programming and satisfiability modulo theories. In *Proceedings of GTTV'11*, 1–13.

JUNKER, U. 2001. QuickXPlain: Conflict detection for arbitrary constraint propagation algorithms. *IJCAI'01 Workshop on Modelling and Solving problems with constraints*.

MELLARKOD, V. AND GELFOND, M. 2008. Integrating answer set reasoning with constraint solving techniques. In *Proceedings of FLOPS'08*, Springer, 15–31.

MELLARKOD, V., GELFOND, M., AND ZHANG, Y. 2008. Integrating answer set programming and constraint logic programming. *Annals of Mathematics and Artificial Intelligence 53*, 1-4, 251–287.

metasp. `http://www.cs.uni-potsdam.de/metasp`.

MOHR, R. AND HENDERSON, T. 1986. Arc and path consistency revisited. *Artificial Intelligence 28*, 2, 225–233.

NIEMELÄ, I. 2008. Stable models and difference logic. *Annals of Mathematics and Artificial Intelligence 53*, 1-4, 313–329.

NIEUWENHUIS, R., OLIVERAS, A., AND TINELLI, C. 2006. Solving SAT and SMT: From an abstract Davis-Putnam-Logemann-Loveland procedure to DPLL(T). *JACM 53*, 6, 937–977.

SCHULTE, C., TACK, G., AND LAGERKVIST, M. 2012. Modeling. In *Modeling and Programming with Gecode*, C. Schulte, G. Tack, and M. Lagerkvist. Corresponds to Gecode 3.7.2.

SYRJÄNEN, T. Lparse 1.0 user's manual. `http://www.tcs.hut.fi/Software/smodels/lparse.ps.gz`.

TAMURA, N., TANJO, T., AND BANBARA, M. 2008. System description of a SAT-based CSP solver Sugar. In *Third International CSP Solver Competition*. 71–75.

VAN LOON, J. 1981. Irreducibly inconsistent systems of linear inequalities. In *European Journal of Operational Research*. 3. Elsevier Science, 283–288.

YU, Y. AND MALIK, S. 2006. Lemma learning in SMT on linear constraints. In *Theory and Applications of Satisfiability Testing - SAT 2006*, Springer, 142–155.