

# OBJECT-ORIENTED MODELING OF HYBRID SYSTEMS

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## ABSTRACT

A new methodology for the object-oriented description of models consisting of a mixture of continuous and discrete components is presented. The object-oriented paradigm enables the user to describe such models in a modular fashion that permits the reuse of these models independently of the environment in which they are to be embedded. The paper explains the basic mechanisms needed for object-oriented modeling of hybrid systems by means of language constructs available in the object-oriented modeling language Dymola. It then addresses more advanced concepts such as variable structure models containing e.g. ideal electrical switches, ideal diodes and dry friction.

## INTRODUCTION

Hybrid models contain both continuous and discrete parts. In simulation programs, the continuous parts are described by sets of differential equations and algebraic equations in either explicit form (ODE) or implicit form (DAE). Traditionally, the discrete parts are expressed with event descriptions. A numerically sound methodology for simulating hybrid models was developed about 15 years ago (Cellier 1979). Unfortunately, this methodology requires a compact description of all differential equations in a single monolithic continuous block, and a description of the accompanying events in one or several separate discrete blocks. This model structure does not support the reuse of models.

Object-oriented programming has evolved to support the independent development and reuse of software components. This programming paradigm was first developed in the context of discrete-event simulation (Birthwistle et al. 1973) and carried over to continuous system modeling about 15 years ago. Dymola, an object-oriented modeling language for continuous systems was designed by Elmqvist for this purpose, and a prototypical implementation of Dymola

was made available (Elmqvist 1978). Dymola represented an important step forward towards the reuse of continuous system models in a truly environment-independent fashion. Recently, Dymola has been upgraded from a mere university prototype to a fully-supported commercial software tool (Elmqvist 1993).

A continuous system modeling methodology that does not allow for descriptions of discontinuities is not generally useful, at least not in the context of engineering applications. All but the most trivial engineering models of dynamic systems contain some sorts of discontinuities.

This paper discusses a recent extension of the Dymola language definition to allow descriptions of models of dynamic systems with discontinuous behavior in a truly reusable object-oriented fashion. Several novel high-level constructs are introduced that are much easier to use, closer to the physical system description, and less error prone than corresponding constructs in today's simulation languages such as ACSL (Mitchell & Gauthier 1991). A different approach to object-oriented hybrid system modeling is described in (Andersson 1992), in which facilities for discrete event modeling in the object-oriented modeling language Omola (Andersson 1990) are discussed. Event handling in Omola supports the (explicit) definition of events and corresponding actions. Contrary, in Dymola the (explicit) definition of events is replaced by higher-level constructs.

Generators have been developed that automatically translate Dymola models into either an ACSL (Mitchell & Gauthier 1991) program or a DSblock (Otter 1992) Fortran subroutine. While Dymola also supports some other simulation languages such as Desire (Korn 1989), Simnon (Elmqvist et al. 1990), and SimuLink (MathWorks 1992), hybrid models cannot be translated into these languages, since they don't offer full support of (time- and state-) event descriptions.

## OBJECT-ORIENTED MODELING IN DYMOLA

The behavior of a physical object is characterized on the one hand by physical laws relating its properties, and on the other hand by the interactions of these properties with its environment. For example, an electrical resistor is characterized by two important electrical properties: the potential difference across the resistor, and the current flowing through it. These two properties are linked with each other through Ohm’s law:

$$u = R \cdot i \quad (1)$$

The laws of Newtonian physics describe interrelations between various physical properties. They are static in nature, i.e., they are always valid. They do not by themselves dictate causes and their effects. Newtonian physics is totally impartial with respect to the question whether current flowing through the resistor causes a difference in the potentials at the two ends, or whether an existing potential difference causes current to flow through the resistor.

Dymola allows to describe models in an object-oriented manner. Consequently, equations describing physical laws are declarative in nature. Although Eq.(1) looks like an assignment statement of a conventional computer program, it must not be interpreted in this way. Eq.(1) could equally well have been solved for  $i$ , or for  $R$ , or it could have been written in the form:

$$0 = u - R \cdot i \quad (2)$$

The effect would have been just the same. If the resulting simulation program is to be generated in explicit ODE form, physical laws must be turned by the compiler (Dymola) into assignment statements of the resulting simulation program, and a computational causality will be dictated upon each of these equations in order that derivatives will be calculated from the known state variables. It is one of Dymola’s foremost tasks to automatically determine the desired causality of each equation, and to solve each of the equations for its desired variable by means of symbolic formula manipulation.

It is quite evident that this “causality” is not a physical phenomenon, but only a numerical artifact. However, since most of today’s simulation languages are based on an explicit ODE formulation of the models to be simulated, this artifact turns out to be rather important.

Consistent with its philosophy of requiring the user to only specify the properties of physical objects and their interactions with their environment, Dymola’s *terminal* variables, i.e., the variables that connect an object to its environment, are nondirectional also. As with the

causality of equations, the causality of terminal variables, i.e., the question which of them are to be considered *inputs* and which are *outputs*, is only decided during the compilation into a simulation program, and not during model formulation.

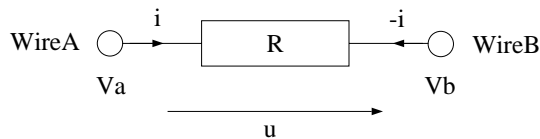


Figure 1: Electrical resistor.

In the light of these considerations, a model of an electrical resistor as shown in Fig.1 can be formulated in Dymola as follows:

```

model class Resistor
  cut WireA (Va/i), WireB(Vb/-i)
  main cut PlugC [WireA,WireB]
  main path PathAB < WireA - WireB >
  local u
  parameter R
    u = Va - Vb
    u = R * i
end

```

Cuts are a way to group terminal variables. When a physical object interacts with its environment, a single interaction may involve more than one variable. For example, if one of the wires of an electrical resistor is soldered into a circuit, two of the variables of the resistor interact simultaneously with the circuit environment through this wire, namely the current that flows into the resistor, and the potential at the wire. In order to be able to describe connections of objects in an object-oriented fashion, the variables involved in the connection must be grouped into a connection object that is called a *cut* in Dymola. Variables can interact in two different ways with their environment. Variables to the left of the “/” operator are *across* variables (the potentials around a node in an electrical circuit are equal), those to the right of the “/” operator are *through* variables (the currents flowing into a node in an electrical circuit add up to zero). Cuts can be hierarchically structured (individual pins can be connected into plugs). One cut can be declared the *main cut*. The main cut does not need to be referenced by name in a connection. Paths are a way to declare logical flows through cuts. They are not essential, but they simplify the connection of several objects in series or in parallel.

The *Resistor* class (type) of objects is described by two equations. One is the physical law (Ohm’s law) that relates the local variable  $u$  to the terminal variable  $i$ , the other is a topological equation that relates the local variable  $u$  to the two terminal variables  $Va$  and  $Vb$ .

Since many of the properties of a resistor are shared by many different types of two-port elements, it makes

sense to declare a super class as a container for all properties of two-port elements, and specialize the resistor by inheriting from the two-port elements. In Dymola, this can be accomplished as follows:

```

model class TwoPort
  cut WireA (Va/i), WireB(Vb/-i)
  main cut PlugC [WireA,WireB]
  main path PathAB < WireA - WireB >
  local u
    u = Va - Vb
end

model class (TwoPort) Resistor
  parameter R
    u = R * i
end

```

Capacitors and inductors could then be made other specializations of two-port elements:

```

model class (TwoPort) Capacitor
  parameter C
    C * der(u) = i
end

model class (TwoPort) Inductor
  parameter L
    L * der(i) = u
end

```

## THE CAUSALITY ASSIGNMENT PROBLEM

As shown in (Cellier and Elmqvist 1993), index 0 problems are characterized by the fact that the causality assignment problem can be solved in a unique fashion. The reverse is true also. Any model, for which the causality assignment problem can be solved in a unique fashion, is of index 0.

Index 1 problems, on the other hand, lead to partly free choices in causality selection. Sets of equations and variables where each equation contains at least two of the unassigned variables, and each of the unassigned variables appears in at least two of the equations form algebraic loops, which are the identifying characteristic of index 1 problems. These algebraic loops must either be solved simultaneously at run time using some sort of iteration or elimination algorithm, or they must be solved symbolically at compile time by means of formula manipulation. Dymola contains mechanisms to solve linear algebraic loops symbolically and (optionally) numerically, thereby reducing an index 1 problem to an index 0 problem.

Higher index problems are characterized by the fact that the causality assignment problem does not have any solution that would allow to keep all integrators in the model as state variables. Such problems always

occur when, in a connection, the system order (or the number of degrees of freedom) gets reduced, i.e., if the system order of the overall system is smaller than the sum of the system orders of its subsystems. Dymola offers mechanisms to automatically reduce higher index problems down to index 1.

The causality assignment problem has been dealt with in much detail in (Cellier and Elmqvist 1993). The main results of this discussion were repeated here, since they will prove important subsequently in this paper.

## DISCONTINUOUS EQUATIONS

The possibility to describe discontinuities in otherwise continuous models is very important in all sorts of engineering applications. We might describe, e.g., a limiter function, as shown in Fig. 2, in a continuous system

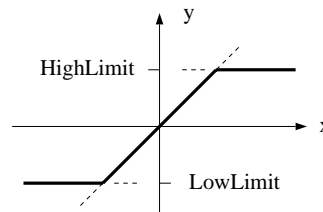


Figure 2: Limiter function.

simulation language such as ACSL (Mitchell & Gauthier 1991) as follows:

```

macro Limiter (y, x, LowLimit, HighLimit)
  if (x .gt. HighLimit) then y = HighLimit
  else if (x .lt. LowLimit) then y = LowLimit
  else y = x end if
macro end

```

However, as shown in (Cellier 1979; Cellier et al. 1993), this will lead to poor numerical behavior of the simulation program, since it will necessitate the numerical integration algorithm to integrate the model across discontinuities with small step-sizes.

A numerically superior way to describe this model is by use of state events. A correct ACSL model would be<sup>1</sup>:

```

program LimiterTest
  constant HighLimit = 1.0, LowLimit = -1.0
  logical High, Low

  initial
    x = 2 * sin(t)
    High = x .ge. HighLimit
    Low = x .le. LowLimit
  end ! of initial

  dynamic

```

<sup>1</sup>Since it is difficult to describe such a limiter function as macro with schedule-statements, a complete ACSL program is given.

```

derivative
  x = 2 * sin(t)

  if (High) then y = HighLimit
  elseif (Low) then y = LowLimit
  else y = x end if

  schedule HighEvent .xz. x - HighLimit
  schedule LowEvent .xz. x - LowLimit
end ! of derivative

discrete HighEvent
  High = .not. High
end ! of discrete HighEvent

discrete LowEvent
  Low = .not. Low
end ! of discrete LowEvent

term t .ge. 10.0
end ! of dynamic
end ! of program

```

This formulation solves the numerical problems. *High* and *Low* are logical variables that change their values only at event times. Thus, during the numerical integration, no discontinuity ever takes place. Demons (in the form of two *schedule* statements) were installed to watch over domain boundaries and initiate a root finder algorithm if one of the watched variables crosses its boundary.

One problem with this notation is its defiance of every principle of object orientation. One and the same physical object leaves its residues in at least three different places of the simulation program. In the case of one single limiter function, the price paid for this separation is not very high. The program displayed above is still simple and understandable. However, if the model contains dozens of discontinuous functions, as will be invariably the case in complex engineering applications, the program will become very messy, hard to read, and even more difficult to maintain.

The Dymola solution is much simpler and more elegant:

```

model class Limiter
  terminal x, y
  parameter LowLimit, HighLimit
  y = if x > HighLimit then HighLimit
      else if x < LowLimit then LowLimit
      else x
end

```

This description looks very similar to the original attempt using an ACSL macro. Yet, the Dymola compiler is much smarter than its ACSL counterpart. It recognizes the discontinuous nature of the *if* statement, and translates it into a description at the level of the simulation language containing, e.g., *schedule* statements in case of ACSL.

For the compilation process, there is one serious difficulty with regard to *schedule* statements: In the generated code, a relation like  $x > HighLimit$  is replaced

by a boolean variable, say *L*. *L* changes its value only at an event instant which is triggered by the corresponding zero crossing function  $x - HighLimit$ . In ACSL, as well as in other simulation environments, an event occurs if a crossing function either crosses zero or hits zero *exactly*. Assume for example, that *x* decreases until it becomes identical to *HighLimit*. In this situation an event occurs and the *else* branch could be chosen for the integration restart. Depending on the model, it is possible that *x* either remains equal to *HighLimit* for a while, or becomes larger or smaller than *HighLimit*. In neither case a new event is triggered, because no zero crossing appeared.

Dymola solves this undesirable situation by using *two* zero crossing functions, i.e., *schedule* statements, for every relation. If the residuum *z* of a relation (e.g.  $z = x - HighLimit$ ) is not equal to zero at an integration restart, the two crossing functions *zp* (positive direction) and *zn* (negative direction) are identical to the residuum *z*. If *z* is identically zero at an integration restart, the two crossing functions are set to  $zp = z + eps$ ,  $zn = z - eps$ , i.e., a small interval around  $z = 0$  is defined. It is very important to set *eps* to an application independent value. By choosing *eps* in the order of the smallest machine number (e.g.  $eps = 10^{-38}$ ), the root finder will determine the time of leaving this interval in accordance with the demanded *time* accuracy for event detection. As long as *z* remains in the interval, it is regarded as zero and the corresponding branch of the *if*-expression is used. When *z* crosses one of the interval boundaries an event is triggered and *zp*, *zn* are reset to *z*. Since *z* is now either positive or negative, the correct branch of the *if*-expression can be chosen.

## INSTANTANEOUS EQUATIONS

Up to this point, the discussion has focused on events that occur as an indirect consequence of selecting the correct branch in *if*-expressions. However, this view is too limited. Mechanisms are also needed to describe sudden changes in the model structure, changes that cannot be properly reflected by merely altering the expression that assigns a value to a state derivative or an algebraic model variable. Such mechanisms are needed, for example, to describe what happens at the boundaries of a hysteresis function, or to model computer algorithms that are used as part of sampled data systems.

An *instantaneous equation* takes the form:

```

when < condition > then
  < equations >
endwhen

```

The equations are evaluated only at the instant when the boolean condition *becomes* true. Again, this time

instant is triggered by an appropriate event, i.e., Dymola maps the condition to e.g. ACSL schedule statements.

For example, a hysteresis function can be modeled by:

```
when u > H or u < -H then
  y = if u > H then 1 else -1
endwhen
```

The output  $y$  changes its value only when  $u$  becomes greater than  $H$  or becomes smaller than  $-H$ .

*Difference equations* are evaluated at certain time instants only and define *discrete variables*. The *next* value of a discrete state variable  $x$  is referred to as **new**( $x$ ), as seen in the following example:

```
when Time >= NextTime then
  new(NextTime) = NextTime + SamplingRate
  new(x) + a * x = b * u
endwhen
```

It is also possible to change continuous state variables when a certain condition is met <sup>2</sup>:

```
when Height <= 0.0 then
  new(Velocity) = -c * Velocity
endwhen
```

It is essential to be able to *propagate* and *synchronize* events. Boolean variables and Boolean equations are used for this purpose: Boolean variables change their values due to some relation becoming *true* or *false* or indirectly due to some instantaneous equation. For example in the following equation

$$a = b \text{ and } c \text{ or } d$$

the boolean variable  $a$  is related to some other boolean variables  $b$ ,  $c$  and  $d$ , i.e., “event”  $a$  is triggered, if the specified conditions on “events”  $b$ ,  $c$  and  $d$  are fulfilled. Events are *propagated* from one submodel to another by using appropriate boolean variables as terminal variables. Instantaneous equations, i.e., boolean equations or “when”-equations, are treated just like continuous equations. In particular, all three types of equations are *sorted* together. This procedure will ensure that event dependencies are recognized and sorted correctly, that loops among instantaneous equations will be detected, and that all variables are always up-to-date when they are needed.

To summarize, there is nothing like an *event* available in the Dymola language, as it is traditionally used in hybrid simulation languages. Instead, Dymola provides high level elements for the definition of discontinuous and instantaneous equations controlled by boolean variables and expressions. The Dymola compiler maps these language constructs to appropriate time and state

<sup>2</sup>as before **new**(*variable*) characterizes the value of *variable* directly after the event

events. Events are low level elements useful for the control of integrators but not appropriate for comfortable definition of hybrid models.

## VARIABLE STRUCTURE MODELS

Discrete events might be so drastical that the structure of the model changes. To illustrate this, let us discuss the case of an electrical switch element. A switching element has been proposed by (Strömberg et al. 1993) as an extension to bond graphs for handling of variable causalities.

The switch is a two-port element like e.g. a resistor. However, it exhibits very peculiar physical laws. The switch has two positions. When the switch is open, no current flows through the switch. When the switch is closed, the voltage across the switch is zero.

In one case, the switch provides an equation to compute the current and none to compute the voltage, whereas in the other case, it provides an equation for the voltage but none for the current.

These types of problems could be solved by generating a different model for the two switch positions, and by changing the model whenever the switch changes its value. However, this solution is impractical. Envisage a circuit with 10 switches. Since all combinations of switch positions are possible, we would need  $2^{10} = 1024$  different models.

We can view the switch as a two-port element with an additional terminal boolean variable *OpenSwitch* to denote the switch position. We can easily write down the switching law as one equation involving the three variables  $u$ ,  $i$ , and *OpenSwitch*:

```
model class (TwoPort) Switch
  terminal OpenSwitch
  0 = if OpenSwitch then i else u
end
```

Dymola is able to solve *if* expressions if both the *then*- and *else*-branches are linear in the unknown variable and the *if*-condition is independent of the unknown. Since the result of such a transformation is less obvious, an alternative formulation is initially used, hereby replacing the boolean variable *OpenSwitch* by the discrete variable *OpenSw* which is either 0 or 1:

```
model class (TwoPort) Switch
  terminal OpenSw
  0 = OpenSw * i + (1 - OpenSw) * u
end
```

Note, that the computational *causality* of the switch equation changes, not as a function of the structure of the environment in which it is embedded, but merely as a function of the numerical value of one of its terminal variables. This is an entirely new situation.

Let us place the switch in series with an inductor. The current through the inductor is a natural state variable, thus  $i$  is known, and therefore, the switch equation will be solved for  $u$ :

$$u = \frac{OpenSw}{OpenSw - 1} \cdot i \quad (3)$$

However, Eq.(3) is only valid as long as the switch is closed. Once the switch opens, the denominator becomes zero, and the simulation will terminate due to the division by zero.

Yet, this result is understandable. The current through the inductor cannot stop immediately. Thus, when we open the switch, an arc will occur. This arc represents a resistor that grows with the distance. The resistor will drive the current to zero, and only then the arc will break and the switch will really open. However, this physical phenomenon is not represented in our model at all, which explains why our model is no longer valid when the switch opens. A similar situation occurs, if a capacitor is placed in parallel with the switch.

What this discussion teaches us is the following. Since the causality of the switch element changes as a function of the numerical value of the variable  $OpenSw$ , both causalities must be structurally compatible with the environment. Thus, if the switch element is used in a physically meaningful configuration, its causality may not be determined by its environment. However, from our earlier discussion, we know that a model, the computational causality of which is not completely determined, always indicates an index 1 problem. Consequently, the switch equation, when used correctly, must always end up in an algebraic loop.

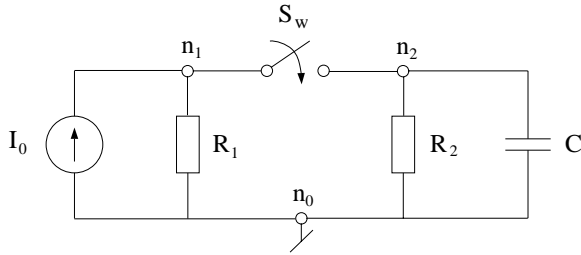


Figure 3: Switch circuit.

Let us illustrate the use of the switch element by means of the simple circuit of Fig.3. The Dymola model of this circuit can be described as follows:

```

model Circuit
  submodel (CSource) I0
  submodel (Resistor) R1(R=100.0), R2(R=20.0)
  submodel (Capacitor) C(C=0.1E-6)
  submodel (Switch) Sw
  submodel Common

  input i
  output y
  node n0, n1, n2

  connect I0 from n1 to n0, R1 from n1 to n0,
           Sw from n1 to n2, R2 from n2 to n0,
           C from n2 to n0, Common at n0

  Sw.OpenSw = if Time < 1.0E-6 then 1
               else if Time < 2.5E-6 then 0
               else if Time < 5.0E-6 then 1
               else if Time < 9.0E-6 then 0 else 1

  I0.I0 = i
  y = C.u
end

```

Looking at the *sorted equations* generated by Dymola:

```

Common. [C.Vb] = 0
C.      u = [Va] - Vb

| R1.    [u] = Sw.Va - C.Vb
|         R * [i] = u
| Circuit. [Sw.i] + R1.i = i
| Sw.     0 = OpenSw * i + (1 - OpenSw) * [u]
|         u = [Va] - C.Va

| I0.    [u] = Sw.Va - C.Vb
| R2.    [u] = C.Va - C.Vb
|         R * [i] = u
| Circuit. [C.i] + R2.i = Sw.i
| C.     C * [deru] = i

```

Those equations that are marked by a “|” belong to an algebraic loop. As expected, the switch equation is inside the loop.

At this point in time, there is no difference any more between this algebraic loop and any other type of algebraic loop, and it can be solved symbolically to:

SOLVED SYSTEM OF EQUATIONS

```

Q105 = OpenSw - 1
Q106 = R1.R * Q105
Q107 = Q106 + OpenSw
Q108 = R1.R * OpenSw
R1.u = (Q106 * C.Va - Q106 * C.Vb + Q108 * i) / Q107
R1.i = (Q105 * C.Va - Q105 * C.Vb + OpenSw * i) / Q107
Sw.i = (Q105 * C.Vb + Q106 * i - Q105 * C.Va) / Q107
Sw.u = (OpenSw * C.Vb + Q108 * i - OpenSw * C.Va) / Q107
Sw.Va = (OpenSw * C.Vb + Q108 * i + Q106 * C.Va) / Q107

```

END OF SYSTEM OF SIMULTANEOUS EQUATIONS

It can be easily verified that the determinant  $Q107$  is different from zero in both switch positions. If our model contains 10 switch elements, we may end up with 10 algebraic loops, but finally obtain one single simulation program that is valid for all combinations of switch positions.

## THE IDEAL DIODE – A VARIABLE STRUCTURE MODEL

The switch model can be used to describe an ideal diode if the switch condition is controlled by internal variables instead of time. An ideal diode is characterized by the additional facts that  $i \geq 0$  and  $u \leq 0$  (see Fig. 4). The

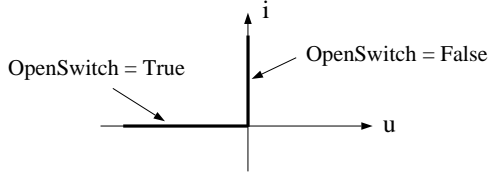


Figure 4: Ideal diode characteristics

switch is open as long as  $u$  is negative or zero and  $i$  is not positive. This can be expressed in the following way:

```
model class (Switch)Diode
  OpenSw = if u <= 0 and not i > 0 then 1 else 0
end
```

A rectifier with load, Fig. 5, can be modeled as follows:

```
model Rectifier
  submodel (VSource) U0
  submodel (Resistor) Ri(R = 10), RL(R = 50)
  submodel (Capacitor) C(C = 0.001)
  submodel (Diode) Diode
  submodel Common

  output y
  parameter f = 50

  connect Common - ((U0 - Ri - Diode)//C//RL)

  U0.u0 = sin(2 * 3.14159 * f * Time)
  y = C.u
end
```

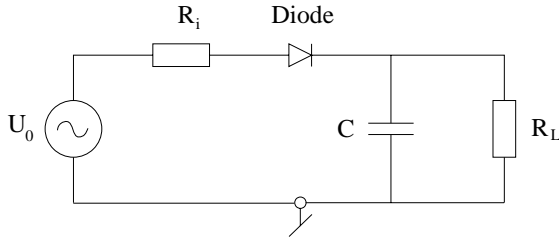


Figure 5: Rectifier with load

When Dymola is requested to output solved equations, the following is reported:

```
SYSTEM OF 5 SIMULTANEOUS EQUATIONS
UNKNOWN VARIABLES
Ri.u
Ri.Vb
Diode.u
Diode.OpenSw
U0.i
```

## EQUATIONS

```
Ri. R * U0.i = [u]
u = U0.Vb - [Vb]
Diode. [u] = Ri.Vb - Vb
[OpenSw] = if u <= 0 and not U0.i > 0
           then 1 else 0
OpenSw * [U0.i] + (1 - OpenSw) * u = 0
```

-- The system of equations is nonlinear.

The variables *OpenSw* and *U0.i* are multiplied and *U0.i* is involved in a relation, i.e., this is not a linear system of equations that can be solved symbolically.

The problem can be handled by noting that *OpenSw* only changes its value at a state event when either  $u$  or  $i$  crosses zero. If a consistent value of *OpenSw* is provided after an event, it could be treated as a constant until the next event. Dymola offers a notation for this by means of discrete state variables. They only change their values at events.

The boolean version of the switch equation will now be used:

```
0 = if OpenSwitch then i else u
```

*OpenSwitch* will thus be a discrete boolean state. The updating equation in model class *Diode* is

```
new(OpenSwitch) = u <= 0 and not i > 0
```

i.e., essentially the same as above. The operator **new** signals to Dymola that *OpenSwitch* is a discrete state. Dymola will recognize that state events for  $u$  and  $i$  crossing zero need to be detected and will generate code for that. After a state event has occurred the equation “**new**(*OpenSwitch*) = ...” is processed and *OpenSwitch* gets a new value.

Outputting solved equations gives the following result:

## SORTED AND SOLVED EQUATIONS

```
common. Diode.Vb = 0
C. U0.Va = Diode.Vb + u
circuit. U0.u0 = -sin(2 * 3.14159 * f * Time)
U0. Vb = Va - u0
```

## SYSTEM OF 4 SIMULTANEOUS EQUATIONS SOLVED SYSTEM OF EQUATIONS

```
Q101 = if Diode.OpenSwitch then 1 else 0
Q102 = if Diode.OpenSwitch then 0 else 1
Q103 = Ri.R * Q102 - Q101
Q104 = Ri.R * Q102
Ri.u = (Q104 * U0.Vb - Q104 * Diode.Vb) / Q103
Ri.Vb = (Q104 * Diode.Vb - Q101 * U0.Vb) / Q103
Diode.u = (Q101 * Diode.Vb - Q101 * U0.Vb) / Q103
U0.i = (Q102 * U0.Vb - Q102 * Diode.Vb) / Q103
```

## END OF SYSTEM OF SIMULTANEOUS EQUATIONS

```
RL. u = U0.Va - Diode.Vb
i = u / R
circuit. C.i = -(U0.i + RL.i)
C. deru = i / C
QRel(1) = Diode.u
If (Init .OR. Event) Then
```

```

    Call HandleEvent('Diode.u',
    QM(1), Qp(1), Qn(1), QRel(1),
    Init, PrintEvent, Time, AnyEvent)
  End If
  QZp(1) = QRel(1) - Qp(1)
  QZn(1) = QRel(1) - Qn(1)
  QL(1) = QM(1).LE.0

  QRel(2) = U0.i
  If (Init .OR. Event) Then
    Call HandleEvent('U0.i',
    QM(2), Qp(2), Qn(2), QRel(2),
    Init, PrintEvent, Time, AnyEvent)
  End If
  QZp(2) = QRel(2) - Qp(2)
  QZn(2) = QRel(2) - Qn(2)
  QL(2) = QM(2).GT.0

  Diode.    newOpenSw = QL(1).AND.(.NOT.QL(2))
END OF SORTED AND SOLVED EQUATIONS

ELIMINATED STATE DERIVATIVES AND OUTPUTS
y = C.u

```

The central parts of the code for ACSL and DSblock look essentially the same. The vector  $QRel(i)$  contains the residue for the  $i$ 'th relation. A relation  $a + b > c$  thus generates  $QRel(i) = a + b - c$ . The subroutine HandleEvent is called initially and at events.  $QM(i)$  is a discrete status variable for the  $i$ 'th relation. It is either  $-1$ ,  $0$  or  $1$  depending on whether  $QRel(i)$  is negative, zero or positive respectively.  $QM$  only changes its value at events. This ensures that the integration routine can interpolate smoothly to find event times.  $Qp(i)$  and  $Qn(i)$  are both zero unless  $QRel(i)$  became exactly zero. In such a case  $Qp$  and  $Qn$  represent a small interval around zero to facilitate triggering of events when  $QRel(i)$  leaves zero.  $QZp(i)$  and  $QZn(i)$  are the zero crossing functions in positive and negative directions respectively. They are used in schedule statements for ACSL and reported in the subroutine interface of DSblock.

It must be checked that the new value of discrete boolean states are consistent with continuous states and algebraic variables. It might be necessary to iterate in order to find consistent restart conditions. Dymola thus produces code to iterate until no relations change any more. Event propagation often involves continuous equations. Dymola thus sorts all equations together and makes event-related equations conditional.

Code is generated to print an (optional) event log. The beginning of the event log for the rectifier simulation is shown below.

```

Expression Diode.u is initially 0.000E+00 ( == 0 ).
Expression Diode.i is initially 0.000E+00 ( == 0 ).
Variable OpenSw changed to True
Iterating to find consistent restart conditions.
Continuing after event.

Expression Diode.u became 1.5717E-07 (> 0) at time = 4.999E-10.
Variable OpenSw changed to False
Iterating to find consistent restart conditions.
Expression Diode.i became 1.571E-07 (> 0) at time = 4.999E-10.
Iterating to find consistent restart conditions.
Continuing after event.

```

```

Expression Diode.i became -1.106E-08 (< 0) at time = 8.708E-03.
Variable OpenSw changed to True
Iterating to find consistent restart conditions.
Expression Diode.u became -1.106E-08 (< 0) at time = 8.708E-03.
Iterating to find consistent restart conditions.
Continuing after event.

```

The results of the simulation, i.e., the voltage over the load is plotted together with the voltage of the voltage source in Fig. 6.

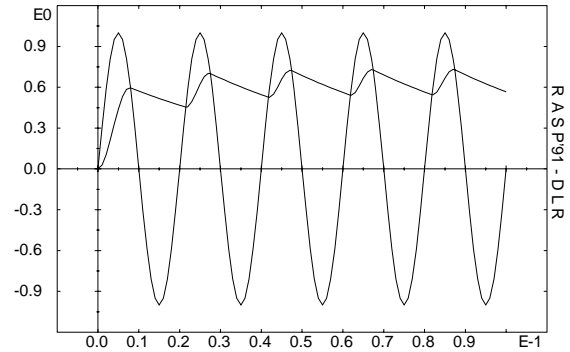


Figure 6: Result of rectifier simulation

## DRY FRICTION – A VARIABLE STRUCTURE MODEL

Let us now look at another frequently modeled phenomenon: friction. To be able to discuss all the nasty effects and the corresponding solution strategies, the example shown in Fig.7 is considered. Two blocks are

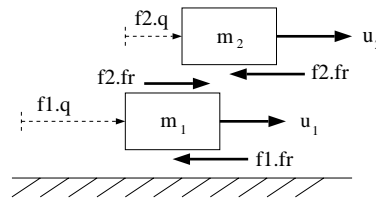


Figure 7: Two block problem with dry friction.

sliding on each other and the environment. Between the environment and block 1 as well as between block 1 and block 2 dry friction is present, according to the friction model shown in Fig.8. A valid Dymola model is presented below.

```

model TwoBlock
  submodel (TransBody) m1 (m = 1), m2 (m = 2)
  submodel (Friction) f1 (R0 = 10, Rm = 8)
  submodel (Friction) f2 (R0 = 5, Rm = 4)
  submodel (ExtForce) fe1, fe2
  submodel (Inertial) i
  input u1, u2

  connect i to f1 to m1, m1 to f2 to m2,
  fe1 at m1, fe2 at m2

```



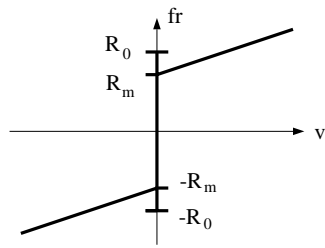


Figure 8: Friction model.

```

fe1.f = u1
fe2.f = u2
end

model class TransBody
  main cut c (p v a/f)
  parameter m

  m * a = f
end

model class Inertial
  main cut c (p v a/f)
  p = 0
  v = 0
  a = 0
end

model class ExtForce
  main cut c (p v a/f)
end

model class TransForce
  cut c1 (p1 v1 a1/f), c2 (p2 v2 a2/ -f)
  main path p < c1 - c2 >
  local p, v, a

  p = p2 - p1
  v = v2 - v1
  a = a2 - a1
  v = der(p)
  a = der(v)
end

model class (TransForce) Friction
  ...
end

```

In model *TwoBlock*, two translational bodies  $m1$  and  $m2$  are connected to each other and the environment  $i$  via two friction elements  $f1$  and  $f2$ . At the bodies external forces  $fe1$  and  $fe2$  are applied. The bodies are objects of class *TransBody* which defines one-dimensional translational bodies. The dynamics of a body is described by Newton's law for the linear momentum of the two blocks. This part of the model poses no problems at all. The nasty part is hidden in the general friction element *Friction*, especially how the friction forces ( $f1$ ,  $f2$ ) are calculated. Before the corresponding Dymola model is given, the friction element will be carefully analyzed.

According to Fig. 8, the friction force is a known applied force if the velocity  $v$  is not zero. When the

velocity becomes zero, the two bodies, between which the friction force is acting, become stuck. In this situation, the model changes its structure: The new equation " $v=0$ " and a new unknown force  $fc$  are added. The constraint force  $fc$  is determined such that the new condition " $v=0$ " is fulfilled. This is a new situation as compared to the electrical switch, because the electrical switch switches between *two different equations*. Contrary, the friction element *adds one equation and one variable* when  $v$  becomes 0 and removes the two, when  $abs(fc)$  becomes larger than the threshold value  $R0$ . Simulation environments do not usually allow to remove a variable during integration. Therefore, a *dummy* equation is added, which becomes active, when the constraint equation " $v=0$ " is removed. The dummy equation is used to provide a unique (but arbitrary) value for  $fc$  (e.g. zero). To summarize, the friction-force  $fr$  is defined by the following equations<sup>3</sup>:

$$\begin{aligned}
 fr &= \text{if } v > 0 \text{ then } Rv * v + Rm \text{ else} \\
 &\quad \text{if } v < 0 \text{ then } Rv * v - Rm \text{ else } fc \\
 0 &= \text{if } Stuck \text{ then } v \text{ else } fc
 \end{aligned}$$

As we have seen before, neither of the variables involved in a switch equation may be a state variable, since otherwise, the causality of the switch equation is predetermined. Unfortunately, velocities are either state variables or are calculated from state variables. Therefore, they are always input quantities. However, if the velocity  $v$  is zero in the "stuck"-state, then also the acceleration  $a = der(v)$  is zero. Accelerations aren't state variables in mechanical models. Thus, we can simply replace the velocity by the acceleration in the switch equation.

This procedure leads to a (minor) complication. Due to limited precision, the velocity  $v$  will not be exactly zero, when switching from sliding to sticking mode. Therefore  $v$  will drift away from zero, if the element remains in the sticking phase for a sufficiently long period of time, because the switch equation will no longer take care of the velocity. Usually this effect can be neglected. If  $v$  is a state variable in the sliding phase, it is easy to provide a better solution, by using  $v = 0$  as new initial condition after the element becomes stuck. The drifting effect will become much smaller. Even an exact solution is possible in this case by switching to two dummy differential equations and keeping the actual value of the relative position and velocity as long as the element is stuck.

It must now be defined, how the switching between the sliding and the sticking phases takes place. For this, it is advantageous to split up the friction force law into the following 5 different regions:

<sup>3</sup>For simplicity, the applied friction force is simply described as a piecewise linear function. It is easy to provide a more complicated law.

<b>region:</b>	<b>region conditions:</b>
<i>Forward</i>	$v > 0$ and $fr = Rv * v + Rm$
<i>StartForward</i>	$v = 0$ and $a > 0$ and $fr = Rm$
<i>Stuck</i>	$v = 0$ and $a = 0$ and $-R0 \leq fr \leq R0$
<i>StartBackward</i>	$v = 0$ and $a < 0$ and $fr = -Rm$
<i>Backward</i>	$v < 0$ and $fr = Rv * v - Rm$

Regions *Forward* and *Backward* describe the sliding phase and are defined by a non-zero velocity. Region *Stuck* is the sticking phase and is defined by an identically vanishing velocity and acceleration. Regions *StartForward* and *StartBackward* define the transition from sticking to sliding. They are defined by a zero velocity. The difference to the sticking phase is that the acceleration is no longer fixed to zero. The above 5 regions cannot be used directly in a Dymola model, because the equality relation “=” appears in the definition. Due to finite precision it is not possible to fulfil such a relationship and therefore an approximate strategy is necessary. Here, an indirect approach is used: The switching between the 5 regions can be described by a *deterministic finite state machine* (DFSM). For the properties of a DFSM see e.g. (Aho et al. 1987). The state transition diagram of the DFSM is shown in Fig. 9.

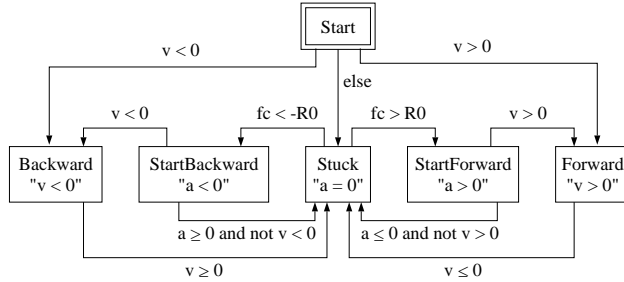


Figure 9: State transition diagram of friction model.

The DFSM has 6 states corresponding to the 5 regions and a *Start* state. Starting from one state of the DFSM and using one of the mutually exclusive conditions, a new state of the DFSM is uniquely determined. None of the switching conditions contains the equality relation. By construction it is guaranteed, that the velocity remains in the neighborhood of zero in the *Stuck*, *StartForward* and *StartBackward* states. The actual value of the velocity in these three states is determined by the stopping condition of the crossing function iteration of the used integrator, when the velocity crosses zero from either the positive or the negative side. The two states *StartForward* and *StartBackward* are used as “waiting” states, see Fig. 9. Assume for example, that the friction element is in state *Stuck* and that the friction force becomes bigger than  $R0$ . Hence, the DFSM switches to state *StartForward*. In this state, the sign of the velocity is *not* known ( $v$  is small, but may be negative). Therefore, the DFSM remains in this state,

until the velocity becomes positive due to a positive acceleration  $a$ .

A valid Dymola program can be easily derived from a DFSM by defining a boolean variable for every state of the DFSM and by using the following transformation rule:

```
new(state) = pre-state-1 and in-condition-1 or
            pre-state-2 and in-condition-2 or
            ... or
            state and not (out-condition-1 or
                          out-condition-2 ...)
```

Now, all the pieces can be put together to construct the following Dymola model for the friction element<sup>4</sup>:

```
model class (TransForce) Friction
parameter R0, Rm, Rv = 0
local fr, fc, Stuck, Start = True,
      Forward = False, StartForward = False,
      Backward = False, StartBackward = False

fr = if Forward then Rv * v + Rm else
      if Backward then Rv * v - Rm else
      if StartForward then Rm else
      if StartBackward then -Rm else fc
f = -fr
0 = if Stuck or Start then a else fc

Stuck = not (Start or
            Forward or StartForward or
            Backward or StartBackward)

new(Forward) = Start and v > 0 or
             StartForward and v > 0 or
             Forward and not v <= 0

new(Backward) = Start and v < 0 or
              StartBackward and v < 0 or
              Backward and not v >= 0

new(StartForward) = Stuck and fc > R0 or
                  StartForward and not
                  (v > 0 or a <= 0 and not v > 0)

new(StartBackward) = Stuck and fc < -R0 or
                   StartBackward and not
                   (v < 0 or a >= 0 and not v < 0)

new(Start) = False

when Stuck and not Start then
  new(v) = 0
endwhen
end
```

Remember, that at an event instant the model equations are evaluated iteratively as long as the boolean variables change their values. In the above case this usually means, that several state transitions of the underlying DFSM take place. E.g. to switch from *Backward* to *Forward*, at least 3 iterations are necessary. At an integration restart it is guaranteed that e.g. “ $\text{new}(\text{Forward}) = \text{Forward}$ ”. There is no guarantee

<sup>4</sup>For states *Stuck* and *Start*, the general transformation rule of a DFSM is not applied, but an obvious (simpler) definition is used.

however, that the iteration will always converge, if several friction elements or other discontinuous elements change their states at the same event point. In a recent paper by (Glocker and Pfeiffer 1993) it is shown that the general planar friction problem (the above form is a special case of it) can be transformed to a so called *linear complementarity* problem for which a rich theory about existence and uniqueness of the solution as well as numerical algorithms exist. But even with the help of this theory, no algorithm is presently known that is able to determine a restart-solution in all possible situations. The reason is that the “system matrix” in the *linear complementarity* formulation does not have the properties, for which convergence proofs exist.

The discussed friction element can be used not only in simple systems such as the “two-block” problem, but in general multibody systems as well. In (Otter et al. 1993) the multibody library of Dymola is described. Here it is shown, that the additional algebraic equations due to friction will introduce nearly no computational overhead, at least for systems in tree structure. We have meanwhile simulated a six degree of freedom robot with friction in all the joints to test the above procedure at a more complicated model and encountered no difficulties. Note, that this system corresponds to  $2^6 = 64$  models with a different number of degrees of freedom.

## CONCLUSIONS

A new methodology was introduced to describe hybrid systems in a truly reusable object-oriented fashion *without* using any explicit event definitions. The novel language constructs, i.e., discontinuous and instantaneous equations controlled by boolean variables and expressions, can be used more comfortably and safely than corresponding language elements in nowadays simulation languages, which are based on explicit definitions of events and event actions. In the traditional “event-oriented” approach, difficulties are often encountered if events occur at the same time instant. In the newly introduced methodology this is avoided by sorting *all* equations together, which provides automatic synchronization of events.

The power of the new language constructs have been demonstrated on the non-trivial class of variable structure systems. It was shown that the exponential growth in the number of different models to be considered can be successfully prevented by transforming the variable structure model into one single simulation program containing systems of simultaneous equations with different zero/non-zero patterns for the different model structures.

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