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User Manual

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Abstract

Multivariable H-infinity Control Design Toolbox : User manual
H.M. Falkus and A.A.H. Damen

A MATLAB toolbox is presented for solving the multivariable H_∞ control design problem. Algorithms are available (Robust control toolbox of MATLAB) which solve the problem, once the control design configuration including process model and weighting functions has been rewritten into a standard H_∞ control problem. In this report a general package is described that facilitates the controller design for various control configurations, the standard H_∞ control problem and the closed-loop system evaluation. Because no solution is known for translating design specifications such as desired behaviour, robustness, performance etc. directly into weighting functions in the frequency domain, the necessarily iterative design procedure has been implemented in a flexible, menu driven way.

Keywords : Robust control, Multivariable control systems, Control simulation software.

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Contents

1

Introduction

In the last few years, there has been much interest in the design of feedback controllers for linear systems that minimize the H_∞ norm of a specified closed-loop transfer function. Since 1988, a state-space solution for general H_∞ problems based on a "2-Riccati" approach, derived by Glover, K. and J.C. Doyle (1988), has been available for the representation of all stabilizing controllers that satisfy an H_∞ norm bound :

$$\left\| \mathscr{F}(G,K) \right\|_\infty \leq \gamma \tag{1.1}$$

A more detailed explanation and a proof of its validity is outlined in Doyle, J.C. *et. al.* (1989). Standard program packages (Robust control toolbox of MATLAB) together with some numerical variations and extensions of the basic solution are available now and can be applied once the original problem has been translated into the standard control H_∞ problem.
The generalized plant G contains what usually is called the plant in a control problem and includes all weighting functions. The signal $w \in \mathbb{R}^{m1}$ represents all external inputs, including disturbances, sensor noise and commands; the output $z \in \mathbb{R}^{p1}$ is the error vector; $y \in \mathbb{R}^{p2}$ is the observation vector; and $u \in \mathbb{R}^{m2}$ is the control input. The generalized plant G can be partitioned according to the dimensions of the signals :

$$G = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} (sI - A)^{-1} \begin{bmatrix} B_1 & B_2 \end{bmatrix} \tag{1.2}$$

which results in the following closed-loop transfer function from w to z :

$$\mathcal{F}(G,K) := G_{11} + G_{12} K \left(I - G_{22} K \right)^{-1} G_{21} \tag{1.3}$$

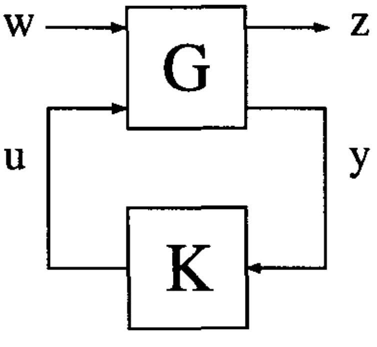

Fig. 1.1 : Standard H_∞ Problem.

The standard assumptions are :

- The triplet (A,B_2,C_2) - namely the plant transfer G_{22} - can be stabilized and detected, so that stabilizing controllers exist.
- $\mathrm{rank}(D_{12}) = m_2$ and $\mathrm{rank}(D_{21}) = p_2$ in order to ensure realizability of the controllers.
- No zeros on the imaginary axis, $p_1 \geq m_2$ and $m_1 \geq p_2$ ensures that the solution to the corresponding LQG problem is closed-loop asymptotically stable.

The main problem, however, is that every control problem has a different configuration because of different design constraints and control objectives. This implies that every new control problem has to be rewritten again into the standard H_∞ control problem.
In this report a MATLAB toolbox is presented which enables us (using computer routines) to transform every multivariable control problem into the standard H_∞ control problem. After selecting the control setup, design constraints and objectives, the design configuration is defined in a fairly simple way. Because no general solution is known for translating design specifications such as desired behavior, robustness, performance etc. directly into weighting functions in the frequency domain, the H_∞ control design is menu driven to ensure easy input

of variables, controller calculation, and analysis of the results by computing both time and frequency responses. In this way the necessary iterative design procedure for optimizing the H_∞ control design problem becomes much easier. All tools in this toolbox are implemented in MATLAB by means of standard .m-files.

In Section 2 the basic setup of the toolbox is presented using the process block diagram of a floating platform laboratory process as an example. The floating platform with rotating crane has been built on laboratory scale to evaluate identification and control theories. This particular process was chosen because it is an essentially linear MIMO system. It can be well described by three decoupled, second order SISO systems. The model errors are then mainly due to unmodeled waves, caused by the movement of the floats, which lead to linear transfers which are however difficult to model. The fact that H_∞ control is said to be particularly suited for robust control in cases of unmodeled linear dynamics, makes this laboratory process an excellent example for testing the H_∞ control synthesis procedure. On the platform, a crane has been mounted rotating a load and thereby tilting the platform. The control to be designed should prevent this tilting of the platform A detailed description of the process together with the physical modeling, identification and control design can be found in Bouwels, J.P.H.M. (1991) and Damen, A.A.H. *et. al.* (1994).

A detailed menu description of the toolbox together with several design options is given in Section 3.

2

General H_∞ Control Design Framework

In this section the most important parts of the general framework will be explained. Fig. 2.1 depicts the H_∞ control design configuration for the floating platform. The solid part illustrates the basic control configuration, while the dashed part is added for the H_∞ control design. The main objectives in the design are disturbance attenuation ($V_d n_d$ to prevent tilting of the platform due to the rotating crane) and robustness (model errors represented by $V_v n_v$ due to unmodeled waves). In addition saturation of the actuators ($W_u u_p$) should be avoided.
The transfers of Fig. 2.1 are described as follows :

P : Platform Dynamics	W_y : Weighting Process Output
R : Disturbance Dynamics	W_u : Weighting Control Input
V_d : Shaping Crane Disturbance	C_{fb} : Feedback Controller
V_v : Shaping Model Disturbance	C_{ff} : Feedforward Controller

where we can define the following standard signals :

$$z = \begin{pmatrix} \tilde{y} \\ \tilde{u} \end{pmatrix} \quad , \quad y = \begin{pmatrix} y_m \\ d \end{pmatrix} \quad , \quad w = \begin{pmatrix} n_d \\ n_v \end{pmatrix} \quad , \quad u = u_p \qquad (2.1)$$

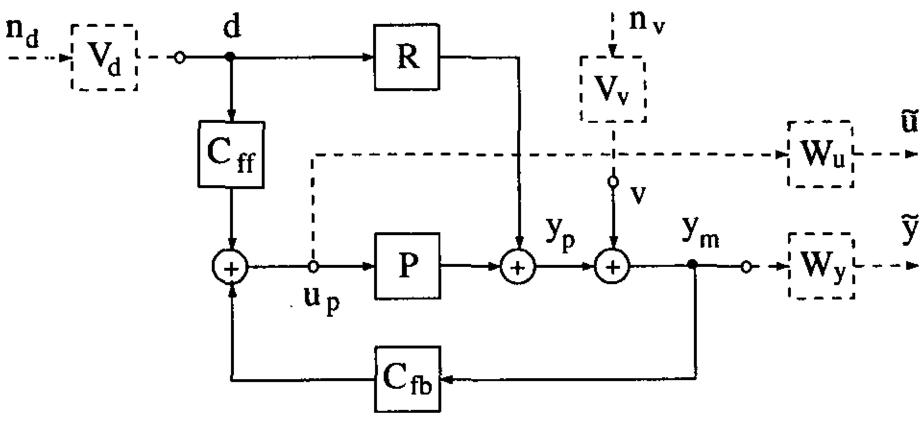

Fig. 2.1 : Multivariable Control Design Configuration Floating Platform

After deriving the various relations between the inputs and outputs, it follows that, in terms of the standard H_∞ problem, the generalized plant G is defined by :

$$\begin{pmatrix} z \\ y \end{pmatrix} = \left(\begin{array}{c} \tilde{y} \\ \tilde{u} \\ \hline y_m \\ d \end{array}\right) = \left(\begin{array}{cc|c} W_y V_v & W_y R V_d & W_y P \\ 0 & 0 & W_u \\ \hline V_v & R V_d & P \\ 0 & V_d & 0 \end{array}\right) \left(\begin{array}{c} n_v \\ n_d \\ \hline u_p \end{array}\right) \tag{2.2}$$

$$= \left(\begin{array}{c|c} G_{11} & G_{12} \\ \hline G_{21} & G_{22} \end{array}\right) \begin{pmatrix} w \\ u \end{pmatrix}$$

The control acting on the outputs y is represented by :

$$u = u_p = \left(C_{fb} \mid C_{ff} \right) \begin{pmatrix} y_m \\ d \end{pmatrix} = K\ y \tag{2.3}$$

This yields the closed-loop system $\mathscr{F} := \mathscr{F}(G,K)$ mapping $w \rightarrow z$ which has H_∞ norm :

$$\|\mathcal{F}\|_\infty = \left\| \begin{matrix} \mathcal{F}_{11} & \mathcal{F}_{12} \\ \mathcal{F}_{21} & \mathcal{F}_{22} \end{matrix} \right\|_\infty$$
$$= \left\| \begin{matrix} W_y(I-PC_{fb})^{-1}V_v & W_y(I-PC_{fb})^{-1}(R+PC_{ff})V_d \\ W_uC_{fb}(I-PC_{fb})^{-1}V_v & W_u(I-PC_{fb})^{-1}(C_{fb}R+C_{ff})V_d \end{matrix} \right\|_\infty \tag{2.4}$$

Here the various transfer functions are named as follows :

$\mathcal{F}_{11}$: Sensitivity $\mathcal{F}_{12}$: Disturbance Attenuation
$\mathcal{F}_{21}$: Control Sensitivity $\mathcal{F}_{22}$: Saturation

Until now we have only described the standard approach to define the H_∞ control problem for the floating platform to achieve disturbance attenuation and avoiding saturation of the control input. This results in the closed-loop system $\mathcal{F}(G,K)$ of Eq. 2.4. This configuration however is specific to the floating platform and can be significantly different for other control problems. To avoid this procedure of building (and implementing) different configurations every time when new control problems occur, we will generalize this setup.

2.1 Structure Definition

Basicly, the design configuration (Fig. 2.1) can be built up of four major blocks (Fig. 2.2) :

1,2) Process models P_1 & P_2
3) Shaping filters V for the input signals w
4) Weighting filters W for the output signals z.

The extra process block P_2 is sometimes necessary if there exists already a known feedback. Further, these four blocks are somehow connected. Rearranging of Fig. 2.1 into the blocks P_1, P_2, V and W is carried out in Fig. 2.3 where IM_1 and IM_2 define the structure of the control design configuration. IM_1 and IM_2 reflect the interconnection structure of the various blocks. These are constant matrices with entries ±1 and 0, each entry corresponding to a specific adding, subtracting or no connection of signals. The matrices I_1 to I_4 define the feed-through of signals which are necessary to build the state-space representation of the generalized plant G.
We emphasize that this structure is general. That is, every control configuration of the form shown in Fig. 2.1 can be represented by a configuration of Fig. 2.2 resulting in Fig. 2.3.

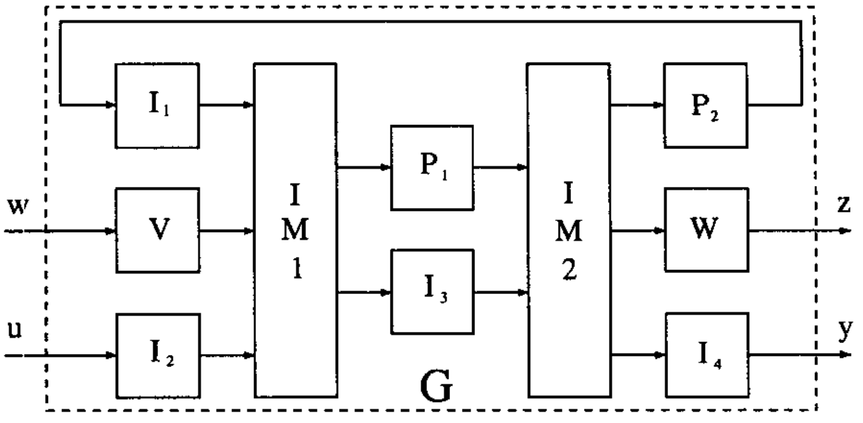

Fig. 2.2 : Basic structure generalized plant G.

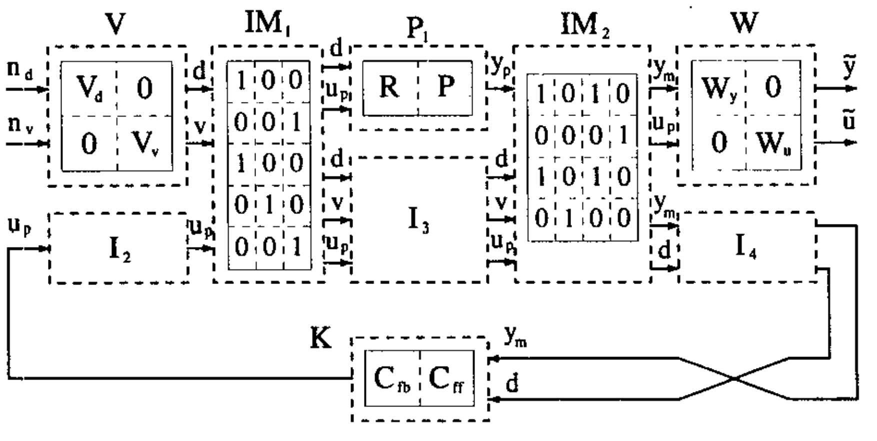

Fig. 2.3 : Floating platform configuration for MHC toolbox.

The control design configuration of the floating platform can now be described in a simple way by defining the dimensions of V (V_d & V_v), P_1 (R & P), P_2 (∅), W (W_y & W_u) and I_1 to I_4 together with the structure of the configuration using the interconnection blocks IM_1 and IM_2. This is effectively all input which is needed for the toolbox to convert it into a standard H_∞ control problem.

2.2 Minimal Realization Generalized Plant

The blocks V, P_1, P_2 and W are represented in the toolbox either as MIMO state-space representations or, entry wise, as SISO transfer functions of a matrix. However, before building the generalized plant G, minimum state-space realizations of these blocks have to be obtained.

Therefore we use the approach outlined in Munro, N. *et. al.* (1971). For every row in a transfer function matrix, the smallest common denominator is determined and the numerators are updated if necessary. The new MISO transfer functions can be transformed into an observer canonical state-space representation which is minimal. Combining the state matrices of the MISO systems for every row in a block-diagonal form and adding the input, output and feed-through matrices correctly results in an overall state-space representation which is observable but not necessarily controllable. In Dooren, P.M. v. (1981) it is proven that if the controllability matrix of (A,B) has rank $r \leq n$, where n is the size of A, then there exists a similarity transformation T such that :

$$\bar{A} = TAT^T \quad , \quad \bar{B} = TB \quad , \quad \bar{C} = CT^T \quad , \quad \bar{D} = D$$

and the transformed system has a staircase form with the uncontrollable modes (being the eigenvalues of A_{uc}), if any, in the upper left-hand corner.

$$\bar{A} = \begin{pmatrix} A_{uc} & 0 \\ A_{21} & A_c \end{pmatrix} \quad , \quad \bar{B} = \begin{pmatrix} 0 \\ B_c \end{pmatrix} \quad , \quad \bar{C} = \begin{pmatrix} C_{uc} & C_c \end{pmatrix} \tag{2.5}$$

where (A_c,B_c) is controllable, $(A_{uc},B_{uc}{=}0)$ is uncontrollable and $C_c(sI-A_c)^{-1}B_c = C(sI-A)^{-1}B$. If the process P_1, P_2 and the design blocks V and W are given in transfer function matrices, this approach can be used to derive a minimum state-space representation of every block.
The dual approach for realizing a minimum state-space representation can also be used. In that case a controllable but not necessarily observable state-space representation can be derived and all unobservable states have to be removed. So if the observability matrix of (A,C) has rank $r \leq n$, there exists again a similarity transformation such that the transformed system has a staircase form with the unobservable modes, if any, in the upper left-hand corner.

$$\bar{A} = \begin{pmatrix} A_{uo} & A_{12} \\ 0 & A_o \end{pmatrix} \quad , \quad \bar{B} = \begin{pmatrix} B_{uo} \\ B_o \end{pmatrix} \quad , \quad \bar{C} = \begin{pmatrix} 0 & C_o \end{pmatrix} \tag{2.6}$$

Because the blocks V, P_1, P_2 and W are now available as minimum state-space realizations, straightforward matrix computations for connecting state-space systems in series or parallel can be used to build the generalized plant :

1) Build I_1, V and I_2 parallel (System 1).
2) Build P_1 and I_3 parallel (System 2).
3) Build P_2, W and I_4 parallel (System 3).
4) Connect system 1 in series with IM_1 (System 4).

5) Connect system 2 in series with IM_2 (System 5).
6) Connect system 4 in series with system 5 (System 6).
7) Connect system 6 in series with system 3 (System 7).
8) Partition system 7 according to the defined inputs/outputs.
9) Close the loop around P_2 and I_1.

The state-space system of the generalized plant might not be a minimum realization because of common modes in the various blocks. Removing again all uncontrollable (2.5) and unobservable (2.6) modes will yield a minimum state-space realization of the generalized plant. This approach has been selected because obtaining the same minimal realization after building the generalized plant using the non-minimal state-space realizations of the various blocks and applying Eq. 2.5 and 2.6 only once, might not be achievable due to numerical problems (e.g. round-off errors).
The constructed minimal state-space realization of the augmented plant might be badly conditioned depending on the design filters and process behaviour. This can result in numerical problems when calculating the H_∞ controller. Balancing of the augmented plant is therefore often desired to improve numerical reliability. The balancing approach described in Weiland, S. (1993) is used in order to handle unstable as well as stable systems.

2.3 Controller Calculation

Because a minimum state-space representation is available, the standard solution method based on solving two Algebraic Riccati equations and implemented in the Robust Control Toolbox of MATLAB, Chiang, R.C. *et. al.* (1988), can be applied. The methods available :

1) Safonov/Limebeer/Chiang loop-shifting formulae ; Safonov, M.G. *et. al.* (1989).
2) Glover/Doyle all-solution formulae ; Glover, K. *et. al.* (1988).
3) Limebeer/Kasenally all-solution formulae ; Limebeer, D.J.N. *et. al.* (1988).

are only different in circumventing some of the numerical problems which generally arise when a design approaches its H_∞ performance limits.
The solutions to the Riccati equations can be solved either by an eigenvalue or Schur decomposition. The eigenvalue approach is the fastest but for design filters close to the H_∞ performance limits the Schur approach is numerically more reliable.
These routines calculate a controller, if one exists, only for a fixed value of γ. That is, a controller is computed achieving $\| \mathcal{F} \|_\infty \leq \gamma$. However, we are interested in γ_{opt} for which a stabilizing controller still exists. Therefore the basic routine has been extended as follows with an iterative search procedure :

- A start value γ_o and a step size α ($\alpha > 1$) are defined.
- An interval [γ_{min}, γ_{max}] is computed which contains the optimal solution.
  1) If a solution exists for γ_o ($\gamma_{opt} \leq \gamma_o$) define $\gamma_{max} = \gamma_o$. The lower bound of the interval can be found by decreasing γ ($\gamma_{k+1} = \gamma_k/\alpha^k$) until no solution exists defining $\gamma_{min} = \gamma_{k+1}$.
  2) If no solution exists for γ_o ($\gamma_{opt} > \gamma_o$) define $\gamma_{min} = \gamma_o$. The upper bound of the interval can be found by increasing γ ($\gamma_{k+1} = \alpha^k.\gamma_k$) until a solution exists defining $\gamma_{max} = \gamma_{k+1}$.
- Bisection search is used to find γ_{opt} within a certain tolerance margin for which a stabilizing solution exists.
  1) Define $\gamma_k = (\gamma_{max} + \gamma_{min})/2$.
  2) If a solution exists for γ_k adjust the upper bound $\gamma_{max} = \gamma_k$. If no solution exists for γ_k adjust the lower bound $\gamma_{min} = \gamma_k$.
  3) Repeat 1 & 2 until $(\gamma_{max} - \gamma_{min})/\gamma_{min} \leq$ tol.

When starting the controller design, no information is available about γ_{opt} which depends of course on the design filters and the process. Because the final goal is to achieve $\gamma \uparrow 1$, it is recommended to start with $\gamma_o = 1$ and $\alpha = 2$ to reduce the number of iterations. This approach has the advantage that it is reasonably fast (7 to 15 iterations depending on the tolerance margin) and that independent of the start value γ_o a sub-optimal solution is found. The variable tolerance margin has been introduced to speed up the design (fewer iterations) and because of the fact that if this margin becomes too small the Riccati equations cannot be solved properly anymore. Using the method proposed in Bruinsma, N.A. (1990), the H_∞ norm of the closed-loop system can be used to check the solution γ_{opt} of the search procedure a postiori. Since the standard solution is only available for the continuous-time case, it should be mentioned here that the discrete-time case is solved via bilinear transformation. In Stoorvogel, A.A. *et. al.* (1993) and Iglesias, P.A. *et. al.* (1993), it is shown that designing a discrete-time controller via a bilinear transformation to the continuous-time domain might introduce an implicit and undesirable additional weighting function. A simple free stable contraction map is added to eliminate this additional weighting.

In general, the resulting H_∞-controllers are of high order because the order is equal to the order of the generalized plant (process & all design filters). To reduce the order of the controller, the following reduction techniques can be applied to the resulting controller :

1) Minimal state-space realization (reduces within a predefined tolerance margin).
2) Optimal Hankel reduction.
3) Schur reduction.
4) Relative Schur reduction.

For the reduction methods 2 to 4 an additional option can be selected to reduce the controller with variable order and fixed error bound or fixed order and variable error bound. A detailed description and more references for these reduction techniques can be found in Chiang, R.C. *et. al.* (1988).

2.4 Evaluation Controller Design

After calculating the H_∞ controller, the closed-loop system is derived (without shaping and weighting functions) in order to evaluate the controller design. For this purpose time as well as frequency responses can be calculated. Time simulations can be performed to check the closed-loop behaviour with respect to design objectives and constraints in the time domain like disturbance attenuation, reference tracking and/or input saturation. Frequency response analysis can be used to verify sensitivity and complimentary sensitivity functions. Whenever the design functions V and W are defined as diagonal blocks, which is recommended to keep the design as simple as possible, the closed-loop behaviour from every input to every output can be compared with the corresponding inverse weighting functions (scaled with the H_∞-norm γ). This can simplify the iterative controller design because the Bode plots indicate which function in which frequency range is the limiting factor and where and how the design can be improved.
If the controller design is not satisfactory, the menu driven structure of the toolbox ensures that the design filters can be changed fast and a new controller can be calculated easily in order to optimize the controller design. At every stage of the design procedure, the controller configuration as well as the actual results can be saved to ensure continuation if necessary. The toolbox is built up in such a way that the input required from the user is minimized and that correct data transfer between the various functions is guaranteed.

2.5 Installation and Requirements

All names of the .m-files in the toolbox start with "mhc" (App. B) and can be copied (e.g. copy mhc*.m) to the working directory of MATLAB. If the files are copied to a directory different than the workspace of MATLAB, this directory has to be added to the matlabpath. The routines in this toolbox make use of standard MATLAB functions and the following MATLAB toolboxes :

- Signal processing toolbox
- Control system toolbox
- Robust control toolbox

3

Menu Description

Before starting the H_∞ control design, the specific control problem, including design filters (Fig. 2.1) must be transformed into the standard configuration defined for this toolbox (Fig. 2.2) resulting in the required input information (Fig. 2.3). The Multivariable H_∞ Control design toolbox (MHC) is menu driven to ensure easy input of variables, controller calculation and analysis of the results. All menus of the toolbox will be described briefly and the controller design for a floating platform will be used as an example. Any of the menu options can be selected by typing the correct number and pressing ENTER. The previous menu will appear again by pressing just ENTER. Every menu is provided with a help screen (menu option 0) describing briefly the several menu options.

Startup : **MHC**

To start the controller design procedure, execute **MHC** from inside MATLAB. The main menu, which is depicted in Fig. 3.1 will appear on the screen.

```
  - - -  Continuous-time MIMO H-infinity Control Design  - - -

                         Main menu

             1) Structure initialization
             2) Input matrix functions P1, P2, V, or W
             3) Controller design
             4) System evaluation
             5) Options
             6) Disk functions

             0) Help

       Please enter menu option or press ENTER to Exit :
```

Fig. 3.1 : Main menu.

Fig. 3.2 depicts the help screen of the main menu.

```
                Help Main Menu H-infinity Control Design

This menu structured H-infinity control design package can be used to
ensure easy definition of the design configuration, input of variables,
controller calculation and analysis of the results by computing time
and frequency responses.

Structure Initialization : Defines the design configuration including
process and weighting filters as a standard problem. The structure is
fixed by defining the dimensions and two interconnection matrices.

Input Matrix Functions V, P and W : Definition of process and weighting
filters via SISO transfer functions or MIMO state-space matrices.

Controller Design : H-infinity control design parametrizing all
stabilizing controllers such that a specified closed-loop transfer
function has H-infinity norm less than a given scalar. This involves
the solution to two algebraic Riccati equations, each with the same
order as the system, and further gives feasible controllers also with
this order.

                        Press any key to continue
```

Fig. 3.2a : Help main menu.

```
                Help Main Menu H-infinity Control Design

System Evaluation : Analysis of closed-loop system by computing time
and frequency responses.

Options : Definition of general options for design package.

Disk Functions : Menu to load and save variables. WARNING all variables
are initialized in a standard form when starting up the design package.

                        Press any key to continue
```

Fig. 3.2b : Help main menu (cont.).

3.1 Options

Before starting the actual controller design, several options must be defined in the options menu. The default menu is shown in Fig. 3.3.

```
    - - -  Continuous-time MIMO H-infinity Control Design  - - -

                   Options

    1)  Mode                                        : Continuous
    2)  Tolerance margin for minimization procedure : 1e-010
    3)  Selected input signal                       :
    4)  Generating META files                       : No
    5)  Lower frequency bound                       : -2 rad
    6)  Upper frequency bound                       : 2 rad
    7)  Number of frequency points                  : 50
    8)  End of time interval                        : 1 sec

    0)  Help

      Select menu option or press ENTER to Exit :
```

Fig. 3.3 : Options menu.

Change for the floating platform design example the following options into :

1) *Mode :* **Discrete**

2)	*Tolerance margin :*	**1e-6**
5)	*Lower frequency bound :*	**-4** (10^{-4})
7)	*Frequency points :*	**100**
8)	*Sample time :*	**0.1**

The help screen provides a brief explanation of the several options.

```
                              Help Options Menu

Mode : The H-infinity control design can be done either in the
continuous time or discrete time. Because the algorithms are only
valid for the continuous time problem, the discrete time is solved by
transforming the discrete augmented plant via bilinear transformation
to the continuous time, calculating the controller and applying the
inverse transformation to obtain the discrete time controller.

Tolerance Margin for Minimization Procedure : To solve the control
design problem properly, it is required that the constructed augmented
plant has a minimal state-space realization. This might not be the case
because of pole-zero cancellations in the design filters. When no minimal
realization has been obtained, this tolerance margin is used to detect
and eliminate the unobservable/uncontrollable modes.

Selected Input Signal : The variable name of the input signal is defined
which is used for the time simulation of the closed-loop system.
Whenever the variable name does not exist in the workspace, a time
signal must be defined by the user.

                          Press any key to continue
```

Fig. 3.4a : Help options menu.

```
                              Help Options Menu

Generating META Files : By selecting this option, a filename will be
requested after every plot, to save the plot as a META file for later
processing using GPP.

Lower/Upper Frequency Bound and Number of Frequency Points : These
options define the frequency range for the magnitude plots.

Time : For the continuous time this defines the length of the time
simulation while for the discrete time the sample time is defined.

                          Press any key to continue
```

Fig. 3.4b : Help options menu (cont.).

3.2 Structure Initialization

The structure initialization menu is depicted in Fig. 3.5. The general structure as well as IM1 & IM2 can be changed.

```
      - - -  Discrete-time MIMO H-infinity Control Design  - - -

                        Structure initialization

                 1)  General structure initialization
                 2)  Change interconnection matrix IM1
                 3)  Change interconnection matrix IM2

                 0)  Help

      Please enter menu option or press ENTER to Exit :
```

Fig. 3.5 : Structure initialization menu.

The generalized plant is described by defining the 11 signal dimensions of the blocks within the basic structure (Fig. 3.6). For the floating platform the transformation of the control configuration into this structure is shown in Section 2.1. The first step in the design is the defenition of the dimensions.

```
                  - - -  Structure Initialization  - - -

            *********************************************************
            *  +-------------------------------------------------+  *
            *  |                                                 |  *
            *  |  +---+ n1  +---+               +---+  n2 +---+  |  *
            *  +->| I |--/->|   |               |   |--/->| P2|->+  *
            *     +---+     |   | n3  +---+  n4 |   |     +---+     *
            *               |   |--/->| P1|--/->|   |               *
       w    * n5  +---+  n6 | I |     +---+     | I | n7  +---+ n8  *    z
       -----*--/->| V |--/->| M |               | M |--/->| W |--/--*---->
            *     +---+     | 1 | n9  +---+     | 2 |     +---+     *
            *               |   |--/->| I |---->|   |               *
       u    *     +---+ n10 |   |     +---+     |   |n11  +---+     *    y
       -----*---->| I |--/->|   |               |   |--/->| I |-----*---->
            *     +---+     +---+               +---+     +---+     *
            *                 MIMO Augmented Plant                  *
            *********************************************************

Define the dimensions of the MIMO augmented plant n1 ... n11 as shown
above in MATLAB notation ( ' [ n1 n2 n3 n4 n5 n6 n7 n8 n9 n10 n11 ] ' ).

Dimensions :
```

Fig. 3.6 : General structure initialization.

Dimensions : **[0 0 2 1 2 2 2 2 3 1 2]**

The dimensions of the interconnection matrices are fixed now and can be defined row by row. After selecting a row number, this row can be defined as an array in matlab notation. Fig. 3.7 depicts the screen for IM1.

```
             - - -  Build interconnection matrix IM1  - - -

Actual matrix elements of IM1 :

     1        0        0
     0        0        1
     1        0        0
     0        1        0
     0        0        1

Enter row number ( 1 -- 4 ) or press ENTER to Exit :  3

Enter elements row 3 in MATLAB notation : [ 1 0 0 ]
```

Fig. 3.7 : Interconnection matrix IM1

Interconnection matrices : $\mathbf{IM1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\mathbf{IM2} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

Fig. 3.8 shows the help screen of the structure initialization menu.

```
                  Help Structure Initialization Menu

H-infinity standard problem :

     *********************************************************
     *  +-------------------------------------------------+  *
     *  |                                                 |  *
     *  |  +---+  n1 +---+               +---+  n2 +---+  |  *
     *  +->| I |--/->|   |               |   |--/->| P2|--+  *
     *     +---+     |   |  n3 +---+  n4 |   |     +---+     *
     *               |   |--/->| P1|--/->|   |               *
w    * n5 +---+  n6 | I |     +---+     | I |  n7 +---+  n8 *    z
-----*--/--| V |--/->| M |               | M |--/->| W |--/--*---->
     *     +---+     | 1 |  n9 +---+     | 2 |     +---+     *
     *               |   |--/->| I |---->|   |               *
u    *     +---+ n10 |   |               |   | n11 +---+     *    y
-----*-----| I |--/->|   |               |   |--/->| I |-----*---->
     *     +---+     +---+               +---+     +---+     *
     *                  MIMO Augmented Plant                 *
     *********************************************************

                     Press any key to continue
```

Fig. 3.8a : Help structure initialization menu.

```
                  Help Structure Initialization Menu

Before designing an H-infinity controller, the control setup has to be
defined together with the design constraints and objectives. (e.g.
tracking, disturbance rejection, input saturation etc.) This design
configuration must be transformed into the H-infinity standard
problem with an augmented plant that contains what usually is called
the plant in a control problem plus all weighting functions.
The augmented plant consists of four basic blocks : 1,2) P1&P2 -
Process models, 3) V - Shaping the disturbance vector w(t) and 4) W -
Weighting the error vector z(t). Further these four blocks are somehow
connected through the interconnection matrices IM1 & IM2 containing
only the elements +/- 1 and 0 which corresponds with adding, subtracting
or no connection.
Every control configuration can now be described in a simple way by
defining the dimensions n1 ... n11 together with the structure of the
configuration using the interconnection matrices.

                     Press any key to continue
```

Fig. 3.8b : Help structure initialization menu (cont.).

3.3 Input Matrix Functions

After initializing the structure, the blocks P1, P2, V and W must be defined. This menu is depicted in Fig. 3.9. The blocks can be defined either as SISO transfer functions per entry or as MIMO state-space matrices. An additional viewing option has been included to verify the magnitude plots.

```
      - - -  Discrete-time MIMO H-infinity Control Design  - - -

               Input Matrix Functions P1, P2, V or W

          1)   Enter P1 as SISO transfer functions
          2)   Enter P1 as MIMO state-space matrices
          3)   Show magnitude plots of P1

          4)   Enter P2 as SISO transfer functions
          5)   Enter P2 as MIMO state-space matrices
          6)   Show magnitude plots of P2

          7)   Enter V as SISO transfer functions
          8)   Enter V as MIMO state-space matrices
          9)   Show magnitude plots of V

         10)   Enter W as SISO transfer functions
         11)   Enter W as MIMO state-space matrices
         12)   Show magnitude plots of W

          0)   Help

    Please enter menu option or press ENTER to Exit :
```

Fig. 3.9 : Input matrix functions P1, P2, V or W

Fig. 3.10 shows the help screen for the input matrix functions menu.

```
                    Help Input Matrix Functions P1, P2, V or W

As described in the Structure Initialization menu, the augmented plant
of the standard problem consists of three basic blocks :

      1) P1 - The process model (part 1).
      2) P2 - The process model (part 2).
      3) V  - Shaping of the disturbance vector w(t).
      4) W  - Weighing of the error vector z(t).

These four blocks can be entered into the design package either as
SISO transfer functions or as MIMO state-space matrices. For the SISO
case the user must define the filters as numerator and denominator
polynomials for every entry of the matrix function. For the MIMO case
the A,B,C or D matrices must be defined. The number of inputs and
outputs of these blocks depends of course on the dimensions entered in
the Structure Initialization menu.

                          Press any key to continue
```

Fig. 3.10a : Help input matrix functions P1, P2, V or W.

```
                    Help Input Matrix Functions P1, P2, V or W

When entering the filters as SISO transfer functions, the corresponding
state-space representation is derived automatically and vice versa.
A consequence of this representation of the blocks P1, P2, V and W in
transfer function matrices and state-space matrices is that all the
process and design blocks must be proper.
In addition the magnitude of the designed filters can be plotted.

REMARK : To simplify the H-infinity control design it is recommended
to define the shaping and weighting blocks, V and W respectively, as
square functions (equal number of inputs and outputs) with elements
only on the diagonal.

                          Press any key to continue
```

Fig. 3.10b : Help input matrix functions P1, P2, V or W (cont.).

When defining a block as SISO transfer function, the correct element of the matrix must be selected first.

```
                  - - -  Discrete Transfer Matrix P1  - - -

Enter input number ( 1 -- 1 ) or press ENTER to Exit :

Enter output number ( 1 -- 1 ) :
```

Fig. 3.11 : Selecting element of transfer matrix P1.

Every transfer function is defined by its numerator and denominator. Both polynomials can be entered as arrays in MATLAB notation. The polynomials must be defined in powers of 'z' or 's' for the discrete or continuous time respectively

```
          - - -  Define Discrete Transfer Function of P1(11)  - - -

Please define the following polynomials in MATLAB notation.
Example :   z^3 + 2z^2 - 3z + 1   ===>   " [ 1 2 -3 1 ]"

Press ENTER if a polynomial should not be changed !!!!!

Old numerator : [ 0 ]

New numerator :

Old denominator : [ 1 ]

New denominator :
```

Fig. 3.12 : Defining SISO transfer function.

The menu to define the state-space matrices A, B, C & D is depicted in Fig. 3.13. These matrices can be entered exactly the same way as the interconnection matrices IM1 & IM2 (Fig. 3.7).

```
- - -  Define Discrete state-space matrices of P1  - - -

     1) Show / Change state matrix A
     2) Show / Change input matrix B
     3) Show / Change output matrix C
     4) Show / Change feed-through matrix D
     5) Change state dimension

     0) Help

Please enter menu option or press ENTER to Exit :
```

Fig. 3.13 : Defining MIMO state-space matrices.

The help screen to define state-space systems is shown in Fig. 3.14.

```
                Help Define State-Space Matrices Menu

The state-space matrices [A,B,C,D] can be defined/changed by selecting
one matrix and then entering the values row by row. The number of inputs
and outputs has been defined in the Structure Initialization menu. Only
the number of states can be defined/changed.

WARNING : When changing the number of states, all matrices are
          initialized to zero matrices and previously entered
          information will be lost !!!!!

                     Press any key to continue
```

Fig. 3.14 : Help define state-space matrices menu.

Before describing the actual H_∞ control design, the following block information for the floating platform should be entered using the input menus for transfer functions and state-space matrices (Fig. 3.11, 3.12 & 3.13). More detailed information about the modeling, identification and H_∞ filter design can be found in Bouwels, J.P.H.M (1991) and Damen, A.A.H. *et. al.* (1994).

Process :

$$A_{p1} = \begin{bmatrix} 1 & 0.1 & 0 \\ 0 & 1 & 0.1 \\ 0 & -3.794215 & 0.4044 \end{bmatrix} \qquad B_{p1} = \begin{bmatrix} -0.0017 & 0.0045 \\ 0.0311 & -0.0007 \\ -0.1213 & 0.1529 \end{bmatrix}$$

$$C_{p1} = [\,1 \;\; 0.1 \;\; 0\,] \qquad D_{p1} = [\,0 \;\; 0\,]$$

$$A_{p2} = B_{p2} = C_{p2} = [\,] \qquad D_{p2} = [\,0\,]$$

Design filters :

$$V_d = \frac{-0.3104\,(z^2 - 0.2z)}{z^2 - 1.9944z + 0.9950}$$

$$V_v = \frac{(z+0.4172)\,(z-0.5727-0.5725i)\,(z-0.5727+0.5725i)}{60\,(z-1)\,(z-0.7022-0.5392i)\,(z-0.7022+0.5392i)}$$

$$W_y = \frac{0.1\,(z-0.8-0.01i)\,(z-0.8+0.01i)}{(z-0.999-0.001i)\,(z-0.999+0.001i)}$$

$$W_u = \frac{40\,(z-0.995-0.0099i)\,(z-0.995+0.0099i)}{(z+0.9)^2}$$

The numerator and denominator polynomials can be defined in several ways in the transfer function entry of Fig. 3.12. For example :

Numerator $V_{d'}$: `-0.3104*[ 1 -0.2 0 ]`

Denominator V_v : `60*poly([1;0.7022+0.5392*i;0.7022-0.5392*i])`

Note that MATLAB commands can be used as well to define the transfer functions.

3.4 Controller Design

Once the complete generalized plant has been defined, the controller design becomes fairly simple.

```
       - - -  Discrete-time MIMO H-infinity Control Design  - - -

                         Controller design

                    1)  H-infinity controller options
                    2)  Calculate H-infinity controller
                    3)  Controller reduction
                    4)  Load original controller

                    0)  Help

          Please enter menu option or press ENTER to Exit :
```

Fig. 3.15 : Controller design.

The help screen for the controller design menu is depicted in Fig. 3.16.

```
                     Help Controller Design Menu

In this menu the actual controller design is performed. All stabilizing
controllers such that a specified closed-loop transfer function has
H-infinity norm less than a given scalar. This characterization involves
the solution to two algebraic Riccati equations, each with the same
order as the system, and further gives feasible controllers also with
this order.

H-infinity Controller Options : In this menu some controller relevant
settings are defined, like type of H-infinity solution, type of Riccati
equation solution and a tolerance margin indicating the accuracy of
the closed-loop H-infinity norm with respect to the optimal gamma. In
addition an option for balancing of the augmented plant can be selected.
This can improve the numerical stability of the controller design. Also
a reduction technique can be selected to reduce the order of the
H-infinity controller.

                      Press any key to continue
```

Fig. 3.16a : Help controller design.

```
                    Help Controller Design Menu

Calculate H-infinity controller : After checking the conditions for the
existence of a stabilizing controller satisfying an H-infinity norm of
the closed-loop system, like whether the system is stabilizable and
detectable, rank conditions to ensure that the controllers are proper,
and no zeros on the imaginary axis of some transfers, a start value
and step size for gamma must be defined. After deriving an interval
containing the optimal solution, the nearly optimal gamma will be derived
using bisection search until the required accuracy has been obtained.

Controller Reduction : Because the order of the controllers will be the
same as the order of the system, controller reduction is often required
to obtain lower order controllers.

Load Original Controller : The controller reduction step can result in
less accurate closed-loop performance, which makes it necessary to
use the original high order controllers.

                    Press any key to continue
```

Fig. 3.16b : Help controller design (Cont.).

The options described in Section 2.3 can be defined in the controller options menu shown in Fig. 3.17.

```
    - - -  Discrete-time MIMO H-infinity Control Design  - - -

                    Controller options

        1)  Type of H-infinity approach          :  GD
        2)  Type of Riccati solution approach    :  eigen
        3)  Tolerance margin optimizing gamma    :  0.01
        4)  Balancing augmented plant            :  No
        5)  Controller reduction method          :  Minreal

        0)  Help

    Please enter menu option or press ENTER to Exit :
```

Fig. 3.17 : Controller options menu

Change for the floating platform design example the following options into :

3) *Tolerance margin optimizing gamma :* **0.001**

4) *Balancing augmented plant :* **Yes**

The help screen for the controller options menu is depicted in Fig. 3.18.

```
                    Help Controller Options Menu

Type of H-infinity solution : Several routines are available which
solve the H-infinity control problem in different ways according to
Safonov/Limebeer/Chiang loop-shifting formulae, Glover/Doyle all-
solution formulae or Limebeer/Kasenally all-solution formulae

Type of Riccati Equation Approach : The calculation of the H-infinity
involves the solution to two algebraic Riccati equations. These two
equations can be solved either by eigenvalue or Schur decomposition.
The eigenvalue decomposition is faster but the Schur decomposition is
numerically robuster for badly conditioned design problems.

Tolerance Margin Optimizing Gamma : The H-infinity controller will be
designed in such a way that the H-infinity norm of the closed-loop
system will be less than gamma. Because the optimal gamma can only be
approximated, a tolerance margin must be defined indicating when the
iterative design procedure optimizing gamma can be stopped.

                    Press any key to continue
```

Fig. 3.18a : Help controller options.

```
                    Help Controller Options Menu

Balancing Augmented Plant : After building the augmented plant from
the several blocks, the overall system might be badly conditioned.
Balancing the augmented plant for badly conditioned problems can
improve the final solution.

Controller Reduction Method : The order of the feasible controllers
will be the same as the order of the augmented plant containing the
process model and all shaping/weighting filters. For more complex MIMO
design problems the order will increase rapidly and therefore controller
reduction methods are often required to obtain lower order controllers
(Minimal State-space realization, Optimal Hankel Reduction, Schur
Reduction and Relative Schur Reduction).

                    Press any key to continue
```

Fig. 3.18b : Help controller options (Cont.).

The following options are available to solve the H_∞ control design problem based on the "2-Riccati" equation approach.

```
   - - -  Discrete-time MIMO H-infinity Control Design  - - -

                     H-infinity type approach

   1)  SLC (Safonov/Limebeer/Chiang) loop-shifting formulae
   2)  GD (Glover/Doyle) all-solution formulae
   3)  LK (Limebeer/Kasenally) all-solution formulae

    Please enter menu option or press ENTER to Exit :
```

Fig. 3.19 : Type of H_∞ approach.

For controller reduction the options depicted in Fig. 3.20 are available.

```
   - - -  Discrete-time MIMO H-infinity Control Design  - - -

                  Controller Reduction Methods

              1)  Minimal state space realization
              2)  Optimal Hankel reduction
              3)  Schur reduction
              4)  Relative Schur reduction

          Please enter menu option or press ENTER to Exit :
```

Fig. 3.20 : Reduction methods.

After selecting the control design options, the actual controller calculation is started (option 2 of he controller design menu). In this case some information will scroll over the screen describing the minimal realization of the generalized plant, bilinear transformation, balancing and ...

```
                 - - -  Prepare Augmented Plant  - - -

Size state matrix : 12

Observability index : 9

Controllability index : 12

... Working on state-space minimization ... Please wait ...

3 states removed.

... Working on inverse bilinear transformation ... Please wait ...

... Working on balancing augmented plant ... Please wait ...
```

Fig. 3.21 : Preparing generalized plant.

checking the conditions to solve the H_∞ control problem :

- G_{22} : (A,B_2) stabilizable and (A,C_2) detectable .
- D_{12} full column rank.
- D_{21} full row rank.
- G_{12} : (A,B_2,C_1,D_{12}) full column rank.
- G_{21} : (A,B_1,C_2,D_{21}) full row rank.

Only if a condition is violated an error message will appear.

```
              - - -  Check solution conditions  - - -

----------------------------------------------------------------
Transfer G22 from the control input vector u(t) to the
observation vector y(t) is stabilizable and detectable.
----------------------------------------------------------------

----------------------------------------------------------------
Transfer G12 from the control input vector u(t) to the error
vector z(t) has full column rank at infinity.
----------------------------------------------------------------

----------------------------------------------------------------
Transfer G21 from the disturbance vector w(t) to the observation
vector y(t) has full row rank at infinity.
----------------------------------------------------------------

----------------------------------------------------------------
No transmission zeros have been detected in the transfer G12
from the control input vector u(t) to the error vector z(t).
----------------------------------------------------------------

----------------------------------------------------------------
No transmission zeros have been detected in the transfer G21
from the disturbance vector w(t) to the observation vector y(t).
----------------------------------------------------------------
```

Fig. 3.22 : Check solution conditions.

For γ minimization a new start value and step size must be defined :

Start Value : `1`
Step size : `1.1`

```
          - - -  Calculate H-infinity Controller  - - -

Old value of gamma : 1

Please enter new start value of gamma or press ENTER :

Old step size : 2

Please enter new step size or press ENTER :
```

Fig. 3.23 : Initializing γ.

The tolerance margin defined in the controller options determines the number of γ iterations (Fig. 3.24). For an optimal design, γ should be slightly smaller than 1. The H_∞ norm of the closed-loop system can be used to verify γ and the difference should be of the same order as the tolerance margin.

```
        - - -  End of iteration  - - -

  # Iter        Gamma              Comment

    1        1.0000e+000        Solution OK
    2        9.0909e-001        No stabilizing controller !!!
    3        9.5455e-001        No stabilizing controller !!!
    4        9.7727e-001        Solution OK
    5        9.6591e-001        Solution OK
    6        9.6023e-001        No stabilizing controller !!!
    7        9.6307e-001        No stabilizing controller !!!
    8        9.6449e-001        Solution OK
    9        9.6378e-001        No stabilizing controller !!!
   10        9.6449e-001        Solution OK

  Optimal gamma : 9.6449e-001

  Check H_infinity norm : 9.6458e-001

  Gamma iteration has been terminated succesfully !!!

                Press any key to continue
```

Fig. 3.24 : Gamma iteration.

Whenever the Riccati equations are not solved properly (large residuals or other numerical problems), the closed-loop system might not be stable although the γ iteration has been terminated successfully. Redefining the design filters and reducing the constraints and objectives can often help to overcome this problem. Because in general H_∞ controllers are of high order (order generalized plant), some controller reduction options (option 3 of the controller design menu) have been included to realize lower order controllers. If this reduction results in an unstable closed-loop system, the original high-order controller can be loaded again without new calculations (option 4 of the controller design menu).

3.5 System Evaluation

Fig. 3.25 shows the menu to analyze the closed-loop behavior by computing time and frequency responses.

```
- - -  Discrete-time MIMO H-infinity Control Design  - - -

                 System Evaluation

          1)  Show bode plots controller
          2)  Plot closed-loop transfer function
          3)  Show time simulation

          0)  Help

  Please enter menu option or press ENTER to Exit :
```

Fig. 3.25 : System evaluation menu.

Help screen of the system evaluation menu.

```
                Help System Evaluation Menu

In this menu the closed-loop system behaviour can be evaluated by
computing time and frequency responses

Show Bode Plots Controller : Shows the magnitude plot of the controller
from a selected observation signal y(t) to a selected input signal u(t)
for the frequency range defined in the Option menu.

Plot Closed-loop Transfer Function : The magnitude plot of a selected
closed-loop transfer function from the disturbance vector w(t) to the
error vector y(t) without shaping/weighting filters is shown for the
specified frequency range. Whenever the shaping and weighting blocks
of the augmented plant are defined as diagonal functions, the magnitude
plot of the inverse shaping/weighting filter scaled with the H-infinity
closed-loop norm gamma is plotted as well. This inverse function defines
an upper bound of the corresponding closed-loop transfer function over
the whole frequency range. This can be used to determine which transfer
function is the limiting factor in the controller design and in which
frequency range.

                 Press any key to continue
```

Fig. 3.26a : Help system evaluation.

```
                Help System Evaluation Menu

Show Time Simulation : A closed-loop time simulation is performed using
the input signal defined in the Option menu. The simulated outputs can
be plotted by selecting the required output.

                 Press any key to continue
```

Fig. 3.26b : Help system evaluation (Cont.).

The single transfers in the closed-loop system are shown by selecting the corresponding input and output.

```
- - -  Show Discrete Closed-loop Bode plots  - - -
Please enter number of input ( 1 -- 2 ) or press Enter to Exit :
Please enter number of output ( 1 -- 2 ) :
... Working ... Please wait ...
```

Fig. 3.27 : Plot closed-loop transfer.

Only the magnitude plots for the closed-loop evaluation will be shown here.

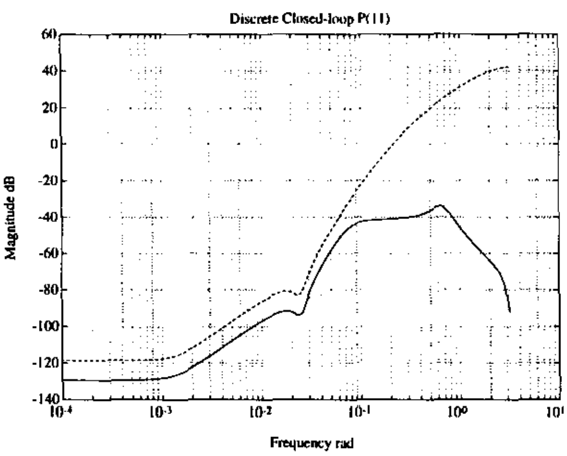

Fig. 3.28 : Closed-loop transfer 1 → 1.

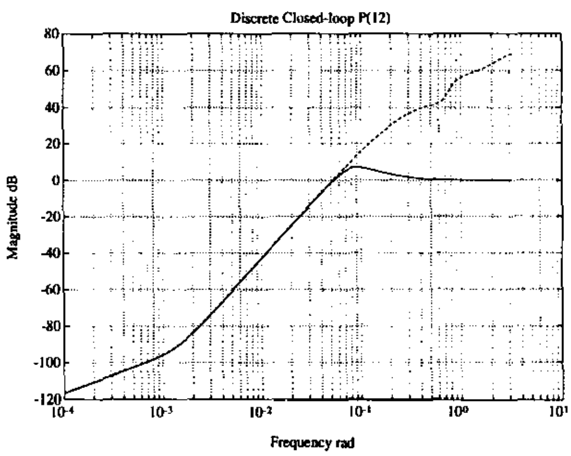

Fig. 3.29 : Closed-loop transfer 2 → 1.

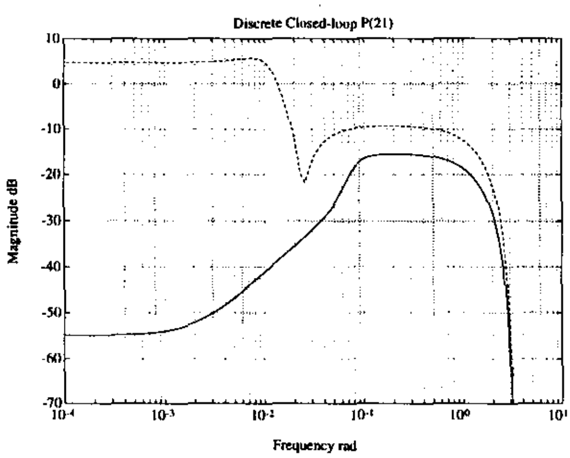

Fig. 3.30 : Closed-loop transfer 1 → 2.

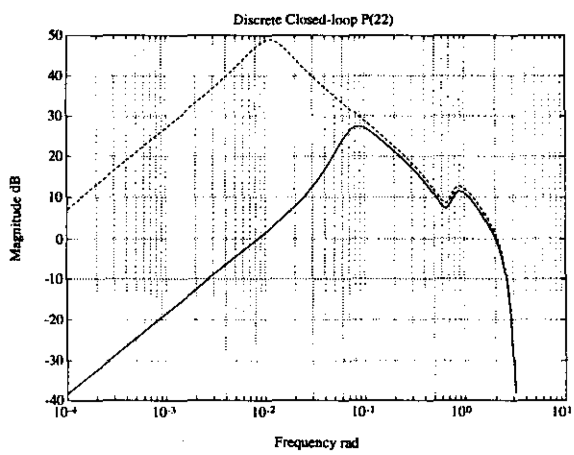

Fig. 3.31 : Closed-loop transfer 2 → 2.

Before we can evaluate the time simulations, an input signal must be defined in the options menu (Fig. 3.3). The variable name of the input signal matrix can be entered. If the variable name exists in the workspace (for example generated before starting up MHC) and the number of columns correspond to the defined input dimensions, the time simulations can be performed. However, if the variable name does not exist in the workspace, the input matrix must be defined first. Also, the variable name of the output signal matrix must be defined. For the floating platform example a disturbance signal can be generated corresponding with 3 rotations of the crane (rotation frequency 0.04 Hz) and a load of 1 kg. After 10 seconds (100 samples, sample time = 0.1 sec) the crane starts rotating and zeros have been added to create a time simulation of 100 seconds.

Name input : `dis`
Input signal : `[[zeros(100,1);-9.81*sin(2*pi*0.04*(0:0.1:75))';`
`zeros(149,1)] zeros(1000,1) ]`
Name output : `dis_out`

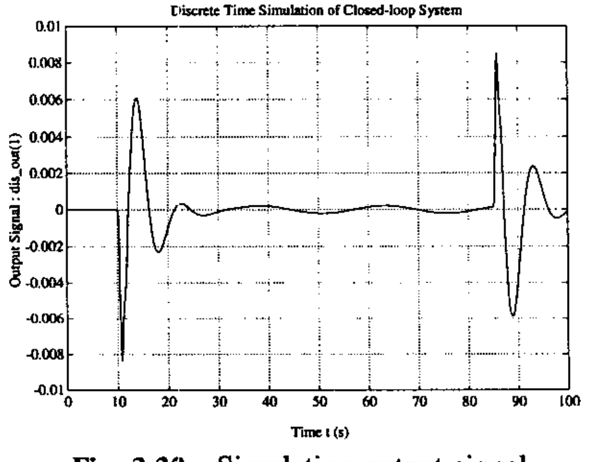

Fig. 3.32 : Simulation output signal.

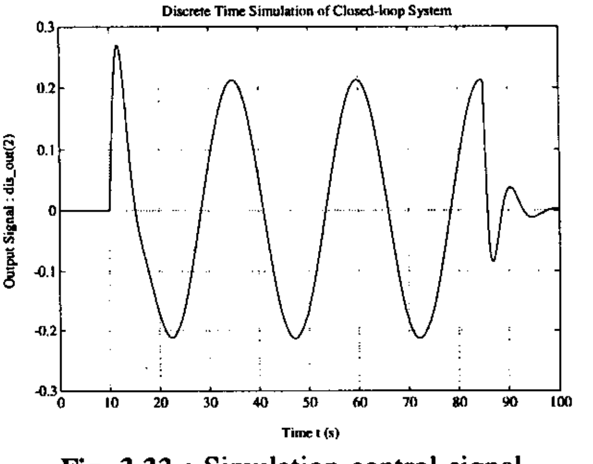

Fig. 3.33 : Simulation control signal.

The corresponding time simulations are depicted in Fig. 3.32 & 3.33. The relative bad disturbance rejection in Fig. 3.32 after 10 and 85 sec. are caused by starting and stopping the rotation of the crane. Note that compared to the designs described in Bouwels, J.P.H.M. (1991) and Damen, A.A.H. *et. al.* (1994) the scaling of the filters has been adjusted such that H_∞-norm γ becomes smaller than 1.

3.6 Disk Functions

All information can be saved and loaded using the disk options menu (Fig. 3.34). Information should be stored regularly during the design because MATLAB errors due to numerical problems can terminate **MHC**.

```
        - - -  Discrete-time MIMO H-infinity Control Design  - - -

                            Disk Options

                     1)  Save all variables
                     2)  Load all variables
                     3)  Contents disk
                     4)  Change directory

                     0)  Help

       Please enter menu option or press ENTER to Exit :
```

Fig. 3.34 : Disk options menu.

The help screen for the disk options menu is depicted in Fig. 3.35.

```
                        Help Disk Options Menu

This menu should be used to SAVE and LOAD variables correctly. To avoid
extensively checking of variables for existence, dimensions etc. in all
functions, this menu has been included. When starting up the design
package, all variables are initialized in a standard way. Therefore
variables can be entered either directly through the menus or by loading
them from the workspace. The available options are :

            Save All Variables.

            Load All Variables.

            Contents Disk.

            Change Directory.

                        Press any key to continue
```

Fig. 3.35 : Help disk options.

The most important features of the H_∞ control design toolbox have been described together with the menus which will appear on the computer screen. The exact screen input has not been described because the control design is rather straightforward and the required user input is fairly simple. The example of the floating platform should be sufficient to guide the user through all menus of the design procedure. It is not the intention to show with this example a complete H_∞ control design procedure for all shaping and weighting filters. This is described in more detail in Bouwels, J.P.H.M. (1991).

4

Conclusions

Any control configuration can be rewritten in the presented basic structure which is automatically transformed into a standard H_∞ control problem. The menu driven structure of the toolbox makes the necessarily iterative design procedure fast, due to easy input of variables and simple analysis of the results by calculating time and frequency responses.
The H_∞ control design of a laboratory process has been used to show the user how to define the basic control structure. An extensive description of all menus and help-facilities should guide the user through the design and explain all options.

A

Menu Overview

The menus of the multivariable H_∞ control design (MHC) toolbox are presented in one scheme to provide an overview of the most important functions. This overview can be used as quick reference guide by the user during the H_∞ control design.

Menus Multivariable H_∞ Control Design

```
- - - Discrete-time MIMO H-infinity Control Design - - -

          Structure initialization

      1)  General structure initialization
      2)  Change interconnection matrix IM1
      3)  Change interconnection matrix IM2

      0)  Help

  Please enter menu option or press ENTER to Exit :
```

```
- - - Discrete-time MIMO H-infinity Control Design - - -

          System Evaluation

      1)  Show bode plots controller
      2)  Plot closed-loop transfer function
      3)  Show time simulation

      0)  Help

  Please enter menu option or press ENTER to Exit :
```

1 4

```
- - - Discrete-time MIMO H-infinity Control Design - - -

      Input Matrix Functions P1, P2, V or W

    1)  Enter P1 as SISO transfer functions
    2)  Enter P1 as MIMO state-space matrices
    3)  Show magnitude plots of P1

    4)  Enter P2 as SISO transfer functions
    5)  Enter P2 as MIMO state-space matrices
    6)  Show magnitude plots of P2

    7)  Enter V as SISO transfer functions
    8)  Enter V as MIMO state-space matrices
    9)  Show magnitude plots of V

   10)  Enter W as SISO transfer functions
   11)  Enter W as MIMO state-space matrices
   12)  Show magnitude plots of W

    0)  Help

  Please enter menu option or press ENTER to Exit :
```

```
- - - Discrete-time MIMO H-infinity Control Design - - -

                    Main menu

      1)  Structure initialization
      2)  Input matrix functions P1, P2, V, or W
      3)  Controller design
      4)  System evaluation
      5)  Options
      6)  Disk functions

      0)  Help

  Please enter menu option or press ENTER to Exit :
```

```
- - - Discrete-time MIMO H-infinity Control Design - - -

              Options

1)  Mode                                         : Discrete
2)  Tolerance margin for minimization procedure  : 1e-06
3)  Selected input signal                        :
4)  Generating META files                        : No
5)  Lower frequency bound                        : -4 rad
6)  Upper frequency bound                        : 3.14 rad
7)  Number of frequency points                   : 100
8)  End of time interval                         : 0.1 sec

0)  Help

    Select menu option or press ENTER to Exit :
```

6 3

```
- - - Discrete-time MIMO H-infinity Control Design - - -

             Disk Options

         1)  Save all variables
         2)  Load all variables
         3)  Contents disk
         4)  Change directory

         0)  Help

  Please enter menu option or press ENTER to Exit :
```

```
- - - Discrete-time MIMO H-infinity Control Design - - -

            Controller design

       1)  H-infinity controller options
       2)  Calculate H-infinity controller
       3)  Controller reduction
       4)  Load original controller

       0)  Help

  Please enter menu option or press ENTER to Exit :
```

```
- - - Discrete-time MIMO H-infinity Control Design - - -

              Controller options

  1)  Type of H-infinity approach         :  GD
  2)  Type of Riccati solution approach   :  eigen
  3)  Tolerance margin optimizing gamma   :  0.001
  4)  Balancing augmented plant           :  Yes
  5)  Controller reduction method         :  Minreal

  0)  Help

  Please enter menu option or press ENTER to Exit :
```

B

Program Structure

The global program structure including all mhc-functions is depicted in Fig. B.1. Note that the standard MATLAB functions (including the toolboxes) which are used in the multivariable H_∞ control design toolbox are not mentioned in the overview. The required toolboxes are described in Section 2.5.

```
mhc.m ─┬─ mhc_hm.m
       ├─ mhc_stm.m ─┬─ mhc_h1.m
       │             ├─ mhc_stin.m
       │             └─ mhc_im.m
       ├─ mhc_imf.m ─┬─ mhc_h2.m
       │             ├─ mhc_rtf.m ─┬─ mhc_dtf.m
       │             │             └─ mhc_tfss.m ─┬─ mhc_c2o.m
       │             │                            └─ mhc_mss.m
       │             ├─ mhc_ssr.m ─┬─ mhc_h21.m
       │             │             ├─ mhc_cm.m
       │             │             └─ mhc_pzc.m
       │             └─ mhc_sbp.m ── mhc_meta.m
       ├─ mhc_hcm.m ─┬─ mhc_h3.m
       │             ├─ mhc_hco.m ── mhc_h31.m
       │             ├─ mhc_hcb.m ─┬─ mhc_map.m ── mhc_mss.m
       │             │             ├─ mhc_rbal.m
       │             │             ├─ mhc_csc.m
       │             │             ├─ mhc_slrc.m ─┬─ mhc_are.m
       │             │             │              └─ mhc_d2ss.m
       │             │             ├─ mhc_kgjd.m ─┬─ mhc_are.m
       │             │             │              └─ mhc_d2ss.m
       │             │             ├─ mhc_djnl.m ─┬─ mhc_are.m
       │             │             │              └─ mhc_d2ss.m
       │             │             └─ mhc_hin.m
       │             ├─ mhc_crm.m ── mhc_mss.m
       │             └─ mhc_ccl.m
       ├─ mhc_sem.m ─┬─ mhc_h4.m
       │             ├─ mhc_sbp.m ── mhc_meta.m
       │             ├─ mhc_pcl.m ── mhc_meta.m
       │             └─ mhc_sim.m ── mhc_meta.m
       ├─ mhc_opt.m ── mhc_h5.m
       └─ mhc_disk.m ── mhc_h6.m
```

Fig. B.1 : Global program structure.

C

Function Description

A brief description of all functions (in alphabetical order) presented in the overview of Appendix A will be given.

Function name	*Description*
mhc.m	Initialization script file showing the main menu.
mhc_are.m	Computes the algebraic Riccati equation for the H_∞ control problem
mhc_c2o.m	State-space transformation from controller canonical form to observer canonical form.
mhc_ccl.m	Calculates the closed-loop system consisting of the process blocks P_1 & P_2 and the H_∞ controller.
mhc_cm.m	Function to change rows in a matrix.

Function name	*Description*
mhc_crm.m	Computing controller reduction according to several methods.
mhc_csc.m	Checks the conditions to solve the H_∞ control problem.
mhc_disk.m	Disk options menu.
mhc_djnl.m	H_∞ all solution formulae derived by Limebeer and Kasenally.
mhc_dtf.m	Define transfer function.
mhc_h1.m	Help screen for structure initialization menu.
mhc_h2.m	Help screen for input matrix functions menu.
mhc_h21.m	Help screen for define state-space matrices menu.
mhc_h3.m	Help screen for controller design menu.
mhc_h31.m	Help screen for controller options menu.
mhc_h4.m	Help screen for system evaluation menu.
mhc_h5.m	Help screen for options menu.
mhc_h6.m	Help screen for disk options menu.
mhc_hcb.m	H_∞ controller basic function which prepares the variables for the general MIMO configuration and minimizes γ to calculate the optimal controller.
mhc_hcm.m	Script file to generate H_∞ control menu.
mhc_hco.m	Shows H_∞ controller options menu.
mhc_hin.m	Routine to calculate H_∞-norm of a state-space system which is the maximum over all frequencies of the maximum singular value.

Function name	*Description*
mhc_hm.m	Help screen for main menu.
mhc_im.m	Function to build interconnection matrices.
mhc_imf.m	Script file to generate input matrix functions menu.
mhc_kgjd.m	H_∞ all solution formulae derived by Glover and Doyle.
mhc_map.m	Function to construct minimal realization of the augmented plant for the basic structure.
mhc_meta.m	This function file generates a meta file using a filename defined by the user and writes the current graph to for late processing.
mhc_mss.m	Routine to calculate minimal state-space realization.
mhc_opt.m	Script file to generate options menu.
mhc_pcl.m	Function to plot closed-loop transfer function. If the design filters V and W are diagonal matrices the inverse design function is also plotted.
mhc_pzc.m	Function to check pole-zero cancellations in SISO transfer functions.
mhc_rbal.m	Returns the LQG or Riccati balanced state-space representation of stable and unstable systems.
mhc_rtf.m	Function to replace an element in a transfer function matrix.
mhc_sbp.m	Routine to show Bode plot of a SISO transfer function.
mhc_sem.m	Script file to generate system evaluation menu.
mhc_sim.m	Function to calculate and show time simulation.
mhc_slrc.m	H_∞ loop-shifting formulae derived by Safonov, Limebeer and Chiang.

Function name	*Description*
mhc_ssr.m	Routine to show and define state-space representation of a system.
mhc_stin.m	Structure initialization function for H_∞ control design.
mhc_stm.m	Script file to generate structure menu to define augmented plant.
mhc_tfss.m	MIMO transfer function matrix to state-space conversion.

D

List of Variables

In this appendix a list of variables in alphabetical order with a short description is given which are used as input/output arguments of the MHC functions described in Appendix C.

Variables	*Description*
Ac, Bc, Cc, Dc	State-space matrices of controller in continuous time domain.
Acl, Bcl, Ccl, Dcl	State-space matrices of closed-loop system without design functions.
Acon, Bcon, Ccon, Dcon	State-space matrices of final controller (discrete/continuous time, high/low order depending on design options).
Acor, Bcor, Ccor, Dcor	State-space matrices of original controller in continuous time domain (backup of Ac, Bc, Cc, Dc if controller reduction fails).
Ap1, Bp1, Cp1, Dp1	State-space representation of process block P1.

Variables	*Description*
Ap2, Bp2, Cp2, Dp2	State-space representation of process block P2.
Av, Bv, Cv, Dv	State-space representation of design block V.
Aw, Bw, Cw, Dw	State-space representation of design block W.
IM1, IM2	Interconnection matrices for basic MHC structure
alpha	Step size
dim	Dimension array for basic MHC structure.
flag	Information array about current status : 1) Mode; 1 = Continuous, 2 = Discrete 2) Configuration structure ; 1 = Known, 0 = Unknown 3) Process block P1 4) Process block P2 ; 0 = Unknown, 1 = Transfer function 5) Shaping block V ; 2 = State-space matrices 6) Weighing block W 7) Generating META files ; 1 = Yes, 0 = No 8) Valid controller design ; 1 = Yes, 0 = No 9) Valid controller reduction ; 1 = Yes, 0 = No 10) H_∞ type approach : 1 = Safonov/Limebeer/Chiang loop-shifting formulae 2 = Glover/Doyle all-solution formulae 3 = Limebeer/Kasenally all-solution formulae 11) Type of Riccati solution approach ; 1 = Eigen, 2 = Schur 12) Balancing augmented plant ; 1 = Yes, 0 = No 13) Controller reduction method : 1 = Minimal realization 2 = Optimal Hankel method, 3 = Schur reduction method 4 = Relative Schur reduction 14) Type of controller reduction for method 2, 3 & 4 : 1 = Variable order & Fixed error bound 2 = Fixed order & Variable error bound

Variables	*Description*
freq	Array defining frequency information : 1) Lower bound 2) Upper bound 3) Number of frequency points
gamtol	Tolerance margin for γ minimization
num_P1, den_P1	Numerator/denominator transfer matrix of process block P1
num_P2, den_P2	Numerator/denominator transfer matrix of process block P2
num_V, den_V	Numerator/denominator transfer matrix of design block V
num_W, den_W	Numerator/denominator transfer matrix of design block W
time	Time information : End of time interval (continuous mode) or Sample time (discrete mode)
tol	Tolerance margin for minimal state-space realization

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