TPS3 **Programmer's Guide**

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Preface

The following is a TEX(actually, LATEX) version of the TPS3 Programmer's Guide. The original version is in Scribe format.

CHAPTER 1

Introduction

TPS3 has been developed over several decades by a number of people, some of whom never actually met each other. Attempts have been made to maintain documentation for the program, but research progress was generally a higher priority, and obsolete documentation was not always corrected or deleted. Therefore, this manual should be used with discretion. The guidance it provides may be very helpful at times, but there is no claim that it is generally adequate, and some of it may be misleading or incorrect.

> "The *Guide*... is an indispensable companion to all those who are keen to make sense of life in an infinitely complex and confusing Universe, for though it cannot hope to be useful or informative in all matters, it does at least make the reassuring claim, that where it is inaccurate it is at least *definitively* inaccurate. In cases of major discrepancy it's always reality that's got it wrong."
>
> Douglas Adams, *The Restaurant at the End of the Universe*

1. Guidelines

In addition to the information in this guide, fragmentary documentation of the TPS3 code can be found in the *tpsjobs-done* file. This is included in the TPS3 distribution.

This guide assumes that the reader is familiar with Common Lisp, and does not attempt to explain or summarize information that is available elsewhere about the workings of that language, in particular, in Steele's *Common Lisp the Language, (2nd ed.)*.

There are three major rules which should be followed whether maintaining TPS3 code, or just fooling around with it:

(1) Always keep a backup copy of the files you are changing, so that when you realize how badly you goofed, you can put things back the way they were.
(2) Don't get too tricky. Clever hacks may be amusing, and may indeed give some increase (usually modest) in efficiency, but within weeks you will have no idea how they work, and others will be even more mystified. Those who follow you in your task will curse and despise

you; consequently, your cute programs will probably be completely rewritten anyway.
(3) Don't panic.

See section 9 for more minor guidelines.

2. Tps3 Conventions

2.1. Filenames. The extension of a filename should indicate what it contains: *.lisp* for Lisp source code; *.exp* for export statements (*vide infra*); *.rules* for deduction rule definitions (`defirule` statements); *.mss* for Scribe formatted documentation; *.tex* for TEXformatted documentation; *.vpw* for vpwindow output; *.work* for work files; *.prf* for proofs.

Filenames should be descriptive of their contents, without being too long. For example, *functions.lisp* would be a stupid name, because from the name no one would know what its purpose was. If you have several related files, it is a good idea to give them a common prefix, so that it is clear just from their names that they are related.

2.2. Lisp packages and export files. Tps3 creates and uses several different Lisp packages. (If you don't know what I mean by Lisp package, read the chapter on packages in *Common Lisp the Language, (2nd ed).*) These packages are created when Tps3 is compiled or built, by `make-package` forms in the files `tps-build.lisp` and `tps-compile.lisp`. The package structure is set up so that common functions are placed in the package `CORE`, which is used by each of the other packages. The package `MAINT` contains functions useful to the maintainer. The package `AUTO` contains the automatic portions of Tps3 , and the package `ML` contains the inference rules used in the mathematical logic system. The `TEACHER` package contains files relevant to GRADER.

Within each Lisp package are several Tps3 modules, groups of related source files which are clumped together. These are defined in the file *defpck.lisp*.

The `CORE` package contains functions such as those dealing with wff parsing and printing, proof manipulation, and operating system interfaces (such as basic file operations). It also contains functions for dealing with Scribe, vertical paths, editing, windows, review, etc... The other packages include: `TEACHER`, for functions relating to the Grader subsystem; `AUTO` for functions relating to automatic proof procedures such as mating search; `ML` for things specific to the Math Logic courses, such as exercises and proof rules.

Etps contains part of the `CORE` package, part of the `OUTLINE` package and part of the `RULES` package.

The idea is that only those symbols that are needed by other packages are exported from their home package. In order to specify which symbols should be exported, the files *core.exp*, *auto.exp*, etc. These files, one for each Lisp package, are loaded at the beginning of the compilation process, before any code is loaded. This way, any package conflicts are detected immediately.

There is a special export file, called *special.exp*. This file contains export statements for symbols which may already exist in certain Lisp implementations. For example, some implementations already contain a symbol `EXIT`, while others do not. Why is this a problem? Because if the `CORE` packages uses a package from an implementation (e.g., Allegro's `EXCL` package), and that package already exports the symbol `EXIT` (so that `EXIT` is imported by `CORE`), then an error will result if we try to export `EXIT` from the `CORE` package, i.e., you can't export a symbol from a package other than its *home* package. *special.exp* uses the standard `#+` and `#-` macros to specify in which implementations such symbols should be exported from the `CORE` package. Generally, these nuisance symbols are found by trial and error when first porting TPS3 to a Lisp implementation, and some symbols may have to be moved from *core.exp* to *special.exp*. (Another symbol in $t(CORE) that can cause problems with some Lisps is `date`.)

Note that when TPS3 starts up, the `USER` (soon to be `COMMON-LISP-USER`, as the changes in Common Lisp suggested by the X3J13 committee are implemented) is the value of the variable `*package*`. What this means is that any symbols typed in by the TPS3 user will be interned in the `USER` package. Thus, any symbols that could be inputted by the user as, say, a flag value, should be exported from the package in which they were defined, otherwise TPS3 will not realize they are supposed to be the same. As an example, the flag `RULEP-MAINFN` can be given the value `RULEP-SIMPLE`. Since `RULEP-SIMPLE` is defined in the `CORE` package, it must be exported in *core.exp*, so that when it is inputted, the symbol `CORE::RULEP-SIMPLE` is interned, not `USER::RULEP-SIMPLE`. Of course, this presumes that the `USER` package uses the `CORE` package (which it always does in TPS3).

2.3. Implementation-specific differences. Not all Lisp implementations are alike. This is particularly true in the areas of Common Lisp which are intentionally unspecified, including things like how the Lisp top level works, how file pathnames are represented, how the user exits the Lisp or saves a core image.

For this reason, certain TPS3 source files contain `#+` and `#-` directives. We try to keep the number of these files to a minimum, so that when porting to new implementations, work is minimized. When using `#+` and `#-`, you should try to use as specific a feature of the implementation as possible (but avoid using the machine type unless that is the reason you have to make a change). For example, the feature `:allegro-v3.1` is probably better than `:allegro`, as I have found out to my dismay when Allegro 4.0 came out. Look at the lisp variable *features* to find what features that version of lisp recognizes. A few examples of features are listed below:

- `:allegro` (Allegro Common Lisp)
- `:clisp` (Gnu Common Lisp)
- `:cmu` (CMU Common Lisp)
- `:mswindows` (Microsoft Windows)

There is one feature (`:andrew`) that is added when we compile TPS3 /ETPS for use on the Andrew workstations (machines in the domain andrew.cmu.edu). This is because on those machines, lisp implementations have problems interfacing with the operating system and getting the proper home directory of a user. Thus special measures are taken in this case. This feature is added in the `.sys` files for the Andrew editions.

There are another two features which are added in the relevant `.sys` files for ETPS and TPS3 ; these are :TPS and :ETPS. This allows programmers to specify slightly different behaviour for the two systems (for example, when using the editor, you may have a window that shows the vpform in TPS3, but not in ETPS).

The files which use `#+` and `#-` are principally *special.exp*, *boot0.lisp*, *boot1.lisp*, *tops20.lisp*, *tps3-save.lisp*, and *tps3-error.lisp*.

2.4. TPS3 **modules.** TPS3 source files are organized into TPS3 modules. Basically, a TPS3 module is just a list of source files, in the sequence in which they are to be compiled/loaded. All TPS3 modules are defined in the file *defpck.lisp*. Each source file should be in some TPS3 module, and that module should be indicated in the file. (Conceivably, one might define two different TPS3 modules which had files in common, but we have never done that.)

Some files are designated as macro-files in the definition of the module in *defpck.lisp*. When a module is compiled [loaded], the macro-files are compiled [loaded] first. Also, there is code for loading the macro-files for a module without loading the other files. When adding a file *foo.lisp* to a module, designate it as a macro-file if many of the other files in that module use structures, macros, or variables defined in *foo.lisp*.

In addition to the files it contains, the definition of a TPS3 module also specifies the other modules which must also be present when it is used.

The TPS3 module structure breaks up the source files into chunks, each which has some particular purpose or purposes. Then to build a version of TPS3 which has certain capabilities, one need only load the modules required. This is how the files *tps-build.lisp* and *tps-compile.lisp* specify TPS3 is to be built. Note that *etps-build.lisp* and *etps-compile.lisp* just load fewer modules than the build/compile files for TPS3. Likewise, there are *grader-compile.lisp* and *grader-build.lisp* files for building a Grader core image.

By using the module mechanism, a module may be modified by adding, deleting, or modifying its constituent files, and other users don't have to know; all they need to know is what the module provides.

Functions such as `LOAD-MODULE` are provided to load modules, making sure that any modules they require are also loaded.

2.5. File format. In general, programmers should use only lower-case. Why? For two reasons. It is easier to read, and in case-sensitive operating systems like Unix, it is easier to use utilities such as fgrep and gnu-emacs tags (*vide infra*) to search for occurrences of symbols.

Each TPS3 source file should contain certain common elements. First is a copyright notice, whose purpose is self-explanatory (just copy it from another source file). Make the copyright date current for any new code.

The first line of the file, however, should be something like:

```
;;; -*- Mode:LISP; Package:CORE -*-
```

The gnu-emacs editor will use this line to put the buffer in Lisp mode, and if you are using one of the gnu-emacs interfaces to the lisp, it will use the package information appropriately. See the documentation for such gnu-emacs/lisp interfaces.

The first non-comment form of each source file should be an `in-package` statement, to tell the compiler what package the file should be loaded in. Recent implementations of lisp will object if there is an `in-package` command anywhere else in the file.

Next, the TPS3 module of the file should be indicated, by a `part-of` statement, like `(part-of` *module-name*`)`. This should match the entry given in *defpck.lisp*.

Don't forget `context` statements. Basically they just reset the variable `current-context`, which is used by other functions to organize the documentation and help messages.

3. Maintenance

3.1. Porting TPS3 to a new Lisp. As discussed above, the lisp-implementation-dependent parts of TPS3 are confined to a few files. See the discussion above (Section 2.3) and the portion of the user's manual on setting things up for more details. The following is a list of steps for compiling TPS3 under a new lisp.

(1) Modify the line in the Makefile of the form

    `lisp = <lisp-executable>`

    so that `<lisp-executable>` is the executable for the new lisp.

(2) Try to perform `make tps`. If you are very lucky, this will work. However, probably the first problem you will encounter involves conflicts with respect to exporting symbols. We explain how to resolve these conflicts in step (3). Once these conflicts are resolved, go on to step (4)

(3) Suppose the implementation of lisp complains about a conflict with an exported symbol `tps-symbol`. We try to keep such conflicts localized to the file special.exp. If `tps-symbol` is exported in the file special.exp, then add a compiler directive `#+` (or `#-`) to the export for `tps-symbol`. If `tps-symbol` is exported in some other export file, then move this export to special.exp and add an appropriate compiler directive. (See Section 2.3 for more information about compiler directives and the *features* lisp variable.)

(4) The next thing you will likely need to do is add definitions for certain functions and macros. For example, the functions tps3-save,

linelength, exit-from-lisp, status-userid, call-system, setup-xterm-window, setup-big-xterm-window, make-passive-socket, make-passive-socket-port, connect-socket, accept-socket-conn and pass-socket-local-pass are defined for each implementation in the file tops20.lisp. The definitions of these functions for other versions of lisp should indicate what the definition should be for the new implementation of lisp. If you really do not know how to define the function, you can always define the function as the following example indicates:

```
#+:newlisp
(defun call-system ()
  (throwfail ‘‘call-system undefined in lisp <newlisp>’’))
```

Such definitions will limit some of the capabilities of TPS.

Among these functions, tps3-save is the most vital for getting started. The function tps3-save should create a core image file which will be used when starting TPS3 .

(5) Call `make tps`. If all goes well, a core image file will be created and you are ready to run TPS3 . Check the documentation for the implementation for lisp to find out how to start lisp with a given image file.

If you are using Allegro Common Lisp version 4.1 or later, `--` is used to separate user options from lisp options, and hence the standard way of starting up the Grader program in X-windows becomes:

```
   xterm -geometry 80x54-14-2 ’#723+0’ -fn vtsingle -fb vtsymbold-sb
-n CTPS-Grader -T CTPS-Grader -e /usr/theorem/bin/run-tps
-- -grader &+
```

Use of defconstant: Since files may be loaded more than once during compilation and building of TPS3 , two identical uses of `defconstant` may occur. As the behaviour of the second occurrence of `defconstant` is not specified in the ANSI standard, and as some Lisps, such as Steel Bank Common Lisp, do not consider this use as correct, it is better to use the macro `defconstnt`, which has been implemented to fit `defconstant` and still be used with SBCL. (Note that ’defconstnt’ has no ’a’ at the end.)

Obsolete In 2004: Also, if you are using Kyoto Common Lisp, you will find that the way it represents directories is a little unusual: all paths are relative unless specified not to be. So, for example, *tps3.sys* should be changed to read:

```
(setq news-dir ’(:root "usr" "tps"))
(setq source-path ’((:root "usr" "tps" "bin")
                    (:root "usr" "tps" "lisp")))

(setq compiled-dir ’(:root "usr" "tps" "bin"))
(setq patch-file-dir ’(:root "usr" "tps"))
```

(assuming the main TPS3 directory is */usr/tps/*).

3.2. Building TPS3**.** See the user's manual for a description of how to set up and build a new version of TPS3/ETPS .

The global variable core-name currently contains "TPS3"; it is defined in *tps3.sys*, which is generated by the Makefile. All files (news, note, ini, sys, patch, exe) use core-name as their 'name'.

File names and extensions should be strings rather than quoted symbols, to avoid any ambiguity with the package qualifiers.

Changes to the code are put in the patch file *tps3.patch* until TPS3 is rebuilt. ETPS and Grader have separate patch files. When you change the file *nat-etr.lisp* (for example), put the line `(qload "nat-etr")` into *tps3.patch*. In general, don't put `(qload "nat-etr.lisp")` into the patch file, or the uncompiled version of the file will be loaded. However, the export files **.exp* do need their extension.

Entries such as `(qload "auto.exp")` which load exp files should come before those loading lisp files. `(qload "core.exp")` should come before loading other export files. Macro files should come before other files in the same module.

Putting the line `(setq core::*always-compile-source-if-newer* T)` near the beginning of the *tps3.patch* file, and
`(setq core::*always-compile-source-if-newer* NIL)` at the end of the same file will cause files to be compiled automatically whenever appropriate as one is starting up TPS3, but then restores the default value of `*always-compile-source-if-newer*` so that you will be able to decide whether or not to compile other files as you load them.

Example: when ms91-6 and ms91-7 were introduced, *tps3.patch* contained:

```
(qload "core.exp")
(qload "auto.exp")
(qload "defpck")
(qload "contexts-auto")
(load-module 'ms91)
(qload "diy")
```

3.3. Memory Management. TPS3 uses a huge amount of memory in the course of a long search, and it may be necessary to rearrange either the internal memory available in your computer or the maximum space occupied by your version of Lisp. Both of these things vary; the former by system (type `sys` to find out what system you are using) the latter by the variety of Lisp. You can tell roughly how much memory is being used in most versions of Lisp by turning on garbage collection messages and watching the numbers they report.

After a long search, TPS3 may fail with an error message that mentions not having enough heap space, or stack space, or swap space. Allegro Lisp is very good about indicating the real cause of the problem. CMU lisp turns off errors while it garbage collects, and unfortunately that's when most of

these errors occur, so if your CMU-based TPS3 seizes up in mid-garbage collect and refuses to stop even for ^C, then you've probably run out of memory somewhere. Lucid Lisp turns off garbage collection when it approaches the internal memory limits (there is a good reason for this; see the Lucid manual), so if you get a message about garbage collection being off then the real problem is probably a lack of memory. (TPS3 never switches garbage collection off itself.)

3.3.1. *Heap Size and Stack Size.* On a Unix system, type `limit` into a C-shell (or whatever shell you're using) to see a list of the upper limits on various things stored in memory. The ones you're most interested in will be datasize and stacksize. If you are superuser, you can remove these restrictions temporarily by typing `unlimit datasize stacksize`, or possibly `unlimit -h datasize stacksize`.

To increase these limits permanently, you need superuser privileges. You will need to reconfigure the kernel and reboot your system. On anything except an HP, write to `gripe@cs` and ask them to do it, unless you're confident about being able to do such things. On an HP, you can use their SAM program (when nobody else is logged in, since you're going to do a reboot), as follows:

(1) Log in as superuser, and type `sam`.
(2) Double-click on "Kernel Configuration"
(3) Double click on "Configurable Parameters"
(4) Highlight the parameter "maxdsiz" and select "Modify" from the "Actions" menu. Increase the value as high as you want. On our machines, it was initially 0x04000000 and we increased it to 0x7B000000. If you choose too high a number, it will be rejected and you can try again.
(5) Check that the "value pending" column shows your new value for maxdsiz. If not, pick "Refresh Screen" from the "Options" menu and do the last step again.
(6) Now do the same for "maxssiz"; we increased it from 0x00800000 to 0x04FB0000.
(7) Choose "Exit" from the "File" menu. You will get a barrage of questions, say yes to all of them. (They will be something like: create the kernel now? replace the old kernel? reboot the system?)

When the reboot is done, type `limit` to check that the values have increased.

3.3.2. *Swap Space.* Swap space is that part of the memory (usually on disk) where the operating system stores parts of the programs that are supposed to be in memory. This is how you can get away with running more programs than your RAM has space for. Clearly, the amount of swap space you need will depend not only on how big your TPS3 grows, but also on what else is running at the same time.

Again, on anything but an HP it's time to go whining to `gripe@cs` and get them to do it. On an HP, start SAM as in the last section, and double-click on "Disks and File Systems". Now double-click on "Swap". There are two sorts of swap space, device (`dev`) and file system (`fs`). The former is faster and should be given priority over the latter.

Here is where I don't quite understand what's going on, so if this information ever becomes crucial it would be a good idea to check it. I believe that device swap space is simply a partition of the internal disk drive, and that it might be possible to create more space simply by rearranging the partition. I have no idea how to do this.

For the time being, then, we'll restrict ourselves to filesystem swap space. You can mount one filesystem swap space on each disk you've got, so take a look at the list that SAM has given you. If there are no `fs` swap space listed, or there is a disk that doesn't have one, then you can create one by selecting "Add Filesystem Swap" from the "Actions" menu. Give it a reasonable number (you can use `du` and `df` to find out how much space there is on the disk at the moment, and then choose some large fraction of that), and allocate a priority that is lower (which is to say, a larger number; 0 is highest-priority) than the priorities of the `dev` swapspace (so that you will use the fast swap space before the slow one). New swap space takes effect right away.

If you already have `fs` swapspace on all disks, you can highlight the one you want to change and then choose "Modify Swap Space" from the "Actions" menu. Increase the size as you want. Modifications only take place after the next reboot, but it is not necessary to reboot right away as it is for the heap and stack space.

3.3.3. *Internal Limits in Lisp.* As if all that wasn't enough, your version of Lisp may also have some constraints on how large it can grow.

(1) CMU Lisp has no such limits, as far as I know.
(2) Lucid Lisp has them, and they are user-modifiable; type `(room t)` into a Lucid TPS3 to see what the current settings are. Look for "Memory Growth Limit"; if it seems too small, type (for example) `(change-memory-management :growth-limit 2048)` into the TPS3 to allocate 128Mb (2048 64kb segments). You can also make this permanent by adding `#lucid(change-memory-management :growth-limit 2048+)` to your *tps3.ini* file. Other parameters besides the overall size limit can also be changed; see the Lucid manual for details.
(3) Allegro Lisp also has a limit, but in this case it is set at the initial building of Lisp. Here you'll have to retrieve the build directory for allegro (which is */afs/cs/misc/allegro/build/* followed by the name of your system). We have a copy of this on *tps-1*, called *allegro4.2hp_huge*, but it requires some hacking to make it build properly. Follow the instructions in the README to build yourself a new Lisp core image with more than the standard 60Mb data

limit. If you aren't up to the hacking, once again the solution is to whine at `gripe@cs`, who will forward your mail to the Allegro maintainer.

3.4. Tps3 distribution.

3.4.1. *Making a tar file for Distribution.* You can execute `/afs/andrew/mcs/math/TPS/admin/tps-` from `/home/theorem/project/dist` (The date is computed automatically.)
The tar file is placed in /home/ftp/pub.

```
KEY STEPS:
     Login to gtps as root (so that you can write a file in /home/ftp/pub).
su
     (You may need to do a klog, e.g.,
klog pa01 -c andrew.cmu.edu
      or
klog cebrown -c andrew.cmu.edu
         so you appropriate permissions.)
cd /home/theorem/project/dist
/afs/andrew/mcs/math/TPS/admin/tps-dist/make-tar.exe

    [This does
     tar cvhf /home/ftp/pub/tps3-date.tar .
     gzip /home/ftp/pub/tps3-date.tar
     ln -sf /home/ftp/pub/tps3-date.tar.gz /home/httpd/html/tps3.tar.gz
      ]
    (It's important that the tar file is not put into the same directory
      as is being tarred, or it will try to work on itself.)

cd /home/ftp/pub
move old tar file to the subdirectory old. Delete the older one.
```

3.4.2. *Distribution of* Tps3 *via http.* There is a perl script `/home/httpd/cgi-bin/tpsdist.pl` which is used to distribute Tps3 via the gtps web site. This perl script displays the distribution agreement and asks for information from the remote user (name, email, etc.). Once this information is given, the perl script updates the log file `/home/theorem/tps-dist-logs/tpsdist_log`. (For this to work, `apache` should be the owner of this log file.) Then, the perl script outputs instructions and a link to the tar file.

3.4.3. *Obsolete Information about Making tar tapes of* Tps3. To make a tar archive onto a big round mag tape (the kind that is seen in science fiction movies of the sixties and seventies, always spinning aimlessly, supposedly to suggest the immense computing power of some behemoth machine):

(1) Go to the CS operator's room on the third floor, at the end of the Wean Hall 3600 hallway.
(2) Tell the operator that you wish to write a tar tape from the machine K.GP. Give her the tape and go back to the terminal room.

(3) Log in on the K.
(4) At the Unix prompt, enter `assign mt`. This gives you control of the mag tape units.
(5) Enter `cd /home/theorem/project/dist`. This puts you in the proper directory.
(6) Clean up /home/theorem/project/dist and its subdirectories by deleting each backup, postscript, or dvi file.
(7) Determine which device *devname* you wish to use. This depends on the density which you wish to write. For 6250 bpi, let *devname* be `/dev/rmt16`, for 1600 bpi, let *devname* be `/dev/rmt8`, and for 800 bpi, let *devname* be `/dev/rmt0`. Generally, you can go for 6250 bpi unless the intended recipient has indicated otherwise.
(8) Execute the following command at the Unix prompt: `tar rvhf` *devname* `.`
(9) A list of file names should pass by on the screen. Just watch until you get a new Unix prompt.
(10) To check the tape, enter `tar tvf` *devname*. The same list of file names should pass by. These are the names of the files which are now on the tape. If there were already files on the tape, you will see all of them listed as well.
(11) If all is well, call the operator and tell them that you are done with the tape and that they can dismount it. Then execute `exit` at the Unix prompt, to give up control of the tape drive, and log out as usual.
(12) Go back to the operator's room and pick up the tape.

To make a tar archive onto a Sun cartridge tape:

(1) Take the tape to the CS operator, and ask her to put it on the machine O.GP. That is a Sun3 running Mach. Go back to a terminal and log in on any machine. Again, you want to be in the directory /home/theorem/project/dist.
(2) Now, you need to make sure that you can write to the O's tape drive. You want to check the owner of the file /../o/dev/rst8, and make sure it's `rfsd`. If not, call the CS operator and ask them to assign it to rfsd so that you can make the tar tape.
(3) Now, execute the following command at the Unix prompt: `tar cvhf /../o/dev/rst8 .`
(4) A list of file names should pass by on the screen. It will be very slow.
(5) After you get a new Unix prompt, wait a few seconds for the tape to rewind, then check it by entering `tar tvf /../o/dev/rst8`. The same list of file names should pass by. These are the names of the files which are now on the tape.
(6) Go back to the operator and ask for the tape.

4. How to locate something?

Sometimes you will be looking at code, and will come across a function or variable whose purpose is not familiar to you. If it is not a standard Common Lisp function, for which the Lisp functions `documentation` and `apropos` may be useful, as well as reference books and user manuals, there are three ways to find where it is defined.

The first method uses the gnu-emacs tags mechanism. Periodically, we run the `etags` program on the *.lisp* files in the source directory. One does this by entering the TPS3 lisp directory and then running the etags program; usually, this is done by typing `M-x shell-command etags *.lisp`. This generates a file called *TAGS*, with entries for each line of code which begins with `(def...`. Then you can use the gnu-emacs `find-tag` function (ESC-. , unless you've rearranged the emacs keys) to look for the first occurrence of the symbol, and the `tags-loop-continue` function (ESC-,) to find the each subsequent occurrence. This can be slow if there are many symbols which begin with the prefix for which you are searching, or if the symbol is overloaded by defining it for different purposes (e.g., `LEAVE` is a matingsearch command, a review command, a unification toplevel command). See the gnu-emacs documentation.

Certain functions, such as eproof-statuses, are defined implicitly, and you won't find their definitions using the tags mechanism. If you look at the definition of the structure eproof in the file etrees-flags.lisp, however you will find:

```
(defstruct (eproof (:print-function print-eproof))
...
  (statuses (make-hash-table :test #'eq))
```

This defines the function eproof-statuses.

The second method is to use the TPS3 export files. Try examining the files with a *.exp* extension. Generally, comments tell which file each symbol comes from. This method will fail, however, if the symbol is not exported, or if the symbol has been moved from the file in which it was originally defined without the *.exp* having been updated.

TPS3 has many global lists; the master list is called global-definelist, and in general each sort of TPS3 object will have an associated global list.

The last method is to use operating system utilities like grep and fgrep to find all occurrences of the symbol.

5. Utilities

Utilities are commonly-used Lisp functions/macros. The functions (or macros) themselves are defined in the normal way, and then a defutil command is added into the code beside the function definition. The point of adding the `defutil` command is that utilities have their own TPS3 category, you can get online help on them, and their help messages are printed

into the Facilities Guide; this will help other TPS3 programmers to find them in the future.

Examples are such functions as msg and prompt-read; see the facilities guide for a complete list.

There aren't really very many utilities at the minute, although it would be useful if more were defined, since then we could avoid duplicating code in different places. So, if you write a useful macro or function *foo*, or discover one already written, please add a utility definition next to it in the code. This should look like:

```
(defutil foo
   (Form-Type function)
   (Keywords jforms printing)
   (Mhelp "Some useful words of wisdom about the function foo."))
```

`Form-Type` should be either `function` or `macro`. `Keywords` can be anything you want, since it is currently ignored by TPS3. `Mhelp` is, of course, a help message. Note: if your useful function is actually an operation on wffs, it should be defined as a wffop or wffrec (recursive wffop) rather than as a utility; utilities are really intended to be functions that are useful to TPS3 programmers but which do not fall into any other TPS3 category.

6. Overloading Commands

There are certain symbols in TPS3 that been *overloaded*, that is they have been defined to have more than one meaning: they may be simultaneously a matingsearch command, review command, and unification command. This is done so that same symbol can have similar effect in different top-levels. For example, *LEAVE* should leave the current top-level, as opposed to having a different exiting command for each top-level, which would make things more difficult for the user to remember.

This can cause problems in TPS3 unless programmers are careful. You see, we currently use the symbol's property list extensively to store things. When a matingsearch command (such as `LEAVE`) is defined, the actions that are to be taken when the user inputs the command are stored on `LEAVE`'s property list. It is important, therefore, that each category use different property names, so that there is never a clash. For example, if we used the property `ACTION` for both review commands and matingsearch commands, then `LEAVE`'s property list could not hold both simultaneously, but merely one or the other. Better property names would be `REVIEW-ACTION` and `MATE-ACTION`.

7. Output

Some general tips for keeping the output as neat as possible:

- Avoid using the lisp function y-or-n-p, and stick to the TPS3 function prompt-read, so that the responses will go into work files correctly.

- msg and msgf (which is like `msg` but adds a linefeed if necessary) are TPS3 functions for producing output. These functions take a sequence of arguments, and evaluate and print out each argument in an appropriate format; an argument t means go to a new line. See `defutil msg`.
- `(msg (gwff1 . gwff))` will print out the correct representation of the gwff, whereas `(princ gwff1)` will just print its internal representation.
- To insert a call to runcount in the code: `msgf (runcount)`
- stringdt gives the time and date. stringdtl also inserts linefeeds
- `princ` often puts messages into a buffer. To get them to print out, add the command `finish-output`. You may also have to do this when you use other output commands, including msg.
- Windows (proofwindows, edwindows, vpwindows) all work by issuing a Unix shell command which runs an xterm which, in turn, runs the Unix "tail" command recursively on an output file that TPS3 creates by temporarily redirecting `*standard-output*`. (Compare such commands as SCRIPT and SAVE-WORK, which *permanently* redirect `*standard-output*`.) See the files tops20.lisp, prfw.lisp, edtop.lisp and vpforms.lisp for more information.

8. Compiling as much as possible

In defining new TPS3 objects, we often define as a side-effect new functions. For example, when defining a new argument type, we define a testfn and a getfn for that type, based on the values for those properties that are given in the `deftype%` form.

Currently, all such functions are compiled, by cleverly defining the definition macros so that `defun` forms are created during the compilation of a file. If you define new categories that will create such functions, you will want to do something similar, so that you aren't always running interpreted code. See the files *argtyp.lisp* and *flavoring.lisp* for examples of how this can be done.

9. Writing New Code Without Making A Nuisance of Yourself

- Programmers should avoid referencing internal symbols of different LISP packages. If you are doing this, think about why it is necessary. Perhaps it is better to export the symbols, or rethink the package structure.
- Symbols should be exported before files containing them are compiled. Otherwise you stand the risk of having those symbol-occurrences interned in the wrong package.
- Lisp macros can be very useful, but it is easy to overuse them. It can be very difficult to debug code that uses many macros, and because there is no guarantee that macros will not be expanded when code is

loaded (and they are always expanded when compiled), modifying a macro means recompiling every file in which it appears, which is quite a nuisance.

- There are a multitude of functions in TPS3, so one must be careful not to inadvertently redefine a function or macro. With the Lisp function `APROPOS`, you can check to see whether a function name is already being used. Use the TAGS table. See section 4, above.
- Try not to re-invent the wheel; look in all the likely places to see if some of the code you need has already been written. If your new construct is similar to an existing one, use `grep -i` in the directory */afs/andrew/mcs/math/TPS/lisp/* to find and examine all uses of the existing construct.
- Remember that rules of inference should be written as *.rules* files and compiled with the ASSEMBLE-RULE command; if you modify the *.lisp* files directly, you run the risk of having your modifications accidentally overwritten by future users.
- When modifying copies of existing files, prior to installing them, rename the file temporarily (for example, preface the filename with your initials) so that if you compile your new code it won't overwrite the existing compiled file.
- Don't install code until you've tested it! After installation, keep backup copies of the old files in the */home/theorem/project/old-source/* directory on gtps, and change their extensions from *.lisp* to *.lisp-to-1997-jan-3* (or whatever). Delete all Emacs backup files from the main lisp directory. Compile new code using the CMU Common Lisp version of TPS3 since that compiler is fussier than most.
- Try to make sure that online documentation is included in all user functions, argument types, etc. that you define. Also, you should at the very least put comments in your code; better yet, write some documentation for the manuals. Note that online help can be associated with any symbol using the definfo command.
- If a new subject has been created, and this subject contains flags important for automatic search, the function mode flagging.lisp should be updated. The code starts as

```
(defun mode (mode)
  (let ((already-set nil))
    (unless (eq mode (gettype 'tps-mode 'maint::quiet))
      (dolist (subject '(IMPORTANT MATING-SEARCH MS88 . . .
 UNIFICATION PRIMSUBS MTREE-TOP))
```

The list in the dolist contains all the subjects important for automatic search. The new subject should be added to this list. The purpose of the list is to make sure important flags have their default value if they are not explicitly set by the mode.

- When a new part of TPS3 is developed, an appropriate module should be defined in */afs/andrew/mcs/math/TPS/lisp/defpck.lisp*. If a new file is being added to an existing module, just add it to the list in defpck.lisp, make sure the correct heading is on the file, and export the filename from */afs/andrew/mcs/math/TPS/lisp/<package>.exp*. (Actually, the exporting should be done automatically by TPS3, but it won't hurt to do it manually as well.)
- If a new package or module has been added, it must go into all the build and compile files for ETPS and TPS3. (See, for example, */afs/andrew/mcs/math/TPS/common/tps-compile.lisp*.) In general, it should go into the ends of the list of modules, so that definitions it depends upon will be loaded first. If a new module is added, be sure to add it to the *facilities.lisp* and *facilities-short.lisp* files, otherwise it won't show up in the facilities guide.
- After installing new code, remember to change the patch files, the *tpsjobs* file and the *tpsjobs-done* file, and to send a mail message to the other people working on the program.

10. Debugging Hints

- Insert print commands in a temporary version of a file to see either which parts of the code are being used or what the current values of some variables are.
- Compile the file in several common lisps, especially in cmulisp (or tps3cmu), and see if the error messages are helpful.
- Try to reproduce the bug in a simpler form.
- See in how many different contexts (such as different matingsearch procedures) it arises, so you can isolate its essential features.
- Use the debugging features of your version of lisp (e.g. step and trace).
- Change the values of the flags QUERY-USER, MATING-VERBOSE, UNIFY-VERBOSE, TACTIC-VERBOSE, OPTIONS-VERBOSE, etc... to get more output.
- Use the monitor. (See chapter 10.)
- Use the lisp function `plist` to inspect the property list of an object. Use `inspect` to see the values of the slots in a structure.
- Errors in translation, or errors during verification of a merged jform ("The formula is not provable as there is no connection on the following path") are usually caused by merging. See the chapter on merging, and in particular the note about using merge-debug.
- The code in the file etrees-debug can be useful for tracking down bugs involving etrees. (See subsection 1.1.)
- Errors of the form "Wff operation <wffop> cannot be applied to labels of flavor <label>" are almost always caused by attempting

to use a wffop on a flavor for which the corresponding property is undefined. See the section on flavors for more details.

- Errors in structure-slot-accessor are often of the form "Structure for accessor <foo-slot> is not a <foo>". For every structure <foo>, there is a test <foo-p>; use it! Of course, you should also work out how something that wasn't a <foo> managed to turn up at that point in the program anyway; often, it's an exceptional case that you forgot to handle.
- In Allegro, the function dumplisp can be used to save a Lisp image. For example, `(excl:dumplisp :name ‘‘saved-image.dxl’’)` will create a (large) file named "saved-image.dxl". Then one can use `lisp -I saved-image.dxl` to start lisp specifying this as the image file. This will start lisp in the same state (e.g., the global variables will have the same values) as when dumplisp was called. This is especially useful if the bug shows up after running for a long time.
- If the bug is new (for example, if you know it wasn't there last month), don't forget that the *tpsjobs-done* file lists all of the files which have been changed, along with the reasons for each change and the date of each change. The *tps/tps/old-source/* directory should contain backup copies of the changed files. Failing that, snapshots of the entire lisp directory (in the form of gzipped tar files made after each rebuild) are stored in the *tps/tps/tarfiles/* directory. Use cload to restore the old copies of the most likely culprit files into a core image, until the bug disappears; then use ediff to compare the old and new files.

11. Miscellaneous

11.1. Counting Flags. One can count the number of flags in TPS3 as follows:

```
[btps]/afs/andrew/mcs/math/TPS/lisp% grep -i defflag *.lisp > flagcount
[btps]/afs/andrew/mcs/math/TPS/lisp% ls -l flagcount
{\it Edit the file flagcount to eliminate lines which do not define flags}
[btps]/afs/andrew/mcs/math/TPS/lisp% wc flagcount
     210     421    8327 flagcount
{\it The number of lines (210} is the number of flags.)
[btps]/afs/andrew/mcs/math/TPS/lisp% rm flagcount
```

The above counts the number of flags defined in the source code. The number currently present in a particular version of TPS3 can be found as follows:

```
(defun discard (list)
(if (null list) nil
  (if (or (listp (car list)) (memq (car list) (cdr list)))
;; if it’s a list, it’s a subject name, and we don’t want to count them.
;; if it appears later on, we don’t want to count it twice.
```

```
;; (may need to use franz:memq rather than memq)
    (discard (cdr list))
      (cons (car list) (discard (cdr list))))))

(msg "TPS has " (length (discard global-flaglist)) " flags.")
```

11.2. Dealing with X Fonts.

- To enable your computer to find the fonts, put into an appropriate (such as *.Xclients*) file: `xset` fp /tps/fonts/+, using the appropriate pathname in place of */tps/fonts/*. The fp+ adds the new directory at the start of the path, because we've had trouble in the past with old fonts with the same name being earlier on in the path.
- `xset q` shows the fonts.
- `xset fp-` takes them out of the fontpath.
- `xlsfonts` lists the fonts available Because the font list is usually very long, you may prefer to use `xlsfonts | grep <fontname>` to check whether the font <fontname> is available.
- `xfd -fn <fontname> &` shows all the characters in <fontname>

Dan Nesmith built the symbol fonts by starting with the vtsingle font, because xterm requires a font that is exactly the same size as vtsingle. However, every character is now different from the original; they were created by hand-colouring the pixels. It is, however, easier to edit an existing font than to create one from scratch.

The exact duplicates of the Concept ROM fonts are found in symfont1.bdf and symfont2.bdf, for a total of 256 characters (including a normal-sized epsilon). Unfortunately, because of the limits below on xterm and lisp, it was necessary to leave out some of these characters when making a single font, now called vtsymbold. The galsymbold font was created by splitting the font vtsymbold into 128 bitmaps, then using an X10 program that would automatically blow each bitmap to the proper size, then manually adjusting a few of the characters.

There is a real limitation of the xterm program in that you get only two fonts, one for normal text and one for bold text. We use the bold text font for symbols, and switch back and forth between the fonts by sending the appropriate escape symbols.

The code for the "appropriate escape symbols" are in xterm.lisp. In this file functions xterm-bold-font, xterm-normal-font and pptyox use codes to switch between bold and normal font. There is currently some confusion about how one switches to bold font. The code in xterm.lisp switches to bold by sending (in ASCII) `<ESC> [ 5 m`. However, it appears the official ANSI code for switching to bold is `<ESC> [ 1 m` while `<ESC> [ 5 m` is for blinking text:

```
  (see http://members.tripod.com/~oldboard/assembly/ansi_codes.html)

  ESC[n;n;...nm
```

```
Set Graphics Rendition is used to set attributes as
well as foreground and background colors.  If multiple
parameters are used, they are executed in sequence, and
the effects are cumulative. 'n' is one of the following
attributes or colors:

0  All attributes off
1  Bold
2  Dim
4  Underline
5  Blink
7  Reverse Video
8  Invisible
```

The working hypothesis (of Chad) at the moment is that the implemenations of xterm we have used for TPS3 render "blinking text" as bold, so that switching to blink is the same as switching to bold. In 2005, while using xterm version `X.Org 6.7.0(192)` for TPS3 , the symbols were not displaying as symbols. Instead, what should have been symbols were blinking normal text. This can be fixed by changing the codes in xterm.lisp to send `<ESC> [ 1 m` instead of `<ESC> [ 5 m`.

It is possible there was good reason why the original programmer (Dan Nesmith?) used blink instead of bold. Instead of explicitly making the change, there is a system flag XTERM-ANSI-BOLD with default value 53 (the ASCII code for 5). The value of this flag can be changed to 49 (the ASCII code for 1) when needed. Perhaps in the future the flag could be deleted and the change hard-coded once someone is confident of what the number should always be.[1]

It would be nice if xterm could render more special symbols. Though xterm would probably allow using more than the "printable" characters, this appears very hard to get lisp to do in general, that is, it's hard to figure out how to get lisp to send these characters to the terminal. This will probably never be completely implementation-independent, because character sets are a very unstable part of the lisp specification.

In TPS3 you will only be able to get the symbols between 32 and 127, basically because the lisp allows only those (without some kind of great hackery).

A few lines of attack for getting more characters suggest themselves:

(1) Hack xterm to allow more than 2 fonts. (In fact, Dan has done it to allow one more font, and thinks it would be possible to add up to two more, for a total of one normal font and three symbol

[1] In 2005 (and previously), a value of 53 worked for XTERM-ANSI-BOLD at CMU while using xterm version XFree86 4.2.0(165), and a value of 49 worked at Saarbrucken while using xterm version X.Org 6.7.0(192).

fonts.) The disadvantage here would be having to distribute the new version of xterm, and worrying about portability problems (which actually should be minimal, but with lots of different machines and versions of X out there, not predictable). But this approach is fairly easy to get to from the current state.

(2) Give up on xterm. There is a new version of gnu-emacs, version 19, which allows the use of more than one font in a buffer. You can then run TPS3 in a gnu-emacs buffer, and at the same time build in support for command-completion, hypertext documentation, etc. You can also get several windows from a single emacs now, so you can still have the editor stuff pop up a separate window. It might also be easier to bind in support for automatically running tex or scribe and displaying it. This requires someone who can hack gnu-emacs lisp, but a lot of this stuff has already been done by somebody, and it's just a matter of putting it together. In this case you could probably roll the fonts together into a single large one.

CHAPTER 2

TPS Structures

Notice that Tps3 has a command TLIST which outputs the same information as the Lisp command *plist*, but formatted more readably. So, for example, `TLIST X2108` will show all of the slots in the structure X2108 (which is a proof).

1. TPS Modules

See the introductory chapter for a discussion of what Tps3 modules are.

1.1. The Tps3 Module Structure. All modules are defined in one central file, called DEFPCK. You may want to look at this file to see examples of module definitions and also a current list of all module known to Tps3.

There is a partial order of modules in Tps3. One whole set of modules called `BARE` is distinguished from the others. All files in the module `BARE` and all of its submodules must always be present in a `TPS3` core image.

When Tps3 is built from **Lisp**, some of the files in the `BARE` module can not be loaded with a module-loading command, since it has not been defined. Thus, even though every file for Tps3 belongs to a proper module, not all modules are loaded the same way because of the "bootstrapping" problem.

Another quirk should be mentioned here. A module called `WFFS` defines the basic operations of wffs. The modules `WFF-PRINT` and `WFF-PARSE` depend on `WFFS.` The module `WFFS,` however, cannot exist alone: the modules `WFF-PRINT` and `WFF-PARSE` must be present also, even though this fact can not be deduced from the module structure.

1.2. Defining a New Module. To define a new module for Tps3, use the `DEFMODULE` macro. Its format is

```
(defmodule {\it name}
  (needed-modules {\it module} {\it module} ...)
  (macro-files {\it mfile} {\it mfile} ...)
  (files {\it file} {\it file} ...)
  (mhelp "{\it help-string}"))
```

`needed-modules:` These are all modules that must be loaded for the module *name* to work. Because of the transitive structure of modules only the direct predecessors of the new module need to be listed.

`macro-files`: These are the files the compiler needs, before it can compile any of the files in the module. It is generally a good idea to make a file with all the macro definitions (e.g. argument types, flavors of labels, etc.) and separate it from the functions, commands, etc. in the module. This means clearer program structure, but also minimal overhead for the compiler.

`files`: These are the rest of the files in the module. When the module is loaded, first the `macro-files` are loaded, then the `files`.

The new module should also be added into *defpck.lisp* at an appropriate point, and should be added into whichever of *tps-build.lisp*, *tps-compile.lisp*, *etps-build.lisp* and *etps-compile.lisp* are appropriate (these files are in the same directory as the *Makefile*, not the main TPS directory).

2. Categories

TPS3 categories are in a sense data types. A category is a way to characterize a set of similar objects which have properties of the same types, use the same auxiliary functions, are acted on by the same functions, etc.

Categories are orthogonal to the package/module structure, i.e. a category may have members which are defined in many different packages and modules. Categories group objects by functionality (how they behave) whereas packages and modules group objects by purpose (why they exist).

Categories are defined using the defcategory macro. For example, the definition of the category of TPS3 top levels is:

```
(defcategory toplevel
  (define deftoplevel)
  (properties
   (top-prompt-fn singlefn)
   (command-interpreter singlefn)
   (print-* singlefn)
   (top-level-category singlefn)
   (top-level-ctree singlefn)
   (top-cmd-interpret multiplefns)
   (top-cmd-decode singlefn)
   (mhelp single))
  (global-list global-toplevellist)
  (mhelp-line "top level")
  (mhelp-fn princ-mhelp))
```

This shows a category whose individual members are defined with the `deftoplevel` command, and whose properties include the prompting function, a command interpreter, and so on. There is a global list called `global-toplevellist` which will contain a list of all of the top levels defined, and an mhelp line "top level" (so that when you type `HELP MATE`, TPS3 knows to respond "MATE is a top level".) The mhelp-fn is the function that will be used to print the

help messages for all the objects in this category. (See chapter 8 for more information.)

The chapters of the facilities guide correspond to categories. Within each chapter, the sections correspond to contexts. In TPS3, global-categorylist contains a list of all the currently defined categories.

3. Contexts

Contexts are used to provide better help messages for the user. Each context is used to partition the objects in a category into groups with similar tasks. For example, the objects in the category `MEXPR` are grouped into contexts such as `PRINTING` and `EQUALITY RULES`. (Contexts are themselves a category, of course: the definition is in *boot0.lisp*.)

New contexts are defined with the defcontext command, and are invoked with the single line `(context` *whatever*`)` in the code (all this does is to set a variable `current-context` to *whatever*).

Here is a sample use of `defcontext`:

```
(defcontext tactics
  (short-id "Tactics")
  (order 61.92)
  (mhelp "Tactics and related functions."))
```

The only property which is not immediately self-explanatory is `order`; this is used to sort the contexts into order before displaying them on the screen (or in manuals).

Contexts are used in the facilities guide (for example) to divide chapters into sections. For example, the line `(context unification)` occurs prior to the definition `(defflag max-utree-depth ...)` of the flag MAX-UTREE-DEPTH in the file *node.lisp*, and so this flag occurs in the section on unification in the chapter on flags in the facilities guide.

To see the contexts into which the commands for a given top-level are divided, just use the ? command at that top-level. Look at global-contextlist in TPS3 to see all the contexts.

4. Flavors

Some TPS structures (in particular, all expansion tree nodes, expansion variables, skolem terms and jforms) are defined as flavors; see the file *flavoring.lisp* for the details. These structures have many attached properties which allow wffops to be used on them as though they were gwffs; for example, the flavor `exp-var` in *etrees-exp-vars.lisp* has the properties

```
  (type (lambda (gwff) (type (exp-var-var gwff))))
  (gwff-p (lambda (gwff) (declare (ignore gwff)) T))
```

which state that the type of an `exp-var` structure is the type of its variable, and all `exp-vars` are gwffs. Errors of the form "Wff operation <wffop> cannot be applied to labels of flavor <label>" are almost always caused by attempting to use a wffop on a flavor for which the corresponding property

is undefined; for example, if we deleted the lines above and recompiled TPS, any attempt to find the type of an expansion variable would result in the error "Wff operation TYPE cannot be applied to labels of flavor EXP-VAR".

Flavors that are defined within TPS will also have the slot `bogus-slot`; this slot is tested for by TPS to confirm that the flavor was defined by TPS, but the contents of this slot are never examined. This means that there is always one empty slot in each node of an expansion tree or jform which the programmer can use to store information while a program is being tested (whereas if you define a new slot, you have to recompile all instances of a structure, which can be a nuisance). Obviously, once the new code is working, you should define a new slot, change all references to bogus-slot and recompile TPS!

For examples of flavors of gwffs, see page 41.

CHAPTER 3

Top-Levels

1. Defining a Top Level

Top levels are a TPS3 category, whose definition is given in section 2. For an example, let's look at the editor top level:

```
(deftoplevel ed-top
  (top-prompt-fn ed-top-prompt)
  (command-interpreter ed-command-interpreter)
  (print-* ed-print-*)
  (top-level-category edop)
  (top-level-ctree ed-command-ctree)
  (top-cmd-decode opdecode)
  (mhelp "The top level of the formula editor."))
```

This says that the top level `ed-top` identifies itself by the function `ed-top-prompt`, which is one of the more complicated prompt functions in TPS3 ; its only purpose is to print the `<ed34>` messages at the start of each line in the editor, but the complications are necessary because the editor can be entered recursively.

The next line of the toplevel definition gives the name of the command interpreter function. The `print-*` function is a function that gets called after every line; in this case, it's the `ed-print-*` function, which prints out the current wff if it has changed due to the last command. The top level category is `edop`, which is defined as follows:

```
(defcategory edop
  (define defedop)
  (properties
   (alias single)
   (result-> singlefn)
   (edwff-argname single)
   (defaultfns multiplefns)
   (move-fn singlefn)
   (mhelp single))
  (global-list global-edoplist)
  (shadow t)
  (mhelp-line "editor command")
  (scribe-one-fn
    (lambda (item)
```

```
      (maint::scribe-doc-command
       (format nil "@IndexEdop(~A)" (symbol-name item))
       (remove (get item 'edwff-argname)
       (get (get item 'alias) 'argnames))
       (or (cdr (assoc 'edop (get item 'mhelp)))
   (cdr (assoc 'wffop (get (get item 'alias) 'mhelp)))))))
  (mhelp-fn edop-mhelp)))
```

This category defines the sort of command found in the editor top level (compare the above definition with that of `mexpr`, for example). So all the commands that can only be seen from the editor top level are defined with the `defedop` command, as follows:

```
(defedop o
  (alias invert-printedtflag)
  (mhelp "Invert PRINTEDTFLAG, that is switch automatic recording of wffs
in a file either on or off.  When switching on, the current wff will be
written to the PRINTEDTFILE. Notice that the resulting file will be in
Scribe format; if you want something you can reload into TPS, then use
the SAVE command."))
```

The `top-command-ctree` is used for command completion, and the `mhelp` property is obvious. This leaves `top-cmd-decode`, which is the name of the function that is called by the command interpreter to, for example, fill in the default arguments for an edop.

2. Command Interpreters

Each top level has its own command interpreter. The actual command interpreters in much of the code are older versions; the code has since been simplified considerably. New command interpreters, which may in time replace the older versions, and which should certainly be used as the models for the command interpreters of any new top levels, are in the two files command-interpreters-core.lisp and command-interpreters-auto.lisp.

CHAPTER 4

MExpr's

TPS3 provides its own top-level. It allows for default arguments and provides a way of giving arguments (e.g. wffs) in some external representation which is converted before the "real" function is called. All this is also available in an interactive mode, where the user is prompted for arguments after he has been told what the defaults are and which alternatives are open. The way all this has been implemented is through MExpr's, which constitute special functional objects analogous to Expr's or FExpr's in LISP. Every TPS3 command should be an MExpr so that the facilities of TPS3 ' top-level can be utilized.

1. Defining MExpr's

Mexprs are special functional objects that are recognized by the top level of TPS3. They can be defined with the `defmexpr` macro, which has a number of optional arguments. The general format is (indicate optional arguments)

```
(defmexpr {\it name}
 {(ArgTypes {\it type1} {\it type2} ...)}
 {(ArgNames {\it name1} {\it name2} ...)}
 {(ArgHelp {\it help1} {\it help2} ...)}
 {(DefaultFns  {\it fnspec1} {\it fnspec2} ...)}
         {(EnterFns {\it fnspec1} {\it fnspec2} ...)}
 {(MainFns {\it fnspec1} {\it fnspec2} ...)}
 {(CloseFns {\it fnspec1} {\it fnspec2} ...)}
         {(Print-Command {\it boolean})}
         {(Dont-Restore {\it boolean})}
 {(MHelp "{\it comment}")}
```

There are actually two other possible entries, `Wffop-Typelist` and `WffArgTypes`; these are only used in mexprs which are generated automatically by the Rules package.

In the following a *function specification* is either a symbol naming a function, or an expression of the form `(Lambda` *arglist* `.` *body*`)`. We also assume that the main function which is to perform the command has n arguments. Then the phrases in the above definition have the following meaning.

name: This is the name of the MExpr as called by the user.

`ArgTypes`: This is a list which must have as many elements as the function arguments, i.e. *n*. *type1*, *type2*, ..., *typen* have to be valid types, which means that they have to have a non-`NIL` `ArgType` property. Each argument supplied by the user on the command line will be processed first by the corresponding `GetFn`. In case an FExpr is to be called, each element of the argument list is presupposed to be of the same type. This type is specified in parentheses. If `ArgTypes` is omitted, the function has no arguments.

`ArgHelp`: This has to be a list of length *n*. Each element is a string describing the argument, or `NIL`. These quick helps for arguments can be accessed via the `?` when being prompted for the argument value. For an FExpr, there should be only one string.

`DefaultFns`: The *fnspecs* declared in this place are being processed in a left-to-right order, where the result of one *fnspec* is passed on to next. A *fnspec* can signal an error (a `THROW` with a `FAIL` label) if the arguments seem to be contradictory (e.g. if a planned line and a term is supplied for a `P`-rule, but the term does not appear in the proof), but it can count on the arguments being of the correct type and in internal representation.

In detail, each default *fnspec* must be either a symbol denoting a function of *n* arguments, where *n* is the number of mexpr arguments, or else a lambda expression of *n* arguments. Each *fnspec* must return a list of length *n*. This list will then be handed on and processed by the next *fnspec* as if it were the list of arguments supplied by the user. Any entry which is not a `$` should be left unchanged. The function is not allowed to have side-effects. As a general convention, the arguments which are not used by a *fnspec* are not written out with their name, but replaced by `%`*i*. This makes it easier to see at one glance which defaults are filled in by a certain *defaultspec*.

`EnterFns`: *fnspec1*, *fnspec2*, ... is an arbitrary list of function specifications. They are called in succession with the value list returned by the last default *fnspec*, before the `MainFns` are called.

`MainFns`: *fnspec1*, *fnspec2*, ... is an arbitrary list of function specifications. They are called in succession with the value list returned by the last default *fnspec*. If none are specified, it is assumed that there is a function named *name*, which can be called. Notice that at this stage, no defaulted arguments may be left. `ComDeCode` (the command processing function) will refuse to call any function, unless all the defaults are determined. This clearly divides the responsibilities between `GetFn`'s, `DefaultFn`'s and `MainFn`'s. Any *fnspec* may abort with an error by doing a `THROW` with a `FAIL` label. A `THROW` with a `TryNext` label will be handled like a normal return. A `THROW` with a `CutShort` label means that none of the remaining `MainFn`'s will be executed and the value of the `THROW` will be handed on to the `CloseFn`'s.

`CloseFns:` *fnspec1*, *fnspec2*, ... is a list of function specifications. They are called in succession with the value returned by the last `MainFn`. Even if the `MainFn`'s were FExpr's, each *fnspec* has to describe an Expr.

`Dont-Restore:` *boolean* determines whether or not this command will be restored, if it is saved using SAVE-WORK. For example, commands like HELP and ? should not be restored.

`Print-Command:` *boolean* is used by RESTORE-WORK and EXECUTE-FILE, which both ask "Execute Print-Commands?"; this is how they know which commands are print commands.

`MHelp:` This has to be a string and will be available through `UserHelp` and the `??` if no `QuickHelp` is available.

2. Argument Types

At the top-level of TPS3 explicitly declared argument types are available. Many of the more important ones are all declared in the file `argtyp.lisp`. They can be recognized by their `ArgType` property value, which is `T`. Each of argument type has at least three properties, `GetFn`, `TestFn`, and `PrintFn`. `GetFn` is responsible for translating the user's value into internal representation, `TestFn` tests if some object is of the given type, and `PrintFn` makes the internal representation intelligible to the user.

The defining command for the category `argtype` is actually `deftype%%`, but all definitions of argtypes should be made through the secondary macro `DefType%`. Its format is as follows (enclose optional arguments):

```
(DefType% {\it name}
 (GetFn {\it fnspec})
 (TestFn {\it fnspec})
 (PrintFn {\it fnspec})
{(Short-Prompt {\it boolean})}
{(MHelp "{\it comment}")}
{({\it property1} {\it value1}) ({\it property2} {\it value2}) ...})
```

In the above a *fnspec* is either the name of a one-argument function, or a list of forms which are to be evaluated as an implicit progn. In the latter case, *name* stands for the argument supplied.

name: The name of the argument type. It will get a property value of `T` for the property `ArgType` when the `DefType%` has been executed.

`GetFn:` Here *fnspec* defines the function used to process the argument as supplied by the user on the command line. The value returned by it is then handed on to the main function executing the command. No `GetFn` will ever receive a `$`. It is simply not called, if the corresponding argument in the command line is defaulted. A `GetFn` should signal an error if the argument is not of the correct type. This will be implemented (as it is right now) as a `THROW` with the label `FAIL`. A special case of *fnspec* for a `GetFn` is `TestFn`. This

means the `GetFn` will test if the supplied argument is of the correct type. If yes, the argument will simply be returned, otherwise an error will be signaled. A `GetFn` may have side-effects, but this has to be declared under `Side-Effects`.

`PrintFn`: Here *fnspec* should print the external representation of its only argument. It can expect this argument to be of the correct type. The value returned is ignored. A `PrintFn` may signal an error if printing is not possible (e.g. if the current style does not have a representation of the given data type).

`TestFn`: Here *fnspec* should return `NIL` if its argument is not of type *name*, and return something not `NIL` otherwise.

`Short-Prompt`: *boolean* is only used in *otl-typ.lisp*, but I can't work out what for.

`MHelp`: This is an optional documentation and is accessed during `MHelp` or after a `?` while the user supplies command arguments interactively.

(*property value*): Pairs like this allow for more information about the type. [1]

For example

```
(deftype% anything
  (getfn (lambda (anything) anything))
  (testfn (lambda (anything) (declare (ignore anything)) t))
  (printfn princ)
  (mhelp "Any legal LISP object."))

(deftype% integer+
  (getfn testfn)
  (testfn (and (integerp integer+) (> integer+ -1)))
  (printfn princ)
  (mhelp "A nonnegative integer."))

(deftype% boolean
  (getfn (cond (boolean t) (t nil)))
  (testfn (or (eq boolean t) (eq boolean nil)))
  (printfn (if boolean (princ t) (princ nil)))
  (mhelp "A Boolean value (NIL for false, T for true)."))
```

No `TestFn` or `PrintFn` is allowed to have any side-effects, since they may be called arbitrarily often. No `GetFn` needs to expect `$` as an argument, since defaults are now figured out elsewhere. This avoids conflicts between different defaults for the same argument type in different functions. Hence `GetFn` never computes the default.

[1] More properties may become useful, so the `Deftype%` macro allows arbitrary property names. Possibilities here include `EdFn` (for editing this argument type) or `OutputFn` (to be able to read back a data object of the specified type.)

2.1. List Types. The macro `deflisttype` defines a list from an existing type:

```
(deflisttype filespeclist filespec)
```

This takes an existing type, `filespec`, and produces a type of lists of filespecs. It is also possible to specify other properties (the same properties as for `deftype%`), in which case these properties override those of the original type. This is typically used to give the list type a different help message from the original type.

2.2. Consed Types. The macro `defconstype` defines a type as a cons of two existing types:

```
(defconstype subst-pair gvar gwff
  (mhelp "Means substitute gwff for gvar."))
```

This takes two existing types, `gvar` and `gwff`, and produces a type `subst-pair` of consed pairs `(gvar . gwff)`. It is also possible to specify other properties (the same properties as for `deftype%`), in which case these properties override those of the original type. This is typically used to give the cons type a different help message from the original type.

CHAPTER 5

Representing Well-formed formulae

1. Types

typeconstant : ::= Type Constant

An identifier with a non-`NIL` `TypeConst` property. For example, `O` and `I`:

```
(def-typeconst o
  (mhelp "The type of truth values."))
```

typevariable : ::= Type Variable

An identifier with a non-`NIL` `TypeVar` property.

It is the parsers responsibility to give the `TypeVar` property to types not previously encountered.

typesymbol : ::= *typeconstant* | *typevariable* | `(`*typesymbol* `.` *typesymbol*`)`

2. Terminal Objects of the Syntax

Before going into detail about the terminal objects of the syntax, some general remarks about type polymorphism in TPS3 are needed.

TPS3 supports polymorphic objects, like $\subseteq$ (subset), which is a relation that may hold between sets of any type. It must be understood, however, that the parser completely eliminates this ambiguity of types, when actually reading a wff. In a given wff every proper subwff has a type! Therefore, there is a class of objects with polymorphic type, which never appear in a wff, but nevertheless may be typed by the user. The instances of those polymorphic abbreviations or polymorphic proper symbols inside the formula will refer, however, to those polymorphic primitive symbols or polymorphic abbreviating symbols.

For reasons of efficiency, binders are handled slightly differently. Binders are also polymorphic in the sense that a certain binder, say $\forall$, may bind variables of any type. The case of binder, however, is slightly different from that of polymorphic abbreviations, since a binder is not a proper subwff. Binders, therefore, are left without having a proper type. We must, however, be able to figure out the type of any given bound wff. Thus each binder carries the information about the type of the scope, the bound variable and the resulting bound wff with it. See Section 5 for more details.

The list below introduces syntactic categories of objects known to the parser only, which are not legal in wffs themselves.

pmprsym : ::= Polymorphic Primitive Symbol

pmprsyms are the STANDS-FOR property of *pmpropsyms*, but cannot appear in *gwffs* themselves. Examples would be PI or IOTA.

pmabbsym : ::= Polymorphic Abbreviating Symbol

pmabbsym are the STANDS-FOR property of *pmabbrevs*, but cannot appear in *gwffs* themselves. Examples are SUBSET, UNION, or IMAGE.

The following categories are the "terminal" objects of proper wffs. The parser may not produce a formula with any other atomic (in the Lisp sense) object then from the list below.

logconst : ::= Logical Constants

For example: AND, OR, IMPLIES, NOT, FALSEHOOD, TRUTH:

```
(def-logconst and
   (type "OOO")
   (printnotype t)
   (infix 5)
   (prt-associative t)
   (fo-single-symbol and)
   (mhelp "Denotes conjunction."))
```

propsym : ::= Proper Symbols

For example: P<OA>, x<A>, y<A>, Q<OB>, x<B> are proper symbols after parsing $\forall x \forall y.\, P_{o\alpha\alpha}\, x\, y \wedge Q_{o\alpha}\, x$. This example demonstrates part of the parser. Since in a given wff, a proper symbol may appear with more than one type, the type of each proper must somehow be encoded in its name. TPS3 does this by appending the type, (and) replaced by < and >, respectively, to the print name of the symbol.

pmpropsym : ::= Polymorphic Proper Symbols

These are just like propsym, except that they also have a STANDS-FOR property, which is the polymorphic primitive symbol (pmprsym) this polymorphic proper symbol was constructed from. Note that this particular instance of the polymorphic primitive symbol always has a specific given type. For example: IOTA<I<OI» is a pmpropsym after parsing $y_\iota = \iota[QQy]$:

```
(def-pmpropsym iota
  (type "A(OA)")
  (typelist ("A"))
  (printnotype t)
  (fo-single-symbol iota)
  (mhelp "Description operator"))
```

abbrev : ::= Abbreviations

For example: @EQUIV. This is separate category from polymorphic abbreviations only for reasons of efficiency. An abbreviation could be thought of as a polymorphic abbreviation with an empty list of type variables. For example:

```
(def-abbrev equiv
  (type "OOO")
  (printnotype t)
  (fo-single-symbol equiv)
  (infix 2)
  (defn "[=(OOO)]"))
```

pmabbrev : ::= Polymorphic Abbreviations

For example: SUBSET<O<OA><OA», SUBSET<O<OB><OB» are polymorphic abbreviations after parsing A@f12(oa) @SUBSET B @or [R@f12(obb) a] @ @SUBSET [R b]. For example:

```
(def-abbrev subset
   (type "O(OA)(OA)")
   (typelist ("A"))
   (printnotype t)
   (infix 8)
   (fo-single-symbol subset)
   (defn "lambda P(OA) lambda R(OA). forall x . P x implies R x"))
```

binder : ::= Variable Binders

For example: $\forall$, $\exists$, λ, $\exists_1$. See the section below.

label : ::= A Label referring to one or more other wffs.

For example: AXIOM1, ATM15, LABEL6. See Section 6.

In principle, the implementation is completely free to choose the representation of the different terminal objects of the syntax. The functions with test whether a given terminal object is of a given kind is the only user visible functions. Once defined, the particular implementation of the object should not be needed or relied upon by other functions.

It is explained more precisely what is meant by “quick” and “slow” predicates to decide whether a given object is in a certain syntactic category in section 1.5. Here is a table of the different syntactic categories with the “slow” test function for it and the properties that are required or must be absent. Keep in mind that the list reflects the current implementation, and may not be reliable.

Category	Predicate	Required Properties	Absent Properties
pmprsym	PMPRSYM-P	TYPE, TYPELIST	DEFN
pmabbsym	PMABBSYM-P	TYPE, TYPELIST, DEFN	
logconst	LOGCONST-P	TYPE, LOGCONST	
propsym	PROPSYM-P	TYPE	LOGCONST, STANDS-FOR
pmpropsym	PMPROPSYM-P	TYPE, POLYTYPELIST, STANDS-FOR (a *pmprsym*)	
pmabb	PMPROPSYM-P	TYPE, POLYTYPELIST, STANDS-FOR (a *pmabbsym*)	
abbrev	ABBREV-P	TYPE, DEFN	TYPELIST
label	LABEL-P	FLAVOR	
binder	BINDER-P	VAR-TYPE, SCOPE-TYPE, WFF-TYPE	

3. Explanation of Properties

The various properties mentioned above are as follows:

- `TYPE :` The type of the object. Common are `"OOO"` for binary connectives and `"I"` for individual constants.
- `PRINTNOTYPE :` In first-order mode, this is insignificant, but if specified and `T`, TPS3 will never print types following the object. It is almost always appropriate to specify this.
- `INFIX :` The binding priority of an infix operator. This will declare the connective to be infix. The absolute value of `INFIX` is irrelevant, only the relative precedence of the infix and prefix operators matters. If two binders have identical precedence, association will be to the left. For example, if R1 and R2 are operators with `INFIX` equal to 1 and 2, respectively, `"p R1 q R2 r R2 s"` will parse as `"[p R1 [[q R2 r] R2 s]]"`.
- `PREFIX :` The binding priority of a prefix operator. Binders are considered prefix operators (see about binders below) and thus have a binding priority. The main purpose of a prefix binding priority is to allow formulas like `" a=b"` to be parsed correctly as `" [a = b]"` by giving = precedence over .
- `PRT-ASSOCIATIVE :` indicates whether to assume that the operator is left associative during printing. You may want to switch this off (specify `NIL`) for an infix operator like equivalence, say `<=>`, since

`"p <=> q <=> r"` is often considered to mean `"p <=> q & q <=> r"`.

`FO-SINGLE-SYMBOL` **:** this is meaningful only in first-order mode and declares the object to be a “keyword” in the sense that it may be typed in all upper or lower case. Moreover, the printer will surround it by blanks if necessary to set off surrounding text. Also the parser will expect that the symbol is delimited by spaces, dots, brackets, unless the symbol just consists of one letter, in which case it doesn’t matter. You **MUST** use this attribute in first-order mode for an identifier with more than one character.

`MHELP` **:** An optional help string.

Properties specific to binders are described in the section below about binders. Here are some more examples. These examples do not actually exist under these names in TPS3.

```
(def-logconst &
   (type "OOO")
   (printnotype t)
   (infix 5)
   (prt-associative t)
   (fo-single-symbol &)
   (mhelp "Conjunction."))
```

Note that the `(fo-single-symbol &)` will make sure that spaces are printed around `&` in formulas.

In the next example the character `/` is used to make sure that the disjunction is printed in lowercase, that is as `v` instead of `V`.

```
(def-logconst /v
   (type "OOO")
   (printnotype t)
   (infix 4)
   (prt-associative t)
   (fo-single-symbol /v)
   (mhelp "Disjunction."))

(def-logconst =>
   (type "OOO")
   (printnotype t)
   (infix 3)
   (fo-single-symbol =>)
   (mhelp "Implication."))
```

We do not like spaces after negation. So we do not declare it to be a `fo-single-symbol`. That works only because `-` consists of only one character.

```
(def-logconst -
   (type "OO")
```

```
(printnotype t)
(prefix 6)
(mhelp "Negation."))
```

4. Non-terminal Objects of the Syntax

[lsymbol] ::= *logconst*
| *propsym*
| *pmpropsym*
| *abbrev*
| *pmabbrev*

lsymbol roughly corresponds to what was called hatom for Huet-atom from Huet's unification algorithm in the old representation.

Generalized WFFs

gwff ::= *lsymbol*
| `((`*propsym* `.` *binder*`)` `.` *gwff*`)` ; Generalized binder
| `(`*gwff1* `.` *gwff2*`)` where `(cdr (type` *gwff1*`))` = `(type` *gwff2*`)`
| *label*

5. Binders in TPS

In the discussion about the internal representation of wffs the issue of binders has been neglected so far. Currently, TPS allows three binders, λ, $\forall$, $\exists$ (plus some "buggy" fragments of support for the $\exists_1$ binder).

Since most binders are inherently polymorphic, there is only one kind of binder. Notice that the definition is formulated such that a binder may have a definition, but need not.

In order to determine the type of a bound wff, the type of the scope of the binder must be matched against the type stored in the `SCOPE-TYPE` property. Also, the type of the bound variable must match the type in the `VAR-TYPE` property. These matches are performed, keeping in mind that all types in the `TYPELIST` property are considered to be variables. Then the bindings established during the match are used to construct the type of the whole bound wff, using the `WFF-TYPE` property of the binder.

An example may illustrate this process. The binder `LAMBDA` has the following properties

TYPELIST	(A B)
VAR-TYPE	B
SCOPE-TYPE	A
WFF-TYPE	(A . B)

When trying to determine the type of $\lambda x_\iota . R_{o\iota\iota} x$, TPS3 determines that `A` must be ι, and that `B` must be $o\iota$. The type of the original formula is (A . B) which then turns out to be $o\iota\iota$.

Note that `TYPELIST` may be absent, i.e. could be (), which amounts to stating that the binder has no variable types. Currently, we are not using

such binders. An example would be *Foralln* , which can bind only variables of type σ.

In addition to the properties mentioned above, a binder (except λ) would have a definition. One can then instantiate a binder just as a definition can be instantiated. The definition is to be written with two designated variables, one for the bound variable and one for the scope. For example *THAT has definition*

$$\iota_{\alpha(o\alpha)}.\lambda b_\alpha S_o$$

Here the `TypeList` would be (α), designation for the bound variable would be b_α, designation for the scope would be S_o.

The internal representation for a binder inside a wff is always the same and simply `((bdvar . binder) . scope)`, but all of the above information must be present to determine the type of a wff, or to check whether formulas are well-formed.

Fancy "special effects" such as $\forall x \in S.A$ must be handled via special flavors of labels and are not treated as proper binders themselves.

Here are some examples of binders:

```
(def-binder lambda
   (typelist ("A" "B"))
   (var-type "A")
   (scope-type "B")
   (wff-type "BA")
   (prefix 100)
   (fo-single-symbol lambda)
   (mhelp "Church's lambda binder."))

(def-binder forall
   (typelist ())
   (var-type "I")
   (scope-type "O")
   (wff-type "O")
   (prefix 100)
   (fo-single-symbol forall)
   (mhelp "Universal quantifier."))
{\it The above definition is for math-logic-1, where forall can only bind individual
variables. In math-logic-2, the definition is as follows:}

(def-binder forall
   (typelist ("A"))
   (var-type "A")
   (scope-type "O")
   (wff-type "O")
   (prefix 100)
```

```
(fo-single-symbol forall)
(mhelp "Universal quantifier."))
```

5.1. An example: How to See the Wff Representations. You can see examples of how wffs are represented by comparing the output of the editor commands P and edwff:

```
<44>ed x2106
<Ed45>p

FORALL x(I) [R(OI) x IMPLIES P(OI) x] AND FORALL x [~Q(OI) x IMPLIES R x]
 IMPLIES FORALL x.P x OR Q x

<Ed46>edwff
((IMPLIES (AND (|x<I>| . FORALL) (IMPLIES R<OI> . |x<I>|) P<OI> . |x<I>|)
 (|x<I>| . FORALL) (IMPLIES NOT Q<OI> . |x<I>|) R<OI> . |x<I>|)
 (|x<I>| . FORALL) (OR P<OI> . |x<I>|) Q<OI> . |x<I>|)
```

Another way to do this is as follows:

```
<3>ed x2106
<Ed4>cw
LABEL (SYMBOL):  [No Default]>x2106a
<Ed5>(plist 'x2106a)
(REPRESENTS ((IMPLIES (AND (|x<I>| . FORALL) (IMPLIES R<OI> . |x<I>|)
 P<OI> . |x<I>|) (|x<I>| . FORALL) (IMPLIES NOT Q<OI> . |x<I>|) R<OI> . |x<I>|)
 (|x<I>| . FORALL) (OR P<OI> . |x<I>|) Q<OI> . |x<I>|) FLAVOR WEAK)
```

And another way:

```
<2>(getwff-subtype 'gwff-p 'x2106)

((IMPLIES (AND (|x<I>| . FORALL)
                (IMPLIES R<OI> . |x<I>|) P<OI> . |x<I>|) (|x<I>| . FORALL)
              (IMPLIES NOT Q<OI> . |x<I>|) R<OI> . |x<I>|)
    (|x<I>| . FORALL) (OR P<OI> . |x<I>|) Q<OI> . |x<I>|)
```

And finally a way that only works at type O (the 0 below is a zero, not a capital O):

```
<3>(get-gwff0 'x2106)

((IMPLIES (AND (|x<I>| . FORALL)
                (IMPLIES R<OI> . |x<I>|) P<OI> . |x<I>|)
                 (|x<I>| . FORALL)
                  (IMPLIES NOT Q<OI> . |x<I>|) R<OI> . |x<I>|)
                    (|x<I>| . FORALL)
                     (OR P<OI> . |x<I>|) Q<OI> . |x<I>|)
```

6. Flavors and Labels of Gwffs

It is sometimes desirable to be able to endow a gwff with additional properties. For example, one may wish to be able to refer to a gwff by a short tag, or to specify that a particular gwff is actually a node in an expansion tree. For this purpose, TPS3 provides the facility of *labels* and *flavors* (see page 23). A *label* is an object which, as far as TPS3 is concerned, is merely a special case of gwff. Labels thus stand for gwffs, but may have additional properties and distinct representations.

Flavors are the classes into which labels are divided. The definition of a flavor specifies some common properties of a class of labels, usually the behavior of wffops and predicates. Also, a flavor's definition should specify what attributes each label of that flavor should have, and how it should be printed.

6.1. Representation. Each flavor is represented in TPS3 by a Lisp structure of type `flavor`, which has the following slots: `wffop-hash-table`, which stores the properties common to each instance of the flavor, in particular, how wffops are to behave; `constructor-fun`, which is the name of the function to be called when a new label of the flavor is to be created; `name`, the flavor's name; and `mhelp`, a description of the flavor. The values of these slots are automatically computed when TPS3 reads a `defflavor` declaration. The flavor structures are stored in a central hash table, called `*flavor-hash-table*`, keyed on the flavor names. This also is updated automatically whenever a flavor is defined (or redefined).

There are two ways to represent labels (instances of flavors), and the choice is made during the definition of the flavor. The first, and more traditional, way is to have each label be a Lisp symbol, with the attributes of the label being kept on the symbol's property list. The second way is to make each label a Lisp structure. The type of the structure is the name of the flavor; thus an object's type can be used to determine that it is a label of a certain flavor.

If one wishes to have labels be symbols, nothing must be done; this is the default. A flavor's labels will be structures only if one of two things is declared in the `defflavor`. The first is that the property `structured` appears. The second is if another flavor whose instances are structures is specified to be *included* in the new flavor.

When a flavor's labels are to be structures, one will usually wish to specify the `printfn` property so that the labels will be printed in a nice way. This function must be one which is acceptable for use in a `defstruct`. It is also required that one specify the slots, or attributes, the structures are to have, by including a list of the form

```
(instance-attributes (slot1 default1) ... (slotN defaultN))
```

in the flavor definition.

6.2. Using Labels. The function `define-label` is a generic way to create new labels of a specified flavor. The function call `(define-label sym flavor-name)` will do one of two things. If `flavor-name` is a flavor whose labels are symbols, then the property list of `sym` will be updated with property `FLAVOR` having value `flavor-name`. If on the other hand, `flavor-name` is a flavor having structures for labels, then `sym` will be setq'd to the value of the result of calling the constructor function for `flavor-name`, which will create a structure of type `flavor-name`.

To access the attributes of a label which is a symbol, use `get`, since all attributes will be on the symbol's property list. The attributes of a label which is a structure of type `flavor-name` can be accessed by using the standard Lisp accessing functions for structures. Thus, if one of the label's attributes is `represents`, the attribute can be accessed by calling the function `flavor-name-represents`.

Flavors can be redefined or modified at any time. This may be done if, for example, one wished to extend a flavor's definition into a Lisp package which was not always loaded. Merely put another `defflavor` statement into the code. You need only put the new or changed properties in the redefinition. If, however, you wish to change the attributes of a flavor which is a structure, you should put in all of the attributes you desire, not just the new ones, and be sure to declare any included flavor as well. Note: it is possible to change a flavor which uses symbols as labels into one which uses structures, but if you fail to redefine code which depends on property lists, the program will be smashed to flinders.

6.3. Inheritance and Subflavors. Some flavors may be similar in many ways; in fact, some flavors may be more specialized versions of other flavors. One may wish a new flavor's labels to be operated upon by most wffops in the same way as an existing flavor's labels; this we will call inheritance of properties. In addition, one may wish a new flavor to actually be a subtype (in Lisp terms) of an existing flavor, and have the attributes of the existing flavor's labels be included in the attributes of the new flavor's labels; this we will call inclusion of attributes. The `defflavor` form allows either or both types of sharing to be used.

Inheritance of properties is signalled in the `defflavor` by a form such as `(inherit-properties` *existing-flavor1* ... *existing-flavorN*`)`. This will cause the properties in the `wffop-hash-table` of the existing flavors to be placed into the `wffop-hash-table` of the new flavor. If any conflict of properties occurs, e.g., if *existing-flavorI* and *existing-flavorJ*, $I < J$, have a property with the same name, then the value which *existing-flavorJ* has for that property will be the one inherited by the new flavor. A new flavor may inherit properties from any number of existing flavors.

In contrast, attributes may be included from only one other flavor. This can be done by using the form `(include` *existing-flavor*`)`. The existing flavor must be a flavor whose instances are structures, and the new flavor's

instances will also be structures whose slots include the attributes of the existing flavor. Thus the same accessing functions for those slots will work on labels of both flavors. To define default values for those slots, add them to the `include` form as if it were an `:include` specifier to a `defstruct`; e.g., `(include` *existing-flavor* `(`*slot1 default1*`))`.

6.4. Examples. Here are some examples of flavor definitions.

```
(defflavor etree
  (mhelp "Defines common properties of expansion tree nodes.")
  (structured t)
  (instance-attributes
   (name '|| :type symbol)
   components ; a node's children
   (positive nil ) ; true if node is positive in the
;formula
   (junctive nil :type symbol) ; whether node acts as neutral,
;conjunction, or disjunction
   free-vars ; expansion variables in whose scope
; node occurs,
; used for skolemizing
   parent ; parent of the node
   ;;to keep track of nodes from which this node originated when copying a
   ;;subtree
   (predecessor nil)
   (status 1))
   (printfn print-etree)
   (printwff (lambda (wff bracket depth)
       (if print-nodenames (pp-symbol-space (etree-name wff))
 (printwff
  (if print-deep (get-deep wff)
    (get-shallow wff))
  bracket depth))))
   ...many more properties...)
```

`Etree` labels will be structures, with several attributes. The function used to print them will be `print-etree`.

```
(defflavor leaf
  (mhelp "A leaf label stands for a leaf node of an etree.")
  (inherit-properties etree)
  (instance-attributes
   shallow)
  (include etree (name (intern-str (create-namestring leaf-name)))))
```

`Leaf` labels will also be structures, with attributes including those of `etree`, as well as a new one called `shallow`. Note that the `name` attribute is given a default in the `include` form. `Leaf` inherits all of the properties of

`etree`, including, for example, its print function, unless they are explicitly redefined in the definition of `leaf`.

CHAPTER 6

Printing and Reading Well-formed formulas

1. Parsing

Frank has implemented a type inference mechanism based on an algorithm by Milner as modified by Dan Leivant. Type inference is very local: The same variable, say "x" will get different type variables assigned, when used in different formulas. Since multiple use of names with different types is rare, the default could be changed, so that after the first occurrence of an "x" during a session core image, the type inferred the first time is remembered.

There are only a total of 26 type variables, so you may run out during a session. The function INITTYPES reset the way type variables are assigned and treats everything except O and I as type variables. Normally, a type variable once mentioned or assigned automatically becomes a type constant.

If TYPE-IOTA-MODE is `NIL`, then TPS will assign type variables starting with Z and going backwards, as more are needed. TYPE-IOTA-MODE defaults to `T`.

Polymorphic abbreviations like SUBSET now may be given a type, so as to fix the type of other variables. E.g. the following is legal: "FORALL x . P x IMPLIES [Q x] IMPLIES . P SUBSET(O(OC)(OC)) Q" Note that "x" will be typed "C" (Gamma). The same typing could have been achieved by "FORALL x(C) . P x IMPLIES [Q x] IMPLIES . P SUBSET Q" If all the types were omitted and TYPE-IOTA-MODE were `NIL`, "x" would have been typed with the next available typevariable.

Using the same name for two variables of distinct type is legal, but not recommended. Consider, for example, "FORALL x . P x(I) AND . Q . x(II) a" Here the type of the very first occurrence of "x" will be assumed as "II". Leaving out the type of the third occurrence of "x" would have led to an error message: Rather than assume that "x(II)" was really meant, TPS assumes instead that the scoping must have been incorrect, which seems much more likely.

All remaining type variables (after a parse) are automatically assumed to be of *base-type* unless the flag `TYPE-IOTA-MODE` is set in which case they are assumed to be of type ι. In first-order mode identifiers have only single characters (Thus "not Pxy" is parsed as "NOT . P x y").

When a wff is read in and parsed, each input token (where the number of characters in a token is dependent on whether you are reading in first-order-mode or not) is made into a lisp symbol which incorporates the token's

printed representation and type. For example, entering "x(A)" will result in a symbol being created whose print-name is "x<A>". When you try to print a symbol like this, first the part without the type information is printed, then the type (if necessary) is printed. E.g., first we print "x", then print "(A)". But the information necessary to print "x" is really on the property list of the symbol whose print-name is "x". So all wffs of the form "x<...>" will be printed the same way (except for the type).

So, if you enter "x1(A)", you get the symbol "x1<A>", but no information about a superscript is put on the symbol "x1". Thus when you print it, you get no superscript, just "x1". Where do superscripts come from, then? Well, when TPS renames a variable in order to get a new one (such as alpha-normalizing a wff), it puts the superscript information on the new symbol's property list. I.e., if we rename "x1<A>", we may get the symbol "x2<A>", and on the property list of "x2", we get the superscript information. Thus, the next time the user types in "x2(A)" or even "x2(I)", the symbols created will have the superscript information.

This can be a little confusing, because the "x1(A)" that you originally entered still isn't superscripted, but the renamed variables "x2", "x3", etc., will be.

2. Printing of formulas

2.1. The Basics. In this section we will talk about how a formula in internal representation is printed on different output devices. There are two main points to take into consideration: how will the parts of the formula appear, and where will they appear. For the latter refer to section 3, the former we will discuss now.

2.2. Prefix and Infix. Since we deal with formulas of type theory, we can regard every formula as built by application and λ-abstraction from a few primitives. In order to make formulas more legible and closer to the form usually used to represent formulas from first order logic, we furthermore have quantification and definitions internally, and quantification, definitions, and infix operators for the purpose of input and output.

The application of a function to an argument is printed by simply juxtaposing the function and its argument. As customary in type theory, we do not have an explicit notation for functions of more than one argument. Predicates are represented as functions with truth values as their codomain.

Infix operators have to be declared as such. Only conjunction, disjunction, and implication are automatically declared to be infix operators. In general, infix operators will be associated to the left, if explicit brackets are missing. For example
$A \wedge B \wedge C$ will be $[[A \wedge B] \wedge C]$
Internally every infix operator has a property Infix which is a number. This number is the relative binding strength of this infix operator. You will have

to specify it, if you define a new connective to be infix. The higher the priority, the stronger the binding. As usual, '$\wedge$' binds stronger than '$\vee$' which has precedence over '$\supset$' (implication).

(As an aside, if you don't want conjunctions bound more tightly than disjunctions, but want brackets to appear, make the `INFIX` property of `OR` the same as `AND`. Thus, do: `(GET 'AND 'INFIX)`, to find it is 5, and then `(PUTPROP 'OR 5 'INFIX)`)

Unfortunately prefix operators like negation, do not currently have a binding strength associated with them and will always be associated to the left. This has to be kept in mind, when formulas are typed in.

Definitions can be infix or prefix and the same rules hold for them. There are flags which control whether a definition or its instantiation will be printed. Similarly, logical atoms can appear as names or as values (or both). In general the appearance of a formula and in particular of a definition very much depends on which output device is used. See section 2.5 for more detail, but remember that this only affects the way the primitive or defined symbols appear, but not how the formula is assembled from its parts.

2.3. Parameters and Flags. The flags listed below are global parameters which can be set by the user to control the way formulas are printed. These settings can be overridden if specific commands are given.

PrintTypes: = `T` causes all types to be printed. If a typed symbol occurs more than once, only the first occurrence will have a type symbol, unless the same symbol name appears in the same formula with a different type.
= `NIL` suppresses type symbols.

PrintDepth: This is a parameter which determines how deep the recursion which prints the formula will go. Subformulas located at a lower level will simply be replaced by an &. A `PrintDepth` of 0 means that everything will be printed, regardless of its depth. `PrintDepth` has to be an integer. It is initialized to 0. The most useful application of this parameter is in the formula-editor, where one usually does not like to see the whole formula.

AtomValFlag: This flag should usually not be touched by the user. If it is true, under each atom its value will appear.

AllScopeFlag: This flag should be `NIL` most of the time. If it is `T` brackets and dots will always be inserted, i.e. no convention of associativity to the left is followed. The precedence values of infix operators are also ignored. It can be forced to `T` by calling the function PWScope GWff.

2.4. Functions available. There are of course a variety of occasions to print wffs, For example in plans, as lines, after the `P` or `PP` -command in the editor etc. Associated with these are different printing commands given by the user. Some of these commands override globally set parameters

or flags. Internally, however, there is only one function which prints wffs. This function PrtWff is called whenever formulas have to be printed. The various flags controlling the way printed formulas will appear, will either be defaulted to the global value, or be passed to this function as arguments. The general form of a call of `PrtWff` is as follows

`(PrtWff Wff (Parameter`$_1$ `Value`$_1$`) ... (Parameter`$_n$ `Value`$_n$`) )`

Before the actual printing is done Parameter$_1$... Parameter$_n$ will be set to Value$_1$ ... Value$_n$, resp. If a parameter of the following list is not included in the call of the function, its global value will be assumed. Possible parameters with their range and the section they are explained in are

PrintTypes	T,NIL	2.3
PrintDepth	0,1, ...	2.3
AllScopeFlag	T,NIL	2.3
AtomValFlag	T,NIL	2.3
PPWfflag	T,NIL	3.1
LocalLeftFlag	T,NIL	3.1
FilLineFlag	T,NIL	3.1
FlushLeftFlag	T,NIL	3.1
Leftmargin	1 ... Rightmargin	2.6
Rightmargin	1, 2 ...	2.6
Style	XTERM, SCRIBE, CONCEPT, GENERIC, SAIL, TEX ...	2.5

2.5. Styles and Fonts. TPS3 can work with a variety of different output devices, producing special characters like ∀ or ∧ where possible, and spelling them out (as `FORALL` and `AND`) where not. Details of how to produce output files for various purposes are in the ETPS and User's Manuals.

At no point does the user actually make a commitment whether to work with special characters or not, since she can easily switch back and forth. The internal representation is completely independent of these switches in the external representation.

A few commands, such as VPForm and VPDiag have an argument Style which specifies the style in which a file is produced. Furthermore there is a flag, STYLE, which TPS3 will use in the absence of any other indication as to the appropriate form of output.

Along with the style the user can usually specify an appropriate line-length by using the LEFTMARGIN and RIGHTMARGIN flags. Some commands (most notably SETUP-SLIDE-STYLE) will change both the style and the default line length.

CONCEPT, CONCEPT-S : this is the style used for a Concept terminal, which might also occasionally also be useful to produce a file which can be displayed on the Concept terminal with `CAT` or `MORE`. The difference between `CONCEPT` and `CONCEPT-S` is that the latter assumes that your Concept is equipped with special characters and the former does not. If special characters are available,

you will then get types as greek subscripts, the universal quantifier as ∀, etc. The default linelength is 80.

GENERIC : this style assumes no special features and defaults the linelength to 80. For example the existential quantifier shows up as EXISTS and types are enclosed in parentheses.

GENERIC-STRING : is much like `GENERIC`, but prints in a format that can be re-read by TPS3.

SCRIBE : corresponds to the style used by the Scribe text processor. A file produced in this style has to be processed by `SCRIBE` before it can be printed. All special characters, superscripts and subscripts, etc. are available. The main drawback of a `SCRIBE`-file is that precise formatting as necessary for vertical path diagrams is impossible. The font used is 10-point, except when doing SLIDEPROOF, when an 18-point font is used.

TEX : is the output style used by the TEXtext processor. A file produced in this style has to be processed by TEXbefore it can be printed. All special characters, superscripts, etc. are available, and vertical path diagrams are correctly formatted (although often too wide to print).

XTERM : produces the special characters used by X-windows. You should set the value of RIGHTMARGIN to reflect the width of the window containing TPS3.

SAIL : `SAIL` is a style (now all but obsolete) used for printing on a Dover printer. The font used is 10-point, with 120 characters per line in landscape format (used for vertical path diagrams), and 86 in portrait format (used for all other applications). When you dover the file , you have to remember size and orientation and specify it in the switches of your call of `DOVER`. A `SAIL` file does not have subscripts, but has as variety of other special characters.

From the information about the style, the low-level printing functions determine which sequence of characters, including control characters, to send to the selected output device. If a symbol expands to a list of known symbols with different names (e.g. EQUIVS expands to an `EQUIV` symbol with a superscript `S`), then it has a property FACE which contains this information. Various other properties give the way that the character is to be printed in different styles. The CFONT property is a pair `(KSet . AsciiValue) . Kset` can be 0,1,2, or 3, although currently only the character sets 0, 1, and 3 are used; this gives the appropriate character for a Concept terminal. Similarly, the DFONT property is a string `"whatever"` which will be printed into Scribe files as `@whatever`. The TEXNAME property does the same for the TEXoutput style. There are some special fonts that are declared in the file tps.mss. A list of the available special characters for the Concept and for the Dover (in a `SCRIBE`-file) are explicitly stored in the files cfont.lisp and dfont.lisp and loaded into TPS3 at the time the system is being built.

Consider the following example:
SIGMA1 is a binder. It has a property FACE of value (CAPSIGMA SUP1).

CAPSIGMA is a tex special character, a scribe special character, and a concept special character. It has a property CFONT of value (3 . 83). It has a property DFONT of value "gS". It has a property TEXNAME of value "Sigma".

SUP1 is a tex special character, a scribe special character, and a concept special character. It has a property CFONT of value (1 . 49). It has a property DFONT of value "+1". It has a property TEXNAME of value "sup1".

In a scribe or tex file, or on a Concept with special characters, `SIGMA1` will appear as Σ^1; elsewhere it will be written as `SIGMA1`. The actual Scribe output produced will be

```
@g{S}@\;@^{1}@\;
```

; the actual TEXoutput will be

```
\Sigma^{1}
```

.

2.6. More about Functions. In this section some more details of the functions which are used to do the printing are given.

As mentioned earlier, the main connection with the rest of TPS3 is the MACRO PrtWff. It expands into a `PROG` in which all the parameters given as arguments are `PROG`-variables. In the body of the `PROG`, all parameters are set to the value specified in the call, then the function PWff is called, just with `Wff` as its argument. All the other parameters and flags are now global, or, in `LISP` terminology, special variables.

The function `PWff` performs two main tasks. First a few special variables are set to the correct value. After this is done, `PWff` checks whether pretty-printing is desired, i.e. whether `PPWfflag` is `T`. For an explanation of what happens during pretty-printing see section 3 and in particular 3.4. Otherwise the recursive function PrintWffPlain is called with the appropriate arguments.

At this point the current style is available to the functions in the flag STYLE. The calling function has to make sure that LEFTMARGIN and RIGHTMARGIN will be bound. They are important for the printing functions in order to determine where to break lines, and where to start formulas on the line. This holds, whether pretty- printing is switched on or off.

Below `PWff` two functions appear. `PrintWffPlain` prints a formula without any delimiting symbols around it. For example (with `STYLE SCRIBE`)
((x<I> . FORALL) . ((OR . (P<OI> . x<I>)) . q<O>)) appears as
$\forall x_\iota.[P_{o\iota}x] \vee q_o$ if BRACKETS = T and as
$\forall x_\iota[[P_{o\iota}x] \vee q_o]$ if BRACKETS = NIL .
PrintWffScope delimits a composite formula with a preceding dot, if the argument BRACKETS is `T` , and with brackets around it , if `BRACKETS` is `NIL`.

Other than that the functions are identical. In the above example we would get
$\forall x_\iota.[P_{o\iota} x] \vee q_o$ if Brackets = T and
$[\forall x_\iota.[P_{o\iota} x] \vee q_o]$ if Brackets = NIL
Both `PrintWffPlain` and `PrintWffScope` call PrintWff, where the real work of distinguishing the different kinds of formulas and symbols is being done. The distinction between `PrintWffPlain` and `PrintWff` is only made for the sake of pretty-printing (see 3.4).

At an even lower level is the function (actually a macro) PCALL, which determines the appropriate way to print a particular symbol in the current style, and prints an error if the relevant function is undefined. `PCALL` actually applies to printing functions, rather than characters, so each function will have a different definition for different styles. For example, in style scribe the print-symbol function is called PP-SYMBOL-SCRIBE, whereas in style xterm it's called PP-SYMBOL-XTERM. (Examine the plists of `SCRIBE` and `XTERM` to verify this, if you like.)

3. Pretty-Printing of Formulas

The most commonly used way of printing formulas, such as lines or plans, is to pretty-print them. This is a feature quite similar to the way LISP pretty-prints functions. Formulas which are too long to fit on one line of the current output device, are broken at the main connective and printed in several lines. The main difference to the LISP pretty-printing is that we have to consider infix operators.

The general structure of the functions doing the pretty-printing allows future changes to the way printing in general is done without making changes to the pretty-printer. Whenever a formula is to be pretty-printed the usual printing functions as described above are called, but instead of printing the characters, they will be appended to a list. Later this list is used to actually output the characters after the decision where to break the formula has been made. From this structure it is clear that all the parameters and flags controlling the appearance of a formula on the several printing devices still work in the way described before. There are however, a few additional flags which determine how subformulas will be arranged within a line.

3.1. Parameters and Flags.

As new flags particularly for pretty-printing we have

PPWfflag : = `T` means that formulas will usually be pretty printed. This is the default value.
= `NIL` means that formulas never will be pretty printed unless the command is given explicitly.

LocalLeftFlag : =`T` will cause the left hand side of an infix expression to be aligned with the operator and not with the right hand side.

= `NIL` is the default and prints left and right hand side of an infix expression with the same indentation.

FilLineFlag : =`T` will try to fill a line as much as possible before starting a new one. This only makes a difference for associative infix operators.

= `NIL` starts a new line for each of the arguments of an infix operator even if only one of several arguments would be too long to fit on the remainder of the line.

FlushleftFlag : = `T` switches off indentation completely, i. e. every line will be aligned with the left margin.

= `NIL` indents the arguments of infix operators.

3.2. Creating the PPlist. The pretty-printing is achieved in two steps. During the first phase printing will be done without any formatting and the characters are not actually printed, but appended to a list, called `PPlist`. In the second phase, this list will then be printed. The decisions, when to start a new line, how to indent etc. are only made in this second stage.

The `PPlist` is of the following syntactical structure.

pplist ::=: ((aplicnlist . (pdepth . pgroup)) . plength)
| ((gencharlist . (pdepth . pgroup)) . plength)
aplicnlist ::=: (aplicn . aplicnlist) | `NIL` | (aplicn . `MARKATOM`)
aplicn ::=: (pplist . pplist)
plength ::=: `0` | `1` | `2` | ...
pgroup ::=: `BRACKETS` | `DOT` | `NIL`
pdepth ::=: `0` | `1` | `2` | ...
gencharlist ::=: (genchar . gencharlist) | `NIL`
genchar ::=: char | (ascnumber) | (gencharlist)
char ::=: <any non-control character>
ascnumber ::=: `0` | `1` | ... | `127`

The `PPlist` contains a list of all the top-level applications , along with the grouping (pgroup), its print-depth (pdepth) and its print-length (plength). If the grouping is `BRACKETS` brackets will be printed around the formula. A grouping `DOT` means that a dot will precede the formula, otherwise the formula will just be printed without any delimiting symbols. The plength is the total length of the formula if printed in one line, including spaces, brackets, a.s.o., but not control characters which are used to denote character sets, or `SCRIBE` -commands.

The pdepth is recursively defined as the maximum pdepth of the left-hand sides plus the maximum pdepth of the right-hand sides of the applications, if the `PPlist` contains applications, and the plength of the generalized-character list (gencharlist) otherwise. The plength of a gencharlist is its length after all members of the form '(gencharlist)' have been deleted. This means that characters that have to be sent to the selected output device but do not occupy space (in the final document) will simply be enclosed in parentheses. By this convention the function which then formats and

actually prints the formula from the `PPlist` can keep track of the vertical position within a line. The pdepth associated with each subformula is used to decide the amount of indentation, as described below.

The list of applications, aplicnlist, typically contains contains only one pair with the left-hand side a function, and the right-hand side the argument the function is applied to. In case we have infix operators or multiple conjunctions or disjunctions, like $A \equiv B$, $A \wedge B \wedge C \wedge D$, or $E \vee F$, aplicnlist will contain a different pair for each argument. The left-hand side contains the infix operator, if one has to be printed in front of the argument, the right-hand side contains the argument itself. Quantifiers are regarded as single applications, where the left-hand side is the quantifier plus the quantified variable, while the right-hand side is its scope. Consider the following examples.
$A \equiv B$
will be translated to
aplicnlist = ((<> . <A>) (<EQUIV> . <B>))

$A \wedge B \wedge C \wedge D$
will be translated to
aplicnlist = ((<> . <A>) (<AND> . <B>) (<AND> . <C>) (<AND> . <D>))

$E \vee F$
will be translated to
aplicnlist = ((<> . <E>) (<OR> . <F>))

$\forall x_\iota G$
will be translated to
aplicnlist = ((<FORALL X<I» . <G>))

where <x> denotes the PPlist corresponding to the subformula x,
and <> stands for the empty PPlist ((NIL . (0 . NIL)) . 0)

A generalized character, genchar, is defined to be an arbitrary non-control ASCII character, the number of an ASCII character in parentheses, or another generalized character list in double parentheses. When an ASCII character is printed it is assumed that the cursor advances one position, while everything in the sub-gencharlist is assumed not to appear on the screen or in the document after being processed by SCRIBE.

An aplicnlist with the structure (aplicn . `MARKATOM`) signals that the aplicn is the internal representation of a logical atom (For example `ATM15`). In case AtomValFlag is `T`, the program notes the cursor position, whenever it encounters such an aplicnlist during printing and prints the name of the atom in the next line at this position.

3.3. Printing the PPlist. After the `PPlist` is created by the function PWff, the actual output is done by the function PrintPPlist. This function takes a `PPlist` and `INDENT` as arguments and has the following basic structure.

(1) : Does the formula fit on the remainder of the line (from `INDENT` to `RightMargin`) ? If yes, just print it from the `PPlist`. If not, go to (2).

(2) : Is the formula composed of subformulas ? If not, go to the next line and print it at the very right. If yes, go to (3).

(3) : Is the formula a single application ? If yes, call `PrintPPlist` recursively, first with the function then with the argument such that the function will appear at `INDENT` and the argument right after the function. If not, go to (4).

(4) : Print each application in the application list in a new line, the operators at the vertical position `INDENT` and the arguments at the position `INDENT` + maximal length of the operators.

This algorithm will be slightly different if the flags described above do not have their default values. See section 3.1 for a description.

Some heuristics are employed to avoid the pathological case where the formula appears mostly in the rightmost 10% of each line. Used in these heuristics is the print-depth (pdepth), which is equal to the furthest extension of the formula to the right if printed with the above algorithm. Whenever the pdepth is greater than the remainder of the line, the indentation will be minimized to two spaces. This is most useful if special characters are not available, for example if '∀' is printed as 'FORALL'.

3.4. Pretty-Printing Functions. Most of the functions used for the first phase of pretty-printing, i.e. for building the `PPlist` are already described in section 2.6. The internal flag `PPVirtFlag` controls whether functions like `PrintFnTTY` will actually produce output or create a `PPlist`. Here it is now of importance, what the different printing functions return, something that was completely irrelevant for direct printing.

The general schema can be described as follows. PrintWffPlain and PrintWffScope return a `PPlist`. If called from PrintWff, these `PPlists` are assembled to an aplicnlist and returned. In this case `PrintWff` returns an aplicnlist. The lower level functions, PrintFnDover and PrintFnTTY return the gencharlist which contains the characters that would be printed in direct mode. Note that therefore `PrintWff` will sometimes return a gencharlist instead of an aplicnlist. These two are interchangeable as far as the definition of the `PPlist` is concerned, and can hence be treated identically by `PrintWffPlain` which constructs a `PPlist` from them.

The special parameters PPWfflist and PPWfflength keep track of the characters "virtually printed" and the length of the formula "virtually printed", respectively.

On the very lowest level PPrinc and PPTyo perform a `PRINC` or `TYO` virtually by appending the appropriate characters to the `PPWfflist`. Characters that do not appear in the final document or on the screen, are virtually printed by PPrinc0 and PPTyo0. They prevent the counter `PPWfflength` from being incremented. Similar functions are PP-Enter-Kset and PPTyos which correspond to `Enter-Kset` and `TYOS`.

In the second phase of pretty-printing as described in the previous section `PrintPPlist` is the main function. If the remainder of a `PPlist` fits on the rest of the current line, SPrintPPlist is called which just prints the `PPlist` without any counting or formatting.

3.5. JForms and Descr-JForms. A JForm is an alternative way of representing well-formed formulas and is used by the matingsearch package and for printing vertical path diagrams. In JForms multiple conjunction are represented as lists and not as trees. Consider the following example.
$A \wedge B \wedge C \wedge [D \vee E \vee F]$
As a wff in internal representation this will be
((AND . ((AND .((AND . A) . B)) . C)) . ((OR . ((OR . D) . E)) . F))
Obviously this is not a very suitable form for vertical path diagrams. As a JForm, however, the above wff would read as
(AND A B C (OR D E F))
which is already close to what we would like to see.

The function Describe-VPForm takes a JForm like the one above as an argument and returns a Descr-JForm, where we have the information about the height and width of the subformulas, which we need in order to format the output, explicitly attached to the parts of the JForm.

Quantifiers are handled similarly. Multiple identical quantifiers are combined in a list whose first element is the quantifier and the rest is the list of variables which are quantified.
$\forall x \forall y \exists z \exists u A$
is in internal representation
((x . FORALL) . ((y . FORALL) . ((z . EXISTS) . ((u . EXISTS) . A)))),
and as a JForm it looks like
((FORALL x y) ((EXISTS z u) A)) .
The following is a formal description of what a JForm and a Descr-JForm are. Note that a descr-jform is entirely an internal concept, used by the file vpforms.lisp for working out how to format a vpform; a jform is a concept which is accessible to users (e.g. users have commands to translate from gwffs to jforms and back)

> *JForm ::=: Literal* | *SignAtom* | (`OR` $[JForm]_2^n$) |(`AND` $[\mathrm{JForm}]_2^n$)
> | ((`FORALL` $[\mathrm{Var}]_1^n$) JForm) | ((`EXISTS` $[\mathrm{Var}]_1^n$) JForm)
> *Literal ::=:* `LIT1 | LIT2 | ...`
> *SignAtom ::=:* (`NOT` *Atom*) | (*Atom*)
> *Var ::=:* < any logical variable >
> *Atom ::=:* < any logical atom >

It should be noted here that some programs might expect the arguments of a JForm starting with `OR` not to start itself with an `OR`, the argument of a JForm starting with `FORALL` not to start with another `FORALL` etc., but this is by no means essential for vertical path diagrams.

Desc-Jform ::=: ($\{{}^{Literal}_{SignAtom}\}$ Height Width (Width Width) (GenCharList PPlist)
((`OR` [Desc-JForm]$_2^n$) Height Width ([Cols]$_2^n$))
((`AND` [Desc-JForm]$_2^n$) Height Width ([Rows]$_2^n$))
((($\{{}^{\texttt{FORALL}}_{\texttt{EXISTS}}\}$ [Var]$_1^n$) Desc-JForm) Height Width Width GenCharList)
Height ::=: `0` | `1` | `2` | ...
Width ::=: `0` | `1` | `2` | ...
Cols ::=: `0` | `1` | `2` | ...
Rows ::=: `0` | `1` | `2` | ...

In a Descr-JForm the second and third element (Height and Width) contain the height and width of the JForm that is described by the Descr-JForm. In case the JForm was a literal or a signed atom the next two elements are lists. The left element of each of these sublists gives the width or print-representation of the literal or atom, the right element gives the width or print-representation of the literal's or atom's value.

If the JForm was a conjunction or disjunction, the last element of the corresponding Descr-JForm is a list of the rows or columns in which the conjuncts or disjuncts begin.

If we deal with a top-level quantifier in our JForm, the last two elements contain the width and the print-representation of the quantifier together with the quantified variables. For a description of a GenCharList or PPlist see section 3.2.

3.6. Some Functions. The function which is called by `VPForm` and `VPDiag` is %VPForm. The handling of the comment and the different files that have to be opened is done here. The main function which translates a JForm into a Descr-JForm is Describe-VPForm. SignAtoms and Literals are described by Describe-VPAtom and Describe-VPLit , respectively. The virtual printing functions used for this process are FlatSym and FlatWff.

`FlatSym` takes an arbitrary `LISP` identifier as an argument and returns a pair (gencharlist . length) for this identifier. `FlatWff` takes a wff as argument and returns a `PPlist` for it.

The main function which then prints the Descr-JForm is Print-VPForm. It takes the line of the Descr-JForm which should be printed as an additional argument. On lower levels %SPrintAplicn and %SPrintPPlist print an aplicn or a `PPlist` much in the same fashion `SPrintAplicn` and `SPrintPPlist` do, except that %%PRINC takes the role of `PRINC` and `TYO`. This is necessary from the way the actual output is handled. If the vertical path diagram does not fit on one page, several temporary files are opened and each file contains the information for one of the pages. This means that the characters have to be counted and a new file to be selected as the current ouput file, whenever

the character count exceeds the global parameter VPFPage. The counting as well as the change of the current output file is done by the function %%PRINC. The argument has to be either a `LISP`-atom, in which case it will be `PRINC`'ed , or a single element list, in which case this element will be `TYO`'ed.

4. How to speed up pretty-printing (a bit)

Pretty printing in TPS or ETPS is slow, for various reasons. One of them is the tremendous amount of temporary list space used, which takes time and more time through garbage collection. Another is the forgetfulness of the printing routine which recomputes length and other information over and over again. Below we will try to explore ways to improve the performance of the pretty printer without sacrificing any of the niceness of the output.

Let us recount which factors make pretty-printing wffs more difficult than pretty-printing Lisp S-Expressions. For once, Lisp does not have infix operators and can therefore get by with a significantly smaller amount of lookahead. Moreover, the lookahead can be done during the printing, where the extra time delay is hardly noticeable, while TPS' lookahead must all be done ahead of time, before the first character is printed. Secondly, Lisp does not deal with a variety of output devices, which makes counting symbol lengths as well as printing symbols much faster and more transparent.

The result of a first attempt at pretty-printing is described earlier in this chapter. The solution is nicely recursive and a lot of information is made available for deciding where to break and how to indent lines. It is a sad fact that the algorithm does not reuse any information whatsoever. For example, the printed representation of identifiers is recomputed over and over again. Even worse, the characters comprising the printed representation of an identifier are stored in a list, copies of which typically occur in many places in the *pplist* of a single wff.

Let us now look at some of the problems and possible solutions of the pretty-printing problem.

4.1. Static and Dynamic Parameters. Crucial to finding a good solution is to understand which factors affect the appearance of wffs when printed. These can be divided into two classes.

- *Static Parameters.* Static parameters are not changed during the printing of a given wff. In particular their values are identical for a wff and their subformulas. Of course, they may be changed from one printing task to another, but not within printing a particular wff. Examples of such static parameters are `AllScopeFlag`, `Style`, `KsetsAvailable`, `PrintAtomnames`, etc. One other characteristic of static parameters is that one frequently would like to (and sometimes does) expand the number of static parameters.
- *Dynamic Parameters.* Dynamic parameters are the ones which change from a wff to a subwff. They are highly context-dependent and are

often not explicitly available as flags, but implicitly computed. Examples of such parameters are "*should I print a type for this identifier?*", `PrintDepth`, "*should I print brackets or a dot?*". An example for the last question would be that we can sometimes write $Q_{o\iota}.f_{\iota\iota}\ x_\iota$ and sometimes $Q_{o\iota}[f_{\iota\iota}\ x_\iota]$ depending on the brackets in wff containing this as a subformula.

One can easily see that static parameters can be handled fairly easily, while dynamic parameters can become a headache if we are trying to save information about the appearance of wffs and symbols.

4.2. A grand solution, and why it fails. A first stab at a solution could be briefly described as follows:

During the printing of a wff we permanently attach relevant printing information like length, depth, or printing characters to each label and symbol in the wff. When the label or symbol appears again somewhere else, the information does not have to be recomputed.

We would then have to somehow code the information about the current static and dynamic parameters into the property of the label or symbol which stores this information.

With the aid of a hashing function this is straightforward for the static parameters, since we can compute the name of the relevant property once and for all for the printing of a wff. For dynamic parameters this is still in theory possible, but in practice unfeasible. We would have to recompute (rehash) the values of the dynamic and static parameters for each subformula. To see that this is very difficult, if not impossible, consider the following example.

The simple wff $P_{o\alpha\alpha}\ x_\alpha\ y_\alpha$ may appear as Pxy, $P_{o\alpha\alpha}\ xy$, $Px_\alpha\ y_\alpha$, Pxy_α, etc., with almost endless possibilities for larger wffs. All the information about which symbols should have types etc. would have to be coded into the property name for, say, the printing length of a label.

This clearly demonstrates that a grand solution is infeasible.

4.3. A modest solution, and why it works. Everything would work out fine if we could limit the number of dynamic parameters. This can be achieved very simply by restricting ourselves to saving information about symbols only, and not about labels in general.

Of the various dynamic parameters, only one survives this cut. "*Do I put a type on this identifier*" is the only question that can be solved from the context only. This simplification also reduces the number of static parameters, For example *AllScopeFlag* is irrelevant to the printing of symbols (wffs without proper subwffs).

However, care must be taken when the appearance of identifiers is changed. We will return to this problem later in the section about other issues.

4.4. Implementation. All printing requests go through the function `PWFF`. When `PWFF` is entered all static parameters have their final value.

Inside `PWFF` we will set two more special (global) variables: `Hash-Notype` and `Hash-Type`.

`Hash-Type` and `Hash-Notype` will have as value of the name of the property, which contains the symbol's *pplist*. When constructing the *pplist* for the given wff (the first pass during pretty-printing), it is checked whether symbols have the appropriate property. If yes, the symbol itself stands for a *pplist*. (We are thus modifying the recursive definition of *pplist*.) If not, the *pplist* will be computed and stored under the appropriate name on the property list of the symbol. In this case, too, the symbol itself will appear in the *pplist*.

During the actual printing phase of the *pplist*, the necessary information about symbols is retrieved from the property lists of the identifiers.

This presents one additional problem: we have to preserve the information about the dynamic parameters in the *pplist* itself, so that the correct property can be accessed. This could be done in a very general way (but for specific problems maybe wasteful way) namely by including the name of the relevant property in the *pplist*. Alternatively we may use the special circumstance that there are usually more identifiers without type. We would then only mark those identifiers with type, while all others are assumed to be printed without types.

The solution above requires some auxiliary data structures. There should be a global variable, say `static-printing-flags`, which contains a list of all flags affecting the printing of symbols. Then there must be a function `hash-printing-flags` which takes one argument (signifying whether types are to be printed) and returns an identifier coding the value of the `static-printing-flags` and the argument.

4.5. Other Issues. In the solution proposed above it is left open, whether the actual ASCII character representation of a symbol should be computed once and for all (for each set of static and dynamic parameters) and saved in a list which is part of the *pplist*, or simply recomputed every time the identifier is printed. The first solution would require significantly more permanently occupied list space, the second solution would take more time during each printing.

Notice, that the time required for the printing is not that long, since the identifier will have to be printed only during the actual printing phase, not during the virtual printing phase. The length is already known through the symbols property list. It therefore seems to be much better only to save the printing length of the identifier.

Another issue arises, when we allow that the printing appearance of identifiers be changed. Since all the length information attached to the identifier will be wrong, it is necessary to remove that information. In order to be able to do this, we need to recognize the properties which stem from the printing algorithm sketched above. The simplest way to achieve this is to declare a global variable `hash-properties`, which is a list of all the

properties that have been used for printing so far. This must be updated, whenever `PWFF` is called. The hope is that due to the limited number of static and dynamic parameters this list remains manageable in size. An alternative would be to write the hashing function in such a way that all names produced by it start with a unique pattern, say `*@*`. One can then systematically look for properties whose name starts with `*@*`.

4.6. How to save more in special cases. There is a straightforward generalization of this to case where we would like to save information about the appearance of arbitrary labels. The most general solution fails, as demonstrated above, but if we restrict ourselves to cases where the number of dynamic parameters is limited, we can get somewhere.

We could make a case distinction of the kind: save and use printing info for labels only if `PrintDepth` is `0`, `PrintTypes` is `NIL`, `AllScopeFlag` is `NIL`. The only remaining dynamic parameter that comes to mind is the bracketing information (which can take two different values). This is what makes this fragment of the grand solution feasible.

Notice that this is not just of academic interest. ETPS in first-order mode satisfies all the criteria above.

5. Entering and printing formulas

5.1. Parsing of Wffs. Wffs can be specified in TPS3 in a variety of ways, e.g. as strings and with or without special characters. Regardless how a wff is specified there are general rules of syntax which always apply. Sometimes one has to distinguish between first-order mode and higher-order mode with slightly different syntactic rules. If the global variable First-Order-Mode is `T`, all parsing will be done in first-order mode. Similarly, the global variable First-Order-Print-Mode determines whether wffs are printed as first-order or higher-order formulas. It is important to note that wffs printed in higher-order mode can only be parsed in higher-order mode, and formulas printed in first-order mode can only be parsed in first-order mode.

- **Operator precedence** - The parser for wffs is a standard operator precedence parser. The binding priority of an infix or prefix operator is a simple integer and conforms with the usual conventions on how to restore brackets in formulas. "`[`" and "`]`" serve as brackets and a period "`.`" is to be replaced by a left bracket and a matching right bracket as far right as consistent with the brackets already present, when brackets are restored from left to right. For operations of equal binding priority, association to the left is assumed. In order of ascending priority we have
  $\equiv$ or `EQUIV` (2)
  $\supset$ or `IMPLIES` (3)
  $\vee$ or `OR` (4)
  $\wedge$ or `AND` (5)
  $\neg$ or `NOT` or `~` (100)

applications (like Pxy or $[\lambda\ x\ x]t$)
binders (λ,$\forall$,$\exists$)

- **Types** - Function types are built from single letter primitive types. Grouping is indicated by parentheses "`(`" and "`)`". The basic types are @subomicron or `O` for truth values and @subiota or `I` for individuals. Any letter (except `T`, i.e. $_\tau$) may serve as a typevariable. A pair $_{(\alpha\beta)}$ or `(AB)` is the type of a function from elements of type `B` to type `A`. E.g. `(O(OI))` or $_{o(o\iota)}$ is the type of a collection of sets of individuals. Association to the left is assumed, so `(OAAA)` or $_{o\alpha\alpha\alpha}$ is the type of a three place predicate on variables of type `(A)`.
- **Identifiers in higher-order mode** - In higher-order mode identifiers may consists of any string of ASCII and special characters. Greek subscripts are reserved for type symbols and superscripts may only appear at the end of the identifier. The following symbols terminate identifiers: "`<Space> [ ] ( ) . <Return> <Tab>`". They may not appear inside an identifier. Reserved for special purposes package are "`: ; ‘ < >`" and should therefore not be used. Also with special characters $\forall$, $\exists$, and λ are also single character identifiers. In strings, superscripted numbers are preceded by "`~` ".
- **Identifiers in first-order mode** - In first-order mode all identifiers consist of a single letter. Upper and lower case letters denote distinct identifiers. In addition there is a set of keywords, currently `AND, OR, IMPLIES, NOT, FORALL, EXISTS, LAMBDA, EQUIV`, which are multi-letter identifiers and are always converted to all uppercase. They have to be delimited by one of the terminating characters listed above, while all other identifiers may be typed without spaces in between.
- **Type inference** - TPS3 implements a version of Milner's algorithm to infer the most general type of a wff with no or incomplete type information. Internally every identifier in a wff is typed. Only the first occurrence of an identifier will be typed in printing, unless the same identifier occurs with different types in the same wff.

6. Printing Vertical Paths

There are a number of operations available in the editor and mate top levels for printing vertical path diagrams. Also, the following wff operation is available for printing vertical diagrams of jforms:

- VPFORM JFORM {FILE} {STYLE} {PRINTTYPES} {BRIEF} {VPFPAGE}

The default values are:

- `File` defaults to `TTY:`, the terminal.
- `Style` defaults to the value of the flag STYLE.
- `PrintTypes` defaults to the value of the flag PRINTTYPES.

- `Brief` has three possible settings: `T` means that only the names of logical atoms will be printed, and not their values, `NIL` means that under each atom its value will appear, and
- `L` means that just the atomnames will be printed in the diagram but a legend which contains every atom with its value will be appended to the first page of output.
- VpfPage is the number of characters which fit on one line.
- AndHeight is an optional global variable which is equal to the number of blank lines to be left for a conjunction. It defaults to 1.
- ForallIndent is another optional global variable, containing the number of columns the quantifier is set off its scope. The default is 1.

`BRIEF` can assume the values `T` for printing the diagram in brief format, `L` for a brief diagram, but with a legend (atomnames with their associated values) at the end of the first page, `LT` for a legend with type symbols forced to print and `NIL` which gives the the full diagram.

Both of these functions will prompt you for a comment after a few statistics about the diagram are given. The comment will be spread across the top lines of the diagram with carriage returns placed where you type them.

7. Global Parameters and Flags

The following Lisp identifiers are either flags or values used by the functions which read or write formulas.

CFontTable: This is a two dimensional array which is used to translate between special characters on the Concept screen and their internal name. For example, `(CFontTable 1 91)` is `AND`.

FIRST-ORDER-PRINT-MODE: If `T` wffs will be printed in first-order mode, otherwise in higher-order mode.

FIRST-ORDER-MODE-PARSE: If `T`, wffs will be parsed in first-order mode, otherwise higher-order parsing mode is in effect. See the section on parsing for a more detailed explanation.

LOWERCASERAISE: If this identifier is set to `T` then lower case letters will be converted to their upper case equivalents. This conversion is done when the formula is first parsed. The default value is `NIL`.

PC: A variable used by the formula printing functions. It stores the previous character printed. It is used to help determine spacing within the formula. Set to `NIL` in prt.lisp. Not important to the user.

PRINTDEPTH: When a formula is printed, subformulas at a depth of more that `PrintDepth` are not printed, but replaced by a `"&"`. In the formula editor, it is set to EDPRINTDEPTH. A `PRINTDEPTH` of `0` means that the formula will be printed up to arbitrary depth.

PRINTTYPES: If this is set to `T`, type symbols will be printed at least once on all primitive symbols. Otherwise, no types are

printed. This defaults to `T`, and can be toggled with the command shownotypes.

SailCharacters: This is a list of pairs, `(SYMBOL . NUM)`. Here `NUM` is the position in the `SAIL` character set for `SYMBOL`.

The following flags are used to control the way formulas are printed. Usually the default setting of all these flags will be adequate. For more information see the section on pretty-printing in the TPS3 user manual.

PPWFFLAG: if `T`, formulas will be pretty-printed. This is the default setting, except in the editor, where you can achieve pretty-printing with the @Ited(PP) command.

FLUSHLEFTFLAG: If `T`, no line of a pretty-printed formula will be indented. The default is `NIL`

FILLINEFLAG: If `NIL`, every argument of an associative infix operator will have a separate line. The default in `NIL`.

LOCALLEFTFLAG: If `T`, arguments of infix operators start in the same column as the operator. The default is `NIL`.

ATOMVALFLAG: If `T`, the name of every atom will be printed below its value.

ALLSCOPEFLAG: If `T`, all punctuations ("`[]`", "`.`") will appear in the formulas to be printed. No association to the left or precedence of logical connectives will be assumed.

8. Simple MetaWffs in TPS3

Even though in TPS3 the principle metalanguage is of course Lisp, it is often convenient to be able to use simple notations from the metalanguage and include them directly in the input format for Wffs. In TPS3 this is achieved by providing a notation for certain kinds of `WFFOPS` inside an external specification of a wff. This method is not perfect, but has other advantages as well, as we shall see.

8.1. The Notation. The motivation behind the notation is an analogy to Lisp: we use the backquote to introduce some Lisp form which is to be evaluated and inserted into the Wff. One restriction is that the wffop must return a *gwff* (or a subtype, like a *gvar*). The other is that TPS3 must have certain pieces of knowledge about the *wffop* used, in order to be able to determine the type of the result of applying the *wffop*.

Some examples of external format and what they are parsed to:

```
"forall x. `(lcontr [[lambda x. P x x x] [f x]])"
```

to

$$\forall x.P\,[f\ x]\,[f\ x]\,[f\ x]$$

```
"forall x exists y.
  `(lexpd z [f x] `(lexpd z [f x] [Q [f x] [f x] y] `(1)) `t)"
```

to

$$\forall x\exists y.[\lambda z\,[\lambda z^1\ Q\ z^1\ z\ y]\ z]\,[f\ x]$$

```
"‘(substitute-types ‘((A . (O . I))) [P(OA) subset Q(OA)])"
```

to

$$P_{o(o\iota)} \subseteq Q_{o(o\iota)}.$$

(The latter could have been more easily specified as

```
(substitute-types (("A" "OI")) "[P(OA) subset Q]")
```

but that is no longer possible when the formula is to be embedded in another.)

Here are the general rules:

- In an ordinary wff, a backquote may precede something of the form (*wffop arg ... arg*), where *wffop* has all the necessary type information. The typecase of *wffop* is irrelevant.
- Among (*arg ... arg*), each argument is either a gwff (and may contain other backquoted expressions) or a Lisp expression, which is considered a constant. This is necessary to supply arguments which are not gwffs to a *wffop*. Notice, that it must be the internal representation of the argument![1]

9. More about Jforms

Much of the code for handling jforms is in *jforms-labels.lisp*; see `defflavor jform` in this file for the definition.

In the same file we see:

```
(eval-when (load compile eval)
  (defflavor disjunction
    (mhelp "A disjunction label stands for a disjunction of wffs.")
    (inherit-properties jform)
    (include jform (type 'disjunction))
```

This tell us that a jform can be a disjunction.

10. Printing Proofs

Proofs printed in Scribe or TEXare preceded by preambles which are defined by the variables SCRIBE-PREAMBLE and VPFORM-TEX-PREAMBLE. The values of these flags are set in the *tps3.ini* file. Since these preambles source files in the directory *.../doc/lib*, things must be done carefully to make sure that SCRIBEPROOF and TEXPROOF will insert the appropriate pathname when tps is distributed to other locations. Note that the Makefile creates the file *tps3.sys*, which contains the variable sys-dir which shows where the tps was built.

When the Scribe preamble was changed to add

```
@@LibraryFile(KSets)
@@LibraryFile(Mathematics10)
```

[1] At some point one could work at removing this restriction, if types are handled properly.

some of the hacks in *tps.mss* may have become obsolete (but harmless). Mathematics10 is a file from the standard Scribe library; KSets is a file belonging to TPS3.

CHAPTER 7

Well-formed formulae operators

1. Operations on Wffs

By definition, operations on wffs differ from commands in that they return a meaningful value, usually another wff or a truth value. While commands are usually given at the top-level, operations are usually used inside the editor. In other respects, operations on wffs are very similar to commands in TPS3. The types of the arguments and the type of the result must be specified in the declaration of a *wffop*. Moreover, help for the arguments and help for the wffop itself is available. Arguments for wffops may be typed exactly the way arguments for commands are: one at a time after a short help message.

You may frequently have to refer to chapter 3, since it will be assumed below that you have a general idea of how the TPS3 top-level interprets commands.

1.1. Arguments to Wffops. In principle, arguments to (or results of) wffops can have any type defined inside TPS3. There are some argument types which are mainly used for wffops and rarely or not at all for commands. They are the following

GWFF: A generalized wff.

BOOLEAN: `NIL` for "false", anything else for "true". Internally these are converted `NIL` and `T` first. In particular, if a wffop has been declared to return an object of type `BOOLEAN`, this wffop may return anything, but `NIL` is printed as `NIL`, while everything else is printed as `T`.

TYPESYM: A type symbol (in string representation). This is extremely useful for error messages (inside THROWFAIL). For example, the type inference program may contain a line

```
(throwfail "Type " (t1.typesym) " does not match " (t2.typesym))
```

For most settings of the STYLE flag, this will print the types as true greek subscripts.

GVAR: A general variable. This is only one of a whole class of possible subtypes of wffs (`GWFF`). The `GETFN` for these special kinds of wffs can easily be described using the function GETWFF-SUBTYPE, which takes a predicate as the first argument, an `RWFF` as the second.

As an example for the definition of a subtype of `GWFF` serves the definition of `GVAR`:

```
(deftype gvar
  (getfn (getwff-subtype 'gvar-p gvar))
  (testfn gvar-p)
  (printfn printwffhere)
  (side-effects t)
  (no-side-effects edwff)
  (mhelp " A gwff which must be a logical variable"))
```

1.2. Defining Wffops. The format for defining a wffop is very similar to that for defining a MExpr. The function that does the definition is called DEFWFFOP. The general format is (enclose optional arguments)

```
(DefWffop <name>
{(ArgTypes <type1> <type2> ...)}
(ResultType <type>)
{(ArgNames <name1> <name2> ...)}
{(ArgHelp <help1> <help2> ...)}
{(Applicable-Q <fnspec>)}
{(Applicable-P <fnspec>)}
        {(WffArgTypes <type> ... <type>)}
        {(Wffop-Type <type>)}
        {(Wffop-Typelist (<typesymbol> ... <typesymbol>))}
        {(DefaultFns <fnspec1> <fnspec2> ...)}
        {(MainFns <fnspec1> <fnspec2> ...)}
        {(Replaces <wffop>)}
        {(Print-Op <boolean>)}
        {(Multiple-Recursion <boolean>)}
{(MHelp "<comment>")})
```

The keywords `ArgTypes`, `ArgNames`, `ArgHelp`, `DefaultFns`, `MainFns` and `MHelp` have the same meaning as for commands (MExprs). See Section 1. You have to mention `ArgNames` before `Applicable-P`, if you want to make use of the argnames without explicitly using lambda. The other keywords are as follows:

- `RESULTTYPE:` is the only non-optional part of the declaration and is used for printing the result of the wffop.
- `APPLICABLE-Q:` is a "quick" predicate (see Section 1.5) to decide whether the wffop is applicable to a given set of arguments. If omitted (or explicitly stated to be TRUEFN), it means that the wffop can always be applied.
- `APPLICABLE-P:` is a "slow" predicate which is supposed to check thoroughly whether the wffop is applicable. Again, if one wants to state explicitly that a wffop is always applicable, use TRUEFN.

`WFFARGTYPES`: There must be exactly as many *type* entries, as there are arguments to the *wffop*. Each *type* entry may be either a type (in string format) or `NIL`, which is used for arguments which are not `gwffs`.

`WFFOP-TYPE`: specifies a *type* in string format, which is the type of the result the *wffop*, or `NIL`, if the result is not a *gwff*.

`WFFOP-TYPELIST`

A list of type symbols which are to be considered type variables in the definition of the *wffop*.

`REPLACES`: The wffop being defined is to replace some previously defined wffop. This is used extremely rarely.

`PRINT-OP`: This is set to `T` for printing operations (which are usually defined using the macro DEFPRTOP, which sets this property automatically). By default, this property has value `NIL`.

`MULTIPLE-RECURSION`: seems to be set to `T` for most tests of equality and `NIL` everywhere else. I'm not entirely sure what it's for.

Here are some example which may shed more light onto the subject.

```
(defwffop substitute-l-term-var
  (argtypes gwff gvar gwff)
  (resulttype gwff)
  (argnames term var inwff)
  (arghelp "term" "var" "inwff")
  (wffargtypes "A" "A" "B") ; TERM and VAR are of type A
  (wffop-type "B") ; INWFF and result of type B
  (wffop-typelist "A" "B") ; where A and B may be any types.
  (mhelp "..."))

(defwffop lexpd
  (argtypes gvar gwff gwff occ-list)
  (resulttype gwff)
  (argnames var term inwff occurs)
  (arghelp "lambda variable" "term to be extracted" "contracted form"
   "occurrences to be extracted")
  (wffargtypes "A" "A" "B" NIL) ; TERM and VAR are of type A,
; INWFF is of type B, OCCURS is not
  (wffop-type "B") ; a gwff, result is of type B,
  (wffop-typelist "A" "B") ; where A and B may be any types.
  (applicable-p (lambda (var term inwff occurs)
  (declare (ignore inwff occurs))
  (type-equal term var)))
  (mhelp "..."))

(defwffop substitute-types
```

```
  (argtypes typealist gwff)
  (resulttype gwff)
  (argnames alist gwff)
  (arghelp "alist of types" "gwff")
  (mhelp "Substitute for types from list ((old . new) ...) in gwff."))
```

1.3. Defining Recursive Wffops. The category wffrec% is for recursive wff functions. Such operations are defined with the defwffrec function; they have only three properties: `ARGNAMES`, `MHELP` and `MULTIPLE-RECURSION`.

The point of this is that we needed a way of saving the `ARGNAME` information for functions which use an APPLY-LABEL, but are not wffops themselves. These are defined as wffrecs.

Some examples:

```
(defwffrec gwff-q
  (argnames gwff))

(defun gwff-q (gwff)
  (cond ((label-p gwff) (apply-label gwff (gwff-q gwff)))
((lsymbol-p gwff) t)
((atom gwff) nil)
((and (boundwff-p gwff) (gvar-p (caar gwff)) (gwff-q (cdr gwff))))
((and (gwff-q (car gwff)) (gwff-q (cdr gwff))))))

(defwffrec wffeq-def1
  (argnames wff1 wff2 varstack switch)
  (multiple-recursion t))

; the function wffeq-def1 is pages long, so it's not quoted here. Look
; in file wffequ2.lisp for details.
```

1.4. Defining a Function Performing a Wffop. There are some necessary restrictions on how to define proper wffops, other conventions are simply a matter of style. The following are general guidelines, which do not address the definition of flavors (see Section 6).

(1) All arguments to a wffop may be assumed to be of the correct type, when the function is invoked. This does not mean, that the function never should check for an error, but at least the function does not have to check whether an argument is well-formed, or whether an argument is a logical variable and not an application.
(2) Most user-level wffops get by without using any "slow" predicates for constituents of a gwff. Use the "quick" predicate and assume that the argument is a gwff.

(3) Make the name of a wffop as descriptive as possible. The user will rarely have to type this long name, since he will normally invoke wffops in the editor, where they can be given short aliases. See section 2.
(4) When using auxiliary functions, make sure their name can be easily related to the name of the main function.
(5) Check the wff operations in the TPS3 Facilities Guide for Programmers and Users before defining new functions. In particular, you should often use GAR and GDR instead of car and cdr to manipulate wffs, since the wffs may have labels.
(6) Always make sure you are invoking the "quick" test in the correct order, since later tests rely on the fact that earlier tests failed.

1.5. Quick Test versus Slow Test. Most predicates which test for certain kinds of subformulas come in two incarnations: as a "quick" test and a "slow" test. As a general convention that should never be violated, both functions have the same name except for the last character, which is `-Q` for the quick test and `-P` for the slow test.

As a rule of thumb, quick predicates may assume a very restricted kind of argument (e.g. a literal atom), but may not work recursively down into the formula. Slow predicates, however, may assume nothing about the argument (they should always work), and often have to do a recursion to see whether the predicate is true of the argument.

Quick predicates are most useful when in recursive functions that implement a wffop. Slow predicates are chiefly called inside the editor to test that certain transformations or applications will be legal, *before they are performed*. Speed is usually not important when working in the editor, but wffops in general should be optimized for speed, since time does make a difference in automatic mode.

A list of the most useful quick predicates in the order in which they must be called is supplied here. See the comments attached to the predicates in the source file if this list is unclear or ambiguous.

It is absolutely essential to understand the role of quick predicates and the order of their invocation to write bug-free code!

LABEL-Q *gwff***:** tests for a label. The standard action in this case is `(APPLY-LABEL GWFF (`*wffop arg1* `...` *argn*`))` where *wffop* is the wffop we are defining and *arg1* through *argn* are its arguments. Always call this first, since any given argument may be a label.

LSYMBOL-Q *gwff***:** tests for a logical symbol. This could either be a variable, constant, or abbreviation. This must come after the test for *label*, but does not assume anything else. There are several subtypes of *lsymbol* which assume that their argument is a *lsymbol* and must be called in the following order:

LOGCONST-Q *gwff***:** a logical constant, which must have been declared with `DEF-LOGCONST`.

PROPSYM-Q *gwff***:** a proper symbol, that is something that has not been declared a constant or abbreviation.

PMPROPSYM-Q *gwff***:** a polymorphic proper symbol (higher-order mode only).

PMABBREV-Q *gwff***:** a polymorphic abbreviation (higher-order mode only).

ABBREV-Q *gwff* **:** an abbreviation.

BOUNDWFF-Q: Test whether the wff starts with a binder (of any type) and assumes that we already know that it is neither *label* nor a *lsymbol* (in Lisp terms: it must be a `CONS` cell). Access the bound variable with `CAAR`, the binder with `CDAR`, the scope of the binder with `CDR`. Construct a new bound formula with `(CONS (CONS` *bdvar binder*`)` *scope*`)`.

`T`: This is the "otherwise" case, i.e. we have an application. Access the "function" part with `CAR`, the "argument" part with `CDR`. Construct a new application with `(CONS` *function argument*`)`. Remember also that all functions and predicates are curried.

Examples of Wffops

The following examples are taken from actual code[1].

```
The following are two different substitution functions
SUBSTITUTE-TERM-VAR (currently in wffsub1.lisp)
substitutes a term for a variable, but gives
and error if the term is not free for the variable in the wff.
SUBSTITUTE-L-TERM-VAR (currently in wffsub2.lisp)
also substitutes a term for a variable,
but renames bound variables if a name conflict occurs.
There may be a global variable, say SUBST-FN, whose value is
the function used for substitution by default, or there may be a function
SUBSTITUTE, which checks certain flags to determine which function
to call.

(defwffop substitute-term-var
  (argtypes gwff gvar gwff)
  (wffargtypes "A" "A" "B")
  (resulttype gwff)
  (wffop-type "B")
  (wffop-typelist "A" "B")
  (argnames term var inwff)
  (arghelp "term" "var" "inwff")
  (applicable-p (lambda (term var inwff) (free-for term var inwff)))
  (mhelp
   "Substitute a term for the free occurrences of variable in a gwff."))
```

[1]As of July 7th, 1994

```
(defun substitute-term-var (term var inwff)
  "This function should be used with extreme caution. There’s an underlying
  assumption that TERM is free for VAR in INWFF (which is true if TERM is
  a new variable)."
  (or (subst-term-var-rec (intern-subst term var) var inwff)
      inwff))

(defun subst-term-var-rec (term var inwff)
  (cond ((label-q inwff)
 (apply-label inwff (subst-term-var-rec term var inwff)))
((lsymbol-q inwff) (if (eq var inwff) term nil))
((boundwff-q inwff)
 (if (eq (caar inwff) var) nil
     (let ((new-wff (subst-term-var-rec term var (cdr inwff))))
       (if new-wff (cons (car inwff) new-wff) nil))))
(t (let ((left (or (subst-term-var-rec term var (car inwff))
   (car inwff)))
 (right (or (subst-term-var-rec term var (cdr inwff))
    (cdr inwff))))
     (unless (and (eq left (car inwff)) (eq right (cdr inwff)))
     (cons left right))))))

(defwffop substitute-l-term-var
  (argtypes gwff gvar gwff)
  (wffargtypes "A" "A" "B")
  (resulttype gwff)
  (wffop-type "B")
  (wffop-typelist "A" "B")
  (argnames term var inwff)
  (arghelp "term" "var" "inwff")
  (mhelp
   "Substitute a term for the free occurrences of variable in a gwff.
Bound variables may be renamed, using the function in the global
variable REN-VAR-FN."))

(defun substitute-l-term-var (term var inwff)
  (or (subst-l-term-rec (intern-subst term var) var inwff) inwff))
```

LCONTR (currently in wfflmbd2.lisp)
does a Lambda-contraction. Notice the use of
THROWFAIL and the use of general predicates like LAMBDA-BD-P
rather than testing directly whether a given wff is bound by
Lambda. This way, the function works, even if the CAR fo
the application is a label!

```
(defwffop lcontr
  (argtypes gwff)
  (wffargtypes "A")
  (resulttype gwff)
  (wffop-type "A")
  (wffop-typelist "A")
  (argnames reduct)
  (arghelp "gwff (reduct)")
  (applicable-p reduct-p)
  (mhelp "Lambda-contract a top-level reduct.
Bound variables may be renamed using REN-VAR-FN"))

(defun lcontr (reduct)
  (cond ((label-q reduct) (apply-label reduct (lcontr reduct)))
((lsymbol-q reduct)
 (throwfail "Cannot Lambda-contract " (reduct . gwff)
    ", a logical symbol."))
((boundwff-q reduct)
 (throwfail "Cannot Lambda-contract " (reduct . gwff)
    ", a bound wff."))
(t (if (lambda-bd-p (car reduct))
       (substitute-l-term-var (cdr reduct) (gar (car reduct))
      (gdr (car reduct)))
       (throwfail "Top-level application " (reduct . gwff)
  " is not of the form [LAMBDA x A]t.")))))
```

FREE-FOR is a simple example of a predicate on wffs.
Here, the type of the result is declared to be BOOLEAN.

```
(defwffop free-for
  (argtypes gwff gvar gwff)
  (resulttype boolean)
  (argnames term var inwff)
  (arghelp "term" "var" "inwff")
  (applicable-q (lambda (term var inwff) (declare (ignore inwff))
(type-equal term var)))
  (applicable-p (lambda (term var inwff) (declare (ignore inwff))
(type-equal term var)))
  (mhelp "Tests whether a term is free for a variable in a wff."))

(defun free-for (term var inwff)
  (cond ((label-q inwff)
 (apply-label inwff (free-for term var inwff)))
   ((lsymbol-q inwff) t)
```

```
((boundwff-q inwff)
 (cond ((eq (caar inwff) var) t)
       ((free-in (caar inwff) term)
(not (free-in var (cdr inwff))))
       (t (free-for term var (cdr inwff)))))
(t (and (free-for term var (car inwff))
(free-for term var (cdr inwff))))))
```

TYPE (currently in wffprim.lisp)
returns the type of the argument. The name is a very
troublesome one and we may eventually need to change it globally so as not
to conflict with Common Lisp.

```
(defwffop type
(argtypes gwff)
(resulttype typesym)
(argnames gwff)
(arghelp "gwff")
(mhelp "Return the type of a gwff."))

(defun type (gwff)
  (cond ((label-q gwff) (apply-label gwff (type gwff)))
((lsymbol-q gwff) (get gwff 'type))
((boundwff-q gwff) (boundwfftype gwff))
(t (type-car (type (car gwff))))))
```

The following are a sequence of functions which instantiate abbreviations.
One can either instantiate a certain abbreviation everywhere
(INSTANTIATE-DEFN), instantiate all abbreviations (not recursively)
(INSTANTIATE-ALL), or instantiate the first abbreviates, counting
from left to right (INSTANTIATE-1).
The functions are implemented by one master function, one of whose
arguments is a predicate to be applied to an abbreviation. This
predicate should return something non-NIL, if this occurrence is to be
instantiated, NIL otherwise.
Notice the subcases inside LSYMBOL-Q and the order of the quick
predicates in the OR clause.

```
(defwffop instantiate-defn
  (argtypes symbol gwff)
  (resulttype gwff)
  (argnames gabbr inwff)
  (arghelp "abbrev" "inwff")
  (applicable-p (lambda (gabbr inwff) (declare (ignore inwff))
```

```
(or (abbrev-p gabbr) (pmabbsym-p gabbr))))
  (mhelp "Instantiate all occurrences of an abbreviation.
The occurrences will be lambda-contracted, but not lambda-normalized."))

(defun instantiate-defn (gabbr inwff)
  (instantiate-definitions
   inwff #'(lambda (abbsym chkarg) (eq abbsym chkarg)) gabbr))


(defwffop instantiate-all
  (argtypes gwff symbollist)
  (resulttype gwff)
  (argnames inwff exceptions)
  (arghelp "inwff" "exceptions")
  (defaultfns (lambda (&rest rest)
(mapcar #'(lambda (argdefault arg)
    (if (eq arg '$) argdefault arg))
'($ NIL) rest)))
  (mhelp "Instantiate all definitions, except the ones specified
in the second argument."))

(defun instantiate-all (inwff exceptions)
  (instantiate-definitions
   inwff #'(lambda (abbsym chkarg) (not (memq abbsym chkarg))) exceptions))

(defwffop instantiate-1
  (argtypes gwff)
  (resulttype gwff)
  (argnames inwff)
  (arghelp "inwff")
  (mhelp "Instantiate the first abbreviation, left-to-right."))

(defun instantiate-1 (inwff)
  (let ((oneflag nil))
    (declare (special oneflag))
    (instantiate-definitions
     inwff #'(lambda (abbsym chkarg)
       (declare (ignore abbsym chkarg) (special oneflag))
       (prog1 (not oneflag) (setq oneflag t)))
     nil)))

(defwffrec instantiate-definitions
  (argnames inwff chkfn chkarg))

(defun instantiate-definitions (inwff chkfn chkarg)
```

```
  (cond ((label-q inwff)
 (apply-label inwff (instantiate-definitions inwff chkfn chkarg)))
((lsymbol-q inwff)
 (cond ((or (logconst-q inwff) (propsym-q inwff) (pmpropsym-q inwff))
inwff)
       ((pmabbrev-q inwff)
(if (funcall chkfn (get inwff 'stands-for) chkarg)
    (get-pmdefn inwff) inwff))
       ((abbrev-q inwff)
(if (funcall chkfn inwff chkarg) (get-defn inwff) inwff))))
((boundwff-q inwff)
 (if (and (anyabbrev-q (binding inwff))
  (funcall chkfn (binding inwff) chkarg))
     (get-def-binder (binding inwff) (bindvar inwff) (gdr inwff))
     (cons (car inwff)
   (instantiate-definitions (gdr inwff) chkfn chkarg))))
(t (let ((newcar (instantiate-definitions (car inwff) chkfn chkarg)))
     (if (and (lambda-bd-p newcar) (not (lambda-bd-p (car inwff))))
 (lcontr (cons newcar
       (instantiate-definitions (cdr inwff)
chkfn chkarg)))
 (cons newcar
       (instantiate-definitions (cdr inwff) chkfn chkarg)))))))
```

2. The formula editor

The formula editor is in many ways very similar to the top-level of TPS3. The main difference is that we have an entity called "current wff" or edwff, which can be operated on. All the regular top-level commands can still be executed, but we can now also call any *wffop* directly. If we want the *wffop* to act on the *edwff*, we can specify `EDWFF` which is a legal *gwff* inside the editor.

This process is made even easier through the introduction of *edops*. An edop is very similar to a *wffop*, but it ties into the structure of the editor in two very important ways: One argument can be singled out, so that it will always be the *edwff*, and secondly the *edop* will specify what happens to the result of the operations, which is often the new *edwff*. This is particularly useful for operations which take one argument and return one wff as a value, like lambda-normalization. It helps to give edops and wffops different names; the name of a wffop should be longer and more descriptive than the name of the edop for which it is an alias.

3. Example of Playing with a Jform in the Editor

```
<Ed9>sub x2115
```

```
<Ed10>neg
<Ed13>cjform
(AND ((FORALL x<I>) (OR ((FORALL y<I>) LIT0) ((FORALL z<I>) LIT1)))
 ((FORALL u<I>) ((EXISTS v<I>) (OR LIT2 (AND LIT3 LIT4))))
 ((FORALL w<I>) (OR LIT5 LIT6)) ((EXISTS u<I>) ((FORALL v<I>) (OR LIT7 LIT8))))

<Ed14>edwff
(AND ((FORALL x<I>) (OR ((FORALL y<I>) LIT0) ((FORALL z<I>) LIT1)))
 ((FORALL u<I>) ((EXISTS v<I>) (OR LIT2 (AND LIT3 LIT4))))
 ((FORALL w<I>) (OR LIT5 LIT6)) ((EXISTS u<I>) ((FORALL v<I>) (OR LIT7 LIT8))))

<Ed15>(setq aa edwff)
(AND ((FORALL x<I>) (OR ((FORALL y<I>) LIT0) ((FORALL z<I>) LIT1)))
 ((FORALL u<I>) ((EXISTS v<I>) (OR LIT2 (AND LIT3 LIT4))))
 ((FORALL w<I>) (OR LIT5 LIT6)) ((EXISTS u<I>) ((FORALL v<I>) (OR LIT7 LIT8))))

<Ed16>aa
(AND ((FORALL x<I>) (OR ((FORALL y<I>) LIT0) ((FORALL z<I>) LIT1)))
 ((FORALL u<I>) ((EXISTS v<I>) (OR LIT2 (AND LIT3 LIT4))))
 ((FORALL w<I>) (OR LIT5 LIT6)) ((EXISTS u<I>) ((FORALL v<I>) (OR LIT7 LIT8))))

<Ed17>(auto::jform-parent aa)
NIL
vp
<Ed19>(setq bb (auto::conjunction-components aa))
(((FORALL x<I>) (OR ((FORALL y<I>) LIT0) ((FORALL z<I>) LIT1)))
 ((FORALL u<I>) ((EXISTS v<I>) (OR LIT2 (AND LIT3 LIT4))))
 ((FORALL w<I>) (OR LIT5 LIT6)) ((EXISTS u<I>) ((FORALL v<I>) (OR LIT7 LIT8))))

<Ed20>(length bb)
4
<Ed21>(car bb)
((FORALL x<I>) (OR ((FORALL y<I>) LIT0) ((FORALL z<I>) LIT1)))

<Ed22>(cadr bb)
((FORALL u<I>) ((EXISTS v<I>) (OR LIT2 (AND LIT3 LIT4))))

<Ed23>(auto::jform-parent (car bb))
(AND ((FORALL x<I>) (OR ((FORALL y<I>) LIT0) ((FORALL z<I>) LIT1)))
 ((FORALL u<I>) ((EXISTS v<I>) (OR LIT2 (AND LIT3 LIT4))))
 ((FORALL w<I>) (OR LIT5 LIT6)) ((EXISTS u<I>) ((FORALL v<I>) (OR LIT7 LIT8))))

<Ed24>(setq cc '((FORALL x<I>) (OR ((FORALL y<I>) LIT0) ((FORALL z<I>) LIT1))))
((FORALL X<I>) (OR ((FORALL Y<I>) LIT0) ((FORALL Z<I>) LIT1)))
```

```
<Ed26>bb
(((FORALL x<I>) (OR ((FORALL y<I>) LIT0) ((FORALL z<I>) LIT1)))
 ((FORALL u<I>) ((EXISTS v<I>) (OR LIT2 (AND LIT3 LIT4))))
 ((FORALL w<I>) (OR LIT5 LIT6)) ((EXISTS u<I>) ((FORALL v<I>) (OR LIT7 LIT8))))

<Ed27>(car bb)
((FORALL x<I>) (OR ((FORALL y<I>) LIT0) ((FORALL z<I>) LIT1)))

<Ed28>cc
((FORALL X<I>) (OR ((FORALL Y<I>) LIT0) ((FORALL Z<I>) LIT1)))

(These look the same, but they are quite different.)
```

4. Defining an EDOP

An *edop* does not define an operation on wffs, it simply **refers** to one. Thus typically we have a *wffop* associated with every *edop*, and the *edop* inherits almost all of its properties from the associated *wffop*, in particular the help, the argument types, the *applicable* predicates etc.

A definition of an *edop* itself then looks as follows (enclose optional arguments)

```
(DefEdop <name>
{(Alias <wffop>)}
(Result-> <destination>)
{(Edwff-Argname <name>)}
        {(DefaultFns <fnspec1> <fnspec2> ...)}
        {(Move-Fn <fnspec>)}
{(MHelp "<comment>")})
```

In the above definition, the properties have the following meanings:

- `ALIAS`: This is the name of the *wffop* this *edop* refers to. It must be properly declared using the `DEFWFFOP` declaration.
- `RESULT->`: This provides part of the added power of *edops*. *destination* indicates what to do with the result of applying the *wffop* in `ALIAS` to the arguments. *destination* can be any of the following:
  - *omitted:* If omitted, the appropriate print function for the type of result returned by the `ALIAS` *wffop* will be applied to the result.
  - `EDWFF`: This means that the result of the operation is made the new current wff (*edwff*) in the editor.
  - `EXECUTE`: This means that the result of the operation is a list of editor commands which are to be executed. This may seem strange, but is actually very useful for commands like `FI` (find the first infix operator), or `ED?` (move to edit the first ill-formed subpart). The argument type ED-COMMAND was introduced for this purpose only.

fnspec: If the value is none of the above, but is specified, it is assumed to be an arbitrary function of one argument, which is applied to the result returned by the *edop*.

`EDWFF-ARGNAME:` This is the name of the argument that will be filled with the *edwff*; see the `ARGNAME` property of MExprs, in section 1, for more information.

`DEFAULTFNS:` See the arguments for MExprs, in section 1.

`MOVE-FN:` This means that the result of the operation will be the new current wff and moreover that the operation qualifies as a "move", namely that we should store what we currently have before executing the command, and then use *replace-fn* on the value returned after then next `0` or ˆ. For example, the editor command `A` moves to the "function part" of an application. Moreover, when we return via `0` or ˆ, we need to replace this "function part".

5. Useful functions

A useful function in defining *edops* is EDSEARCH. EDSEARCH *gwff predicate* will go through *gwff* from left to right and test at every subformula, whether *predicate* is true of that subformula. If such a subformula is found, *EDSEARCH* will return a list of editor moving commands which will move down to this subformula. If the predicate is true of the *gwff* itself, `EDSEARCH` will return (P), the command to print the current wff. If no subformula satisfying *predicate* is found, `EDSEARCH` will return `NIL`. For example

```
(defedop fb
  (alias find-binder)
  (result-> execute)
  (mhelp "Find the first binder (left to right)")
  (edwff-argname gwff))

(defwffop find-binder
  (argtypes gwff)
  (resulttype edcommand)
  (argnames gwff)
  (arghelp "gwff")
  (mhelp "Find the first binder (left to right)"))

(defun find-binder (gwff) (edsearch gwff (function boundwff-p)))
```

6. Examples

Consider the following examples.[2]

[2]As taken from the code, 7th July 1994.

```
(defedop ib
  (alias instantiate-binder)
  (result-> edwff)
  (edwff-argname bdwff))

(defwffop instantiate-binder
  (argtypes gwff gwff)
  (resulttype gwff)
  (argnames term bdwff)
  (arghelp "term" "bound wff")
  (applicable-p (lambda (term bdwff)
  (and (ae-bd-wff-p bdwff) (type-equal (gar bdwff) term))))
  (mhelp
   "Instantiate a top-level universal or existential binder with a term."))

(defun instantiate-binder (term bdwff)
  (cond ((label-q bdwff)
 (apply-label bdwff (instantiate-binder term bdwff)))
((lsymbol-q bdwff)
 (throwfail "Cannot instantiate " (bdwff . gwff)
    ", a logical symbol."))
((boundwff-q bdwff)
 (cond ((ae-bd-wff-p bdwff)
(substitute-l-term-var term (caar bdwff) (cdr bdwff)))
       (t
(throwfail "Instantiate only existential or universal quantifiers," t
   "not " ((cdar bdwff) . fsym) "."))))
(t (throwfail "Cannot instantiate an application."))))

(defedop subst
  (alias substitute-l-term-var)
  (result-> edwff)
  (edwff-argname inwff))

(defedop db
  (alias delete-leftmost-binder)
  (result-> execute)
  (edwff-argname gwff))

(defwffop delete-leftmost-binder
  (argtypes gwff)
  (resulttype ed-command)
  (argnames gwff)
  (arghelp "gwff")
```

```
  (mhelp "Delete the leftmost binder in a wff."))

(defun delete-leftmost-binder (gwff)
  (let ((bdwff-cmds (find-binder gwff)))
    (append (ldiff bdwff-cmds (member 'p bdwff-cmds))
    `(sub (delete-binder edwff)))))


(defwffop delete-binder
  (argtypes gwff)
  (resulttype gwff)
  (argnames bdwff)
  (arghelp "bound wff")
  (applicable-q ae-bd-wff-p)
  (applicable-q ae-bd-wff-p)
  (mhelp "Delete a top-level universal or existential binder."))

(defun delete-binder (bdwff)
  (cond ((label-q bdwff)
 (apply-label bdwff (delete-binder bdwff)))
((lsymbol-q bdwff)
 (throwfail "Cannot delete binder from " (bdwff . gwff)
    ", a logical symbol."))
((boundwff-q bdwff)
 (cdr bdwff))
(t (throwfail "Cannot delete binder from an application."))))
```

6.1. Global Parameters and Flags. The following are the flags and parameters controlling the output of the editing session. Note that there are also editor windows, which have separate flags; type `SEARCH "EDWIN" T` to see a list of these.

- **PRINTEDTFILE:** The name of the file in which wffs are recorded.
- **PRINTEDTFLAG:** If `T`, a copy of the current editing in `ED` will be printed into the file given by `PRINTEDTFILE`. The prompt will also be changed to `-ED` or ED+.
- **PRINTEDTFLAG-SLIDES:** As `PRINTEDTFLAG`, but the output is in Scribe 18-point style.
- **PRINTEDTOPS:** contains the name of a function which tests whether or not to print a particular wff to the `PRINTEDTFILE`.
- **VPD-FILENAME:** is the equivalent of `PRINTEDTFILE` for vertical path diagrams.
- **PRINTVPDFLAG:** is the equivalent of `PRINTEDTFLAG` for vertical path diagrams.

The flags and parameters listed below are the counterparts of flags described in full detail on page 7. They have the identical meaning, except that they are effective in the editor, while their counterparts are effective on the top-level of TPS3.

EDPPWFFLAG: If `T`, wffs in the editor will generally be pretty-printed. Default is `NIL`.

EDPRINTDEPTH: The value used as PRINTDEPTH within the formula editor. It is initialized to `0`.

7. The formula parser

7.1. Data Structures.

ByteStream: This list stores essentially the printing characters which are in its input. `CR`, `LF`, and `TAB` characters are replaced with a space. The ending `ESC` does not appear in this list. All elements are `INTERN` identifiers. See the function bytestream-tty in *wffing.lisp*.

RdCList: This data structure appears to be all but obsolete; the last remnants of it are in the file *wffing.lisp*. `RdC` refers to the Concept terminal. This list contains either integers between 0 and 127, lists containing precisely one of 0, 1, or 3, or the identifier CRLF. The lists represent character set switches, the integers represent characters, and `CRLF` represents a carriage return/line feed combination.

LexList: This is a list of lexical objects, i.e. it contains name for logical objects which will appear in the fully parsed formula. It also contains the brackets "[","]", and ".". It also contains the type symbols from the initial input. These are distinguishable from the other items in the list since they are stored as lists. Hence, LexList is a "flat" list of these three things.

TypeAssoc: This is an association list which associates to those identifiers in the LexList which got a type, that type. This is necessary so that an identifier which is typed explicitly at one place in the formula can have that type attributed to it at other non-typed occurrences.

GroupList: This is essentially the same as `LexList`, except that the bracket identifiers are removed, and nested s-expressions are used to denote groupings. Type symbols are also "attached" to the identifier preceding it. Hence a GroupList contains only logical identifiers - some with types and some without - grouped in a hierarchical fashion.

PreWff: This data structure is like that of the wff structure, except that not all items are correctly typed yet. The full prefix organization is present in this formula. The types for polymorphic definitions, however, are not yet computed.

7.2. Processing. Input is first processed into `ByteStream`s and then into `LexList`s by the function LexScan.

GroupScan now operates on `LexList` in order to construct the `GroupList`. This function has no arguments and uses a special variable, called LexList, to communicate with recursive calls to itself. `GroupScan` is also responsible for building the `TypeAssoc` list.

InfixScan converts a `GroupList` into a `PreWff`. This requires using the standard infix parser. MakeTerm is used to build the prefix subformulas of the input.

Now that all logical items appear in their final positions, the actual types of polymorphic abbreviations can be determined. This is the job of FinalScan. This function takes a `PreWff` and returns with a `WFF`.

This is not a very efficient algorithm. A few of the passes could be joined together, and a few might be made more efficient by using destructive changes. The parser, however, is rather easy to upgrade.

CHAPTER 8

Help and Documentation

1. Providing Help

When the user types the command HELP object, TPS3 will first try to determine which category `object` is in (it may be in several, in which case it will produce a list of categories and then print the help for each separately).

Recall from the entry about categories (section 2) that each category has `mhelp-line` and `mhelp-fn` properties. The `mhelp-line` is a short phrase that describes each object in the category (for example the category `PMPROPSYM` has the mhelp line "polymorphic proper symbol"). The mhelp-fn is a function that will print out the help for a specific object. For many simple categories (e.g. `context`), the function `princ-mhelp` is sufficient; this simply prints the `mhelp` string attached to the object in question. Other categories need more complex help (for example `mexpr`), and so have their own specially-defined mhelp functions.

An example of a `mexpr` which has some automatically constructed help information is the function EXTRACT-TEST-INFO in maint.lisp.

When writing help messages or mhelp functions, keep in mind that the information given should contain all the information that a *user* would want to know. More detailed help for maintainers and programmers should be written down and incorporated into this manual, not added into the online documentation.

1.1. Mhelp and Scribe. The online documentation can be used to generate a facilities guide, so it is important that you be aware that the mhelp properties and mhelp functions you define for new objects or categories will be used to generate Scribe files. Take a look at the files *mhelp.lisp* and *scrdoc.lisp* and see how this works. You may need to set things up properly so that the entries you are introducing are put into the index of such guides. Look at the file *tpsdoc.mss* in the doc/lib area to see how the indexing is done.

1.2. Mhelp and LaTeX. The online documentation can also be used to generate a facilities guide using LaTeX. Take a look at the files *mhelp.lisp* and *latexdoc.lisp* and see how this works. Because of the restrictions of LaTeXconcerning special command characters, such as "$ " or "\", the help processing may need special care: an alternative to the usual `princ-mhelp` function is provided to handle such characters in *latexdoc.lisp* as `princ-mhelp-latex`.

Many specific LaTeXcommands are defined in the *tps.tex* and *tpsdoc.tex* files in the doc/lib area. The User's guide describes how the processing of such manuals is done. You may need to set things up properly so that the entries you are introducing are put into the index of such guides. Look at the file *latexdoc.lisp* in the lisp area to see how the indexing is done.

2. The Info Category

There is a category of objects called INFO which is used solely for providing help on symbols that would otherwise not have help messages (for example, the various settings of some of the flags, such as PR97 or APPLY-MATCH-ALL-FRDPAIRS). You can attach a help message to any symbol *foo* with:

```
(definfo foo (mhelp "Help text."))
```

3. Printed Documentation

The directories with root */home/theorem* mentioned below are on gtps.

/home/theorem/project/doc/files.dir contains information about TPS3 documentation files.

/home/theorem/project/doc/etps/tps-cs.mss describes how to access TPS3 in the cmu cs cell.

/home/theorem/project/doc/etps/etps-andrew.mss describes how to access ETPS in the andrew cell.

/home/theorem/project/doc/<topic>/manual.mss is the main file for the manual on <topic> (one of: `char`, `etps`, `facilities`, `grader`, `prog`, `teacher` and `user`).

See the TPS3 User Manual for additional information.

When new facilities are added to ETPS , copy the information about them from the automatically produced *facilities.mss* and *facilities.tex* into the appropriate ETPS mss/tex file.

4. Indexing in the Manuals

The basic mechanisms are in */home/theorem/project/doc/lib/index.lib* and */home/theorem/project/doc/lib/indexcat.mss*. Note the comment on the use of `@IndexCategory` in the former file. In the TEXversion of the Programmer's Guide, there are indexing commands defined which mimic the role of the corresponding Scribe commands.

`@indexotherDIY-TAC` in the text on page <pagenumber> puts `DIY-TAC <pagenumber>` into the index.

`@index*X*(WORD)` in the text on page <pagenumber> puts `WORD, *Y* <pagenumber>` into the index.

Example: `@indexcommandDO-GRADES` in the text on page <pagenumber> puts `DO-GRADES, System Command <pagenumber>` into the index. Here is a partial list of of possible values for `*X*` and `*Y*`, where the complete list is in */home/theorem/project/doc/lib/indexcat.mss*.

- `*X*` = command gives `*Y*` = System Command
- `*X*` = edop gives `*Y*` = Editor Command
- `*X*` = flag gives `*Y*` = flag
- `*X*` = function gives `*Y*` = function
- `*X*` = style gives `*Y*` = style
- `*X*` = mexpr gives `*Y*` = mexpr

See "quitting" in the index of the ETPS manual to see the effect of the following lines in the file */home/theorem/project/doc/etps/system.mss*:

```
@@seealso[Primary="Quitting",Other="@{\tt EXIT}"]
@@seealso[Primary="Quitting",Other="@{\tt END-PRFW}"]
@@seealso[Primary="Quitting",Other="@{\tt OK}"]
@@indexentry[key="Quitting",entry="Quitting"]
```

5. Other commands in the manuals

Any other Scribe commands may be used in the manuals; for example we use the `typewriter` font given by @t for command names, and the *italic* font given by @i for file names.

In the TEXversions of the manuals, one uses the corresponding TEXcommands.

We also have @TPS in Scribe (and `\TPS` in TEX) to print the string "TPS3 ", and @HTPS in Scribe to do the same in headers.

6. Converting Scribe to LATEX documentation

The aime of this section is to provide helpful information on how to program a new documentation device.

6.1. The *latexdoc.lisp* file. This file was written as an equivalent to *scrdoc.lisp*. Functions and macros are essentially equivalent. Nevertheless, while the Scribe documentation system contains several calls to special function, such as `scribe-one-fn`, which are internal properties of special objects (e.g. tactics), *latex-doc.lisp* contains every LATEX-specific help-formatting function.

Some of these functions uses the tex style in the TPS3 system, which is described in *deftex.lisp*. As the style properties were thought to be used with Scribe, some of them cannot be easily translated in LATEX. This obstacle occurs for instance when using the Scribe `@Tabset` function, which has no strict equivalent in LATEX, where it is usually replaced by the `Description` environment.

6.2. Special Characters. The mhelp properties of many TPS3 objects present special characters: when Scribe prevents the use of , a lot of characters have to be escaped, using a prefixed "\", when generating a LATEX document. The most common are: ~ , # , $, % , and & . Note that the backslash character in LATEX is `\textbackslash` and the character "^" is `\textasciicircum`.

In order to prevent the programmer from editing every mhelp property, a function `princ-mhelp-latex` is used instead of the regular `princ-mhelp`. This function simply replaces every occurence of a protected character by the correspondant LaTeX sequence.

6.3. LaTeX Macros. To facilitate the conversion from Scribe to LaTeX commands, a great number of new commands and macros for LaTeX are defined in *tps.tex* and *tpsdoc.tex*. These commands enable us to use commands such as `\greeka` instead of the regular `\alpha`.

CHAPTER 9

Flags

Here is an example of how to add the flag *neg-prim-sub* to TPS3 :

Insert into the file prim.lisp the code:

```
(defflag neg-prim-sub
  (flagtype boolean)
  (default nil)
  (subjects primsubs)
  (mhelp "When T, one of the primitive substitutions will introduce negation."))
```

Actually, that code almost worked, but changing the flag did not have the desired effect. The primsubs were stored in a hashtable which, once computed, was never changed again, so the code had to be replaced by:

```
(defflag neg-prim-sub
  (flagtype boolean)
  (default nil)
  (change-fn (lambda (a b c)
       (declare (ignore a b c))
       (ini-prim-hashtable)))
  (subjects primsubs)
  (mhelp "When T, one of the primitive substitutions will introduce negation."))
```

Also put into the file auto.exp the line

```
(export '(neg-prim-sub))
```

There are two ways to update flags. One is to do it manually. This is supported by function update-flag. The other way is to set flags automatically. For example, you may have to do this in your .ini file. If XXX is a flag and you want to set it to YYY, then you can add a line (set-flag 'XXX YYY) in your .ini file. Sometimes, you may use (setq XXX YYY), but this is highly discouraged because XXX may have a "change-fn" associated with it, which should be called whenever you set XXX. Flag HISTORY-SIZE is such an example. (Note that if the variable being set is just a variable, and not a TPS3 flag, then the setq form is correct.)

1. Symbols as Flag Arguments

If your new flag accepts symbols as arguments, and only certain symbols are acceptable (as in, for example, PRIMSUB-METHOD or APPLY-MATCH), the symbols which can be used should have help messages attached somehow. This can either be done by defining a new category for the arguments, such as ORDERCOM or DEV-STYLE, or it can be done using the definfo command: `(definfo foo (mhelp "Help text."))` attaches the given text to the symbol `foo`.

2. Synonyms

It is possible to define two flags with different names which are synonymous to each other, using the defsynonym macro. The advantage of this is that it allows the name of a flag to be changed (from the user's point of view) without requiring either a change in the code or extensive editing of all the modes saved in the library.

For example:

```
(defsynonym SUBNAME
            (synonym TRUENAME)
            (replace-old T)
            (mhelp "SUBNAME is a synonym for the flag TRUENAME."))
```

defines a new synonym for `TRUENAME`. The *replace-old* property determines whether or not the new synonym is to be regarded as the "new name" of the flag, if replace-old is `T` (and so to be recorded in the library, etc.) or merely as an alias, if replace-old is `NIL`.

3. Relevancy Relationships Between Flags

When defining a new flag, one can specify relevancy relationships between the flag and the values of other flags. For example, if the flag **DEFAULT-MS** is set to **MS90-3**, then the flag **MS98-NUM-OF-DUPS** is irrelevant. On the other hand, if **DEFAULT-MS** is set to **MS98-1**, then the flag **MS98-NUM-OF-DUPS** is relevant. Since we expect the relevancy information to be incomplete at any point in time, it makes sense to explicitly record relevancy and irrelevancy information separately.

The slots used to record these relationships are

- irrelevancy-preconditions
- relevancy-preconditions
- irrelevant-kids
- relevant-kids

If we want to record the irrelevancy relationship in the **DEFAULT-MS**/**MS98-NUM-OF-DUPS** example above, there are two ways. The first is to record this in the definition of **DEFAULT-MS** using the *irrelevant-kids* slot as shown below.

```
(defflag default-ms
  (flagtype searchtype)
  (default ms90-3)
  (subjects mating-search . . .)
  (change-fn (lambda (flag value pvalue)
       (when (neq value pvalue) (update-otherdefs value))))
  (irrelevant-kids ((neq default-ms 'ms98-1) '(ms98-num-of-dups)))
  (mhelp . . . ))
```

The format for this slot is a list of elements of the form `(<pred> <sexpr>)` where `<pred>` is a condition on the value of the flag being defined and `<sexpr>` is an s-expression which should evaluate to a list of flags.

Alternatively, we can specify the relationship when we define **MS98-NUM-OF-DUPS** using the slot *irrelevancy-preconditions*

```
(defflag ms98-num-of-dups
  (default nil)
  (flagtype null-or-posinteger)
  (subjects ms98-1)
  (irrelevancy-preconditions (default-ms (neq default-ms 'ms98-1)))
  (mhelp . . .))
If some positive integer n, we reject any component using more than n
of the duplications."))
```

The format for this slot is a list of pairs `(<flag> <pred>)` where `<flag>` is a flag and `<pred>` is a condition on the value of the flag `<flag>`.

Relevancy relationships can be specified in an analogous way.

```
(defflag default-ms
  (flagtype searchtype)
  (default ms90-3)
  (subjects mating-search . . .)
  (change-fn (lambda (flag value pvalue)
       (when (neq value pvalue) (update-otherdefs value))))
  (irrelevant-kids ((neq default-ms 'ms98-1) '(ms98-num-of-dups)))
  (relevant-kids ((eq default-ms 'ms98-1) '(ms98-num-of-dups)))
  (mhelp . . . ))
```

or

```
(defflag ms98-num-of-dups
  (default nil)
  (flagtype null-or-posinteger)
  (subjects ms98-1)
  (irrelevancy-preconditions (default-ms (neq default-ms 'ms98-1)))
  (relevancy-preconditions (default-ms (eq default-ms 'ms98-1)))
  (mhelp . . .))
```

The conditions given are compiled into a relevancy graph and an irrelevancy graph. The graphs are labelled directed graphs, where the nodes are

flags and the arcs are labelled by the conditions. These graphs are currently used in the following two ways.

(1) The relevancy graph is used by the **UPDATE-RELEVANT** command. The user specifies a flag to update. Based on the value given for that flag, the user is then asked to specify values for the target flags for which the condition on the arc is true. For example, consider the following session:

```
<0>update-relevant default-ms
DEFAULT-MS [MS98-1]>ms98-1
. . .
MS98-NUM-OF-DUPS [NIL]>2
. . .
<1>update-relevant default-ms
DEFAULT-MS [MS98-1]>ms90-3
. . .
<2>
```

(2) The irrelevancy graph is used to warn the user when an irrelevant flag is being set. A flag F_0 is *never irrelevant* if there are no arcs with F_0 as the target. A flag F_1 is *irrelevant* when there is a path from a flag F_0 to F_1, where F_0 is never irrelevant and at least one of the conditions on the path evaluates to true. Consider the following session.

```
<2>default-ms
DEFAULT-MS [MS90-3]>

<3>ms98-num-of-dups
MS98-NUM-OF-DUPS [NIL]>3
WARNING: The setting of the flag DEFAULT-MS makes
     the value of the flag MS98-NUM-OF-DUPS irrelevant.

<4>default-ms
DEFAULT-MS [MS90-3]>ms98-1

<5>ms98-num-of-dups
MS98-NUM-OF-DUPS [3]>2

<6>
```

3.1. Automatically Generating Flag Relevancy. In addition to the relevancy information directly specified in the `defflag` declarations in the code, there is now code (in flag-deps.lisp) to read and analyze the code in the lisp files to determine flag relevancy. This code first reads the lisp files and records all `defun` and `defmacro` definitions. Then, it computes easy flag conditions in which flags and calls to other functions occur. At present

an *easy flag condition* is built from atoms of the form *easy-flag-term easy-operator easy-flag-term* using boolean operations and `IF`. An *easy-operator* is one of $=$, $<$, $>$, $<=$, $>=$, eq, eql, equal, or neq. An *easy-flag-term* is a flag, NIL, T, a number, or any quoted term. The important property these conditions should satisfy is that their values are static, i.e., they do not depend on the dynamic environment. (Note that some flags, such as FIRST-ORDER-MODE-MS are often dynamically set by the code. This flag, for example, is removed from the list of flags, along with any flag, such as RIGHTMARGIN, that is never relevant for automatic search. The code does attempt to recognize when flags are dynamically bound in a certain context and take this into consideration.)

A user is given the option of computing this flag relevancy information when calling UPDATE-RELEVANT or SHOW-RELEVANCE-PATHS. Another option is to load the relevance information from a file. Such a file could have been created in a previous TPS3 session using SAVE-FLAG-RELEVANCY-INFO.

CHAPTER 10

The Monitor

The monitor is designed to be called during automatic proof searches; its basic operation is described in the User Manual. There are three basic steps required to write a new monitor function, which are described below, using the monitor function monitor-check as an example. More examples are in the file *monitor.lisp*.

1. The Defmonitor Command

The command defmonitor behaves just like `defmexpr`, the only difference being that the function it defines does not appear in the list when the user types `?`. This command will be called by the user before the search is begun, and should be able to accept any required parameters (or to calculate them from globally accessible variables at the time the command is called).

So, for example, the `defmonitor` part of monitor-check looks like this:

```
(defmonitor monitor-check
  (argtypes string)
  (argnames prefix)
  (arghelp "Marker string")
  (mainfns monitor-chk)
  (mhelp "Prints out the given string every time the monitor is called,
followed by the place from which it was called."))

(defun monitor-chk (string)
  (setq *current-monitorfn* 'monitor-check)
  (setq *current-monitorfn-params* string)
  (setq *monitorfn-params-print* 'msg))
```

Note that this accepts a marker string as input from the user (other monitor functions may look for a list of connections, or flags, or the name of an option set; it may be necessary to define a new data type to accommodate the desired input). It then calls a secondary function, which in this case needs to do very little further processing in order to establish the three parameters which are *required* for every such function: `*current-monitorfn*` contains a symbol corresponding to the name of the monitor function, `*current-monitorfn-params*` contains the user-supplied parameters (in any form you like, since your function will be the only place where they are used) and `*monitorfn-params-print*` contains the name of a function that can print out `*current-monitorfn-params*`

in a readable way, for use by the commands monitor and nomonitor. The latter should be set to `nil` if you can't be bothered to write such a function.

2. The Breakpoints

In the relevant parts of the mating search code, you should insert breakpoints of the form:

```
(if monitorflag
    (funcall (symbol-function *current-monitorfn-params*)
             <place> <alist>))
```

The value of *place* should reflect what part of the code the breakpoint is at. So, for example, it might be `'new-mating`, `'added-conn` or `'duplicating`.

The value of *alist* should be an association list of local variables and things that your monitor function will need. For example, *alist* might be `(('mating . active-mating) ('pfd . nil))`; it might equally well be just `nil`.

All breakpoints should have exactly this pattern. By typing *grep "(if monitorflag (funcall" *.lisp* in the *tpslisp* directory, you can get a listing of all the currently defined breakpoints.

3. The Actual Function

This is the function which will actually be called during mating search. By convention, it has the same name as the `defmonitor` function. Normally, it will first check the value of *place*, to see if it has been called from the correct place; it can then use the `assoc` command to retrieve the relevant entries from *alist*. Theoretically, it should be completely non-destructive so as to ensure that the mating search continues properly; of course, you may be as destructive as you like, provided you understand what you're doing...

The function for `monitor-check` is as follows; notice that this does not check *place* since it is intended to act at every single breakpoint.

```
(defun monitor-check (place alist)
  (declare (ignore alist))
  (msg *current-monitorfn-params* place t))
```

CHAPTER 11

Writing Inference Rules

Information on how to write new rules using the rules module is in the User Manual.

The chapter about ETPS (chapter 12) has some discussion of how default line numbers and default hypotheses are generated in the various `OUTLINE` modules. This should correspond fairly closely to the way in which the automatically-generated rules generate their defaults. The same chapter also has a discussion of support transformations which illustrates the way in which rules are defined.

Do not edit the automatically-generated files. This means any file *whatever.lisp* that has a companion file *whatever.rules*. If you edit the lisp files directly, your changes will be lost if the rules are ever recompiled.

There are some rules which are defined directly as commands (mexpr's). Examples are RULEP, TYPESUBST, ASSERT, ADD-HYPS and DELETE-HYPS. In order for these commands to show up when LIST-RULES is called, the programmer should modify the definition of list-rules in otl-help.lisp to explicitly include them. For example, RULEP is explicitly included by the following code:

```
(when (get 'ML::rulep 'mainfns)
  (push 'RULEP gsrl)
  (push '(ML::RULES-2-PROP) gsrl))
```

This includes RULEP in the context ML::RULES-2-PROP.

CHAPTER 12

ETPS

1. The Outline Modules

The `OUTLINE` modules in TPS have two main subparts; the bookkeeping functions, and the GO command which gives sophisticated help or constructs a proof automatically. They are collected in the modules of the form `OTL*`.

In ETPS only the bookkeeping functions are present.

The discussion below is aimed at understanding the `OUTLINE` modules independently of the system, but we generally assume we are working in ETPS . If TPS3 differs, this is noted.

We often talk about *proofs*, even though they are properly only incomplete proofs or *proof outlines*. It is assumed that the reader knows what planned lines (*plines*) and deduced lines (*dlines*) are. This and general familiarity with ETPS are necessary to understand this discussion.

1.1. Proofs as a Data Structure. Proofs in ETPS are represented by a single atom with a variety of properties. The global variable `DPROOF` has as value the name of the current proof. In case you are working, say on exercise `X6200`, `DPROOF` will have the value `X6200`. The current proof name then has a variety of properties.

`LINES:` `(`*line line* `...)`. This is simply an ordered list of all lines in the current proof without repetition. The order is such that lines with a lower number appear first in the list.
WARNING: This property is frequently changed destructively. As a consequence it may never be empty and should be used for other purposes only in a copy.

`LINEALIASES:` `((`*line* `.` *no*`)(`*line* `.` *no*`) ...)`. This is an unordered association list correlating lines with their numbers. No line should ever appear in more than one pair, and neither should a number. Try to think of arguments for and against this representation, compared to one where the number of a line is stored on the line's property list.
WARNING: This property is frequently changed destructively. As a consequence it may never be empty and should be used for other purposes only in a copy.

`PLANS:` `((`*pline* `.` *supportlist*`)(`*pline* `.` *supportlist*`) ...)`. This stores the important *plan-support structure* of the current proof. *pline* is a still unjustified line in the current proof, *supportlist* is a list of

deduced lines supporting a *pline*. A *pline* may never have a justification other than `PLAN`i, a *sline* (support line) must always be completely justified, i.e. may not ultimately depend on a planned line.

The association list is ordered such that the most recently affected *pline* is closer to the front of the list. The order can be changed explicitly with the `SUBPROOF` command.

WARNING: This property is frequently changed destructively. As a consequence it may never be empty and should be used for other purposes only in a copy.

`GAPS:` (*gap gap ...*). This is a list of the gaps between lines in the proof. Each gap has the properties (`MIN-LABEL` *line*) and (`MAX-LABEL` *line*).

`NEXTPLAN-NO:` *integer*. This is just the next number that will be used for a planned line.

`ASSERTION:` *gwff*. This is the assertion being proven.

When a proof is saved using the SAVEPROOF command, a checksum may be generated. This is used by ETPS to verify that the saved proof has not been manually edited by a student (otherwise it would be possible to edit out the planned lines and convince ETPS to issue the DONE command). Since it takes time to generate the checksum, it is only generated if the flag EXPERTFLAG is NIL. This means that proofs written by TPS3 with EXPERTFLAG T cannot be read into ETPS with EXPERTFLAG NIL.

1.2. Proof Lines as a Data Structure. Proof lines in ETPS have a variety of properties:

`REPRESENTS:` This is the wff asserted by the line. In the original TPS this had to be an atomized wff of very particular structure, which lead to numerous problems in higher-order logic. In ETPS this has been maintained for the present. Our goal, of course, is to allow arbitrary *gwffs* as `REPRESENTS` of lines.

`HYPOTHESES:` This is a list of lines assumed as hypotheses for the line. The list of hypotheses is ordered (lowest numbered line first), but to my knowledge no function assumes this. It simply looks better in the output. No line should appear twice as an hypothesis (this fact may actually be used here and there).

`JUSTIFICATION:` *RULE gwfflist linelist* The line can be inferred by an inference rule *RULE*from *linelist*. *gwfflist* has somehow been used to infer the line.

`LINENUMBER:` The line number associated with the line.

2. Defaults for Line Numbers - a Specification

There will never be an absolutely correct way of assigning default for line numbers; we can merely make sure that the result will always be logically correct - the rest is often a matter of style and the kind of heuristics used.

Below we give a description of the tasks to be done by a function LINE-NO-DEFAULTS which is called during the application of every inference rule in interactive mode. We will set the stage by giving some, not necessarily exhaustive, examples of what meaning to assign to the data structures and what output to expect from the function.

2.1. The *support* data structure. At each stage in a proof, we have associated with it a *support* structure, which, for any given planned line (*pline*), tells us which deduced lines (*dlines*), we expect to use in the proof of the *pline*.

Thus the support structure is of the form

$$((p_1\ d_{11} \ldots d_{1x_1}) \ldots (p_p\ d_{p1} \ldots d_{px_p}))$$

One may assume the following:

(1) The p_i are pairwise distinct.
(2) The d_{ik} are pairwise distinct for every fixed i and $1 \leq k \leq x_i$.
(3) For each i, $d_{ik} < p_i$ for all $1 \leq k \leq x_i$.
(4) The planned lines p_i are ordered such that the ones the user is expected to work on first appear closer to the front. In particular, p_1 is the planned line worked on most recently.
(5) Similarly, for a given i, the d_{ik} are ordered such that the one the user is expected to use first appear earlier.

With each rule definition, there will be a description of how the *support* structure changes. This is given as two *support* structure templates, using the name given to the lines in the rule specification.

2.2. Examples. The examples below are not complete, in the sense that not the full description of the rule (for the rules module) is given, we have merely extracted what is important in our context. p and d are placeholders for a *pline* or any number of *dlines*, respectively, which are found in the support structure of the current proof, but are merely copied in the application of the particular rule described.

```
Rule of Cases

*(D1)  H      !A(O) OR B(O)
 (H2)  H,H2   !A(O)                              Case 1: D1
 (P3)  H,H2   !C(O)
 (H4)  H,H4   !B(O)                              Case 2: D1
 (P5)  H,H4   !C(O)
*(P6)  H      !C(O)                          Cases: D1 P3 P5

Support Transformation: (P6 D1 ss) ==> (P3 H2 ss) (P5 H4 ss)
```

Note that the specified support transformation tells TPS3 what lines it expects to be there, when the rule is applied, and which lines should be new. In this case, `P` and `Dab` are expected to be new, the others are to be

constructed. Of course, these are only defaults, and the user can apply the rule with any combination of lines present or absent.

```
Induction Rule

 (D1) H      ! P 0
 (H2) H,H2   ! P m                            Inductive Assumption on m
 (D3) H,H2   ! P . Succ m
*(P4) H      ! FORALL n . NAT n IMPLIES P n             Induction: D1 D3

Support Transformation: (P4 ss) ==> (D1 ss) (D3 H2 ss)
```

```
  Forward Conjunction Rule

 (P1)  H      !A(O)
 (P2)  H      !B(O)
*(P3)  H      !A(O) AND B(O)                                  Conj: P1 P2

Support Transformation: (P3 ss) ==> (P1 ss) (P2 ss)
```

```
Backward Conjunction Rule

*(D1)  H      !A(O) AND B(O)
 (D2)  H      !A(O)                                 Conj: D1
 (D3)  H      !B(O)                                 Conj: D1

Support Transformation: (pp D1 ss) ==> (pp D2 D3 ss)
```

2.3. The LINE-NO-DEFAULTS functions. There are two functions whose job it is to determine defaults for line numbers. The reason we need two functions is, that some of the lines which appear on the left-hand side of the *support-transformation*, may reappear on the right. The way we handle these connections, is that we first determine the defaults for lines which are supposed to exist (the left-hand side of the *support-transformation*), then substitute those values into the right-hand side and call the second default function.

The function LINE-NO-DEFAULTS-FROM is called with one argument line-no-defaults-from *default-exist* and LINE-NO-DEFAULTS-TO is called with two arguments line-no-defaults-to *default-exist default-new* where

default-exist **:** is the left hand side of the support transformation specified for the rule, with lines that we need the default replaced by a $, while the other lines are numbers (which means they either have been figured out by an earlier default function or specified by the user). Something which is neither $ nor a number is one of the "variables" *d* or *p* standing for other *dlines* or *plines* in the current *support* structure. They must simply be returned (in the proper place, of course).

default-new **:** is the right hand side of the support transformation specified for the rule, with the same interpretation as for *default-exist*.

The output of `LINE-NO-DEFAULTS-FROM` should be a *default-exists-figured*, the output of `LINE-NO-DEFAULTS-TO` is a list (*default-exists-figured default-new-figured*) in which all `$` of the arguments have been filled in. These functions may also do a `THROWFAIL`, if one of the requirements R for logical correctness cannot be satisfied in the given proof structure.

Also note that all lines in *default-exists* have already been determined, when *default-new* is called.

The specification which must be meet by the `LINE-NO-DEFAULTS-`x functions can be grouped into three classes: requirements which ensure the logical correctness of the rule application (R), requirements which make the defaults sensible for the "usual" application of the rule (D) and should never be deviated from, and desired properties, which need not be satisfied, but approximate what the user would like to see most of the time.

Note that the scope in this function is restricted by the fact that it does not examine the logical structure assertions or hypotheses of the lines in the proof. This is accomplished by a completely different mechanism and is not the responsibility of the function. For instance, it is perfectly sensible for `LINE-NO-DEFAULTS` to suggest the first *pline* in the current support structure for the backwards conjunction rule, even though it may not be a conjunction at all! [This may cause mayhem in rule tactics. The latter assumes that if there is a correct default, the default function will choose it. Since a new line, properly located, is always a correct, and possibly a useful default, tactics may miss an opportunity to apply a rule.]

Subsequently, we will assume that

default-exist **is :** $((p_1\ d_{11} \ldots d_{1x_1}) \ldots (p_p\ d_{p1} \ldots d_{px_p}))$
default-new **is :** $((q_1\ e_{11} \ldots e_{1y_1}) \ldots (q_q\ e_{q1} \ldots e_{qy_q}))$

Requirements for Logical Correctness

R_1 **:** $q_j < p_i$ for all $1 \leq i \leq p$, $1 \leq j \leq q$.
R_2 **:** $e_{jk} < q_j$ for all $1 \leq j \leq q$, $1 \leq k \leq y_j$
R_3 **:** $d_{ik} < p_i$ for all $1 \leq i \leq p$, $1 \leq k \leq x_i$

Sensible Defaults Requirements

The requirements below only make sense, if the lines specified by the user do not already violate them. In that case, they must be relaxed to apply only to the remaining unspecified lines.

D_1 **:** A *plan-support* pair suggested for an element of *default-exist* must always match a *plan-support* pair in the current *support* structure of the (incomplete) proof.

D_2 **:** A *plan-support* pair suggested for an element of *default-new* must consist of entirely new lines, and no two lines among all the suggested defaults may have the same number.

Wishful Thinking

The following are constraints we would like to met, but is of course not always possible.

W_1 **:** $q_j < q_{j+1}$ for all $1 \le j < q$
W_2 **:** $q_j < e_{j+1,k}$ for all $1 \le j < q$ and $1 \le j \le q_{j+1}$
W_3 **:** $d_{ik} < e_{jl}$ for all $1 \le i \le p$, $1 \le k \le x_i$, $1 \le j \le q$, $1 \le l \le y_j$.
W_4 **:** $\neg\exists \mathtt{L}.\max e_{jk} < \mathtt{L} < q_j$ for all $1 \le j \le q$.
W_5 **:** Let $gap_j = q_j - \max e_{jk}$ for $1 \le j \le q$, if $\{e_{jk} | 1 \le k \le y_j\}$ is non-empty, otherwise let $gap_j = q_j - \max\{\mathtt{L} | \mathtt{L} < q_j \vee \exists n \ne j.\mathtt{L} = q_n\}$. Then maximize gap_j, giving equal "weight" to all $1 \le j \le q$.
W_6 **:** Minimize $b = \min e_{jl} - \max d_{ik}$, with an alternative similar to W_5 in case any of the sets is empty.
W_7 **:** Minimize $t = \min p_i - \max q_j$, with an alternative similar to W_5 in case any of the sets is empty.

3. Updating the *support* structure

Part of the execution of a rule application, is updating the plan structure; this is one of the reasons why with every rule there comes a description of how the plan structure should be updated. Below we will give a description of what the function UPDATE-PLAN is supposed to accomplish, even in cases when the rule is used in a way different from the defaults. Again, we assume the `UPDATE-PLAN` is called as in

`(update-plan` *default-exist default-new*`)`

where

default-exist **is :** $((p_1\ d_{11} \ldots d_{1x_1}) \ldots (p_p\ d_{p1} \ldots d_{px_p}))$
default-new **is :** $((q_1\ e_{11} \ldots e_{1y_1}) \ldots (q_q\ e_{q1} \ldots e_{qy_q}))$

Recall that there may be variables appearing in place of a line number. The following restrictions should be noted:

- For each *plan-support* pair $(p_i\ d_{i1} \ldots d_{ix_i})$ in *default-exist*, there is at most one occurrence of a variable among $d_{i1}, \ldots, d_{ix_i}$.
- Any occurrence of a variable in *default-exist* is unique.

When `UPDATE-PLAN` is called, all arguments are filled in, that is each place is occupied either by a variable or a line number.

3.1. ***support* Structure Transformation in the Default Case.** If the rule is used completely in the default direction, i.e. all *plan-support* pairs in *default-exist* exist in the current *support* structure and all pairs in *default-new* consist of new lines, then the effect of the rule application on the *support* structure is straightforward:

- Delete all pairs matching $(p_i\ d_{i1} \ldots d_{ix_i})$ from the *support* structure and attach to the front the pairs $(q_j\ e_{j1} \ldots e_{jy})$.
- A variable in place of a p_i matches **any** *plan-support* pair in the current proof, as long as the d_{ik} match the corresponding support lines.

- A variable in place of a d_{ik} matches the lines which are not matched by any of the line numbers. If p_i is a variable, every match for p_i produces a corresponding match of d_{ik}.
- A variable in place of q_j must occur as some p_i and as many copies of $(q_j\ e_{j1} \ldots e_{jy_j})$ are produced as there are matches of p_i.
- A variable in place of e_{jl} must occur as some d_{ik} and the matched list of lines in filled in.

3.2. What if ...? We will go through all cases which differ from the default application of the rule and specify what should happen to the *support* structure. Of course, TPS3 can not always correctly predict what the user had in mind, when applying a rule, so the following must partly be considered heuristics, but they will not always implement the user's devious intentions.

(1) **What if . . .** a p_i exists, but is not a *pline?* This case is delicate and perhaps frequently occurs, if the user does not bother deleting some lines before backtracking after some previous mistake. Here execute a `PLAN-AGAIN` (which may become smart about support lines)[1]. This will make p_i into a planned line and we can handle it the usual way.
(2) **What if . . .** a p_i does not exist? Then, very likely, a rule meant to be used backwards, was applied in a forward way. We can't do much here: just ignore the relevant part of *default-exist* completely.
(3) **What if . . .** a p_i is a variable, but d_{ik} don't match anything in the current *support* structure. This is already a special case of something discussed in the previous section.
(4) **What if . . .** a d_{ik} does not exist? Then we must enter it as a planned line, collecting $\{d_{il} | l \lneq k\}$ as its support lines.
(5) **What if . . .** a d_{ik} does exist, but does not support p_i (p_i not a variable)? Then somehow d_{ik} was improperly erased form the supports of p_i. Just treat d_{ik}, as if it were supporting p_i.
(6) **What if . . .** a q_j is a variable (thus exists as a *pline*) and matched a line number identical to a e_{jk}? Then we are closing a gap with a forward rule: Do not enter the jth *plan-support* pair into the *support* structure.
(7) **What if . . .** a q_j already exists as a *pline*? In this (probably very rare case) we are reducing the proof of one planned line to the proof of another planned line. Add the e_{jk} as additional support lines (also, of course, pulling it to the front of the *support* structure).
(8) **What if . . .** a q_j exists as a *dline*? Here we already proved what we need, so leave this *plan-support* pair out when constructing the new *support* structure.
(9) **What if . . .** a e_{jk} exists as a *dline*? Here we may be in a situation similar to 1. The justification of e_{jk} will be changed according to the

[1] This is a small project in itself!

current rule applied. As far as the *support* structure is concerned, we don't treat it specially.

(10) **What if . . .** a e_{jk} exists as a *pline*? Here we are justifying a planned line. Delete the *plan-support* pair for e_{jk} from the current *support* structure. The justification of e_{jk} will be changed appropriately.

(11) **What if . . .** a e_{jk} exists as a *hline*? If `TREAT-HLINES-AS-DLINES` is `T`, do what you would do to a *dlines* (see 9). Otherwise, nothing special is done.

3.3. Entering Lines into the Proof Outline. The descriptions in the previous section can, when read carefully, also serve as a guide to what should happen when entering a line into the proof outline. Of course, what should be done is clear, if we are in the all-default case. Otherwise we may have to change some justifications as indicated in the previous section, but otherwise existing lines are left alone.

Entering lines into the proof could be taken over by the same function, if we handed it linelabels instead of line numbers in *default-exist* and *default-new*.

4. Defaults for Sets of Hypothesis

In TPS3, the user will rarely ever have to deal explicitly with sets of hypothesis. However the detail can be controlled by a flag called AUTO-GENERATE-HYPS. If this flag is `T`, TPS3 will not only generate smart defaults for sets of hypothesis, but make them strong defaults, which means that the user will never be asked to specify hypotheses for a line.

There some restrictions on what the user of the `RULES` module may specify as hypothesis in a rule. Ignoring for the moment the problem of fixed hypotheses, like sets representing axioms of extensionality of an axiom of infinity, the hypotheses for each line l may have the form $H, s_1, \ldots, s_n$, where H is a meta-notation for a set of lines and the s_i are labels for lines present elsewhere in the rule specification. Let us use H_l for this set of specified hypotheses for line l.

Note the restriction that there may be only one variable standing for "arbitrary" sets of lines in any single rule description.

Defaults strong or not for the hypotheses of lines are only calculated after all line numbers have been specified. This includes existent and non-existent lines equally. The algorithm below will always generate legal applications of the rule, at the same time generating the "correct" set of hypotheses for each line. The algorithm will almost always be adequate, in the sense that the user will almost never need to explicitly add hypotheses to a deduced line or drop hypotheses from a planned line. There are cases, however, where this may still be necessary (see discussion below).

4.1. The Algorithm. Here, unlike in other parts of the `OUTLINE` modules, we do not need to refer to the *support* structure. Instead let us view the rule as if we were to infer the *plines* from all the *dlines* specified in the rule, and let us disregard hypothesis lines (*hlines*) for the moment.

For a given line l (in the rule specification) we now let S_l stand for the set of lines in the hypotheses which were explicitly specified in the rule description (corresponds to $s_1, \ldots, s_n$ above) and let L_l the actual list of hypotheses for the line, which must either be matched or constructed (depending on whether the line existed or not). Furthermore let H stand for the unique name for an "arbitrary" set of lines which appears in zero or more of the lines in the rule description.

Let us first consider the case that the hypotheses specified in the rule description do not contain H. For *dlines* d we must check

$L_d \subseteq S_d$

and for *plines* p we need to check

$S_p \subseteq L_p$

For *dlines* d which contain H among their hypotheses, we must satisfy

$L_d \subseteq H \cup S_d$

and, if we are filling in hypotheses for a new line, we would like to choose L_d as large as possible, so it satisfies this equation. From another point of view, namely when we match existent lines, we find out some constraint on H:

$L_d \setminus S_d \subseteq H$

On the other hand, for any given *pline* p, we obtain

$H \cup S_p \subseteq L_p$

or equivalently

$H \subseteq L_p$ and $S_p \subseteq L_p$

Here we would like to make L_p as small as possible (the fewer hypotheses we used, the stronger the statement result). Alternatively, the second line can again be viewed as a constraint on H when matching an existent *pline*.

This leads to the following algorithm for determining set of hypothesis:

(1) Let D_{exist} be the set of *dlines* which exist in the current proof. Then set

$$H_{lower} = \bigcup \{L_d - S_d | d \in D_{exist} \wedge H \in H_d\}.$$

Also let L_d be the strong default for the hypotheses of line d for each $d \in D_{exists}$.

(2) Let P_{exist} be the set of *plines* which exist in the current proof. Then set

$$H_{upper} = \bigcap \{L_p | p \in P_{exist} \wedge H \in H_p\}.$$

Also let L_p be the strong default for the hypotheses of line p for each $p \in P_{exists}$.

(3) If *not* $H_{lower} \subseteq H_{upper}$ the application of the inference rule is illegal. (Do a `THROWFAIL` with proper message.)

(4) If both, H_{lower} and H_{upper} are undefined (empty intersection or union, respectively), do not fill in any further defaults.
(5) If exactly one of H_{lower} and H_{upper} is undefined, let $H_{lower} := H_{upper}$ or vice versa.
(6) For non-existent *dlines* d, we let $L_d = H_{upper} \cup S_d$. If `AUTO-GENERATE-HYPS` is `T`, make L_d the strong default for that argument, otherwise just a regular default.
(7) For non-existent *plines* p, we let $L_p = H_{lower} \cup S_p$. If `AUTO-GENERATE-HYPS` is `T`, make L_d the strong default for that argument, otherwise just a regular default.

This algorithm is coded in a separate function for each rule. For the rule *rule*, the function is called *rule*-HYP-DEFAULTS and is called (when appropriate) form within *rule*-DEFAULTS.

4.2. When the Algorithm is not Sufficient. We must of course consider the case, when a restriction like "x not free in H" is imposed upon applications of the inference rule. Since we fill in H_{upper} for the hypotheses of the *dlines* which do not exist, we must check whether "x not free in H_{upper}". It may be the case, however, that all *dlines* actually already existed. In this case, it would be sufficient for the validity of the rule application, to check whether "x not free in H_{lower}". To see this may think of the rule as first a legal application of the inference rule, leaving out the extra hypotheses, then enlarging the set of hypotheses of the inferred line, possibly with lines which contain "x" free.

This situation can also come up, when not all the *dlines* are specified. Then we may have been able to make the inference rule application legal, by leaving out the lines `H` from H_{upper}, which violate the condition "x not free in (the assertion of) `H`".

This leads to a simple modification of the algorithm above, which would need much more information about the rule (namely the restrictions), where we modify the definition of H_{upper} in step 2 by

2^*. $H_{upper} = \bigcap\{L_p | p \in P_{exist} \wedge H \in H_p \wedge L_p \text{ satisfies any restriction on } H\}$.

It seems more reasonable, however, not to place that restriction, but rather give an error message. Otherwise the user may only find out much later, that some of the hypotheses he expected to be able to use, have not been included in the *dlines*, since they violated a restriction. This makes it necessary, however, to give the user explicit rules which allow adding hypotheses to a deduced line or dropping hypotheses from a planned line.

4.3. Hypothesis Lines. There are two principal ways hypothesis lines (*hlines*) can be treated in TPS3 and since there is very little extra work required, both are provided for. The flag TREAT-HLINES-AS-DLINES controls how hypotheses lines are handled.

If `TREAT-HLINES-AS-DLINES` is `T`, an *hline* may have more hypotheses than simply *hline*. Also, *hlines* may have descriptive justifications like "Case

a" or "Ind. Hyp. for n". The price you pay is that hypotheses lines become unique to a subproof and should not be used elsewhere. In this case, *hlines* are truly treated as *dlines*, and in the above algorithm for determining default for lines, we mean *dline* or *hline* whenever we say *dline*.

If `TREAT-HLINES-AS-DLINES` is `NIL`, every *hline* has exactly one hypothesis: itself. Also the justification for any *hline* will be the same, namely the value of the flag HLINE-JUSTIFICATION (by default `Hyp`). What you gain in this case is, that the same hypothesis line may used many different places in the given proof. The default for the hypotheses of an *hline* will always be strong and equal to `(`*hline*`)`, anything else will result in an error, even if perhaps logically correct. Also, in this case, if CLEANUP-SAME is `T`, then CLEANUP will eliminate unnecessary hypotheses.

CHAPTER 13

Mating-Search

The top level files for matingsearch are: *mating-dir.lisp* for ms88, *ms90-3-top.lisp* for ms90-3, *option-tree-search.lisp* for ms89 and ms90-9, *ms91-search.lisp* for ms91-6 and ms91-7, and *ms92-9-top.lisp* for ms92-9 and *ms93-1.lisp* for ms93-1. The lisp files with prefix *ms98* are those used by ms98-1. The code for GO in *mating-top.lisp* shows what the main functions are. Mating search with extensional expansion dags is different in many respects than mating search with expansion trees. We delay this discussion until section 12.

There are a lot of comments about the workings of the code embedded in the lisp files; in particular there is an outline of ms90-3 at the top of *ms90-3-top.lisp*.

1. Data Structures

See the section on flavors and labels (section 6) for a discussion of some relevant information about the data structures below. Among other things, that section has the definition of the flavor "etree".

1.1. Expansion Tree. The data structure etree, defined in etrees-labels.lisp, has the following properties:

(1) name: the name of the etree. We can use this attribute to identify which kind of structure an etree is.
(2) components: is a list which contains all children of the etree. The children of an etree are also etrees. We could use this attribute to check whether an etree is a leaf, true, or false.
(3) positive: tells us whether the formula which an etree represents (which is the formula given by get-shallow) appears positively or negatively in the whole formula. This will be used to compute the vpform of the whole formula. (The vpform of a subformula may be not the same as the corresponding part of it in the whole formula because the "positive" property of the subformula is dependent on the context.)
(4) junctive: can be used for printing the vpform. This attribute is linked tightly with the "positive" attribute, and has to do with whether the node acts as neutral, conjunction or disjunction.
(5) free-vars: is a list, containing the free variables in whose scope the node appears. When you skolemize a formula, you should use this attribute.

(6) parent: is the parent of this etree.
(7) predecessor: this slot tells you the leaf name from which the current etree was deepened. It is mainly used for handling skolem constants.
(8) status: has little to do with the system as currently implemented, but you should be careful when you are creating commands which will change the variable `current-topnode`. You have two choices:
    (a) Change the value of `*ignore-statuses*` to T. Then you need not worry about this attribute. Of course, what you are doing may then not be compatible with the future versions of the system. This is highly discouraged.
    (b) When you want to create new nodes or change some nodes in the `current-topnode` make the corresponding changes in the attribute `statuses` of `current-eproof`, which is a hash-table. Don't forget this, otherwise your new commands won't work.

Actually, according to an old email from Dan Nesmith about status, the status of etrees is *not* stored in the status slot of an etree. Instead the status of an etree is in a hash table associated with the current-eproof. (So, update-statuses depends on the value of current-eproof.)

The same email contains information on the predecessor slot. For reference, here is the email:

```
To: Peter.Andrews@K.GP.CS.CMU.EDU
Cc: issar@K.GP.CS.CMU.EDU, hwxi@K.GP.CS.CMU.EDU, mbishop@K.GP.CS.CMU.EDU
Subject: Re: STATUS and PREDECESSOR
In-Reply-To: Your message of "Fri, 25 Sep 92 16:05:44 EDT."
             <1992.9.25.20.0.41.Peter.Andrews@K.GP.CS.CMU.EDU>
Date: Tue, 29 Sep 92 16:34:36 +0100
Message-Id: <29531.717780876@js-sfbslc10.cs.uni-sb.de>
From: "Dan Nesmith" <nesmith@cs.uni-sb.de>

Your message dated: Fri, 25 Sep 92 16:05:44 EDT
>Can you please explain what the slots STATUS and PREDECESSOR
>in the structure current-topnode are for? (Of course, maybe we should
>have a meeting with Sunil to have a discussion about this.)

Sorry it took so long to reply.  At first glance, I thought this was
something to do with unification.

current-topnode is a variable whose value, while you are in
the mating-search top-level, is the node of the expansion tree that you
are currently looking at.  Commands like D (down), UP, ^ (move to root),
use and change the value of this variable (see etrees-wffops, mating-top,
mating-move).

Actually, the unification top-level uses this same variable name, of course
```

rebinding it during the duration of the top-level (so there's no real
conflict).

Now, STATUS and PREDECESSOR are actually slots in every expansion tree
node (defined in etrees-labels).

Each etree node has a status, which is a nonnegative integer. 0 means
that the etree node should be ignored (as if it and its descendants
were not in the tree), while positive values indicate the node is active,
and (potentially) the higher the value, the more important it is.
By the etree's status, you could rank certain expansions as more interesting
than others. I don't think that is now being used anywhere.
Originally, this status was kept in a slot in each etree node. I didn't
really like this, because then you can't share etree nodes among different
"virtual" expansion trees. For example, during the MS90-9 search procedures,
there is really just one expansion tree, which contains all of the
expansions. There are, however, many "virtual" expansion trees, that is,
expansion trees with the same root, but with different subsets of the
expansions "turned on". Each one of these virtual trees is kept in a
separate eproof structure. For this reason, the statuses are actually
kept in a hashtable in the eproof structure as well, so changing the status
of a node in one virtual tree doesn't affect its status in other trees.
E.g.,
Assume we have a tree with root expansion node EXP0, and children LEAF1, LEAF2.
Then we have potentially 3 virtual trees: one where LEAF1 has positive status
and LEAF2 has 0 (is not there); one where LEAF2 positive status and LEAF1 has 0;
and one where both LEAF1 and LEAF2 have positive status (are thus both
considered in the proof process). Functions that do things like create the
jform use the status to decide which nodes belong and which don't.

Because statuses are now kept separate from the nodes themselves, the STATUS
slot is an anachronism, and actually can now be removed (delete the form
"(status 1)" from the file etrees-labels.lisp).

PREDECESSOR is related. This is a symbol, the name of the etree node from
which this node originated. For example, suppose we have a leaf node LEAF0.
If we deepen this node, then we will get something like EXP0 as a result. Its
PREDECESSOR slot will be LEAF0. If we then change the status of all its
expansions to 0, then this node is effectively a leaf node again, and it
will be printed out with the name LEAF0 as before. E.g.

```
<34>mate "exists y P y"

DEEPEN (YESNO): Deepen? [Yes]>no

<Mate35>etd
```

```
LEAF0   EXISTS y(I) P(OI) y

<Mate36>dp

EXP0
<Mate37>etd

EXP0   LEAF1    y^0(I)
LEAF1   P(OI) y^0(I)

<Mate38>1

LEAF1
<Mate39>mod-status 0


<Mate40>up

LEAF0
<Mate41>etd

LEAF0   EXISTS y(I) P(OI) y


PREDECESSOR is also used in case a node's name is not found in the
statuses hashtable; so effectively a node can inherit the status of the
node from which it was created.

Dan
```

The file etrees-debug contains functions useful for debugging code dealing with etrees. The function check-etree-structure recursively checks structural properties of an etree, and the function check-etree-structure-break calls check-etree-structure and calls a break if the etree fails the structural test. The idea is that one can temporarily insert

```
(check-etree-structure-break <etree>)
```

in suspicious parts of the code to find out when an etree loses its integrity. If the etree does not have structural integrity, a break is called, sending the user (programmer) to the debugger. If one wants to insert this in several places in the code, one may want to include a message as in

```
(check-etree-structure-break <etree> "unique identifying message")
```

to identify which caused the break.

1.2. The Expansion Proof. In the mate toplevel, we have an expansion proof stored in the special variable current-eproof, which is an eproof-structure. current-eproof has a attribute etree, whose value is often used to update variable current-topnode.

Actually, a whole formula is represented by a tree, each node of which is an etree. At first, current-topnode is the root of the tree. Each node in the tree can be one of the following structures, all of which are derived from the structure etree, described above. We note only the differences between these structures and etrees.

(1) econjunction is just an etree without any additional new attributes. components is a list containing two elements, and junctive should be `dis` or `con`.
(2) edisjunction is like econjunction.
(3) implication is like econjunction
(4) negation is just an etree. components contains one element and junctive is `neutral`.
(5) skolem is an etree with two additional attributes:
    (a) shallow: contains the shallow formula that the attribute skolem represents. Never forget to make the corresponding changes in it if you have changed some other parts of this node; otherwise the proof cannot be transformed into natural deduction style by etree-nat since the function get-shallow would not work normally.
    (b) terms: is a skolem-term structure, containing a term replacing the original variable, and something else.
(6) selection is also an etree with attributes shallow and terms, just as skolem etree nodes. Whether the etree contains selection or skolem nodes depends on the values of SKOLEM-DEFAULT.
(7) expansion is an etree with three additional properties:
    (a) shallow: is the same as in skolem.
    (b) terms: is an exp-var structure, containing the expansion variable for this expansion.
    (c) prim-vars
(8) rewrite is an etree that rewrites the wff in some way. Rewrite nodes have the following four additional attributes: shallow, justification, ruleq-shallow, and reverse. The justification attribute is a symbol which can currently be one of the following values (this list may not be exhaustive):
    (a) EQUIVWFFS: Usually means there have been some definition expansions. In case dual instantiation is being used, it may mean the wff has been rewritten to a conjunction or disjunction of the wff and the instantiated form.
    (b) LAMBDA, BETA, ETA: The wff is the result of the appropriate normalization.

(c) EQUIV-IMPLICS, EQUIV-DISJS: An equivalence was expanded.
(d) LEIBNIZ=: Rewrites an equational wff using the Leibniz definition of equality.
(e) EXT=: Rewrites an equational wff between terms of functional type using extensionality.
(f) REFL=: Rewrites an equational wff of the form "a = a" to TRUTH.
(g) RULEQ: The only time this appears to be used is when MIN-QUANTIFIER-SCOPE is set to T, in which case the quantifiers in the wff are pushed in as far as possible.
(h) ADD-TRUTH, TRUTHP: May conjoin the wff with TRUTH, or disjoin the wff with (NOT . TRUTH). See the flags ADD-TRUTH and TRUTHVALUES-HACK.

(9) leaf is an etree with the additional attribute shallow, as in skolem, above. The components, junctive and predecessor attributes of leaf are all `nil`.
(10) true
(11) false
(12) empty-dup-info is an etree used by the NAT-ETREE translation code, not by the mating search.

There is also an eproof stored in the global variable master-eproof. In my experience, this has been set to the same value as current-eproof. The only place I can find in the code where it may have a different value is when using option sets (search procedures MS91-6 and MS91-7, see the files ms91-basic.lisp and ms91-search.lisp). In particular, there are option-set structures which have an eproof slot. These are set to *copies* of the eproof structure (as opposed to the identical structure) in master-eproof. Then, in finish-up-option-search, current-eproof is set to the value of such an eproof slot.

In addition to the etree slot, there are numerous other slots associated with an eproof:

(1) jform: Contains the jform associated with the etree (see section 5).
(2) all-banned: A list of expansion vars and a list of selected vars whose selection node occurs beneath the expansion term. This is needed to check acyclicity condition. The value is set by the function fill-selected.
(3) inst-exp-vars-params: An association list of expansion variables that occur instantiated in the etree and the selected variables that occur in the term. This is needed to check the acyclicity condition when there are substitutions (e.g., set variable substitutions made in a preprocessing stage) that have contain selected variables. (See section 4.)
(4) dissolve: An alist of symbols representing connections between nodes in the etree which we assume will be in the final solution. The code

for building jforms from etrees (etree-to-jform, etree-to-prop-jform) will use this, as well as the flag DISSOLVE, to dissolve vertical paths from the jform. (Dissolution is described in [MR93]. The current dissolution code dissolves one connection at a time, iterating the procedure for each connection.)

(5) free-vars-in-etree: This is an association list between free expansion variables which occur in the eproof (i.e., those which have not be instantiated), and the corresponding expansion node in which the variable was introduced. Note that if expansion variables are EXP-VAR structures. An expansion variable is uninstantiated with the VAR slot is the same as its SUBST slot. When an expansion variable p (introduced in expansion node EXP_j) is instantiated with a term which introduces new expansion variables $\{q_i\}$ (e.g., a PRIM-SUB), the pair $(p\ .\ EXP_j)$ is removed from this slot and the pairs $(q_i\ .\ EXP_j)$ is included in the list. (Also, in such a case, the value of the slots substitution-list, inst-exp-vars-params, all-banned may change to reflect this instantiation.)
(6) skolem-constants: An association list of skolem constants and integers representing their arities. (Note: If SKOLEM-DEFAULT is set to NIL, then all skolem constants will have arity 0.)
(7) substitution-list: A list of expansion variables which have been instantiated.
(8) leaf-list: A list of the leaf nodes occurring in the etree.
(9) skolem-method: Corresponds to the value of SKOLEM-DEFAULT.
(10) max-cgraph-counter
(11) bktrack-limit
(12) connections-array
(13) incomp-clists-wrt-etree
(14) mating-list
(15) incomp-clists
(16) cgraph
(17) skolem-node-list: A list of the skolem nodes which occur in the etree.
(18) stats
(19) max-incomp-clists-wrt-etree
(20) symmetry: A hash table with etree nodes as keys and symmetry-holder structures as values. This information is built when the etree is deepened, but does not appear to be used anywhere in the code.
(21) merged: This is true if the etree has been merged.
(22) statuses: A hashtable of nodes in the etree and their statuses. See the discussion above, and Dan's email in section 1.1.
(23) name: This symbol is the name of the eproof.
(24) lemmas: This slot contains information about the lemma structure in the expansion proof. See section 11.

1.3. Relevant Global Variables. In addition to the eproof slots, there are some global variables which store information relevant to the current etree.

The following global variables are used by dual instantiation (when REWRITE-DEFNS or REWRITE-EQUALITIES is set to LAZY2 or, equivalently, DUAL).

(1) *instantiated-defs-list* The value is an association list of symbols and the shallow formula of the rewrite in which the dual instantiation was performed. For example, the value might be

```
((#:G162733 ((SUBSET<O<OI><OI>> . c<OI>) . d<OI)
(#:G162731 . ((=<OII> . a<I>) . b<I>)))
```

The symbols have no apparent meaning, but are used internally as an identifier. Note that this list contains both abbreviations and equations which have been instantiated using dual instantiation. The value of this global is built during deepening (see section 2.1)

(2) *instantiated-eqs-list* This global's value during deepening is an association list of symbols and the shallow formula of the equation being rewritten (so its elements are a subset of the elements of *instantiated-defs-list*). However, the elements of this list are removed during deepening so that the final value after deepening an etree to literals seems to always be NIL.

(3) *hacked-rewrites-list* Its value is a list of elements of the form

```
(<rewrite node> .
    (<instantiated wff-or-symbol> . <leaf with uninstantiated form>))
```

(4) *banned-conns-list* This is an association list of leaves which are not to be mated, e.g.,

```
((L4 . L8) (L4 . L9) (L11 . L14) (L11 . L15))
```

The value appears to be leaves which correspond to an uninstantiated definition and the leaves which appear beneath the instantiated form. Since these leaves share a vertical path, they could be mated. Apparently, the intuition is that we never want to mate a wff with an uninstantiated defn with a subformula of the instantiated form. However, it is not clear that we *can* really rule out such connections, since higher order quantifiers might cause a wff to be the negation of a subformula of itself. However, we can legally ban these connections if only for the reason that we do not *need* to use dual instantiation at all. The value of this flag is used by quick-unification-connection to rule out some connections.

(5) *ho-banned-conns-list* Similar to *banned-conns-list*, but with a slightly different representation. Instead of pairs of leaves, each leaf corresponding to an uninstantiated definition is associated with a list of the leaves that occur beneath the instantiated form. For example,

```
((L4 . (L9 L8)) (L11 . (L15 L14)))
```

(6) *unsubst-exp-vars* This is a list of the expansion variables in the etree which are not instantiated. This should usually be a list of the car's from the current-eproof eproof slot free-vars-in-etree. Note that this variable is only set if some node was rewritten using dual instantiation. Otherwise its value will be whatever it was the last time an etree was deepened using dual instantiation.
(7) *rew-unsubst-exps* This is a list of expansion variables which occur free in some leaf corresponding to an uninstantiated definition.

These are some other global variables.

(1) *leibniz-var-list* An association list of variables and rewrite nodes. The variables are expansion variables (actually, only the symbol for the variable is used) introduced by the Leibniz definition of equality (i.e., the q in $\forall q \,.\, q\, A \supset q\, B$). The rewrite node is the rewrite node in which the equality was rewritten using the Leibniz definition. Note that the q is only an expansion variable if the rewrite node is positive.
(2) *after-primsub* This is just a toggle that is temporarily set to T after a primsub has been done, so that it will be T while the new etree is deepened.

There may be other global variables. Needless to say, it is difficult to build an expansion tree in any way other than using the deepening code that is already written (see the file etrees-wffops.lisp and section 2.1 of this chapter) because all these global variables and eproof slots need to be maintained.

1.4. Functional Representation of Expansion Trees. The implementation of expansion trees described above is well-designed for mating search on a single tree. An expansion tree has a great deal of global information associated with it such as what expansion variables and selected variables occur in the tree. Also, expansion trees contain circular references, since each node is associated with both its children and its parent. (In particular, since a node is associated with its parent, we cannot coherently share a node in two expansion trees.) Unfortunately, this makes it very difficult to build and modify expansion trees using recursive algorithms. One must constantly update global information.

There is an alternative representation called "ftrees" implemented in the file ftrees. Ftrees are designed for functional programming (the "f" is for "functional"). Operations on ftrees are never destructive, and the information carried at each node is minimal. Functions etree-to-ftree and ftree-to-etree translate between the two representations.

Finally, I was convinced that we needed a different representation designed for functional programming. I have implemented this alternative representation ("ftrees") and translations between the two representations.

The new representation also allowed me to write code to save and restore expansion trees. The commands are SAVE-ETREE and RESTORE-ETREE.

1.5. Other Structures.

- A mating has certain attributes:
  1. A set of connections
  2. A unification tree
- A unification tree is a tree of nodes.
- A uni-term is an attribute of a node (which is a structure); it is a set of disagreement pairs.
- A failure record is a hashtable. MS88 (and the other non-path-focused procedures) uses the failure record. MS90-3 (and the other path-focused procedures) does not use it (this is one reason why, when TPS3 abandons a mating and later returns to the partially completed eproof, ms91-6 continues approximately where it left off and ms91-7 does not). Links (which all occur in the connection graph) are represented as numbers, so sets of links are just ordered lists of numbers, and one can efficiently test for subsets. Given a new partial mating M, TPS3 just looks at all the entries in the failure record to see if any of them are subsets of M.

2. Operations on Expansion Trees

2.1. Deepening. The code for deepening an expansion tree is in the file etrees-wffops.lisp. The idea behind deepening an expansion tree is to find the leaves, then destructively replace the leaves with a new node depending on the structure of the shallow formula and the setting of many, many flags. The key function to try to understand is deepen-leaf-node-real. Suppose A is the shallow wff of the leaf in question. Here is a quick outline of what this function does:

1. If A can be λ-reduced, create a rewrite node with a leaf of the reduced form as its child.
2. If A is $\neg B$, then create a negation node with a leaf of B as its child (except sometimes when ADD-TRUTH is set to T).
3. If A is $B \wedge C$, then create an econjunction node with leaves for B and C as children.
4. If A is $B \vee C$, then create an edisjunction node with leaves for B and C as children.
5. If A is $B \supset C$, then create an implication node with leaves for B and C as children.
6. If A is $B \equiv C$, then create a rewrite node, rewriting the equivalence either as a conjunction of implications or disjunction of conjunctions (depending on the values of MIN-QUANTIFIER-SCOPE and REWRITE-EQUIVS and the parity of the leaf).

(7) If A is a positive existential formula or negative universal formula, create a skolem node or selection node (depends on the value of SKOLEM-DEFAULT) with a leaf of the scope of the quantifier as its child.
(8) If A is a positive universal formula or negative existential formula, create an expansion node with a leaf of the scope of the quantifier as its child.
(9) If A is an equation of the form $t = t$, and REWRITE-EQUALITIES is not set to NONE, then create a rewrite node with a leaf of TRUTH as its child.
(10) If A is an equation, and REWRITE-EQUALITIES is not set to NONE, then rewrite the equation (depending on REWRITE-EQUALITIES) and create an appropriate rewrite node.
(11) If A is a symbol introduced by dual instantiation of an equality, replace it with the instantiated formula.
(12) Finally, if there is a definition, then there is a very complicated case which creates an appropriate rewrite node. Anyone trying to figure out this part of the code needs to pay close attention to the value of REWRITE-DEFNS. Ordinarily, REWRITE-DEFNS is a flag whose value is a list of a form such as

```
(DUAL (EAGER TRANSITIVE) (NONE INJECTIVE SURJECTIVE))
```

However, at the beginning of deepen-leaf-node-real, the value of REWRITE-DEFNS is dynamically set to a form

```
((DUAL SUBSET REFLEXIVE) (EAGER TRANSITIVE) (NONE INJECTIVE))
```

where all the abbreviations appearing the A are explicitly in the list, and those not appearing in A are removed. To make matters more confusing, rewrite-defns is dynamically set in this case to a simple list of abbreviations which may be rewritten, before calling the function contains-some-defn. This would be a value such as

```
(SUBSET REFLEXIVE TRANSITIVE)
```

The deepening code also sets many global variables as well as eproof slots in current-eproof. The code really assumes we are deepening the etree in the etree slot of current-eproof.

3. Skolemization

There are three skolemization procedures in TPS3 ; SK1, SK3 and NIL. (Actually, the latter is not skolemization at all, but the selection nodes method from Miller's thesis. However, it still uses skolem constants internally.) The flag SKOLEM-DEFAULT decides which one will be used in a proof, and the help message for that flag explains the difference.

We assume familiarity with the way that SK1 is handled in TPS. SK3 is broadly similar; the only difference between the two is in the function create-skolem-node, where the skolem variables are chosen differently.

NIL, the selection node method, is very different. Selections are represented as Skolem constants with no arguments, and we now describe the additional machinery needed to make the search procedure work in this case.

During simplification (in unification), the requirement that a certain relation should be acyclic is checked. The exact statement of this relation is given in Miller's thesis; we implement it (roughly speaking) as a requirement that no substitution term for an expansion variable should contain any of the selections which occur below that variable.

Extra slots, called `exp-var-selected` on expansion variables and `universal-selected` on universal jforms, are used to record all of the selections below each quantifier. This is used in the unification check, and more crucially in path-focused duplication (since skolem terms are stripped out of the jform, this is our only way to remember where they were). See the section on selected variables for more information about checking acyclicity of the dependency relation.

In SK1 and SK3, duplicating a quantifier above a skolem term produces a new skolem term consisting of the same skolem constant applied to different variables. In NIL, we obviously can't use the same skolem constant everywhere (consider EXISTS X FORALL Y . P X IMPLIES P Y ; if we persistently select the same Y every time we duplicate X, the proof will fail). This has two major consequences:

- Path-focused duplication has to be changed. We can no longer duplicate implicitly by changing the name of the quantified variable; we must now make a copy of the entire scope of the quantifier and descend into it, renaming all the selections as we go. These copies are stored in a chain using the universal-dup slot of the jform (so the universal-dup of the top jform contains the first copy, whose universal-dup contains the second copy, and so on). These duplications are preserved during backtracking, in case they are needed again later; we use universal-dup-mark to remember how many of them are "really" there.
- The procedure for expanding the etree after ms90-3 finishes a search has to be completely replaced. The old procedure relied on the fact that the names of skolem constants never changed, and so it was possible to attach all of the expansion terms to the jform and then duplicate and deepen the etree while applying the appropriate substitutions. The names of selections *do* change; this makes the substitutions incorrect (because they will contain the names of old selections). So we use ms90-3-mating and dup-record to duplicate the etree directly, and then add the correct connections to it using ADD-CONN. This procedure is probably still buggy.

4. Checking Acyclicity of the Dependency Relation

4.1. The Dependency Relation. An expansion proof is given by an expansion tree and a mating. One of the conditions on the expansion tree

is that the so-called "dependency relation" is acyclic (irreflexive). For an expansion tree Q, let S_Q be the set of selected variables in Q and let Θ_Q be the set of occurrences of expansion terms. The following definition can be found in Miller's thesis.

Definition. Let Q be an expansion tree. Let $<^0_Q$ be the binary relation on Θ_Q such that $t <^0_Q s$ if there exists $y \in S_Q$ so that y is selected in a node dominated by t and y is free in s. The transitive closure $<_Q$ of $<^0_Q$ is called the *dependency relation*.

While the relation above is defined in terms of expansion terms, the way TPS actually searches for a proof is as follows. Expansion variables are used in place of expansion terms, and TPS finds instantiations for these variables. These instantiations arise from two sources: pre-processing (usually giving set variable instantiations) and unification. The instantiations made during pre-processing may contain expansion variables which will later be instantiated by unification.

During the mating search, unification constructs substitutions for expansion variables, and checks the acyclicity condition for those substitutions. To be sound, TPS should include the substitutions made in pre-processing as well. Until now, TPS did not include these substitutions in the check. This did not cause a problem with soundness because the omitted substitutions (all PRIMSUBS/GENSUBS) did not contain any selected variables.

The most obvious way to ensure soundness is to include all the substitutions in the acyclicity check, not only those arising from unification. However, since the instantiations made before search begins will not change during search, we should be able to find a more efficient method. The idea is to start with an expansion tree obtained after pre-processing, i.e., after instantiations have been made and the resulting tree has been deepened.

Definitions. Let Q be a given expansion tree.

- Let $\Sigma_Q \subseteq \Theta_Q$ be the set of occurrences of expansion terms in Q which are *not* expansion variables.
- Let V_Q be the set of expansion variables that occur in Q. Note that V_Q and Σ_Q are disjoint sets.
- For each expansion variable v, let $T_v \subseteq \Theta_Q$ be the set of expansion terms in which v occurs free. Note that T_v is always nonempty.
- For each $t \in \Sigma_Q$, let $T_t = \{t\} \subseteq \Theta_Q$.
- For each $q \in \Sigma_Q \cup V_Q$, let $B(q)$ be the set of selected variables y whose selection node is dominated by the arc corresponding to t for some $t \in T_q$.
- For each $t \in \Sigma_Q$, let $S(t)$ be the set of selected variables which occur free in t.

The set $B(t)$ is the set of "banned" selection variables for $t \in \Sigma_Q \cup V_Q$.

The following definitions depend upon a substitution θ. This corresponds to the substitution found by unification during mating search.

Definitions. Let θ be a given substitution for the expansion variables in V_Q. (Note that we allow $\theta(v) = v$ for some $v \in V_Q$ in order to make $dom(\theta) = V_Q$.)

- For each $v \in V_Q$, let $S(v)$ be the set of selected variables which occur free in $\theta(v)$.
- Define a relation $<^0_{Q,\theta}$ on $\Sigma_Q \cup V_Q$ by $q <^0_{Q,\theta} r$ iff there exists a $y \in B(q) \cap S(r)$.
- Let $<_{Q,\theta}$ be the transitive closure of $<^0_{Q,\theta}$.
- The substitution θ is acyclic with respect to Q if the relation $<_{Q,\theta}$ is acyclic (irreflexive).

Remark 1. It is important to note that θ is a substitution which does not create any new nodes in Q. (This is in contrast to substituting and then deepening the tree.) In particular, the set of selected variables is the same, i. e., $S_Q = S_{\theta(Q)}$. Also, for the same reason, for each $t \in \Sigma_Q$, $B(t)$ is the set of selected variables y whose selection node is dominated in $\theta(Q)$ by the expansion term occurrence $\theta(t)$. Similarly, for $v \in V_Q$, $y \in B(v)$ means there is some expansion term $t \in \Theta_Q$ such that v occurs free in t and y is dominated in $\theta(Q)$ by $\theta(t)$.

Remark 2. Note that we do not consider λ-terms equivalent up to λ-conversion. The expansion tree must have explicit rewrite nodes to λ-normalize formulas. For this reason, if y is free in a term t in Q, it will also be free in $\theta(t)$ in $\theta(Q)$. (That is, we can never project the y away because this would require not just substitution, but also deepening $\theta(Q)$.)

Lemma 1. Let an expansion tree Q and a substitution θ for V_Q be given. Suppose $q <_{Q,\theta} q'$. Then there exists a $t \in T_q$ such that for every $t' \in T_{q'}$

$$\theta(t) <_{\theta(Q)} \theta(t').$$

Proof. By induction on the number of transitivity steps.

For the base case, suppose $q <^0_{Q,\theta} q'$. Then, there is some $y \in B(q) \cap S(q')$. Since $y \in B(q)$, there is some $t \in T_q$ such that y is dominated by t in Q. So, y is dominated by $\theta(t)$ in $\theta(Q)$. Now, suppose $t' \in T_{q'}$. We consider two cases.

If $q' \in \Sigma_Q$, then $t' = q'$ and $y \in S(q')$. So, y is free in t'. By Remark 2, y is free in $\theta(t')$ and we are done.

If $q' \in V_Q$, then q' is free in t'. Also, $y \in S(q')$ implies y is free in $\theta(q')$. From this we have y is free in $\theta(t')$ and we are done.

For the induction step, suppose we have $q <_{Q,\theta} q_1 <_{Q,\theta} q'$. By induction, we have some $t \in T_q$ such that for any $t_0 \in T_{q_1}$, $\theta(t) <_{\theta(Q)} \theta(t_0)$. Also, we have some $t_1 \in T_{q_1}$ such that for any $t' \in T_{q'}$, $\theta(t_1) <_{\theta(Q)} \theta(t')$. In particular, we have $\theta(t) <_{\theta(Q)} \theta(t_1) <_{\theta(Q)} \theta(t')$ for any $t' \in T_{q'}$. □

Lemma 2. Let an expansion tree Q and a substitution θ for V_Q be given. Suppose we have $t, t' \in \Theta_Q$ with $\theta(t) <_{\theta(Q)} \theta(t')$. There is a $q' \in \Sigma_Q \cup V_Q$ with $t' \in T_{q'}$ such that for any $q \in \Sigma_Q \cup V_Q$ with $t \in T_q$ we have $q <_{Q,\theta} q'$.

Proof. By induction on the number of transitivity steps.

For the base case, suppose $\theta(t) <^0_{\theta(Q)} \theta(t')$. Let y be a selected variable dominated by $\theta(t)$ and free in $\theta(t')$. Since y is free in $\theta(t')$, we must either have y free in t' or y free in $\theta(v)$ for some $v \in V_Q$ free in t'. In the first case, let $q' = t'$. In the second case, let q' be some $v \in V_Q$ where y is free in $\theta(v)$ and v is free in t'. So, we have $y \in S(q')$. Suppose we have any q with $t \in T_q$. Since y is dominated by t, we have $y \in B(q)$ and we are done.

For the induction step, suppose $\theta(t) <^0_{\theta(Q)} \theta(t_1) <^0_{\theta(Q)} \theta(t')$. By induction there is a q' with $t' \in T_{q'}$ such that for any q_0 with $t_1 \in T_{q_0}$, we have $q_0 <_{Q,\theta} q'$. Also by induction there is a q_1 with $t_1 \in T_{q_1}$ such that for any q with $t \in T_q$, we have $q <_{Q,\theta} q_1$. Together we have $q <_{Q,\theta} q_1 <_{Q,\theta} q'$ for any q with $t \in T_q$. □

Proposition. Given an expansion tree Q and a substitution θ for V_Q, the dependency relation for $\theta(Q)$ is acyclic iff θ is acyclic with respect to Q.

Proof. Suppose we have $q <_{Q,\theta} q$ for some $q \in \Sigma_Q \cup V_Q$. By Lemma 1, there is a $t \in T_q$ such that for any $t' \in T_q$, $\theta(t) <_\theta (Q)\theta(t')$. In particular, $\theta(t) <_\theta (Q)\theta(t)$.

Suppose we have $s <_{\theta(Q)} s$ for some expansion term s in $\theta(Q)$. Since $\theta(Q)$ is obtained from Q by substitution (and no deepening), there is a unique expansion term occurrence $t \in \Theta_Q$ such that $s = \theta(t)$. By Lemma 2, there is a $q' \in \Sigma_Q \cup V_Q$ with $t \in T_{q'}$ such that for any $q \in \Sigma_Q \cup V_Q$ with $t \in T_q$ we have $q <_{Q,\theta} q'$. In particular, $q' <_{Q,\theta} q'$. □

After pre-processing, we can compute the set $\Sigma(Q)$ and V_Q, as well as the sets $B(t)$, $B(v)$, and $S(t)$ for each $t \in \Sigma(Q)$ and $v \in V_Q$. So, to check that the acyclicity condition is satisfied when unification generates a substitution θ, it suffices to compute $S(v)$ for each $v \in V_Q$ with respect to θ, and check for a $<^0_{Q,\theta}$-cycle.

Efficiency Refinement. Clearly, if for some $s \in \Sigma(Q)$, either $B(s)$ or $S(s)$ is empty, then s cannot be part of a cycle with respect to any substitution θ, so we may disregard any such term.

5. Expansion Tree to Jform Conversion

In *ms90-3-node*, the jform is computed directly from the etree without using the jform which may be stored with the etree. (It is not clear where or whether that jform is used; it might be part of the interactive system.)

msearch does the search. It returns `(dup-record ms90-3-mating unif-prob)`. `unif-prob` represents the solution to the unification problem, perhaps as a substitution stack. This triple is then handed to the processes that translate things back to an expansion proof, call merging, and then translate to a natural deduction proof. msearch looks at the flag order-components; read the help message for this flag for more information.

Each literal is a jform. One of its attributes is a counter which gets adjusted to count how many mates that literal has; this is compared with max-mates. The current jform being worked on is essentially represented as a stack which is passed around as an argument. Indices are associated with

outermost variables which are implicitly duplicated. These indices are also associated with literals in the scope of these quantifiers to keep track of what copy of the literal is being mated. It is only when unification is called that these indices are actually attached to the variables to construct the terms unification must work on. (The functions check-conn and conn-unif-p in the file *ms90-3-path-enum*, and related functions in that file, may be relevant here.)

The original code only created literals for leaves of the etree. However, it is possible to mate arbitrary nodes of an etree (that share a vertical path) if we include literals corresponding to these nodes. Currently, the user may use the flag ALLOW-NONLEAF-CONNS to specify which nodes to include in the jform. The flag ALLOW-NONLEAF-CONNS takes a list of symbols as its value. If this list contains the symbol ALL, then every node will have a literal in the jform. If this list the symbol REWRITES is in the list, then every rewrite node will have a literal in the jform (giving a jform similar to the one dual instantiation produces, though dual instantiation affects the structure of the etree). If the name of any particular etree node is in the list, then that etree node will have a literal in the jform. The code also uses the slot allow-nonleaf-conns in the current-eproof to decide which nonleaf etree nodes to include as literals in the jform.

After converting an etree into a jform, TPS3 will perform dissolution (see [MR93]) iteratively on each connection in the flag DISSOLVE and in the dissolve slot of the current-eproof. The resulting jform will not have any vertical paths that pass through these connections.

6. Path-Enumerator

6.1. Duplication Order. Along a path the procedure stops at the first eligible universal jform. A slot *dup-t* in a universal jform tells whether it is eligible. Then it starts from there to find the innermost universal jform on the path under the currently picked one, and uses the innermost one as its candidate for next duplication. This is fulfilled by calling function *find-next-dup*.

When testing, please set flag max-dup-paths to an appropriate value so that you can suppress some unnecessary quantifier duplications. It may save a lot of your searching time and make you aware if you are on the right track. Always duplicating innermost quantifiers has the following advantages. 1) producing shorter and clearer proofs, and 2) lowering the values of flags max-search-depth, max-mates, and num-of-dups, sometimes.

6.2. Backtracking. When bactracking starts, the search procedure removes the last added connection. A path attached to the connection tells the procedure where it should pick up the search. This works efficiently since the following claim is almost always true: With the help of disjunction heuristic, the number of paths used to block jform is often a very small fraction of the whole paths in the jform. This means that it is not a big burden to carry

the paths around all the time during searching. The advantage is that the procedure knows exactly where it is without having to do heavy computation by using the information given by the current mating. To make this work, also carried with a path is an environment, which stores the indices and (partial) substitutions for the variables in the path.

7. Propositional Case

In the file *mating-dir.lisp*, you can see that the function ms-director checks whether there are any free variables in current-eproof (seealso eproof, current-eproof) in order to decide whether to call ms or ms-propositional; if there are no free variables in current-eproof, ms-propositional is called.

7.1. Sunil's Propositional Theorem Prover. The original files for Sunil's fast propositional calculus theorem-prover are in */home/theorem/project/tps-variants/si-prop/*. `qload` these files, go into the editor, and make the edwff the example you wish to run. Within the editor `(test)` runs the program using edwff as argument. When it is done, `(test1)` shows the mating it found.

Most of the code in the above directory is now a permanent part of Tps3 ; the function prop-msearch can be called from the `mate` top level, and will display a correct mating for propositional jforms. The way to call it is: `(auto::prop-msearch (auto::cr-eproof-jform))` (the latter function is the internal name for CJFORM). For some reason, the propositional theorem prover is never used, except to reconstruct the mating after a path-focused duplication procedure has found it.

8. Control Structure and Interface to Unification

The non-path-focused-duplication (npfd) search procedures have a connection graph, but the pfd procedures do not; the latter just apply simpl to decide whether literals may be mated. Sunil's disjunction heuristic (see below) is implemented for pfd search procedures, but not for npfd.

The non-path-focused-duplication search procedures break a jform with top-level disjuncts into separate problems, but the path-focused-duplication search procedures do not.

When searching for a way to span a path, Tps3 runs down the path from the top, and considers each literal. As a mate for that literal, it considers each literal which precedes it on the path.

When Tps3 considers adding an essentially ffpair (pair of literals which each start with a variable when one ignores any negations) to the mating, it simultaneously considers both orientations (choices for which literal will be negative and which positive) of the ffpair. Roughly speaking, it does this by putting a disagreement pair corresponding to the ffpair into the leaves of the unification tree, and proceeding with the unification process. If this process encounters a disagreement pair of the form <**A**, ~**B**>, where **A** starts with a constant but **B** does not, it replaces this pair with **<~A, B>**

and continues. In this way it finds whichever substitution works in a very economical fashion. When a success node is found for a complete mating, the associated substitution determines the orientation of the ffpair in the mating.

Here is some more detail about how this is actually implemented. When TPS decides to mate a pair L, K of literals (which it considers as an unordered pair), it seeks to unify ~ L with K, where L occurred before K on the path. Whenever the unification process encounters a double negation, it deletes it. (Thus, in the case of a first-order problem, TPS quickly starts to unify the atoms of the mated literals.)

When the unification process encounters a flexible-rigid pair (which we designate by <...f... , ...~ H...>) where the flexible term has head variable f and the rigid term has a head of the form ~ H, the following substitutions for f are generated:

(1) Projections
(2) $\lambda w^1 \cdots \lambda w^k.{\sim}f^1 \ldots$, where information is attached to f^1 which does not permit a substitution of this same type (i.e., introducing a negation) to be applied to f^1.
(3) $\lambda w^1 \cdots \lambda w^k.f^2 \ldots$, where information is attached to f^2 which does not permit the first two of these types of substitution to be applied to it.

(The information is stored by putting the variables into the lists neg-h-var-list and imitation-h-var-list.) The restrictions on f^2 assure that the dpair which is essentially <...f^2... , ...~ H...> can only be used to generate new substitutions for f^2 if other substitutions reduce ~H to a form which does not start with a negation.

8.1. Sunil's Disjunction Heuristic.

(1) If a matrix contains $[A \vee B]$, and A has no mate, then no mate for B will be sought.
(2) If a matrix contains $[A \vee B]$, and A has a mate, but no mate for B can be found, then the search will backtrack, throwing out the mate for A and all links which were subsequently added to the mating.

Remark: This heuristic is also used by Matt Bishop's search procedure ms98-1. See his thesis [Bis99] for more details.

9. After a Mating is Found

Here is the sequence of events for pfd:

(1) An expansion proof has been found. A record (probably called DUP-RECORD) of indices for duplicated variables, leaves, and connections is maintained by the search process. The unification tree associated with the mating has a record of the substitutions for variables.

(2) Construct jform with final substitutions applied. This uses all copies of variables needed for the final mating.
(3) Duplicate expansion tree from the jform
(4) Attach expansion terms to the expansion tree
(5) Call propositional search to reconstruct the mating
(6) Reorganize mating in ms88 form
(7) Merge expansion proof
(8) Translate expansion proof

10. How MIN-QUANT-ETREE Works

After a proof is found, TPS3 constructs an expansion proof tree. The implementation of flag MIN-QUANT-ETREE consists of the following steps.

(1) TPS3 searches through the expansion proof tree to find if there are primsubs which are not in minimized-scope form. If TPS3 finds some, it goes to step (2).
(2) First, TPS3 transform all the primsubs into their minimized-scope forms. In order to make sure that the expansion proof tree is still a correct one, TPS3 has to modify it. This is done by calling two functions, namely, one-step-mqe-bd and one-step-mqe-infix. Now TPS3 goes to step (3).
(3) Since the expansion proof tree is still a correct one, TPS3 can use a propositional proof checker to search for a mating. This mating will be used to construct a proof in natural deduction style.

There are still potential bugs in the procedure, since various rewrite nodes in an expansion proof can interfere with flag MIN-QUANT-ETREE. This has to be dealt with case by case.

11. Lemmas in Expansion Proofs

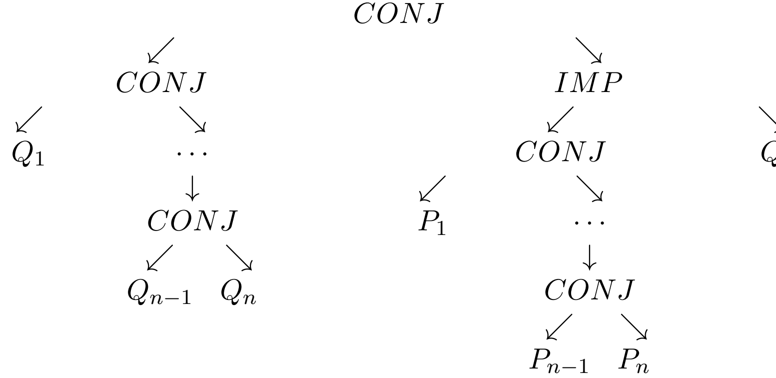
There are facilities to allow an expansion proof of a theorem A to depend on lemmas. For the simplest case, if a theorem A depends on a lemma B, then the expansion tree has shallow formula $B \wedge [B \supset A]$. So, a complete mating for this expansion tree gives a proof of B and a proof of $B \supset A$. If an expansion proof does contain lemmas, merging and translation must take this into account. In general, lemmas can themselves depend on lemmas. The value of lemmas slot is a list structure containing symbols. We can recursively describe these values as

```
<LEMMAS>::((<SYMBOL> . <LEMMAS>) . . . (<SYMBOL> . <LEMMAS>))
```

We can describe how these values correspond to lemmas in the expansion tree inductively. An expansion tree proving A with lemmas corresponding to the value

```
((<SYM1> . <LEMMAS1>) . . . (<SYMn> . <LEMMASn>))
```

where $n > 0$ is of the form



where Q has shallow formula A, each P_i has shallow formula B_i, and each Q_i is an expansion tree proving B_i with lemmas corresponding to the value `LEMMASi`. Note that if `LEMMASi` is NIL, then the shallow formula of Q_i will be B_i, the same as that of P_i. Otherwise, the shallow formula of Q_i will be of the form $[C \wedge [D \supset B_i]]$.

We may use lemmas to handle some extensionality reasoning. Consider the example

$$P_{o(o(o\iota))}[\lambda x_{o\iota}. A_{o(o\iota)}\, x \vee \bot] \supset\; P\, A$$

The jform for this example is of the form

```
|              L1              |
|P [LAMBDA x .A x OR FALSEHOOD]|
|                              |
|              L2              |
|             ~P A             |
Number of vpaths: 1
```

We would like to mate `L1` with `L2`, but we cannot since they are not unifiable. We need to use the fact that A and $\lambda x.Ax \vee \bot$ are extensionally equivalent. The mate command ADD-EXT-LEMMAS finds pairs of such propositional or set or relation terms embedded inside literals and includes an extensionality lemma for any two such terms (occurring in literals of opposite polarity). In this example, there are two extensionality lemmas added to the expansion tree:

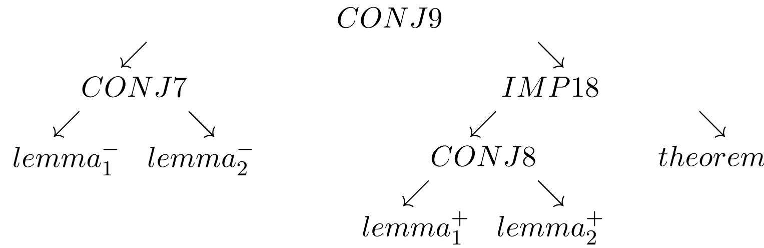
- $\forall x_{o\iota}[\bot \equiv\; A_{o(o\iota)}\, x] \supset \lambda\, x \bot \;=\; A$
- $\forall x_{o\iota}[\, A_{o(o\iota)}\, x \vee \bot \equiv\; A\, x] \supset \lambda\, x[\, A\, x \vee \bot] \;=\; A$

This second lemma can be used to prove the theorem. Both lemmas have an easy proof by expanding equality using extensionality. For example, the part of the expansion tree corresponding to the proof of the second lemma

is of the form

$$
\begin{array}{ccc}
 & IMP1 & \\
\swarrow & & \searrow \\
EXP0 : \forall x [A_{o(o\iota)}\, x \lor \bot \equiv A\, x] & & (EXT =)REW2 : \lambda x [A\, x \lor \bot] = A \\
\downarrow x^0 & & \downarrow \\
LEAF4 : A_{o(o\iota)}\, x^0 \lor \bot \equiv A\, x^0 & & (\lambda)REW1 : \forall x_{o\iota}.[\lambda x. A_{o(o\iota)}\, x \lor \bot]\, x = A\, x \\
 & & \downarrow \\
 & & SEL0 : \forall x_{o\iota}.[A_{o(o\iota)}\, x \lor \bot] = A\, x \\
 & & \downarrow x^0 \\
 & & (EXT =)REW0 : A_{o(o\iota)}\, x^0 \lor \bot = A\, x^0 \\
 & & \downarrow \\
 & & LEAF3 : A_{o(o\iota)}\, x^0 \lor \bot \equiv A\, x^0
\end{array}
$$

with complete mating `((L3 . L4))`. The full expansion tree with the two lemmas has the form



The $lemma_i^-$ etrees correspond to the proofs of the lemmas, and the $lemma_i^+$ etrees correspond to the lemmas that can be used to show the theorem. The value of the `lemmas` slot in the expansion proof has a value such as `((EXT-LEMMA-1) (EXT-LEMMA-2))` indicating that there are two lemmas neither of which depend on lemmas of their own. The jform, after dissolving the connections corresponding to the proofs of the lemmas, is

```
|             L14      FORALL q^9                                     |
|            A x^18 OR  |           L15                L16 |          |
|                       |~q^9 [LAMBDA x FALSEHOOD] OR q^9 A|          |
|                                                                     |
||  L5   |     |  L7   |                                              |
||A x^15 |     |A x^15 |    FORALL q^7                                |
||       | OR  |       | OR  |               L9                  L10 ||
||  L6   |     |  L8   |     |~q^7 [LAMBDA x .A x OR FALSEHOOD] OR q^7 A||
||~A x^15|     |~A x^15|                                              |
|                                                                     |
|                                 L1                                  |
|                  P [LAMBDA x .A x OR FALSEHOOD]                     |
|                                                                     |
|                                 L2                                  |
|                                 ~P A                                |
```

Here we can find a complete mating by connecting `((L1 . L9) (L2 . L10))` (corresponding to the mating of `L1` and `L2` that we wanted to make), and `((L5 . L6) (L7 . L8))` (corresponding to the proof that $\lambda x.Ax \lor \bot$ and A really are extensionally the same).

Given this complete expansion proof, we can merge the proof and translate to a natural deduction proof. During the merging process, the first two lemmas are recognized as unused. The translation to natural deduction depends on the value of the flag ASSERT-LEMMAS. If ASSERT-LEMMAS is set to T, then the expansion proof translates into two natural deduction proofs. The first is a proof of the lemma. The second is a proof of the theorem using the lemma. The name of the natural deduction proof of the lemma is given by the corresponding symbol in the value of the `lemmas` slot. For example, the name could be EXT-LEMMA-2. In the proof of the theorem, there would be a line justified by "Assert: EXT-LEMMA-2". If ASSERT-LEMMAS is NIL, the expansion proof translates to a single natural deduction proof. First, the outline

	......	
(5)	$\vdash \forall x_{o\iota}[A_{o(o\iota)}\, x \lor \bot \equiv A\, x] \supset \lambda x[A\, x \lor \bot] = A$	PLAN4
(6)	$\vdash \forall x_{o\iota}[A_{o(o\iota)}\, x \lor \bot \equiv A\, x] \supset \lambda x[A\, x \lor \bot] = A$	Same as: 5
	......	
(104)	$\vdash P_{o(o(o\iota))}[\lambda x_{o\iota}. A_{o(o\iota)}\, x \lor \bot] \supset P\, A$	PLAN2

is formed, then these gaps are filled in using the corresponding parts of the expansion proof.

Extensionality examples can of this form can be proven automatically using DIY if the flag USE-EXT-LEMMAS is set to T.

12. Extensional Expansion Dags

Extensional expansion dags are a generalization of expansion trees which represent proofs in extensional type theory (see Chad E. Brown's thesis [Bro04]). There are four lisp structures used to represent extensional expansion dags: ext-exp-dag, ext-exp-arc, ext-exp-open-dag and ext-exp-open-arc. These structures are defined in ext-exp-dag-macros.lisp. The intention is that ext-exp-dag (ext-exp-arc) structures represent nodes (arcs) in ground dags with no expansion variables and should only be constructively manipulated. On the other hand, ext-exp-open-dag (ext-exp-open-arc) structures represent nodes (arcs) may contain expansion variables and can be destructively manipulated (e.g., via substitution). The global variable ext-exp-dag-verbose causes the structures to be printed with a huge amount of verbosity and should be set to NIL unless debugging. Similarly, setting the global variable ext-exp-dag-debug to T causes a lot of extra sanity checking to aid debugging.

The EXT-MATE top level can be used to manipulate extensional expansion dags. The code implementing the EXT-MATE is in the file ext-mate-top.lisp.

The automatic search procedures ms03-7 and ms04-2 use extensional expansion dags. The code for ms03-7 is in ext-search.lisp. The code for ms04-2 is in ms04-search.lisp.

13. Printing

CHAPTER 14

Merging

Once a complete mating is found, we enter a merging process. The merging process performs the following steps, some of which are described in more detail in separate sections.

Note: merging still contains bugs, although not very many. If a correct mating is merged and produces a translation error, or a message of the form "The formula is not provable as there's no connection on the following path: <path>", then it's likely that a bug in merging is the culprit. Within merging, the routines for REMOVE-LEIBNIZ, CLEANUP-ETREE, and PRETTIFY are the most likely causes of problems. The first only applies for formulae with equality, and can be checked by trying again with the flag REMOVE-LEIBNIZ set to NIL. For the other two, you need to use merge-debug; type `setq auto::merge-debug t` before calling merging, and you can step through the process, inspecting the etree at each step and omitting the optional steps. This can be a great help in discovering which part of the merging process is causing the bug.

Note: In November, 2000, merging was changed to handle the case when a mating contains nonleaf nodes. The changes were to REMOVE-LEIBNIZ and RAISE-LAMBDA-NODES. Also, the final phase was separated into a cleanup phase and a prettify phase. The prettify code was for the most part rewritten. In the process of making these changes, this section of the Programmer's Guide was extended to reflect the current state of merging.

- The expansion tree is processed by the function etr-merge (see section 1) which applies the substitutions for expansion variables in the expansion tree and merges duplicate expansion nodes. It returns both the new etree and an alist of nodes corresponding to the mating. This is the part that actually corresponds to the "merging" algorithm (Algorithm 84) in Frank Pfenning's thesis[Pfe87].
- Duplicate connections and connections between nodes that do not occur in the tree are deleted from the mating (actually, connection list, in the local variable `new-conn-list`).
- If dual instantiation[BA98] is used, modify-dual-rewrites is called (see section 3). Then, connections between nodes no longer in the tree are removed from the mating.
- prune-unmated-branches is called. If MERGE-MINIMIZE-MATING is set to T, this function removes children of expansion nodes which are not needed to have a complete mating. The function also calls

replace-non-leaf-leaves on the etree, which replaces empty expansion nodes with leaves. (Note: leaf-p* returns T on any node that has no kids, except true and false nodes.) See section 4.

- If the skolem method (determined by SKOLEM-DEFAULT) used is not NIL, then subst-skol-terms is called. This replaces terms such as XMN with skolem-terms SK whenever there is a skolem-term SK with `TERM` slot XMN. See section 5.
- If the top of the tree is a conjunction whose first child is an $ADD-TRUTH$-rewrite, then delete the conjunction leaving only the second child as the expansion tree. (See TRUTHVALUES-HACK and ADD-TRUTH.)
- If REMOVE-LEIBNIZ is set to T, then *Leibniz* =-rewrite nodes are removed. This is a somewhat complicated process based on an algorithm described in Frank Pfenning's thesis[Pfe87]. See section 6.
- subst-vars-for-params is called. This replaces skolem-terms with the variable bound by the corresponding quantifier, if this is possible. If this is not possible, then we replace the skolem-term with the value of its `PARAMETER` slot.
- λ-rewrite nodes are raised over any propositional connectives and skolem/selection nodes. This lifting stops at expansion nodes and rewrite nodes other than equiv-implics, equiv-disjs and lambda. This also moves connections to lambda nodes, with the result that no connection in the mating involves a λ-rewrite node after this step is performed. See section 7.
- The etree is converted to a propositional jform (including any non-leaf nodes in the mating) and the current set of connections is used to set active-mating.
- The etree is cleaned up by calling cleanup-etree. This λ-normalizes expansion terms, may remove some λ-rewrite nodes, and may modify *Subst* =-rewrite nodes. See section 8
- The etree is prettified by calling prettify-etree. See section 9. This renames bound variables and free variables in the etree that do not occur in the original wff. We must be careful to avoid variable capture when doing this renaming. (There were bugs with the old code because of variable capturing.)

1. Applying Substitutions and Merging Duplicate Expansions

The functions etr-merge and merge-all are in the file *mating-merge.lisp.* These functions are used to preprocess the expansion tree in order to make the rest of merging more efficient. For a discussion of why this preprocessing is done first, see section 2.

The function etr-merge calls make-mating-lists to create the alist of mated nodes and the substitution for expansion variables corresponding to the mating. Then the function prune-status-0 deletes children of expansion

nodes which have status zero. The function substitute-in-etree is used to apply the substitution to the etree (this puts the appropriate terms into the SUBST slot of the expansion variables). Then the functions strip-exp-vars-for-etree and strip-exp-vars are used to replace all expansion variables by their SUBST slot. Finally, merge-all is called.

The function merge-all takes an expansion tree and a mating, and descends into the tree. At each expansion node, if two expansion terms are identical, their corresponding trees are merged. The resulting tree replaces the two original ones, and the substitution returned is applied to the terms and trees. The resulting tree and mating are returned.

The actual merging of two children of expansion nodes is carried out by treemerge. The algorithm is described as Algorithm 84 in Frank Pfenning's thesis[Pfe87]. The algorithm also must build a substitution replacing some selected variables with other selected variables and apply this to the tree. The function returns three values: the new etree, the substitution `merge-theta` for selected variables, and the new mating.

2. Detecting Unneeded Nodes

The function unneeded-node-p is used both by modify-dual-rewrites and prune-unmated-branches. It is defined in the file *mating-merge.lisp*. The purpose of unneeded-node-p is to determine if a node is needed to have a complete mating. If the node has zero status, then it is not needed. If the flag MERGE-MINIMIZE-MATING is set to NIL, then we insist that it is needed. Otherwise, we temporarily set the status to zero (essentially removing the node from the tree), and use SPANS to check if the mating still spans all paths.

The function SPANS calls SPANNING-CLIST-PATH, which calls FIND-CHEAPEST-CLIST-SPANNING-PATH, which FIND-ALT-CHEAPEST-CLIST-SPANNING-PATH. Note that even in x5207, which is relatively small, 12 calls to UNNEEDED-NODE-P result in 295 calls to FIND-CHEAPEST-CLIST-SPANNING-PATH; these functions are the main reason why merging can be so slow, especially in proofs created by MS90-3 or MS90-9.

You could possibly (as was done at one time) not test this spanning condition, and just check to see if every expansion actually has a connection below it. The problem here is that in ms90-3, by the time we get to the merging process, we have mated every possible pair in the tree, whether the connection is necessary or not. That is why unneeded-node-p was modified to be more rigorous, because otherwise it was almost useless. Additionally, there may be embedded falsehood nodes below it, which are required to close some paths, even if there are no mated nodes below it.

A better spanning function should be used, though actually the one used is already propositional, but of an earlier generation than Sunil's propositional search function. One should realize, however, that the procedure

should use the mating provided (and not the eager "mate-everything", because our mating might *not* be that big). In fact, Dan wrote such a SPANS that uses a variant of PROP-MSEARCH, and the time used in X5207 by SPANS went from 1 second to about .3 sec. Unfortunately, PROP-MSEARCH (or rather, PROP-FIND-CHEAPEST-PATH) appears to have the "empty disjunction causes confusion" bug. (Try MS90-3 on the formula "falsehood implies A").

More drastic changes were tougher to implement. There were a few suggestions:

- What this is doing is a lot of duplicated effort, so perhaps it would be possible to cache some results. This would be pretty space-intensive; e.g. THM131 has an astronomical number of vpaths when it begins merging. It turned out that attempts to make SPANS better by caching the results were pretty silly, because the way it is invoked, you can't tell the difference between sets of arguments. The differences are made by changing the status of various lower-level expansion nodes. So that attempt was abandoned.
- Perhaps it would be possible to check the paths which the suspect node was on. It's not clear how to do this.
- Of course, it might be possible to avoid some of the calls to spans in the first place (though possibly not with MS90-3), but even eliminating half would only save 3 days in the wolf-goat-cabbage problem, without changing what it does.

In the end, the solution used was as follows: when path-focused duplication has been used, the expansion proof will often have a great deal of redundancy in the sense that the same expansion term will be used for a given variable many times. More precisely, if one defines an expansion branch by looking at sequences of nested expansion nodes, attaching one expansion term to each expansion node in the sequence, there will be many identical expansion branches. So one can start by merging the tree in the sense of eliminating this redundancy (see section 1), and then apply to this much simpler tree the procedure for deleting unnecessary expansion terms which we think is using so much time. It turned out to be easiest to do this by throwing away the mating, and reconstructing it by propositional search after the tree has been cut down to size. Of course, one could also preserve the original mating by "merging" it appropriately as one collapsed the tree.

The precise way in which this was done, in the file *mating-merge.lisp*, was:

(1) Don't do pruning of unnecessary nodes at the beginning of the merge, when the tree is its greatest size.
(2) Instead, *do* prune all branches that couldn't possibly have been used They are those that have a zero status. This is probably not necessary, but certainly makes debugging easier and doesn't cost much. (See section 1.)

(3) After merging of identical expansions has been done, call the original pruning function, prune-unmated-branches (see Section 4).

Note that the merge process does (or should, anyway) merge the mating appropriately as the tree collapses. On THM131 this takes the time spent on merging from 7 days down to 12 minutes. This is not so surprising, because it begins with 113 million paths, and after the merging of duplicate expansions, it's down to around 442 thousand.

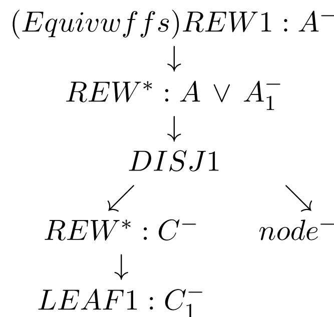
The matingstree top level has its own approach to merging, which is essentially step (2) above, in which all unused expansions are simply thrown away, followed by a regular merge as detailed above. Putting step (2) first here is necessary because the master expansion tree has many nodes which are irrelevant to any particular proof.

3. Modify-Dual-Rewrites

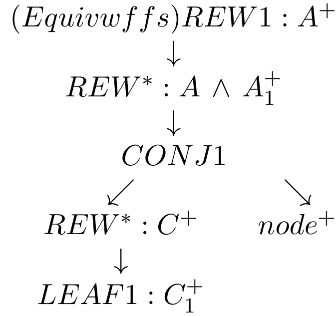
The functions for MODIFY-DUAL-REWRITES are in the file *mating-merge.lisp*. This is only called when dual instantiation is used. The main function, modify-dual-rewrites, is described in this section. First, this function uses the global variable *hacked-rewrites-list*. The value of the global is a list of elements of the form

```
(<rewrite node> .
    (<instantiated wff-or-symbol> .
                  <leaf with uninstantiated form>))
```

This list is sorted so that the names of the rewrites are increasing. The main body is a dolist considering each element of `*hacked-rewrites-list*`. For each subtree of the form



or



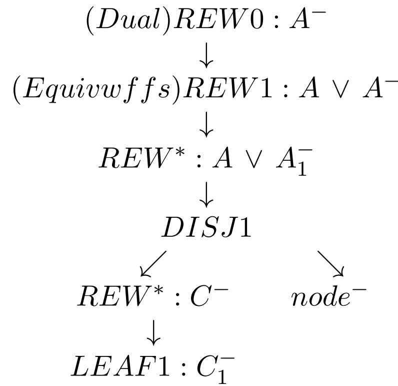
is converted to either only use one branch or a slightly modified tree with an explicit dual rewrite (separate from expanding the definition).

The local variables in the loop are

- `junct` $DISJ1$ or $CONJ1$
- `gsym` A symbol standing in for the uninstantiated formula A.

First, we check to make sure the rewrite node $REW1$ is a rewrite still in the etree. Assuming it is in the tree, we consider several cases

(1) If the left child of `junct` is not needed in the mating, then the uninstantiated definition is not needed. We replace the `junct` node with its second child and call fix-shallow-chain to change the shallow formulas of the rewrites between $REW1$ and `junct` to be the second conjunct.
(2) If the right child of `junct` (`realrew`) is not needed in the mating, then the instantiated definition is not needed. We replace the `junct` node with its first child and call fix-shallow-chain to change the shallow formulas of the rewrites between $REW1$ and `junct` to be the first conjunct, replacing `gsym` by A when necessary.
(3) Otherwise, both are needed. In this case, we change the tree to have a form like



This makes the dual rewrites easier to recognize and handle in the cleanup code.

4. Prune-Unmated-Branches

The purpose of the function prune-unmated-branches in the file *mating-merge.lisp* is to delete some children of expansion nodes by checking if it is really needed in the proof. Each node is checked using unneeded-node-p (see section 2).

Before returning, this function calls replace-non-leaf-leaves on the etree, which replaces all empty expansion nodes with leaves. (Note: leaf-p* returns T on any node that has no children, except true and false nodes.)

5. Subst-Skol-Terms

The function subst-skol-terms in the file *mating-merge.lisp* is used to replace terms in the etree which correspond to skolem terms by the skolem-term itself. For example, the etree might contain a skolem-term SK with slots

- `PARAMETER` : for example, $X_{o\beta}$
- `TERM` : for example, $X^2_{o\beta\alpha\alpha}MN$

The shallow formulas and expansion terms in the etree might contain subterms of the form $X^2_{o\beta\alpha\alpha}MN$ (up to λ-conversion). These are replaced by the skolem-term SK.

The function subst-skol-terms-main actually does the work. It takes an argument `skol-terms`, which is an list of pairs `(<term> . <skolem-term>)` where each `<skolem-term>` is the skolem term for a skolem node, and the `<term>` is the gwff in the `TERM` slot of the `<skolem-term>`. The function traverses each term doing the (destructive) replacement in the shallow formula and/or expansion terms.

6. Remove-Leibniz

The functions for REMOVE-LEIBNIZ are in the *mating-merge-eq.lisp*. The functions described here are

- remove-leibniz-nodes
- pre-process-nonleaf-leibniz-connections
- remove-leibniz
- cleanup-leibniz-expansions
- remove-spurious-connections
- check-shallow-formulas
- apply-thm-146

(1) **remove-leibniz-nodes** This is the main function. It collects negative and positive *Leibniz* =-rewrite nodes. The connection list is pre-processed (by pre-process-nonleaf-leibniz-connections) so that any mates to nonleaf nodes strictly below a negative *Leibniz* =-rewrite is replaced by mating the leaves below the node. Next, the function remove-leibniz is called on each negative *Leibniz* =-rewrite

node, possibly changing the connection list. This has the result of changing negative subtrees of the form

$$
\begin{array}{ccc}
 & (Leibniz =)REW1 : A = B^- & \\
 & \downarrow & \\
 & SEL1 : \forall q \,.\, q\, A \supset q\, B^- & \\
 & \downarrow q_0 & \\
 & IMP1 & \\
\swarrow & & \searrow \\
\lambda/EquivwffsREW^* : q_0\, A^+ & & \lambda/EquivwffsREW^* : q_0\, B^- \\
\downarrow & & \downarrow \\
LEAF1 : q_0\, A_0^+ & & LEAF2 : q_0\, B_0^-
\end{array}
$$

by trees of the form

$$
\begin{array}{c}
\lambda REW^* : A = B^- \\
\downarrow \\
EquivwffsREW^* : A_1 = B_1^- \\
\downarrow \\
(Refl =)REW1 : C = C^- \\
\downarrow \\
TRUE1^-
\end{array}
$$

or simply a leaf

$$LEAF2 : A = B^-$$

Remark: The notation $\begin{array}{c} REW^* : C \\ \downarrow \\ \ldots \end{array}$ indicates a chain of rewrites starting with shallow formula C. This chain may be empty, a single node, or several nodes.

In such cases, connections to $LEAF1^+$ are deleted from the connection list. The function deepen-negated-leaves is called, but this (apparently) has no effect unless make-left-side-refl returns NIL (and it currently always returns T).

Finally, cleanup-leibniz-expansions is called in order to change positive *Leibniz* =-rewrites to *Subst* =-rewrites, destructively changing a positive subtree of the form

$$\begin{array}{c}
(Leibniz =)REW1 : A = B^+ \\
\downarrow \\
REW^* : \forall q \,.\, q\,A \supset q\,B^+ \\
\downarrow \\
EXP1 : \forall q \,.\, q\,A_1 \supset q\,B_1^+ \\
Q_1 \swarrow \qquad\qquad \downarrow * \quad \searrow Q_n \\
\lambda REW^* : Q_1\,A_1 \supset Q_1\,B_1^+ \quad \cdots \quad \cdots \\
\downarrow \\
IMP1 \\
\swarrow \quad \searrow \\
\cdots \quad \cdots
\end{array}$$

to a subtree of the form

$$\begin{array}{c}
REW^* : A = B^+ \\
\downarrow \\
(Subst =)REW1 : A_1 = B_1^+ \\
\downarrow \\
EXP1 : \forall q \,.\, q\,A_1 \supset q\,B_1^+ \\
Q_{i_1} \swarrow \qquad\qquad \downarrow * \quad \searrow Q_{i_m} \\
IMP1 \qquad\qquad \cdots \quad \cdots \\
\swarrow \qquad \searrow \\
\lambda REW^* \qquad \lambda REW^* \\
\downarrow \qquad\qquad \downarrow \\
\cdots \qquad\qquad \cdots
\end{array}$$

where $1 \leq i_1 \leq \cdots \leq i_m \leq n$.

(2) **pre-process-nonleaf-leibniz-connections** Since the remove-leibniz function may replace negative subtrees of the form

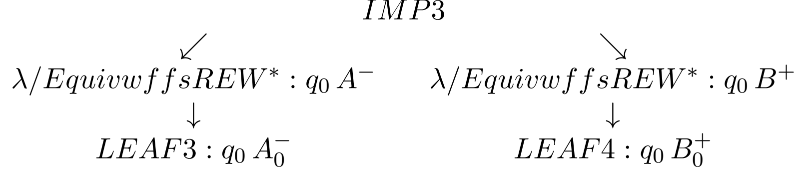
$$\begin{array}{c}
(Leibniz =)REW1 : A = B^- \\
\downarrow \\
SEL1 : \forall q \,.\, q\,A \supset q\,B^- \\
\downarrow q_0 \\
IMP1 \\
\swarrow \qquad\qquad\qquad \searrow \\
\lambda/EquivwffsREW^* : q_0\,A^+ \qquad \lambda/EquivwffsREW^* : q_0\,B^- \\
\downarrow \qquad\qquad\qquad\qquad \downarrow \\
LEAF1 : q_0\,A_0^+ \qquad\qquad LEAF2 : q_0\,B_0^-
\end{array}$$

with a leaf of the form

$$LEAF2 : A = B^-$$

we must have some way of dealing with connections to nonleaf nodes such as $SEL1$ and $IMP1$. This pre-processing function replaces a

connection such as $(IMP1 \,.\, IMP3)$ with connections to the leaves $(LEAF1 \,.\, LEAF3)\,(LEAF2 \,.\, LEAF4)$ where $IMP3$ is a positive subtree of the form

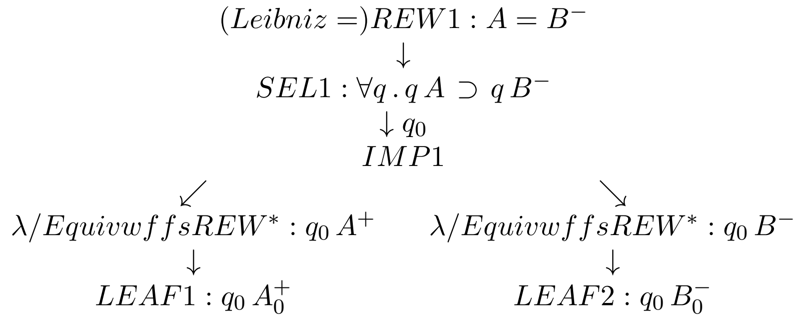


Such connections between leaves are dealt with in the function remove-leibniz.

Note that we do allow connections to the node $REW1$ to remain in the mating. This connection may also be modified in the function remove-leibniz.

(3) **remove-leibniz** This function basically corresponds to the proof of Theorem 138 in Frank Pfenning's thesis[Pfe87]. Also, Remark 140 in Frank Pfenning's discusses the choice between using the formula (`subwff`) $[\lambda x \,.\, A_0 = x]$ or $[\lambda x \,.\, \neg x = B_0]$ in the algorithm below. One may lead to a more elegant proof. In TPS3 the choice is made by a call to the function make-left-side-refl, which currently always returns T.

Suppose we are given a negative subtree of the form



(Actually, the selection node $SEL1$ might be a Skolem node, but this is treated the same way.) The local variables in the function are given the following values:

- `param-node` $SEL1$
- `param` q_0
- `imp-node` $IMP1$
- `new-refl-node` $LEAF1$ (or, $LEAF2$ if make-left-side-refl were to return NIL)
- `subwff` $[\lambda x \,.\, A_0 = x]$ (or, $[\lambda x \,.\, \neg x = B_0]$ if make-left-side-refl were to return NIL)
- `new-non-refl-node` $LEAF2$ (or, $LEAF1$ if make-left-side-refl were to return NIL)

- `non-refl-branch` the second son of $IMP1$, which is a rewrite node or $LEAF2$ (or, the first son of $IMP1$ if make-left-side-refl were to return NIL)
- `mated-to-refl-node` list of nodes mated to $LEAF1$

We consider two cases

(a) If $LEAF1$ is connected to $LEAF2$, then A_0 and B_0 must be identical. Let `lhs` be A and `rhs` be B. If these are identical wffs, then simply change the etree to be

$$\begin{array}{c} (Refl =)REW1 : A = B^- \\ \downarrow \\ TRUE1^- \end{array}$$

Otherwise, let A_1 (`lhs*`) be the λ-normal form of A and B_1 (`rhs*`) be the λ-normal form of B. If these are contain no abbreviations, they should be identical, so we change the etree to be

$$\begin{array}{c} (\lambda)REW2 : A = B^- \\ \downarrow \\ (Refl =)REW3 : A_1 = B_1 \\ \downarrow \\ TRUE1^- \end{array}$$

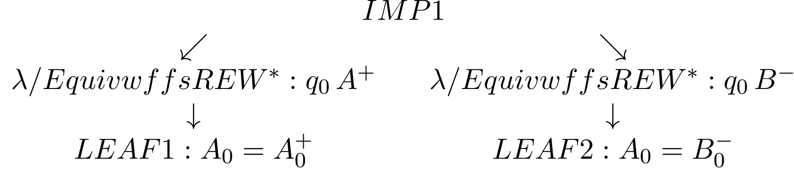
Otherwise, let A_2 and B_2 be the result of instantiating all definitions (except equiv) in A_1 and B_1, resp. These should be identical, so we can change the subtree to be

$$\begin{array}{c} (\lambda)REW2 : A = B^- \\ \downarrow \\ (Equivwffs)REW3 : A_1 = B_1^- \\ \downarrow \\ (Refl =)REW4 : A_2 = B_2^- \\ \downarrow \\ TRUE1^- \end{array}$$

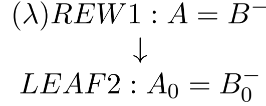
Finally, we remove all connections involving $LEAF1$, $LEAF2$, or $REW1$. We still have a complete mating without these connections. First note that any path which would have passed through any of these nodes would have passed through all of them. Now, the corresponding path in the jform for the new tree must pass through $TRUE1^-$.

Possible Bug: The point of Theorem 138 in Frank Pfenning's thesis[Pfe87] is to remove all Leibniz selected variables q_0. However, in this case we are not substituting for the q_0, so there may still be references to it in the tree. It's unclear, however, if this causes a problem in this special case. If it does turn out to be a bug, probably the fix is to substitute the value of `subwff` for q_0.

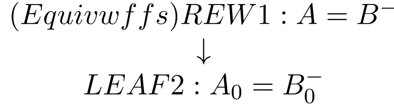
(b) If $LEAF1$ and $LEAF2$ are not mated, substitute the value of `subwff` for q_0 in the etree. Assume `subwff` has value $[\lambda x . A_0 = x]$ (or, $[\lambda x . \neg x = B_0]$. (Note that this substitution automatically λ-normalizes and puts λ-rewrites above the leaves if they are needed.) So, the subtree starting at $IMP1$ now has the form



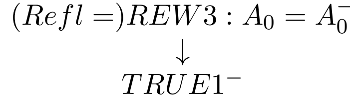
If $A = B$ is the same as $A_0 = B_0$ up to α-conversion, we replace $REW1$ with $LEAF2$. Otherwise, replace $REW1$ with a subtree of the form



or

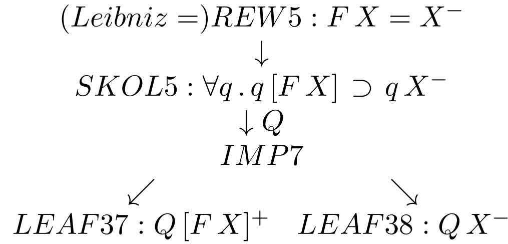


Since $LEAF1$ is no longer in the tree, we must delete any connection $(LEAF1 . LEAF3)$. We replace each such $LEAF3$ with



and delete any connection with $LEAF3$.

Consider the following examples, which arise in a proof of THM15B using mode MODE-THM15B-C.



with connections

$$(LEAF28 . LEAF109)\,(LEAF38 . LEAF33)\,(LEAF37 . LEAF32)$$
$$(LEAF29 . LEAF22)\,(LEAF21 . LEAF108)\,(LEAF105 . LEAF101)$$
$$(LEAF104 . LEAF100)\,(LEAF94 . LEAF93)$$

becomes

$$LEAF38 : F\,X = X^-$$

removing the connection

$$(LEAF37\,.\,LEAF32)$$

$$\begin{array}{c}
(Leibniz =)REW15 : [\lambda w\,.\,F\,.\,w\,X]\,F = [\lambda u\,.\,u\,.\,F\,X]\,F^- \\
\downarrow \\
SKOL5 : \forall q\,.\,q\,[[\lambda w\,.\,F\,.\,w\,X]\,F] \supset q\,[[\lambda u\,.\,u\,.\,F\,X]\,F]^- \\
\downarrow Q^{18} \\
IMP13 \\
\swarrow \qquad\qquad\qquad\qquad \searrow \\
(\lambda)REW19 : Q^{18}\,[[\lambda w\,.\,F\,.\,w\,X]\,F]^+ \qquad (\lambda)REW20 : Q^{18}\,[[\lambda u\,.\,u\,.\,F\,X]\,F]^- \\
\downarrow \qquad\qquad\qquad\qquad\qquad \downarrow \\
LEAF93 : Q^{18}\,.\,F\,.\,F\,X^+ \qquad\qquad LEAF94 : Q^{18}\,.\,F\,.\,F\,X^-
\end{array}$$

with connections

$$(LEAF94\,.\,LEAF93)\,(LEAF38\,.\,LEAF33)\,(LEAF29\,.\,LEAF22)$$

$$(LEAF21\,.\,LEAF108)\,(LEAF105\,.\,LEAF101)$$

becomes

$$\begin{array}{c}
(\lambda)REW38 : [\lambda w\,.\,F\,.\,w\,X]\,F = [\lambda u\,.\,u\,.\,F\,X]\,F^- \\
\downarrow \\
(Refl =)REW39 : F\,.\,F\,X = F\,.\,F\,X^- \\
\downarrow \\
TRUE3^-
\end{array}$$

removing the connection

$$(LEAF94\,.\,LEAF93)$$

(4) **cleanup-leibniz-expansions** Start with a positive subtree (`eq-rew-node`) of the form

$$\begin{array}{c}
(Leibniz =)REW1 : A = B^+ \\
\downarrow \\
REW^* : \forall q\,.\,q\,A \supset q\,B^+ \\
\downarrow \\
EXP1 : \forall q\,.\,q\,A_1 \supset q\,B_1^+ \\
Q_1 \swarrow \qquad\qquad \downarrow * \quad \searrow Q_n \\
\lambda REW^* : Q_1\,A_1 \supset Q_1\,B_1^+ \qquad \cdots \qquad \cdots \\
\downarrow \\
IMP1 \\
\swarrow \quad \searrow \\
\cdots \quad \cdots
\end{array}$$

A do loop early in the function pushes the justifications for the initial rewrites up one step, and changes the shallow formulas from

$$\forall q \,.\, q\, A^* \supset q\, B^*$$

to be the uninstantiated equation

$$A^* = B^*.$$

Also, the last rewrite in the chain is changed to have justification $Subst =$. So, the intermediate tree has the form

$$\begin{array}{ccccc}
 & REW^* : A = B^+ & & & \\
 & \downarrow & & & \\
 & (Subst =)REWn : A_1 = B_1^+ & & & \\
 & \downarrow & & & \\
 & EXP1 : \forall q \,.\, q\, A_1 \supset q\, B_1^+ & & & \\
 Q_1 \swarrow & & \downarrow * & \searrow Q_n & \\
 \lambda REW^* : Q_1\, A_1 \supset Q_1\, B_1^+ & & \cdots & \cdots & \\
 \downarrow & & & & \\
 IMP1 & & & & \\
 \swarrow \quad \searrow & & & & \\
 \cdots \quad \cdots & & & &
\end{array}$$

After the do loop, the local variable `exp-node` has value $EXP1$.

Next, for each child of the expansion node of the form

$$\begin{array}{c}
\lambda REW^* : Q_i\, A_1 \supset Q_i\, B_1^+ \\
\downarrow \\
IMPi \\
\swarrow \quad \searrow \\
\cdots \quad \cdots
\end{array}$$

we remove the initial λ-rewrites, possibly adding a λ-rewrite beneath $IMPi$. (The function check-shallow-formulas does part of this work.) This replaces the corresponding son of $EXP1$ with a subtree of the form

$$\begin{array}{ccc}
 & IMPi & \\
\swarrow & & \searrow \\
\lambda REW^* : Q_i\, A_1^- & & \lambda REW^* : Q_i\, B_1^+ \\
\downarrow & & \downarrow \\
\cdots & & \cdots
\end{array}$$

Next, we call the function apply-thm-146. This may replace some $Subst$ =-rewrite nodes with leaves. There is a description of this function later in this section. The function corresponds to Theorem 146 in Frank Pfenning's thesis[Pfe87].

As a final step (which occurs in the code in the return portion of the outermost dolist loop), the function remove-spurious-connections is called. This function cleans the mating and expansion tree, and returns the new mating.

(5) **remove-spurious-connections** This function finds connections between nodes with shallow formula $A = A$ (`bad-conn`). One of these nodes must be negative so we can deepen it (if necessary) to replace it with a new tree of the form

$$\begin{array}{c} (Refl =) REW1 : A = A^{-} \\ \downarrow \\ TRUE1^{-} \end{array}$$

Then, we remove this connection from the mating. After simplifying the mating in this way, we delete any children of expansion nodes immediately beneath *Subst* =-rewrite nodes which are not used in the mating. Then we call apply-thm-146 because we may have simplified some *Subst* =-rewrite node to be of the appropriate form. Finally, we return the connection list.

(6) **check-shallow-formulas** This function takes a positive equational rewrite node $REW1$ with shallow formula $A = B$, an expansion node $EXP1$ which is a child of $REW1$, and an implication node $IMPi$ which is a child of $EXP1$. Suppose $IMPi$ has shallow formula $C \supset D$. This function checks if the D can be obtained from C by replacing occurrences of A by B. If not, the relevant expansion term Q_i must have the form $[\lambda z\, .\, P]$. So, we add λ-rewrites are added beneath the implication to make the implication node have the form

$$\begin{array}{ccc} & IMPi & \\ \swarrow & & \searrow \\ (\lambda) REW2 : [A/z]P & & (\lambda) REW3 : [B/z]P \\ \downarrow & & \downarrow \\ node_1 & & node_2 \end{array}$$

Note that we can clearly obtain $[B/z]P$ formula from $[A/z]P$ by replacing some occurrences of A by B. So, the tree has the appropriate form.

(7) **apply-thm-146** This function corresponds to Theorem 146 in Frank Pfenning's thesis[Pfe87]. If the subtree passed to the function has

the form

$$\begin{array}{c} (Subst =)REWn : A = B^+ \\ \downarrow \\ EXP1 : \forall q \,.\, q\, A \supset q\, B^+ \\ \downarrow Q \\ IMP1 \\ \swarrow \qquad\qquad \searrow \\ \cdots \qquad\qquad \cdots \\ \downarrow \\ LEAF1 : A = B^+ \end{array}$$

we can replace this subtree with $LEAF1$. If the subtree passed to the function has the form

$$\begin{array}{c} (Subst =)REWn : A = B^+ \\ \downarrow \\ EXP1 : \forall q \,.\, q\, A \supset q\, B^+ \\ \downarrow Q \\ IMP1 \\ \swarrow \qquad\qquad \searrow \\ \cdots \qquad\qquad \cdots \\ \downarrow \\ LEAF2 : A = B^+ \end{array}$$

we can replace this subtree with $LEAF2$.

7. Raise-Lambda-Nodes

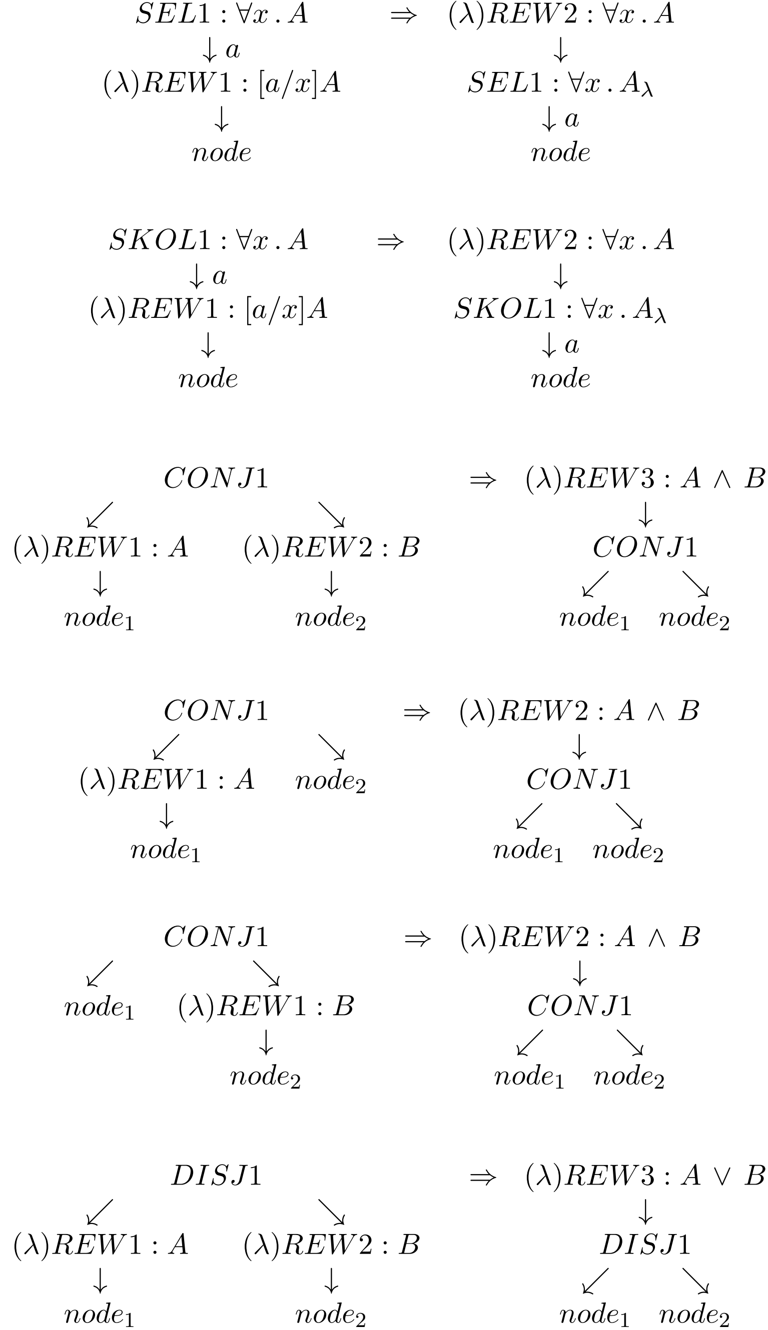
The functions of RAISE-LAMBDA-NODES are in the file *mating-merge2.lisp*. The main function is raise-lambda-nodes which calls the following auxiliary functions:

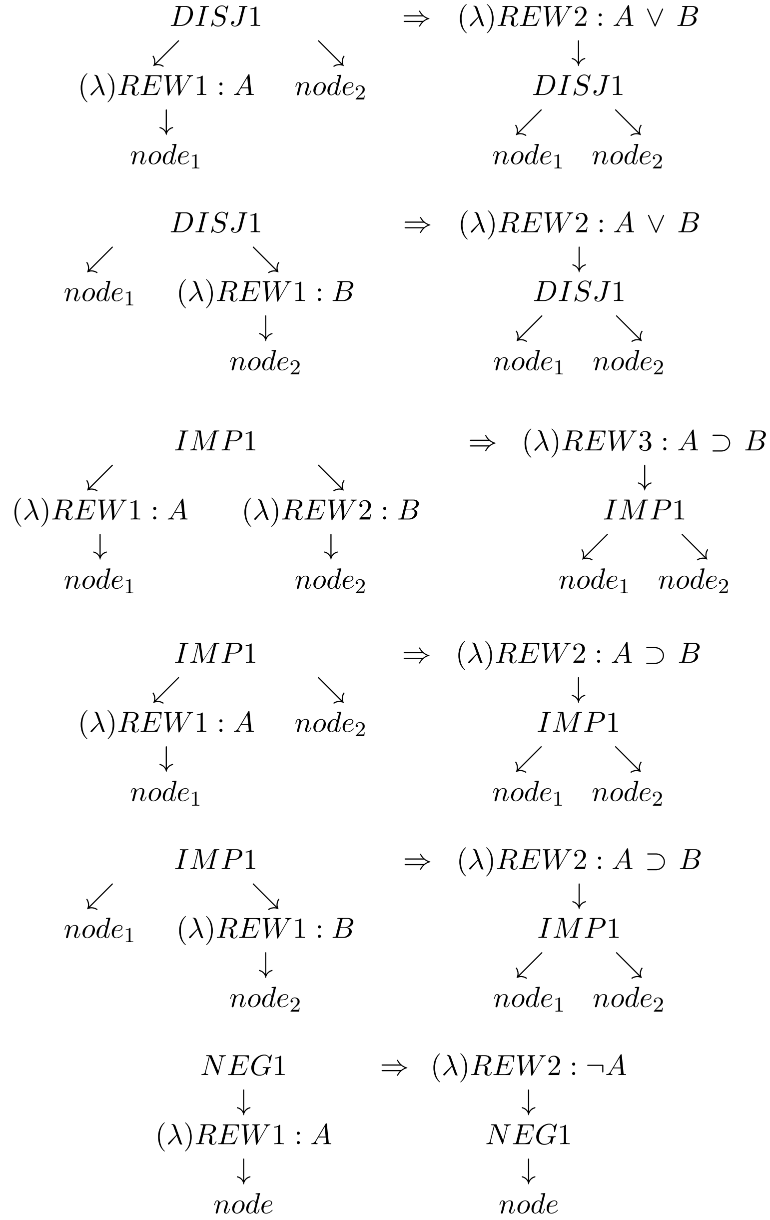
- raise-lambda-nodes-skol (commutes a λ-rewrite with a selection or skolem node)
- raise-lambda-nodes-aux1 (commutes one or two λ-rewrites with a conjunction, disjunction, or implication node)
- raise-lambda-nodes-neg (commutes a λ-rewrite with a negation node)
- raise-lambda-nodes-ab (commutes a λ-rewrite with an AB-rewrite by destructively changing the justifications of the two rewrites and the shallow formula of the lower rewrite)
- raise-lambda-nodes-equiv (commutes a λ-rewrite with an $EQUIV-IMPLICS$-rewrite or $EQUIV-DISJS$-rewrite by destructively changing the two rewrites)

Since λ-rewrite nodes may be destroyed during this process, we may need to change the mating. In fact, we maintain the following invariant.

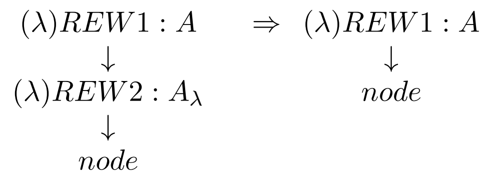
Invariant: Once a tree has been processed, there are no connections to any λ-rewrite nodes in that tree. (Note that it is legal to mate two nodes even if the shallow formulas are only the same up to λ-normal form.)

The function raise-lambda-nodes processes each child of a subtree (unless the node is a *Subst* =-rewrite, which is handled differently), then simply returns the resulting tree and connection list, except in certain cases. These cases correspond to the trees on the left of the following diagrams. We return the tree on the right. (Here, A_λ is the λ-normal form of A.)





The rest of the transformation rules are for two rewrite nodes. Note that these are destructive operations.



$$\begin{array}{ccc} (AB)REW1 : A & \Rightarrow & (\lambda)REW1 : A \\ \downarrow & & \downarrow \\ (\lambda)REW2 : B & & (AB)REW2 : A_\lambda \\ \downarrow & & \downarrow \\ node & & node \end{array}$$

$$\begin{array}{ccc} (Equiv - Implics)REW1 : A & \Rightarrow & (\lambda)REW1 : A \\ \downarrow & & \downarrow \\ (\lambda)REW2 : B & & (Equiv - Implics)REW2 : A_\lambda \\ \downarrow & & \downarrow \\ node & & node \end{array}$$

$$\begin{array}{ccc} (Equiv - Disjs)REW1 : A & \Rightarrow & (\lambda)REW1 : A \\ \downarrow & & \downarrow \\ (\lambda)REW2 : B & & (Equiv - Implics)REW2 : A_\lambda \\ \downarrow & & \downarrow \\ node & & node \end{array}$$

After applying the transformation, if the result is a λ-rewrite node REW, then we move any connection from REW to its child. (Actually, in the code we only do this if the original tree is a rewrite, since in all other cases the top node of the resulting etree could only be a *new* λ-rewrite. This is true because only the rewrite transformation rules are destructive.)

We need to make sure the invariant holds. If we are given a $Subst$ =-rewrite node $REW1$ to process, we start by pushing connections to λ-rewrite nodes below $REW1$ to the child of the λ-rewrite. This forces the invariant to hold (no translation applies to a $Subst$ =-rewrite).

In all other cases, the invariant holds for the children because of the recursive call.

So we have a situation in which there are no connections to λ-rewrite nodes which are subtrees of the node N of interest. Consider the following cases:

(1) Suppose N is not a rewrite. In this case, there are no connections to λ-rewrite nodes in N. Since all the transformations for non-rewrites can only create a new λ-rewrite node, there will be no connections to λ-rewrites in the result (connections can only involve nodes in the tree before the transformation).

(2) Suppose N is a rewrite node. Again, there are no connections to λ-rewrite nodes in N. However, there may be connections to N itself. Since the AB, $Equiv{-}Implics$, and $Equiv{-}Disjs$ transformations are destructive, the node N may become a λ-rewrite (if it was not already). In these cases, we have pushed the connections from N to the child of N. We can see the child of N is not λ-rewrite nodes by examining the transformation rules.

Remark about Subst=: *Subst* =-rewrites are processed further during the CLEANUP-ETREE stage (in the function cleanup-rewrite-node) described in section 8.

8. Cleanup-Etree

The code for CLEANUP-ETREE is in the file *mating-merge-eq.lisp*. This (terribly complicated) procedure comes after merging, because we assume that all exp-vars and skolem-terms have been removed, leaving just ordinary wffs.

First, a general description of the procedure:

(1) At each expansion term, normalize it and reduce superscripts on the bound variables, and make a new expansion which is a "copy",
    (a) remove unnecessary lambda-norm steps.
    (b) make the leaves the same name, so mating still holds
(2) Remove original expansion.

In reality, we just create a whole new expansion tree, not sharing with original tree at all.

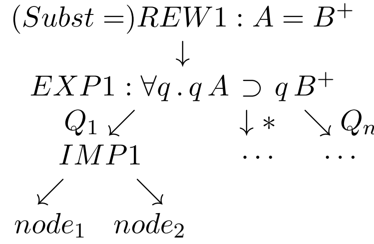
The main functions described below are

(1) cleanup-etree
(2) cleanup-all-expansions
(3) cleanup-expansion
(4) cleanup-rewrite-node

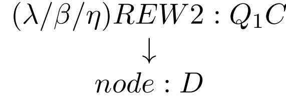
(1) **cleanup-etree** This is the function called by merge-tree-real. It calls cleanup-all-expansions to rebuild the expansion tree cleaning up along the way.
(2) **cleanup-all-expansions** Despite the name, this actually builds a completely new copy of the etree, with special attention paid to expansion and rewrite nodes. The arguments are
    - `etree` the old etree node
    - `shallow` the shallow formula for the new node, which may be only λ-equal to the old shallow
    - `parent` the parent for the new node being created
    - `lambda-normal-p` a boolean indicating if `shallow` is λ-normal

    For each expansion term and corresponding kid, call cleanup-expansion to obtain the new (λ-normal) term and new kid. For rewrite nodes, call cleanup-rewrite-node.
(3) **cleanup-expansion** We λ-normalize the expansion term t to obtain t'. If the new shallow formula is $\forall u \,.\, A$ or $\exists u \,.\, A$, then the new shallow (`newshallow`) for the kid is $[t'/u]A$. If `newshallow` is not λ-normal, then rewrite nodes may need to be included between the expansion node and this kid. Then we recursively call cleanup-all-expansions on the kid with the new shallow formula B.
(4) **cleanup-rewrite-node** There are cases for the different kinds of rewrite nodes.

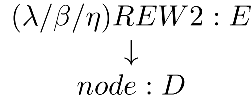
- λ, β, η: We can skip this rewrite if the new shallow formula is already λ-normal. Otherwise, we copy the node, normalize the new shallow formula, and recursively call cleanup-all-expansions on the kid.
- *Subst* =, *Leibniz* =, *Ext* = (and *Both* =, which is not currently fully supported): We copy the node, expand the equality in the new shallow formula and recursively call cleanup-all-expansions on the kid.
  If the node is a positive *Subst* =-rewrite, we process the new tree further. We start with a tree of the form



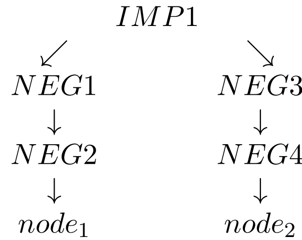
If $node_1$ or $node_2$ is of the form



where D is the λ-normal form (or, β- or η-normal form) of Q_1C, then reduce-rewrites modifies the node to be of the form



(and they both, in fact, should be) where E is $[C/z]F$ and F is the λ-normal form (or, β- or η-normal form) of Q_1z. If in fact, E and D are α-equal, we simply replace the node with $node : D$, removing the rewrite altogether. That is, we change the children of the implications so that the shallow formulas are normalized, until all redexes are in A or B, or are of the form $[A\,M]$ or $[B\,M]$.
Next, if the implication node is of the form



then remove-double-negations-merge deletes the four negation nodes iteratively until the tree does not have this form.
There is also code to make sure $IMP1$ is the son of $EXP1$, but this should already be true because of the function cleanup-leibniz-expansions, described in section 6.
Finally check-shallow-formula (described in section 6) is called in case we need to adjust the children of $IMP1$ to have the appropriate form.

- $Equiv{-}Implics$, $Equiv{-}Disjs$: In these cases, we simply copy the node, rewrite the equivalence in the new shallow formula in the same way as the old, and recursively call cleanup-all-expansions on the kid.
- $Refl =$: If the new shallow formula is of the form $A = A$, we simply copy the node, and recursively call cleanup-all-expansions with new shallow $TRUTH$ on the kid. If the new shallow formula does not have this form, we build a chain of λ, AB, $EQUIVWFFS$ rewrites until it does have the form $A = A$, and end the chain with a true node. (An example where the new shallow formula does not have the form $A = A$ is THM144A with mode MODE-THM144A.)
- $Dual$: The shallow changes from A to $A \vee A$ or $A \wedge A$ depending on the polarity of the node, and make the recursive call.
- $Equivwffs$: In this case, essentially just copy the node, replace the shallow formula with `shallow` as usual, and recursively call cleanup-all-expansions on the kid. The tricky part is computing the new shallow formula for the kid. We used to mimic deepening, in particular, using the value of the flag REWRITE-DEFNS. Now, we find a chain of explicit steps (expanding defns, lambda normalization, etc) from the new shallow to the old shallow of the child of the rewrite.
- $Ruleq$, $Ruleq - Univ$: We again copy the node, replace the shallow formula with `shallow` as usual, and recursively call cleanup-all-expansions on the kid. And again, the tricky part is calculating what the new shallow formula of the kid should be. What it does is let A (`newshallow`) be the gwff in the `RULEQ-SHALLOW` slot of the node. If this slot is empty, it lets A be the min-quant-scope of `shallow` if the justification is $Ruleq$, or just the same `shallow` if the justification is $Ruleq - Univ$. The new shallow of the kid is $Qx_1 \cdots Qx_n \,.\, A$ where Q is $\forall$ if the node is positive and $\exists$ if the node is negative, and the x_i's are the variables introduced in this node. **Possible Bug:** It is not clear why this case is handled the way it is.
- $Truthp$: We copy the node replacing the shallow formula as usual. Then recursively call cleanup-all-expansions on the kid and a shallow formula $A^* \vee \neg TRUTH$ obtained from the old

shallow formula A by replacing occurrences of $FALSEHOOD$ with $\neg TRUTH$ to obtain A^*. **Possible Bug:** One would expect the new shallow formula of the kid to be something like $B^* \vee \neg TRUTH$ where B is the new shallow formula of the rewrite. It's not clear that this causes a problem though.

9. Prettify

The code for PRETTIFY is in the file *mating-merge-eq.lisp*. Merging used to prettify the variables in the etree during the CLEANUP-ETREE phase, but there were some examples where prettify would lead to illegal variable captures. To fix this, the two phases have been separated, and all PRETTIFY does is rename all the free and bound variables in the etree. We go to great lengths to ensure that the renaming does not lead to variable capture.

First, we should work out the theory of such renamings. We would like to find two substitutions θ and α taking variables to variables. The intention is to use θ for free variables and α for bound variables. Let R^θ_α be the renaming function on etrees and wffs. On an etree Q, $R^\theta_\alpha(Q)$ is simply the result of applying R^θ_α to all shallow formulas, expansion terms, and selected variables. Next we define R^θ_α inductively on wffs:

- $R^\theta_\alpha(x) = \alpha(x)$ if x is bound.
- $R^\theta_\alpha(y) = \theta(y)$ if y is free.
- $R^\theta_\alpha(c) = c$ if c is a constant.
- $R^\theta_\alpha(B\,x\,.\,M) = B\,\alpha(x)\,R^\theta_\alpha(M)$ where B is a binder.
- $R^\theta_\alpha([M\,N]) = [R^\theta_\alpha(M)\,R^\theta_\alpha(N)]$.

We would like the renamed wff to be the same as the result of doing some α-conversions and substituting for the free variables. So, with respect to all the wffs M in the etree Q, we need to know there is an M' with $M =_\alpha M'$ such that θ is a legal substitution for M' (avoiding any variable capture), and so that $R^\theta_\alpha(M) \equiv \theta(M')$ (identical wffs).

We can guarantee this if we have θ and α satisfy two conditions with respect to the etree Q.

(*) For distinct variables x and z, and any subwff $[B_1\,x\,.\cdots[B_2\,z\,.\,M]\cdots]$ of any wff in Q where x is free in M, we must have $\alpha(x) \neq \alpha(z)$.

(**) For any subwff $[B_2\,z\,.\,M]$ of any wff N in Q where a free occurrence of y in N is in M, we must have $\alpha(z) \neq \theta(y)$.

In order to prove the result we want, for any collection of variables Γ, define an ordinary substitution ϕ^Γ by

- $\phi^\Gamma(x) = \alpha(x)$ if $x \in \Gamma$
- $\phi^\Gamma(y) = \theta(y)$ if $y \notin \Gamma$

Clearly, ϕ depends on Γ, θ, and α. We will omit the Γ superscript when possible. The idea, of course, is that Γ is the context of bound variables. Note that if Γ is empty, then $\phi^{\{\cdot\}} \equiv \theta$.

Proposition. Suppose Q is an etree and θ and α are variable renamings satisfying conditions (*) and (**) with respect to Q. Let M be any occurrences of a subwff of a wff in Q. Let Γ be the collection of variables bound in the context M occurs. Let ϕ^Γ be defined as above. Then, there is an M' satifying

- $M =_\alpha M'$,
- for each $x \in \Gamma$, $\alpha(x)$ is free for x in M',
- for each $y \notin \Gamma$, $\theta(y)$ is free for y in M' (these conditions together give that ϕ is a legal substitution for M'),
- $R^\theta_\alpha(M) \equiv \phi(M')$.

In particular, if M is a shallow formula, expansion term, or selected variable (so that Γ is empty), we have $R^\theta_\alpha(M) \equiv \theta(M')$ as desired.

Proof. We can prove this by induction on M.

- Suppose M is a variable or constant. Let M' be M.
- Suppose M is $[N\,P]$. By induction we have N' and P' with $N =_\alpha N'$, $P =_\alpha P'$, and satisfying the other conditions. It is easy to see that letting M' be $[N'\,P']$ works.
- Suppose M is $[B\,z\,.\,N]$ for some binder B. By induction on N and $\Gamma \cup \{z\}$, we have an N' satisfying
  - $N =_\alpha N'$
  - for each $x \in \Gamma$, $\alpha(x)$ is free for x in N'
  - $\alpha(z)$ is free for z in N'
  - for each $y \notin \Gamma \cup \{z\}$, $\theta(y)$ is free for y in N'
  - $R^\theta_\alpha(N) \equiv \phi^{\Gamma\cup\{z\}}(N')$

  Let M' be $[B\,\alpha(z)\,.\,[\alpha(z)/z]N']$. Note that $M =_\alpha M'$ since $\alpha(z)$ is free for z in N'.

  Suppose we have $x \in \Gamma$. To check if $\alpha(x)$ is free for x in M', we need to know if x occurs free in N' in the scope of a binder for $\alpha(x)$. By the induction hypothesis, we know such a binder cannot be in N' itself, so the binder would have to be the outermost binder of M', the one for $\alpha(z)$. That is, we must have $\alpha(x) = \alpha(z)$. In such a case, we need to ensure that x does not occur free at all in M'. Now, condition (*) ensures us that M is not of the form $[B\,z\,.N]$ where x (bound in context) occurs free in N. So, x cannot occur free in the α-equivalent M'. So, $\alpha(x)$ is free for x in M'.

  Suppose $z \notin \Gamma$. We need to show $\theta(z)$ is free for z in M'. But this is immediate since z does not occur free in $[B\,\alpha(z)\,.\,[\alpha(z)/z]N']$.

  Suppose we have $y \notin \Gamma$, where y is not z. We need to show $\theta(y)$ is free for y in M'. So, we need to show no free occurence of y in M' occurs withen the scope of a binder for $\theta(y)$. By the induction hypothesis, we know no such binder occurs in N'. So, the binder would have to be the outermost binder of M', the one for $\alpha(z)$. That is, we must have $\alpha(z) = \theta(y)$. In this case, we need to ensure

y does not occur free at all in M'. Equivalently, y should not be free in M. But this is precisely what condition (**) ensures.

Finally, we have

$$R^\theta_\alpha(M) \equiv [B\,\alpha(z)\,.\,R^\theta_\alpha(N)] \equiv [B\,\alpha(z)\,.\,\phi^{\Gamma\cup\{z\}}(N')] \equiv \phi(M').$$

□

Now, the algorithm should build θ and α for Q so that they satisfy (*) and (**). We also want θ to be an injective renaming, so that no two selected variables will be identified. Given partial renamings θ and α satisfying (*) and (**), we need to know if an extension will continue to satisfy the conditions. These tests are implemented by `prettify-free-legal-p` and `prettify-bound-legal-p` and are described below.

The actual PRETTIFY functions are

- **prettify-etree** This is the main function called by **merge-tree-real**. This calls **prettify-process-vars-in-etree** to collect the names of free variables and bound variables in the etree. We distinguish the ones which occur in the topmost shallow formula since these were supplied by the user and should not be renamed. (We also do not rename frees which are introduced by a rewrite. This is a bit unusual, but can happen. An example is a rewrite instantiating the abbreviation `PLUS` which introduces the free `S`. Here we are really thinking of `S` as being part of the signature, but there's nothing to explicitly indicate that `S` is not a variable.) We start off by sending each such free to itself and each such bound to itself. (It is easy to see that conditions (*) and (**) are satisfied by any partial identities for θ and α.) Then we extend θ and α in stages. First, whenever y is a selected variable corresponding to a bound variable x in the original wff, we let $\theta(y) = x$ if this is legal. Next, whenever y is a selected variable associated with a bound variable x, we try to send both of these to the same "pretty" (no superscript) variable, if this is possible. (The function **prettify-identify-free-bound** is used to try to send a free and a bound to the same pretty variable.) Third, we check for bound variables z_i which occur in a subwff of the form

$$[[\lambda z_1 \cdots \lambda z_n\,.\,M]\,A_1\,\cdots\,A_n]$$

where A_i is a free or bound variable (such A_i are stored in the properties `bound-try-to-equate-free` and `bound-try-to-equate-bound`) . In this case, we try to send A_i and z_i to the same pretty variable, if possible (see the functions **prettify-identify-free-bound** and **prettify-identify-bound-bound**). Finally, we choose the rest of θ and the rest of α using the functions **get-best-alt-free-name** and **get-best-alt-bound-name**. This completes the computation of θ and α, so we call **rename-all-vars-in-etree** to actually do the

renaming. Finally, **remove-unnecessary-ab-rews** eliminates α-rewrite nodes where the shallow does not change (this may happen since we renamed variables).

- **prettify-process-vars-in-etree** This collects the frees and bounds in the etree into the variables `fixed-frees`, `fixed-bounds`, `frees-to-rename`, and `bounds-to-rename`. Each free has the property `free-must-avoid` which is eventually set to all bound vars z such that y occurs free in the scope of a binder for z. Each bound variable has a similar property `bound-must-avoid`. (Note that we need to use two different names for the properties since a variable may occur both free and bound in the etree.) If a free variable y is a selected variable in the etree, it has the property `sel-var-bound` which is set to the bound variable corresponding to the outermost binder at the selection node for y. Bound variables z also have properties `bound-try-to-equate-bound` and `bound-try-to-equate-to-free`. A variable x will be on one of these lists if there is a subwff of the form

$$[[\lambda z_1 \cdots \lambda z_n \,.\, M]\, A_1 \,\cdots\, A_n]$$

where z is z_i and x is A_i for some i. In such cases, we will try to send these variables to the same (pretty) renamed variable.
- **prettify-free-rename** This extends θ to include $\theta(y) = y'$. We also must propagate information about this commitment by including y' in the list `used-frees`, representing the codomain of θ and by including y' in the property `not-alpha-image` for any b in the property `free-must-avoid` for y. We will use this to ensure no such b will later have $\alpha(b) = y'$.
- **prettify-bound-rename** This extends α to include $\alpha(z) = z'$. Again, we propagate information by including z' in the property `not-alpha-image` for any b in the property `bound-must-avoid` for z. We will use this to ensure no such b will later have $\alpha(b) = z'$.
- **prettify-free-legal-p** This checks if it is legal to extend θ to include $\theta(y) = y'$. First, we check to make sure $\theta(y)$ is not already defined and that y' is not in the codomain of θ (`used-frees`), since we want θ to be injective. Next, to ensure condition (**) will hold, we make sure there is no bound z with $\alpha(z) = y'$ and z in the list `free-must-avoid` for y.
- **prettify-bound-legal-p** This checks if it is legal to extend α to include $\alpha(z) = z'$. First, we check to make sure $\alpha(z)$ is not already defined. Of course, we do not mind if many bound variables are mapped to the same variable, because this is often how we make the proof pretty, so we do not need to check the codomain of α. We must make sure that z' is not on the list in the property `not-alpha-image` for z. If it is, then there is some free or bound x which is sent to z' and occurs free in some subwff where it would be captured

by a binder $B\,z$ if this binder were renamed to $B\,z'$. Also, we must make sure there is no bound b on the list in the property `bound-must-avoid` for z such that $\alpha(b) = z'$. In such a case, there would be an occurrence of z which would be captured by a binder for b upon renaming. These checks ensure condition (*) will hold.

- **prettify-free-bound-legal-p** Checks if we can send both y and z to a variable v. This involves a bit more checking than just checking that both commitments are independently legal. Comments in the code explain the check.
- **prettify-bound-bound-legal-p** Checks if we can send both x and z to a variable v. This involves a bit more checking than just checking that both commitments are independently legal. Comments in the code explain the check.
- **prettify-identify-free-bound** Given a free y and bound z, if both are already committed, do nothing. If one is committed to a pretty variable and the other is not committed, send the other to the same pretty variable, if this is legal. If neither are committed, compute alternative names for each. If either have a pretty alternative v which is legal for the other, send both to this v.
- **prettify-identify-bound-bound** Similar to **prettify-identify-free-bound** except with a bound x and another bound z.
- **pretty-var-p** Returns T if the var does not have a superscript.
- **get-best-alt-name** Given a variable and a legality test, finds a new legal variable to replace it. If the old variable is w, w^n, h, or h^n, then the new variable will be given a name based on whether the type is of a proposition, predicate, relation, function, or individual. (This depends on the values of the globals `proposition-var` `predicate-var` `relation-var` `function-var` `individual-var`. The values of some of these globals were being *randomly changed* by a call to randomvars at the beginning of **merge-tree-real**. This may make prettify bugs difficult to reproduce. We have decided to comment out this call to randomvars.) If the old variable is anything else, say x or x^n, then the new variable will be of the form x or x^m.
- **get-best-alt-free-name** Returns the nicest legal alternative for a free y, using **get-best-alt-name** and **prettify-free-legal-p**.
- **get-best-alt-bound-name** Returns the nicest legal alternative for a bound z, using **get-best-alt-name** and **prettify-bound-legal-p**.
- **rename-all-vars-in-etree** This corresponds to R^{θ}_{α} on etrees.
- **scope-problem-p** Checks to make sure the new var and old var are either both free in context, or were both bound by the same binder. If not, return T. This will cause `rename-all-vars-in-wff` to throw a failure, indicating a bug in PRETTIFY.
- **rename-all-vars-in-wff** This corresponds to R^{θ}_{α} on wffs described above. We do check to make sure we are avoiding variable capture.

If a variable capture does occur, there is a bug in PRETTIFY and a failure is thrown.

10. Merging Extensional Expansion Proofs

The code for merging extensional expansion proofs is completely different than the corresponding code for merging expansion proofs. Essentially we translate from an open dag (ext-exp-open-dag) to a ground dag (ext-exp-dag) via the function eeod-to-eed-node (see ext-exp-open-dags.lisp). This translation process deletes any unnecessary parts of the extensional expansion proof.

Prettify for extensional expansion proofs is performed by ext-exp-dag-prettify in ext-exp-dags.lisp. The code is similar to the prettify code in mating-merge-eq.lisp.

CHAPTER 15

Unification

The relevant files are: *ms90-3-node.lisp*, *ms90-3-unif*.lisp*, *node.lisp*, *unif*.lisp*

TPS3 has four unification algorithms, two for first-order logic and two for the full type theory. Here we are mainly concerned with the two type theory ones, which differ as follows:

- UN88 is called by those procedures which do not use path-focused duplication, and by the TPS3 UNIFY top level. Each variable is a symbol. We use lazy reduction. Head normal form. General implementation. Can use different strategies for searching the unification tree. Default breadth-first. Requires storing almost the entire tree. When called from mating-search, we search for a success node or generate the tree to a pre-determined maximum depth.
- UN90 is called by those procedures which do use path-focused duplication. No interactive interface exists now. Each variable has the form (symbol . number) Terms are reduced to λ-normal form as in Huet's paper. Depth-first search. Stores only non-failure leaf node. Does not store the entire unification tree. When called from mating-search, we search for the first non-failure node within the pre-determined maximum depth. Search for a success node only when the mating is complete. Major drawback: Needs modification to implement subsumption.

1. Data Structures

2. Computing Head Normal Form

3. Control Structure

4. First-Order Unification

5. Subsumption Checking

There is a subsumption checker for UN88 which uses the slot `subsumed` in each node of the unification tree; this is implemented in the file *unif-subs.lisp*. The subsumption-checker is passed the new node and a list of other nodes which might subsume it. If SUBSUMPTION-CHECK is NIL, it returns immediately. Otherwise, it first checks the flags SUBSUMPTION-NODES and SUBSUMPTION-DEPTH and eliminates all nodes from the list that do not fit the criteria established by these two flags (so it might, for example,

pick out just those nodes at a depth of less than ten which lie either on the path to the new node or at the leaves of the current unification tree). Since it is possible to add new disagreement pairs to the leaves of the tree under some conditions, it also rejects any nodes that do not represent the same original set of disagreement pairs as the new node.

Then it computes a hash function, somewhat similar to Goedel-numbering, by considering each wff in the set of disagreement pairs at a node. The hash function has to ignore variables, because we want to catch nodes that are the same up to a change in the h-variables that have been introduced. These hash numbers are calculated once and then stored in the `subsumed` slot in the following format: for a dpairset

```
((A1 . B1) (A2 . B2) ...)
```

we first calculate the hash numbers for each wff, and generate the following list:

```
(((#A1 . #B1) (A1 . B1)) ((#A2 . #B2) (A2 . B2)) ...)
```

Then, for each disagreement pair, if #Bi < #Ai we replace it with ((#Bi . #Ai) (Bi . Ai)). Finally, we sort the list lexicographically by the pairs of hash numbers and store it in the `subsumed` slot. In future, if we return to this node, we can just read off the hash function without recalculating it.

Now TPS3 compares the dotted pairs of numbers from the hash functions of the new and old node. If those for the new node are equal to, or a superset of, those for the old node, then we need to do some more detailed checking. This is the point at which TPS3 prints a "?", if UNIFY-VERBOSE is not SILENT. Otherwise we know there is no subsumption and proceed to the next node.

If there is still a possibility of subsumption, the next thing to do is to enumerate all the ways in which the old node might be considered a subset of the new one. If we are lucky, each dotted pair of numbers in a given node will be different from each other and from all other dotted pairs at that node, and there will only be one way in which this could happen. If we aren't so lucky (if there are several disagreement pairs that get the same pair of hash numbers, or if there is a disagreement pair where the hash numbers for both wffs are the same), there may be multiple ways to think about. For each possible way, we output two disagreement pair lists, which will be the entire old node and that subset of the new node to which it might correspond, ordered so that the nth element of one is supposed to compare to the nth element of the other, for all n.

Next, for each one of these possible ways, we take the two disagreement pair sets given, and begin to rename the h-variables in them. We start at the left of both sets, and build up a substitution as we move rightwards, comparing each term to the other symbol-by-symbol. (Note that we are only replacing variables with other variables.) If we reach the end of the term without contradicting ourselves, we output a "!" and the new node is subsumed. If we fail (because the substitution is inconsistent, or because

we reach two different variables neither of which is an h-variable), we fail immediately and go on to the next arrangement, if there is one.

Subsumption-checking can be very slow; set the flag SUBSUMPTION-DEPTH with care. Because of this, it was necessary to add time-checking to unification (it was previously only done between considering connections). The functions `unify`, `unify-ho-rec` and `subsumption-check` now check the time if they are called from within a procedure that uses time limits (and in order to implement this, many other unification functions have been given optional "start-time" and "time-limit" arguments that they do nothing with except passing them on to the next function).

6. Notes

The code that TPS3 uses to handle double-negations is part of the unification code. See imitation-eta in the file unif-match.lisp

CHAPTER 16

Set Variables

Inductively, a set type is either the type of propositions o or a function type $\alpha\beta$ where α is a set type and β is any type. A term of a set type represents either a proposition, a set, or a relation.

When we refer to set variables, we generally mean an expansion variable of a set type. To find an expansion proof with expansion variables we may need to instantiate these variables. Propositional variables are relatively easy to instantiate, since there are only two possible truth values, $\top$ and $\bot$. It is far more difficult to instantiate set variables which are not of propositional type. Semantically, these set types may correspond to infinite domains. Syntactically, we have logical constants and quantifiers which can be used to create terms of set types. In some cases, but certainly not in all cases, the instantiations can be find using higher-order unification. For instantiations which require logical constants or quantifiers, the original method used by TPS3 is that of PRIMSUBS (see section 1).

For references on PRIMSUBS, see [And89], [ABI+96], and [And01]. Some work regarding instantiating set variables in other contexts include [Ble77], [Ble79], [Ble83], [BF93], and [BBP93]. The SCAN algorithm (see) reduces some second-order formulas to equivalent first-order formulas, avoiding the need to instantiate the set variables.

1. Primitive Substitutions

Set variables can be instantiated in a pre-processing stage using Primitive Substitutions. This depends on the value of several flags. There is a subject PRIMSUBS. The command `LIST PRIMSUBS` in TPS3 will list the relevant flags. Some of the main flags that determine if and how primitive substitutions are generated are DEFAULT-MS, DEFAULT-EXPAND, and PRIMSUB-METHOD.

Some examples, in principle, might require applying primsubs beneath primsubs. An example discussed in [ABB00] is the injective version of Cantor's Theorem, **X5309**.

2. Using Unification to Compute Setsubs

If DEFAULT-MS is set to MS98-1, MS98-INIT is set to 2 or 3, and PRIMSUB-METHOD is set to PR00, then unification is used to compute instantiations during pre-processing.

3. Set Constraints

An alternative to instantiating set variables in a pre-processing step is to intertwine instantiating set variables with the mating search.

Let v be a set variable occurring at the head of some literal.

We write constraints in sequent form. A constraint is, in practice, a list of positive and negative literals (or expansion tree nodes). Usually, positive literals are written on the left of the sequent and negative literals are written on the right. In some cases, we write a positive literal on the right or a negative literal on the left and interpret it as the negation of the literal. A sequent corresponds to a subset of a vertical path on a jform.

Minimal constraints for v are a collection of sequents (these correspond to subsets of vertical paths in the jform) of the form

$$\Psi|\Gamma(v) \rightarrow [v\,\overline{t}]$$

where $\Gamma(v)$ is a collection of literals are not negative literals with v at the head, and Ψ is a list of selection variables which occur in the sequent and are banned from occurring in the instantiation for v (see section 4). In general, v can occur inside the body of the literals in Γ, and this case will be discussed below. We do not allow v to occur in the argument terms $\overline{t}$.

Maximal constraints can be defined and handled in a dual way. We concentrate on minimal constraints for the present.

It is very easy to see that any collection of minimal constraints can be simultaneously solved, since $v = \lambda\overline{z}\top$ is a solution. In interesting cases, this instantiation will fail other conditions we might need v to satisfy. What we would prefer to have is an "optimal" solution to the constraints. In the case of minimal constraints, "optimal" means a minimal solution.

First, consider the case of a single minimal constraint of the form

$$\Psi|\Gamma \rightarrow [v\,\overline{z}].$$

where the arguments $\overline{z}$ are distinct variables that occur in Ψ. Also, assume that there are no other variables in Ψ. In this case, we can directly define the minimal solution as an intersection:

$$\lambda\overline{z}\bigwedge\Gamma.$$

The notation $\bigwedge(\Gamma)$ means $A_1 \wedge \cdots \wedge A_n$ where each A_j is either a positive literal in Γ or A_j is $\neg B_j$ where B_j is a negative literal in Γ. The case where $\Psi = \overline{z}, \overline{w}$ for some extra selected variables $\overline{w}$ is only slightly more complicated. We can directly define the minimal solution in this case as

$$\lambda\overline{z}\exists\overline{w}\bigwedge\Gamma.$$

This is a legal instantiation for v since all the variables in Ψ are bound.

Next, consider the more general case in which the minimal constraint is of the form

$$\Psi|\Gamma \rightarrow [v\,\overline{t}]$$

where the arguments $\overline{t}$ need not be distinct variables from Ψ. Let n be the length of $\overline{t}$. In this case, the easiest way to directly define the minimal solution is

$$\lambda x^1 \cdots \lambda x^n \exists \overline{w}. x^1 = t^1 \wedge \cdots \wedge x^n = t^n \wedge \bigwedge(\Gamma)$$

for new variables x^j of the same type as t^j, and $\overline{w} = \Psi$. However, in practice it is easier if we distinguish between arguments t^j which are actually variables in Ψ and those which are not. So, let us write $\Psi = \overline{z}, \overline{w}$ where each $z \in \overline{t}$. For each $j = 1, \ldots, n$, if $t^j \in \overline{z}$ and $t^k \neq t^j$ for $k < j$, then let $x^j = t^j$. Otherwise, let x^j be a new variable of the same type as t^j. Let $Eqns = \{x^j = t^j | t^j \text{ is not the variable } x^j\}$. In this case, we can directly define the minimal solution as

$$\lambda x^1 \cdots \lambda x^n \exists \overline{w}. \bigwedge(Eqns) \wedge \bigwedge(\Gamma).$$

Clearly, if $\overline{t}$ actually is the list $\overline{z}$, then this solution is

$$\lambda \overline{z} \exists \overline{w}. \bigwedge(\Gamma)$$

as before.

Now, consider the case in which there are several minimal constraints of the form

$$\Psi_i | \Gamma_i \to [v\, \overline{t^i}]$$

for $i = 1, \ldots, n$. Again, we assume v does not occur in Γ_i, so we can directly define the minimal solution. First, let M_i be the minimal solution for each of the constraints individually defined as above:

$$\lambda \overline{x_i} \exists \overline{w_i}. \bigwedge(Eqns_i) \wedge \bigwedge(\Gamma_i).$$

Then we take the union of these

$$\lambda \overline{x} \bigvee_i [M_i \overline{x}]$$

to get the minimal solution for the combined constraints.

Rather than diving into the general case in which v may occur in Γ, first consider a familiar example in which we have the two constraints

$$\to [v\, 0]$$

and

$$w | [v\, w] \to [v\, [S\, w]].$$

The minimal solution to this is the least set containing 0 and closed under the function S. Since we have the full power of higher-order logic, we can define such a solution by

$$\lambda x \forall p\, .\, [[p\, 0] \wedge [\forall z\, .[v\, z] \supset [v\, [S\, z]]]] \supset px.$$

It should be clear at this point that the terms defining these solutions can become quite large. Making such instantiations can be prohibitive in theorem proving, because we may need to perform unification with the solution in some other part of the problem. This suggests it may be simpler to use the

instantiation to prove there is a set satisfying the conditions we want. Then using this lemma, the set that exists is represented by a selected variable (a *very* small term). Another motivation for using such lemmas is that we gain more control over what properties of the sets are included in the lemma. See section 11 for more on the implementation of expansion proofs using lemmas.

Suppose we have a general set of minimal constraints

$$\Psi_i|\Gamma_i(v) \to [v\,\overline{t_i}].$$

Consider what properties of v an existence lemma should include. The most obvious is the condition $C_i(v)$:

$$\forall \overline{w_i}.\Gamma_i(v) \to [v\,\overline{t_i}]$$

where $\Psi_i = \overline{w_i}$. Of course, as mentioned above, $C_i(\lambda \overline{x} \top)$, so we certainly need to include more conditions. Two other conditions are *inversion principles* and *inductive principles*. An inversion principle for the set of constraints would be of the form

$$\forall \overline{x}.v\overline{x} \supset D(v, \overline{x})$$

for some formula D. In words, this principle says that any element $\overline{x}$ of v must be of a form satisfying $D(v, \overline{x})$. An induction principle is a statement that the solution really is minimal. This would be a higher-order statement of the form

$$\forall \overline{p}. \bigwedge_i (C'_i(p)) \supset v \subseteq p$$

where C'_i is a condition similar to C_i defined above. The general case below will explain the difference between C'_i and C_i.

Consider the minimal constraints

$$y|[A\,y] \to [v\,y]$$

and

$$z|[B\,z] \to [v\,z].$$

Here $C_1(u)$ is

$$\forall y\,.\,[A\,y] \supset [v\,y]$$

and $C_2(u)$ is

$$\forall z\,.\,[B\,z] \supset [v\,z].$$

The inversion principle here would be

$$\forall x\,.[v\,x] \supset\,.\,[A\,x] \vee [B\,x].$$

The induction principle here would be

$$\forall p\,.C_1(p) \wedge C_2(p) \supset v \subseteq p.$$

In such cases where none of the minimal constraints are of the form

$$\Psi|\Gamma(v) \to [v\,\overline{t}]$$

where v *does* occur in Γ, the set is fully determined by the conjunction of the C_i constraints and the inversion principle, since these amount to an extensional definition of v. In the general case, the induction principle determines

the set. Though the inversion principle follows from the induction principle, it may be helpful to have the inversion principle when we are trying to use the lemma to prove the theorem. The flag INCLUDE-INDUCTION-PRINCIPLE controls whether the induction principle is included in set existence lemmas.

Let us return to the familiar example of the constraints

$$\to [v\,0]$$

and

$$w|[v\,w] \to [v\,[S\,w]].$$

In this case, $C_1(v)$ is $[v\,0]$, and $C_2(v)$ is $\forall w\,.\,[v\,w] \supset [v\,[S\,w]]$. The inversion principle is the familiar statement

$$\forall x\,.\,[v\,x] \supset [[x = 0] \vee \exists w\,.\,[x = [S\,w]] \wedge [v\,w]].$$

The induction principle is the equally familiar statement

$$\forall p\,.\,[[p\,0] \wedge [\forall w\,.\,[p\,w] \supset [p\,[S\,w]]]] \supset \forall w\,.\,[v\,w] \supset [p\,w].$$

The inversion principle follows from the induction principle by instantiating p with

$$\lambda x\,.\,[[x = 0] \vee \exists w\,.\,[x = [S\,w]] \wedge [v\,w]],$$

but it is clearly easier, if we need the inversion principle, to have it in the lemma rather than needing to prove it by instantiating the new set variable p.

In the general case in which some constraints contain a $\Gamma(v)$ with v free in $\Gamma(v)$, we need machinery to show that there is a solution v satisfying the constraints, the inversion principle, and the induction principle. This machinery is provided by the Knaster-Tarski Fixed Point Theorem.

3.1. Knaster-Tarski Fixed Point Theorem. The Knaster-Tarski Fixed Point Theorem states that monotone set functions have fixed points. There are also versions showing there are least and greatest fixed points (in fact, there are a lattice of such fixed points).

Definitions: Suppose $K : \wp(A) \to \wp(A)$ for a power set $\wp(A)$. A *pre-fixed point* of K is a set v such that $K(v) \subseteq v$. A *post-fixed point* of K is a set v such that $v \subseteq K(v)$. A *fixed point* of K is a set v satisfying $K(v) = v$.

Knaster-Tarski Fixed Point Theorem: Suppose $K : \wp(A) \to \wp(A)$ for a power set $\wp(A)$. Further suppose K is monotone function in the sense that for every $v \subseteq w \subseteq A$, $K(v) \subseteq K(w)$. Then there is a fixed point u of K.

Proof: Let $u = \bigcap\{v \in \wp(A) | K(v) \subseteq v\}$. That is, we define u to be the intersection of all the pre-fixed points of K. We need to show $K(u) \subseteq u$ and $u \subseteq K(u)$.

First, we show $K(u) \subseteq u$. Suppose $z \in K(u)$. To show $z \in u$, we need to show $z \in v$ for every pre-fixed point v. Let v be a pre-fixed point. By the definition of u, we have $u \subseteq v$. Since K is monotone, $K(u) \subseteq K(v)$, so

$z \in K(v)$. But v is a pre-fixed point, so $z \in K(v) \subseteq v$. Thus, u is itself a pre-fixed point. (In fact, it is clearly the least pre-fixed point.)

Since u is a pre-fixed point and K is monotone, we have $K(K(u)) \subseteq K(u)$. So, $K(u)$ is a pre-fixed point. Since u is the least pre-fixed point, we have $u \subseteq K(u)$. □

This proof actually shows the following form of the theorem:

Knaster-Tarski Fixed Point Theorem (Least): Suppose $K : \wp(A) \rightarrow \wp(A)$ for a power set $\wp(A)$. Further suppose K is monotone function in the sense that for every $v \subseteq w \subseteq A$, $K(v) \subseteq K(w)$. There is a least pre-fixed point u of K which is also a fixed point of K.

A dual proof shows

Knaster-Tarski Fixed Point Theorem (Greatest): Suppose $K : \wp(A) \rightarrow \wp(A)$ for a power set $\wp(A)$. Further suppose K is monotone function in the sense that for every $v \subseteq w \subseteq A$, $K(v) \subseteq K(w)$. There is a greatest post-fixed point u of K which is also a fixed point of K.

These statements and proofs have a straightforward representation in type theory. The same proof idea works regardless of the arity of the set type of u. The functions make-knaster-tarski-lemma, make-knaster-tarski-negf, make-knaster-tarski-leastfp-lemma, make-knaster-tarski-leastfp-negf, make-knaster-tarski-gfp-lemma, make-knaster-tarski-gfp-negf, and others with similar names, construct (an ftree representation of) an expansion proof of these different versions of the Knaster-Tarski Theorem for a given set type. A proof of the Knaster-Tarski Theorem for the set type of u can be used as a lemma to show a set existence lemma for u which includes the constraint properties, the inversion principle, and the induction principle.

THM2 in TPS3 is a version of the Knaster-Tarksi Theorem.

When we apply the Knaster-Tarski theorem, we need to instantiate K with a set function we know is monotone. This can be ensured syntactically.

Definition: We can define a set of terms *positive* and *negative* with respect to u by induction.

- $[u\,\overline{t}]$ is positive with respect to u, so long as u does not occur free in $\overline{t}$.
- $P_1 \vee P_2$ is positive (negative) with respect to u if both P_1 and P_2 are.
- $P_1 \wedge P_2$ is positive (negative) with respect to u if both P_1 and P_2 are.
- $P_1 \supset P_2$ is positive (negative) with respect to u if P_1 is negative (positive) with respect to u and P_2 is positive (negative) with respect to u.
- $\neg P$ is positive (negative) with respect to u if P is negative (positive) with respect to u.
- $\forall x P$ is positive (negative) with respect to u if P is, or if x is the same variable as u.

- $\exists x P$ is positive (negative) with respect to u if P is, or if x is the same variable as u.

Proposition: Suppose P is positive with respect to u. Then $\lambda u \lambda \overline{z} P(u, \overline{z})$ represents a monotone function of u.

Proof: We can prove this by induction on P. $\square$

The function mon-fn-negf generates an expansion proof that a particular $\lambda u \lambda \overline{z} P(u, \overline{z})$ is monotone where P is positive with respect to u. This function returns an ftree and pushes new connections on the special variable clist.

We will solve some constraints by building a monotone function K from the constraints. Some constraints do not directly give a monotone function of the set variable. In these cases, we may want to find a best approximating.

Definition: Given a function $F : \wp(A) \rightarrow \wp(A)$, let $K_F : \wp(A) \rightarrow \wp(A)$ and $K^F : \wp(A) \rightarrow \wp(A)$ be defined by

$$K_F(u) = \{z | \forall w \, [u \subseteq w \supset z \in F(w)]\}$$

and

$$K^F(u) = \{z | \exists w \, [w \subseteq u \wedge z \in F(w)]\}.$$

Proposition: K_F and K^F are monotone set functions. For all u, $K_F(u) \subseteq F(u) \subseteq K^F(u)$. Futhermore, if $L : \wp(A) \rightarrow \wp(A)$ is a montone set function and forall u, $L(u) \subseteq F(u)$, then forall u, $L(u) \subseteq K_F(u)$. Similarly, if $R : \wp(A) \rightarrow \wp(A)$ is a montone set function and forall u, $F(u) \subseteq R(u)$, then forall u, $K^F(u) \subseteq R(u)$. So, K_F and K^F are the best monotone upper and lower aproximations of F.

Proof: Suppose $u \subseteq v$ and $z \in K_F(u)$. We need to show that $z \in F(w)$ for every $w \supseteq v$. Given such a w, apply the definition of K_F to the set w to obtain $[u \subseteq w] \supset z \in F(w)$. Since $u \subseteq v \subseteq w$, we have $z \in F(v)$. So, K_F is monotone. Similarly, suppose $u \subseteq v$ and $z \in K^F(u)$. Apply the definition of $K^F(u)$ to obtain a set $w \subseteq u$ with $z \in w$. Now, since $w \subseteq u \subseteq v$, this w can be used to witness that $z \in K^F(v)$. So, K^F is monotone.

Let $u \in \wp(A)$ be given. We need to show $K_F(u) \subseteq F(u)$ and $F(u) \subseteq K^F(u)$. Suppose $z \in K_F(u)$. Apply the definition of K_F to u. Since $u \subseteq u$, we have $z \in F(u)$. Next, suppose $z \in F(u)$. Then u can witness that $z \in K^F(u)$.

Now suppose L is a monotone function such that for every $u \in \wp(A)$, $L(u) \subseteq F(u)$. We need to show for every $u \in \wp(A)$, $L(u) \subseteq K_F(u)$. Let $u \in \wp(A)$ be given and suppose $z \in L(u)$. Since L is monotone, for every $w \supseteq u$, $z \in L(w)$. So, $z \in F(w)$. This shows $z \in K_F(u)$ and we are done.

Now suppose R is a monotone function such that for every $u \in \wp(A)$, $F(u) \subseteq R(u)$. We need to show for every $u \in \wp(A)$, $K^F(u) \subseteq R(u)$. Let $u \in \wp(A)$ be given and suppose $z \in K^F(u)$. So, there is a $w \subseteq u$ with $z \in F(w)$. So, $z \in R(w)$. Since R is monotone and $w \subseteq u$, we have $z \in R(u)$ and we are done. $\square$

3.2. Tracing Through An Example. Consider again the two minimal constraints

$$\rightarrow [v\,0]$$

and

$$w|[v\,w] \rightarrow [v\,[S\,w]]$$

where v has type $o\iota$. Let us name the literals

L1 $v\,0$
L2 $v\,w$
L3 $v\,[S\,w]$

In the function ftree-solve-constraint-set, consider some of the local variables:

- `f` is an ftree containing the nodes L1, L2, and L3, and the expansion variable v.
- `constrs` has the value ((L1) (L3 L2))
- `v` has the value v.
- `vsel` is a new variable v^1 of the same type as v.
- `f3` is `f` with v^1 (not an expansion variable) substituted for the expansion variable v.
- `banned-occurs` ends up having the value ((w) NIL), representing Ψ_1 and Ψ_2.
- `misc-occurs` is NIL, assuming S and 0 are constants.
- `paths`: ((LF3 LF2) (LF1)) where each LFi is the ftree node in `f3` corresponding to Li.

Then make-ftree-setvar-soln is called with

- `vsel`: v^1
- `kind`: MIN
- `paths`: ((LF3 LF2) (LF1))
- `banned-occurs`: ((w) NIL)
- `misc-occurs`: NIL
- `rec-flag`: T

A special dynamic variable `clist` is used to collect connections created in the process of building the lemma. The function make-min-inv-princ constructs the inversion principle

$$\forall\, x .\, v^1\, x \supset \exists\, w[\, x \,=\, S\, w \wedge\, v^1\, w] \vee\, x \,=\, 0.$$

Next, make-min-setvar-lemma-posf constructs the full lemma

$$\begin{aligned} &\exists\, v^2 .[\forall\, w^1[\, v^2\, w^1 \supset\, v^2 .\, S\, w^1] \wedge\, v^2\, 0] \\ \wedge\, \forall\, x^1[\, &v^2\, x^1 \supset \exists\, w[\, x^1 \,=\, S\, w \wedge\, v^2\, w] \vee\, x^1 \,=\, 0] \\ \wedge \forall p\, .\, [\forall\, w^1[\, p\, w^1 \supset\, &p .\, S\, w^1] \wedge\, p\, 0] \supset \forall x^2 \,.\, [v^1\, x^2] \supset\, [p\, x^2]. \end{aligned}$$

and the positive ftree that will correspond to how the lemma can be used. In particular, connections to the nodes LF1, LF2, and LF3 are created which solve the constraints (i.e., block every verticle path through LF1, and every vertical path through LF2 and LF3).

In simpler examples which do not require a recursive definition using the Knaster-Tarski Fixed Point Theorem, the function make-min-setvar-lemma-negf constructs an ftree proof of the lemma. In this case, the lemma does require a recursive definition, so the function make-clos-setvar-lemma-negf is called. If INCLUDE-INDUCTION-PRINCIPLE is T, we need a strong form of the Knaster-Tarski Theorem, in which we know the set u is the least pre-fixed point. The function make-knaster-tarski-leastfp-lemma is called to construct an ftree proof for this lemma. Otherwise, make-knaster-tarski-lemma gives an ftree proof of the simpler version. The function make-clos-setvar-lemma-negf-0 uses the Knaster-Tarski lemma to prove the set existence lemma.

Now, let us examine each of these steps in more detail.

(1) **make-min-inv-princ** Given v^1, the paths (LF1 (LF3 LF2)), and the banned variable information ((w) NIL), use the type of v^1 to create a formula

$$\forall x. [v^1 x] \supset InvP_2$$

where $InvP_2$ is formed by make-min-inv-princ-2. $InvP_2$ needs to be positive with respect to v^1 so that we can apply the Knaster-Tarski Theorem later.

- **make-min-inv-princ-2** Given the bound list (`zl`) (x), the atomic formula (`vzl`) vx, as well as the information above, form a disjunction of inversion principles corresponding to each path:

$$InvP_4^{(LF3\,LF2)} \vee InvP_4^{(LF1)}$$

  where each $InvP_4$ formula (positive with respect to v^1) is constructed by a call to make-min-inv-princ-3.
- **make-min-inv-princ-3** Given one of the constraints, construct equations and a substitution sending some banned variables (those occurring as an argument of the main literal of the constraint) to the variables constructed by make-min-inv-princ (in this case, x). In this case we have two constraints.
  First consider, (LF1), i.e., $v^1\,0$. In this case we only have one literal, the main literal of the constraint. Since 0 is a constant, and not a banned selected variable, we make an equation $x = 0$ between the argument of $v^1\,x$ and the argument of the main constraint literal $v^1\,0$. In general, there may be several arguments, giving several equations. Given these equations, make-min-inv-princ-4 constructs the formula $InvP_4^{(LF1)}$.
  Second consider, (LF3 LF2), i.e., $w|v^1\,w \rightarrow v^1\,[S\,w]$. In this case, the main literal is $v^1\,[S\,w]$. If the argument $[S\,w]$ were w, we could replace w by x in the constraint for the purpose of computing the inversion principle. Instead, we make an

equation $x = [S\,w]$ and call make-min-inv-princ-4 to construct the formula $InvP_4^{(LF3\,LF2)}$.

- **make-min-inv-princ-4** Constructs a formula existentially binding the remaining banned variables in the constraint. Since (LF1) has no banned variables, $InvP_4^{(LF1)}$ is $InvP_5^{(LF1)}$ constructed by make-min-inv-princ-5. Since (LF3 LF2) has the banned variable w, $InvP_4^{(LF3\,LF2)}$ is of the form

  $$\exists w\,.\,InvP_5^{(LF3\,LF2)}$$

  where $InvP_5^{(LF3\,LF2)}$ is constructed by make-min-inv-princ-5.
- **make-min-inv-princ-5** Constructs a conjunct of the equations generated in make-min-inv-princ-3 corresponding to the constraint.
  For the constraint (LF1), $InvP_5^{(LF1)}$ is $x = 0$. Since in this constraint, the constraint has no literals in this constraint other than the main literal LF1, make-min-inv-princ-6 makes no contributation.
  For the constraint (LF3 LF2), $InvP_5^{(LF3\,LF2)}$ is $x = [S\,w] \wedge InvP_6^{(LF2)}$ where $InvP_6^{(LF2)}$ is constructed by make-min-inv-princ-6 using the extra literals of the constraint, in this case (LF2).
- **make-min-inv-princ-6** Constructs a conjunct of formulas for each extra literal of the constraint. In this case, there is the one literal LF2, giving $InvP_6^{(LF2)}$ as $v^1\,w$.
  In the general case, when a literal LF does not contain the set selected variable (`vsel`) v^1, $InvP_6^{(LF\,.\,<LIST>)}$ will be of the form $A \vee InvP_6^{<LIST>}$ where A is the shallow formula of LF or its negation (if LF is negative). This will also be the form if LF is a positive literal only containing v^1 at the head.
  The more complicated case is when v^1 occurs inside the body of the literal. Suppose $A(v^1)$ is the shallow formula of such a literal LF (or its negation if the literal is negative). Since we will want a formula which is positive with respect to v^1, we let $InvP_6^{(LF\,.\,LEAF)}$ be of a form such as

  $$\exists w^i_{o\iota}\,[[\forall x\,.\,[w^i\,x] \supset [v^1_{o\iota}\,x]] \wedge A(w^i)].$$

  Of course, in general $o\iota$ may be any set type and x may be a list of variables. See section 3.1 for a semantic description of this as a least monotone upper approximation. **THM2** is an example where such an approximation is necessary.

In the end, we have constructed the inversion principle as follows:

$$\forall\,x.\,[v^1\,x] \supset InvP_2$$

$$\forall x . [v^1\, x] \supset . InvP_4^{(LF3\, LF2)} \vee InvP_4^{(LF1)}$$

$$\forall x . [v^1\, x] \supset . \exists w\, InvP_5^{(LF3\, LF2)} \vee InvP_5^{(LF1)}$$

$$\forall x . [v^1\, x] \supset . \exists w\, [x = [S\, w] \wedge InvP_6^{(LF2)}] \vee [x = 0]$$

$$\forall x . [v^1\, x] \supset . \exists w\, [x = [S\, w] \wedge [v^1\, w]] \vee [x = 0].$$

(2) **make-min-setvar-lemma-posf** Constructs the set existence lemma

$$\exists\, v^2 . \forall\, w^1 [\, v^2\, w^1 \supset v^2 .\, S\, w^1] \wedge v^2\, 0$$
$$\wedge \forall\, x^1 [\, v^2\, x^1 \supset \exists\, w [x^1 = S\, w \wedge v^2\, w] \vee x^1 = 0]$$
$$\wedge \forall p\, . [\forall\, w^1 [\, p\, w^1 \supset p .\, S\, w^1] \wedge p\, 0] \supset \forall x^2 . [v^2\, x^2] \supset [p\, x^2].$$

and a positive ftree used to solve the constraints. In general, this would start by universally quantifying any extra expansion variables and selected variables occurring in the constraints (`misc-occurs`). In this case, there are none. We create a fresh variable (`v2`) v^2 to play the role of the set variable in the formula. make-min-setvar-lemma-posf-1 is called to construct a positive ftree $POSF_1$. The shallow formula of $POSF_1$ is $LEM_1(v^1)$ the body of the set existence lemma, using the selected variable v^1. So, we return a selected node $POSF$ with shallow $\exists v^2 . LEM_1(v^2)$ and child $POSF_1$.

- **make-min-setvar-lemma-posf-1** The main part of the lemma we need to construct is the part that solves the constraints. The function make-min-setvar-lemma-posf-3 returns a postive ftree $POSF_2$ with shallow formula $Main(v^1)$. It also adds connections between literals in $POSF_2$ and the literals LF1, LF2, and LF3 of the constraints.
  We have already constructed the inversion principle $InvP$. If INCLUDE-INDUCTION-PRINCIPLE is NIL, we construct a positive ftree for $Main(v^1) \wedge InvP$. If INCLUDE-INDUCTION-PRINCIPLE is T, we construct a positive ftree for $Main(v^1) \wedge [InvP \wedge IndP]$ where $IndP$ is an induction principle. In this case, the induction principle is

  $$\forall p\, . Main(p) \supset \forall x^2 . [v^1\, x^2] \supset [p\, x^2].$$

  The positive ftrees for the inversion principle and the induction principle are simply constructed by expanding the formula as an ftree, duplicating the outermost quantifier NUM-OF-DUPS times. These are combined using conjunction nodes with $POSF_2$.
- **make-min-setvar-lemma-posf-2** For each constraint P, we make a conjunction of $POSF_3^P$ obtained by calling make-min-setvar-lemma-posf-3. In this case, we have a conjunction of $POSF_3^{(LF3\, LF2)}$ and $POSF_3^{(LF1)}$.
- **make-min-setvar-lemma-posf-3** If there are banned variables in the constraint, we will make an expansion node. In this case, we will have one child that corresponds expanding

using the banned variables (so we can mate to the literals in the constraints). In case we will later want to use this part of the lemma elsewhere in the proof, we also duplicate NUM-OF-DUPS times.

The constraint (LF1) does not contain any banned variables and has no extra literals. So, we simply let $POSF_3^{(LF1)}$ be a positive leaf with shallow $[v^1\, 0]$ (the same shallow as LF1). This leaf is connected to the negative node LF1, solving this constraint.

The constraint (LF3 LF2) contains the banned variable w. So, $POSF_3^{(LF3\, LF2)}$ is an expansion node. (We create a fresh variable w^1 to use as the bound variable in the shallow formula.) One child of this expansion node is $POSF_4^{(LF3\, LF2)}$ with expansion term w. This child is constructed and is used to solve the constraint by calling make-min-setvar-lemma-posf-4. If NUM-OF-DUPS is greater than 0, we also have NUM-OF-DUPS many other children of $POSF_3^{(LF3\, LF2)}$ expanded using expansion variables. These children could be used in the proof of the theorem.

- **make-min-setvar-lemma-posf-4** This function creates more expansion nodes corresponding to the rest of the banned variables of the constraint. Since (LF3 LF2) only has one banned variable, we skip directly to constructing an implication node $POSF_4^{(LF3\, LF2)}$ where the first child is a negative ftree $POSF_5^{(LF2)}$ and the second child is a positive leaf with shallow formula $[v^1\, [S\, w]]$ (the same as LF3). This is connected to the node LF3. The negative ftree $POSF_5^{(LF2)}$ is constructed by make-min-setvar-lemma-posf-5 and used to block LF2.
- **make-min-setvar-lemma-posf-5** This function constructs a conjunction corresponding to the extra literals in a given constraint. In this case, we only have the one extra literal LF2. So, $POSF_5^{(LF2)}$ is $POSF_6^{LF2}$ constructed by make-min-setvar-lemma-posf-6.
- **make-min-setvar-lemma-posf-6** Given a literal in a constraint, this creates a corresponding leaf and mates it to the literal in the constraint. Since LF2 is positive, $POSF_6^{LF2}$ is a negative leaf with shallow formula $[v^1\, w]$ and this is mated to LF2.

So, to sum up, we created a positive ftree for the lemma along with connections to the nodes in the constraints. We can follow the construction by the noting that the construction of the shallow formulas proceeded as

$$\exists v^2 \,.\, LEM_1(v^2)$$

$$Main(v^1) \wedge InvP \wedge IndP$$

where $Main(v^1)$ was constructed as

$$Sh(POSF_3^{(LF3\,LF2)}) \wedge Sh(POSF_3^{(LF1)})$$

$$\forall w^1 Sh(POSF_4^{(LF3\,LF2)}) \wedge [v^1\,0]$$

$$\forall w^1 [Sh(POSF_5^{(LF2)}) \supset [v^1\,[S\,w^1]]] \wedge [v^1\,0]$$

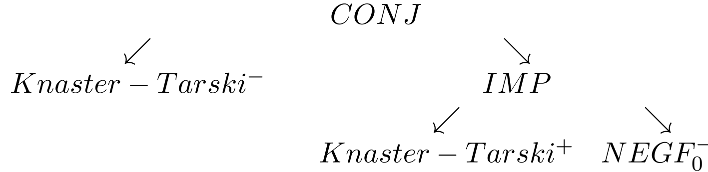
$$\forall w^1 [[v^1\,w^1] \supset [v^1\,[S\,w^1]]] \wedge [v^1\,0]$$

(where $Sh(N)$ means the shallow formula of the node N).

(3) **make-clos-setvar-lemma-negf** This produces a negative ftree with connections giving the proof of the set existence lemma. Let us assume INCLUDE-INDUCTION-PRINCIPLE is set to T. The function make-knaster-tarski-leastfp-lemma constructs an ftree proof $Knaster-Tarski^-$ of the least fixed point version of the Knaster-Tarski Theorem for the type of the set variable v. In this case, v has type $o\iota$, so the Knaster Tarski Theorem generated is

$$\begin{array}{r} \forall K_{o\iota(o\iota)} . \forall u_{o\iota} \forall v_{o\iota} [\forall z . u\, z \supset v\, z] \\ \supset \exists u_{o\iota} . \forall z\, [K\, u\, z \supset u\, z] \wedge \forall z\, [u\, z \supset K\, u\, z] \\ \forall v_{o\iota} . \forall z\, [K\, v\, z \supset v\, z] \supset \forall z . u\, z \supset v\, z \end{array}$$

The function make-clos-setvar-lemma-negf-0 does the work of constructing the negative ftree node $NEGF$ giving the proof of the set existence lemma. The special variable `expf` is set to a positive ftree $Knaster-Tarski^+$ for the Knaster-Tarski Lemma which is used to prove the set existence lemma. Then we return the ftree



The new connections are added to the dynamic variable `clist`.

- **make-clos-setvar-lemma-negf-0** Our goal is to construct a negative ftree $NEGF_0^-$ for the set existence lemma

$$\begin{array}{r} \exists\, v^2 . \forall\, w^1 [\, v^2\, w^1 \supset v^2 . \, S\, w^1] \wedge v^2\, 0 \\ \wedge \forall\, x^1 [\, v^2\, x^1 \supset \exists\, w [\, x^1 = S\, w \wedge v^2\, w] \vee x^1 = 0] \\ \wedge \forall p\, . [\forall\, w^1 [\, p\, w^1 \supset p . \, S\, w^1] \wedge p\, 0] \supset \forall x^2 . [v^2\, x^2] \supset [p\, x^2]. \end{array}$$

  In general, the set existence lemma universally binds the variables in `misc-occurs`. In this case there are no such variables. We also need to construct the positive $Knaster - Tarski^+$ node (`expf`) and add connections to `clist` between nodes in these two ftrees.
  We proceed to the most important step, instantiating the K in the Knaster-Tarski Theorem. The monotone function we want

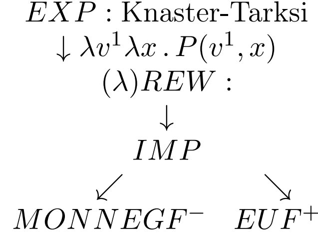
can be extracted from the inversion principle. The inversion principle (`inv-princ`) is

$$\forall x. v^1\, x \supset \exists\, w[\, x \,=\, S\, w \wedge\, v^1\, w] \vee\, x \,=\, 0.$$

and we construct the monotone function (`monfn`):

$$\lambda v^1 \lambda x\, .\exists\, w[\, x \,=\, S\, w \wedge\, v^1\, w] \vee\, x \,=\, 0.$$

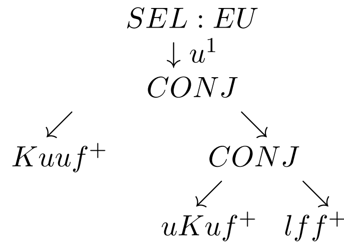
Let us write $\lambda v^1 \lambda x\, .\, P(v^1, x)$ for this term. Note that $P(v^1, x)$ is positive with respect to v^1, so the function will be monotone. We substitute this for K so that $Knaster - Tarski^+$ is an expansion node



with one child using the monotone function as the expansion term. The child of this node is a λ-rewrite passing to the normal form. Below this is an implication node with two children $MONNEGF^-$ providing a proof that $\lambda v^1 \lambda x\, .\, P(v^1, x)$ is monotone and EUF^+ a positive node with shallow formula (`eu`)

$$\exists u_{o\iota}\, .\, \forall z\, [P(u, z) \,\supset\, u\, z] \,\wedge\, \forall z\, [u\, z \,\supset\, P(u, z)]$$
$$\forall v_{o\iota}\, .\, \forall z\, [P(v, z) \,\supset\, v\, z] \,\supset\, \forall z\, .\, u\, z \,\supset\, v\, z.$$

The node $MONNEGF^-$ is constructed by the function monfn-negf. This function implements the proof that set functions defined by positive formulas are monotone.
Let u^1 be a new selected variable to use in the selection node EUF^+:



where EU is the right side of the top implication of the Knaster-Tarski Theorem. The node $Kuuf^+$ is has shallow formula Kuu:

$$\forall z\, .\, [\exists\, w[\, z \,=\, S\, w \wedge\, u^1\, w] \vee\, z \,=\, 0] \supset u^1\, z$$

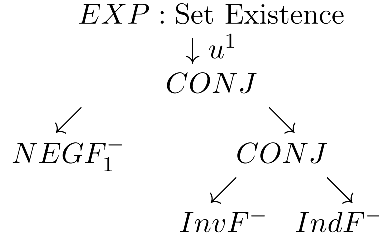
and is constructed during the process of constructing $NEGF_0^-$ below. We start with $Kuuf^+$ as a leaf. The node $uKuf^+$ is a leaf that corresponds directly to the inversion principle in the set existence lemma

$$\forall z\,[u^1\,z \supset \exists\,w[\,z\,=\,S\,w \wedge\,u^1\,w] \vee\,z\,=\,0.$$

Finally, we will construct a node lff^+ has shallow formula

$$\forall p\,\forall z\,[[\exists\,w[\,z\,=\,S\,w \wedge\,u^1\,w] \vee\,z\,=\,0] \supset u^1\,z] \supset \forall z\,.\,u^1\,z \supset p\,z$$

obtained from the fact that u^1 is the least pre-fixed point. The selected variable u^1 is used as the expansion term $NEGF_0^-$ we want to construct for the set existence lemma. So, $NEGF_0^-$ will have the form



where $NEGF_1^-$ (`f`) is constructed by make-clos-setvar-lemma-negf-1, $InvF^-$ (`l`) is a leaf corresponding to the inversion principle, and $IndF^-$ (`indf`) constructed by make-clos-setvar-ind-negf corresponds to the induction principle. The leaves $InvF^-$ and $uKuf^+$ are mated on `clist`.

- **make-clos-setvar-lemma-negf-1** This function constructs $NEGF_1^-$ with shallow formula

  $$\forall\,w[\,u^1\,w \supset\,u^1.\,S\,w] \wedge\,u^2\,0.$$

  Along with this, we will construct $Kuuf^+$ and put connections onto `clist`. So, we build $NEGF_1^-$ as a conjunction $NEGF_2^{(LF3\,LF2)} \wedge NEGF_2^{(LF1)}$ where $NEGF_2^{(LF3\,LF2)}$ and $NEGF_2^{(LF1)}$ corresponding to the two constraints is constructed by make-clos-setvar-lemma-negf-2. We also use an integer `n` to keep up with which constraint we are considering.
- **make-clos-setvar-lemma-negf-2** Given a constraint C (`paths`), this function builds a negative ftree $NEGF_2^C$ and changes $Kuuf^+$ and `clist`.
  First, consider the constraint (LF1). The shallow formula of $NEGF_2^{(LF1)}$ should be $u^1\,0$. Here, we let make-clos-setvar-lemma-negf-3 do the work of constructing an ftree which will be merged with $Kuuf^+$. Then make-clos-setvar-lemma-negf-4 creates the negative leaf $NEGF_2^{(LF1)}$ mated with part of the ftree generated in make-clos-setvar-lemma-negf-4.

The second constraint we need $NEGF_2^{(LF3\,LF2)}$ to have shallow formula

$$\forall w\,.\,u^1\,w \supset u^1\,.\,S\,w.$$

First, we create a new selected variable w^2 and let $NEGF_2^{(LF3\,LF2)}$ be

$$\begin{array}{c} SEL : \forall w\,.\,u^1\,w \supset u^1\,.\,S\,w \\ \downarrow w^2 \\ IMP \\ \swarrow \qquad \searrow \\ NEGF_3^{(LF2)} \quad LEAF3 \end{array}$$

where $NEGF_3^{(LF2)}$ is constructed by make-clos-setvar-lemma-negf-3 (along with an ftree to be merged with $Kuuf^+$) and $LEAF3$ is a negative leaf with shallow $u^1\,.\,S\,w^2$ (mated to part of the new part of $Kuuf^+$).

- **make-clos-setvar-lemma-negf-3** Suppose we are given a constraint C $(\Gamma \rightarrow A)$ and the arguments (`args`) of the main literal of the constraint. This function constructs three positive ftrees. The first is an ftree $Kuuf_3^C$ which will be merged with $Kuuf^+$. The second is $NEGF_3^\Gamma$ where L is the list of extra literals L of the constraint, or NIL if there are no extra literals. If $NEGF_3^\Gamma$ is not nil, it is a positive ftree corresponding to a conjunct of the extra literals L of the constraint. The third is a leaf inside the first ftree which will be used in the mating.
  Recall that Kuu is

  $$\forall z\,.\,[\exists\, w[\,z\,=\,S\,w \wedge u^1\,w] \vee z\,=\,0] \supset u^1\,z,$$

  and this will be the shallow formula of $Kuuf_3^C$. The third ftree returned will be the leaf corresponding to $u^1\,z$ beneath $Kuuf_3^C$.
  In the first constraint (LF1), the args are (0). We use the list of arguments as expansion terms to create $Kuuf_3^{(LF1)}$. Here, $Kuuf_3^{(LF1)}$ will be

  $$\begin{array}{c} EXP : Kuu \\ \downarrow 0 \\ IMP \\ \swarrow \qquad \searrow \\ Kuuf_5^{(LF1)} \quad LEAF : u^1\,0^+ \end{array}$$

  where $LEAF : u^1\,0^+$ will be the third ftree returned. $Kuuf_5^{(LF1)}$ is a negative ftree constructed by make-clos-setvar-lemma-negf-5. In this case there are no extra literals, so the second return value is NIL.

In the second constraint (LF3 LF2), the one argument is $S\,w^2$. In this case, $Kuuf_3^{(LF3\,LF2)}$ is

$$\begin{array}{c} EXP : Kuu \\ \downarrow S\,w^2 \\ IMP \\ \swarrow \qquad \searrow \\ Kuuf_5^{(LF3\,LF2)} \quad u^1 \,.\, S\,w^2 \end{array}$$

where $Kuuf_5^{(LF3\,LF2)}$ is constructed along with $NEGF_3^{(LF2)}$ by make-clos-setvar-lemma-negf-5.

- **make-clos-setvar-lemma-negf-4** This simply makes a negative leaf and mates it to a given positive leaf.
- **make-clos-setvar-lemma-negf-5** Given a constraint C $(\Gamma \rightarrow A)$, this function returns two values. The first is a negative ftree $Kuuf_5^C$, and the second is a positive ftree $NEGF_8^\Gamma$ (or NIL if Γ is empty) constructed later.

  For the constraint (LF1) $Kuu_5^{(LF1)}$ should have shallow formula

  $$[\exists\, w[\, z \,=\, S\,w \wedge u^1\, w] \vee 0 \,=\, 0].$$

  So, is $Kuu_5^{(LF1)}$ of the form

  $$\begin{array}{c} DISJ \\ \swarrow \qquad \searrow \\ UNUSEDLEAF \quad Kuu_6^{(LF1)} \end{array}$$

  where $Kuu_6^{(LF1)}$ is constructed by make-clos-setvar-lemma-negf-6.

  For the constraint (LF3 LF2) $Kuu_5^{(LF3\,LF2)}$ should have shallow formula

  $$[\exists\, w[S\,w^2 \,=\, S\,w \wedge u^1\, w] \vee [S\,w^2] \,=\, 0].$$

  In this case, $Kuu_5^{(LF3\,LF2)}$ is of the form

  $$\begin{array}{c} DISJ \\ \swarrow \qquad \searrow \\ Kuu_6^{(LF3\,LF2)} \quad UNUSEDLEAF \end{array}$$

  where $Kuu_6^{(LF3\,LF2)}$ is constructed by make-clos-setvar-lemma-negf-6 (along with $NEGF_8^{(LF2)}$).
- **make-clos-setvar-lemma-negf-6** This function constructs Kuu_6^C using the selected variables created in make-clos-setvar-lemma-negf-2 as expansion terms, then calls make-clos-setvar-lemma-negf-7 to construct the rest of Kuu_6^C and $NEGF_8^C$. At this

step we distinguish between arguments (`args2`) that correspond to these selected variables, and remove one of the arguments corresponding to each such selected variable. The arguments sent to make-clos-setvar-lemma-negf-7 correspond to equations in the formula `wff` (originating with the inversion principle).

Since the constraint (LF1) has not banned variables, $Kuu_6^{(LF1)}$ is constructed as $Kuu_7^{(LF1)}$ by make-clos-setvar-lemma-negf-7. The constraint (LF3 LF2) has the single banned variable w. The corresponding selected variable w^2 is used to create $Kuu_6^{(LF3\,LF2)}$ as

$$\begin{array}{c} EXP : [\exists\, w[S\, w^2 \;=\; S\, w \wedge\, u^1\, w] \vee\, [S\, w^2] \;=\; 0] \\ \downarrow w^2 \\ Kuu_7^{(LF3\,LF2)} \end{array}$$

where $Kuu_7^{(LF3\,LF2)}$ by make-clos-setvar-lemma-negf-7 (along with $NEGF_8^{(LF2)}$).

- **make-clos-setvar-lemma-negf-7** The arguments that are sent to this function are those from the main literal which were not the first argument corresponding to the selected variables generated in make-clos-setvar-lemma-negf-2. These arguments correspond to a conjunction of reflexive equations. If there is more than one literal in the constraint,more conjuncts are formed when make-clos-setvar-lemma-negf-8 is called.

  The first constraint (LF1) has no extra literals, so when this function is called $Kuu_7^{(LF1)}$ with shallow (`wff`) $0 = 0$ is simply constructed as

$$\begin{array}{c} (REFL =)REW : 0 = 0 \\ \downarrow \\ TRUE \end{array}$$

  The second constraint (LF3 LF2) does have the extra literal LF2. When this is called $Kuu_7^{(LF3\,LF2)}$ which shallow (`wff`)

$$[[S\, w^2] \;=\; [S\, w^2]] \wedge [u^1\, w^2]$$

  is constructed as

$$\begin{array}{ccc} & CONJ & \\ \swarrow & & \searrow \\ (REFL =)REW : & & Kuu_8^{(LF2)} \\ \downarrow & & \\ TRUE & & \end{array}$$

  where $Kuu_8^{(LF2)}$ is constructed by make-clos-setvar-lemma-negf-8.

- **make-clos-setvar-lemma-negf-8** Given the left side Γ of a constraint, a negative ftree Kuu_8^Γ and positive ftree $NEGF_8^\Gamma$ are constructed along with connections between them. These are constructed as conjuncts for each literal in Γ using make-clos-setvar-lemma-negf-9 to construct the children of the conjuncts.
  In our example, $Kuu_8^{(LF2)}$ is simply Kuu_9^{LF2} since there is only one literal. Similarly, $NEGF_8^{(LF2)}$ is $NEGF_9^{LF2}$.
- **make-clos-setvar-lemma-negf-9** Usually this function is given two wffs which are α-equal. A positive leaf and a negative leaf are created, mated, and returned. Another case is when set variable occurs embedded in the literal of the constraint. In this case, the first wff `wff1` is of the form

$$\exists w^i_{o\iota}\,[[\forall \overline{x}\,.\,[w^i\,\overline{x}] \supset [u^1_{o\iota}\,\overline{x}]] \wedge A(w^i)].$$

  where the second wff `wff2` is $A(u^1)$. In this case, we create a positive ftree of the form

$$\begin{array}{c} EXP : \exists w^i_{o\iota}\,[[\forall \overline{x}\,.\,[w^i\,\overline{x}] \supset [u^1_{o\iota}\,\overline{x}]] \wedge A(w^i)] \\ \downarrow u^1 \\ CONJ \\ \swarrow \qquad\qquad\qquad \searrow \\ SEL : \forall \overline{x}\,.\,[w^i\,\overline{x}] \supset [u^1_{o\iota}\,\overline{x}] \qquad\qquad LEAF3 : A(u^1) \\ \downarrow \overline{a} \\ IMP \\ \swarrow \qquad \searrow \\ LEAF1 : [u^1\,\overline{a}] \quad LEAF2 : [u^1\,\overline{a}] \end{array}$$

  and a negative leaf with shallow formula $A(u^1)$ which we mate to $LEAF3$. Also, $LEAF1$ and $LEAF2$ are mated. These two ftrees are returned.
  In our example, the literal is LF2. The formulas are both $u^1\,w^2$, so we create Kuu_9^{LF2} as a positive leaf and $NEGF_9^{LF2}$ as a negative leaf. The two are mated and returned.
- **make-clos-setvar-ind-negf** This function is called with the least pre-fixed point property formula (`lfwff`), the induction property formula (`indwff`), and the list of constraints (`paths`). The goal is to prove the induction property from the pre-fixed point property by constructing a negative ftree $IndF^-$ for the induction property, a positive ftree lff^+ for the least pre-fixed point property, and a complete set of connections between nodes in them. The dynamic variable `lff` is set to lff^+. $IndF^-$ is returned.
  In our example, the least pre-fixed property is

$$\forall v_{o\iota}\,.\,\forall z\,[[\exists\,w[\,x\,=\,S\,w \wedge\,v\,w] \vee\,x\,=\,0] \supset\,v\,z] \supset\,\forall z\,.\,u^1\,z \supset\,v\,z$$

and the induction property is

$$\forall p\,.\,[\forall\, w^1[p\, w^1 \supset\, p.\, S\, w^1] \wedge\, p\, 0] \supset \forall x^2\,.\,[u^1\, x^2] \,\supset\, [p\, x^2].$$

The first thing we do is choose a new selected variable $p^1_{o\iota}$ (`p`). Let P_1 (`lfpre`) be

$$\forall z\,[[\exists\, w[\, z \,=\, S\, w \wedge\, p^1\, w] \vee\, z \,=\, 0] \supset p^1\, z],$$

P_2 be

$$\forall z\,.\, u^1\, z \,\supset\, p^1\, z,$$

I_1 (`indhyp`) be

$$\forall\, w^1[\, p^1\, w^1 \,\supset\, p^1.\, S\, w^1] \wedge\, p^1\, 0,$$

and I_2 be

$$\forall x^2\,.\,[u^1\, x^2] \,\supset\, [p^1\, x^2].$$

We construct the positive ftree lff^+ as

$$\begin{array}{c} EXP : \text{Least Pre-fixed Point} \\ \downarrow p^1 \\ IMP \\ \swarrow \qquad\qquad \searrow \\ lff_1 : P_1^- \quad LEAF1 : P_2^+ \end{array}$$

and the negative ftree $IndF^-$ as

$$\begin{array}{c} SEL : \text{Induction Principle} \\ \downarrow p^1 \\ IMP \\ \swarrow \qquad\qquad \searrow \\ IndF_1 : I_1^+ \quad LEAF2 : I_2^- \end{array}$$

where lff_1 and $IndF_1$ are constructed by make-clos-setvar-ind-negf. Since P_2 and I_2 are α-equal, we can mate $LEAF1$ and $LEAF2$.

- **make-clos-setvar-ind-negf-1** We start with two wffs: `wff1` of the form $C_1 \wedge \cdots \wedge C_n$ `wff2` of the form

$$\forall \overline{z}\,.\,[D_1(\overline{z}) \,\vee\, \cdots \,\vee\, D_n(\overline{z})] \,\supset\, [p\,\overline{z}].$$

where n is the number of constraints. Let $\overline{a}$ be new selected variables and $LEAF1$ be a negative literal with shallow formula $[p\,\overline{a}]$. We return two values, a negative ftree of the form

$$\begin{array}{c} SEL : \forall \overline{z}\,.\,[D_1(\overline{z}) \,\vee\, \cdots \,\vee\, D_n(\overline{z})] \,\supset\, [p\,\overline{z}] \\ \downarrow \overline{a} \\ NF_2 \end{array}$$

and a positive ftree PF_2 with shallow formula `wff1` where PF_2 and NF_2 are constructed by make-clos-setvar-ind-negf-2.

In our example, `wff1` is

$$\forall w^1 [p^1 w^1 \supset p^1 . S w^1] \wedge p^1 0,$$

and `wff2` is

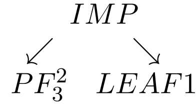
$$\forall z [[\exists w [z = S w \wedge p^1 w] \vee z = 0] \supset p^1 z].$$

We choose a new selected variable z^1_ι and create a negative leaf $LEAF1 : p^1 z^1$. We call make-clos-setvar-ind-negf-2 with

$$\forall w^1 [p^1 w^1 \supset p^1 . S w^1] \wedge p^1 0,$$

$$\exists w [z^1 = S w \wedge p^1 w] \vee z^1 = 0,$$

and $LEAF1$ to get two positive ftrees, PF^1_3 and PF^2_3. We return the positive ftree PF^1_3 and the negative ftree



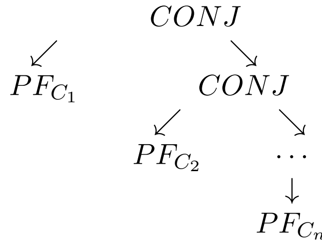
- **make-clos-setvar-ind-negf-2** We start with n constraints (given by `paths` and `banned-occurs`), and two wffs, one (`wff1`) of the form $C_1 \wedge \cdots \wedge C_n$, and the other (`wff2`) of the form

  $$D_1(\overline{z}) \vee \cdots \vee D_n(\overline{z}).$$

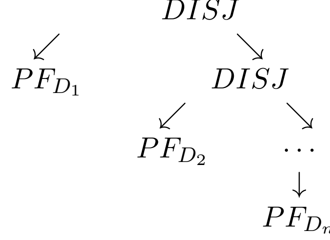
  We consider each constraint

  $$\Psi_1 | \Gamma_i \to A_i,$$

  conjunct C_i and disjunct D_i. We distinguish between variables $z \in \Psi_i$ which occur as arguments in A_i and those $w \in \Psi_i$ which do not. When $z \in \Psi_1$ occurs as an argument in A_i, the corresponding argument in the shallow formula of $LEAF1$ is a variable a created in the previous step. Let θ_1 (`psi-z-assoc`) be the substitution sending $\theta(z) = a$ for each such z and a. Those $w \in \Psi_i$ which do not occur as arguments are stored on Ψ^0_i. This information is passed to make-clos-setvar-ind-negf-3 which returns two positive ftrees PF_{D_i} and PF_{C_i} and returns



and



In our example, the two constraints are of the form

$$v^1\,0$$

and

$$w|v^1\,w \to v^1\,[S\,w].$$

Neither 0 nor $[S\,w]$ is a variable in the corresponding Ψ's, so θ_1 is empty. Ψ^0 is NIL in one case and (w) in the other.

- **make-clos-setvar-ind-negf-3** Given two wff's C (`wff1`) and D (`wff2`) and $\Psi^0 = \overline{w}$ where `wff2` is of the form $\exists\overline{w^1}D'(\overline{w^1})$, we create new selected variables $\overline{w^2}$. Let θ_2 (`psi-w-assoc`) send each w to w^2. We call make-clos-setvar-ind-negf-4 with C and $D'(\overline{w^2})$ to obtain two positive ftrees PF_4^1 and PF_4^2 with shallow formulas C and $D'(\overline{w^2})$. We then return PF_4^1 and

$$\begin{array}{l} SEL : \exists\overline{w^1}D'(\overline{w^1}) \\ \quad\downarrow \overline{w^2} \\ \quad PF_4^2 \end{array}$$

The constraint $\emptyset|\cdot \to v^1\,0$ has an empty Ψ, so we proceed directly to make-clos-setvar-ind-negf-4 with $p^1\,0$ and $z^1 = 0$ to get the two positive ftrees.

The constraint $w|v^1\,w \to v^1\,.\,S\,w$ has the variable w in Ψ, and D is

$$\exists w[\,z^1 = S\,w \wedge p^1\,w].$$

We create a new selected variable w^2 and call make-clos-setvar-ind-negf-4 with

$$\forall\,w^1[\,p^1\,w^1 \supset p^1.\,S\,w^1]$$

and

$$z^1 = S\,w^2 \wedge p^1\,w^2.$$

The second positive ftree PF_4^2 constructed has shallow $z^1 = S\,w^2 \wedge p^1\,w^2$. We return the first positive ftree along with

$$\begin{array}{l} SEL : \exists w[\,z^1 = S\,w \wedge p^1\,w] \\ \quad\downarrow w^2 \\ \quad PF_4^2 \end{array}$$

- **make-clos-setvar-ind-negf-4** We are given a constraint $\Psi|\Gamma \to A$ and two wff's C and D where C is of the form $\forall\overline{y}C' \supset A'$ (or $\forall\overline{y}A'$ if Γ is empty) where $\Psi = \overline{y'}$. Define a substitution θ with $dom(\theta) = \overline{y}$ as follows. Either $y' \in \overline{y'}$ is a $z \in \Psi$ which occurs as an argument in A, and so is in the domain of θ_1 (`psi-z-assoc`), or y is a $w \in \Psi$ in the domain of θ_2 (`psi-w-assoc`). In the first case, let $\theta(y) = \theta_1(y')$. In the second case, let $\theta(y) = \theta_2(y')$. We use these association lists to determine the expansion term for y. Then make-clos-setvar-ind-negf-5 is called with $\theta(C')$ (or NIL) (`hyp`) and $\theta(A')$ (`conc`). This constructs two positive ftrees, PF_5^1 with shallow $\theta(C' \supset A')$ and a positive ftree PF_5^2 with shallow D. We return

$$\begin{array}{c} EXP : C \\ \downarrow \theta(\overline{(}y)) \\ PF_5^1 \end{array}$$

and PF_5^2.
Again, the constraint $\emptyset|\cdot \to v^1\,0$ has an empty Ψ, so we proceed directly to make-clos-setvar-ind-negf-5 with $p^1\,0$ and $z^1 = 0$ to get the two positive ftrees.
For the constraint $w|v^1\,w \to v^1\,.\,S\,w$, C is

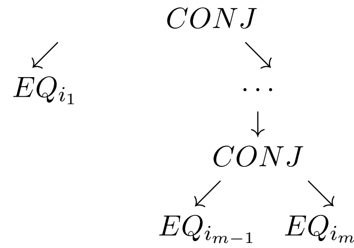
$$\forall\, w^1[p^1\,w^1 \supset p^1.\,S\,w^1].$$

$\theta(w^1) = \theta(w) = w^2$ by definition. make-clos-setvar-ind-negf-5 then constructs two positive ftrees, PF_5^1 with shallow formula $[p^1\,w^2 \supset p^1.\,S\,w^2]$ and PF_5^2 with shallow formula $z^1 = S\,w^2 \wedge p^1\,w^2$. We return

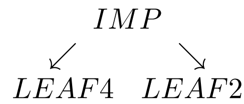
$$\begin{array}{c} EXP : \forall\, w^1[p^1\,w^1 \supset p^1.\,S\,w^1] \\ \downarrow w^2 \\ PF_5^1 \end{array}$$

and PF_5^2.
- **make-clos-setvar-ind-negf-5** This function starts with the negative leaf $LEAF1$ (`leaf`) constructed in make-clos-setvar-ind-negf-1. This function compares the arguments in the shallow formula $p(b_1, \ldots, b_n)$ of $LEAF1$ `leaf` and the given formula $p(a_1, \ldots, a_n)$ (`conc`).
We recursively compare the arguments. Suppose we have a negative leaf `leaf` with shallow formula $p(a_1, \ldots, a_{i-1}, b_i, \ldots, b_n)$ If b_i and a_i are syntactically equal, then these correspond to some $z \in \Psi$. Otherwise, there should be an equation at the beginning of `wff2`. That is, either `wff2` is $[b_i = a_i]$ or $[b_i = a_i] \wedge D'$. The function make-ftree-subst creates a positive ftree with shallow $[b_i = a_i]$, a negative leaf $LEAF2$ with

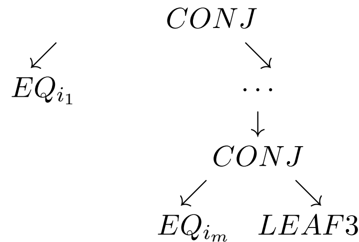
shallow formula $p(a_1, \ldots, a_i, b_{i+1}, \ldots, b_n)$ (and some new connections to add to the connection list). Let EQ_{i_j} be these positive ftrees for these equations.
So, after n steps, we have a negative leaf (`leaf`) with shallow formula $p(a_1, \ldots, a_n)$. We create a positive leaf $LEAF2$ (`leaf2`) with this shallow and mate the two leaves. If Γ is empty, we return this positive ftree along with the positive ftree



If Γ is nonempty, then the rest of D (after removing the equations) should be α-equal to C (`hyp`). We create a negative leaf $LEAF3$ and a positive leaf $LEAF4$ with these shallows and return the positive ftrees



and the positive ftree



In the constraint (LF1), Γ is empty and D is $z^1 = 0$. The shallow formula of $LEAF1$ is $p^1 z^1$. We construct EQ_1 as

$$\begin{array}{c}
(z^1 = 0) REW : Leibniz = \\
\downarrow \\
([\lambda x \forall y . \forall q . q\, x \supset q\, y]\, z^1\, 0) REW : \lambda \\
\downarrow \\
EXP : \forall q . q\, z^1 \supset q\, 0 \\
\downarrow \lambda x . \neg [p^1\, x] \\
([\lambda x . \neg [p^1\, x]]\, z^1 \supset [\lambda x . \neg [p^1\, x]]\, 0) REW : \lambda \\
\downarrow \\
IMP \\
\swarrow \qquad \searrow \\
NEG \qquad NEG \\
\downarrow \qquad \downarrow \\
LEAF3^+ \qquad LEAF1'^-
\end{array}$$

where $LEAF3^+$ mates to $LEAF1^-$. We also create a positive leaf $LEAF2$ with shallow formula $p^1\, 0$ and mate this to $LEAF1'^-$. We return $LEAF2$ and EQ_1.

In the constraint (LF3 LF2), D is $z^1 = S\, w^2 \wedge p^1\, w^2$. As above, a positive ftree EQ_2 for the equation $z^1 = S\, w^2$ giving a mate for $LEAF1^-$ and a negative leaf $LEAF3^-$ with shallow formula $p^1 . S\, w^2$. We also create a positive leaf $LEAF2^+$ to mate with $LEAF3$. The "leftover" part of D, $p^1\, w^2$ corresponds to Γ. We create positive and negative leaves $LEAF4^+$ and $LEAF5^-$ with this shallow formula and mate the two. We return the two positive ftrees

$$\begin{array}{c}
IMP \\
\swarrow \qquad \searrow \\
LEAF5^- \quad LEAF2^+
\end{array}$$

and

$$\begin{array}{c}
CONJ \\
\swarrow \qquad \searrow \\
EQ_2 \quad LEAF4^+
\end{array} .$$

(4) **make-knaster-tarski-leastfp-lemma**

CHAPTER 17

Tactics and Tacticals

Modify tactics.tex

1. Overview

Ordinarily in TPS3, the user proceeds by performing a series of atomic actions, each one specified directly. For example, in constructing a proof, she may first apply the deduct rule, then the rule of cases, then the deduct rule again, etc.. These actions are related temporally, but not necessarily in any other way; the goal which is attacked by one action may result in several new goals, yet there is no distinction between goals produced by one action and those produced by another. In addition, this use of small steps prohibits the user from outlining a general procedure to be followed. A complex strategy cannot be expressed in these terms, and thus the user must resign herself to proceeding by plodding along, using simple (often trivial and tedious) applications of rules.

Tactics offer a way to encode strategies into new commands, using a goal-oriented approach. With the use of tacticals, more complex tactics (and hence strategies) may be built. Tactics and tacticals are, in essence, a programming language in which one may specify techniques for solving goals.

Tactics are called partial subgoaling methods by [GMW79]. What this means is that a tactic is a function which, given a goal to be accomplished, will return a list of new goals, along with a procedure by which the original goal can be achieved given that the new goals are first achieved. Tactics also may fail, that is, they may not be applicable to the goal with which they are invoked.

Tacticals operate upon tactics in much the same way that functionals operate upon functions. By the use of tacticals, one may create a tactic that repeatedly carries out a single tactic, or composes two or more tactics. This allows one to combine many small tactics into a large tactic which represents a general strategy for solving goals.

As implemented in TPS3, a tactic is a function which takes a goal as an argument and returns four values: a list of new goals, a message which tells what the tactic did (or didn't do), a token indicating what the tactic did, and a validation, which is a lambda expression which takes as many arguments as the number of new goals, and which, given solutions for the new goals, combines the solutions into a solution for the original goal. It is

possible that the validation is used nowhere in the code, and that it should be phased out.

Consider this example. Suppose we are trying to define tactics which will convert an arithmetic expression in infix form to one in prefix form and evaluate it. One tactic might, if given a goal of the form "A / B", where A and B are themselves arithmetic expressions in infix form, return the list ("A" "B"), some message, the token "succeed", and the validation `(lambda (x y) (/ x y))`. If now we solve the new goals "A" and "B" (i.e., find their prefix forms and evaluate them), and apply the validation as a function to their solutions, we get a solution to the original goal "A / B".

When we use a tactic, we must know for what purpose the tactic is being invoked. We call this purpose the *use* of the tactic. Some examples of uses are `nat-ded` for carrying out natural deduction proofs, `nat-etree` for translating natural deduction proofs to expansion proofs (not yet implemented), and `etree-nat` for translating expansion proofs to natural deductions. A single tactic may have definitions for each of these uses. In contrast to tactics, tacticals are defined independent of any specific tactic use; some of the auxiliary functions they use, however, such as copying the current goal, may depend upon the current tactic use. For this purpose, the current tactic use is determined by the flag tacuse. Resetting this flag resets the default tactic use. Though a tactic can be called with only a single use, that tactic can call other tactics with different uses. See the examples in the section "Using Tactics".

Another important parameter used by a tactic is the *mode*. There are two tactic modes, `auto` and `interactive`. The definition of a tactic may make a distinction between these two modes; the current mode is determined by the flag , and resetting this flag resets the default tactic mode. Ideally, a tactic operating in `auto` mode should require no input from the user, while a tactic in `interactive` mode may request that the user make some decisions, e.g., that the tactic actually be carried out. It may be desirable, however, that some tactics ignore the mode, compound tactics (those tactics created by the use of tacticals and other tactics) among them.

One may wish to have tactics print informative messages as they operate; the flag tactic-verbose can be set to T to allow this to occur, and tactics can be defined so that messages are printed when `tactic-verbose` is so set. Each tactic should call the function *tactic-output* with two arguments. The first argument should be a string containing the information to be printed, and the second argument T if the tactic succeeds, and NIL otherwise. *tactic-output* will, depending on the second argument and the current value of `tactic-verbose`, either print or not print the first argument.

2. Syntax for Tactics and Tacticals

The TPS3 category for tactics is called `tactic`. The defining function for tactics is `deftactic`. The variable `auto::*global-tacticlist*` contains a list of all tactics. Each tactic definition has the following form:

```
(deftactic tactic
  {(<tactic-use> <tactic-defn> [<help-string>])}+
)
```

with components defined below:

tactic-use ::=	`nat-ded	nat-etree	etree-nat`
tactic-mode ::=	`auto	interactive`
tactic-defn ::=	*primitive-tactic* | *compound-tactic*
primitive-tactic ::=	`(lambda (goal)` *form*`*)`
	This lambda expression should return four values of the form:
	goal-list msg token validation.
compound-tactic::=	(*tactical tactic-exp**)
tactic-exp::=	`tactic` a tactic which is already defined
	| (*tactic* `[:use` *tactic-use*`]` `[:mode` *tactic-mode*`]` `[:goal` *goal*`]`)
	| *compound-tactic*
	| (`call` *command*) ; where *command* is a command which could be given at the TPS3 top level
goal ::=	a goal, which depends on the tactic's use,
	e.g., a planned line when the tactic-use is `nat-ded`.
goal-list ::=	(*goal**)
msg ::=	*string*
token ::=	`complete` meaning that all goals have been exhausted
	`| succeed` meaning that the tactic has succeeded
	`| nil` meaning that the tactic was called only for side effect
	`| fail` meaning that the tactic was not applicable
	`| abort` meaning that something has gone wrong, such as an undefined tactic

Tacticals are kept in the TPS3 category `tactical`, with defining function `deftactical`. Their definition has the following form:

```
(deftactical tactical
  (defn <tacl-defn>)
  (mhelp <string>))
```

with

tacl-defn ::=	*primitive-tacl-defn* / *compound-tacl-defn*
primitive-tacl-defn ::=	`(lambda (goal tac-list)` *form*`*)` This lambda-expression, where `tac-list` stands for a possibly empty list of tactic-exp's, should be independent of the tactic's use and current mode. It should return values like those returned by a *primitive-tac-defn*.
compound-tacl-defn ::=	`(tac-lambda (`*symbol*`*)` *tactic-exp*`)` Here the tactic-exp should use the symbols in the tac-lambda-list as dummy variables.

Here is an example of a definition of a primitive tactic.

```
(deftactic finished-p
 (nat-ded
  (lambda (goal)
    (if (proof-plans dproof)
(progn
 (when tactic-verbose (msgf "Proof not complete." t))
 (values nil "Proof not complete." 'fail))
(progn
 (when tactic-verbose (msgf "Proof complete." t))
 (values nil "Proof complete." 'succeed))))
  "Returns success if all goals have been met, otherwise
returns failure."))
```

This tactic is defined for just one use, namely `nat-ded`, or natural deduction. It merely checks to see whether there are any planned lines in the current proof, returning failure if any remain, otherwise returning success. This tactic is used only as a predicate, so the goal-list it returns is nil, as is the validation.

As an example of a compound tactic, we have

```
(deftactic make-nice
  (nat-ded
   (sequence (call cleanup) (call squeeze) (call pall))
   "Calls commands to clean up the proof, squeeze the line
numbers, and then print the result."))
```

Again, this tactic is defined only for the use `nat-ded`. `sequence` is a tactical which calls the tactic expressions given it as arguments in succession.

Here is an example of a primitive tactical.

```
(deftactical idtac
  (defn
    (lambda (goal tac-list)
      (values (if goal (list goal)) "IDTAC" 'succeed
      '(lambda (x) x))))
  (mhelp "Tactical which always succeeds, returns its goal
unchanged."))
```

The following is an example of a compound tactical. `then` and `orelse` are tacticals.

```
(deftactical then*
  (defn
    (tac-lambda (tac1 tac2)
      (then tac1 (then (orelse tac2 (idtac)) (idtac)))))
  (mhelp "(THEN* tactic1 tactic2) will first apply tactic1; if it
fails then failure is returned, otherwise tactic2 is applied to
each resulting goal.  If tactic2 fails on any of these goals,
```

```
then the new goals obtained as a result of applying tactic1 are
returned, otherwise the new goals obtained as the result of
applying both tactic1 and tactic2 are returned."))
```

3. Tacticals

There are several tacticals available. Many of them are taken directly from [GMW79]. After the name of each tactical is given an example of how it is used, followed by a description of the behavior of the tactical when called with `goal` as its goal. The newgoals and validation returned are described only when the tactical succeeds.

(1) `idtac: (idtac)`
Returns `(goal), (lambda (x) x)`.
(2) `failtac: (failtac)`
Returns failure
(3) `call: (call command)`
Executes command as if it were entered at top level of TPS3. This is used only for side-effects. Returns `(goal), (lambda (x) x)`.
(4) `orelse: (orelse tactic1 tactic2 ... tacticN)`
If N=0 return failure, else apply `tactic1` to `goal`. If this fails, call `(orelse tactic2 tactic3 ... tacticN)` on `goal`, else return the result of applying `tactic1` to `goal`.

(5) `then: (then tactic1 tactic2)`
Apply `tactic1` to `goal`. If this fails, return failure, else apply `tactic2` to each of the subgoals generated by `tactic1`.
If this fails on any subgoal, return failure, else return the list of new subgoals returned from the calls to `tactic2`, and the lambda-expression representing the combination of applying `tactic1` followed by `tactic2`.
Note that if `tactic1` returns no subgoals, `tactic2` will not be called.
(6) `repeat: (repeat tactic)`
Behaves like `(orelse (then tactic (repeat tactic)) (idtac))`.
(7) `then*: (then* tactic1 tactic2)`
Defined by:
`(then tactic1 (then (orelse tactic2 (idtac)) (idtac)))`. This tactical is taken from [Fel86].
(8) `then**: (then** tactic1 tactic2)`
Acts like `then`, except that no copying of the goal or related structures will be done.
(9) `ifthen: (ifthen test tactic1)` or
`(ifthen test tactic1 tactic2)`
First evaluates `test`, which may be either a tactic or (if user is an expert) an arbitrary LISP expression. If test is a tactic and does

not fail, or is an arbitrary LISP expression that does not evaluate to nil, then `tactic1` will be called on `goal` and its results returned. Otherwise, if `tactic2` is present, the results of calling `tactic2` on `goal` will be returned, else failure is returned. `test` should be some kind of predicate; any new subgoals it returns will be ignored by `ifthen`.

(10) `sequence:  (sequence tactic1 tactic2 ...  tacticN)`
Applies `tactic1, ...  , tacticN` in succession regardless of their success or failure. Their results are composed.

(11) `compose:  (compose tactic1 ...  tacticN)`
Applies `tactic1, ..., tacticN` in succession, composing their results until one of them fails. Defined by:
`(idtac)` if `N`=0
`(then* tactic1 (compose tactic2 ...  tacticN))` if `N` > 0.

(12) `try:  (try tactic)`
Defined by: `(then tactic (failtac))`. Succeeds only if tactic returns no new subgoals, in which case it returns the results from applying `tactic`.

(13) `no-goal:  (no-goal)`
Succeeds iff goal is nil.

4. Using Tactics

To use a tactic from the top level, the command use-tactic has been defined. Use-tactic takes three arguments: a *tactic-exp*, a *tactic-use*, and a *tactic-mode*. The last two arguments default to the values of tacuse and tacmode, respectively. Remember that a *tactic-exp* can be either the name of a tactic or a compound tactic. Here are some examples:

```
<1> use-tactic propositional nat-ded auto

<2> use-tactic (repeat (orelse same-tac deduct-tac))
               $ interactive

<3> use-tactic (sequence (call pall) (call cleanup) (call pall)) !

<4> use-tactic (sequence (foo :use nat-etree :mode auto)
                         (bar :use nat-ded :mode interactive)) !
```

Note that in the fourth example, the default use and mode are overridden by the keyword specifications in the tactic-exp itself. Thus during the execution of this compound tactic, `foo` will be called for one use and in one mode, then `bar` will be called with a different use and mode.

Remember, setting the value of the flag tactic-verbose to T will cause the tactics to send informative messages as they execute.

4.1. Implementation of tactics and tacticals.

(1) Main files (in order of importance): tactics-macros, tacticals, tacticals-macros, tactics-aux. These files contain tactic-related functions of a general nature. Most tactics are actually contained in other Lisp packages.
(2) When a tactic is executed, two global variables affect its execution: tacuse (the tactic's use), and tacmode (the current mode). Tacuse determines for what reason the tactic is being called. Current uses are etree-nat (translation of eproof to natural deduction) and nat-ded (construction of a natural deduction proof without any mating information). A single tactic may be defined for more than one use. Tacmode can have the value of either auto or interactive. Each tactic should take this value into account during operation. In general, this means that when the value is interactive, the user should be advised that the tactic is about to be applied and should be allowed to abort it. When the value is auto, the tactic should just be carried out if applicable.
(3) For each use, a number of auxiliary functions needed by the tacticals must be defined.
    (a) get-tac-goal: if a goal has not been specified, get the next one, e.g., the active planned line.
    (b) copy-tac-goal: copy the current goal into a new goal, so that subsequent actions can be performed without destroying the current goal. Allows later backtracking if necessary.
    (c) save-tac-goal: put the current goal into a form suitable for saving. This is not actually used by current tactics.
    (d) restore-tac-goal: backtrack to the previous goal. This is not actually used by current tactics.
    (e) update-tac-goal: given the old (saved) goal, and the new goal on which some progress has been made, update the old goal to reflect the progress made.

Tacticals must be independent of the value of tacuse. They cannot make any assumptions about the structure of the goals, etc.

The main function used in applying tactics is apply-tactic. This is a function that takes a tactic as argument, and allows keyword arguments of :goal, :use and :mode. If not specified, the use and mode default to the global values of tacuse and tacmode. If they are specified, the values given then override the global values of tacuse and tacmode. apply-tactic and (every tactic) returns four values. The first is a list of goals, the second a string with some kind of message, the third a token which indicates the result of the tactic and the fourth a validation, which, if non-nil, should be a function which specifies how solutions to the returned goals can be combined to solve the original goal.

apply-tactic works as follows:

(a) Checks that tactic is a valid tactic.

(b) If a goal has not been specified, calls get-tac-goal.
(c) If the tactic is an atom:
   (i) gets the tactic's definition for the use.
   (ii) if the definition is primitive, and the goal is nil, return (nil "Goals exhausted." 'complete nil). If the goal is non-nil, apply the tactic to the goal.
   (iii) if the definition is compound, call apply-tactical on the definition and the goal.
(d) If the tactic's definition begins with a tactic, call apply-tactic recursively, using those optional arguments.
(e) If the tactic begins with a tactical, call apply-tactical.

Whenever a tactic begins with a tactical, the function apply-tactical is used. It takes two arguments, a goal and a tactic. It is assumed that the tactic begins with a tactical. apply-tactical works as follows:
(a) Get the definition for the tactical.
(b) If the definition is primitive (a lambda expression), funcall the definition on the goal and the remainder (cdr) of the tactic.
(c) If the definition is compound (i.e., is defined in terms of other tacticals and begins with tac-lambda), expand the definition, substituting the arguments provided in the tactic's definition for the dummy arguments in the tactical's definition. Then call apply-tactical recursively.
(d) Otherwise abort, returning abort as the token (third value returned).

(4) Validations: Though the validation mechanism is in place, no use is made of them in the current tactic uses, since any changes in the constructed proofs are made immediately, not saved. Validations must be modified as tacticals are executed, since during their execution, the order of goals may be changed. For example, a tactical may repeatedly apply a tactic to a goal, then to all the new goals created, etc., until it fails on all of them. When it succeeds on a successor goal, the validation returned must be integrated into the validation which was returned for the first application of the tactic on the original goal. The function make-validation is used for this purpose.

CHAPTER 18

Proof Translations

1. Data Structures

2. EProofs to Nproofs

Here is a summary of what happens after matingsearch has terminated with a proof. The functions involved are located in the files *mating-merge.lisp*, *mating-merge2.lisp* and *mating-merge-eq.lisp*.

1. Apply Pfenning's Merge algorithm (`etr-merge`), put resulting etree and mating in variable current-eproof
   - (a) Get a list of the connections from mating
   - (b) Get a list of substitutions required
     - (i) Extract substitutions from unification tree
     - (ii) Replace occurrences of PI and SIGMA by quantifiers
     - (iii) Lambda-normalize each substitution
     - (iv) Alpha-beta-normalize each substitution
   - (c) Prune any unused expansions from the tree
   - (d) Make substitutions for variables
   - (e) Carry out merging (`merge-all`)
2. Replace skolem terms by parameters (if applicable) (`subst-skol-terms`)
3. If remove-leibniz is T, apply Pfenning's algorithm for removing the Leibniz equality nodes for substitution of equality nodes (`remove-leibniz-nodes`)
4. Try to replace selected parameters by the actual bound variables (`subst-vars-for-params`), not always possible because of restriction that a parameter should appear at most once
5. Raise lambda rewrite nodes, so that in natural deduction the lambda normalization occurs as soon as possible. (`raise-lambda-nodes`)
6. Clean up the etree (`cleanup-etree`). For each expansion term in the tree,
   - (a) Lambda-normalize it
   - (b) Minimize the superscripts on bound variables
   - (c) Make a new expansion with the new term
   - (d) Deepen the new expansion like the original, but removing unnecessary lambda-norm steps.
   - (e) Remove the original expansion

   Begin natural deduction proof, using current-eproof
   - Set up planned line
     - (a) Use shallow wff of the current-eproof's etree

(b) Give it the tree as value for its NODE property
(c) Give it the current-eproof's mating (list of pairs of node names) as its MATING property

Call `use-tactic` with tactic desired

(a) Each line in the proof will correspond to a node in the etree; the natural deduction proof is stored in the variable dproof.
(b) Here is an important property which should remain invariant during the translation process: It should always be the case that the line-mating of the planned line is a p-acceptable mating for the etree that one could construct by making an implication whose antecedent is the conjunction of the line-nodes of the supports, and whose consequent is the line-node of the planned line. This will assure us that we have sufficient information to carry out the translation.

It was observed that when path-focused duplication had been used, the expansion proof would often have a great deal of redundancy in the sense that the same expansion term would be used for a given variable many times. More precisely, if one defines an expansion branch by looking at sequences of nested expansion nodes, attaching one expansion term to each expansion node in the sequence, there would be many identical expansion branches.

In response to this, *mating-merge.lisp* was modified in the following ways:

- Don't do pruning of unnecessary nodes at the beginning of the merge, when the tree is its greatest size.
- Instead, prune all branches that couldn't possibly have been used; they are those that have a zero status. This is probably not necessary, but certainly makes debugging easier and doesn't cost much.
- After merging of identical expansions has been done, call the original pruning function.

3. NProofs to Eproofs

There are three versions of NAT-ETREE, the command for translating natural deductions into expansion tree proofs. The user can choose between the three by setting the flag NAT-ETREE-VERSION to one of the following values:

(1) **OLD** (the original version)
(2) **HX** (Hongwei Xi's version, written in the early to mid 1990's)
(3) **CEB** (Chad E. Brown's version, written in early 2000)

Also, note that setting the flag NATREE-DEBUG to T is useful for debugging the **HX** and **CEB** versions. The subsections that follow describe each of these versions in greater detail.

After using NAT-ETREE to translate to an expansion proof, the user can use this expansion proof to suggest flag settings (via the mate commands ETR-INFO and ETREE-AUTO-SUGGEST) or to trace MS98-1 (using the

flag MS98-TRACE. See the User's Manual for a description of these facilities. The User's Manual also has examples.

3.1. Chad's Nat-Etree. To use this version of NAT-ETREE, set NAT-ETREE-VERSION to **CEB**. The main functions for this version are in the files ceb-nat-etr.lisp and ceb-nat-seq.lisp. The relevant functions are:

ceb-nat-etree: This is the main function. It preprocesses the proof to
- remove applications of *Subst=* and *Sym=*,
- expand applications of *RuleP* and other propositional rules (e.g., *Assoc*) in terms of more primitive inference rules,
- attempt to expand any applications of *RuleQ*,
- replace instances of *Assert* by hypotheses which are discharged at the end of the proof,
- and replace applications of the *Cases* rule using more than two disjuncts by multiple applications of the *Cases* rule using two disjuncts.

The function then calls ceb-proof-to-natree to build the natree version of the natural deduction proof, calls natree-to-ftree-main to build the ftree representation of the expansion tree and a complete mating. Finally, this is converted into an ordinary expansion proof which may optionally be merged. (Merging is appropriate if the user plans to translate back to a natural deduction proof, but inappropriate if the user is trying to gather information about a potential automatic proof.)

ceb-proof-to-natree: This is a modification of Hongwei's proof-to-natree (see hx-natree-top.lisp). This function builds the natree, changing some justifications to *RuleP* or *RuleQ*, and changing the variables in applications of *UGen* and *RuleC* so they are unique in the entire proof (i.e., the natree rules satisfy a global eigenvariable condition, since the etree selection variables must be distinct).

natree-to-ftree-main: This function calls natree-to-ftree-seq-normal to build a sequent calculus derivation (see 3.1.5) from the natree. Then ftree-seq-weaken-early modifies the derivation so that the weaken rule is applied eagerly. This may eliminate certain unnecessary cuts and simplify the derivation. Then the cut elimination function ftree-seq-cut-elim (see section 4.2) is used to make the derivation cut-free. Finally, cutfree-ftree-seq-to-ftrees is used to obtain an ftree and a complete mating from the cut-free derivation.

natree-to-ftree-seq-normal, natree-to-ftree-seq-extraction: These functions are mutually recursive and provide the main algorithm for constructing the sequent calculus derivation. A description of the algorithm is below. The function natree-to-ftree-seq-normal is called on natree nodes which are considered normal. (These would be annotated with a $\Uparrow$.) The function natree-to-ftree-seq-extraction

is called on natree nodes which are considered extractions. (These would be annotated with a $\downarrow$.)

Frank Pfenning's ATP class contained notes on annotating (intuitionistic first-order) normal natural deduction proofs, and gave a constructive proof (algorithm) that every natural deduction proof translates into a sequent calculus proof. Also, normal natural deduction proofs translate to cut-free sequent calculus proofs. The idea of using annotations carries over to classical higher-order logic.

3.1.1. *Normal Deductions.* The idea of a normal deduction is that the proof works down using elimination rules, and up using introduction rules, meeting in the middle. We can formalize this idea by saying that a natural deduction proof is normal if its assertions can be annotated, so that the assertions involved in the applications of rules of inference are as described below. Technically, we are defining normal natural deductions by mutually defining normal deductions ($\Uparrow$) and extraction deductions ($\downarrow$).

3.1.2. *Annotations of the Assertions in a Proof.* First, the basic rules which allow one to infer normal deductions ($\Uparrow$).

$$\frac{A \Uparrow}{\forall x A \Uparrow} UGen$$

where x is not free in any hypotheses.

$$\frac{\exists y A \downarrow \qquad \begin{array}{c}\overline{[x/y]A \downarrow}\\ \vdots \\ C \Uparrow\end{array}}{C \Uparrow} RuleC$$

where x is not free in any hypotheses.

$$\frac{[t/x]A \Uparrow}{\exists x A \Uparrow} EGen$$

$$\frac{A \Uparrow}{A \vee B \Uparrow} IDisj-L \quad \frac{B \Uparrow}{A \vee B \Uparrow} IDisj-R$$

$$\frac{A \Uparrow \qquad B \Uparrow}{A \wedge B \Uparrow} Conj$$

$$\frac{\bot \downarrow}{A \Uparrow} Absurd$$

$$\frac{\begin{array}{c}\overline{\neg A \downarrow}\\ \vdots \\ \bot \Uparrow\end{array}}{A \Uparrow} Indirect \quad \frac{\begin{array}{c}\overline{A \downarrow}\\ \vdots \\ \bot \Uparrow\end{array}}{\neg A \Uparrow} NegIntro \quad \frac{\begin{array}{c}\overline{A \downarrow}\\ \vdots \\ B \Uparrow\end{array}}{A \supset B \Uparrow} Deduct$$

$$\frac{\neg A \downarrow \qquad A \Uparrow}{C \Uparrow} NegElim$$

$$\frac{A \vee B \downarrow \qquad \begin{matrix} \overline{A \downarrow} \\ \vdots \\ C \Uparrow \end{matrix} \qquad \begin{matrix} \overline{B \downarrow} \\ \vdots \\ C \Uparrow \end{matrix}}{C \Uparrow} Cases$$

TPS also has rules *Cases3* and *Cases4* which may be used to eliminate disjunctions with three or four disjuncts, resp. Such rule applications are replaced by iterations of the binary *Cases* rule in a preprocessing step using expand-cases.

Next, the basic rules which allow one to infer extraction deductions ($\downarrow$).

$$\frac{A \wedge B \downarrow}{A \downarrow} Conj \qquad \frac{A \wedge B \downarrow}{B \downarrow} Conj$$

$$\frac{\forall x A \downarrow}{[t/x]A \downarrow} UI$$

$$\frac{A \supset B \downarrow \qquad A \Uparrow}{B \downarrow} MP$$

Notice that hypothesis lines are always considered extraction derivations. Such lines may be justified by any of the following: Hyp, $Choose$, $Assumenegation$, $Case1$, $Case2$, $Case3$, $Case4$.

We need a coercion rule, as every extraction is a normal derivation:

$$\frac{A \downarrow}{A \Uparrow} coercion$$

In a TPS natural deduction style proof, this coercion step will not usually be explicit. Instead, a single line will be given the property of being a coercion, in which case we know it has both annotations $\downarrow$ and $\Uparrow$, and that these annotations were assigned in a way consistent with the coercion rule above. Often, when interactively constructing a natural deduction proof in TPS, one finds that a planned line is the same as a support line, and finishes the subgoal using SAME. This would correspond to the coercion rule above.

The backward coercion rule

$$\frac{A \Uparrow}{A \downarrow} bcoercion$$

is used to pass from normal deductions to extractions. Backwards coercions correspond to instances of cut. A separate, interesting project in TPS would be to program a normalization procedure. Such a procedure would find instances of the backward coercion rule when annotating a proof, identify to what kind of "redex" the backward coercion rule corresponds, and perform the reduction. For this to work we would need to define the notion of redex

so that every proof which needs the backward coercion rule to be annotated (proofs that are not normal) must have a redex. Also, we could not prove that reduction terminates – a task equivalent to constructively proving cut-elimination in classical higher-order logic. Instead, the current code translates backwards coercion as an application of cut, and then uses a cut elimination procedure (which may not terminate) to obtain a cut-free proof. (See section 4.2)

3.1.3. *Some Nonstandard ND Rules.* There is code to replace "fancy" propositional rules like *RuleP* with subderivations using primitive rules. See the commands ELIMINATE-ALL-RULEP-APPS, ELIMINATE-RULEP-LINE, and ELIMINATE-CONJ*-RULEP-APPS. The command ELIMINATE-CONJ*-RULEP-APPS only expands those *RuleP* applications which can be replaced by applications of *Conj* rules. The other two commands use Sunil's fast propositional search to find an expansion proof and uses the tactic BASIC-PROP-TAC to translate back to natural deduction to fill in the gap using only primitive propositional rules.

Of course, the *Same* rule just propagates the annotation. So, with respect to annotations, there are two versions of this rule:

$$\frac{A\downarrow}{A\downarrow}\,SameAs \qquad \frac{A\Uparrow}{A\Uparrow}\,SameAs$$

When converting normal natural deduction to expansion tree proofs, we only consider formulas up to α-conversion, so we can ignore the corresponding ND rule. But effectively, we allow this rule to be annotated in either of two ways, as with the *Same* rule:

$$\frac{\forall x A\downarrow}{\forall y[y/x]A\downarrow}\,AB \qquad \frac{\forall x A\Uparrow}{\forall y[y/x]A\Uparrow}\,AB$$

Neg, NNF, and $NNF-Expand$ can be used to make small first-order inferences (from $\neg\forall x.A$ to $\exists x.\neg A$, etc.). Since we only care about formulas up to $\alpha\beta$ and negation-normal-form, we can treat these rules the same way as the *Same* and AB rules.

Applications of *Assert* (other than $AssertRefl =$) are replaced by explicit hypotheses in a preprocessing step by the function make-assert-a-hyp. So, when building the natree, there should be no instances of *Assert* other than $Refl =$. $AssertRefl =$ is annotated as a normal deduction:

$$\frac{}{A = A\Uparrow}\,AssertRefl =$$

The idea is that we work backwards to an instance of reflexivity.

Definitions can be eliminated or introduced, and the annotations reflect this. Also, elimination and introduction of definitions includes some β-reduction. Suppose the abbreviation A is defined to be $\lambda x_1 \cdots \lambda x_n.\psi[x_1,\ldots,x_n]$

in the following annotated rule schemas.

$$\frac{\phi[A\ B_1\ \cdots\ B_n] \downarrow}{\phi[\psi[B_1,\ldots,B_n]] \downarrow} Defn$$

$$\frac{\phi[\psi[B_1,\ldots,B_n]] \Uparrow}{\phi[A\ B_1\ \cdots\ B_n] \Uparrow} Defn$$

When annotating λ rules, the arrows point in the direction of normalization.

$$\frac{B \downarrow}{A \downarrow} Lambda \qquad \frac{A \Uparrow}{B \Uparrow} Lambda$$

where A is the $\beta\eta$-normal form of B. There are also rules *Beta* and *Eta* which are treated similarly.

3.1.4. *Equality Rules.* We assume that the proof has been preprocessed to remove applications of substitution of equals, and applications of symmetry, so there is no need to annotate these rules (for now).

As noted above, reflexivity is treated as a normal deduction:

$$\frac{}{A = A \Uparrow} Refl =$$

There are two ways to apply extensionality consistent with the idea of annotations (both correspond to expanding an equation using extensionality in the corresponding expansion tree). Also, there are two kinds of extensionality (functional and propositional).

$$\frac{f_{\beta\alpha} = g_{\beta\alpha} \downarrow}{\forall x_\alpha . fx = gx \downarrow} Ext = \qquad \frac{P_o = Q_o \downarrow}{P_o \equiv Q_o \downarrow} Ext =$$

$$\frac{\forall x_\alpha . fx = gx \Uparrow}{f_{\beta\alpha} = g_{\beta\alpha} \Uparrow} Ext = \qquad \frac{P_o \equiv Q_o \Uparrow}{P_o = Q_o \Uparrow} Ext =$$

Leibniz equality is handled just like definition expansion.

$$\frac{A_\alpha = B_\alpha \downarrow}{\forall q_{o\alpha} . qA \supset qB \downarrow} Equiv - eq$$

$$\frac{\forall q_{o\alpha} . qA \supset qB \Uparrow}{A_\alpha = B_\alpha \Uparrow} Equiv - eq$$

3.1.5. *A Sequent Calculus.* In the file ftree-seq.lisp, a sequent calculus is implemented. The file contains code to convert sequent calculus derivations into expansion proofs, and a cut elimination algorithm. This is a two sided sequent calculus with sequents $\Gamma \to \Delta$. The code refers to formulas in Γ as positive (as opposed to "left") and formulas in Δ as negative (as opposed to "right") to correspond to the parity of expansion tree nodes. Γ and Δ are lists, as opposed to multisets or sets, so order and multiplicity are important.

There are many variations of sequent calculi for classical logic. For example, consider the two variants of the negative rule for $\wedge$:

$$\frac{\Gamma \to A, \Delta \qquad \Gamma \to B, \Delta}{\Gamma \to A \wedge B, \Delta}$$

$$\frac{\Gamma_1 \to A, \Delta_1 \qquad \Gamma_2 \to B, \Delta_2}{\Gamma_1, \Gamma_2 \to A \wedge B, \Delta_1, \Delta_2}$$

Furthermore, there is the issue of the positions of the main formulas (i.e., must A and B be the first formulas on the list?) Different kinds of rules determine what structural rules the sequent calculus should have. The sequent calculus implemented in ftree-seq has the following logical rules:

$$\frac{\Gamma_1 \to A, \Delta_1 \qquad \Gamma_2 \to B, \Delta_2}{\Gamma_1, \Gamma_2 \to A \wedge B, \Delta_1, \Delta_2} \wedge - \qquad \frac{A, B, \Gamma \to \Delta}{A \wedge B, \Gamma \to \Delta} \wedge +$$

$$\frac{\Gamma \to A, B, \Delta}{\Gamma \to A \vee B, \Delta} \vee - \qquad \frac{A, \Gamma_1 \to \Delta_1 \qquad B, \Gamma_2 \to \Delta_2}{A \vee B, \Gamma_1, \Gamma_2 \to \Delta_1, \Delta_2} \vee +$$

$$\frac{A, \Gamma \to B, \Delta}{\Gamma \to A \supset B, \Delta} \supset - \qquad \frac{\Gamma_1 \to A, \Delta_1 \qquad B, \Gamma_2 \to \Delta_2}{A \supset B, \Gamma_1, \Gamma_2 \to \Delta_1, \Delta_2} \supset +$$

$$\frac{A, \Gamma \to \Delta}{\Gamma \to \neg A, \Delta} \neg - \qquad \frac{\Gamma \to A, \Delta}{\neg A, \Gamma \to \Delta} \neg +$$

$$\frac{A(a), \Gamma \to \Delta}{\forall x A(x), \Gamma \to \Delta} SEL+^a \qquad \frac{\Gamma \to A(a), \Delta}{\Gamma \to \forall x A(x), \Delta} SEL-^a$$

$$\frac{A(t), \Gamma \to \Delta}{\forall x A(x), \Gamma \to \Delta} EXP+^t \qquad \frac{\Gamma \to A(t), \Delta}{\Gamma \to \exists x A(x), \Delta} EXP-^t$$

There are also rewrite rules:

$$\frac{\Gamma \to [A \supset B] \wedge [B \supset A], \Delta}{\Gamma \to A \equiv B, \Delta} REW(\equiv)-$$

$$\frac{[A \supset B] \wedge [B \supset A], \Gamma \to \Delta}{A \equiv B, \Gamma \to \Delta} REW(\equiv)+$$

$$\frac{\Gamma \to A^{\lambda,\beta,\eta}, \Delta}{\Gamma \to A, \Delta} REW(\lambda,\beta,\eta)- \qquad \frac{A^{\lambda,\beta,\eta}, \Gamma \to \Delta}{A, \Gamma \to \Delta} REW(\lambda,\beta,\eta)+$$

where $A^{\lambda,\beta,\eta}$ is either the $\beta\eta$-normal form, β-normal form, or η-normal form of A.

$$\frac{\Gamma \to A, \Delta}{\Gamma \to B, \Delta} REW(AB)- \qquad\qquad \frac{A, \Gamma \to \Delta}{B, \Gamma \to \Delta} REW(AB)+$$

where A and B are α-equivalent.

$$\frac{\Gamma \to A, \Delta}{\Gamma \to B, \Delta} REW(EQUIVWFFS)-$$

$$\frac{A, \Gamma \to \Delta}{B, \Gamma \to \Delta} REW(EQUIVWFFS)+$$

where A is the result of expanding some abbreviations in B.

$$\frac{\Gamma \to A, \Delta}{\Gamma \to B, \Delta} REW(Leibniz =)- \qquad\qquad \frac{A, \Gamma \to \Delta}{B, \Gamma \to \Delta} REW(Leibniz =)+$$

where A is the result of expanding some equalities in B using the Leibniz definition of equality.

$$\frac{\Gamma \to A, \Delta}{\Gamma \to B, \Delta} REW(Ext =)- \qquad\qquad \frac{A, \Gamma \to \Delta}{B, \Gamma \to \Delta} REW(Ext =)+$$

where A is the result of expanding some equalities in B using extensionality. (This does not provide a complete calculus for extensionality without a cut rule. So, sometimes cut elimination will fail if these extensionality rules are used.)

The structural rules are:

$$\frac{\Gamma \to \Delta}{\Gamma \to A, \Delta} weaken- \qquad\qquad \frac{\Gamma \to \Delta}{A, \Gamma \to \Delta} weaken+$$

$$\frac{\Gamma \to A, A, \Delta}{\Gamma \to A, \Delta} merge- \qquad\qquad \frac{A, A, \Gamma \to \Delta}{A, \Gamma \to \Delta} merge+$$

$$\frac{\Gamma \to \Delta_1, A, \Delta_2}{\Gamma \to A, \Delta_1, \Delta_2} focus^n+ \text{ where } \Delta_1 \text{ has length } n.$$

$$\frac{\Gamma_1, A, \Gamma_2 \to \Delta}{A, \Gamma_1, \Gamma_2 \to \Delta} focus^n+ \text{ where } \Gamma_1 \text{ has length } n.$$

Finally, we have an initial rule and a cut rule:

$$\frac{}{A \to A} init \qquad\qquad \frac{\Gamma_1 \to A, \Delta_1 \qquad A, \Gamma_2 \to \Delta_2}{\Gamma_1, \Gamma_2 \to \Delta_1, \Delta_2} cut$$

In all these rules, it is important that the formulas appear in the positions as indicated in the diagrams above. The focus rule gives us the only way to reorder the formulas of the sequent. This forces us to do some tedious shuffling in some places, but makes it easier to perform recursion on the sequent derivations, since we have a very good idea of how the rule application looks.

The sequent calculus is similar to the ftree representation of expansion trees (see section 1.4), and the file includes a function cutfree-ftree-seq-to-ftrees which translates a cut-free sequent calculus derivation of $\Gamma \to \Delta$ to two lists of ftrees Γ^* and Δ^*, and a list of connections M. For each wff $A \in \Gamma$, there is a correponding positive ftree $F \in \Gamma^*$ with shallow formula A. For each wff $A \in \Delta$, there is a correponding negative ftree $F \in \Delta^*$ with shallow formula A. The list of connections M gives a complete mating for the ftree $\bigwedge(\Gamma^*) \supset \bigvee(\Delta^*)$. In particular, a cut-free sequent calculus derivation of $\to A$ will be translated into a negative ftree with shallow formula A and a complete mating M.

Regarding this translation to ftrees, the names of the logical and rewrite rules correspond to the construction of the corresponding ftree. The *weaken* and *focus* structural rules are relatively easy to handle. Applications of *merge* require the use of a *merge* algorithm for ftrees (in the file ftrees). The *init* rule corresponds to two mated nodes. And, of course, we cannot translate an application of *cut*.

3.1.6. *Translating from Natural Deduction to Sequent Calculus.* In the sequent calculus described above, the order and multiplicity of formulas is important. However, in describing the algorithm below, we are more interested in sets of formulas. So, let us use the notation $Set(\Gamma)$ to denote the set of fomulas on the list Γ.

Normal natural deductions are converted into the sequent calculus via two mutually recursive algorithms:

(1) natree-to-ftree-seq-normal: Suppose we are given a line $\Gamma \vdash C \Uparrow$. Then we can compute a derivation of a sequent $\Gamma_1 \to C$ where $Set(\Gamma_1) \subseteq Set(\Gamma)$.
(2) natree-to-ftree-seq-extraction: Given a line $\Gamma \vdash B \downarrow$ and a derivation of a sequent $\Gamma_1 \to C$ where $Set(\Gamma_1) \subseteq Set(\Gamma) \cup \{B\}$. Then we can compute a derivation of a sequent $\Gamma_2 \to C$ where $Set(\Gamma_2) \subseteq Set(\Gamma)$. (That is, we have eliminated occurrences of B on the positive side.)

We can show a few cases to demonstrate how the algorithms work.

Case: Coercion.

$$\frac{C \downarrow}{C \Uparrow} coercion$$

with hypotheses Γ. We need a derivation of some $\Gamma_2 \rightarrow C$. We can apply the second induction hypothesis to the initial sequent $C \rightarrow C$ (with Γ_1 empty) to obtain a derivation of such a sequent $\Gamma_2 \rightarrow C$.

Case: Hyp. Suppose the line is

$$\Gamma \vdash B$$

where B is in Γ, and suppose we are given a derivation of a sequent $\Gamma_1 \rightarrow C$ with $Set(\Gamma_1) \subseteq Set(\Gamma) \cup \{B\}$. Since $B \in \Gamma$, we have $Set(\Gamma_1) \subseteq Set(\Gamma)$ and we are done.

Case: Deduct. This case is easy, as are most of the "introduction" rules. Suppose we have

$$\frac{\begin{array}{c}\mathfrak{D}\\ \Gamma, A \vdash B \Uparrow\end{array}}{\Gamma \vdash A \supset B \Uparrow} Deduct$$

By induction we have a derivation of a sequent $\Gamma_1 \rightarrow B$ where $Set(\Gamma_1) \subseteq Set(\Gamma) \cup \{A\}$. Using the structural rules (see the function ftree-seq-merge-focus-all-pos) we obtain a derivation with A at the front: $A, \Gamma_2 \rightarrow B$ where $Set(\Gamma_2) \subseteq Set(\Gamma)$. Applying the $\supset-$ rule, we have a derivation of $\Gamma_2 \rightarrow A \supset B$ as desired.

Case: MP. This case is interesting, because a naive algorithm would be forced to treat this case like a "cut" in the sequent calculus. Suppose we have

$$\frac{\begin{array}{c}\mathfrak{D}\\ \Gamma \vdash A \supset B \downarrow\end{array} \qquad \begin{array}{c}\mathfrak{E}\\ \Gamma \vdash A \Uparrow\end{array}}{\Gamma \vdash B \downarrow} MP$$

Since this is an extraction, we must be given a derivation of $\mathfrak{D}_1$ a sequent $\Gamma_1 \rightarrow C$ where $Set(\Gamma_1) \subseteq Set(\Gamma) \cup \{B\}$. Applying structural rules to $\mathfrak{D}_1$, we have a derivation $\mathfrak{D}_2$ of a sequent $B, \Gamma_2 \rightarrow C$ with $Set(\Gamma_2) \subseteq Set(\Gamma)$.

The first algorithm applied to $\mathfrak{E}$ gives a derivation of some $\Gamma_3 \rightarrow A$ where $Set(\Gamma_3) \subseteq Set(\Gamma)$. If we apply $\supset+$ as follows:

$$\frac{\Gamma_3 \rightarrow A \qquad B, \Gamma_2 \rightarrow C}{A \supset B, \Gamma_3, Gamma_2 \rightarrow C} \supset+$$

then we can call the second algorithm on this derivation and $\mathfrak{D}$ to obtain a derivation of some $\Gamma_4 \rightarrow C$ with $Set(\Gamma_4) \subseteq Set(\Gamma)$.

Case: Backwards Coercion.

$$\frac{\begin{array}{c}DD\\ B \Uparrow\end{array}}{B \downarrow} bcoercion$$

with hypotheses Γ. Suppose we are given a derivation of some $\Gamma_1 \to C$ where $Set(\Gamma_1) \subseteq Set(\Gamma) \cup \{B\}$. Using structureal rules we obtain a derivation of a sequent $B, \Gamma_2 \to C$ where $Set(\Gamma_2) \subseteq Set(\Gamma)$. We want to remove B from the positive side. Applying the first algorithm to $\mathfrak{D}$, we obtain a derivation of a sequent $\Gamma_3 \to B$ with $Set(\Gamma_3) \subseteq Set(\Gamma)$. An application of *cut* gives us the sequent we desire:

$$\frac{\Gamma_3 \to B \qquad B, \Gamma_2 \to C}{\Gamma_3, \Gamma_2 \to C}\,cut$$

Remark: We check equality of wff's up to α-conversion and negation-normal-form. Because we check up to negation-normal-form, applications of *Neg* and *NNF* rules can be treated the same way as the *Same* and *AB* rules.

Note: If NATREE-DEBUG is set to T, then at each step, the code double checks that the derivation is well formed.

After we have a sequent calculus derivation, cut elimination can be used to try to remove applications of cut (see section 4.2. If we obtain a cut-free derivation, this can be translated into an ftree with a complete mating.

3.1.7. *Normalization of Proofs.* There is now a TPS3 command NORMALIZE-PROOF that converts a natural deduction proof (or a natural deduction proof with asserted lemmas which have natural deduction proofs in memory) into a sequent calculus proof (with cuts), then uses the cut-elimination algorithm to obtain a cut-free proof (assuming termination), and finally translates back to natural deduction. The resulting natural deduction proof is normal.

If we decided to normalize natural deduction proofs directly (without passing through a sequent calculus), we would need to identify possible redexes (pairs of rule applications which must use backward coercion to be annotated), and show how to reduce these. There are many such redexes. The following is a typical example:

$$\frac{\dfrac{\begin{matrix}\mathfrak{D} \\ A\end{matrix} \qquad \begin{matrix}\mathfrak{E} \\ B\end{matrix}}{A \wedge B \downarrow}\,Conj}{A \downarrow}\,Conj \to \begin{matrix}\mathfrak{D} \\ A\end{matrix}$$

In first order logic, one can show that some measure on the proof reduces when a redex is reduced, so that the process will terminate with a normal proof. In higher order logic, showing termination is equivalent to showing termination of cut-elimination.

Actually carrying this out is a possible future project. Though this is much less important since we now have a cut elimination algorithm implemented.

3.2. Hongwei's Nat-Etree. This is a brief description of Hongwei's code for NAT-ETREE. To use this code, set NAT-ETREE-VERSION to **HX**.

ATTACH-DUP-INFO-TO-NATREE is the main function, which is called recursively on the subproofs of a given natural deduction. The goal of ATTACH-DUP-INFO-TO-NATREE is to construct an expansion tree, with no mating attached, corresponding to a given natural deduction. The constructed expansion tree contains all the correct duplications done on quantifiers and all substitutions done on variables. A propositional search will be called on the generated expansion tree to recover the mating and generate an expansion proof. Then ETREE-NAT can produce a natural deduction corresponding to the constructed expansion proof.

The following is an oversimplified case.

Given natural deductions N1 and N2 with conclusions A and B, respectively, and N derived from N1 and N2 by conjunction introduction. ATTACH-DUP-INFO-TO-NATREE called on N generates two recursive calls on N1 and N2, and get the expansion proofs corresponding to N1 and N2, respectively, with which it constructs an expansion proof corresponding to N.

An important feature of ATTACH-DUP-INFO-TO-NATREE is that it can deal with all natural deductions, with or without cuts in them. This is mainly achieved by substitution and merge. This essentially corresponds to the idea in Frank Pfenning's thesis, though his setting is sequent calculus. On the other hand, the implementation differs significantly since natural deductions grow in both ways when compared with sequent calculus. This is reflected in the code of ATTACH-DUP-INFO-TO-NATREE which travels through a natural deduction twice, from bottom to top and from top to bottom, to catch all the information needed to duplicate quantifiers correctly.

Overview of the files:

- hx-natree-top contains the definition of the data structure, some print functions and top commands.
- hx-natree-duplication contains the code of ATTACH-DUP-INFO-TO-NATREE and some auxiliary functions such as UPWARD-UPDATE-NATREE. Also many functions for constructing expansion trees are defined here.
- hx-natree-rulep contains the code for handling RULEP. This is done by using hash tables to store positive and negative duplication information. Then cuts are eliminated by substitution and merge. The case in ATTACH-DUP-INFO-TO-NATREE which deals with implication is a much simplified version of this strategy, and helps understand the algorithm.
- hx-natree-aux contains the code of merge functions and the ones handling rewrite nodes. Presumably there are some bugs in handling rewrites, and this can be found in the comments mixed with

the code. Also a new version of ETREE-TO-JFORM-REC is defined here to cope with a modified date structure ETREE.

- hx-natree-cleanup contains the functions which clean up the expansion proofs before they can be used by ETREE-NAT. This is temporary crutch, and should be replaced by some systematic methods. For instance, one could construct brand new expansion proofs according to a constructed one rather than modify it to fit the needs of ETREE-NAT. This yields a better chance to avoid some problems caused by rewrite nodes.
- hx-natree-debug contains some simple debugging facilities such as some display function and some modified versions of the main functions in the code. A suggested way is to modify the code using these debugging functions and trace them. More facilities are needed to eliminate sophisticated bugs.

Selection nodes, not Skolem nodes, are used in the constructed expansion trees. The prevents us from setting the MIN-QUANT-ETREE flag to simplify a proof. It is a little daunting task to modify the code for MIN-QUANT-ETREE, but the benefits are also clear: both NAT-ETREE and non-pfd procedures can take advantage of the modification.

3.3. The Original Nat-Etree. Note: What follows is a description of how NAT-ETREE used to work. To use this code set NAT-ETREE-VERSION to **OLD**.

Legend has it that the code was written by Dan Nesmith and influenced by the ideas of Frank Pfenning. Frank's thesis contains ideas for translating from a cut-free sequent calculus to expansion tree proofs.

(1) Important files: nat-etr (defines functions which are independent of the particular rules of inference used); ml-nat-etr1 and ml-nat-etr2 (which define translations for the rules in the standard TPS).
(2) There are three global variables which are used throughout the translation process: DPROOF, which is the nproof to be translated; LINE-NODE-LIST, which is an association list which associates each line of the proof to the node which represents it in the expansion tree which is being constructed; MATE-LIST, which is a list of connections in the expansion proof which is being constructed.
(3) At the beginning of the translation process, the current proof is copied because modifications will be made to it. (It is restored when the translation is complete.) The copy is stored in the variable DPROOF. Next the function SAME-IFY is called. This attempts to undo the effects of the CLEANUP function, and to make explicit the "connections" in the proof. This is done because, in an nproof, a single line can represent more than one node in an expansion proof. SAME-IFY tries to add lines to the proof in such a way that each line corresponds to exactly one expansion tree node.

(4) After the proof has been massaged by SAME-IFY, the initial root node of the expansion tree is constructed. This node is merely a leaf whose shallow formula is the assertion of the last line of the nproof. LINE-NODE-LIST is initialized to contain just the association of this leaf node with the last line of the proof, and MATE-LIST is set to nil.

(5) Next the function NAT-XLATE is called on the last line of the proof. NAT-XLATE, depending on the line's justification, calls auxiliary functions which carry out the translation, and which usually call NAT-XLATE recursively to translate lines by which the current line is justified. When the justification "Same as" is found, this indicates that the node associated with this line and the node which is associated with the line it is the same as should be mated in the expansion proof.

(6) Example: Suppose we have the following nproof:

```
(1) 1  !  A            Hyp
(2)    !  A implies A  Deduct: 1

SAME-IFY will construct the new proof:

(1) 1  !  A            Hyp
(2) 1  !  A            Same as: 1
(3)    !  A implies A  Deduct: 2
```

Then a leaf node LEAF0 is constructed with shallow formula "A implies A", and LINE-NODE-LIST is set to ((3 . LEAF0)). NAT-XLATE is called, and because line 3 is justified using the deduction rule, LEAF0 is deepened to an implication node, say IMP0, with children LEAF1 and LEAF2. Then LINE-NODE-LIST is updated to be ((1 . LEAF1) (2 . LEAF2) (3 . IMP0)), and NAT-XLATE is called recursively on lines 1 and 2. Since line 1 is justified by "Hyp", NAT-XLATE does nothing. Since line 2 is justified by "Same as: 1", NAT-XLATE updates the value of MATING-LIST to (("LEAF1" . "LEAF2")), a connection consisting of the nodes which represent lines 1 and 2.

(7) In an nproof that is not cut-free, there will exist lines which do not arise from deepening the expansion tree which represents the last line of the nproof. Currently, NAT-XLATE will get very confused and probably blow up. The justification "RuleP" causes other difficulties, because it generally requires that several connections be made, involving lines whose nodes haven't been deepened to the literal level yet. The function XLATE-RULEP attempts to do this, but does not always succeed. This is true because RULEP can also be used to justify a line whose node is actually a child of the justifying line, e.g.:

```
(45)  ! A and B
(46)  ! A          RuleP: 45
```

Though XLATE-RULEP can handle this situation, it cannot handle more complex ones such as:

```
(16)  ! A
(17)  ! A implies B
(18)  ! B            RuleP: 16 17
```

Ideally, SAME-IFY would identify these situations before the translation process is begun, but it does not.

4. Cut Elimination

A cut elimination algorithm is worked out in Frank's Thesis. First he defines a notion of expansion development (a sort of sequent calculus for sets of expansion trees with rules for quantifiers, merging, and cuts). Then he gives reductions on expansion developments with the hope that these reductions result in an expansion tree. Frank's algorithm is not currently implemented as part of TPS. Of course, there are many ways of representing cuts and performing cut elimination.

4.1. An Example of a Loop in a Cut Elimination Algorithm. One approach we tried (Chad, Summer 2001) was to include explicit CUT and MERGE nodes in expansion trees, defining redexes, and doing cut elimination by contracting redexes. This section contains a brief outline of the approach and an example that shows how a loop can occur.

A CUT node is of the form

$$\begin{array}{ccc} & CUT & \\ \swarrow & \downarrow & \searrow \\ C^* & B^* & B^{**} \end{array}$$

where C^* has shallow formula C, and B^* and B^{**} have the same shallow formula, but opposite polarity. The polarity and shallow formula of the CUT node is the same as the polarity of the shallow formula of C^*. The deep formula of the cut node is

$$deep(C^*) \wedge [deep(B^*) \vee deep(B^{**})]$$

A MERGE node is of the form

$$\begin{array}{cc} \multicolumn{2}{c}{MERGE} \\ \swarrow & \searrow \\ A^* & A^{**} \end{array}$$

where A^* and A^{**} have the same polarity and same shallow formula A. The shallow formula of the MERGE node is also A. The polarity is also inherited from the children. The deep formula is given by

$$deep(A^*) \wedge deep(A^{**}).$$

With these nodes, the translation from a natural deduction proof (see subsection 3.1) can now be extended to translate backward coercions. Ignoring hypothesis for simplicity, suppose we have a backward coercion

$$\frac{B \Uparrow}{B \downarrow} bcoercion$$

a negative expansion tree C^* with shallow formula C, and a positive expansion tree B^* with shallow formula B. By induction hypothesis, we can obtain an appropriate positive expansion tree B^{**} with shallow formula B. Then the algorithm returns

$$\begin{array}{ccc} & CUT & \\ \swarrow & \downarrow & \searrow \\ C^* & B^* & B^{**} \end{array}$$

There is one more modification required in the algorithm. When translating a hypothesis line, we used the merge algorithm on expansion trees. However, merge is not defined on expansion trees containing CUT nodes. So, instead we make an explicit MERGE node with the two trees as children. The actual merging would be done during cut/merge elimination. That is, if we are translating a hypothesis line

$$A_1, \ldots, A_n \vdash A_j,$$

then we must start with positive expansion trees A_i^{**}, a positive expansion tree A_i^*, and a negative expansion tree C^*. Instead of associating the hypothesis line with the merge of A_i^* and A_i^{**}, we now associate the line with the node

$$\begin{array}{cc} \multicolumn{2}{c}{MERGE} \\ \swarrow & \searrow \\ A_i^* & A_i^{**} \end{array}$$

The most interesting case is eliminating a CUT between a selection and an expansion node. There may be multiple expansion nodes and the acyclicity of the dependency relation may affect the order in which the expansion terms are processed. We were trying to reduce such a CUT by doing a global merge of two modified expansion trees. Suppose the expansion tree is Q and contains a CUT node of the form

$$\begin{array}{ccc} & CUT & \\ \swarrow & \downarrow & \searrow \\ C & SEL : \forall x.A(x) & EXP : \forall x.A(x) \\ & \downarrow a & t_1 \swarrow \quad \downarrow * \quad \searrow t_n \\ & P(a) : A(a) & P_1 : A(t_1) \quad \cdots \quad P_n : A(t_n) \end{array}$$

Let us use the notation $Q[CUT]$ to indicate this CUT node inside the larger tree Q. Assuming we do not have a problem with acyclicity, the CUT node

should reduce as follows:

$$\begin{array}{ccc} & MERGE & \\ \swarrow & & \searrow \\ \{t_1/a\}Q[CUT1] & & Q[CUT2] \end{array}$$

where $CUT1$ is

$$\begin{array}{ccc} & CUT1 & \\ \swarrow & \downarrow & \searrow \\ C & P(t_1) : A(t_1) & P_1 : A(t_1) \end{array}$$

and $CUT2$ is

$$\begin{array}{ccccccc} & & & CUT2 & & & \\ \swarrow & \downarrow & & & & \searrow & \\ C & SEL : \forall x.A(x) & & & & EXP : \forall x.A(x) & \\ & \downarrow a & & t_2 \swarrow & & \downarrow * & \searrow t_n \\ & P : A(a) & & P_2 : A(t_2) & & \cdots & P_n : A(t_n) \end{array}.$$

However, this leads to a loop as in the following simple example. Consider the following proof of

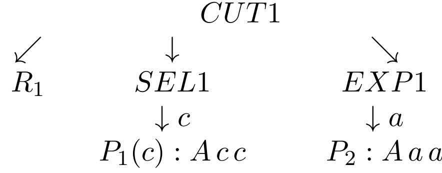
$$\forall x_\iota \forall y_\iota\, A_{o\iota\iota}\, x\, y \supset A\, a_\iota\, a \wedge A\, b_\iota\, b$$

using the lemma

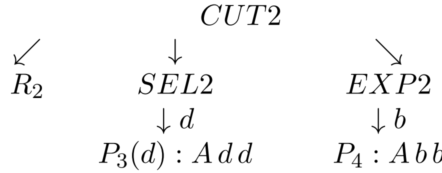
$$\forall z_\iota\, A_{o\iota\iota}\, z\, z.$$

(1)	1	$\vdash \forall x_\iota \forall y_\iota\, A_{o\iota\iota}\, x\, y$	Hyp
(2)	1	$\vdash \forall y_\iota\, A_{o\iota\iota}\, z_\iota\, y$	UI: z_ι 1
(3)	1	$\vdash A_{o\iota\iota}\, z_\iota\, z$	UI: z_ι 2
(4)	1	$\vdash \forall z_\iota\, A_{o\iota\iota}\, z\, z$	UGen: z_ι 3
(5)	1	$\vdash A_{o\iota\iota}\, a_\iota\, a$	UI: a_ι 4
(6)	1	$\vdash A_{o\iota\iota}\, b_\iota\, b$	UI: b_ι 4
(7)	1	$\vdash A_{o\iota\iota}\, a_\iota\, a \wedge A\, b_\iota\, b$	Conj: 5 6
(8)		$\vdash \forall x_\iota \forall y_\iota\, A_{o\iota\iota}\, x\, y \supset A\, a_\iota\, a \wedge A\, b_\iota\, b$	Deduct: 7

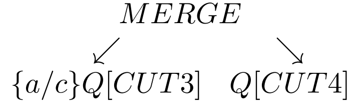
When translating this proof we obtain a tree Q with two cut nodes



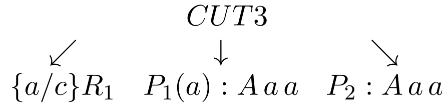
and



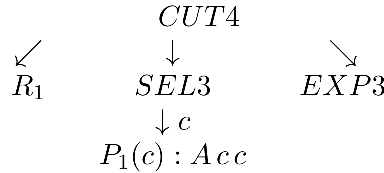
corresponding to the two applications of the lemma in line 4. Contracting $CUT1$ gives



where $CUT3$ and $CUT4$ are



and



Both $CUT3$ and $CUT4$ are easy to eliminate. ($CUT3$ can be replaced by $\{a/c\}R_1$ and $CUT4$ can be replaced by R_1, with appropriate changes to the complete mating.) However, note that $CUT2$ appears in both sides of the merge in

Let us call these two occurrences $CUT2.1$ and $CUT2.2$. Eventually, we will want to reduce one of these cuts in some reduced tree $Q'[CUT2.1][CUT2.2]$. Suppose we reduce $CUT2.1$. Similar to the reduction of $CUT1$ above, this will copy $CUT2.2$ so that there are $CUT2.2.1$ and $CUT2.2.2$. The loop is evident.

It is conceivable that we could get around this loop by developing a notion of "expansion DAG" (directed acyclic graph) so that the CUT would not actually be duplicated. But it isn't clear that this would eliminate all such loops.

Also, there are some technical problems with the reductions above. For instance, a selection node may get copied, which means the result will be a tree with two selection nodes that use *the same* selected variable. This doesn't seem to be a serious problem because we could probably allow such a situation whenever the least common ancestor of two such selection nodes is a MERGE node. But these details would have to be worked out to make this approach work.

4.2. Cut and Mix Elimination in this Sequent Calculus. There is a cut elimination algorithm implemented for the sequent calculus described in section 3.1.5. Suppose we have an instance of the cut rule:

$$\frac{\begin{matrix}\mathfrak{D}_1\\ \Gamma_1 \to A, \Delta_1\end{matrix} \qquad \begin{matrix}\mathfrak{D}_2\\ A, \Gamma_2 \to \Delta_2\end{matrix}}{\Gamma_1, \Gamma_2 \to \Delta_1, \Delta_2}\ cut$$

A cut elimination algorithm should take cut-free derivations $\mathfrak{D}_1$ and $\mathfrak{D}_2$ and return a cut-free derivation $\mathfrak{E}$ of $\Gamma_1, \Gamma_2 \to \Delta_1, \Delta_2$. This is usually done by a recursive algorithm on the two derivations where at each step either one of the derivations gets smaller, or the cut formula gets smaller (while the derivations may get much larger). Since we have an explicit *merge* (contraction) rule, we have the following problematic case. Suppose $\mathfrak{D}_1$ ends with an application of *merge*:

$$\frac{\begin{matrix}\mathfrak{D}_3\\ \Gamma_1 \to A, A, \Delta_1\end{matrix}}{\Gamma_1 \to A, \Delta_1}\ merge-$$

where A is the cut formula. Also, suppose A is the principal formula of $\mathfrak{D}_2$. The most natural way to handle this case is to first perform cut elimination on $\mathfrak{D}_3$ and $\mathfrak{D}_2$, giving a cut-free derivation

$$\begin{matrix}\mathfrak{D}_5\\ \Gamma_1, \Gamma_2 \to A, \Delta_1, \Delta_2\end{matrix}$$

Then we could call cut elimination again with $\mathfrak{D}_5$ and $\mathfrak{D}_2$ to obtain a cut-free derivation

$$\begin{matrix}\mathfrak{D}_6\\ \Gamma_1, \Gamma_2, \Gamma_2 \to \Delta_1, \Delta_2, \Delta_2\end{matrix}$$

With some applications of *focus* and *merge* to shuffle and merge the formulas, we would have the cut-free derivation of

$$\begin{matrix}\mathfrak{D}_7\\ \Gamma_1, \Gamma_2 \to \Delta_1, \Delta_2\end{matrix}$$

we desire. *However,* the derivation $\mathfrak{D}_5$ will in general be bigger than $\mathfrak{D}_3$, while the cut formula A remains the same. So, this recursive call to cut elimination may not terminate (even with a first-order derivation).

We can get around this problem by performing mix-elimination (as did Gentzen [Gen69]) instead of cut-elimination. Because we care about the order of formulas in the sequent, mix elimination is complicated. The main functions are ftree-seq-mix-elim-1 and ftree-seq-mix-elim-principal. To understand mix elimination, first consider a few generic examples:

Suppose we have two cut-free derivations of

$$B \to A, C$$

and

$$D, A \to E$$

where A is the mix formula, in negative position 0 in the first sequent and positive position 1 in the second sequent. Mix elimination might return a cut-free derivation of

$$B, D \to C, E$$

along with two lists of indices

(1) $((t\ .\ 0)\ (nil\ .\ 0))$ (indicating that B corresponds to the 0'th positive formula of the first sequent, and D corresponds to the 0'th positive formula of the second sequent)
(2) $((t\ .\ 1)\ (nil\ .\ 0))$ (indicating that C corresponds to negative position 1 in the first sequent and E corresponds to negative position 0 in the second sequent)

We say "might" because the return value depends, of course, on the given derivations, not just the sequents. Other possible outputs include a cut-free derivation of

$$B \to C, C$$

with two lists of indices

(1) $((t\ .\ 0))$ (again, B corresponds to positive position 0 in the first sequent), and
(2) $((t\ .\ 1)\ (t\ .\ 1))$ (both occurrences of C correspond to negative position 1 in the first sequent).

The point is that we have eliminated the two mix formulas (two occurrences of A) and retain only residues of the other formulas in the two given sequents. The other formulas may occur several times, or not at all. The lists of indices indicate where the formulas originally occured. An "index" is a pair $(< bool >\ .\ < nat >)$ where

- If $< bool >$ is t, the formula is the residue of a formula in the first sequent.
- If $< bool >$ is nil, the formula is the residue of a formula in the second sequent.
- $< nat >$ is a natural number indicating the position of the formula in the first or second sequent.

For another example, given two cut-free derivations of

$$B, C \to A, A, D$$

and

$$E, A, F \to G$$

with mix formulas might return

- – a cut-free derivation of $C, F, E \to G, D$,
  – positive indices $((t\ .\ 1)\ (nil\ .\ 2)\ (nil\ .\ 0))$, and
  – negative indices $((nil\ .\ 0)\ (t\ .\ 2))$
- or,
  – a cut-free derivation of $B, B \to G$,
  – positive indices $((t\ .\ 0)\ (t\ .\ 0))$, and
  – negative indices $((nil\ .\ 0))$
- or,
  – a cut-free derivation of $F, E \to D, D$,
  – positive indices $((nil\ .\ 2)\ (nil\ .\ 0))$, and
  – negative indices $((t\ .\ 2)\ (t\ .\ 2))$.

In general, we start with two derivations

$$\begin{array}{c}\mathfrak{D}_1\\ \Gamma_1 \to \Delta_1\end{array}$$

and

$$\begin{array}{c}\mathfrak{D}_2\\ \Gamma_2 \to \Delta_2\end{array}$$

and two lists $(i_1 \cdots i_k)$ and $(j_1 \cdots j_l)$ of natural numbers. The natural numbers give us the positions of the mix formulas in Δ_1 and Γ_2. Let Δ_1 be a list $(A_1 \cdots A_n)$ and Γ_2 be a list $(B_1 \cdots B_m)$. The mix formulas are A_{i_r} and B_{j_s}. These formulas should have a common reduct with respect to λ-reduction, expanding abbreviations, and expansion equalities using either extensionality or Leibniz. Mix elimination returns a cut-free derivation of

$$\Gamma \to \Delta$$

and two lists of $indl^+$ and $indl^-$ of indices indicating the preimage of the formulas in Γ and Δ. Γ and $indl^+$ are lists of the same length. For each B in Γ there is a corresponding index $(b.j)$ where either b is t and j is the position of B in Γ_1 or b is nil and $j \notin \{j_1, \ldots, j_l\}$ is the position of B in Γ_2. Similarly, Δ and $indl^-$ have the same length. For each A in Δ there is a corresponding index $(a.i)$ where either a is t and $i \notin \{i_1, \ldots, i_k\}$ is the position of A in Δ_2 or a is nil and i is the position of A in Δ_2.

The main function which attempts to eliminate all cuts from a derivation is ftree-seq-cut-elim. This is a simple recursive algorithm which first eliminates cuts from premisses, then either imitates the rule or, if the rule was *cut*, uses mix elimination to obtain a cut-free derivation from the cut-free derivations of the premisses. That is, given two derivations

$$\begin{array}{c}\mathfrak{D}_1\\ \Gamma_1 \to A, \Delta_1\end{array}$$

and

$$\begin{array}{c}\mathfrak{D}_2\\ A,\Gamma_2 \rightarrow \Delta_2\end{array}$$

we call mix elimination with these derivations and two lists of positions (0) and (0). Mix elimination returns a cut-free derivation of $\Gamma \rightarrow \Delta$ and two lists of indices. Using the indices and the *focus*, *weaken*, and *merge* rules, we can shuffle the formulas in Γ and Δ to obtain a derivation of $\Gamma_1, \Gamma_2 \rightarrow \Delta_1, \Delta_2$. This final shuffling is performed by ftree-seq-mix-elim-finish.

4.3. The Mix Elimination Algorithm. The mix elimination algorithm works by recursion on the two given derivations. Suppose we are given two cut-free derivations

$$\begin{array}{c}\mathfrak{D}_1\\ \Gamma_1 \rightarrow \Delta_1\end{array}$$

and

$$\begin{array}{c}\mathfrak{D}_2\\ \Gamma_2 \rightarrow \Delta_2\end{array}$$

and two lists $nl_1 = (i_1 \cdots i_k)$ and $nl_2 = (j_1 \cdots j_l)$ indicating the positions of the mix formulas in Δ_1 and Γ_2.

There are several cases to consider:

- **Mix Formula Does Not Occur**. That is, $nl_1 = nil$ or $nl_2 = nil$. If nl_1 is nil, then we can return
  -
  $$\begin{array}{c}\mathfrak{D}_1\\ \Gamma_1 \rightarrow \Delta_1\end{array}$$
  - $((t \,.\, 0) \; \cdots \; (t \,.\, (n-1)))$ where n is the length of Γ_1
  - $((t \,.\, 0) \; \cdots \; (t \,.\, (m-1)))$ where m is the length of Δ_1

  The indices indicate that every formula is the residual of a formula from the first sequent. If nl_2 is nil, we can return $\mathfrak{D}_2$ and indices which indicate all the formulas are residuals of the second sequent.
- **INIT** Suppose $\mathfrak{D}_1$ is an initial sequent $A \rightarrow A$. (A must be the mix formula, or nl_1 must be nil.) Ideally, we would like to return $\mathfrak{D}_2$, and replace the indices for the mix formula occurrences in Γ_2 by the index $(t \,.\, 0)$ indicating the positive A from $\mathfrak{D}_1$. The only problem is that each occurrence of the mix formula in Γ_2 is a B equal to A only in the sense that the two formulas have a common reduct with respect to λ-reduction and expansions of abbreviations and equalities. The function ftree-seq-replace-equivwffs replaces each such B by A in $\mathfrak{D}_2$. We can handle $\mathfrak{D}_2$ initial similarly.
- **FOCUS, WEAKEN, MERGE** If either $\mathfrak{D}_1$ or $\mathfrak{D}_2$ is a *focus*, *weaken*, or *merge* step, we can simply recursively call with the premiss and the positions shuffled appropriately.

- **REWRITES** If either $\mathfrak{D}_1$ or $\mathfrak{D}_2$ is a λ, $EQUIVWFFS$, *Leibniz* $=$, or *Ext* $=$ rewrite, and the mix formula is the principal formula, we can usually simply recursively call the algorithm with the premiss and the same positions. This is possible because we are only maintaining that the mix formulas are the same up to having a common reduct via such rewriting steps. The actual rewriting is done in the INIT step.

  Exception: Nonanalytic Uses of Extensionality Suppose one occurrence of the mix formula is $A[=]$ and another occurrence of the mix formula is $A[\forall q \,.\, q\, M_o \supset q\, N_o]$. If $A[=]$ is the principal formula of an *Ext* $=$ rewrite, then

  $$A[\equiv]$$

  and

  $$A[\forall q \,.\, q\, M_o \supset q\, N_o]$$

  do not have a common reduct. The recursion fails at such a step. In general this problem occurs when one instance of equality is expanded as Leibniz and another corresponding instance is expanded as (functional or propositional) extensionality. The real problem is that the extensionality rules by themselves without cut do not give a complete calculus for extensional higher-order logic. To get a cut-free extensional calculus, one needs to be able to have initial sequents (i.e., "mate") *modulo equations* and then use decomposition rules and extensionality rules to solve the introduced equations. Benzmuller's thesis gives rules for handling extensionality in the context of resolution. The case for sequent calculi should be described in upcoming technical reports by Benzmuller, Brown and Kohlhase. There should also be information in Chad E. Brown's thesis.
- **NONPRINCIPAL LOGICAL RULE** If either $\mathfrak{D}_1$ or $\mathfrak{D}_2$ ends with a logical rule and the principal formula is not a mix formula, then we can recursively call mix elimination on the premisses and imitate the rule. For example, if $\mathfrak{D}_1$ is

  $$\dfrac{\begin{array}{c}\mathfrak{D}_{11}\\ \Gamma_{11} \to A, \Delta_{11}\end{array} \qquad \begin{array}{c}\mathfrak{D}_{12}\\ \Gamma_{12} \to B, \Delta_{12}\end{array}}{\Gamma_{11}, \Gamma_{12} \to A \wedge B, \Delta_{11}, \Delta_{12}} \wedge-$$

  the function ftree-seq-invert-position-list uses the positions of the mix formulas in

  $$A \wedge B, \Delta_{11}, \Delta_{12}$$

  to find the positions of the mix formulas in Δ_{11} and Δ_{12}. Using this we can call mix elimination twice to obtain

  $$\begin{array}{c}\mathfrak{D}_3\\ \Gamma_3 \to \Delta_3\end{array}$$

and

$$\begin{array}{c}\mathfrak{D}_4 \\ \Gamma_4 \rightarrow \Delta_4\end{array}$$

The function ftree-seq-mix-elim-imitate-rule finishes this case. First, ftree-seq-bring-to-front uses structural rules to bring the residuals of A and B to the front of Δ_3 and Δ_4, so we have

$$\begin{array}{c}\mathfrak{D}'_3 \\ \Gamma_3 \rightarrow A, \Delta'_3\end{array}$$

and

$$\begin{array}{c}\mathfrak{D}'_4 \\ \Gamma_4 \rightarrow B, \Delta'_4\end{array}$$

At this point, we apply the $\wedge-$ rule:

$$\dfrac{\begin{array}{c}\mathfrak{D}'_3 \\ \Gamma_3 \rightarrow A, \Delta'_3\end{array} \qquad \begin{array}{c}\mathfrak{D}'_4 \\ \Gamma_4 \rightarrow B, \Delta'_4\end{array}}{\Gamma_3, \Gamma_4 \rightarrow A \wedge B, \Delta'_3, \Delta'_4} \wedge-$$

and compute the indices of the residuals.

- **PRINCIPAL** The final, and most important case, is when both mix formulas are principal. This is handled by the function ftree-seq-mix-elim-principal.

In each of the cases above, one of the derivations gets smaller in the recursive call. The case when both mix formulas are principal is complicated by the need to perform several recursive calls, requiring us to adjust and compose the indices as we go along. The cases for each connective and quantifier are described next.

- $\top$,$\bot$: This cannot happen, because the only way $\top$ can be positive principal in $\mathfrak{D}_2$ is if the last step is $focus$, $weaken$, or $merge$, which were handled above. Similarly, if $\bot$ is negative principal in $\mathfrak{D}_1$, then it must end with a $focus$, $weaken$, or $merge$.
- $REW(\equiv)$: $\mathfrak{D}_1$ is

$$\dfrac{\begin{array}{c}\mathfrak{D}_{11} \\ \Gamma_1 \rightarrow [A \supset B] \wedge [B \supset A], \Delta_1\end{array}}{\Gamma_1 \rightarrow A \equiv B, \Delta_1} REW(\equiv)-$$

$\mathfrak{D}_2$ is

$$\dfrac{\begin{array}{c}\mathfrak{D}_{21} \\ [A \supset B] \wedge [B \supset A], \Gamma_2 \rightarrow \Delta_2\end{array}}{A \equiv B, \Gamma_2 \rightarrow \Delta_2} REW(\equiv)+$$

First we recursively call mix elimination for all the unexpanded formulas $A' \equiv B'$ with $\mathfrak{D}_1$ and $\mathfrak{D}_{21}$ (smaller than $\mathfrak{D}_2$) giving $\mathfrak{D}_3$:

$$[A \supset B] \wedge [B \supset A], \Gamma_3 \rightarrow \Delta_3$$

Next, recursively call mix elimination for all the unexpanded formulas $A' \equiv B'$ with $\mathfrak{D}_{11}$ (smaller than $\mathfrak{D}_1$) and $\mathfrak{D}_2$ giving $\mathfrak{D}_4$:

$$\Gamma_4 \to [A \supset B] \wedge [B \supset A], \Delta_4$$

Finally, we can call mix elimination for the two occurrences of the "smaller" mix formula $[A \supset B] \wedge [B \supset A]$ with $\mathfrak{D}_4$ and $\mathfrak{D}_3$ giving $\mathfrak{D}_5$:

$$\Gamma_5 \to \Delta_5$$

Of course, with if we weigh $\equiv$ more than $\wedge$ and $\supset$, we can say $[A \supset B] \wedge [B \supset A]$ is "smaller" than $A \equiv B$. So, this case does not cause a problem with termination. The hard part is tracing the residuals to return the proper indices. Each formula in C in Γ_5 is either the residual of some C in Γ_4 or C in Γ_3. If the preimage is in Γ_4, then C is a residual of a C in Γ_1 or Γ_2. Once we compute the preimage of the preimage, we have the proper index.

The following diagram is helpful when trying to compute preimages:

$$\dfrac{\dfrac{\mathfrak{D}_{11} \qquad \mathfrak{D}_2}{\mathfrak{D}_4}\,elim \qquad \dfrac{\mathfrak{D}_1 \qquad \mathfrak{D}_{21}}{\mathfrak{D}_3}\,elim}{\mathfrak{D}_5}\,elim$$

- $EXP-, SEL+$ or $SEL-, EXP+$: Suppose $\mathfrak{D}_1$ is

$$\dfrac{\begin{matrix}\mathfrak{D}_{11}\\ \Gamma_1 \to A(t), \Delta_1\end{matrix}}{\Gamma_1 \to \exists x A(x), \Delta_1}\,EXP-^t$$

and $\mathfrak{D}_2$ is

$$\dfrac{\begin{matrix}\mathfrak{D}_{21}\\ A(a), \Gamma_2 \to \Delta_2\end{matrix}}{\exists x A(x), \Gamma_2 \to \Delta_2}\,SEL+^a$$

As described in the $REW(\equiv)$ case, we must first eliminate all the nonprincipal occurrences of the mix formulas by recursive calls using $\mathfrak{D}_1$ and $\mathfrak{D}_{21}$ to obtain $\mathfrak{D}_3(a)$:

$$A(a), \Gamma_3 \to \Delta_3$$

and with $\mathfrak{D}_{11}$ and $\mathfrak{D}_2$ to obtain $\mathfrak{D}_4$:

$$\Gamma_4 \to A(t), \Delta_4$$

Then we substitute t for a in $\mathfrak{D}_3(a)$ to obtain $\mathfrak{D}_3(t)$, and recursively call to eliminate the two occurrences of $A(t)$ using $\mathfrak{D}_4$ and $\mathfrak{D}_3(t)$.

Remark About Termination: Usually $A(t)$ will be "smaller" than $\exists x A(x)$. But, of course, in higher order logic there are cases where the formula is most certainly not smaller. The most obvious example is when A is $\exists\, x_o\, x$ and t is also $\exists\, x_o\, x$, so that $A(t)$ is

actually the same as $\exists x A(x)$. We don't actually know whether the algorithm always terminates.

As in the $REW(\equiv)$ case, computing the indices involves computing preimages of preimages using the following diagram

$$\dfrac{\dfrac{\mathfrak{D}_{11} \qquad \mathfrak{D}_2}{\mathfrak{D}_4}\,elim \qquad \dfrac{\mathfrak{D}_1 \qquad \mathfrak{D}_{21}}{\mathfrak{D}_3}\,elim}{\mathfrak{D}_5}\,elim$$

The $SEL-, EXP+$ case is similar.

- $\neg$: This case is relatively simple, we first make two recursive calls to eliminate the nonprincipal occurrences of the mix formula $\neg A$ giving $\mathfrak{D}_3$:

$$\Gamma_3 \to A, \Delta_3$$

and $\mathfrak{D}_4$:

$$A, \Gamma_4 \to \Delta_4$$

Finally, we eliminate the two occurrences of A (smaller than $\neg A$) in $\mathfrak{D}_3$ and $\mathfrak{D}_4$ (the order is the opposite of the previous cases). Again, the indices are preimages of preimages, though we must be careful to account for the changes in lengths of the two sides as $\neg A$ in the final sequents and A in the premisses are on opposite sides. The diagram that applies here is

$$\dfrac{\dfrac{\mathfrak{D}_1 \qquad \mathfrak{D}_{21}}{\mathfrak{D}_3}\,elim \qquad \dfrac{\mathfrak{D}_1 \qquad \mathfrak{D}_{21}}{\mathfrak{D}_4}\,elim}{\mathfrak{D}_5}\,elim$$

- $\wedge$: There are three relevant premisses here. $\mathfrak{D}_1$ has the form

$$\dfrac{\begin{matrix}\mathfrak{D}_{11} \\ \Gamma_{11} \to A, \Delta_{11}\end{matrix} \qquad \begin{matrix}\mathfrak{D}_{12} \\ \Gamma_{12} \to B, \Delta_{12}\end{matrix}}{\Gamma_{11}, \Gamma_{12} \to A \wedge B, \Delta_{11}, \Delta_{12}}\,\wedge-$$

and $\mathfrak{D}_2$ has the form

$$\dfrac{\begin{matrix}\mathfrak{D}_{21} \\ A, B, \Gamma \to \Delta\end{matrix}}{A \wedge B, \Gamma \to \Delta}\,\wedge+$$

Recursive calls eliminate the nonprincipal occurrences of the mix formula.

  - $\mathfrak{D}_3$: ($\mathfrak{D}_1$ mix $\mathfrak{D}_{21}$ to eliminate $A \wedge B$) $\Gamma_3 \to \Delta_3$ where Γ_3 contains residuals of A and B.
  - $\mathfrak{D}_4$: ($\mathfrak{D}_{11}$ mix $\mathfrak{D}_2$ to eliminate $A \wedge B$) $\Gamma_4 \to \Delta_4$ where Δ_4 contains residuals of A
  - $\mathfrak{D}_5$: ($\mathfrak{D}_{12}$ mix $\mathfrak{D}_2$ to eliminate $A \wedge B$) $\Gamma_5 \to \Delta_5$ where Δ_5 contains residuals of B

  - $\mathfrak{D}_6$: ($\mathfrak{D}_5$ mix $\mathfrak{D}_3$ to eliminate residuals of B) $\Gamma_6 \rightarrow \Delta_6$ where Γ_6 contains residuals of A
  - $\mathfrak{D}_7$: ($\mathfrak{D}_4$ mix $\mathfrak{D}_6$ to eliminate residuals of A) $\Gamma_7 \rightarrow \Delta_7$ where there are no residuals of A, B, or $A \wedge B$.

  Since there were more recursive calls in this case, we must compute preimages of preimages of preimages in some cases, and preimages of preimages in other cases, as indicated by the following diagram:

$$\dfrac{\dfrac{\mathfrak{D}_{11} \qquad \mathfrak{D}_2}{\mathfrak{D}_4} \qquad \dfrac{\dfrac{\mathfrak{D}_{12} \qquad \mathfrak{D}_2}{\mathfrak{D}_5} \qquad \dfrac{\mathfrak{D}_1 \qquad \mathfrak{D}_{21}}{\mathfrak{D}_3}}{\mathfrak{D}_6}}{\mathfrak{D}_7}$$

  Each wff in the sequent proven by $\mathfrak{D}_7$ can be traced back to a non-mix formula in either $\mathfrak{D}_1$ or $\mathfrak{D}_2$ using this diagram.
- $\vee$: This case is similar to the $\wedge$ case, but $\mathfrak{D}_1$ has one premiss and $\mathfrak{D}_2$ has two premisses. $\mathfrak{D}_1$ is

$$\dfrac{\begin{array}{c}\mathfrak{D}_{11}\\ \Gamma \rightarrow A, B, \Delta\end{array}}{\Gamma \rightarrow A \vee B, \Delta}\ \vee-$$

  and $\mathfrak{D}_2$ is

$$\dfrac{\begin{array}{c}\mathfrak{D}_{21}\\ A, \Gamma_1 \rightarrow \Delta_1\end{array} \qquad \begin{array}{c}\mathfrak{D}_{22}\\ B, \Gamma_2 \rightarrow \Delta_2\end{array}}{A \vee B, \Gamma_1, \Gamma_2 \rightarrow \Delta_1, \Delta_2}\ \vee+$$

  The following diagram indicates the order of the recursive calls.

$$\dfrac{\dfrac{\dfrac{\mathfrak{D}_{11} \qquad \mathfrak{D}_2}{\mathfrak{D}_4} \qquad \dfrac{\mathfrak{D}_1 \qquad \mathfrak{D}_{22}}{\mathfrak{D}_5}}{\mathfrak{D}_6} \qquad \dfrac{\mathfrak{D}_1 \qquad \mathfrak{D}_{21}}{\mathfrak{D}_3}}{\mathfrak{D}_7}$$

- $\supset$: This is similar to the $\vee$ case. The following diagram indicates the order of the recursive calls.

$$\dfrac{\dfrac{\mathfrak{D}_1 \qquad \mathfrak{D}_{21}}{\mathfrak{D}_3} \qquad \dfrac{\dfrac{\mathfrak{D}_{11} \qquad \mathfrak{D}_2}{\mathfrak{D}_4} \qquad \dfrac{\mathfrak{D}_1 \qquad \mathfrak{D}_{22}}{\mathfrak{D}_5}}{\mathfrak{D}_6}}{\mathfrak{D}_7}$$

5. Cut-free Extensional Sequent Derivations to Extensional Expansion Proofs

There is an implementation of the extensional sequent calculus in Chad E. Brown's thesis (cf. [Bro04]) in TPS3 . The structure ext-seq defined

in ext-exp-dag-macros.lisp represents sequent derivations in this extensional sequent calculus. Sequent derivations can be created, manipulated, saved and restored in the EXT-SEQ top level. If a sequent derivation is cut-free, then the command CUTFREE-TO-EDAG (implemented in ext-seq-top.lisp) will translate the sequent derivation to an extensional expansion proof. The proof that this translation works is in Chapter 7 Section 10 of Chad E. Brown's thesis (cf. [Bro04]).

6. Extensional Expansion Proofs to NProofs

TPS3 can translate an extensional expansion proof to a natural deduction proof using the code in the file ext-exp-dags-nd.lisp. This code is automatically called when either of the extensional search procedures MS04-2 or MS03-7 successful find a proof. The translation code can also be explicitly called from the EXT-MATE top level using the command ETREE-NAT.

There is an algorithm described in Chapter 7 Section 9 of Chad E. Brown's thesis (cf. [Bro04]) which translates extensional expansion proofs to extensional sequent derivations. This algorithm is not implemented as part of TPS3 , but the same ideas are used for the algorithm translating from extensional expansion proofs to natural deduction proofs.

CHAPTER 19

Library

The library commands are documented in the user manual.

A library can currently only occupy one directory (i.e. subdirectories may not be used), although users are given the ability to refer additionally to a common directory of basic definitions by using the BACKUP-LIB-DIR flag.

Many library commands are essentially written as two copies of the same function, the first of which checks the default library directory and the second of which checks the backup directory. The second piece of code is surrounded by `unwind-protect` commands in order to make sure that the DEFAULT-LIB-DIR and BACKUP-LIB-DIR flags always end up correct. Users may not write to the backup directory.

The index for each library directory is stored in the *libindex.rec* file in that directory; this file that is read every time the directory is changed. Objects are removed from the library by deleting them from the appropriate *.lib* file and removing their entry from the *libindex.rec* file. This may result in a *.lib* file of zero length, in which case the file is deleted.

Objects which are loaded by the user are re-defined as TPS3 objects; library objects of type MODE or MODE1 become TPS3 modes, whereas gwffs and abbreviations each become both theorems (of type *library*) and abbreviations. Notice that this blurs the distinction between a gwff and an abbreviation. Users are allowed to redefine TPS3 theorems of type *library*, and their corresponding abbreviations; theorems of other types may not be redefined (this is to prevent users from accidentally overwriting standard abbreviations with their own library definitions).

Library definitions are parsed every time they are written, and this involves re-loading all of the needed objects. Since the needed objects are often abbreviations, this will frequently result in their redefinition, and so LISPwill generate warning messages. If a large file is being re-parsed, this can take a long time and produce a huge number of warnings.

1. Converting TPTP Problems to TPS3 library items

Every needed function or flag is set in *library2.lisp*. The utility is made of two library commands: INSERT-TPTP, to insert one TPTP problem into TPS3 , and INSERT-TPTP* to automatically call insert-tptp on an entire directory. These two commands act on .tps files, which are generated

using the TPTP2X utility. One flag, INSERT-LIB-DIR, defines the output directory for the newly created items.

Please note that a modified format.tps file is used, in order to prevent conflit with other objects inside of tps. The original file uses 'const', 'def', 'axiom' and 'thm' as functions: they have been replaced by 'const-insert', 'def-insert', 'axiom-insert' and 'thm-insert'.

The principal issue when converting TPTP problems into TPS items is to avoid using already defined objects. For this reason, every inserted item is suffixed, usually with the name of the destination library file (e.g. 'one' becomes 'one-ALG2684').

More information about how to process this conversion are available in the User's guide.

CHAPTER 20

Teaching Records

1. Events in TPS3

The primary purpose of events in TPS3 is to collect information about the usage of the system. That includes support of features such as automatic grading of exercises and keeping statistics on the application of inference rules.

Events, once defined and initialized, can be signalled from anywhere in TPS3. Settings of flags, ordinarily collected into modes, control if, when, and where signalled events are recorded. Siganlling of events can be suppressed by changing the values of the flags in subject EVENTS. Notice that this, of course, only suppresses the signalling, not the events themselves!

In ETPS , a basic set of events is predefined, and the events are signalled automatically whenever appropriate. If these events are then recorded depends on your ETPS profile.

There are some restrictions on events that should be respected, if you plan to use `REPORT` to extract statistics from the files recording events. Most importantly: **No two events should be written to the same file.** If you would like to record different things into the same file, make one event with one template and allow several kinds of occurrences of the event. For an example, see the event `PROOF-ACTION` below.

1.1. Defining an Event.

If you are using ETPS , it is unlikely that you need to define an event yourself. However, a lot of general information about events is given in the following description.

Events are defined using the `DEFEVENT` macro. Its format is

```
(defevent <name>
  (event-args <arg1> ... <argn>)
  (template <list>)
  (template-names <list>)
  (signal-hook <hook-function>)
  (write-when <write-when>)
  (write-file <file-parameter>)
  (write-hook <hook-function>)
  (mhelp <help-string>))
```

`event-args`: list of arguments passed on by `SIGNAL-EVENT` for any event of this kind.

`template:` constructs the list to be written. It is not assumed that every event is time-stamped or has the user-id. The template must only contain globally evaluable forms and the arguments of the particular event signalled. It could be the source of subtle bugs, if some variables are not declared special.

`template-names:` names for the individual entries in the template. These names are used by the `REPORT` facility. As general conventions, when the template form is a variable, use the same name for the template name (e.g. `DPROOF`). If the template form is `(STATUS` *statusfn*`)` use *statusfn* as the template name (e.g. `DATE` for `(STATUS DATE)` or `USERID` for `(STATUS USERID)`).

`signal-hook:` an optional function to be called whenever the the event is signalled. This should **not** to the writing of the information, but may be used to do something else. If the function does a `THROWFAIL`, the calling `SIGNAL-EVENT` will return `NIL`, which means failure of the event. The arguments of the function should be the same as `EVENT-ARGS`.

`write-when:` one of `IMMEDIATE`, `NEVER`, or an integer n, which means to write after an implementation depended period of n. At the moment this will write, whenever the number of inputs = n * `EVENT-CYCLE`, where `EVENT-CYCLE` is a global variable, say 5.

`write-file:` the name of the global `FLAG` with the filename of the file for the message to be appended to.

`write-hook:` an optional function to be called whenever a number (>0) of events are written. Its first argument is the file it will write to, if the write-hook returns. Its second argument is the list of evaluated templates to be written. If an event is to be written immediately, this will always be a list of length 1.

`mhelp:` The mhelp string for the event.

Remember that an event is ignored, until `(INIT-EVENTS)` or `(INIT-EVENT` *event*`)` has been called.

1.2. Signalling Events. TPS3 provides a function `SIGNAL-EVENT`, which takes a variable number of arguments. The first argument is the kind of event to be signalled, the rest of the arguments are the event-args for this particular event. `SIGNAL-EVENT` will return `T` or `NIL`, depending on whether the action to be taken in case of the event was successful or not. Note that when an event is disabled (see below), signalling the event will always be successful. There are basically three cases in which an event will be considered unsuccessful: if the `SIGNAL-HOOK` is specified and does a `THROWFAIL`, if `WRITE-WHEN` is `IMMEDIATE` and either the `WRITE-HOOK` (if specify) does a `THROWFAIL`, or if for some reason the writing to the file fails (if the file does not exists, or is not accessible because it has the wrong protection, for example).

It is the caller's responsibility to make use of the returned value of `SIGNAL-EVENT`. For example, the signalling of `DONE-EXERCISE` below.

If `WRITE-WHEN` is a number, the evaluated templates will be collected into a list *event*-`LIST`. This list is periodically written out and cleared. The interval is determined by `EVENT-CYCLE`, a global flag (see description of `WRITE-WHEN` above). The list is also written out when the function `EXIT` is called, but not if the user exits TPS3 with `^C`. Note that if events have been signalled, the writing is done without considering whether the event is disabled or not. This ensures that events signalled are always recorded, except for the `^C` safety valve.

Events may be disabled, which means that signalling them will always be successful, but will not lead to a recordable entry. This is done by setting or binding the flag *event*-`ENABLED` to `NIL` (initially set to `T`). For example, the line `(setq error-enabled nil)` in your `.INI` file will make sure that no MacLisp error will be recorded. For a maintainer using expert mode, this is probably a good idea.

1.3. Examples. Here are some examples take from the file `ETPS-EVENTS`. Interspersed is also the code from the places where the events are signalled.

```
(defflag error-file
  (flagtype filespec)
  (default "etps3.error")
  (subjects events)
  (mhelp "The file recording the events of errors."))

(defevent error
  (event-args error-args)
  (template ((status-userid) error-args))
  (template-names (userid error-args))
  (write-when immediate)
  (write-file error-file)    ; a global variable, eg
     ; `((tpsrec: *) etps error)
  (signal-hook count-errors) ; count errors to avoid infinite loops
  (mhelp "The event of a Lisp Error."))
```

`DT` is used to freeze the daytime upon invocation of `DONE-EXC` so that the code is computed correctly. The code is computed by `CODE-LIST`, implementing some “trap-door function”.

```
(defvar computed-code 0)

(defvar dt '(0 0 0))

(defvar score-file)
(defflag score-file
  (flagtype filespec)
  (default "etps3.scores")
  (subjects events)
```

```
  (mhelp "The file recording completed exercises."))

(defevent done-exc
  (event-args numberoflines)
  (template ((status-userid) dproof numberoflines computed-code
     (status-date) dt))
  (template-names (userid dproof numberoflines computed-code date daytime))
  (signal-hook done-exc-hook)
  (write-when immediate)
  (write-file score-file)
  (mhelp "The event of completing an exercise."))

(defun done-exc-hook (numberoflines)
  ;; The done-exc-hook will compute the code written to the file.
  ;; Freeze the time of day right now.
  (declare (special numberoflines))
  ;; because of the (eval `(list ..)) below.
  (setq dt (status-daytime))
  (setq computed-code 0)
  (setq computed-code (code-list (eval `(list ,@(get 'done-exc 'template))))))

(defflag proof-file
  (flagtype filespec)
  (default "etps3.proof")
  (subjects events)
  (mhelp "The file recording started and completed proofs."))

(defevent proof-action
  (event-args kind)
  (template ((status-userid) kind dproof (status-date) (status-daytime)))
  (template-names (userid kind dproof date daytime))
  (write-when immediate)
  (write-file proof-file)
  (mhelp "The event of completing any proof."))

(defflag advice-file
  (flagtype filespec)
  (default "etps3.advice")
  (subjects events)
  (mhelp "The file recording advice."))

(defevent advice-asked
  (event-args hint-p)
  (template ((status-userid) dproof hint-p))
  (template-names (userid dproof hint-p))
```

```
  (write-when 1)
  (write-file advice-file)
  (mhelp "Event of user asking for advice."))
```

Here is how the DONE-EXC and PROOF-ACTION are used in the code of the DONE command. We don't care if the PROOF-ACTION was successful (it will usually be), but it's very important that the user knows when a DONE-EXC was unsuccessful, since it is used for automatic grading.

```
(defun done ()
  ...
  (if (funcall (get 'exercise 'testfn) dproof)
      (do ()
  ((signal-event 'done-exc (length (get dproof 'lines)))
   (msgf "Score file updated."))
(msgf "Could not write score file.  Trying again ... (abort with ^G)")
(sleep 1/2))
      (msgf "You have completed the proof.  Since this is not an assigned exercise,"
    t "the score file will not be updated."))
  (signal-event 'proof-action 'done))
```

2. The Report Package

The REPORT package in TPS3 allows the processing of data from EVENTS. Each report draws on a single event, reading its data from the record-file of that event. The execution of a report begins with its BEGIN-FN being run. Then the DO-FN is called repetitively on the value of the EVENTARGS in each record from the record-file of the event, until that file is exhausted or the special variable DO-STOP is given a non-NIL value. Finally, the END-FN is called. The arguments for the report command are given to the BEGIN-FN and END-FN. The DO-FN can only access these values if they are assigned to certain PASSED-ARGS, in the BEGIN-FN. Also, all updated values which need to be used by later iterations of the DO-FN or by the END-FN should be PASSED-ARGS initialized (if the default NIL is not acceptable in the BEGIN-FN.

NOTE: The names of PASSED-ARGS should be different from other arguments (ARGNAMES and EVENTARGS). Also, they should be different from other variables in those functions where you use them and from the variables which DEFREPORT always introduces into the function for the report: FILE, INP and DO-STOP.

The definition of the category of REPORTCMD, follows:

```
(defcategory reportcmd
  (define defreport1)
  (properties
   (source-event single)
   (eventargs multiple)   ;; selected variables in the var-template of event
```

```
   (argnames multiple)
   (argtypes multiple)
   (arghelp multiple)
   (passed-args multiple) ;; values needed by DO-FN (init in BEGIN-FN)
   (defaultfns multiplefns)
   (begin-fn singlefn)    ;; args = argnames
   (do-fn singlefn)       ;; args = eventargs ;; special = passed-args
   (end-fn singlefn)      ;; args = argnames
   (mhelp single))
  (global-list global-reportlist)
  (mhelp-line "report")
  (mhelp-fn princ-mhelp)
  (cat-help "A task to be done by REPORT."))
```

The creation of a new report consists of a DEFREPORT statement (DEFREPORT is a macro that invokes DEFREPORT1) and the definition of the BEGIN-FN, DO-FN and END-FN. Any PASSED-ARGS used in these functions should be declared special. It is suggested that most of the computation be done by general functions which are more readily usable by other reports. In keeping with this philosophy, the report EXER-TABLE uses the general function MAKE-TABLE. The latter takes three arguments as input: a list of column-indices, a list of indexed entries (row-index, column-index, entry) and the maximum printing size of row-indices. With these, it produces a table of the entries. EXER-TABLE merely calls this on data it extracts from the record file for the DONE-EXC event. The definition for EXER-TABLE follows:

```
(defreport exer-table
  (source-event done-exc)
  (eventargs userid dproof numberoflines date)
  (argtypes date)
  (argnames since)
  (defaultfns (lambda (since)
(cond ((eq since '$) (setq since since-default)))
(list since-default)))
  (passed-args since1 bin exerlis maxnam)
  (begin-fn exertable-beg)
  (do-fn exertable-do)
  (end-fn exertable-end)
  (mhelp "Constructs table of student performance."))

(defun exertable-beg (since)
  (declare (special since1 maxnam)) ;the only non-Nil passed-args
  (setq since1 since)
  (setq maxnam 1))
```

```
(defun exertable-do (userid dproof numberoflines date)
  (declare (special since1 bin exerlis maxnam))
  (if (greatdate date since1)
      (progn
       (setq bin (cons (list userid dproof numberoflines) bin))
       (setq exerlis
     (if (member dproof exerlis) exerlis (cons dproof exerlis)))
       (setq maxnam (max (flatc userid) maxnam)))))

(defun exertable-end (since)
  (declare (special bin exerlis maxnam))
  (if bin
      (progn
       (make-table exerlis bin maxnam)
       (msg t "On exercises completed since ")
       (write-date since)
       (msg "." t))
      (progn
       (msg t "No exercises completed since ")
       (write-date since)
       (msg "." t))))
```

CHAPTER 21

The Grader Program

(Programmers should be aware that the GRADER program has its own manual.)

1. The Startup Switch

In theory, adding the switch `-grader` to the command line which starts up TPS3 should start up the Grader program directly. The code which implements this is in tps3-save.lisp.

In practice, some modifications may be needed depending on the particular Lisp being used. For example:

- When starting up in CMUlisp on an IBM RT, the error `"Switch does not exist"` will be given. This is just Lisp complaining that it doesn't recognize the switch; it passes the switch on to TPS3 anyway, so this is no cause for concern.
- When using Allegro Lisp version 4.1 or later, a - symbol is used to separate Lisp options from user options. So, on early versions of Allegro Lisp the line to start up grader is: `xterm` *<many xterm switches>* `-e /usr/theorem/bin/run-tps -grader &` whereas for later versions it is: `xterm` *<many xterm switches>* `-e /usr/theorem/bin/run-tps - -grader &`

CHAPTER 22

Running TPS With An Interface

There is an interface for TPS3 written in Java. Running TPS3 through such an interface is similar to running TPS3 within an xterm window, except the Java interface supports menus and popup prompts. The TPS3 lisp code now includes general facilities for communicating with such an interface (when running under Allegro).

To start TPS3 with the java interface, one can use the command line argument -javainterface along with other relevant information as shown below:

```
lisp -I tps3.dxl -- -javainterface cd javafiles \;
     /usr/bin/java TpsStart
```

The command line arguments following -javainterface should form a shell command which run the interface. In this case, the shell command would be "cd javafiles; /usr/bin/java TpsStart". Other command line arguments which have meaning for the "java TpsStart" command are listed below.

-big: Use the bigger sized fonts.
-x2: Multiply the font size by 2.
-x4: Multiply the font size by 4.
-nopopups: Do not use "popup" style prompts. Instead, the Java window should behave more like the x-window interface.

The remaining command line arguments should be followed by a non-negative integer.

-screenx: The initial horizontal size of the Java window.
-screeny: The initial vertical size of the Java window.
-rightOffset: The amount of extra room given to the right margin.
-bottomOffset: The amount of extra room given to the bottom margin.
-maxChars: The maximum number of characters to hold in the buffer. This should be large enough that you can scroll back and see previous TPS3 output. The default value of 20000 should usually be enough.

These other arguments should be preceeded by a command line "-other". This tells TPS that the remaining command line information should be passed to the call to "java TpsStart". For example,

```
lisp -I tps3.dxl -- -javainterface cd javafiles \;
   /usr/bin/java TpsStart -other -big -rightOffset 10 -nopopups
```

which tells To send these command line arguments to "java TpsStart", they should For example, "java TpsStart -big" instructs Java to use the big fonts, and "-x2" and "-x4" instruct Java to multiply the size of the fonts by 2 or 4, respectively. The extra argument "-nopopups" will provide an alternative to popup prompts.

Another way to use the java interface is to start TPS3 as usual, then use the command JAVAWIN. This requires the flag JAVA-COMM to be set appropriately.

When TPS3 is started in -javaservice mode, it uses the rest of the command line arguments to start the Java interface and creates sockets connecting TPS3 to the Java interface. Two processes are spawned, one to receive input from the Java interface (either from a prompt or from a menu item selection), and another to actually run TPS3 commands.

The rest of the description does not particularly depend on the Java interface, so I will simply refer to "the interface" and attempt to emphasize that such an interface could be implemented in a variety of ways.

The code to receive input from the interface is written in external-interface.lisp. It listens to the socket stream and collects characters into strings separated by null characters (ASCII 0). There are a few possibilities.

(1) If the string "COMMAND" is received, the next string is a command TPS3 should run (or the response to a prompt if popups are disabled). All the input does with this command string is attach it to the COMMAND symbol as the property RESPONSE. The main process will accept this string as input from linereadp since, when running through the interface, a function read-line-sym will wait for this RESPONSE property to be set. An exceptional case is when the string after "COMMAND" is "INTERRUPT". In this case, the main process is killed and a new process with a top level prompt is created.
(2) If the string "RIGHTMARGIN" is received, the next string received should be an integer giving the new value for the flag RIGHTMARGIN. This allows the interface to change this flag without having to interrupt another command that may be running.
(3) When popups are enabled, some other string starting with "PROMPT" is received. In this case, the next string is put on the RESPONSE property of this "PROMPT" symbol. This should be a response to a particular (popup) prompt.

So, the code in external-interface.lisp handles receiving input from the interface. The other problem is that of sending output to the interface. This is handled by setting *standard-output* to the socket stream and changing the STYLE to ISTYLE ("interface style" defined in interface-style.lisp). This style is similar to the XTERM style, except with more control information. Control information is sent by first sending a null character (ASCII 0) followed by a byte giving information. The current possible byte values

following a null character and their meanings are listed below. There are lisp functions in interface-style.lisp which send these bytes, but anyone coding a new interface will need to know these values.

0 Switch to normal font mode. In normal font mode, each character is communicated by a single byte.
1 Switch to symbol font mode. In symbol font mode, each symbol character is communicated by two bytes (allowing for many more symbol characters than normal font characters).
2 Start a prompt message string.
3 Start a prompt name string.
4 Start a prompt argtyp string. (This allows the interface to recognize some special finite argtyp's such as boolean.)
5 Start a list of prompt-options
6 Start a list giving the prompt's default value.
7 Start a prompt help string.
8 End a prompt.
9 A note that a command has finished executing.
10 Start and end a string specifying the current top level.
11 Open a window (eg, for proof windows or vpforms) and start sending a string giving the port value for a socket to connect to.
12 End a string giving a prompt for a window and start sending a string given a title for the window.
13 End the title of a window and start sending a string giving the width of the window.
14 End sending the width of the window and start sending the height of the window.
15 End sending window information for a window with small fonts.
16 End sending window information for a window with big fonts.
17 Clear the contents of a window.
18 Close a window.
19 Change the color. This should be followed by another byte to indicate the color. For now, this third byte can be 0 (black), 1 (red), 2 (blue), or 3 (green).

1. Generating the Java Menus

There are two categories in the TPS3 lisp code for menus: menu and menuitem. Everytime a programmer adds a command (mexpr), flag, or top level command, a corresponding menuitem should be defined. This menuitem should have a parent menu to indicate where the item lives.

The Java menu code is in the file TpsWin.java between the comment lines:

```
// Menu Code BEGIN
```

and

```
// Menu Code END
```

When you have added or changed menus or menuitems in the lisp code and want the Java interface to reflect these changes, perform the following steps:

(1) Within TPS3 , call the command generate-java-menus. This will prompt for an output file, e.g., "menus.java". This command will create an output file with Java code which should be inserted into TpsWin.java.

(2) Delete the code between

```
// Menu Code BEGIN
```

and

```
// Menu Code END
```

in TpsWin.java.

(3) Insert the contents of the output file of generate-java-menus (e.g., "menus.java") into TpsWin.java between the comment lines

```
// Menu Code BEGIN
```

and

```
// Menu Code END
```

and save TpsWin.java.

(4) On each machine, find the Java directories which contains links to the main java files (e.g., /home/theorem/tps/java/ and /home/theorem/tps/java/tps/). cd to this directory and call "javac TpsWin.java" to compile the new version.

2. Adding a New Symbol

The Java information for the fonts is contained in the files TpsSmallFonts.java and TpsBigFonts.java. The lisp information containing the "code" for the symbol is in interface-style.lisp. To add a new symbol for the Java interface, one should add a new "code" for the symbol to the variable istyle-characters in interface-style.lisp. For example, the epsilon character was added by including

```
(epsilon 2 1)
```

to istyle-characters. This means that epsilon will be communicated to the interface by sending the bytes 2 and 1 in symbol font mode.

Then one needs to add information about how to draw the new symbol to the variables `blank`, `xstartData`, `ystartData`, `widthData`, and `heightData` in TpsSmallFonts.java and TpsBigFonts.java. Each of these variables is set to a multi-dimensional array. The 0'th element of each array corresponds to the normal fonts. The rest are for symbol fonts. For example, the information for epsilon should be put in the (2,1) position of each array. This information (for epsilon in TpsSmallFonts.java) is as follows:

blank: false (the epsilon character is not blank).

xstartData: 3,2,1,1,1,2,3 (This character is drawn using 7 rectangles starting from "x" coordinates 3, 2, 1, 1, 1, 2, 3, resp.)

ystartData: 10,9,8,7,6,5,4 (These are the "y" coordinates of the 7 rectangles.)

widthData: 4,1,1,4,1,1,4 (These are the widths of the 7 rectangles.)

heightData: 1,1,1,1,1,1,1 (These are the heights of the 7 rectangles. Since all heights are 1, the epsilon character is drawn by drawing 7 horizontal lines.)

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