

Online Technology Guide for *Elementary Linear Algebra, 6e* Larson/Falvo

Computer Software Programs and Graphing Utilities

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Computer Software Programs and Graphing Utilities

To help you become acquainted with the rudiments of using a computer software program or graphing utility, the following pages contain certain introductory instructions for the following popular systems: MATLAB, Maple, *Mathematica*, and Derive, as well as information about various graphing utility models.

Introduction to MATLAB

The purpose of this introduction is to illustrate some basic MATLAB commands for linear algebra. It is suggested that you complete the following series of simple examples while actually working at the computer terminal. Once you have signed on to MATLAB, you should type in your commands to the right of the prompt `>>`.

1. Enter the matrix

$$a = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

by typing `a = [1 2 3; 4 5 6; 7 8 0]` and hitting the **return** or **enter** key. Be sure to separate the entries in each row by spaces and to terminate the rows with semicolons. MATLAB should return the following matrix.

$$a = \begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{matrix}$$

2. Perform the following elementary commands on the matrix a . You should obtain the indicated results.

- (a) `inv(a)`

$$\text{ans} = \begin{matrix} -1.7778 & 0.8889 & -0.1111 \\ 1.5556 & -0.7778 & 0.2222 \\ -0.1111 & 0.2222 & -0.1111 \end{matrix}$$

- (b) `a'`

$$\text{ans} = \begin{matrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 0 \end{matrix}$$

- (c) `a*inv(a)`

$$\text{ans} = \begin{matrix} 1.0000 & 0 & -0.0000 \\ -0.0000 & 1.0000 & 0 \\ 0.0000 & -0.0000 & 1.0000 \end{matrix}$$

(d) **2*a**

```

      2   4   6
ans = 8  10  12
      14  16   0

```

(e) **rref(a)**

```

      1   0   0
ans = 0   1   0
      0   0   1

```

(f) **det(a)**

```
ans = 27
```

(g) **rank(a)**

```
ans = 3
```

(h) **diag(a)**

```

      1
ans = 5
      0

```

3. Enter the matrix $\mathbf{b} = [7; 16; 7]$. You can solve the system of linear equations $\mathbf{a}\mathbf{x} = \mathbf{b}$ in different ways:

(a) **a\b**

```

      1
ans = 0
      2

```

(b) **c = [a b]** and then **rref(c)**

```

      1   2   3   7
c = 4   5   6  16
      7   8   0   7

>> rref(c)

      1   0   0   1
ans = 0   1   0   0
      0   0   1   2

```

4. Enter the matrices $\mathbf{A} = [1\ 2; 3\ 4]$ and $\mathbf{B} = [2\ 0; -3\ 5]$ and perform the following matrix operations. Notice that MATLAB distinguishes between upper- and lowercase letters; using the uppercase letter B to label the matrix above is not the same as using the lowercase letter b.

(a) **A + B**

$$\text{ans} = \begin{bmatrix} 3 & 2 \\ 0 & 9 \end{bmatrix}$$

(b) **A - B**

$$\text{ans} = \begin{bmatrix} -1 & 2 \\ 6 & -1 \end{bmatrix}$$

(c) **A * B**

$$\text{ans} = \begin{bmatrix} -4 & 10 \\ -6 & 20 \end{bmatrix}$$

5. What do the following commands do?

- (a) **zeros(3, 5)**
- (b) **eye(5)**
- (c) **rand(6)**
- (d) **hilb(6)**

6. Describe the effects of the following commands on the matrix **a = [1 2 3; 4 5 6; 7 8 0]**.

- (a) **a(2, 3)**
- (b) **a(2, :)**
- (c) **a(:, 3)**

7. You can obtain more information about MATLAB by typing **help**.

8. You can exit MATLAB by typing **exit**.

Introduction to Maple

The purpose of this introduction is to illustrate some basic Maple commands for linear algebra. It is suggested that you complete the following series of simple examples while actually working at the computer terminal. Once you have signed on to Maple, you should type in your commands to the right of the prompt **>**.

Begin by loading the linear algebra package that comes with Maple by typing **with(linalg)**; and hitting the **return** or **enter** key. Notice that all commands in Maple terminate with a semicolon **;**.

1. Enter the matrix

$$a = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

by typing **a = matrix ([[1, 2, 3], [4, 5, 6], [7, 8, 0]])**; and hitting the **return** or **enter** key. Be sure to separate the entries in each row by commas and place each row of the matrix within brackets. Maple should return the following matrix.

$$a := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

2. Perform the following elementary commands on the matrix a . You should obtain the indicated results.

(a) **inverse(a);**

$$\begin{bmatrix} -\frac{16}{9} & \frac{8}{9} & -\frac{1}{9} \\ \frac{14}{9} & -\frac{7}{9} & \frac{2}{9} \\ -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \end{bmatrix}$$

(b) **transpose(a);**

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 0 \end{bmatrix}$$

(c) **multiply(a, inverse(a));**

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(d) **scalarmul(a, 2);**

$$\begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 0 \end{bmatrix}$$

(e) **rref(a);**

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(f) **det(a);**

27

(g) **rank(a);**

3

(h) **diag(a, inverse(a));**

$$\begin{bmatrix} 1 & 2 & 3 & 0 & 0 & 0 \\ 4 & 5 & 6 & 0 & 0 & 0 \\ 7 & 8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{16}{9} & \frac{8}{9} & -\frac{1}{9} \\ 0 & 0 & 0 & \frac{14}{9} & -\frac{7}{9} & \frac{2}{9} \\ 0 & 0 & 0 & -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \end{bmatrix}$$

3. Enter the column matrix $\mathbf{b} := \text{transpose}(\text{matrix}([[7, 16, 7]]))$; You can solve the system of linear equations $\mathbf{ax} = \mathbf{b}$ in different ways:

(a) $\text{linsolve}(\mathbf{a}, \mathbf{b})$;

$$\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

(b) $\mathbf{c} := \text{augment}(\mathbf{a}, \mathbf{b})$; and then $\text{rref}(\mathbf{c})$;

$$\mathbf{c} := \begin{bmatrix} 1 & 2 & 3 & 7 \\ 4 & 5 & 6 & 16 \\ 7 & 8 & 0 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

4. Enter the matrices $\mathbf{A} := \text{matrix}([[1, 2], [3, 4]])$; and $\mathbf{B} := \text{matrix}([[2, 0], [-3, 5]])$; and perform the following matrix operations. Notice that Maple distinguishes between upper- and lowercase letters; using the uppercase letter B to label the matrix above is not the same as using the lowercase letter b.

(a) $\text{matadd}(\mathbf{A}, \mathbf{B})$;

$$\begin{bmatrix} 3 & 2 \\ 0 & 9 \end{bmatrix}$$

(b) $\text{matadd}(\mathbf{A}, \mathbf{B}, 1, -1)$;

$$\begin{bmatrix} -1 & 2 \\ 6 & -1 \end{bmatrix}$$

(c) $\text{multiply}(\mathbf{A}, \mathbf{B})$;

$$\begin{bmatrix} -4 & 10 \\ -6 & 20 \end{bmatrix}$$

5. What do the following commands do?

(a) $\text{matrix}(3, 5, 0)$;

(b) $\text{band}([1], 5)$;

(c) $\text{randmatrix}(4, 6)$;

(d) $\text{hilbert}(6)$;

6. Describe the effects of the following commands on the matrix $\mathbf{a} := \text{matrix}([[1, 2, 3], [4, 5, 6], [7, 8, 0]])$;

(a) $\text{submatrix}(\mathbf{a}, 2..2, 3..3)$;

(b) $\text{submatrix}(\mathbf{a}, 2..2, 1..3)$;

(c) $\text{submatrix}(\mathbf{a}, 1..3, 3..3)$;

7. You can obtain more information about a particular topic, such as eigenvalues, by typing **?eigenvalues**.

8. You can exit Maple by choosing **Exit** from the file menu.

Introduction to Mathematica

The purpose of this introduction is to illustrate some basic *Mathematica* commands for linear algebra. It is suggested that you complete the following series of simple examples while actually working at the computer terminal. Once you have signed on to *Mathematica*, you should type in your commands to the right of the prompt.

1. Enter the matrix

$$a = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

by typing `a = {{1, 2, 3}, {4, 5, 6}, {7, 8, 0}}` and evaluating the expression (cell). Be sure to separate the entries in each row by commas and place each row of the matrix within braces. *Mathematica* should return the following matrix. You can obtain a more natural matrix display by typing `%//MatrixForm`.

`Out[1] =`

`{{1, 2, 3}, {4, 5, 6}, {7, 8, 0}}`

`In[2] =`

`%//MatrixForm`

`Out[2]//MatrixForm =`

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix}$$

2. Perform the following elementary commands on the matrix *a*. You should obtain the indicated results.

- (a) `Inverse[a] //MatrixForm`

`Out[3]//MatrixForm =`

$$\begin{pmatrix} -\frac{16}{9} & \frac{8}{9} & -\frac{1}{9} \\ \frac{14}{9} & -\frac{7}{9} & \frac{2}{9} \\ -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \end{pmatrix}$$

- (b) `Transpose[a]`

`Out[4] =`

`{{1, 4, 7}, {2, 5, 8}, {3, 6, 0}}`

- (c) `a.Inverse[a]`

`Out[5] =`

`{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}`

- (d) `2*a`

`Out[6] =`

`{{2, 4, 6}, {8, 10, 12}, {14, 16, 0}}`

(e) **RowReduce[a]**

$$\begin{aligned} \text{Out}[7] = \\ \{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\} \end{aligned}$$

(f) **Det[a]**

$$\begin{aligned} \text{Out}[8] = \\ 27 \end{aligned}$$

3. Enter the matrix $\mathbf{b} = \{7, 16, 7\}$. You can solve the system of linear equations $\mathbf{ax} = \mathbf{b}$ in different ways:

(a) **LinearSolve[a, b]**

$$\begin{aligned} \text{Out}[10] = \\ \{1, 0, 2\} \end{aligned}$$

(b) $\mathbf{c} = \text{Table}[\text{Append}[\mathbf{a}[[i]], \mathbf{b}[[i]]], \{i, 1, 3\}]$ and then **RowReduce[c]**

$$\begin{aligned} \text{Out}[11] = \\ \{\{1, 2, 3, 7\}, \{4, 5, 6, 16\}, \{7, 8, 0, 7\}\} \end{aligned}$$

$$\begin{aligned} \text{In}[12] = \\ \text{RowReduce}[\mathbf{c}]/\text{MatrixForm} \end{aligned}$$

$$\text{Out}[12]/\text{MatrixForm} =$$

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

4. Enter the matrices $\mathbf{A} = \{\{1, 2\}, \{3, 4\}\}$ and $\mathbf{B} = \{\{2, 0\}, \{-3, 5\}\}$ and perform the following matrix operations. Notice that *Mathematica* distinguishes between upper- and lowercase letters; using the uppercase letter B to label the matrix above is not the same as using the lowercase letter b.

(a) **A + B**

$$\begin{aligned} \text{Out}[15] = \\ \{\{3, 2\}, \{0, 9\}\} \end{aligned}$$

(b) **A - B**

$$\begin{aligned} \text{Out}[16] = \\ \{\{-1, 2\}, \{6, -1\}\} \end{aligned}$$

(c) **A.B** (Matrix multiplication is not $\mathbf{A} * \mathbf{B}$.)

$$\begin{aligned} \text{Out}[17] = \\ \{\{-4, 10\}, \{-6, 20\}\} \end{aligned}$$

5. What do the following commands do?

(a) **Table[0, {i, 1, 3}, {j, 1, 5}]**

(b) **IdentityMatrix[5]**

(c) **Table[Random[Integer, { - 100, 100}], {4}, {6}]/MatrixForm**

(d) **Table[1/(i + j - 1), {i, 1, 6}, {j, 1, 6}]**

6. Describe the effects of the following commands on the matrix $\mathbf{a} = \{\{1, 2, 3\}, \{4, 5, 6\}, \{7, 8, 0\}\}$.
- $\mathbf{a}[[2, 3]]$
 - $\mathbf{a}[[2]]$
 - $\mathbf{Transpose}[\mathbf{a}][[3]]$
7. You can obtain more information about a particular topic, such as determinants, by typing **?Det**.
8. You can exit *Mathematica* by choosing **Exit** from the file menu.

Introduction to Derive

The purpose of this introduction is to illustrate some basic Derive commands for linear algebra. It is suggested that you complete the following series of simple examples while actually working at the computer terminal.

1. Enter the matrix

$$a = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

by using the **Author/Expression** feature—i.e., $\mathbf{a} = [[1, 2, 3], [4, 5, 6], [7, 8, 0]]$ and hitting the **return** or **enter** key. Be sure to separate the entries in each row by commas and place each row of the matrix within brackets. Derive should return the following matrix.

$$a: = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$$

2. Perform the following elementary operations on the matrix a by typing the given expression and **Simplifying** it. You should obtain the indicated results.

- (a) \mathbf{a}^{-1}

$$\begin{bmatrix} -\frac{16}{9} & \frac{8}{9} & -\frac{1}{9} \\ \frac{14}{9} & -\frac{7}{9} & \frac{2}{9} \\ -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \end{bmatrix}$$

- (b) \mathbf{a}^{\backslash}

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 0 \end{bmatrix}$$

(c) $\mathbf{a} * \mathbf{a}^{-1}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(d) $\mathbf{a} * 2$

$$\begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 0 \end{bmatrix}$$

(e) **Row_Reduce(a)**

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(f) **det(a)**

$$27$$

3. Enter the column matrix $\mathbf{b} := [[7], [16], [7]]$. You can solve the system of linear equations $\mathbf{ax} = \mathbf{b}$ in several ways:

(a) You can row reduce the augmented matrix $[a : b]$.**Row_Reduce(a, b)**

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

(b) You can **Simplify** the equation $[[x], [y], [z]] = \mathbf{a}^{-1} * \mathbf{b}$.

$$\begin{bmatrix} x = 1 \\ y = 0 \\ z = 2 \end{bmatrix}$$

(c) You can **Expand** the equation $a[[x], [y], [z]] = \mathbf{b}$. Then **Solve** the **Expression** by choosing solution variables x , y , and z and choosing **Solve**.

$$[x = 1, y = 0, z = 2]$$

4. Enter the matrices “**A**”: = $[[1, 2], [3, 4]]$ and “**B**”: = $[[2, 0], [-3, 5]]$ and perform the following matrix operations.

(a) “**A**” + “**B**”

$$\begin{bmatrix} 3 & 2 \\ 0 & 9 \end{bmatrix}$$

(b) “**A**” - “**B**”

$$\begin{bmatrix} -1 & 2 \\ 6 & -1 \end{bmatrix}$$

(c) “**A**” * “**B**”

$$\begin{bmatrix} -4 & 10 \\ -6 & 20 \end{bmatrix}$$

5. What do the following commands do?
 - (a) **VECTOR(VECTOR(0, j, 1, 5), k, 1, 3)**
 - (b) **Identity_Matrix(5)**
 - (c) **VECTOR(VECTOR(Random(100), j, 1, 6), k, 1, 4)**
 - (d) **VECTOR(VECTOR(1/(j + k - 1), j, 1, 6), k, 1, 6)**
6. Describe the effects of the following commands on the matrix **a** = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{bmatrix}$.
 - (a) **Element(Element(a, 2), 3)**
 - (b) **Element(a, 2)**
 - (c) **Element(a^, 3)^**
7. You can obtain more information about a particular topic by choosing **Contents** from the help menu.
8. You can exit Derive by choosing **Exit** from the file menu.

Graphing Utilities

The following chart lists various graphing utilities* and their built-in matrix capabilities. For example, the first row indicates that all the graphing utilities can perform matrix addition.

| <i>Matrix operation</i> | Voyage 200 | TI-92 | TI-89 | TI-86 | TI-84 Plus | TI-83 Plus | TI-83 |
|---------------------------|---------------|-------|-------|-------|---------------|---------------|-------|
| Addition | x | x | x | x | x | x | x |
| Scalar multiplication | x | x | x | x | x | x | x |
| Multiplication | x | x | x | x | x | x | x |
| Elementary row operations | x | x | x | x | x | x | x |
| Reduced row-echelon form | x | x | x | x | x | x | x |
| Determinant | x | x | x | x | x | x | x |
| Inverse | x | x | x | x | x | x | x |
| Transpose | x | x | x | x | x | x | x |
| Norm | x | x | x | x | | | |
| Power | x | x | x | x | x | x | x |
| Eigenvalues | x | x | x | x | | | |
| LU-decomposition | x | x | x | x | | | |
| Identity matrix | x | x | x | x | x | x | x |
| Random matrix | x | x | x | x | x | x | x |
| Concatenate matrices | x | x | x | x | x | x | x |
| Condition number | | | | x | | | |
| Complex matrices | x | x | x | x | | | |
| Vectors | x | x | x | x | | | |

*For other graphing utility models, see your user’s manual.

Technology Pitfalls

Many matrix computations in linear algebra can be performed by modern graphing utilities and computer software programs. However, you should be aware that these devices are not perfect because of roundoff error—that is, the graphing utility or computer must approximate fractions by decimals either by rounding off the last digit or by truncating.

For example, evaluate $\frac{1}{6}$ using your graphing utility. On the TI-86 you will get 0.16666666667, which is not correct, because the graphing utility has rounded the last digit up. To analyze this more closely, evaluate $\left[\left(\frac{1}{6}\right) \cdot 3\right] - 0.5$. Do you get the expected answer of 0? Normally this kind of rounding will not affect your answers to problems in linear algebra, but it can happen. Here are a couple of simple examples.

1. Calculate the determinant of the matrix $A = \begin{bmatrix} 3 & 11 \\ 2 & 6 \end{bmatrix}$ on the TI-86. Then calculate the greatest integer of the determinant, $\text{int det } A$. Again, roundoff error has produced an error.
2. The Hilbert matrix of order 6 is

$$H = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} \\ \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} \end{bmatrix}.$$

This matrix is notorious for causing poor results in matrix computations. Try calculating $\text{inv}(H) \cdot H$ to see if you obtain the identity matrix.

You might find it interesting to discover other examples for which your graphing utility gives erroneous results. In general, you should always be aware that such errors are possible, and accept the outputs of your graphing utility with some skepticism.

Keystrokes and Programming Syntax for Selected Examples

Selected examples in your text can be solved using a variety of graphing utilities and computer software programs. Keystrokes and programming syntax for these utilities/programs are provided on the following pages, for use with a variety of graphing utilities, MATLAB, Maple, *Mathematica*, and Derive.

Section 1.1, page 7

EXAMPLE 7 Using Elimination to Rewrite a System in Row-Echelon Form

Solve the system.

$$\begin{aligned}x - 2y + 3z &= 9 \\ -x + 3y &= -4 \\ 2x - 5y + 5z &= 17\end{aligned}$$

Keystrokes for TI-83

Enter the system into matrix A.

To rewrite the system in row-echelon form, use the following keystrokes.

MATRIX **→** **(ALPHA)** **[A]** **MATRIX** **(ENTER)** **(ENTER)**

Keystrokes for TI-83 Plus

Enter the system into matrix A.

To rewrite the system in row-echelon form, use the following keystrokes.

(2nd) **[MATRIX]** **→** **(ALPHA)** **[A]** **(2nd)** **[MATRIX]** **(ENTER)** **(ENTER)**

Keystrokes for TI-84 Plus

Enter the system into matrix A.

To rewrite the system in row-echelon form, use the following keystrokes.

(2nd) **[MATRIX]** **→** **(ALPHA)** **[A]** **(2nd)** **[MATRIX]** **(ENTER)** **(ENTER)**

Keystrokes for TI-86

Enter the system into matrix A.

To rewrite the system in row-echelon form, use the following keystrokes.

(2nd) **[MATRIX]** **(F4)** **(F4)** **(ALPHA)** **[A]** **(ENTER)**

Keystrokes for TI-89

Enter the system into matrix A.

To rewrite the system in row-echelon form, use the following keystrokes.

(2nd) **[MATH]** **4 3** **(alpha)** **[A]** **(↓)** **(ENTER)**

Keystrokes for TI-92 and Voyage 200

Enter the system into matrix A.

To rewrite the system in row-echelon form, use the following keystrokes.

(2nd) **[MATH]** **4 3** **(A)** **(↓)** **(ENTER)**

MATLAB, Maple, *Mathematica*, and Derive

These computer software programs cannot produce row-echelon form. (Row-echelon form is not unique.)

Section 1.2, page 16

EXAMPLE 2 Elementary Row Operations

Add -2 times the first row to the third row to produce a new third row.

Original Matrix

$$(c) \begin{bmatrix} 1 & 2 & -4 & 3 \\ 0 & 3 & -2 & -1 \\ 2 & 1 & 5 & -2 \end{bmatrix}$$

Keystrokes for TI-83

Enter the matrix into matrix A.

To perform the elementary row operations, use the following keystrokes.

[MATRX] **[→]** **[ALPHA]** **[F]** **[(-)]** 2 **[.]** **[MATRX]** 1 **[.]** 1 **[.]** 3 **[ENTER]**

Keystrokes for TI-83 Plus

Enter the matrix into matrix A.

To perform the elementary row operations, use the following keystrokes.

[2nd] **[MATRX]** **[→]** **[ALPHA]** **[F]** **[(-)]** 2 **[.]** **[2nd]** **[MATRX]** 1 **[.]** 1 **[.]** 3 **[ENTER]**

Keystrokes for TI-84 Plus

Enter the matrix into matrix A.

To perform the elementary row operations, use the following keystrokes.

[2nd] **[MATRX]** **[→]** **[ALPHA]** **[F]** **[(-)]** 2 **[.]** **[2nd]** **[MATRX]** 1 **[.]** 1 **[.]** 3 **[ENTER]**

Keystrokes for TI-86

Enter the matrix into matrix A.

To perform the elementary row operations, use the following keystrokes.

[2nd] **[MATRX]** **[F4]** **[MORE]** **[F5]** **[(-)]** 2 **[.]** **[2nd]** **[MATRX]** **[F1]** **[F1]** **[.]** 1 **[.]** 3 **[ENTER]**

Keystrokes for TI-89

Enter the matrix into matrix A.

To perform the elementary row operations, use the following keystrokes.

[2nd] **[MATH]** 4 **[alpha]** **[J]** 4 **[(-)]** 2 **[.]** **[alpha]** **[A]** **[.]** 1 **[.]** 3 **[J]** **[ENTER]**

Keystrokes for TI-92 and Voyage 200

Enter the matrix into matrix A.

To perform the elementary row operations, use the following keystrokes.

[2nd] **[MATH]** 4 **[J]** 4 **[(-)]** 2 **[.]** **[A]** **[.]** 1 **[.]** 3 **[J]** **[ENTER]**

Programming Syntax for MATLAB

Enter the matrix into matrix A.

$$A(3, :) = -2*A(1, :)+A(3, :)$$

Hit the **return** or **enter** key.

Programming Syntax for Maple

with(linalg);

Hit the **return** or **enter** key.

Enter the matrix into matrix A.

A:= addrow(A, 1, 3, -2);

Hit the **return** or **enter** key.

Programming Syntax for Mathematica

Enter the matrix into matrix A.

A[[3]] = -2A[[1]] + A[[3]]

Hit **shift** + **enter**.

A // MatrixForm

Hit **shift** + **enter**.

Programming Syntax for Derive

Enter the matrix into matrix A.

Subtract_Elements(A, 3, 1, 2)

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

Section 1.2, page 18

EXAMPLE 4

Row-Echelon Form

$$(f) \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -4 \end{bmatrix}$$

Keystrokes for TI-83

Enter the matrix into matrix A.

To rewrite the matrix in reduced row-echelon form, use the following keystrokes.

(MATRX) **(→)** **(ALPHA)** **[B]** **(MATRX)** **(ENTER)** **(ENTER)**

Keystrokes for TI-83 Plus

Enter the matrix into matrix A.

To rewrite the matrix in reduced row-echelon form, use the following keystrokes.

(2nd) **(MATRX)** **(→)** **(ALPHA)** **[B]** **(2nd)** **(MATRX)** **(ENTER)** **(ENTER)**

Keystrokes for TI-84 Plus

Enter the matrix into matrix A.

To rewrite the matrix in reduced row-echelon form, use the following keystrokes.

(2nd) **(MATRX)** **(→)** **(ALPHA)** **[B]** **(2nd)** **(MATRX)** **(ENTER)** **(ENTER)**

Keystrokes for TI-86

Enter the matrix into matrix A.

To rewrite the matrix in reduced row-echelon form, use the following keystrokes.

(2nd) **(MATRX)** **(F4)** **(F5)** **(ALPHA)** **[A]** **(ENTER)**

Keystrokes for TI-89

Enter the matrix into matrix A.

To rewrite the matrix in reduced row-echelon form, use the following keystrokes.

$\boxed{2\text{nd}}$ $\boxed{[MATH]}$ 4 4 $\boxed{\alpha}$ $\boxed{[A]}$ $\boxed{\text{D}}$ $\boxed{[ENTER]}$

Keystrokes for TI-92 and Voyage 200

Enter the matrix into matrix A.

To rewrite the matrix in reduced row-echelon form, use the following keystrokes.

$\boxed{2\text{nd}}$ $\boxed{[MATH]}$ 4 4 $\boxed{[A]}$ $\boxed{\text{D}}$ $\boxed{[ENTER]}$

Programming Syntax for MATLAB

Enter the matrix into matrix A.

```
rref(A)
```

Hit the **return** or **enter** key.

Programming Syntax for Maple

```
with(linalg);
```

Hit the **return** or **enter** key.

Enter the matrix into matrix A.

```
rref(A)
```

Hit the **return** or **enter** key.

Programming Syntax for Mathematica

Enter the matrix into matrix A.

```
RowReduce[A] // MatrixForm
```

Hit **shift + enter**.

Programming Syntax for Derive

Enter the matrix into matrix A.

```
Row_Reduce(A)
```

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

Section 2.1, page 52

EXAMPLE 5 Matrix Multiplication

$$(c) \begin{matrix} \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} \\ 2 \times 2 & 2 \times 2 \end{matrix}$$

Keystrokes for TI-83

Enter the matrices into matrix A and matrix B.

To multiply the matrices, use the following keystrokes.

$\boxed{[MATRX]}$ 1 $\boxed{[MATRX]}$ 2 $\boxed{[ENTER]}$

Keystrokes for TI-83 Plus

Enter the matrices into matrix A and matrix B.

To multiply the matrices, use the following keystrokes.

$\boxed{2\text{nd}}$ $\boxed{[MATRX]}$ 1 $\boxed{2\text{nd}}$ $\boxed{[MATRX]}$ 2 $\boxed{[ENTER]}$

Keystrokes for TI-84 Plus

Enter the matrices into matrix A and matrix B.

To multiply the matrices, use the following keystrokes.

$\boxed{2\text{nd}}$ $\boxed{[MATRIX]}$ 1 $\boxed{2\text{nd}}$ $\boxed{[MATRIX]}$ 2 $\boxed{[ENTER]}$

Keystrokes for TI-86

Enter the matrices into matrix A and matrix B.

To multiply the matrices, use the following keystrokes.

$\boxed{2\text{nd}}$ $\boxed{[MATRX]}$ $\boxed{[F1]}$ $\boxed{[F1]}$ $\boxed{[F2]}$ $\boxed{[ENTER]}$

Keystrokes for TI-89

Enter the matrices into matrix A and matrix B.

To multiply the matrices, use the following keystrokes.

$\boxed{[\alpha]}$ $\boxed{[A]}$ $\boxed{[\times]}$ $\boxed{[\alpha]}$ $\boxed{[B]}$ $\boxed{[ENTER]}$

Keystrokes for TI-92 and Voyage 200

Enter the matrices into matrix A and matrix B.

To multiply the matrices, use the following keystrokes.

$\boxed{[A]}$ $\boxed{[\times]}$ $\boxed{[B]}$ $\boxed{[ENTER]}$

Programming Syntax for MATLAB

Enter the matrices into matrix A and matrix B.

A*B

Hit the **return** or **enter** key.

Programming Syntax for Maple

with(linalg);

Hit the **return** or **enter** key.

Enter the matrices into matrix A and matrix B.

multiply(A, B);

Hit the **return** or **enter** key.

Programming Syntax for Mathematica

Enter the matrices into matrix A and matrix B.

A.B // MatrixForm

Hit **shift + enter**.

Programming Syntax for Derive

Enter the matrices into matrix A and matrix B.

A * B

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

Section 2.3, page 76

EXAMPLE 3 Finding the Inverse of a Matrix

Find the inverse of the matrix.

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$$

Keystrokes for TI-83

Enter the matrix into matrix A.

To find the inverse, use the following keystrokes.

(MATRX) 1 **(x^{-1})** **(ENTER)**

Keystrokes for TI-83 Plus

Enter the matrix into matrix A.

To find the inverse, use the following keystrokes.

(2nd) **[MATRX]** 1 **(x^{-1})** **(ENTER)**

Keystrokes for TI-84 Plus

Enter the matrix into matrix A.

To find the inverse, use the following keystrokes.

(2nd) **[MATRX]** 1 **(x^{-1})** **(ENTER)**

Keystrokes for TI-86

Enter the matrix into matrix A.

To find the inverse, use the following keystrokes.

(2nd) **[MATRX]** **(F1)** **(F1)** **(2nd)** **(x^{-1})** **(ENTER)**

Keystrokes for TI-89

Enter the matrix into matrix A.

To find the inverse, use the following keystrokes.

(alpha) **[A]** **(\wedge)** **(-)** 1 **(ENTER)**

Keystrokes for TI-92 and Voyage 200

Enter the matrix into matrix A.

To find the inverse, use the following keystrokes.

(A) **(\wedge)** **(-)** 1 **(ENTER)**

Programming Syntax for MATLAB

Enter the matrix into matrix A.

`inv(A)`

Hit the **return** or **enter** key.

Programming Syntax for Maple

`with(linalg);`

Hit the **return** or **enter** key.

Enter the matrix into matrix A.

`inverse(A);`

Hit the **return** or **enter** key.

Programming Syntax for Mathematica

Enter the matrix into matrix A.

```
Inverse[A] // MatrixForm
```

Hit **shift + enter**.

Programming Syntax for Derive

Enter the matrix into matrix A.

```
A ^-1
```

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

Section 3.1, page 127

EXAMPLE 4 The Determinant of a Matrix of Order 4

Find the determinant of

$$A = \begin{bmatrix} 1 & -2 & 3 & 0 \\ -1 & 1 & 0 & 2 \\ 0 & 2 & 0 & 3 \\ 3 & 4 & 0 & -2 \end{bmatrix}.$$

Keystrokes for TI-83

Enter the matrix into matrix A.

To find the determinant, use the following keystrokes.

```
(MATRX) (→) 1 (MATRX) 1 (ENTER)
```

Keystrokes for TI-83 Plus

Enter the matrix into matrix A.

To find the determinant, use the following keystrokes.

```
(2nd) (MATRX) (→) 1 (2nd) (MATRX) 1 (ENTER)
```

Keystrokes for TI-84 Plus

Enter the matrix into matrix A.

To find the determinant, use the following keystrokes.

```
(2nd) (MATRX) (→) 1 (2nd) (MATRX) 1 (ENTER)
```

Keystrokes for TI-86

Enter the matrix into matrix A.

To find the determinant, use the following keystrokes.

```
(2nd) (MATRX) (F3) (F1) (2nd) (M1) (F1) (ENTER)
```

Keystrokes for TI-89

Enter the matrix into matrix A.

To find the determinant, use the following keystrokes.

```
(2nd) (MATH) 4 2 (alpha) [A] (⏏) (ENTER)
```

Keystrokes for TI-92 and Voyage 200

Enter the matrix into matrix A.

To find the determinant, use the following keystrokes.

2nd [MATH] 4 2 **(A)** **()** **(ENTER)**

Programming Syntax for MATLAB

Enter the matrix into matrix A.

det(A)

Hit the **return** or **enter** key.

Programming Syntax for Maple

with(linalg);

Hit the **return** or **enter** key.

Enter the matrix into matrix A.

det(A);

Hit the **return** or **enter** key.

Programming Syntax for Mathematica

Enter the matrix into matrix A.

Det[A]

Hit **shift** + **enter**.

Programming Syntax for Derive

Enter the matrix into matrix A.

Det(A)

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

Section 3.4, page 153

EXAMPLE 2 Finding Eigenvalues and Eigenvectors

Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.

Keystrokes for TI-83, TI-83 Plus, and TI-84 Plus

These graphing utilities cannot find eigenvalues.

Keystrokes for TI-86

Enter the matrix into matrix A.

To find the eigenvalues, use the following keystrokes.

2nd [MATRX] **(F3)** **(F4)** **2nd** [M1] **(F1)** **(ENTER)**

Keystrokes for TI-89

Enter the matrix into matrix A.

To find the eigenvalues, use the following keystrokes.

2nd [MATH] 4 9 **(alpha)** **(A)** **()** **(ENTER)**

Keystrokes for TI-92 and Voyage 200

Enter the matrix into matrix A.

To find the eigenvalues, use the following keystrokes.

$\boxed{2\text{nd}}$ $\boxed{[\text{MATH}]}$ 4 9 $\boxed{[A]}$ $\boxed{)}$ $\boxed{[\text{ENTER}]}$

Programming Syntax for MATLAB

Enter the matrix into matrix A.

`eig(A)`

Hit the **return** or **enter** key.

Programming Syntax for Maple

`with(linalg);`

Hit the **return** or **enter** key.

Enter the matrix into matrix A.

`eigenvalues(A);`

Hit the **return** or **enter** key.

Programming Syntax for Mathematica

Enter the matrix into matrix A.

`Eigenvalues[A]`

Hit **shift + enter**.

Programming Syntax for Derive

Enter the matrix into matrix A.

`Eigenvalues(A)`

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

Section 4.1, page 184

EXAMPLE 4 Vector Operations in R^3

Provided that $\mathbf{u} = (-1, 0, 1)$ and $\mathbf{v} = (2, -1, 5)$ in R^3 , find each vector.

(a) $\mathbf{u} + \mathbf{v}$ (b) $2\mathbf{u}$ (c) $\mathbf{v} - 2\mathbf{u}$

Keystrokes for TI-83, TI-83 Plus, and TI-84 Plus

Store the vectors in lists L1 and L2.

To find each vector, use the following keystrokes.

Part (a) $\boxed{2\text{nd}}$ $\boxed{[L1]}$ $\boxed{+}$ $\boxed{2\text{nd}}$ $\boxed{[L2]}$ $\boxed{[\text{ENTER}]}$

Part (b) 2 $\boxed{2\text{nd}}$ $\boxed{[L1]}$ $\boxed{[\text{ENTER}]}$

Part (c) $\boxed{2\text{nd}}$ $\boxed{[L2]}$ $\boxed{-}$ 2 $\boxed{2\text{nd}}$ $\boxed{[L1]}$ $\boxed{[\text{ENTER}]}$

Keystrokes for TI-86

Enter the vectors into vector U and vector V.

To find each vector, use the following keystrokes.

Part (a) $\boxed{2\text{nd}}$ $\boxed{[\text{VECTR}]}$ $\boxed{F1}$ $\boxed{F1}$ $\boxed{+}$ $\boxed{F2}$ $\boxed{[\text{ENTER}]}$

Part (b) $\boxed{2\text{nd}}$ $\boxed{[\text{VECTR}]}$ 2 $\boxed{F1}$ $\boxed{F1}$ $\boxed{[\text{ENTER}]}$

Part (c) $\boxed{2\text{nd}}$ $\boxed{[\text{VECTR}]}$ $\boxed{F1}$ $\boxed{F2}$ $\boxed{-}$ 2 $\boxed{F1}$ $\boxed{[\text{ENTER}]}$

Keystrokes for TI-89

Store the vectors in U and V.

To find each vector, use the following keystrokes.

Part (a) α [U] + α [V] [ENTER]

Part (b) 2 α [U] [ENTER]

Part (c) α [V] - 2 α [U] [ENTER]

Keystrokes for TI-92 and Voyage 200

Store the vectors in U and V.

To find each vector, use the following keystrokes.

Part (a) [U] + [V] [ENTER]

Part (b) 2 [U] [ENTER]

Part (c) [V] - 2 [U] [ENTER]

Programming Syntax for MATLAB

Enter the vectors into vector u and vector v.

$u + v$

Hit the **return** or **enter** key.

$2 * u$

Hit the **return** or **enter** key.

$v - 2 * u$

Hit the **return** or **enter** key.

Programming Syntax for Maple

with(linalg);

Hit the **return** or **enter** key.

Enter the vectors into vector u and vector v.

evalm(u + v);

Hit the **return** or **enter** key.

evalm(2 * u);

Hit the **return** or **enter** key.

evalm(v - 2 * u);

Hit the **return** or **enter** key.

Programming Syntax for Mathematica

Enter the vectors into vector u and vector v.

$u + v$

Hit **shift + enter**.

$2u$

Hit **shift + enter**.

$v - 2u$

Hit **shift + enter**.

Programming Syntax for Derive

Enter the vectors into vector u and vector v .

$$u + v$$

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

$$2u$$

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

$$v - 2u$$

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

Section 4.7, page 257

EXAMPLE 5 Finding a Transition Matrix

Find the transition matrix from B to B' for the bases for R^2 listed below.

$$B = \{(-3, 2), (4, -2)\} \quad \text{and} \quad B' = \{(-1, 2), (2, -2)\}$$

Keystrokes for TI-83

Enter the matrices into matrix A and matrix B.

To find the transition matrix, use the following keystrokes.

[MATRX] **[→]** **[ALPHA]** **[B]** **[MATRX]** **[→]** 7 **[MATRX]** 2 **[.]** **[MATRX]** 1 **[ENTER]**

Keystrokes for TI-83 Plus

Enter the matrices into matrix A and matrix B.

To find the transition matrix, use the following keystrokes.

[2nd] **[MATRX]** **[→]** **[ALPHA]** **[B]** **[2nd]** **[MATRX]** **[→]** 7 **[2nd]** **[MATRX]** 2 **[.]** **[2nd]** **[MATRX]** 1 **[ENTER]**

Keystrokes for TI-84 Plus

Enter the matrices into matrix A and matrix B.

To find the transition matrix, use the following keystrokes.

[2nd] **[MATRX]** **[→]** **[ALPHA]** **[B]** **[2nd]** **[MATRX]** **[→]** 7 **[2nd]** **[MATRX]** 2 **[.]** **[2nd]** **[MATRX]** 1 **[ENTER]**

Keystrokes for TI-86

Enter the matrices into matrix A and matrix B.

To find the transition matrix, use the following keystrokes.

[2nd] **[MATRX]** **[F4]** **[F5]** **[MORE]** **[F1]** **[2nd]** **[M1]** **[F2]** **[.]** **[F1]** **[ENTER]**

Keystrokes for TI-89

Enter the matrices into matrix A and matrix B.

To find the transition matrix, use the following keystrokes.

[2nd] **[MATH]** 4 4 **[2nd]** **[MATH]** 4 7 **[alpha]** **[B]** **[.]** **[alpha]** **[A]** **[)]** **[)]** **[ENTER]**

Keystrokes for TI-92 and Voyage 200

Enter the matrices into matrix A and matrix B.

To find the transition matrix, use the following keystrokes.

$\text{\textcircled{2nd}}$ $\text{\textcircled{[MATH]}}$ 4 4 $\text{\textcircled{2nd}}$ $\text{\textcircled{[MATH]}}$ 4 7 $\text{\textcircled{B}}$ $\text{\textcircled{.}}$ $\text{\textcircled{A}}$ $\text{\textcircled{)}}]$ $\text{\textcircled{)}}]$ $\text{\textcircled{ENTER}}$

Programming Syntax for MATLAB

Enter the matrices into matrix B and matrix BPRIME.

```
rref([BPRIME B])
```

Hit the **return** or **enter** key.

Programming Syntax for Maple

```
with(linalg);
```

Hit the **return** or **enter** key.

Enter the matrices into matrix B and matrix BPRIME.

```
rref(augment(BPRIME, B));
```

Hit the **return** or **enter** key.

Programming Syntax for Mathematica

Enter the matrices into matrix B and matrix BPRIME.

```
<<LinearAlgebra`MatrixManipulation`
RowReduce[AppendRows[BPRIME, B]] // MatrixForm
```

Hit **shift** + **enter**.

Programming Syntax for Derive

Enter the matrices into matrix B and matrix BPRIME.

```
Row_Reduce(Append_Columns(BPRIME, B))
```

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

Section 5.1, page 279

EXAMPLE 1 The Length of a Vector in R^n

(a) Find the length of $\mathbf{v} = (0, -2, 1, 4, -2)$.

Keystrokes for TI-83, TI-83 Plus, and TI-84 Plus

These graphing utilities cannot find the length of a vector.

Keystrokes for TI-86

Enter the vector into vector V.

To find the length, use the following keystrokes.

$\text{\textcircled{2nd}}$ $\text{\textcircled{[VECTR]}}$ $\text{\textcircled{F3}}$ $\text{\textcircled{F3}}$ $\text{\textcircled{2nd}}$ $\text{\textcircled{[M1]}}$ $\text{\textcircled{F1}}$ $\text{\textcircled{ENTER}}$

Keystrokes for TI-89

Store the vector in V.

To find the length, use the following keystrokes.

$\text{\textcircled{2nd}}$ $\text{\textcircled{[MATH]}}$ 4 $\text{\textcircled{alpha}}$ [H] 1 $\text{\textcircled{alpha}}$ [V] $\text{\textcircled{)}}]$ $\text{\textcircled{ENTER}}$

Keystrokes for TI-92 and Voyage 200

Store the vector in V.

To find the length, use the following keystrokes.

$\boxed{2\text{nd}}$ $\boxed{[\text{MATH}]}$ $\boxed{4}$ $\boxed{[\text{H}]}$ $\boxed{1}$ $\boxed{[\text{V}]}$ $\boxed{[\text{D}]}$ $\boxed{[\text{ENTER}]}$

Programming Syntax for MATLAB

Enter the vector into vector v.

```
norm(v)
```

Hit the **return** or **enter** key.

Programming Syntax for Maple

```
with(linalg);
```

Hit the **return** or **enter** key.

Enter the vector into vector v.

```
norm(v, 2);
```

Hit the **return** or **enter** key.

Programming Syntax for Mathematica

Enter the vector into vector v.

```
<<LinearAlgebra`MatrixManipulation`  
VectorNorm[N[v], 2]
```

Hit **shift + enter**.

Programming Syntax for Derive

Enter the vector into vector v.

```
Abs(v)
```

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

Section 5.1, page 281, Technology Note

Find the unit vector for $\mathbf{v} = (-3, 4)$.

Keystrokes for TI-83, TI-83 Plus, and TI-84 Plus

These graphing utilities cannot find the unit vector.

Keystrokes for TI-86

Enter the vector into vector V.

To find the unit vector, use the following keystrokes.

$\boxed{2\text{nd}}$ $\boxed{[\text{VECTR}]}$ $\boxed{[\text{F3}]}$ $\boxed{[\text{F2}]}$ $\boxed{2\text{nd}}$ $\boxed{[\text{M1}]}$ $\boxed{[\text{F1}]}$ $\boxed{[\text{ENTER}]}$

Keystrokes for TI-89

Store the vector in V.

To find the unit vector, use the following keystrokes.

$\boxed{2\text{nd}}$ $\boxed{[\text{MATH}]}$ $\boxed{4}$ $\boxed{[\text{alpha}]}$ $\boxed{[\text{L}]}$ $\boxed{1}$ $\boxed{[\text{alpha}]}$ $\boxed{[\text{V}]}$ $\boxed{[\text{D}]}$ $\boxed{[\text{ENTER}]}$

Keystrokes for TI-92 and Voyage 200

Store the vector in V.

To find the unit vector, use the following keystrokes.

$\boxed{2\text{nd}} \boxed{[\text{MATH}]} \boxed{4} \boxed{[L]} \boxed{1} \boxed{[V]} \boxed{)} \boxed{[\text{ENTER}]}$

Programming Syntax for MATLAB

Enter the vector into vector v.

`v/norm(v)`

Hit the **return** or **enter** key.

Programming Syntax for Maple

`with(linalg);`

Hit the **return** or **enter** key.

Enter the vector into vector v.

`evalm(v/norm(v, 2));`

Hit the **return** or **enter** key.

Programming Syntax for Mathematica

Enter the vector into vector v.

`<<LinearAlgebra`MatrixManipulation`
v/VectorNorm[N[v], 2]`

Hit **shift + enter**.

Programming Syntax for Derive

Enter the vector into vector v.

`v/Abs(v)`

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

Section 5.1, page 283

EXAMPLE 4 Finding the Dot Product of Two Vectors

Find the dot product of $\mathbf{u} = (1, 2, 0, -3)$ and $\mathbf{v} = (3, -2, 4, 2)$.

Keystrokes for TI-83, TI-83 Plus, and TI-84 Plus

These graphing utilities cannot find the dot product.

Keystrokes for TI-86

Enter the vectors into vector U and vector V.

To find the dot product, use the following keystrokes.

$\boxed{2\text{nd}} \boxed{[\text{VECTR}]} \boxed{[F3]} \boxed{[F4]} \boxed{2\text{nd}} \boxed{[M1]} \boxed{[F1]} \boxed{.} \boxed{[F2]} \boxed{[\text{ENTER}]}$

Keystrokes for TI-89

Store the vectors in U and V.

To find the dot product, use the following keystrokes.

$\boxed{2\text{nd}} \boxed{[\text{MATH}]} \boxed{4} \boxed{[\alpha]} \boxed{[L]} \boxed{3} \boxed{[\alpha]} \boxed{[U]} \boxed{.} \boxed{[\alpha]} \boxed{[V]} \boxed{)} \boxed{[\text{ENTER}]}$

Keystrokes for TI-92 and Voyage 200

Store the vectors in U and V.

To find the dot product, use the following keystrokes.

2nd **[MATH]** 4 **[L]** 3 **[U]** **[.]** **[V]** **[)]** **[ENTER]**

Programming Syntax for MATLAB

Enter the vectors into vector u and vector v.

`dot(u, v)`

Hit the **return** or **enter** key.

Programming Syntax for Maple

`with(linalg);`

Hit the **return** or **enter** key.

Enter the vectors into vector u and vector v.

`dotprod(u, v);`

Hit the **return** or **enter** key.

Programming Syntax for Mathematica

Enter the vectors into vector u and vector v.

`u.v`

Hit **shift** + **enter**.

Programming Syntax for Derive

Enter the vectors into vector u and vector v.

`u.v`

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

Section 5.2, page 297

EXAMPLE 7 Using the Inner Product on $C[0, 1]$ (Calculus)

Use the inner product defined in Example 5 and the functions $f(x) = x$ and $g(x) = x^2$ in $C[0, 1]$ to find

(b) $d(f, g)$.

Keystrokes for TI-83, TI-83 Plus, and TI-84 Plus

To find $d(f, g)$, use the following keystrokes.

2nd **[√]** **[MATH]** 9 **[X,T,θ,n]** **[−]** **[X,T,θ,n]** **[x²]** **[)]** **[x²]** **[.]** **[X,T,θ,n]** **[.]** 0 **[.]** 1 **[)]** **[)]**
[ENTER]

Keystrokes for TI-86

To find $d(f, g)$, use the following keystrokes.

2nd **[√]** **2nd** **[CALC]** 9 **[F5]** **[X-VAR]** **[−]** **[x-VAR]** **[x²]** **[)]** **[x²]** **[.]** **[x-VAR]** **[.]** 0 **[.]**
1 **[)]** **[ENTER]**

Keystrokes for TI-89, TI-92, and Voyage 200

To find $d(f, g)$, use the following keystrokes.

$\boxed{2\text{nd}} \boxed{[\sqrt{\quad}]} \boxed{F3} \ 2 \ \boxed{C} \ \boxed{X} \ \boxed{-} \ \boxed{X} \ \boxed{\wedge} \ 2 \ \boxed{D} \ \boxed{\wedge} \ 2 \ \boxed{-} \ \boxed{X} \ \boxed{\cdot} \ 0 \ \boxed{-} \ 1 \ \boxed{D} \ \boxed{D} \ \boxed{\text{ENTER}}$

Programming Syntax for MATLAB

MATLAB cannot calculate inner products.

Programming Syntax for Maple

```
sqrt(int((x - x^2)^2, x = 0..1));
```

Hit the **return** or **enter** key.

Programming Syntax for Mathematica

```
Sqrt[Integrate[(x - x^2)^2, {x, 0, 1}]]
```

Hit **shift** + **enter**.

Programming Syntax for Derive

```
sqrt(int((x - x^2)^2, x, 0, 1))
```

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

Section 5.4, page 328

EXAMPLE 7 Solving the Normal Equations

Find the solution of the least squares problem

$$A\mathbf{x} = \mathbf{b}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}.$$

Keystrokes for TI-83, TI-83 Plus, and TI-84 Plus

Enter the data into lists L1 and L2.

To find the least squares regression line, use the following keystrokes.

$\boxed{\text{STAT}} \ \boxed{\rightarrow} \ 4 \ \boxed{2\text{nd}} \ \boxed{[L1]} \ \boxed{-} \ \boxed{2\text{nd}} \ \boxed{[L2]} \ \boxed{\text{ENTER}}$

Keystrokes for TI-86

Use $\boxed{2\text{nd}} \ \boxed{\text{STAT}} \ \boxed{F2}$ to enter the data. There should be a 1 in each row of the $fStat$ column. To find the least squares regression line, use the following keystrokes.

$\boxed{2\text{nd}} \ \boxed{\text{STAT}} \ \boxed{F1} \ \boxed{F3} \ \boxed{\text{ENTER}}$

Keystrokes for TI-89

Enter the data into c1 and c2 using the *Data/Matrix Editor*. To find the least squares regression line, use the following keystrokes while in the *Data/Matrix Editor*.

(F5) (→) 5 (↓) (alpha) [c] 1 (↓) (alpha) [c] 2 (ENTER) (ENTER)

Keystrokes for TI-92 and Voyage 200

Enter the data into c1 and c2 using the *Data/Matrix Editor*. To find the least squares regression line, use the following keystrokes while in the *Data/Matrix Editor*.

(F5) (→) 5 (↓) (C) 1 (↓) (C) 2 (ENTER) (ENTER)

Programming Syntax for MATLAB

```
x = [1 2 3];
```

Hit the **return** or **enter** key.

```
y = [0 1 3];
```

Hit the **return** or **enter** key.

```
polyfit(x, y, 1)
```

Hit the **return** or **enter** key.

Programming Syntax for Maple

```
with(Statistics);
```

Hit the **return** or **enter** key.

```
xvalues := vector([1, 2, 3]);
```

Hit the **return** or **enter** key.

```
yvalues := vector([0, 1, 3]);
```

Hit the **return** or **enter** key.

```
Fit(a*x + b, xvalues, yvalues, x);
```

Hit the **return** or **enter** key.

Programming Syntax for Mathematica

```
data = {{1, 0}, {2, 1}, {3, 3}}
```

Hit **shift + enter**.

```
Fit[data, {1, x}, x]
```

Hit **shift + enter**.

Programming Syntax for Derive

```
Fit([x, a*x + b], [[1, 0], [2, 1], [3, 3]])
```

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

Section 5.4, page 332

EXAMPLE 10 Application to Astronomy

Table 5.2 shows the mean distances x and the periods y of the six planets that are closest to the sun. The mean distance is given in terms of astronomical units (where the Earth's mean distance is defined to be 1.0), and the period is provided in years. Find a model for these data. (Source: *CRC Handbook of Chemistry and Physics*)

TABLE 5.2

| Planet | Mercury | Venus | Earth | Mars | Jupiter | Saturn |
|---------------|---------|-------|-------|-------|---------|--------|
| Distance, x | 0.387 | 0.723 | 1.0 | 1.523 | 5.203 | 9.541 |
| Period, y | 0.241 | 0.615 | 1.0 | 1.881 | 11.861 | 29.457 |

Keystrokes for TI-83, TI-83 Plus, and TI-84 Plus

Enter the data into lists L1 and L2.

To find the power model, use the following keystrokes.

[STAT] **[→]** **[ALPHA]** **[A]** **[2nd]** **[L1]** **[,]** **[2nd]** **[L2]** **[ENTER]**

Keystrokes for TI-86

Use **[2nd]** **[STAT]** **[F2]** to enter the data. There should be a 1 in each row of the $fStat$ column. To find the power model, use the following keystrokes.

[2nd] **[STAT]** **[F1]** **[MORE]** **[F1]** **[ENTER]**

Keystrokes for TI-89

Enter the data into c1 and c2 using the *Data/Matrix Editor*. To find the power model, use the following keystrokes while in the *Data/Matrix Editor*:

[F5] **[→]** **8** **[↓]** **[alpha]** **[c]** **1** **[↓]** **[alpha]** **[c]** **2** **[ENTER]** **[ENTER]**

Keystrokes for TI-92 and Voyage 200

Enter the data into c1 and c2 using the *Data/Matrix Editor*. To find the power model, use the following keystrokes while in the *Data/Matrix Editor*:

[F5] **[→]** **8** **[↓]** **[C]** **1** **[↓]** **[C]** **2** **[ENTER]** **[ENTER]**

Programming Syntax for MATLAB

$x = [0.387 \ 0.723 \ 1.0 \ 1.523 \ 5.203 \ 9.541];$

Hit the **return** or **enter** key.

$y = [0.241 \ 0.615 \ 1.0 \ 1.881 \ 11.861 \ 29.457];$

Hit the **return** or **enter** key.

`polyfit(log(x), log(y), 1)`

Hit the **return** or **enter** key.

Programming Syntax for Maple

```
with(Statistics);
Hit the return or enter key.
xvalues := vector([0.387, 0.723, 1.0, 1.523, 5.203, 9.541]);
Hit the return or enter key.
yvalues := vector([0.241, 0.615, 1.0, 1.881, 11.861, 29.457]);
Hit the return or enter key.
Fit(a*x ^b, xvalues, yvalues, x);
Hit the return or enter key.
```

Programming Syntax for Mathematica

```
<<Statistics`NonLinearFit`
Hit shift + enter.
data = {{0.387, 0.241}, {0.723, 0.615}, {1.0, 1.0}, {1.523, 1.881}, {5.203, 11.861},
{9.541, 29.457}}
Hit shift + enter.
NonlinearFit[data, a*x^b, x, {a, b}]
Hit shift + enter.
```

Programming Syntax for Derive

Derive cannot calculate power models.

Section 5.5, page 336

EXAMPLE 1 Finding the Cross Product of Two Vectors

Provided that $\mathbf{u} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, find

(a) $\mathbf{u} \times \mathbf{v}$.

Keystrokes for TI-83, TI-83 Plus, and TI-84 Plus

These graphing utilities cannot find the cross product.

Keystrokes for TI-86

Enter the vectors into vector U and vector V.

To find the cross product, use the following keystrokes.

(2nd) **[VECTR]** **(F3)** **(F1)** **(2nd)** **[M1]** **(F1)** **(.)** **(F2)** **(ENTER)**

Keystrokes for TI-89

Store the vectors in U and V.

To find the cross product, use the following keystrokes.

(2nd) **[MATH]** **4** **(alpha)** **[L]** **2** **(alpha)** **[U]** **(.)** **(alpha)** **[V]** **(J)** **(ENTER)**

Keystrokes for TI-92 and Voyage 200

Store the vectors in U and V.

To find the cross product, use the following keystrokes.

(2nd) **[MATH]** **4** **(L)** **2** **(U)** **(.)** **(V)** **(J)** **(ENTER)**

Programming Syntax for MATLAB

Enter the vectors into vector u and vector v .

`cross(u, v)`

Hit the **return** or **enter** key.

Programming Syntax for Maple

`with(linalg);`

Hit the **return** or **enter** key.

Enter the vectors into vector u and vector v .

`crossprod(u, v);`

Hit the **return** or **enter** key.

Programming Syntax for Mathematica

Enter the vectors into vector u and vector v .

`Cross[u, v]`

Hit **shift** + **enter**.

Programming Syntax for Derive

Enter the vectors into vector u and vector v .

`Cross(u, v)`

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

Section 7.1, page 429

EXAMPLE 6

Finding Eigenvalues and Eigenvectors

Find the eigenvalues of

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -10 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}.$$

Keystrokes for TI-83, TI-83 Plus, and TI-84 Plus

These graphing utilities cannot find eigenvalues or eigenvectors.

Keystrokes for TI-86

Enter the matrix into matrix A.

To find the eigenvalues and eigenvectors, use the following keystrokes.

`(2nd) [MATRX] (F3) (F4) (2nd) [M1] (F1) (ENTER)`

`(2nd) [M3] (F5) (2nd) [M1] (F1) (ENTER)`

Keystrokes for TI-89

Enter the matrix into matrix A.

To find the eigenvalues and eigenvectors, use the following keystrokes.

`(2nd) [MATH] 4 9 (alpha) [A] () (ENTER)`

`(2nd) [MATH] 4 (alpha) [A] (alpha) [A] () (ENTER)`

Keystrokes for TI-92 and Voyage 200

Enter the matrix into matrix A.

To find the eigenvalues and eigenvectors, use the following keystrokes.

(2nd) **[MATH]** 4 9 **(A)** **(>)** **(ENTER)** **(2nd)** **[MATH]** 4 **(A)** **(A)** **(>)** **(ENTER)**

Programming Syntax for MATLAB

Enter the matrix into matrix A.

```
[V, D] = eig(A)
```

Hit the **return** or **enter** key.

Programming Syntax for Maple

```
with(linalg);
```

Hit the **return** or **enter** key.

Enter the matrix into matrix A.

```
eigenvalues(A);
```

Hit the **return** or **enter** key.

```
eigenvectors(A);
```

Hit the **return** or **enter** key.

Programming Syntax for Mathematica

Enter the matrix into matrix A.

```
EigenSystem[A] // MatrixForm
```

Hit **shift + enter**.

Programming Syntax for Derive

Enter the matrix into matrix A.

```
Eigenvalues(A)
```

Hit the **return** or **enter** key.

Choose **Simplify** from the toolbar. Then choose **Basic**.

Derive cannot find eigenvectors.