

**Math 215: Calculus II**  
**Extra credit problems.**

This list will grow throughout the semester.

**guidelines:** When doing an extra credit problem, you should write up your solution carefully, though I will not collect your write up. When you are confident in your solution, come see me. You will need to talk me through whatever work you have.

You may present a partial solution to me for partial credit.

Feel free to ask me questions.

- (1) There is a STEM (Science Technology Engineering Mathematics) Colloquium Friday March 6. Attend some of the talks and write me something about the topic of one talk.
- (2) Let  $C$  be a cone of height  $h$  and radius  $r$ . Compute its volume using calculus.
- (3) Select a geometric figure whose volume formula you might know. Use Calculus to derive the volume of this solid. (Full credit will be given for examples which are interesting and challenging. )
- (4) Think about a **4-dimensional** sphere of radius  $r$ . It is given by the inequality  $x^2 + y^2 + z^2 + w^2 \leq r^2$ .
  - What range of values for  $w$  can occur in the 4-sphere? (Find a domain for integration.)
  - At any given  $w$ , What does the defining inequality say about  $x$ ,  $y$  and  $z$ . What is the  $w$ -cross-section? What is its 3-dimensional volume?
  - Integrate the 3-dimensional volume of cross sections to find the 4-dimensional volume. For full points explain the integration technique used. For partial credit, use a computer.
- (5) This exercise will guide you through an approximation of  $\pi$ .
  - (a) Compute  $\int_0^1 \frac{1}{t^2 + 1} dt$ . Does this have something to do with  $\pi$ ?
  - (b) Using Wolfram alpha, approximate  $\int_0^1 \frac{1}{t^2 + 1} dt$  Use midpoint approximation and  $n = 100$ .
  - (c) Use  $n = 10000$
  - (d) Use  $n = 1000000$
  - (e) If  $f(t) = \frac{1}{t^2 + 1}$ , then what are the maximum value and minimum values of  $f''(t)$  on  $[0, 1]$ ?
  - (f) In the error bound on page 510 of the book, what value can you take for  $K$ ?
  - (g) How large must  $n$  be if you want an approximation with error less than  $10^{-20}$ ? Have wolfram alpha make this approximation.
- (6) Section 7.8 problem 71.
- (7) Plot the direction field for the differential equation  $y' = \cos(x+y)$  and sketch a picture of the solution. Using Maple's online user's manual, or wolfram alpha's documentation, or else your own ingenuity, run Euler's Method to approximate the solution to  $y' = \cos(x + y)$ .