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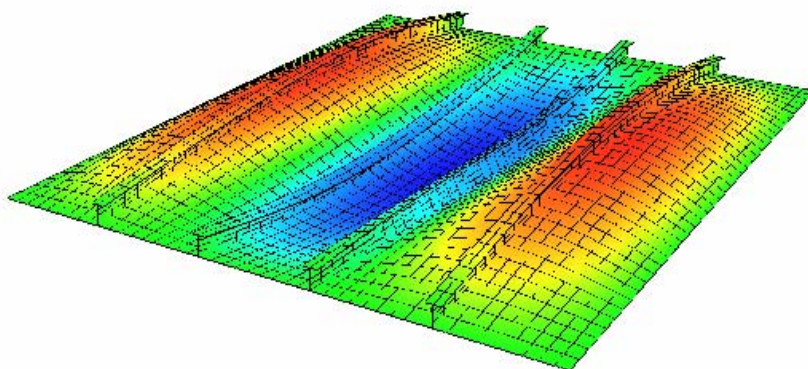
# NAUTICUS HULL

## USER MANUAL

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# PULS

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DET NORSKE VERITAS

# NAUTICUS HULL

## USER MANUAL

# PULS

SEPTEMBER 2006

Valid from Hull program version 10.5

Puls version 2.0.6

Developed and marketed by  
DET NORSKE VERITAS

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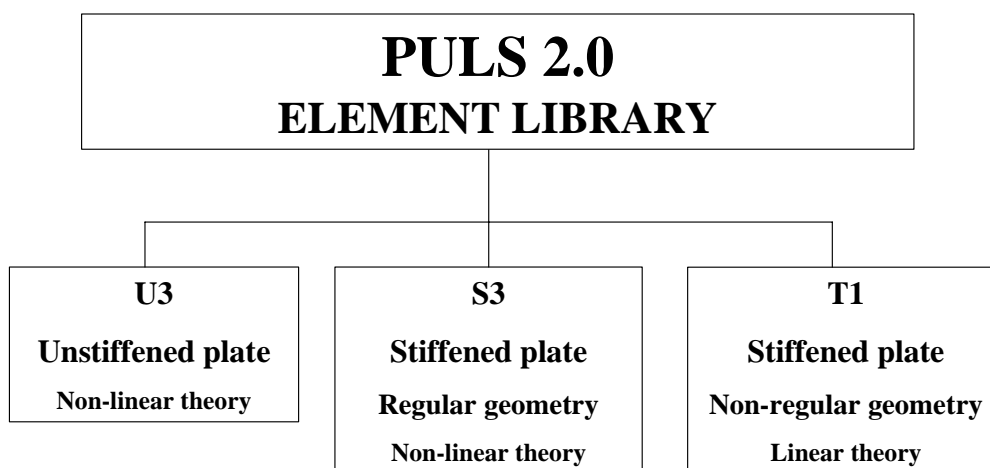
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## **1 INTRODUCTION**

### **1.1 General**

The present report gives a User's guide to the Windows program PULS 2.0 with a brief review of theoretical background, design principles and purpose. Main emphasis is on program features and functionality, input description, results and example illustration.

PULS is a computerised buckling code for thin-walled plate constructions. It assess the elastic buckling stresses and ultimate load bearing capacities under combined loads of stiffened and unstiffened plates used as building blocks in larger plated constructions such as ships and offshore constructions. The PULS element library is illustrated in Fig.1 below



**Fig.1 PULS element library**

The present PULS 2.0 version has three elements, which are briefly summarized in the following. A detailed description of each element is given in separate chapters. For more theoretical details see publications given in the reference list at the end of this report.

**1) Element U3: Unstiffened plate**

Field of application: Integrated hull element between laterally rigid structures such as frames, bulkheads etc. Rectangular plate without stiffeners. Based on non-linear theory.

Material: Steel and aluminium.

Loads: Buckling Strength (BS) and Ultimate Capacity (UC) can be estimated for linearly varying longitudinal compression/tension, linearly varying transverse compression/tension and uniform shear, and for all possible in-plane load combinations of these. Lateral pressure can also be specified.

Boundary conditions: For all load combinations simply supported out-of-plane (free to rotate but laterally fixed) and constrained in-plane (straight but movable in-plane) edges are by default assumed along all four boundaries. Alternative boundary conditions may be specified by the user.

**2) Element S3: Uni-axially stiffened plate**

Field of application: Integrated hull element between laterally rigid structures such as frames, bulkheads etc. Rectangular plate with primary stiffeners in axial direction. Based on non-linear theory.

Primary Stiffeners: Welded open profiles; Angle, T, bulb or flat bar profiles.

Secondary Stiffeners option: Perpendicular to primary stiffeners, simplified theory.

Material: Steel and aluminium.

Loads: Buckling Strength (BS) and Ultimate Capacity (UC) can be estimated for uniform longitudinal compression/tension, linearly varying transverse compression/tension and uniform shear, and for all possible in-plane load combinations. Lateral pressure can be specified and is assumed to act across several bays in the continuous primary stiffener direction.

Boundary conditions: For all load combinations simply supported out-of-plane (free to rotate but laterally fixed) and constrained in-plane (straight but movable in-plane) edges are by default assumed along all four boundaries. When lateral pressure is specified a symmetric (clamped) boundary constraint is introduced at transverse frame supports by adding an extra set of deflection forms on top of the regular asymmetric regular (simply supported) forms. Alternative boundary conditions may be specified by the user.

**3) Element T1: Stiffened plate with non-regular geometry**

Field of application: Hull element between laterally rigid structures such as frames, bulkheads etc. Rectangular plate with arbitrary oriented stiffeners. The model is based on linear theory.

Stiffeners: Welded open profiles; Angle, T, bulb or flat bar profiles.

Material: Steel and aluminium.

Loads: Buckling strength (BS) can be estimated for linearly varying axial and transverse compression/tension and uniform shear, and for all possible in-plane load combinations.



Boundary conditions: For all load combinations simply supported out-of-plane (free to rotate but laterally fixed) edges are by default assumed along all four boundaries. Alternative boundary conditions may be specified by the user.

## 1.2 Approach

The PULS buckling models for the S3 and U3 elements apply the non-linear large deflection plate theory of Marguerre and von Karman. Discretizations of the buckling displacements follow the Rayleigh-Ritz method using Fourier series expansions across the plate and stiffener surfaces. Energy principles are used for establishing the algebraic non-linear equilibrium equations and incremental perturbation techniques are used for solving these. Hot spot stress control using the redistributed membrane stresses in a selection of critical locations determines the ultimate strength allowing for overcritical strength.

For stiffened panels (S3 element) an orthotropic version of the same theory is used for the global buckling mode. The local buckling model treats the plate and stiffeners as discrete elements including all relevant effects such as buckling of the plate between the stiffeners, buckling of the stiffener web plate as well the rotational restraints between plate and stiffeners. The interaction between local and global (lateral) stiffener buckling is coped with using modified orthotropic stiffness coefficients (reduced stiffness/modulus coefficients).

For the T1 element (plates with stiffeners in arbitrary direction), linearized plate theory is used (not overcritical strength). This means that the buckling capacity predicted for this element is maximum the minimum eigenvalue. The incremental approach and hot spot stress control is used also for this element for assessing the strength of geometrical imperfect plates.

The PULS code is programmed in a Windows environment using Visual Basic (VB) tools. Subroutines are programmed in Fortran 2000.

Two different user-interfaces are available

- i) Advanced Viewer (AV). Provides 3D graphical presentation of buckling modes, membrane stress redistributions, load interaction curves etc.
- ii) PULS Excel version with compatible input/output data files with the AV version.

## 1.3 Revision history

The PULS revision history is summarized in the following table:

Prog. Version*	Date	Theory updates	New program functionality	Comments
1.2-4	April 2002	New imperfection model for U1 element	As for 1.2-3	
1.3	May 2002	<b>New local buckling model implemented for stiffened panels: S2.</b> -More correct local buckling	-Local eigenvalues for pure shear and shear in comb. with normal stresses shown. Shown	Aluminium option locked (alu can be specified in

		<p>assessment of slender stiffener webs, rotational restraint effects from stiffeners</p> <p>-Local shear buckling assessment with rotational restraint effects from stiffeners</p> <p>-Improved imperfection modelling as default, user specified imp. as option</p>	<p>in:</p> <p>..detail result window,</p> <p>..in local eigenmode window,</p> <p>..in capacity curves window</p> <p>-Status bar</p> <p>-Recalculate the project option(File menu)</p> <p>-Extended imperfection settings</p> <p>-Increased control of change of input values (cell control)</p>	<p>steel mode but no HAZ effects included.)</p>
1.4	Sept. 2002	<p><b>Improvements of stiffened panel S2 theory:</b></p> <p>i) Generally more degrees of freedom for local buckling description. Consequences:</p> <p>Max allowable aspect ratio plate between stiffeners from 8 to 14.</p> <p>ii) Modified imperfection model. Consequence:</p> <p>Predicts more conservative ULS strength (than 1.3) for some rare geometry's and loading: Typical very high stiffeners and dominating transverse compression.</p> <p><b>Improvements of unstiffened plate U1 theory:</b></p> <p>i) Generally more degrees of freedom for plate buckling description. Consequences:</p> <ul style="list-style-type: none"> <li>- Max allowable aspect ratio plate from 6 to 20.</li> </ul> <p>ii) Modified imperfection model. Consequence:</p> <p>Predicts more smooth bi-axial capacity curves. ULS usage factor estimates marginally changed (compared to 1.3).</p>	<p>- Automatic Profile Table: Bulb profiles, L-profiles etc. (from Nauticus Hull)</p> <p>- Button for switch between ULS estimates (load bearing capacity) and SLS (buckling estimates) w.r.t. usage factor assessment</p> <p>- New application of dialog box for selecting element/panel type</p> <p>-Two options for secondary transv. stiffeners; sniped or tripping stiffeners.</p> <p>-Extended 3D graphics for S2 element, similar to U1 element</p> <p>-All 3D graphics sorted into panel tree menu: eigenmodes, imperfections, ULS modes, ULS stresses.</p> <p>-Automatic Word Report generator: Updated to support both S2 and U1 elements</p>	<p>Aluminium option locked ( alu can be specified in steel mode but no HAZ effects included.)</p> <p>PULS ver. 1.4 supports ver. 1.3 data files</p>
1.5	Feb.	<p><b>Improvements of stiffened panel S2 theory:</b></p>	<p><b>New functionality:</b></p>	<p>Comments:</p>

	2003	<p>i) Improvements of local shear buckling model</p> <p><b>ii) New non-linear global buckling model with lateral pressure option. Consequence: For most geometries new model gives same in-plane ULS strength prediction than version 1.4 for default imperfections settings. However, new model gives improved and more realistic ULS estimates for tolerance exceed.</b></p> <p>iii) lateral pressure included</p> <p>iv) Modified local buckling theory for coping with three types of secondary transverse stiffening; sniped stiffeners, tripping stiffeners and tripping brackets</p> <p>v) Modified imperfection model for high and slender stiffeners; added a short wave local pattern for axially compressed panels having a min. eigenmode in local (1,1) mode. Consequence: Predicts more conservative and realistic ULS strength (than 1.4) for axially compressed panels with very high webs and flanged profiles</p> <p>v) Updated aluminium model with HAZ corrections</p> <p>vi) aspect ratio of plate between stiffeners from 14 ( PULS 1.4) to range 0.25 - 10 for covering cases with closely spaced secondary stiffeners</p> <p><b>Improvements of unstiffened plate U1 theory:</b></p> <p>i) Updated aluminium model with HAZ corrections</p>	<p>- <b>New 3D graphics visualizing cont. stiffeners in S2 element (<math>x_1</math>-direction).</b></p> <p><b>3D graphics in panel tree menu: eigenmodes, imperfection modes, ULS modes/stresses, S2 lateral pressure graphics: pure global pressure deflections</b></p> <p>- New: Three options for secondary transv. stiffeners; sniped stiffeners, tripping stiffeners or tripping brackets.</p> <p>- New: Standard parameter output list – placed in a separate tab strip under “detailed result” folder. It gives linear parameters such as cross-sectional data, moment of inertia of stiffener/plate unit etc.</p> <p><b>- Animation of non-linear buckling response: separate menu on tool bar for animation of non-linear buckling response (NB! may take long time for thin plates and cases with much elastic buckling; 10-15 minutes)</b></p> <p>- Lateral pressure as “preload” in Capacity Curves modus.</p> <p>- Separate option in “capacity curves settings”: GEB cut-off and scaling of ULS capacity curve as allowable usage factor. This gives the capacity curves as “required strength envelope”.</p>	PULS ver. 1.5 supports version 1.3 and 1.4 data files, but not the graphics. Recalculation necessary for updated graphics and detailed result summary
2.0	April 2004	<p><b>Stiffened panel S3:</b></p> <p>i) Linearly varying load in the transverse direction</p> <p>ii) Alternative in-plane boundary</p>		

		conditions iii) Sniped primary stiffener option iv) Tilted primary stiffener option v) Improved imperfection control vi) Improved lateral pressure model vii) Capacity curves prestress option  <b>Unstiffened plate U3:</b> i) Rotational restraint may be specified for each edge individually ii) Corrugated bulkhead option iii) Alternative in-plane boundary conditions iv) Improved imperfection control  <b>Non-regular stiffened plate T1:</b> i) New linear element formulation for plates with arbitrary oriented stiffeners		
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\* sub-versions will include bug fixing, minor relevant improvements etc.

## 1.4 Buckling of hull elements - problem identifications

A thin-walled stiffened panel is the basic building block in ship hulls as illustrated in Fig.2. They are typically located in the bottom, ship sides, longitudinal and transverse bulkheads and in the deck structure. Each stiffened panel is composed of individual component plates joined together along junction lines.

Depending on where in the structure the stiffened panel is located, i.e. bottom, deck, shipside, girder webs etc, it will be subjected to different types of local loads.

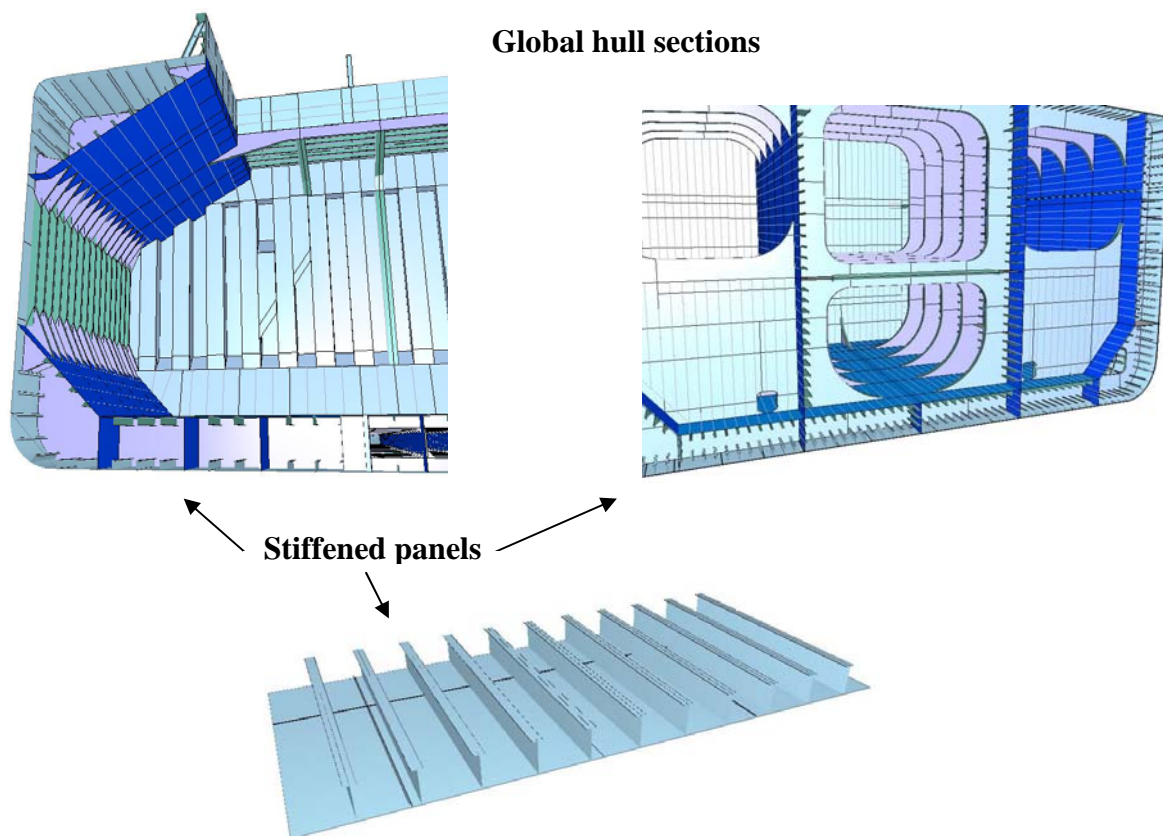


Fig. 2 Ship hulls with stiffened panels as main building block

Typically the main load components acting on a local stiffened panel/plate are

- i) In-plane load in the direction of the stiffener, compression or tension
- ii) In-plane load in the direction perpendicular to the stiffener, compression or tension
- iii) In-plane shear
- iv) Lateral pressure from sea or cargo

Buckling and ultimate strength of plates depends on the nature of the locally applied loads and the boundary conditions enforced from the surrounding structure. The boundary conditions can be categorised in two groups, i.e. out-of-plane support and in-plane support.

Out-of-plane support: In most codes the out-of-plane support along the outer plate edges is assumed to be rigid in the lateral direction while free to rotate. This corresponds to the classical simply supported boundary conditions. This is also the default for the present PULS elements, but alternative boundary conditions may be specified. See also description for each element type for more details.

In-plane support: The in-plane (membrane) support is also important particularly with respect to the elements ability to carry loads beyond the elastic buckling load (LEB and GEB eigenvalues) level, i.e. the nature of in-plane support influences the postbuckling and ultimate capacity behaviour (UC) of thin plates.

The PULS elements U3 and S3 are by default assumed to be **integrated** elements, Fig.2. Integrated elements mean that the plate edges can transfer second order normal membrane stresses to neighbouring elements, compressive as well as tension (i.e. tension fields). In a theoretical language this means that the plate edges are constrained to remain straight but free to move in-plane. This is a constraint that is very much used both in the analytical and numerical published literature on plate buckling. In practise this constraint means that the present PULS elements are relevant for dimensioning and strength control of “internal” plates and panels such as in bottom and deck structures, bulkheads, shipsides etc. Weaker “free to pull in edges” membrane conditions may be specified as an alternative being more relevant for plate girders etc.

For some type of structures and type of loadings (e.g. serviceability loads, SLS) it may be that elastic buckling and thereby large deflections are not acceptable. This philosophy implemented in a design code means that the plate thickness and stiffener proportions are to be increased as compared to a ULS philosophy. For girders in particular such a SLS philosophy will be very reasonable and ensure robust designs with extra margins to take “additional redistributed” loads coming from accepting elastic buckling of surrounding structures, i.e. typically inner and outer bottom plating in ship hulls.

For all elements the present PULS code gives a buckling control (BS) and ultimate capacity control (UC) under a given load combination (nominal applied stresses, load control) as specified by the user. This corresponds to the **standard mode** of program operation. The results are presented as elastic buckling (eigenvalues) and ultimate strength nominal capacities and summarized in a detailed result table. As a single parameter result, the safety margin in the form of **usage factor** is given, both related to buckling (BS) and ultimate capacity (UC). The usage factor provides a measure of the difference between the user specified loads and the corresponding ultimate capacity (UC) or buckling strength (BS).

The PULS AV program also provides **capacity curves** under combined loads. The capacity curves are illustrated in two-dimensional load-spaces. They are to be understood as limit boundaries covering the load-space selected by the user. They inform about the strength of the plates in the different load directions and under any load combination. In this mode the usage factor is not calculated as it is not defined. The term **capacity curve** is demonstrated in Section 5.7.

## 2 U3 – UNSTIFFENED PLATE ELEMENT

### 2.1 General

The U3 unstiffened plate element applies Marguerre's non-linear plate theory (geometrical non-linearity). The elastic buckling (eigenvalues) and non-linear postbuckling problem are solved using a multiple degree of freedom model in terms of Fourier expansions.

The plate can be subjected to combined load situations, and the numerical procedure scale the applied loads up to collapse. Stress control criteria describe the onset of material yielding in the highest loaded position along the plate edges using the **redistributed** membrane stress distribution. The redistributed membrane stresses consist of the external applied nominal stresses added to the second order stress distribution arising due to elastic buckling and due to presence of geometrical imperfections from production.

The values of the proportionally scaled loads, at the onset of first edge membrane yield, is taken as representative for the UC values. UC values based on such first yield criteria for thin-walled designs are close to the real UC values, and on the conservative side. In addition to the UC value, the ideal elastic buckling stress (eigenvalue) is calculated (Linear Elastic Buckling).

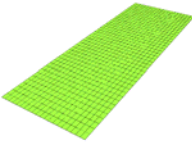
Using default tolerance settings and boundary conditions, and characteristic yield strength as specified in the rules, the code predicts UC strength values representative for integrated plates in **larger** flat plate constructions. Alternatively, the in-plane boundary conditions may be specified as free, so that an isolated unstiffened plate is represented.

User specified tolerance input for the max amplitude is optionally. The default tolerance amplitude is specified in terms of a maximum amplitude ( $\delta = s/200$ ). The tolerance shape is automatically taken to harmonise with the minimum eigenmode with some added trigger modes for safe UC strength assessment. The latter is called imperfection model and is not possible to control by the user.

### 2.2 Overview of element characteristics

The main characteristics of the U3 element are summarized in Table 1.

Table 1 Overview of U3 element characteristics

U3 element – unstiffened plate	
Theory:	Non-linear
Structural configuration: 	Rectangular unstiffened plate - Uniform plate thickness
Loads:	<ul style="list-style-type: none"> <li>- Linearly varying normal stress along long edge</li> <li>- Linearly varying normal stress along short edge</li> <li>- Uniform in-plane shear stress (<math>x_1</math>-<math>x_2</math> plane)</li> <li>- Uniform lateral pressure <math>p</math> (fixed)</li> </ul>
Materials:	<ul style="list-style-type: none"> <li>- Isotropic elastic material (<math>E</math>, <math>\nu</math>)</li> <li>- Steel</li> <li>- Aluminium</li> <li>- HAZ correction option</li> </ul>
Boundary conditions:	<p>Out-of plane support (bending support)</p> <ul style="list-style-type: none"> <li>- All four edges supported laterally in plate plane</li> <li>- Rotational restraint control of each edge; free to rotate, rotationally restrained (spring) or clamped</li> </ul> <p>Two options for in-plane support (membrane support )</p> <ul style="list-style-type: none"> <li>- I : Integrated panel , all four edges restrained to be straight (decks, bottom etc)</li> <li>- G : Girder panel, two opposite plate edges free to pull in (web girders etc.)</li> </ul>
Model imperfections:	<ul style="list-style-type: none"> <li>- Default model imperfections consistent with as welded steel panels and normal production tolerance standards</li> <li>- User defined tolerances: Damages, imperfection sensitivity studies etc</li> </ul>
Output:	<ul style="list-style-type: none"> <li>- Ultimate Capacity (UC)</li> <li>- Buckling Strength (BS) (minimum of UC, LEB)</li> <li>- Usage factor UC (ratio: applied loads/ UC)</li> <li>- Usage factor BS (ratio: applied loads/ BS)</li> <li>- Minimum Eigenvalue (LEB)</li> </ul>
Validity limits:	<p>Plate slenderness: <math>(L_{\text{shortest}}) / t_p &lt; 200</math></p> <p>Max aspect ratio of plate; <math>L_{\text{longest}} / L_{\text{shortest}} &lt; 20</math></p>



## 2.3 Element limitations - slenderness requirements

A limited number of degrees of freedom is used in the PULS code, in order to achieve high computational efficiency. Therefore, some limitations are put on the geometric proportions of the plates that are to be analysed. The limitations set in the PULS 2.0 version are:

Aspect ratio limit:  $L_1/L_2 < 20$  for  $L_1 > L_2$  (or equivalent  $L_2/L_1 < 20$  for  $L_1 < L_2$ )

Plate slenderness ratio:  $L_i/t_p < 200$  ( $L_i$  = minimum of  $L_1$  and  $L_2$ )

## 2.4 General U3 element design principles

The U3 element applied in an ULS design setting is based on the following main principles

- i) **Elastic buckling is accepted.**
- ii) **Permanent buckles are not accepted.**

By ensuring the maximum membrane stresses along the plate edges to stay below the yield stress condition (von Mises), excessive permanent sets and buckles are prevented ( principle ii) above). In plates second order bending stresses adds to the second order membrane stresses. The resulting surface stress is generally accepted to exceed the yield condition in local areas. This is not considered critical for the ULS strength of plates and it will not induce permanent sets beyond what is normally accepted in ship designs.

In some cases, it will from serviceability/functional reasons (SLS), not be acceptable that elastic buckling takes place. In such cases the local elastic buckling stress (LEB cut off) can be used as the upper limit of allowable load application. In effect a SLS type of strength assessment allows no elastic buckling and may prove to be a useful approach in particular for girder webs, webs frames i fore and aft ship etc. and in general for designs that are not accepted to buckle elastically nor plastically.

## 2.5 External prescribed loads

The U3 element can be subjected to a combined external load situation covering linearly varying in-plane loads in bi-axial directions and constant in-plane shear. The five in-plane external nominal stresses are given as

$$\sigma_{1,1} = \Lambda \sigma_{10,1}$$

$$\sigma_{1,2} = \Lambda \sigma_{10,2}$$

$$\sigma_{2,1} = \Lambda \sigma_{20,1}$$

$$\sigma_{2,2} = \Lambda \sigma_{20,2}$$

$$\sigma_3 = \Lambda \sigma_{30}$$

where  $\Lambda$  is the proportional load factor automatically controlled by the program. A subscript 0 on the nominal external stresses indicates the values of the input values, i.e. they corresponds to  $\Lambda = 1$ . This correspond to a proportional load history control up to elastic buckling (LEB) and beyond to final collapse (UC).

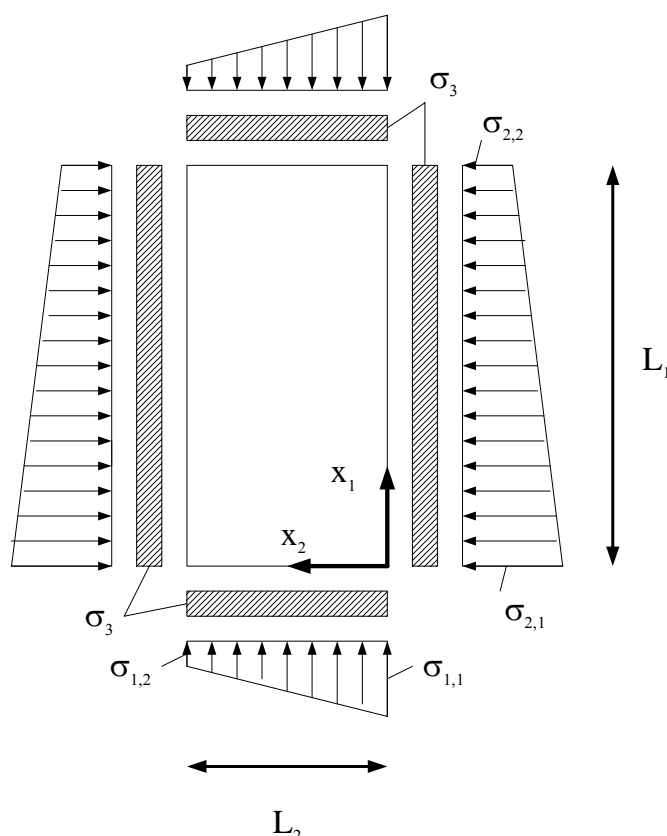
Lateral pressure can also be specified. However, it should be noted that by default the plate is assumed to have simply supported boundary conditions along all four edges, i.e. a type of single span model in both directions. This is not relevant for analysing cases where the lateral pressure is acting across several spans (i.e. several stiffener and girder spans) since then symmetric (clamped) deflection modes needs to be included on top. In order to analyse this case more realistic a multi span plate model is needed. Multi span unstiffened plate model is not available in the current PULS version. Alternatively, the boundary conditions should be prescribed so that all four edges are clamped if this can be documented to be realistic boundary conditions.

In practical cases the stiffeners will be designed to carry the lateral pressure and thus the S3 model will ensure all relevant failure modes.

For strength assessment the lateral pressure  $p$  is kept fixed, equal to the input value, while the in-plane loads are scaled proportionally until elastic buckling and UC strength is identified.

Alternatively, a plate subjected to lateral pressure alone may be analysed. In this case, stress and deflection results are presented for the specified pressure magnitude, and no UC assessment is performed.

The considered external loads typically take the form as illustrated on Fig.3.



**Fig. 3 External load system;**  
linearly varying normal loads in perpendicular directions and constant shear

## 2.6 Boundary conditions

All four edges in the U3 element are supported in the lateral direction. By default, the rotational restraint of the edges is set to zero, representing a plate simply supported on all edges. The rotational restraint may be set for each edge individually. The edges may be specified either as simply supported, clamped, or as a specified spring stiffness.

A corrugated bulkhead option is included. The rotational restraints of the edges are then set to a value that increases the eigenvalue of the plate with 25%, but limited to that of clamped edges. This is equivalent to using a buckling factor from  $C = 4$  for unstiffened plates to  $C = 5$ .

The in-plane boundary conditions are by default so that all four edges are forced to remain straight. This is representative of an integrated plate that is part of a larger structure. Alternatively, two opposite edges may be specified as free to pull in, which is more representative for web girders, stringers, etc.

## 2.7 Available Results

The numerical results from the U3 element are:

- i) Local Elastic buckling stresses; minimum eigenvalue (LEB)
- ii) Ultimate stresses (UC)
- iii) Buckling load (BS) = minimum of ultimate load and minimum eigenvalue
- iv) Usage factor  $\eta$  with ultimate load or buckling load as reference

Graphical results available are plots of the minimum eigenmode, the imperfection mode, and the ultimate limit state deflection mode. Membrane stress plots for the ultimate limit state are also available. For pure lateral pressure computation, it is possible to request bending stress output, rather than membrane stress output.

For a given prescribed load combination, the code calculates the value  $\Lambda_E$  of the load factor at ideal elastic buckling. The elastic buckling stresses follow as

$$\begin{aligned}\sigma_{1E,1} &= \Lambda_E \sigma_{10,1} \\ \sigma_{1E,2} &= \Lambda_E \sigma_{10,2} \\ \sigma_{2E,1} &= \Lambda_E \sigma_{20,1} \\ \sigma_{2E,2} &= \Lambda_E \sigma_{20,2} \\ \sigma_{3E} &= \Lambda_E \sigma_{30}\end{aligned}$$

The corresponding UC strength values are

$$\begin{aligned}\sigma_{1u,1} &= \Lambda_u \sigma_{10,1} \\ \sigma_{1u,2} &= \Lambda_u \sigma_{10,2} \\ \sigma_{2u,1} &= \Lambda_u \sigma_{20,1} \\ \sigma_{2u,2} &= \Lambda_u \sigma_{20,2} \\ \sigma_{3u} &= \Lambda_u \sigma_{30}\end{aligned}$$

## 2.8 Safety margin

The safety margin is presented as the usage factor defined as

$$\eta = \frac{\sqrt{(\sigma_{1u,1})^2 + (\sigma_{1u,2})^2 + (\sigma_{2u,1})^2 + (\sigma_{2u,2})^2 + (\sigma_{3u})^2}}{\sqrt{(\sigma_{10,1})^2 + (\sigma_{10,2})^2 + (\sigma_{20,1})^2 + (\sigma_{20,2})^2 + (\sigma_{30})^2}}$$

The calculated usage factor  $\eta$  is to be measured against the allowable  $\eta_{\text{allow}}$  (or called  $\eta_{\text{max}}$ ) as specified in the relevant Rules and Standards.

## 2.9 Capacity curves

The PULS code can also assess **2D capacity curves** under combined loads. The capacity curves are illustrated in two-dimensional load-spaces. The notation of **capacity curves** and buckling boundaries are illustrated in Section 6.7. Current version handles **pre-stress** for any of the three independent in-plane load component and lateral pressure  $p$ , i.e. 2D capacity curves can be generated within limits set in the program:

- i) Bi-axial load space  $\sigma_1$ - $\sigma_2$  for constant shear say  $\tau_{12} = 10, 20, 30$  MPa and for fixed  $p = 0.0, 0.1$  MPa etc.
- ii) Axial-shear load space  $\sigma_1$ - $\tau_{12}$  for constant transverse compression say  $\sigma_2 = 10, 20, 30$  MPa and for fixed  $p = 0.0, 0.1$  MPa etc.
- iii) Transverse-shear load space  $\sigma_2$ - $\tau_{12}$  for constant axial compression say  $\sigma_1 = 10, 20, 30$  MPa and for fixed  $p = 0.0, 0.1$  MPa etc.

## 2.10 Lateral pressure

Lateral pressure can be prescribed acting uniformly across the whole plate surface. In the buckling analysis the specified lateral pressure is kept fixed, while the in-plane loads are increased until subsequent collapse is reached. Alternatively, a plate subjected to lateral pressure alone may be analysed. In this case, results are presented for the specified pressure magnitude, and no ULS assessment is performed.

It is remarked that the U3 plate element by default assumes all four plate edges to be simply supported out-of plane and constrained to be straight in-plane. For application on, say bottom panels in ships having uniform lateral pressure acting across many plate-spans between stiffeners and frames, this SS assumption is not fully consistent, and clamped boundary conditions may be more appropriate. Option for any rotational edge restraint is available.

When using the PULS program, an indicator/warning always pops up as a dialog box for pressure beyond a fixed limit. This fixed limit is based on a linear clamped plate unit strip model, i.e.  $p_f$  is defined as

$$p_f = 2\sigma_F \left(\frac{t}{s}\right)^2$$

which corresponds to first material yielding in extreme fibre along the long edges due to pure bending stress across the plate thickness.



## 3 S3 - STIFFENED PANEL ELEMENT

### 3.1 General

The S3 element code applies Marguerre's non-linear plate theory (geometrical non-linearity) in combination with stress control criteria for ultimate strength assessment.

The stiffened panel may be subjected to combined loads. The model calculates the ideal elastic buckling loads (eigenvalues) as a separate procedure. A non-linear model is used for assessing the elastic postbuckling strength, while stress control criteria covers the inelastic response and is used for determining the ultimate strength. The stress control criteria describes the onset of material yielding in a selection of critical positions in the panel (hard corners) and are called limit state functions.

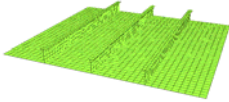
The stresses in the hard corners are calculated as the sum of the direct applied membrane stresses added to the second order membrane stress due to buckling. The second order membrane stresses have contributions from the local buckling of the plate between stiffeners-sideways/torsional buckling of the stiffeners and global buckling of the stiffeners (out-of plate bending of stiffeners). Membrane stresses are mid-plane stresses of each component plate in the cross-section. Bending stresses across any component plate thickness are not included in the limit state yield criteria. The limit states solved explicitly gives the ultimate strength.

Using default tolerance settings and characteristic yield strength as specified in the rules, the code predicts UC strength values as being representative for integrated plates in **larger** flat plate constructions consistently including redistribution of stresses between primary stiffeners and the plate.

User specified tolerance for the maximum amplitude input is optionally. The tolerance shape is always (automatically) taken to harmonise with some critical modes being most influential for the ultimate strength. The latter is called the imperfection model and can not be controlled by the user.

### 3.2 Overview of element characteristics

The main characteristics of the S3 element are summarized in Table 2.

<b>S3 element – stiffened plate – regular geometry</b>	
Theory:	Non-linear
Structural configuration: 	<p>Rectangular stiffened plate</p> <ul style="list-style-type: none"> <li>- Uniform plate thickness</li> <li>- N equal constantly spaced main stiffeners in <math>x_1</math>-dir. Continuous or sniped.</li> <li>- Secondary stiffeners perpendicular to <math>x_1</math>- dir. <ul style="list-style-type: none"> <li>a) Sniped between main stiffeners</li> <li>b) Supporting stiffener sideways</li> </ul> </li> </ul> <p>(Secondary stiffeners = only strengthening the plate between main stiffeners, carry no axial stress)</p>
Loads:	<ul style="list-style-type: none"> <li>- Uniform normal stress in main stiffener direction (<math>x_1</math>- direction)</li> <li>- Linearly varying normal stress perpendicular to main stiffeners (<math>x_2</math>- direction)</li> <li>- Uniform in-plane shear stress (<math>x_1</math>-<math>x_2</math> plane)</li> <li>- Uniform lateral pressure p (fixed)</li> </ul>
Materials:	<ul style="list-style-type: none"> <li>- Isotropic elastic material (E, <math>\nu</math>)</li> <li>- Steel (different yield stress in plate and stiffeners optionally)</li> <li>- Aluminium (different yield stress in plate and stiffeners) <ul style="list-style-type: none"> <li>- HAZ correction option</li> </ul> </li> </ul>
Boundary conditions:	<p>Out-of plane support (bending support)</p> <ul style="list-style-type: none"> <li>- All four edges supported laterally in plate plane</li> <li>- Long edges locally elastically restrained as along primary stiffeners</li> <li>- Transverse edges simply supported</li> </ul> <p>Two options for in-plane support (membrane support )</p> <ul style="list-style-type: none"> <li>- I : Integrated panel , all four edges restrained to be straight (decks, bottom etc)</li> <li>- G : Girder panel, two opposite plate edges free to pull in (web girders etc.)</li> </ul>
Model imperfections:	<ul style="list-style-type: none"> <li>- Default model imperfections consistent with as welded steel panels and normal production tolerance standards</li> <li>- User defined tolerances: Damages, imperfection sensitivity studies etc</li> </ul>
Output:	<ul style="list-style-type: none"> <li>- Ultimate Capacity (UC)</li> <li>- Buckling Strength (BS) (min of UC, LEB, GEB)</li> <li>- Usage factor UC (ratio: applied loads/ UC)</li> <li>- Usage factor BS (ratio: applied loads/ BS)</li> <li>- Local Eigenvalue (LEB)</li> <li>- Global Eigenvalue (GEB)</li> </ul>



Validity limits:	<p>Plate slenderness: <math>s/t_p &lt; 200</math> (s, shortest edge between primary and sec stiffeners)  Max aspect ratio of plate between prim/sec. stiffeners; <math>0.17 &lt; L_1/s &lt; 20</math></p> <p>Primary stiffeners:</p> <p>Stiffener web slenderness for flat bar stiffeners, <math>h/t_w &lt; 35</math>  Stiffener web slenderness Angle and T profiles; <math>h_w/t_w &lt; 90</math>  Free flange slenderness for Angle and T profiles; <math>f_f/t_f &lt; 15</math>  Minimum width of flange for Angle and T profiles <math>b_f/h_w &gt; 0.22</math></p> <p>Secondary stiffeners(AV):</p> <p>Stiffener web slenderness for flat bar stiffeners, <math>h/t_w &lt; 15</math>  Stiffener web slenderness Angle and T profiles; <math>h_w/t_w &lt; 33</math>  Free flange slenderness for Angle and T profiles; <math>f_f/t_f &lt; 15</math></p>
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**Table 2 Overview of S3 element characteristics**

### **3.3 Element limitations- slenderness requirements**

The present PULS S3 element has one local macro material routine with the following characteristics and range of application:

- Geometry:** Uni-axially stiffened plate with open continuous profiles welded to plating. The stiffeners may also be specified as sniped. The stiffeners may have a tilt angle relative to the plate normal.
- Profiles:** Open profile type with standard angle shape, eccentric angle, symmetric T or flat bar shape. Standard profile table included. Profiles run continuously across several spans. Bulb profiles are modelled as equivalent angle profile.
- Sec. stiffeners:** Secondary stiffeners perpendicular to continuous stiffeners; option for sniped stiffeners, continuous stiffeners or tripping brackets.
- Loads:** Uniform membrane stresses in axial  $\sigma_{10}$  direction, linearly varying stresses in transverse  $\sigma_{20}$  direction, and constant shear stress  $\sigma_{30}$ . All combinations of bi-axial compression/tension and shear can be analysed. Uniform lateral pressure acting across several bays is included.
- Edge Support:** All four edges are assumed to be straight, free to move in-plane and free to rotate out-of-plane(simply-supported edges) and constrained to be straight and stay in the original plate plane. This implies full utilisation of normal compressive and

tension fields. Alternatively, two opposite edges may be specified as free to pull in, for analysis of web girders, stringers etc.

In order not to exceed the range of validity of the theory used in the S3 sub-element, the following slenderness requirements of component plate elements in a cross-section are formulated:

i) General:

Web slenderness for flat bar stiffeners:

$$h_w / t_w < 35$$

Web slenderness for L or T profiles:

$$h_w / t_w < 90$$

Free flange for L or T profiles:

$$f_f / t_f < 15$$

Plate between stiffeners:

$$s / t < 200$$

Aspect ratio of plate between stiffeners

$$0.17 < L_1/s < 20 \quad *$$

Flange width for L or T profiles

$$h_f/h_w > 0.22$$

\* NB! Note that the “real” validity limit for the plate between stiffeners may well exceed this recommended limit if the actual load condition analysed enforces a wave pattern sufficiently described by less than  $20+1=21$  half-waves in the axial direction ( $x_1$ -direction). The local minimum eigenmode can give a hint of the “real” validity limit of the ULS capacity calculated by PULS.

These validity limits are checked by the program. An error message is given when limits are exceeded. A summary is given on the status bar shown in the bottom of the PULS AV window.

### 3.4 General S3 element design principles

The S3 element applied in an ULS design application is based on the following main principles:

i) **Elastic local buckling of any of the component plates in a panel section is accepted.**

Local buckling is classified as all modes where the stiffener/plate junction lines act as nodal lines in the buckling pattern.

For open profiles local elastic buckling means buckling of the plating between stiffeners, sideways/torsional buckling of stiffeners, stiffener web plate buckling and interactions between these modes.

ii) **Permanent buckles are not accepted.**

By ensuring the maximum membrane stresses within a panel to stay below the yield stress condition (von Mises), excessive permanent sets and buckles are prevented.

**iii) Global (overall) buckling of the panel is not accepted (GEB cut-off).**

This principle ensures the panel as a whole to have sufficient out-of plane bending stiffness to avoid global buckling (overall stiffener buckling, see Fig.8 for illustration). Sufficient overall bending stiffness of the stiffeners ensures lateral support to the component plates, which is a reasonable requirement for accepting local elastic buckling of these (principle i).

In sum the present ULS philosophy predicts an ultimate strength value accepting elastic local buckling deflections of plates and stiffeners while preventing excessive permanent damages. ULS design for ships corresponds to an extreme load condition typically the probably largest loading experienced in a 20 years period.

These ULS design principles are established in order to constrain the panel designs to have some minimum stiffness properties for efficient in-plane load transfer. The principles are consistent with the present DNV rules and guidelines, even though in the latter they are not explicitly stated and not consistently included for combined load situations.

In some cases, it will from serviceability/functional reasons (SLS), not be acceptable that local elastic buckling takes place. In such cases the local elastic buckling boundary (LEB and GEB cut-off) can be used as the upper limit of allowable load application.

The **ultimate limit state** calculation procedure for the S3 element can be split into three levels:

- i) Local level: Establishment of orthotropic macro material coefficients and assessment of local eigenvalues. Non-linear analysis.
- ii) Global level: Eigenvalue calculation of the global/overall mode (GEB), and nonlinear global deflection analysis, including knock down effects from local buckling, postbuckling and local imperfections/residual stresses.
- iii) Ultimate limit state: Global non-linear analysis with explicit solution of different limit state functions for identifying the most critical failure hot spot location and corresponding loads acting on the panel.

The calculation procedure is illustrated schematically in Fig.4.

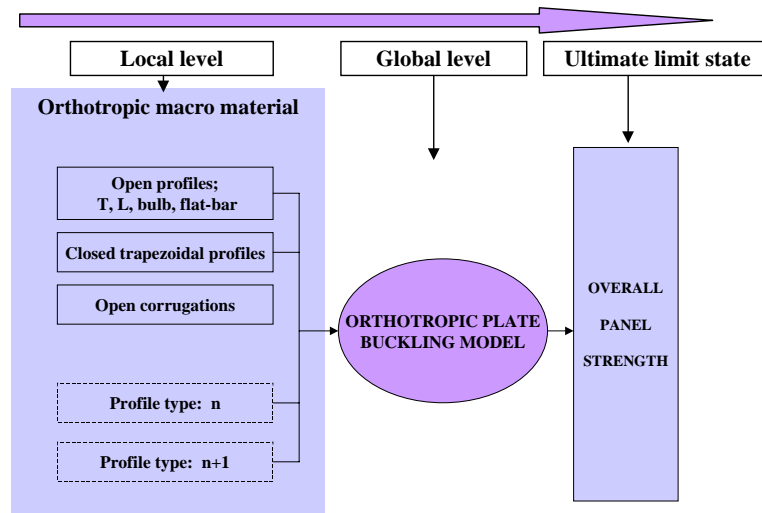


Fig. 4 Calculation procedure in PULS S3 element

A Rayleigh-Ritz expansion of the out-of-plane displacements, using trigonometric functions, is used both at the local and global level. Different types of simplifications are introduced in order to limit the degrees of freedoms. These are not discussed here.

### 3.5 External prescribed loads

The S3 sub-element can be subjected to a combined external load situation covering uniform in-plane load in the axial direction, linearly varying load in the transverse direction, and constant in-plane shear. The four in-plane loads are given as

$$\sigma_1 = \Lambda \sigma_{10}$$

$$\sigma_{2,1} = \Lambda \sigma_{20,1}$$

$$\sigma_{2,2} = \Lambda \sigma_{20,2}$$

$$\sigma_3 = \Lambda \sigma_{30}$$

where  $\Lambda$  is the load factor automatically controlled by the program. A subscript 0 on the nominal external stresses indicates the values of the input values, i.e. they correspond to  $\Lambda=1$ .

For strength assessment the in-plane loads are scaled proportionally until elastic buckling and ULS strength is identified.

The considered external loads typically takes the form as illustrated on Fig.5.

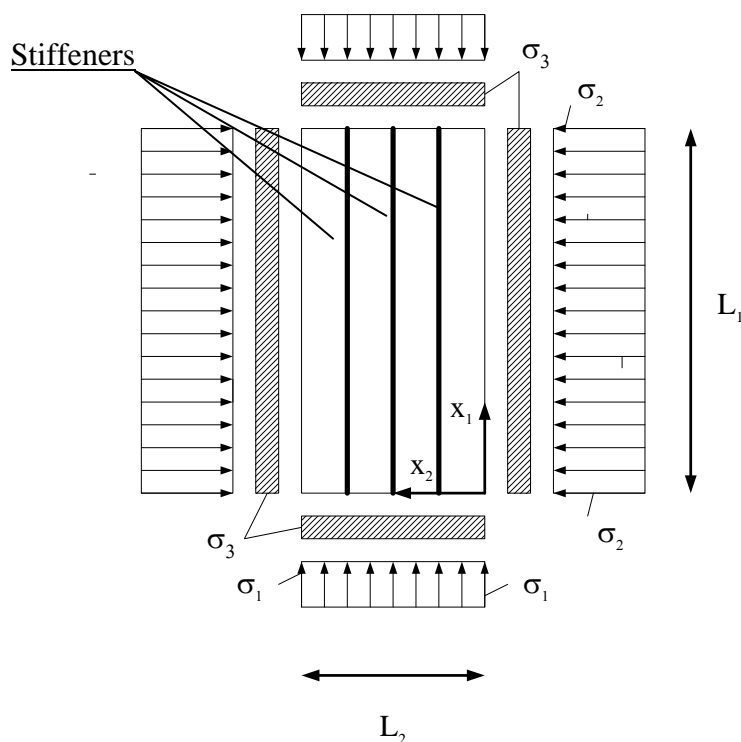


Fig.5 External load system;

Constant axial load and shear, and linearly varying normal loads in transverse direction

### 3.6 Boundary conditions

All four edges in the S3 element are taken as simply supported in the global buckling model. In the local model, the rotational restraint of the plate is determined by the influence from the stiffener. The same restraint is implicitly assumed to act at the ends of the stiffened panel.

The in-plane boundary conditions are by default so that all four edges are forced to remain straight. This is representative of an integrated panel that is part of a larger structure. Alternatively, two opposite edges may be specified as free to pull in, which is more representative for web girders, stringers, etc.

The stiffeners are by default continuous in the axial direction, but may alternatively be specified as sniped. This will reduce the support from the stiffeners to the plate.

### 3.7 General available results

When operated in the **standard mode** the PULS 2.0 code applies a proportional loading history I load space. Along the proportional load history the following strength parameters are calculated:

- i) Local eigenvalue (LEB) (SLS)

- ii) Global eigenvalue (GEB)
- iii) Ultimate capacity stress (UC)
- iv) Buckling load = minimum of eigenvalues and ultimate load
- v) Usage factor  $\eta$  with reference to ultimate load and buckling load
- vi) Orthotropic macro stiffness coefficients;  $C_{\alpha\beta}$ ,  $D_{\alpha\beta}$

i) and ii). The eigenvalue and the corresponding eigenmode are ideal elastic buckling stresses with associated buckling shape for a stiffened panel with perfect flat geometry. They are categorised into local (LEB) and global (GEB) modes. They represent an idealized reference state at which the panel will start buckling (deflecting out of plane).

iii) The ultimate stress (UC) is the maximum nominal stress the panel can carry for the defined proportional load history.

iv) The buckling load is defined as the minimum of the eigenvalues and the ultimate load. Applicable if functional requirements are to be imposed (SLS) with the purpose of avoiding elastic buckling deflections of plates and stiffeners.

v) The UC usage factor describes the margin between the applied loads and the corresponding ultimate capacity stresses. The usage factor BS is measured against the buckling load.

vi) Orthotropic macro coefficients represent the in-plane and out-of-plane stiffness of the panel in an unloaded and loaded state. They are reduced compared to the linear smeared macro coefficients taking into account the non-linear effect of local elastic buckling of plates and stiffeners. They can be used as reduced efficiency elements in linear FE models or as reduced stiffness elements in simple hull girder models.

Graphical results available are plots of minimum eigenmodes, imperfection modes, and ultimate limit state deflection modes. In addition, membrane stress plots are produced.

The calculated parameters are more thoroughly explained in the following sections.

### **3.8 Local level: Elastic Eigenvalue and reduced stiffness properties**

#### **3.8.1 General**

Local buckling is classified as modes associated with pure local deflections of the component plates in the cross-section, i.e. as per definition the stiffener junctions to the continuous plating act as nodal lines in the local buckling pattern. The local level assesses the eigenvalue and the postbuckling strength of panels buckling exclusively into local modes. Local geometrical imperfection effects and residual stress effects are implicitly considered in a set of orthotropic macro material coefficients. From an overall point of view the stiffened panel is then considered as an orthotropic panel in which the orthotropic macro material coefficients have reduced stiffness/efficiency properties (compared to linear elastic values) thus accounting for the local buckling on the overall strength.

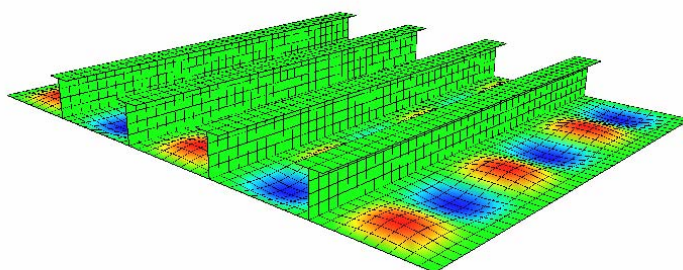
For all buckling and deflection modes the stiffener flanges are considered locally strong and they bend and twist like a beam constrained to follow the stiffener web deflections.

### **3.8.2 Local eigenvalue; LEB**

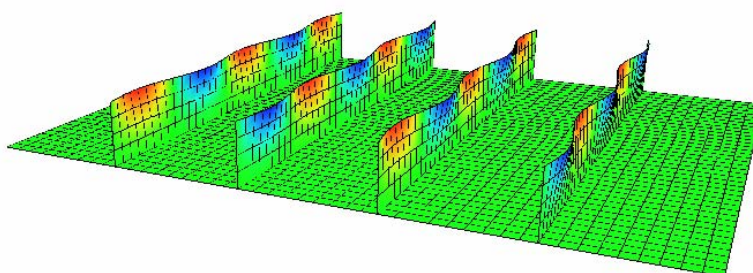
For panels stiffened by open profiles the following three categories of local buckling modes are typical in stiffened ship panels

- i) plate buckling
- ii) torsional stiffener buckling
- iii) stiffener web plate buckling interaction with plate buckling

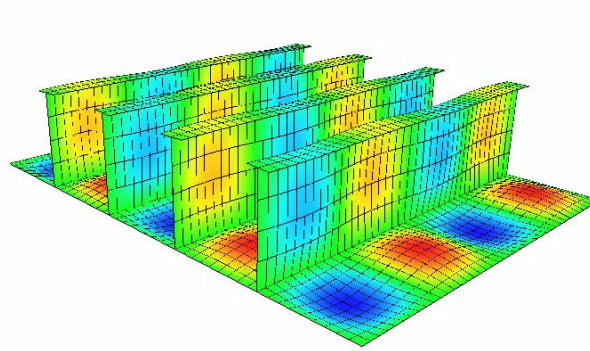
Interactions between these mode categories are typical for stiffened panels. Fig.6 illustrates the three main categories as assessed by PULS



**i) plate buckling (strong stiffener sideways); LEB**



**ii) torsional stiffener buckling (typical for tall profiles and flat bar profiles in particular); LEB**



iii) stiffener web plate buckling interaction with plate buckling and some torsional stiffener buckling; LEB

**Fig. 6: Examples of local buckling modes; junction line between stiffener and plate straight (assessed by PULS)**

The values for the in-plane stresses at the instant where local elastic buckling starts are defined as

$$\begin{aligned}\sigma_{1LE} &= \Lambda_{LE} \sigma_{10} \\ \sigma_{2,1LE} &= \Lambda_{LE} \sigma_{20,1} \\ \sigma_{2,2LE} &= \Lambda_{LE} \sigma_{20,2} \\ \sigma_{3LE} &= \Lambda_{LE} \sigma_{30}\end{aligned}$$

where  $\Lambda_{LE}$  is the eigenvalue of the load proportionality parameter  $\Lambda$  calculated by the program. The values  $\sigma_{1LE}, \sigma_{2,1LE}, \sigma_{2,2LE}, \sigma_{3LE}$  are called the local elastic buckling stresses under a combined load situation.

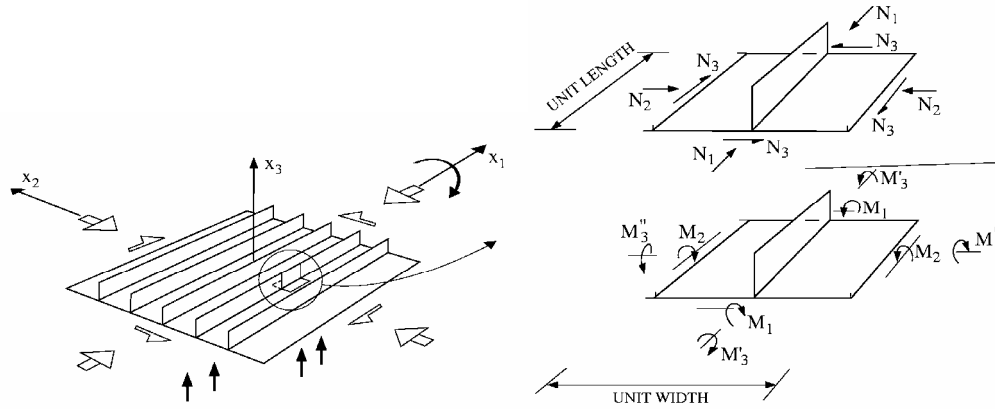
The local elastic buckling stresses are not critical with respect to the ultimate load bearing capacity of integrated panels. Higher loads can be carried due to a positive postbuckling effect. In an ultimate strength context the local buckling stress (LEB) can be viewed as a reference state, beyond which there exist extra load bearing capacity (typically for thin plates or high local slenderness in general).

The local elastic buckling stress can be useful as an upper limit for panels constrained to follow strict functional requirements (SLS), i.e. for design where elastic buckling deflections are not accepted.

### 3.8.3 Orthotropic material coefficients

In a simplified mathematical model, the stiffened panel can be considered to be equivalent to an orthotropic material where the stiffeners are smeared out over the plate surface. The PULS code is based on a six-dimensional orthotropic macro material law, e.g. Brush and Almroth (1975), Ref.[1], Fig.7.





**Fig. 7 Six-dimensional macro model for stiffened panels**

According to non-linear plate theory this macro material law takes the form of an incremental relation between the in-plane loads ( $N_1, N_2, N_3$ ) and moments ( $M_1, M_2, M_3$ ), and the corresponding strains ( $\epsilon_1, \epsilon_2, \epsilon_3$ ) and curvatures ( $\kappa_1, \kappa_2, \kappa_3$ ) of the continuous plating. In mathematical terms the orthotropic macro material law takes the following form

$$\begin{bmatrix} \Delta N_1 \\ \Delta N_2 \\ \Delta N_3 \\ \Delta M_1 \\ \Delta M_2 \\ \Delta M_3 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & Q_{11} & Q_{12} & Q_{13} \\ C_{21} & C_{22} & C_{23} & Q_{21} & Q_{22} & Q_{23} \\ C_{31} & C_{32} & C_{33} & Q_{31} & Q_{32} & Q_{33} \\ Q_{11} & Q_{21} & Q_{31} & D_{11} & D_{12} & D_{13} \\ Q_{12} & Q_{22} & Q_{32} & D_{21} & D_{22} & D_{23} \\ Q_{13} & Q_{23} & Q_{33} & D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} \Delta \epsilon_1 \\ \Delta \epsilon_2 \\ \Delta \epsilon_3 \\ \Delta \kappa_1 \\ \Delta \kappa_2 \\ \Delta \kappa_3 \end{bmatrix}$$

The symbol  $\Delta$  indicates incremental properties. By accepting local elastic buckling in stiffened panels under extreme loads, the panel will behave in a non-linear and more flexible way than under the standard linear response hypothesis. This local panel flexibility is assessed in the PULS code as a set of reduced orthotropic macro stiffness coefficients defined as

$$\begin{aligned} C_{\alpha\beta} &\equiv C_{\alpha\beta}^L + C_{\alpha\beta}^N \\ D_{\alpha\beta} &\equiv D_{\alpha\beta}^L + D_{\alpha\beta}^N \\ Q_{\alpha\beta} &\equiv Q_{\alpha\beta}^L + Q_{\alpha\beta}^N \end{aligned} \quad \alpha, \beta = 1, 2, 3$$

Each of the coefficients in the stiffness matrix has two contributions, i.e. a linear part and a non-linear part. The linear part represents coefficients with the full stiffener rigidities smeared out

over the total plate area. The non-linear corrections  $C_{\alpha\beta}^N, D_{\alpha\beta}^N, Q_{\alpha\beta}^N$  are assessed using a numerical procedure. This and more details about the theory is explained in the Theory Manual, Ref.[3].

The macro model stiffness relation is written on sub-matrix notation as

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} \mathbf{C} & \mathbf{Q} \\ \mathbf{Q}^T & \mathbf{D} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\kappa} \end{bmatrix} \quad \Leftrightarrow \quad \begin{bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\kappa} \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{S} \\ \mathbf{S}^T & \mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix}$$

The bending/extension coupling coefficients  $Q_{\alpha\beta}, (\mathbf{Q})$  enter due to the eccentricity effect of the one-sided welded stiffeners. This coupling effect is eliminated using a simple neutralisation procedure. The resulting uncoupled moment-curvature relation is

$$\mathbf{M} = \tilde{\mathbf{D}} \boldsymbol{\kappa}$$

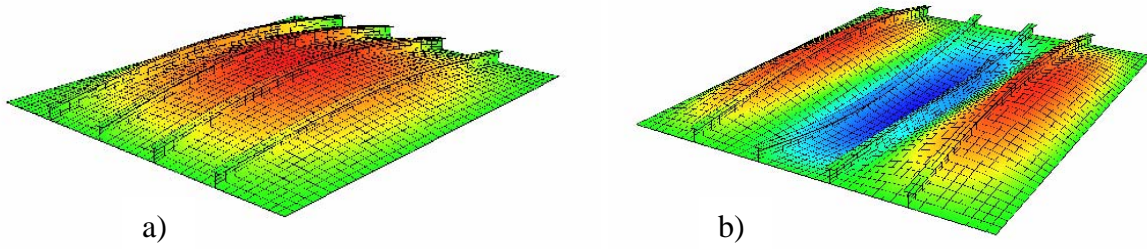
where neutral orthotropic bending stiffness matrix is

$$\tilde{\mathbf{D}} = \mathbf{D} - [(\mathbf{Q})^T (\mathbf{C})^{-1} \mathbf{Q}] \quad (= \mathbf{F}^{-1})$$

The PULS S3 element calculates the macro model submatrices  $\mathbf{C}, \mathbf{Q}, \mathbf{D}, \tilde{\mathbf{D}}$  for the linear state, for the initial unloaded geometrically imperfect state and for an averaged state (secant) being representative for the ultimate strength assessment.

### 3.9 Global level: Overall elastic eigenvalue of panel; GEB

Global buckling is associated with an overall mode lifting the stiffeners out-of-plane together with the continuous plating assuming lateral support along all four outer edges, see Fig. 8 for illustration.



**Fig. 8 Examples of global buckling modes; stiffeners lift out-of-plane together with the plating (assessed by PULS); GEB**

**a) Example for an axially compressed panel**

**b) Example for a transversely compressed and shear loaded panel**

The corresponding global elastic buckling level (GEB eigenvalue) is assessed using the classical orthotropic plate theory, though as explained in chapter 3.6.3, with modified orthotropic macro material coefficients accounting for the local buckling effects.

With reference to the DNV rules, Ref.[4], Ref.[5], the lateral buckling mode (or column buckling) corresponds to the global buckling mode in PULS. However, in the DNV rules, and most other international codes, the lateral buckling mode neglects the plate effect altogether (i.e. the column approach assumes lateral support only at two opposite edges typically being at transverse frame supports). This is the reason while the column approach is not suited for analysing integrated stiffened panels in ship and offshore structures addressing the problem of combined in-plane bi-axial and in-plane shear loads.

According to design principle **iii)**, Section 3.2, the present S3 element takes the global eigenvalue as the upper limit of the buckling capacity of the panel. This is referred to as GEB cut-off in the present code and is consistent the Perry approach much referenced in the literature.

In the standard program mode the global eigenvalues are found by scaling the simultaneously combined loads  $\sigma_{10}, \sigma_{20,1}, \sigma_{20,2}, \sigma_{30}$  in proportion until global buckling takes place. The global buckling loads are accordingly

$$\begin{aligned}\sigma_{1GE} &= \Lambda_{GE} \sigma_{10} \\ \sigma_{2,1GE} &= \Lambda_{GE} \sigma_{20,1} \\ \sigma_{2,2GE} &= \Lambda_{GE} \sigma_{20,2} \\ \sigma_{3GE} &= \Lambda_{GE} \sigma_{30}\end{aligned}$$

where  $\Lambda_{GE}$  is the global eigenvalue of the load parameter  $\Lambda$  found by the program. The nominal stresses  $\sigma_{1GE}, \sigma_{2,1GE}, \sigma_{2,2GE}, \sigma_{3GE}$  are called the global elastic buckling stresses under a combined load situation.

The eigenvalue problem is formulated as

$$(\mathbf{K} - \Lambda \mathbf{K}_g) \mathbf{q} = 0$$

$\mathbf{K}$  is the small displacement stiffness matrix,  $\mathbf{K}_g$  is the geometrical stiffness matrix and  $\mathbf{q}$  is the eigenvector.

### 3.10 Local stress limit states: Ultimate strength evaluation

#### 3.10.1 General

The S3 element includes non-linear elastic models for both the global mode (orthotropic plate theory) and the local mode (isotropic plate theory with plate-stiffener compatibility). These non-linear models are run in sequence with a logging of the response for local stress registration to be used in the subsequent limit state evaluations.

The limit state evaluations are based on the redistributed membrane stress distributions within the stiffened panel. Membrane stresses in this context means stresses in the middle-plane of the individual thin-walled component plates (plating, stiffener web, stiffener flange) of which the stiffened panel is built. Thus membrane stresses can be a purely local effect due to local plate/stiffener buckling or they arise from global bending effects of the stiffener, the latter due to compressive in-plane forces or lateral pressure.

Due to local buckling of the component plates in a cross-section the membrane stress will redistribute as compared to neglecting buckling effects. For thin plates and stiffeners the redistributions will be significant. Typically the stresses in the hard corners/critical positions = along plate edges/plate-stiffener junction lines etc. will be higher than in mid regions of local buckles. The degree of redistributions depends on the slenderness/plate thickness of each component plate and on geometrical imperfections shape/size and residual stresses present.

The current PULS version apply six limit state functions  $f_i$ 's ( $i = 1, 2, 3, 4, 5, 6$ ) for identifying critical conditions in different locations in the panel. A function  $f_i > 0$  corresponds to applied loads less than the critical condition in the corresponding point. Moreover,  $f_i = 0$  solved explicitly give the values for the applied loads corresponding to the ultimate limit state. The ultimate strength is found from the minimum of all defined limit states.

The six limit states are formulated for capturing critical stress conditions in selected critical positions, see Fig. 9.

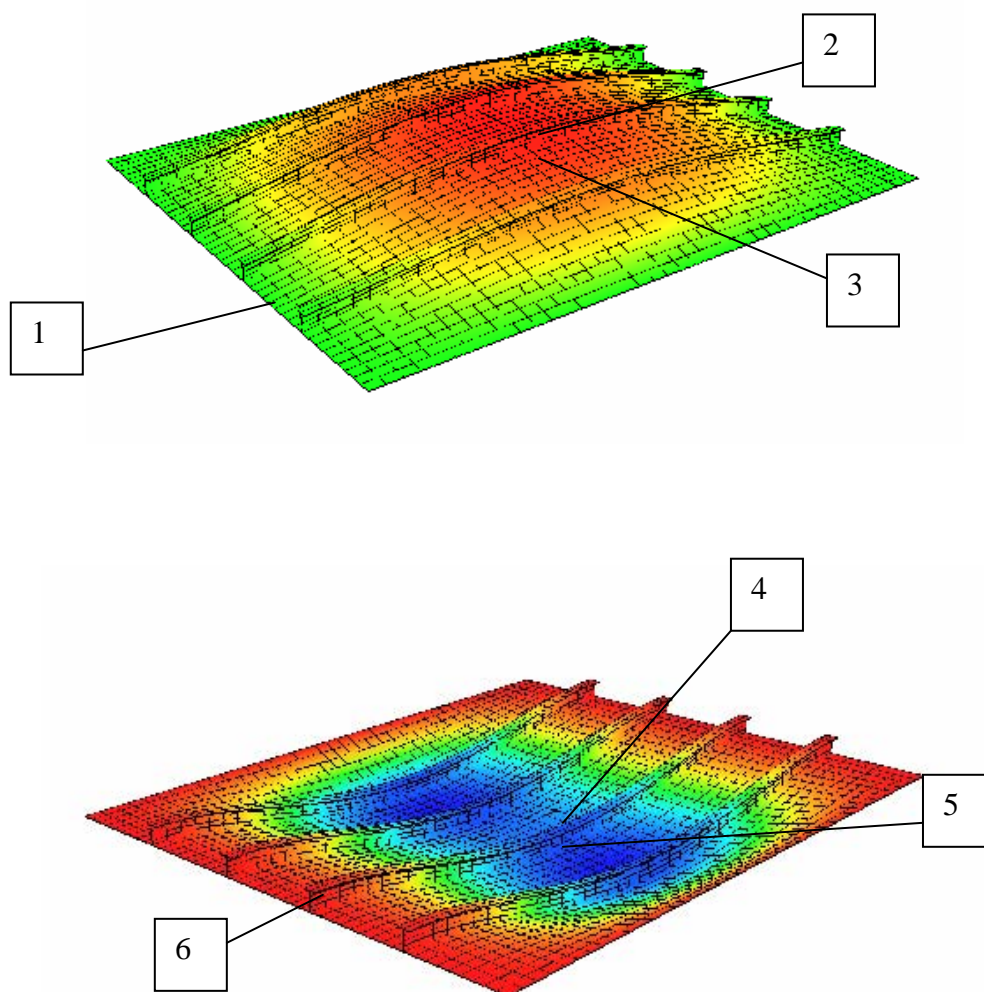


Fig. 9 Stress control points in critical positions in a panel defining the ultimate limit states

The six limit states  $f_i$ 's are stress controls in the following positions:

- $i = 1$ ; **Plate criterion:** Stress control along plate edges – based on max edge stresses along supported edges (typical: transverse load when local buckling dominates)
- $i = 2$ ; **Stiffener tension criterion:** Stress control in stiffener; at midspan  $x_1 = L_1/2$ ; in stiffener flange for global panel deflecting towards stiffener flange, tension criterion - rare for compressive loads, but kicks in for tension loads (will also kick in for transverse compressive loads for panel with small stiffeners, i.e. large global effects)
- $i = 3$ ; **Plate compression criterion:** Stress control in plate; at midspan  $x_1 = L_1/2$ ; in plating for global panel deflecting towards stiffener flange, compression criterion (PI collapse)
- $i = 4$ ; **Stiffener compression criterion:** Stiffener criterion Stress control in stiffener; at midspan  $x_1 = -L_1/2$ ; in stiffener flange for global panel deflecting towards plating, compression criterion (SI collapse) (typical for pure axial load)
- $i = 5$ ; **Plate tension criterion:** Stress control in plate; at midspan  $x_1 = -L_1/2$ ; in plating for global panel deflecting towards plating, tension criterion – rare for compressive loads, but kicks inn for tension loads

(Note that the limit state criteria  $i = 2-5$  is not always evaluated at midspan. Maximum curvature in  $x_1$  – direction, and thereby the highest bending stress, could be closer to the ends for certain geometrical proportions of stiffened panels. Typical are cases with small stiffeners for which the panel behaves more as a plate than a “column”, with a global buckling mode pattern flattening in the mid-regions.)

$i = 6$ ; **Stiffener bending stress criterion at support:** Stress and capacity control at support  $x_1 = 0$ ; compressive or tension criterion, kicks in for cases with lateral pressure. This limit state is used to control the bending and shear capacity of the stiffeners under the influence of combined lateral load and in-plane loads. Yielding in the stiffener flange at the transverse frames is accepted, since stiffeners have significant strength reserves after first yield when subjected to lateral pressure. The panel is loaded until the plastic capacity of the stiffeners is reached. Two criteria are used for this limit state. The first is the capacity of the top and bottom flanges to carry the combined axial force and bending moment resulting from the applied loads, and the second is the capacity of the web to carry the shear force and axial force due to the applied loads.

The value for the four independent in-plane stresses at the point of ultimate strength is defined as

$$\begin{aligned}\sigma_{1u} &= \Lambda_u \sigma_{10} \\ \sigma_{2,1u} &= \Lambda_u \sigma_{20,1} \\ \sigma_{2,2u} &= \Lambda_u \sigma_{20,2} \\ \sigma_{3u} &= \Lambda_u \sigma_{30}\end{aligned}$$

The value of the  $\Lambda_u$  factor is calculated as the minimum explicit solution of the six limit states functions. The solutions are found using a numerical procedure. There exist one  $\Lambda_u$  for each limit state and the minimum of these is used as representative for the ultimate strength. It follows per definition that it is the inverse of the UC usage factor  $\eta$  as defined in Chapter 3.9.

### 3.10.2 Steel

The steel limit states follow the six main stress control points as given above.

Different material yield stresses can be specified in the plate and stiffeners (same in all stiffeners) while only one set of values for the Young's modulus  $E$  and Poisson ratio  $\nu$  is possible.

The geometrical imperfection size and shape effect, and the residual stress effects, are implicitly considered in the ULS values when using default tolerance values. The imperfection values used as default values are typical for welded and fabricated steel plates used in ships and offshore constructions.

### 3.10.3 Aluminium

A special option for analysis of aluminium panels, including HAZ effects for NV 5082 T6, is implemented in PULS 2.0.

A first membrane yield criterion in the HAZ zone is added to the limit states used for steel. This first HAZ yield criterion will be conservative, but reasonable as a design limitation since limited knowledge is available for soft HAZ zones strained beyond the material yield level. Crack initiations and fracture can be the result if the HAZ zones around welds frequently are loaded beyond the first yield level.

### 3.11 Safety margin

The calculated UC usage factor  $\eta$  in the PULS code represents the ratio between the applied combined loads and the corresponding ultimate strength. It is defined as

$$\eta = L_0 / L_u \quad (= 1 / \Lambda_u)$$

where the radius vectors  $L_0$  and  $L_u$  in load space are defined as

$$L_u = \sqrt{(\sigma_{1u}^2 + \sigma_{2u}^2 + \sigma_{3u}^2)}$$

$$L_0 = \sqrt{(\sigma_{10}^2 + \sigma_{20}^2 + \sigma_{30}^2)}$$

see Fig. 10 for illustration

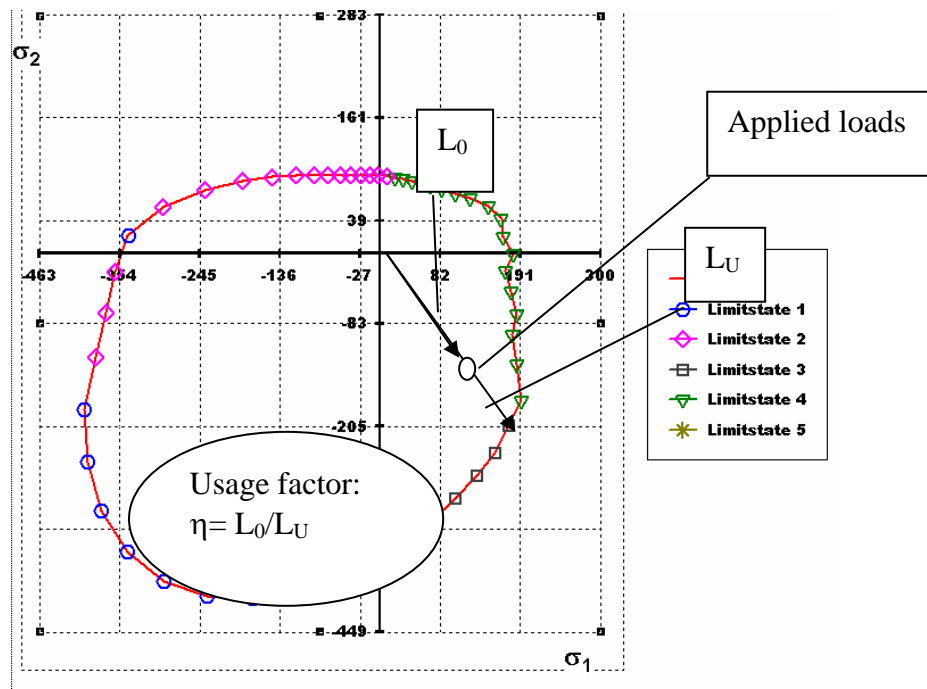


Fig. 10 Definition of safety margin/usage factor; capacity curve example for bi-axial loading

For the single load cases the definition of usage factor becomes

$$\eta = \sigma_{i0} / \sigma_{iu} \quad i = 1, 2, 3$$

as is the familiar form used in the DNV steel ship rules. For the present PULS code approach the final rule acceptance criterion will be in the form

$$\frac{L_0}{L_u} < \eta_{\text{allow}} \quad \text{i.e.} \quad \eta < \eta_{\text{allow}}$$

where  $\eta_{\text{allow}}$  (or also called  $\eta_{\text{max}}$ ) is to be specified in the rules and  $\eta$  is calculated by the PULS code.

In the offshore marked the LRFD format is used, i.e. the acceptance criterion is on the form

$$S_d < R_d$$

where  $S_d$  is the load effect and  $R_d$  is the design resistance. The design resistance is related to the characteristic resistance as

$$R_d = R_k / \gamma_m$$

For offshore application of the PULS code, the following definitions applies:

Characteristic resistance:

$$R_k = \sqrt{(\sigma_{1u}^2 + \sigma_{2u}^2 + \sigma_{3u}^2)} \quad (= L_u)$$

Load effect:

$$S_d = \sqrt{(\sigma_{10}^2 + \sigma_{20}^2 + \sigma_{30}^2)} \quad (= L_0)$$

The offshore strength format is then:

$$\frac{S_d}{R_k} < \frac{1}{\gamma_m}$$

which in Ship rule terminology is

$$\text{i.e.} \quad \eta < \frac{1}{\gamma_m} \quad \text{where} \quad \eta = \frac{S_d}{R_k}$$

The ratio  $S_d / R_k$  is the same as the actual Ship Rule usage factor  $\eta$  and is calculated by the PULS code. Load factors are to be included in load effect parameter  $S_d$  since it represents the actual load situation to be checked against buckling.



### 3.12 Capacity Curves

The PULS code can also assess **2D capacity curves** under combined loads. The capacity curves are illustrated in two-dimensional load-spaces. The notation of **capacity curves** and buckling boundaries are described more in detail in Section 6.7. Pre-loading of pressure and for the in-plane uniform stresses is handled for any of the three in-plane uniform stress components.

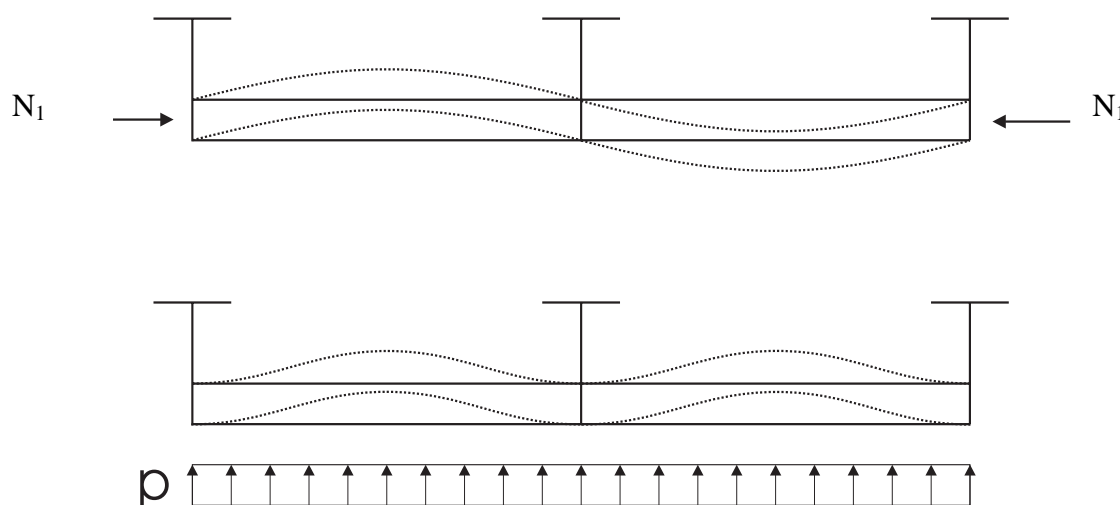
- i) Bi-axial load space  $\sigma_1$ -  $\sigma_2$  for fixed shear  $\sigma_3$  ( $\tau_{12}$ ) and fixed  $p$  (MPa)
- ii) Axial-shear load space  $\sigma_1$ -  $\tau_{12}$  for fixed transverse stress  $\sigma_2$  and fixed  $p$  (MPa).
- iii) Transverse-shear load space  $\sigma_2$  -  $\tau_{12}$  for fixed axial stress  $\sigma_1$  and fixed  $p$  (MPa)

Limits for acceptable lateral pressures are set in the program and capacity curves generation will be aborted if these limits are exceeded. The pressure limits are summarised in the detailed result menu under standard parameters.

### 3.13 Lateral pressure

Lateral pressure can be prescribed acting uniformly across the panel. In the Ultimate Capacity analysis the specified lateral pressure is kept fixed, while the in-plane loads are increased until subsequent collapse is reached.

It is remarked that the S3 stiffened panel element assumes the main stiffeners to be continuous across several bays to match typical ship bottom/side designs and the pressure distribution is uniform all across all bays. Theoretically this means that the model includes both symmetric deflection modes (CS) for assessing the pressure effect and asymmetric modes (SS) for assessing the buckling effect with respect to the global stiffener bending (in  $x_1$ -direction) as illustrated schematically in Fig.11 below.



**Fig. 11. Deflection of a stiffener in the simply supported mode (SS asymmetric modes, top) and the clamped mode (CS symmetric modes, bottom)**

Maximum pressure criteria are introduced representing practical design boundaries for the stiffeners. They kick in and present themselves in dialog boxes with stated limits not to be exceeded. Two different pressure limits are specified; one for controlling the bending stiffness of the stiffener/plate unit and one for controlling the shear area of the stiffener web. They are based on linear beam theory.

The bending stiffness pressure limit  $p_{Fs}$  is

$$p_{Fs} = 12 \sigma_F \frac{W_{\min}}{s L_1^2}$$

where  $W_{\min}$  is the minimum section modulus of the stiffener/plate unit normally being at the stiffener flange position

$$W_{\min} = \frac{I}{h_w + 0.5t_f + 0.5t_p - z_g}$$

where the moment of inertia of the plate/stiffener unit is

$$I = \frac{1}{12} b_f t_f^3 + b_f t_f (h_w + 0.5t_f + 0.5t_p - z_g)^2 + \frac{1}{12} t_w h_w^3 + h_w t_w (0.5h_w + 0.5t_p - z_g)^2 + \frac{1}{12} s t_p^3 + s t_p z_g^2$$

and  $z_g$  is the neutral axis measured from the plate middle-plane. This limit correspond to first bending stress yield at support for a clamped stiffener with span  $L_1$ . The  $p_{Fs}$  value is tabulated in the PULS output detailed result/standard parameter list for info.

The lateral pressure which gives shear stress yielding in the web is

$$p_{\tau s} = \frac{2V}{sL_1}$$

where the shear force which gives shear stress yield is

$$V = \frac{\sigma_{Fs} t_w I}{\sqrt{3} S_p}$$

where the moment of area about the neutral axis, where the maximum shear stress occurs, is

$$S_p = s t_p z_g + 0.5 t_w (z_g - 0.5 t_p)^2$$

This pressure limit correspond to first pure shear yield in the stiffener web as calculated for a clamped stiffener with span  $L_1$ . The  $p_{ts}$  value is tabulated in the PULS output detailed result/standard parameter list for info.

Another pressure limit of practical interest is the first onset of surface yield due to pure local bending of the plate between stiffeners. This limit is simply

$$p_F = 2\sigma_F \left(\frac{t}{s}\right)^2$$

and is based on a clamped plate unit strip formulation. This limit is not included as design limit in the present PULS 2.0 version as it will be too strict for practical applications. For ship bottom designs, pressure level significantly beyond this  $p_F$  is normal. It is also documented using advanced elasto-plastic FE analysis, that pressure levels well beyond  $p_F$  can be carried for plates supported by typical solid stiffener/frame structures. It is also well documented that the in-plane capacity for stiffened panels is not very influenced by local surface yielding along stiffener/plate supports. The  $p_F$  value is tabulated in PULS output detailed result/standard parameter list for info.

**Conclusion:** The present S3 lateral model has practical pressure limits implemented reflecting reasonable design requirements to the stiffener/plate unit. These are based on the philosophy that the stiffeners shall carry the pressure and transfer these to the supporting transverse frame/bulkhead structures without significant plastic deformations. Input pressures beyond these limits are not accepted by the program and warnings are given in dialog boxes.



## **4 T1 – STIFFENED PLATE ELEMENT (NON-REGULAR GEOMETRY)**

### **4.1 General**

The T1 stiffened plate element applies a linearized version of Marguerre's non-linear plate theory. Linearized in this context means that the theory does not assess the non-linear postbuckling strength (no overcritical strength), i.e. loads beyond the ideal elastic buckling level (eigenvalue) are not possible. The elastic buckling (eigenvalue) is found using a multiple degree of freedom model in terms of Fourier expansions of lateral deflections. Stress control criteria checking the onset of material yielding in hot spot stress locations along the plate edges and in the stiffeners are used as method for the buckling strength assessment.

The stiffened plate can be subjected to combined load situations and the numerical procedures implemented scale the applied loads up to buckling. The results are presented in terms of a single parameter called the usage factor. The numerical algorithms are based on the linearized load-deflection solution using the minimum eigenvalue in the amplification factor.

The element can be used for assessing the buckling strength of panels where the stiffener can have variable geometry and arbitrary orientations. This means it is suitable for non-regular stiffener arrangement and triangular plates.

More details concerning the T1 element can be found in ref.(18).

### **4.2 Overview of element characteristics**

The main characteristics of the T1 element are summarized in Table 3.

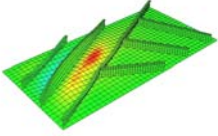
T1 element – stiffened plate – non-regular geometry	
Theory:	Linear
Structural configuration: 	Rectangular stiffened plate, non-regular stiffeners <ul style="list-style-type: none"> <li>- Uniform plate thickness</li> <li>- N arbitrary oriented stiffeners, (N = 1, 2,..., 25)</li> <li>- The stiffeners may have different proportions</li> <li>- Stiffener input:               <ul style="list-style-type: none"> <li>- direct input of scantlings</li> <li>- as springs with lateral and/or rotational stiffness</li> </ul>               (The stiffeners are secondary in the sense that they carry no axial stress)             </li> </ul>
Loads:	<ul style="list-style-type: none"> <li>- Linearly varying normal in <math>x_1</math>- direction</li> <li>- Linearly varying normal stress in <math>x_2</math>-direction</li> <li>- Uniform in-plane shear stress (<math>x_1</math>-<math>x_2</math>) plane</li> </ul>
Materials:	<ul style="list-style-type: none"> <li>- Isotropic elastic material (E, <math>\nu</math>)</li> <li>- Material yield stress in plate used in buckling strength assessment</li> </ul>
Boundary conditions:	Out-of plane support (bending support) <ul style="list-style-type: none"> <li>- All four edges supported laterally in plate plane</li> <li>- Rotational restraint control of each edge; free to rotate, rotationally restrained or clamped</li> <li>- Stiffener with rotational spring stiffness to simulate partly restrained edges</li> </ul>
Model imperfections:	- Automatically set default model tolerances consistent with as welded panels
Output:	<ul style="list-style-type: none"> <li>- Buckling Strength (BS)</li> <li>- Usage factor BS (ratio: applied loads/ BS)</li> <li>- Minimum Eigenvalue</li> </ul>
Validity limits:	Max slenderness $\lambda \leq 4 ; \lambda = \sqrt{\frac{\text{squash von Mises yield}}{\text{min eigenvalue}}}$ Max stiffener slenderness Stiffener web slenderness for flat bar stiffeners, $h/t_w < 35$ Stiffener web slenderness Angle and T profiles; $h_w/t_w < 90$ Free flange slenderness for Angle and T profiles; $f_f/t_f < 15$

Table 3 Overview of T1 element characteristics

### 4.3 Element limitations- slenderness requirements

The present PULS T1 element has the following range of application:

- Geometry: Rectangular stiffened plate, with stiffeners oriented in any direction
- Profiles: Open profile type with standard angle shape, eccentric angle, symmetric T or flat bar shape. Standard Profile table included. Bulb profiles are modelled as equivalent angle profile.
- Loads: Linearly varying membrane stresses in axial  $\sigma_{10}$  and transverse  $\sigma_{20}$  direction, and constant shear stress  $\sigma_{30}$ . All combinations of bi-axial compression/tension and shear can be analysed.
- Edge Support: All four edges are assumed to be supported laterally. Rotational constraint may be specified. In-plane boundary conditions is not necessary to define due to the use of linear theory.

The theory for stiffeners is simplified in sense that they are not considered to carry any axial stress. This means that buckling of stiffener web is not checked and thus a certain max slenderness of the stiffener web has to be ensured. The T1 stiffener model is similar to the secondary stiffener option in the S3 element, and the web proportions should fulfil the table 2 requirements at least if the stiffeners are continuous across the boundaries. For sniped stiffeners the stiffener web requirement can be relaxed. As an upper max slenderness limit the following ratios are implemented:

i) General:

- Web slenderness for flat bar stiffeners: 
$$h_w / t_w < 35$$
- Web slenderness for L or T profiles: 
$$h_w / t_w < 90$$
- Free flange for L or T profiles: 
$$f_f / t_f < 15$$

These validity limits are checked by the program. An error message is given when limits are exceeded. A summary is given on the status bar shown in the bottom of the PULS window.

Another alternative option for modelling stiffeners is also available using directly a user defined effective moment of inertia and rotational spring stiffness. This option enables the user to specify his own preferred stiffener efficiency properties.

#### 4.4 General T1 element design principles

The T1 element applied in an design setting is based on the following main principles

- i) **Elastic buckling of the plate and/or stiffener is not accepted.**
- ii) **Permanent buckles are not accepted.**

By ensuring the maximum membrane stresses along the plate edges and the plate/stiffener connections to stay below the yield stress condition (von Mises), excessive permanent sets and buckles are prevented ( principle ii) above). In plates second order bending stresses adds to the second order membrane stresses. The resulting surface stress is generally accepted to exceed the yield condition in local areas. This is not considered critical for the strength of plates and it will not induce permanent sets beyond what is normally accepted in ship designs.

Since linear theory is used, the ULS strength calculated will always be below the local elastic buckling load.

#### 4.5 External prescribed loads

The T1 element can be subjected to a combined external load situation covering linearly varying in-plane loads in bi-axial directions and constant in-plane shear. The five in-plane external nominal stresses are given as

$$\begin{aligned}\sigma_{1,1} &= \Lambda \sigma_{10,1} \\ \sigma_{1,2} &= \Lambda \sigma_{10,2} \\ \sigma_{2,1} &= \Lambda \sigma_{20,1} \\ \sigma_{2,2} &= \Lambda \sigma_{20,2} \\ \sigma_3 &= \Lambda \sigma_{30}\end{aligned}$$

where  $\Lambda$  is the load factor automatically controlled by the program. A subscript 0 on the nominal external stresses indicates the values of the input values, i.e. they corresponds to  $\Lambda=1$ .

#### 4.6 Available results

The following strength parameters are calculated for the T1 element:

- i) Eigenvalue
- ii) Buckling load
- iii) Usage factor  $\eta$  with reference to buckling load



- i) The eigenvalue and the corresponding eigenmode are ideal elastic buckling stresses with associated buckling shape for a stiffened panel with perfect flat geometry. They represent an idealized reference state at which the panel will start buckling (deflecting out of plane).
- ii) The buckling load is defined as the load at the onset of yield. Since linear theory is used, the buckling load is always lower than the eigenvalue.
- iii) The usage factor describes the margin between the applied loads and the corresponding buckling load.

Graphical results available are plots of the minimum eigenmode, the model imperfection, and the ultimate limit state deflection mode. In addition, membrane stress plots are produced.

## 4.7 Safety margin

The safety margin is presented as the usage factor defined as

$$\eta = \frac{\sqrt{(\sigma_{1u,1})^2 + (\sigma_{1u,2})^2 + (\sigma_{2u,1})^2 + (\sigma_{2u,2})^2 + (\sigma_{3u})^2}}{\sqrt{(\sigma_{10,1})^2 + (\sigma_{10,2})^2 + (\sigma_{20,1})^2 + (\sigma_{20,2})^2 + (\sigma_{30})^2}}$$

The calculated usage factor  $\eta$  is to be measured against the allowable  $\eta_{\text{allow}}$  (or called  $\eta_{\text{max}}$ ) as specified in the relevant Rules and Standards.

## 4.8 Capacity curves

Capacity curves under combined loads are available for the T1 element. The capacity curves are illustrated in two-dimensional load-spaces. The notation of **capacity curves** and buckling boundaries are described more in detail in Section 6.7. Pre-loading is handled for any of the three in-plane load components.

- i) Bi-axial load space  $\sigma_1 - \sigma_2$  for fixed  $\tau_{12}$  shear.
- ii) Axial-shear load space  $\sigma_1 - \tau_{12}$  for fixed transverse  $\sigma_2$  stress.
- iii) Transverse-shear load space  $\sigma_2 - \tau_{12}$  for fixed axial  $\sigma_1$  stress.



## 5 PULS CODE APPLICATION - USAGE HINTS

### 5.1 General assumptions- Boundary conditions

The current U3 and S3 elements by default assess the buckling and ultimate strength of **integrated** thin-walled panels in ship and offshore structures. The strength characteristics of an integrated element in larger structures depend on the structural geometry and profile type, nature of prescribed loads and not at least the boundary conditions. Two sets of boundary conditions are of particular importance for buckling and ultimate strength assessment, i.e. i) out-of plane (bending) boundary conditions and ii) in-plane (membrane) boundary conditions.

i) Out-of-plane (bending) boundary conditions.

The U3, S3, and T1 elements by default assume the plate edges to be rigidly supported out-of-plane along all four edges, with the edges are geometrically (physically) free to rotate (simply supported boundary conditions). This is a conservative set of boundary conditions. For the U3 and T1 elements, alternative rotational restraints may be specified by the user for each edge individually. For the S3 element, the rotational restraint at the outer edges is less important than the restraint from the stiffeners, which is automatically accounted for.

Out-of-plane boundary conditions are important for the buckling load (eigenvalue), but not so much for the elastic postbuckling behaviour which controls the amount of incremental ULS margin beyond elastic buckling.

ii) In-plane (membrane) boundary conditions.

In-plane boundary conditions are important with respect to elastic post-buckling behaviour and they therefore dictate to a large extent the ULS strength margins. In other words they control the available strength beyond ideal elastic buckling (overcritical strength). Plates compressed beyond the minimum eigenvalue develop second order membrane stresses which need to be transmitted to neighbouring plate fields and surrounding structures. By default, both the U3 and the S3 elements are assumed to be integrated elements. Alternatively, two opposite edges may be specified as free to pull in, which may be more representative for girders and stringers. For the T1 element, the in-plane boundary conditions are not relevant, since linear theory is used.

The U3 element is assumed to have uniform plate thickness. The S3 element is assumed to have constant plate thickness and stiffener proportions across the panel. The T1 element is assumed to have uniform plate thickness, but may have stiffeners with different proportions.

The PULS code concept isolates a rectangular panel and prescribes a set of external loads across the panel surface. For the S3 element the prescribed external nominal loads (stresses) have to be uniformly distributed in the axial direction, but may be linearly varying in the transverse direction.

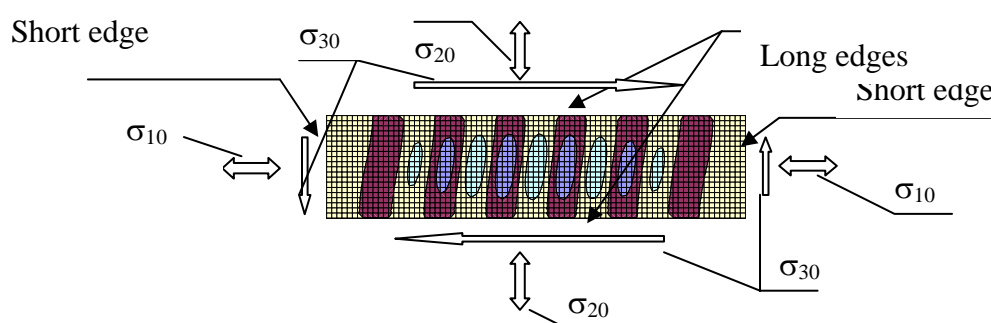


Fig. 12 Rectangular panel; definition of supported edges and applied loads

Limitations and fields of applications of the PULS 2.0 code are summarised in Table 1 in Section 5.2. Usage hints are given in Chapter 6.

## 5.2 Applications

For overview, typical ship hull areas of application for the U3 and S3 elements are tabulated below.

	Short edges Edges 1	Long edges Edges 2	Remark	Misc.	PULS element Application
Deck	Transverse webs	Long. BHDs, or deck girders			S3, ULS OK U3, ULS OK
Inner bottom	Floors	Girders			S3, ULS OK U3, ULS OK
Bottom shell	Floors	Girders			S3, ULS OK U3, ULS OK
BTM girder	Floors	Inner and outer bottom			S3 SLS OK U3 SLS OK
BTM floor	Girders	Inner and outer bottom	Check area by area as thickness varies	Variable thickness? No	S3 SLS OK U3 SLS OK
Side shell	Deck and bottom	Transverse webs		Variable thickness? Impl. in Midship Section model	S3, ULS OK U3, ULS OK
Side shell, bulk carrier	Transverse BHDs	Upper and lower sloping BHD		Variable thickness? Impl. in Midship Section model	S3, ULS OK U3, ULS OK
Long. BHD. (vertical)	Deck and bottom	Transverse webs		Variable thickness? Impl. in Midship Section model	S3, ULS OK U3, ULS OK
Long. BHD. (hopper)	Shell or inner skin and inner bottom	Transverse webs		Variable thickness? Impl. in Midship Section model	S3 SLS OK U3 SLS OK
Transverse BHD	Horizontal girders	Inner skin and Longitudinal BHDs	Check area by area as thickness varies	Variable thickness? No	S3, ULS OK U3, ULS OK

**Table 1. PULS application matrix. ULS OK means that local elastic buckling is accepted; i.e. a GEB cut-off is implemented ensuring sufficient stiffener proportions, but plate between stiffeners can buckle elastically. SLS OK means that not any type of elastic buckling is accepted; i.e. both GEB and LEB cut-off are implemented.**



## 6 USING THE PULS 2.0 ADVANCED VIEWER PROGRAM

### 6.1 Program structure

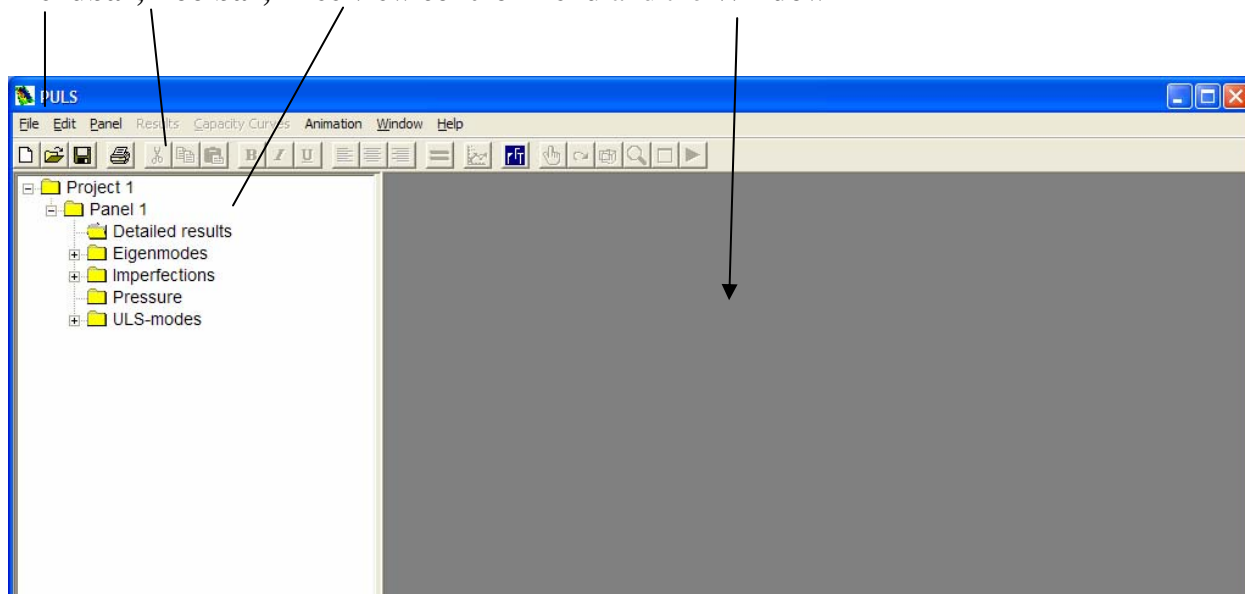
The PULS program has a multi document (MDI) layout for easy input and output. All parameters are given in cells with text identification and unit assignment.

The program has two levels of organising data. The highest level is the “**Project**” typically being a specific ship identification or similar. The level below is the “**Panel**” level typically being a specific panel in the ship. Any number of “**Panels**” can be specified, but it is recommended not to exceed 15 especially when there are generated graphics for most panels.

The “**Project**”, with the corresponding “**Panels**” organised as subfolders, are saved in a file with a format notation of \*.pbp. The **Project** name will automatically be updated to be the same as the name given to the file using the **Save as...** option in the File menu. The **Panel** can be given any name by directly editing in the Tree view control menu or using the identification text box.

The program features and the project control is facilitated using the

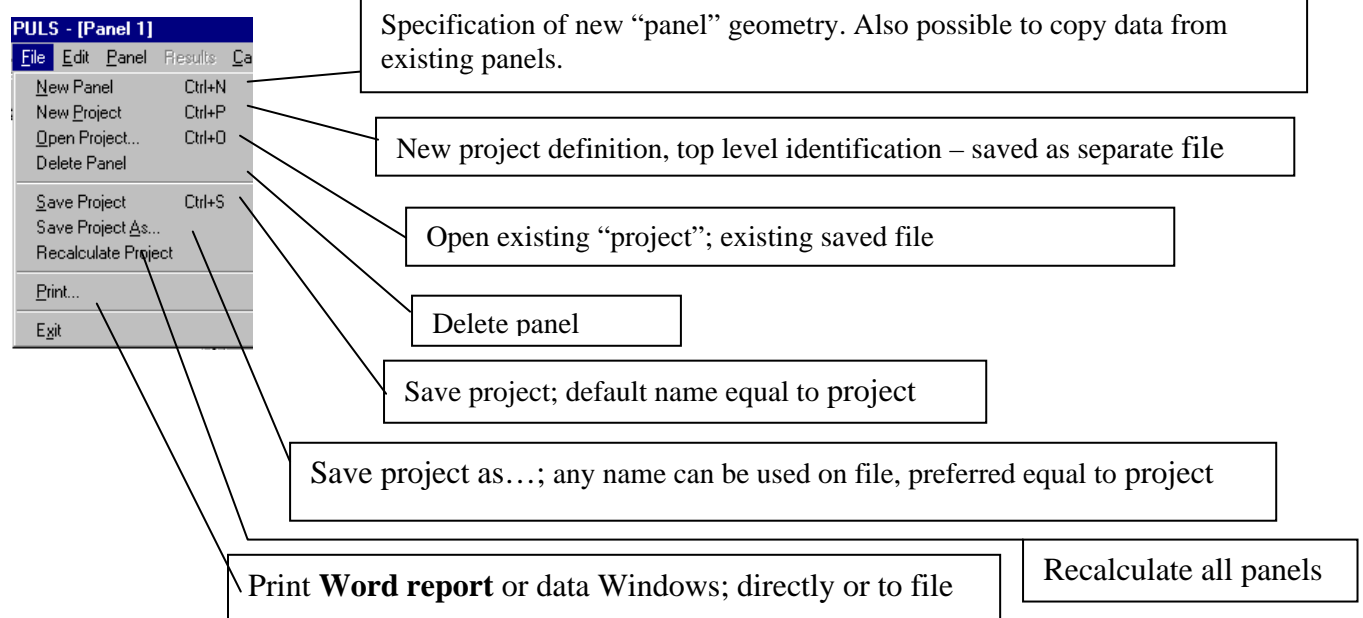
**Menubar, Toolbar, Tree view control menu and the Window**



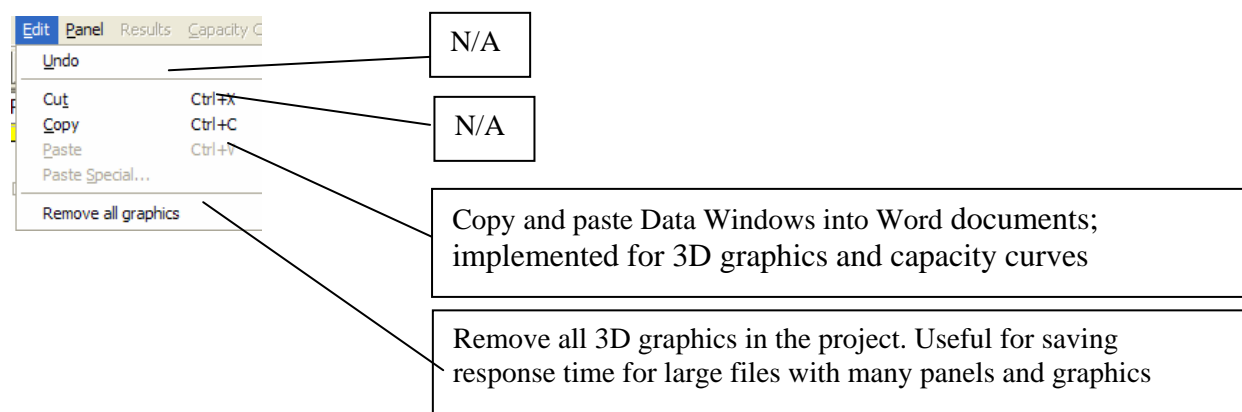
## 6.2 Menu, Toolbars and Tree view control menu

The following roll down curtains are available

### File curtain:

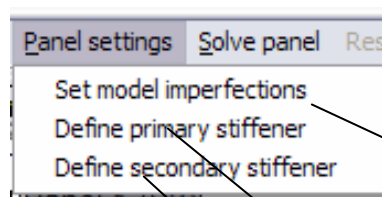


### Edit curtain:





**Panel settings** curtain:

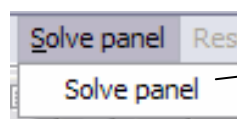


Set tolerances; default or user specified. U3: one plate parameter only. S3: three independent parameters. Default values correspond to acceptable code strength estimates.

Define stiffener from Profile table; Bulb profiles etc.

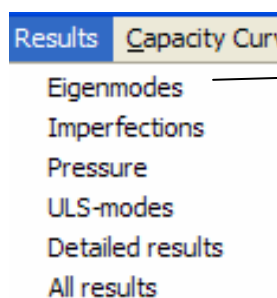
Define secondary stiffeners in transverse direction; sniped stiffener, continuous stiffener or tripping bracket.

**Solve panel** curtain:

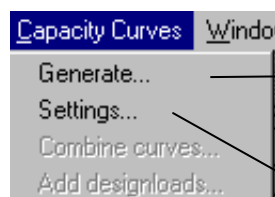


Calculates results for a given load combination; **standard** mode of program operation, external stresses have to be specified different than zero, proportional load history for all in-plane loads assumed. Lateral pressure fixed.

**Result** curtain:

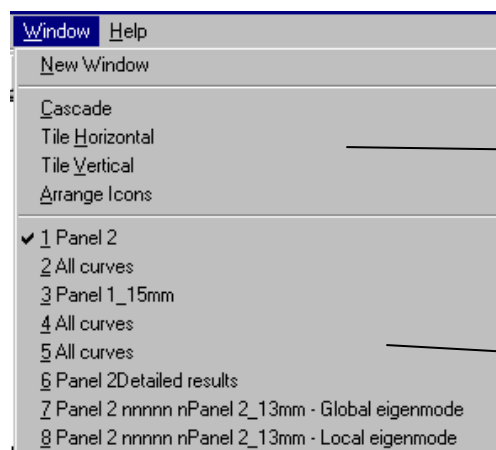


Windows for: Eigenmodes, Imperfection modes, ULS modes (deflections and stresses), Detailed results, Pressure deflections. "All results" activate all available windows.

**Capacity curves curtain:**

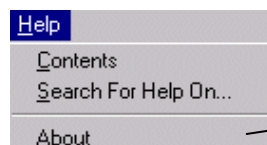
Generate capacity curves in 2D load spaces; local and global eigenvalues and ULS boundaries. Prestress option = third load component fixed, zero or non-zero. S3 and U3 also support lateral pressure.

Curve settings; type of curve, number of points in 2D load space, scaling against accept levels, scaling of axes etc. Also available from dialog box under Generate..

**Window curtain:**

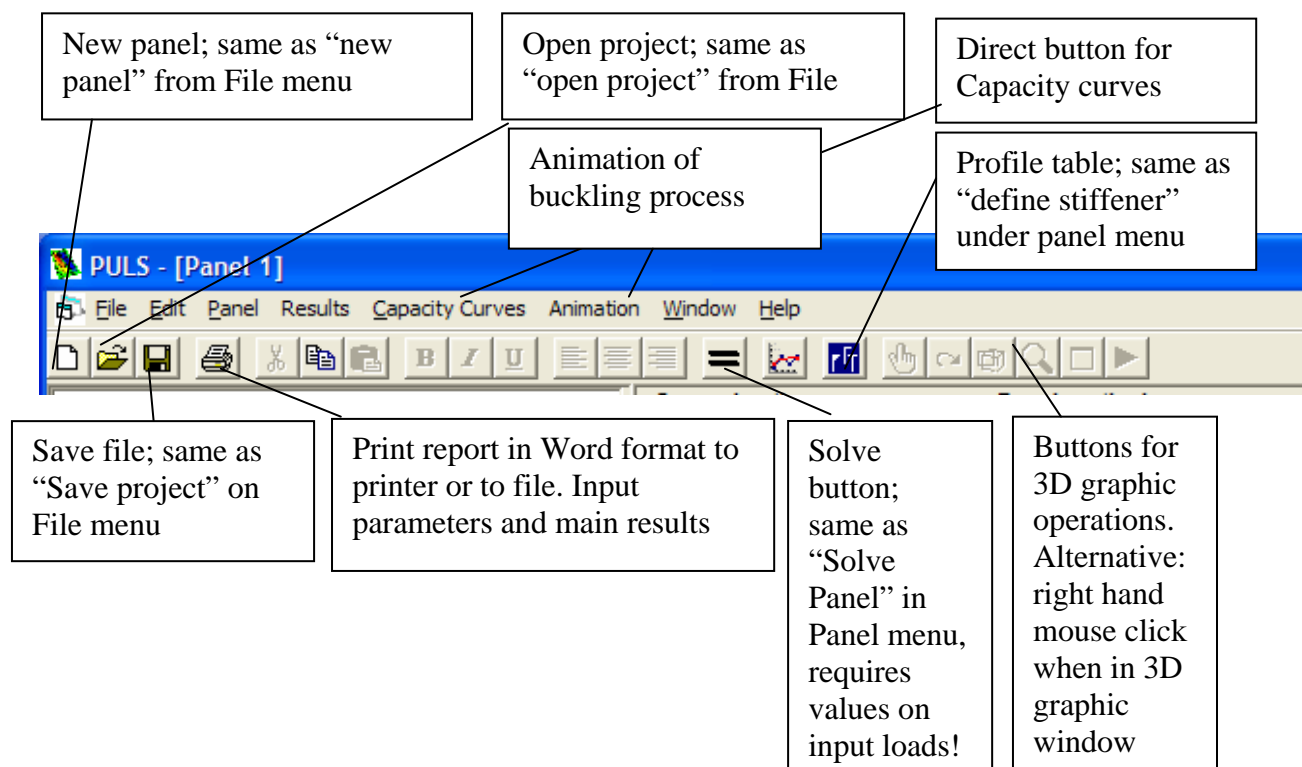
Arrange data windows

Active data windows

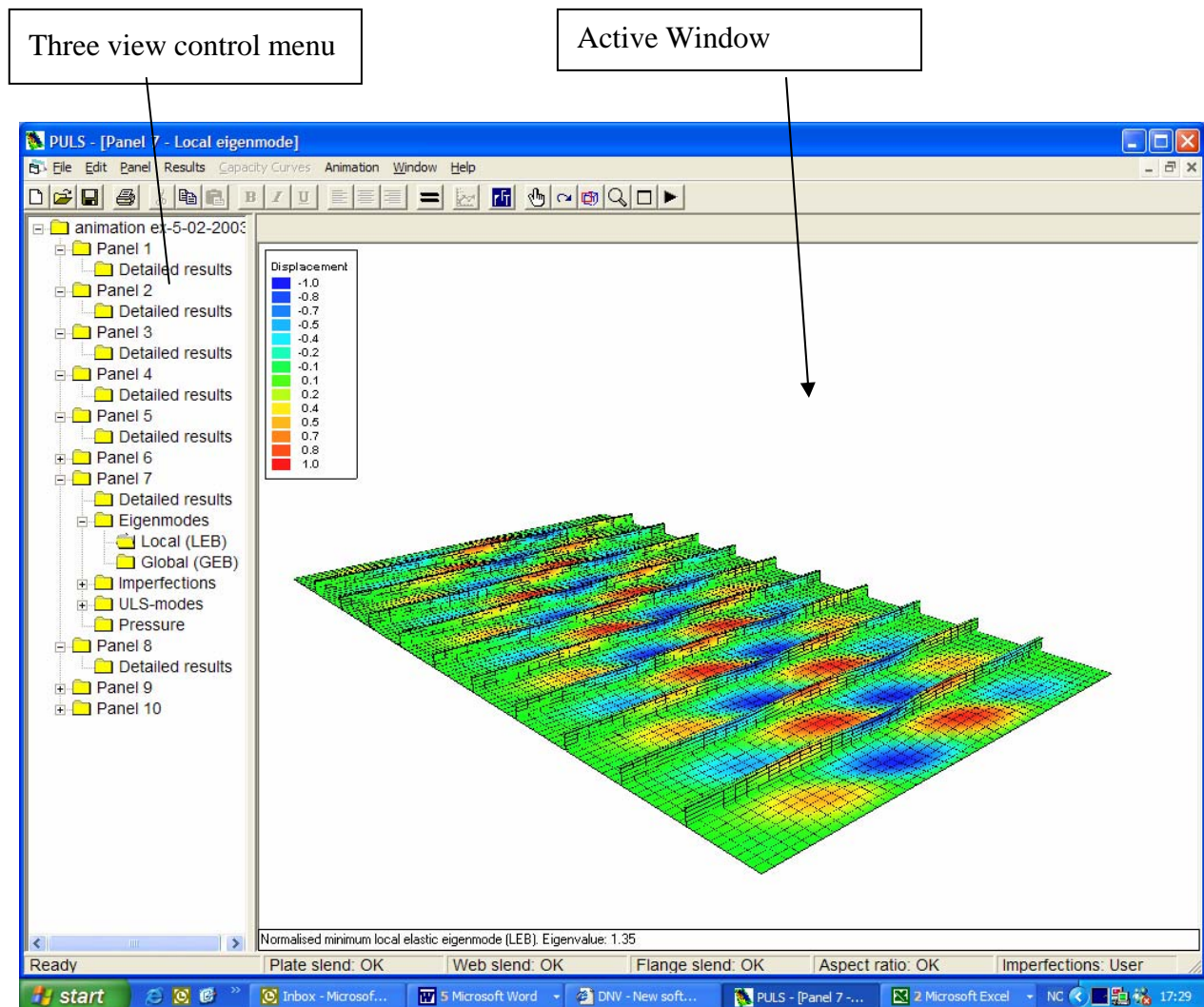
**Help menu not implemented.**

Check of current PULS version: Version 2.0

A toolbar for direct operation of some menu items is implemented as follows

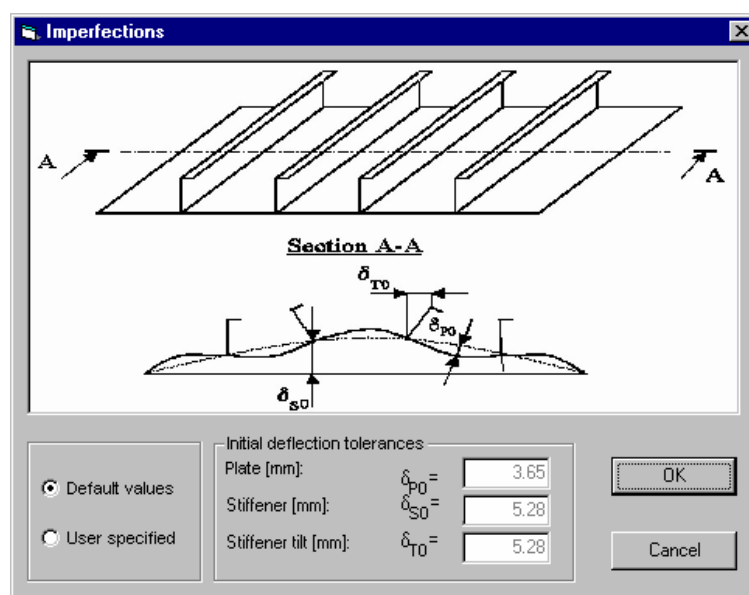


The tree view control menu operates equal as to Windows Explorer. It has the following layout



### 6.3 Tolerance specifications

Explicit geometrical model tolerances can be specified. For the unstiffened element (U3) there is only one model tolerance amplitude available, i.e. the maximum initial plate amplitude  $\delta_{p0}$ . For the stiffened panel (S3) three independent imperfection amplitudes can be specified, i.e. see figure below;  $\delta_{p0}$ ,  $\delta_{s0}$ , and  $\delta_{t0}$ . Default model tolerances correspond to ULS code strength values according to normal fabrication standards of welded **integrated** structures used in the shipbuilding and offshore industry.



The model imperfection shape is not operated by the user, only the imperfection amplitude. Imperfection models combining eigenmodes for constructing relevant imperfection modes are pre-set in the program. They can be viewed in the tree control menu after a calculation has been carried out.

Both amplitude and shape is defined by the program for the T1 element.

## 6.4 Opening and Saving files

Opening an existing file is done by clicking the Open button on the toolbar or by using the Open option in the File menu.

A new blank input page is generated by clicking the New file button on the tool bar or using the New option in the File menu.

After an analysis is run the results are saved by clicking the Save button or using the Save or Save as... in the File menu. Save as... will provide a dialog box for setting of file name, which is automatically set to be the same as the "Project" name.

The "Panel" identification names can be edited and given any name by clicking on the corresponding labels in the Tree view control menu.

## 6.5 Input window- Defining the S3 element (stiffened panel)

**NB!! Note that all input numbers entered has to be followed by a tab entry for proper registration. A tab reminder is given on the status bar at bottom whenever a new number is entered into a cell.**

Having defined a Panel identification name, the input parameters are added most conveniently by stepping through the cells using the tab key on the keyboard. Alternatively the cursor may be used. It is important that the tab key is pressed upon entering new data in a cell, in order for data to be registered.

The Input Window for stiffened panels takes the following form

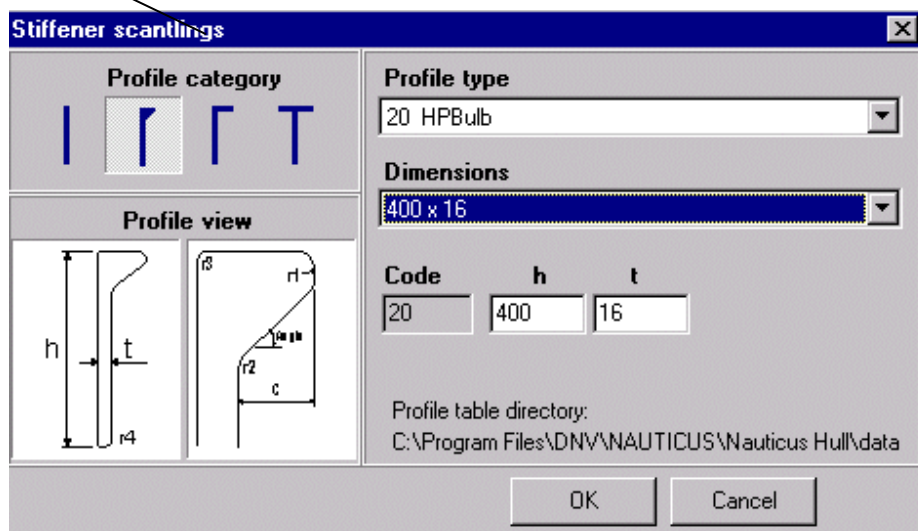
The screenshot shows the PULS software interface for configuring a stiffened panel. The window is divided into several sections:

- General Identification:** Includes fields for 'Panel 1' and 'Allowable usage factor' ( $\eta_{\text{allowable, UC}} = 1.00$ ,  $\eta_{\text{allowable, BS}} = 1.00$ ).
- Actual usage factor:** Shows calculated values for 'Ultimate capacity' ( $\eta = 0.58$ ) and 'Buckling strength' ( $\eta = 0.80$ ), both with 'Status OK'.
- Panel Particulars:** A diagram of a rectangular panel with dimensions  $L_1$ ,  $L_2$ , and stiffener spacing  $S$ . It also shows stress components  $\sigma_{1,1}$ ,  $\sigma_{1,2}$ ,  $\sigma_{2,1}$ , and  $\sigma_{2,2}$ .
- Geometry:** Contains input fields for 'Panel length [mm]' ( $L_1 = 3000$ ), 'Number of secondary stiffeners' ( $N_{\text{sec}} = 0$ ), and 'Plate thickness [mm]' ( $t_p = 10$ ).
- Material:** A tab for specifying material properties.
- Loads:** A tab for specifying applied loads.
- Boundary conditions:** A tab for specifying boundary conditions.
- Primary stiffeners:** A diagram of a T-profile stiffener with dimensions  $b_f$ ,  $t_f$ ,  $h$ , and  $t_w$ .
- Stiffener type:** Options for 'Continuous stiffeners' (selected) and 'Snipped stiffeners'.
- Stiffener dimensions:** Fields for 'Stiffener height [mm]' ( $h = 350$ ), 'Web thickness [mm]' ( $t_w = 10$ ), 'Flange width [mm]' ( $b_f = 90$ ), 'Flange thickness [mm]' ( $t_f = 15$ ), 'Flange eccentricity [mm]' ( $e_f = 0$ ), and 'Stiffener tilt angle [degrees]' ( $\beta = 0$ ).

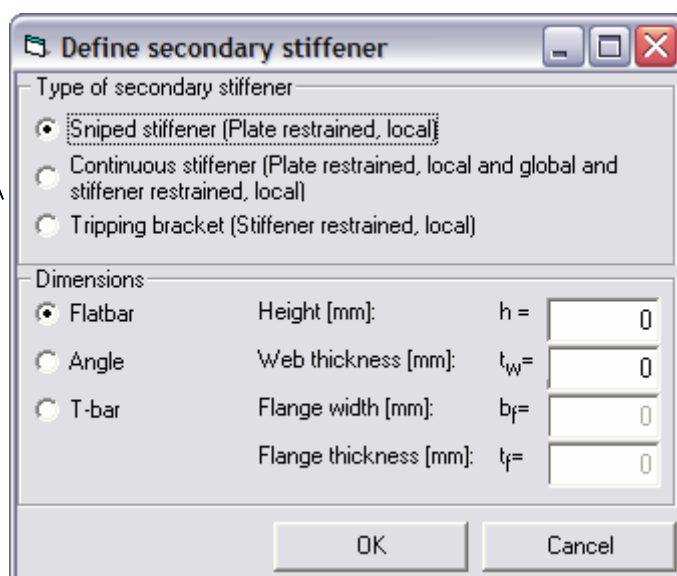
Annotations and callouts provide additional context:

- Profile table, see next page:** Points to the 'Primary stiffeners' diagram.
- Stiffened panel figure - automatically adjusted with correct proportions and number of stiffeners:** Points to the 'Panel Particulars' diagram.
- Input; Allowable usage factor is defined in the rules:** Points to the 'Allowable usage factor' input fields.
- Output/result: Calculated actual usage factor:** Points to the 'Actual usage factor' section.
- Both ultimate capacity and buckling strength usage factor calculation:** Points to the 'Actual usage factor' section.
- Input: Plate dimensions and number of continuous stiffeners  $N_s$ . Secondary transverse stiffener option: own dialog if different from zero.** Points to the 'Geometry' tab.
- Tab strips for material, load and boundary condition specification- see next page:** Points to the 'Material', 'Loads', and 'Boundary conditions' tabs.

Profile table for flat bars, bulbs, angles and T-bars (same as in DNV Nauticus Hull)



Secondary transverse stiffeners; three options: i) Sniped stiffeners ii) continuous stiffeners, iii) tripping brackets.



**Material Properties**

Material classification:

Plate material:  Stiffener material:

Modulus of elasticity [MPa]:  $E =$

Poissons ratio:  $\nu =$

Shear modulus [MPa] (not input):  $G =$

Yield strength plate [MPa]:  $\sigma_{Fp} =$

Yield strength stiffener [MPa]:  $\sigma_{Fs} =$

Material class: Steel or aluminium.

Input: Material parameters.  
NB! G not input.

**Loads**

Axial stress [MPa]:  $\sigma_1 =$

Transverse stress [MPa]:  $\sigma_{2,1} =$    $\sigma_{2,2} =$

Shear stress [MPa]:  $\tau_{12} =$

Lateral pressure\* [MPa]:  $p =$   ☐ Only pressure computation

\* Negative pressure is on the stiffener side.  
Compression: positive

**Only pressure computation:** Linear computation of deflections for specified pressure. No ULS computation.

Input: Applied loads – average nominal stresses; bi-axial and shear. Scaled proportionally for assessing elastic buckling and ULS  
Sign convention:  
Positive normal stresses (“loads”) in compression

Input: Pressure fixed.  
Negative pressure is on stiffener side.

**Location of panel**

☐ Integrated panel, restrained in-plane support, all edges. (Default option)

☒ Girder panel, free in-plane support: ☐ Free upper/lower edges ☐ Free left/right edges

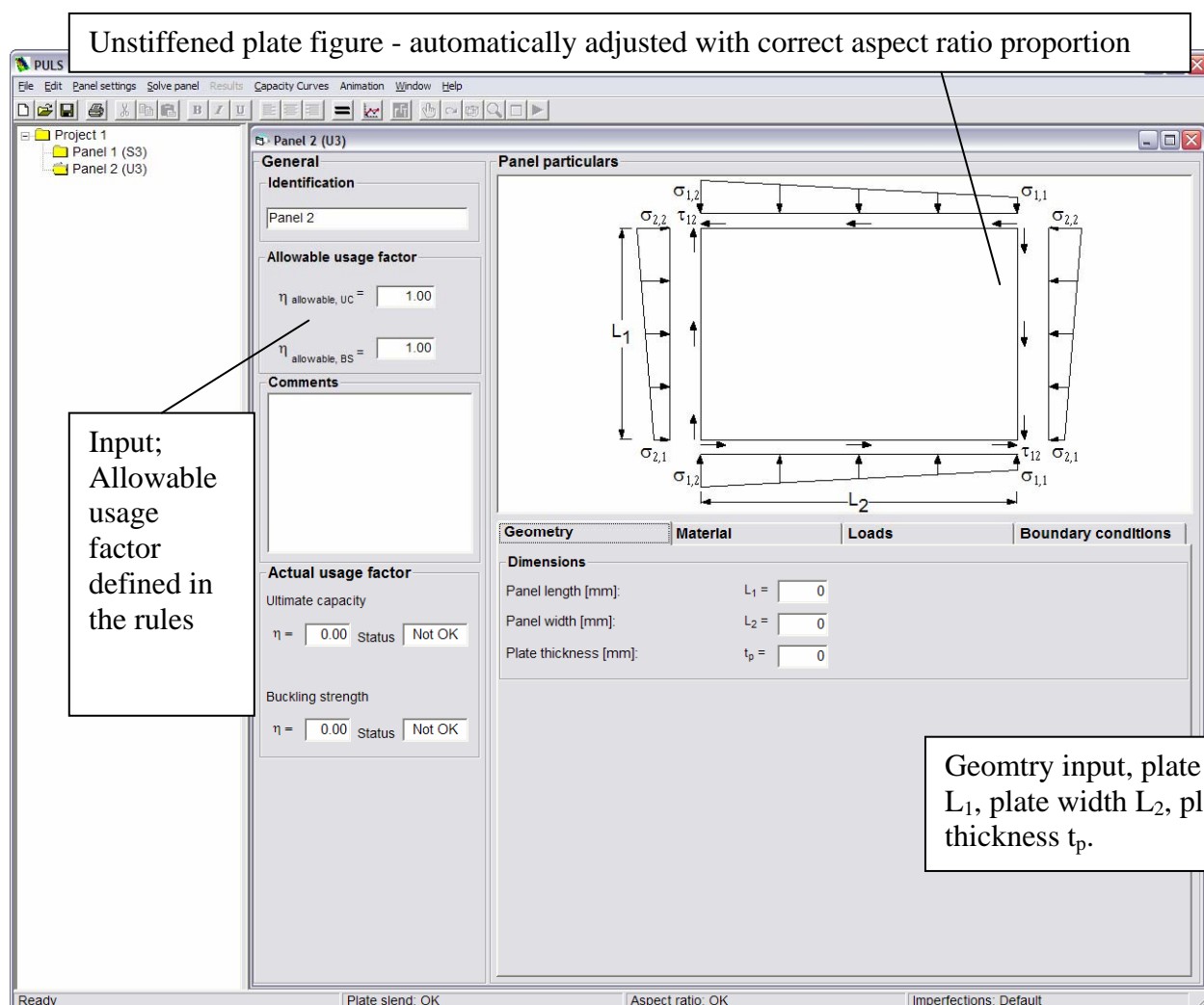
Both the integrated panel and the girder panel are simply supported.

**Location of panel:** Choose between integrated panel or girder panel. Girder panel sets restrictions to which load components that can be non-zero.



## 6.6 Input window- Defining the U3 element (unstiffened plate)

**NB!!** Note that all input numbers entered has to be followed by a tab entry for proper registration.  
 A tab reminder is given on the status bar at bottom whenever a new number is entered into a cell.



**Geometry** | **Material Properties** | **Loads**

Material classification:

Plate material:

Modulus of elasticity [MPa]:  $E =$

Poissons ratio:  $\nu =$

Shear modulus [MPa] (not input):  $G =$

Yield strength plate [MPa]:  $\sigma_{Fp} =$

Specification of material properties. Aluminium properties also available.

**Geometry** | **Material** | **Loads** | **Boundary conditions**

Axial stress [MPa]:  $\sigma_{1,1} =$    $\sigma_{1,2} =$

Transverse stress [MPa]:  $\sigma_{2,1} =$    $\sigma_{2,2} =$

Shear stress [MPa]:  $\tau_{12} =$

Lateral pressure [MPa]:  $p =$

☐ Only pressure computation - membrane  
☐ Only pressure computation - bending

Compression: positive

Specification of applied loads.

Only pressure mode. Same computation method for membrane and bending but different stresses are shown in the stress plots, membrane and bending.

**Location of panel**

☒ Integrated panel, restrained in-plane support, all edges. (Default option)

☐ Girder panel, free in-plane support:

☒ Free upper/lower edges  
☒ Free left/right edges

**Rotational support**

Rotation stiffness [MNm/m] :

Left side :  ☐ Vertical corrugated panel

Right side :  ☐ Horizontal corrugated panel

Upper side :

Lower side :

**Location of panel:** Choose between integrated panel or girder panel. Girder panel sets restrictions to which load components that can be non-zero.

Rotational support of plate: Specify either clamped, simply supported or user specified. Also possible to specify corrugated panel.

## 6.7 Input window- Defining the T1 element (triangular plate)

**NB!!** Note that all input numbers entered has to be followed by a tab entry for proper registration.  
 A tab reminder is given on the status bar at bottom whenever a new number is entered into a cell.

The screenshot shows the 'Panel 2 (T1)' input window. It has a 'General' tab on the left with fields for 'Identification' (Panel 2), 'Allowable usage factor' ( $\eta_{allowable, BS} = 1.00$ ), 'Comments', and 'Actual usage factor' ( $\eta = 0.00$  Status: Not OK). The main area has tabs for 'Geometry', 'Material', 'Loads', and 'Boundary conditions'. The 'Geometry' tab is active, showing 'Plate dimensions' (Panel length [mm]:  $L_1 = 3000$ , Panel width [mm]:  $L_2 = 1000$ , Plate thickness [mm]:  $t_p = 10$ ) and 'Stiffener 3' details. The 'Stiffener 3' section includes 'Orientation' (End point 1:  $X_1 = 2443$ ,  $X_2 = 709$ ; End point 2:  $X_1 = 136$ ,  $X_2 = 414$ ) and 'Scantling' (Stiffener type: T-profile, Stiffener height [mm]:  $h = 300$ , Web thickness [mm]:  $t_w = 10$ , Flange width [mm]:  $b_f = 90$ , Flange thickness [mm]:  $t_f = 15$ , Flange eccentricity:  $e_f = 0$ , Tilt angle [degrees]:  $\beta = 0$ ). There are buttons for 'Define new stiffener' and 'Delete stiffener'. A 'Panel particulars' window is overlaid on the main window, showing a grid with a red line representing a stiffener. Annotations explain the active picture for defining stiffener location, the change between moment of inertia and scantlings input, and the buttons for defining and deleting stiffeners. A status bar at the bottom shows 'Web slend: OK', 'Flange slend: OK', 'Aspect ratio: OK', and 'Imperfections: Default'.

Active picture for easy definition of stiffener location. Define new stiffener by clicking on picture. Change location of stiffener by clicking on relevant end point. The plate dimensions must be set before the stiffeners can be defined.

Change between moment of inertia input or stiffener scantlings input.

Defining the end points of each stiffener.

Buttons for defining and deleting stiffeners. Stiffeners can be defined or deleted with buttons or directly in picture.

Profile data for each stiffener individually.

Geometry	Material	Loads	Boundary conditions
Material classification: <input type="text" value="Steel"/>			
Plate material: <input type="text" value="Steel NVNS"/>			
Modulus of elasticity [MPa]: $E =$ <input type="text" value="208000"/>			
Poissons ratio: $\nu =$ <input type="text" value="0.3"/>			
Shear modulus [MPa] (not input): $G =$ <input type="text" value="80000"/>			
Yield strength plate [MPa]: $\sigma_{Fp} =$ <input type="text" value="235"/>			

Material input.

Geometry	Material	Loads	Boundary conditions
Axial stress [MPa]: $\sigma_{1,1} =$ <input type="text" value="0"/> $\sigma_{1,2} =$ <input type="text" value="0"/>			
Transverse stress [MPa]: $\sigma_{2,1} =$ <input type="text" value="0"/> $\sigma_{2,2} =$ <input type="text" value="0"/>			
Shear stress [MPa]: $\tau_{12} =$ <input type="text" value="0"/>			
Compression: positive.			
Positive shear stress as defined on figure.			
The loads are only acting on the plate. No load transfer through stiffeners.			

Load input. The loads are only acting on the plate and not the stiffeners. Note the difference between positive and negative shear stresses.

Geometry	Material	Loads	Boundary conditions
<b>Boundary conditions</b>			
Rotation stiffness [MN*m / m]:			
Left side: <input type="text" value="Simply supported"/>			
Right side: <input type="text" value="Simply supported"/>			
Upper side: <input type="text" value="Simply supported"/>			
Lower side: <input type="text" value="Simply supported"/>			

Rotational spring support for plate from surrounding structure

## 6.8 Running the program - Standard mode or Capacity Curves

The common use of the program is to use the **Standard** mode. The standard mode gives the results by clicking the “=” button on the toolbar or alternatively using the Solve option in the Panel menu. The “=” button have to be bold (active) for a case to be analysed. A non bold “=” button indicates a finalised analysis ready for saving.

In order for the Solve routine to work, non-zero external loads have to be specified in the input window. Non-zero external loads are automatically used as the reference load combination through which a proportional load history is prescribed. The final result in the **standard** mode is the **actual usage factor**, which is given in the Input Window as well as in the Detailed Window page (see below).

The program can also be run in an alternative mode giving **Capacity Curves**. This option is given in a separate Menu.

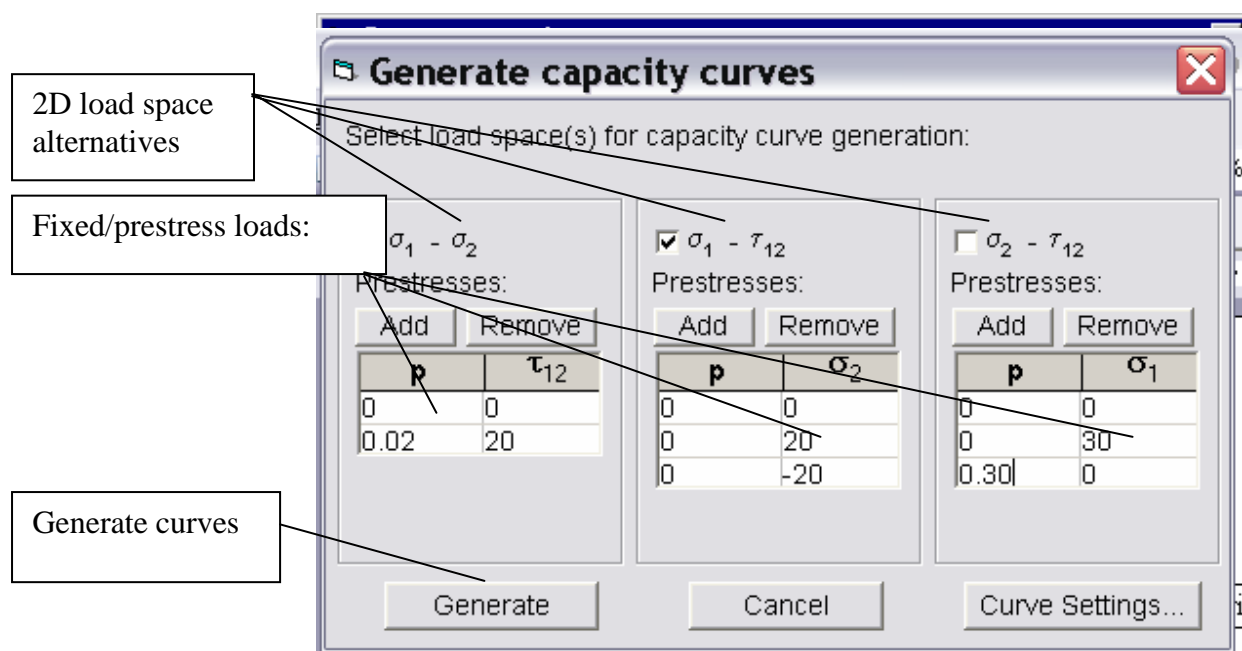
Capacity curves mean elastic local buckling (eigenvalues), elastic global buckling (eigenvalues) and ULS buckling boundaries in 2D load spaces. If this mode is used the actual usage factor is not defined and consequently not calculated.

The 2D load spaces for capacity curve visualisation are

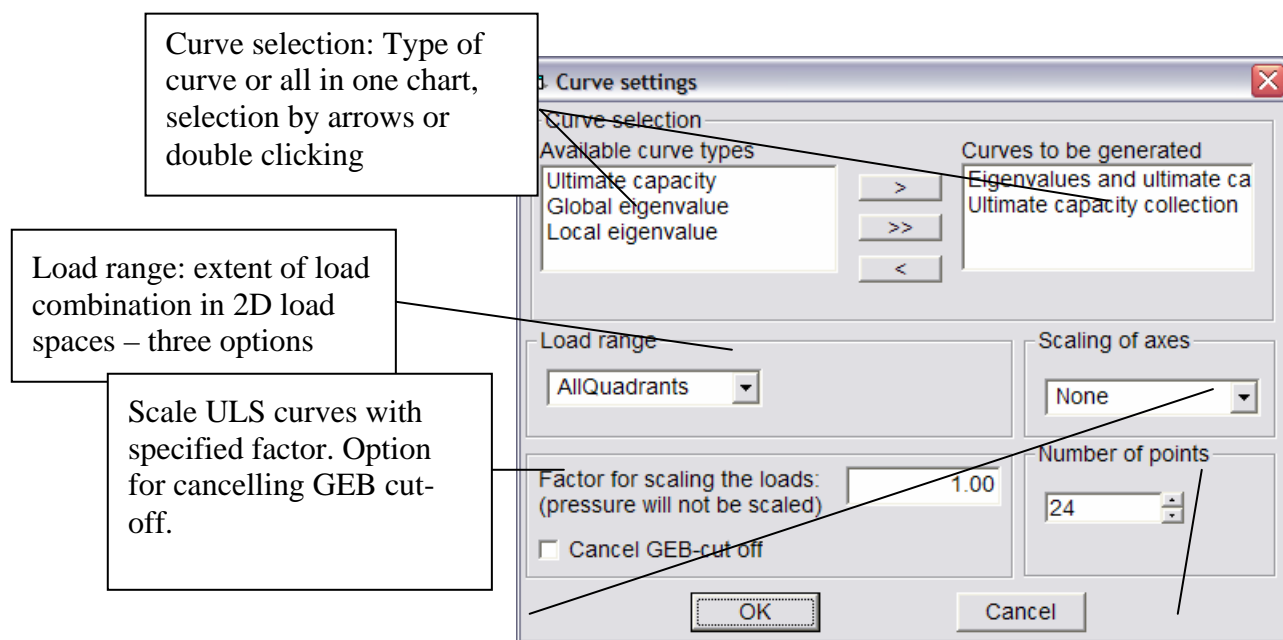
- i) **bi-axial loads**  $\sigma_1 - \sigma_2$  ;
- ii) **axial/shear loads**  $\sigma_1 - \tau_{12}$  ;
- iii) **transv./shear loads**  $\sigma_2 - \tau_{12}$  ;

For pre-stress selection of the “ULS collection” in the dialog box (see next page) will put all ULS curves into one diagram. The S3 and U3 elements assess capacity curves for preloading w.r.t. a set of specified lateral pressure levels and stresses. In girder mode the prestress option is limited. The T1 element supports prestresses, but not lateral pressure.

The dialog box for capacity curves is given under **Capacity Curves** on the menu (or separate button) and it takes the form



For the S3 element the **Curve settings** for the 2D curves are given in the following dialog box (for the U3 and T1 elements the dialog box is similar but no distinction between global and local elastic buckling is necessary)



Scaling of axes:  
None: axes in MPa  
Alternative: non.dim. axes scaled with yield stress

Number of points: Number of points describing the capacity curves in specified Load range (= number of analyses). Max. allowable - 64 points.

A set of capacity curves in the 2D load space (according to **i**) classification above) is shown in Fig.13 for a constructed S3 element example. The red curve indicates the global elastic buckling boundary (GEB), which always will be the very outer curve according to the adopted design principles (see chap.3.3.. design principle iii)). The blue curve indicates local elastic buckling (LEB) and the violet curve is the ULS curve. Note that the local elastic buckling boundary (LEB) is below the ULS boundary in a limited region in load space for this example. This is typical for cases where the plate is thin. For thicker plates the ULS boundary curve will be the inner curve in the whole load space. See Fig.14 for illustration of such a case.

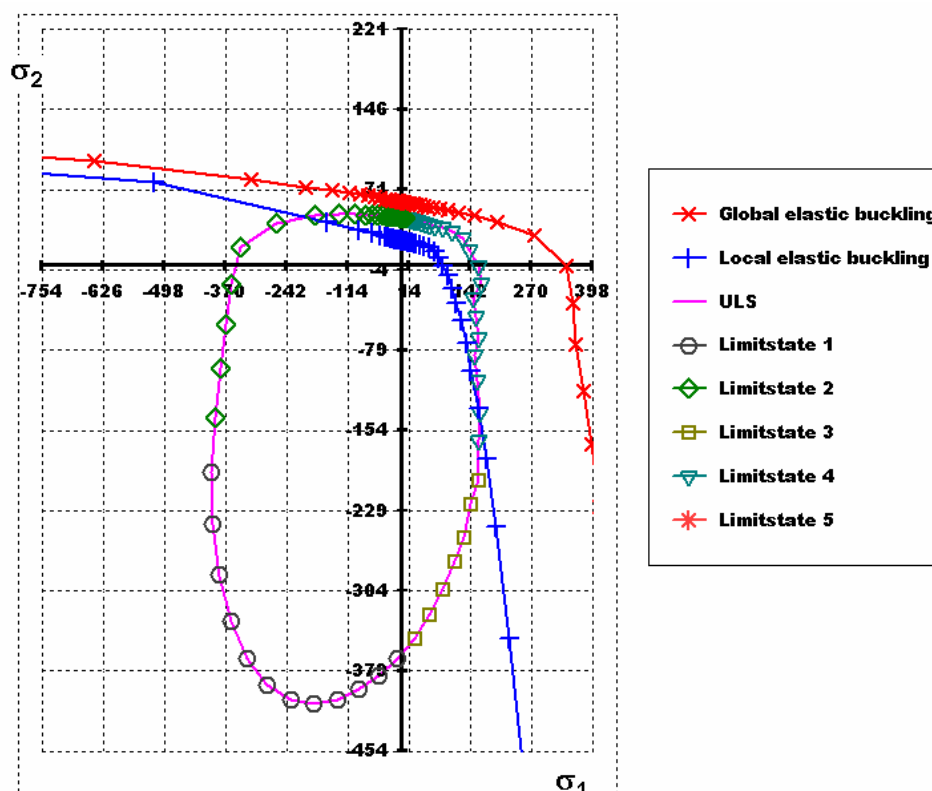


Fig. 13 Capacity Curves in 2D load space; bi-axial loading, all quadrants, thin plate

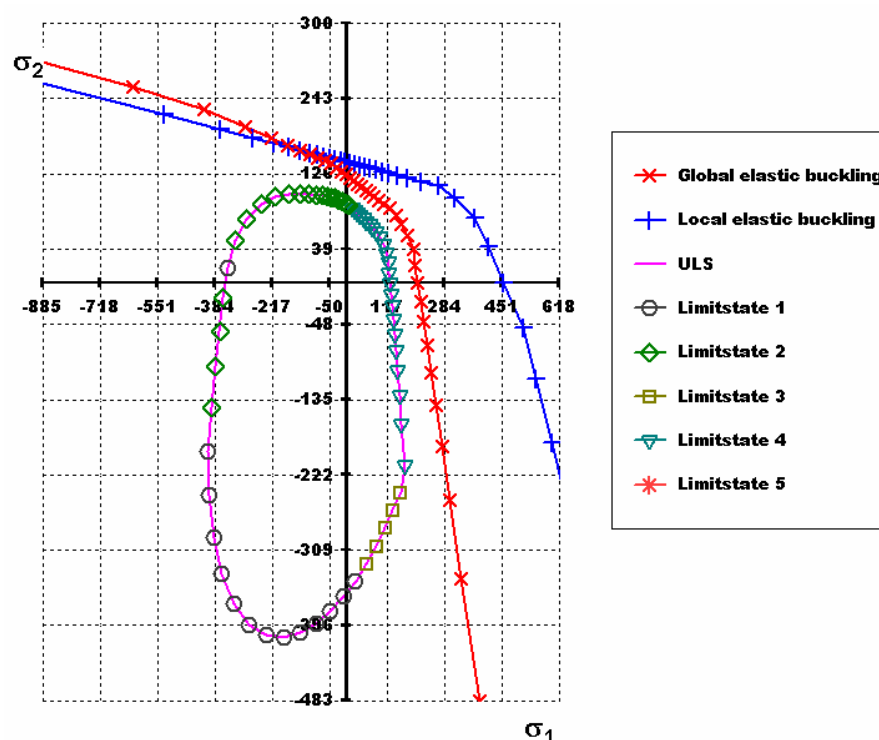


Fig. 14 Capacity Curves in 2D load space; bi-axial loading, all quadrants, thick plate

Similar capacity curves are available in the other two load spaces where shear stresses are one of the load components. Fig. 15 shows an example including shear. Note the symmetry in the shear load  $\tau_{12}$  ( $= \sigma_3$ ).

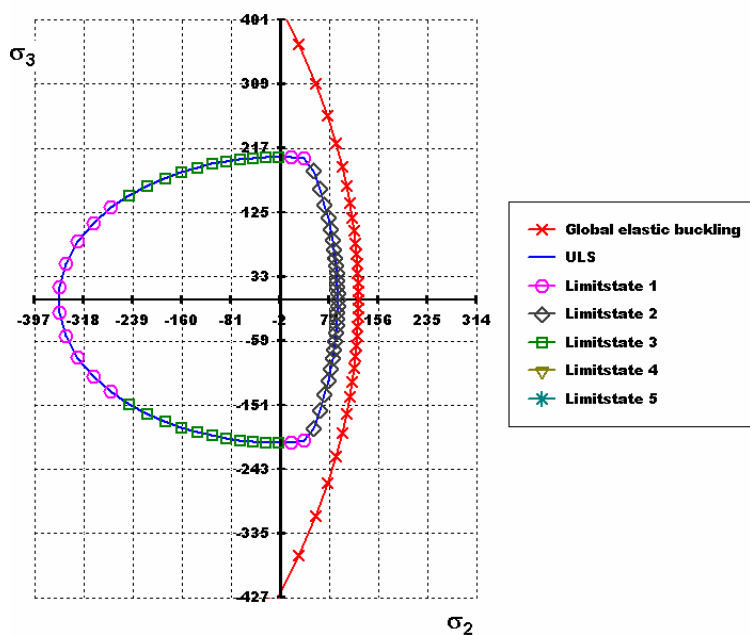


Fig. 15 Capacity Curves in 2D load space; transverse load  $\sigma_2$  and shear  $\sigma_3(\tau_{12})$ , all quadrants.



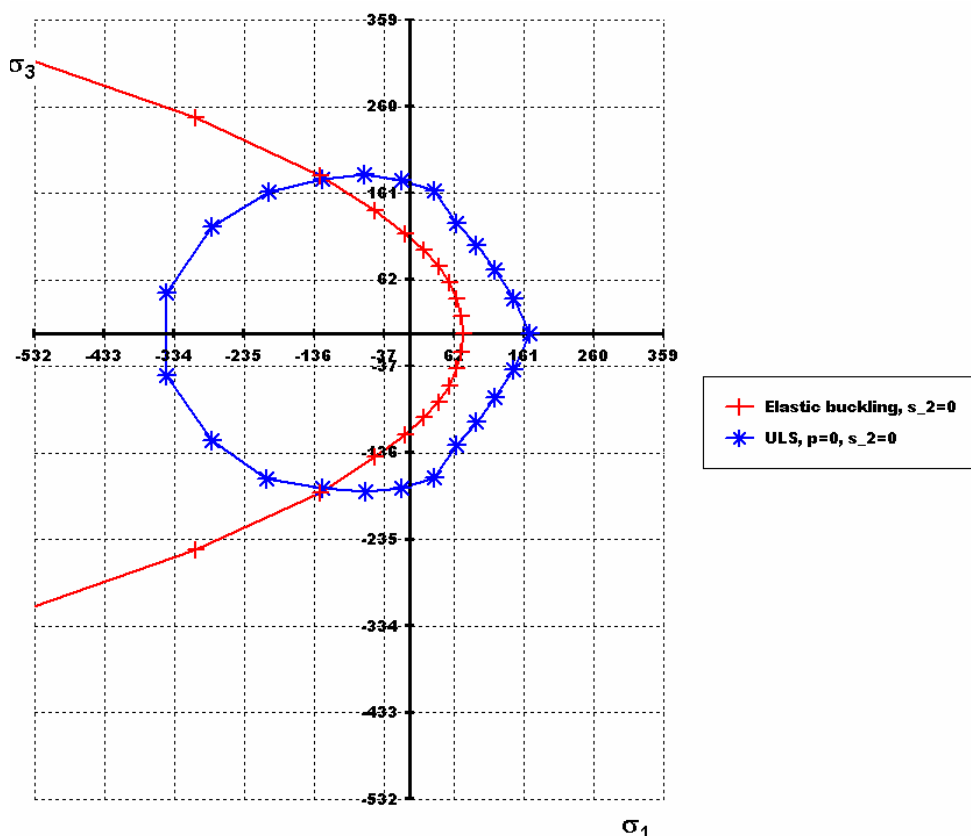


Fig. 16 Capacity Curves in 2D load space for U3 element (unstiffened plate); axial stress  $\sigma_1$  and shear stress  $\sigma_3(\tau_{12})$ , all quadrants.

An example of 2D Capacity curves for the U3 element is given in Fig.16 for combined axial nominal stress  $\sigma_1$  and nominal shear stress  $\sigma_3 (= \tau_{12})$ . As can be seen, the example is for relatively thin plate for which the UC strength is beyond the elastic buckling stress for most of the load combinations.

**NB!! The Capacity Curve Windows has some extra features:**

- The axis can be stretched/compressed and the origin can be moved by a drag option using the mouse. Stretching/compressing axes: Press the shift key on the keyboard and drag the mouse with left button pressed. Moving origin: Press the shift key down and drag the mouse with the right button pressed. The Window frames can be moved and resized in a standard Windows manner.
- The capacity curves can be copied and pasted as numbers into e.g. Microsoft Excel for comparison purposes.

## 6.9 3D plots of buckling modes, UC modes and UC membrane stresses

In the **standard** mode of program operation 3D plots of the eigenmode, model imperfections, pressure plots, ULS mode and ULS membrane stresses is available. These data can be found under the Result menu.

### U3 unstiffened plate element:

For illustration Fig. 17 below shows 3D plots for an U3 element subjected to pure in-plane bending, **a)** elastic buckling mode, **b)** ULS mode and **c)** ULS membrane von Mises stresses.

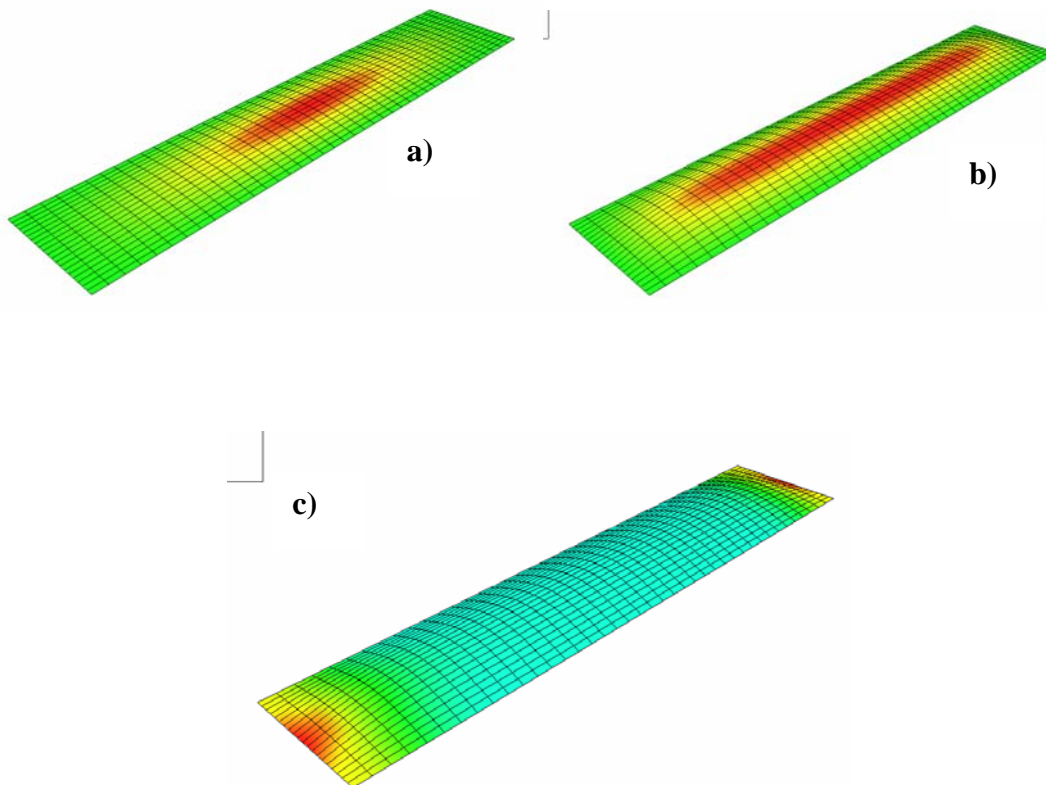


Fig. 17 a) U3 sub-element : LEB; eigenmode, b) UC mode, c) UC von Mises membrane stresses

In particular, the ULS membrane stresses show the non-linear redistributed stress distribution at the first edge membrane yield condition. This is then, as per definition in the PULS approach, the ULS state. Some more capacity may be expected for thin plates involving spread of plasticity, and consequently involving permanent sets, but the extra capacity is marginal.

### S3 stiffened plate element:

The stiffened plate shows similar results as for the unstiffened plate. The difference is with respect to categorisation of modes, i.e. stiffened panels buckle into local and global modes respectively.

#### General:

3D plots of LEB and GEB eigenmodes and ULS modes are available for the **capacity curves**. The 3D visualisations are generated by double clicking on the corresponding point in the **capacity curve** Window figure. 3D plots are shown in Fig.18 for a S3 element example subjected to bi-axial loading in two different points in load space illustrating different types of collapse modes.

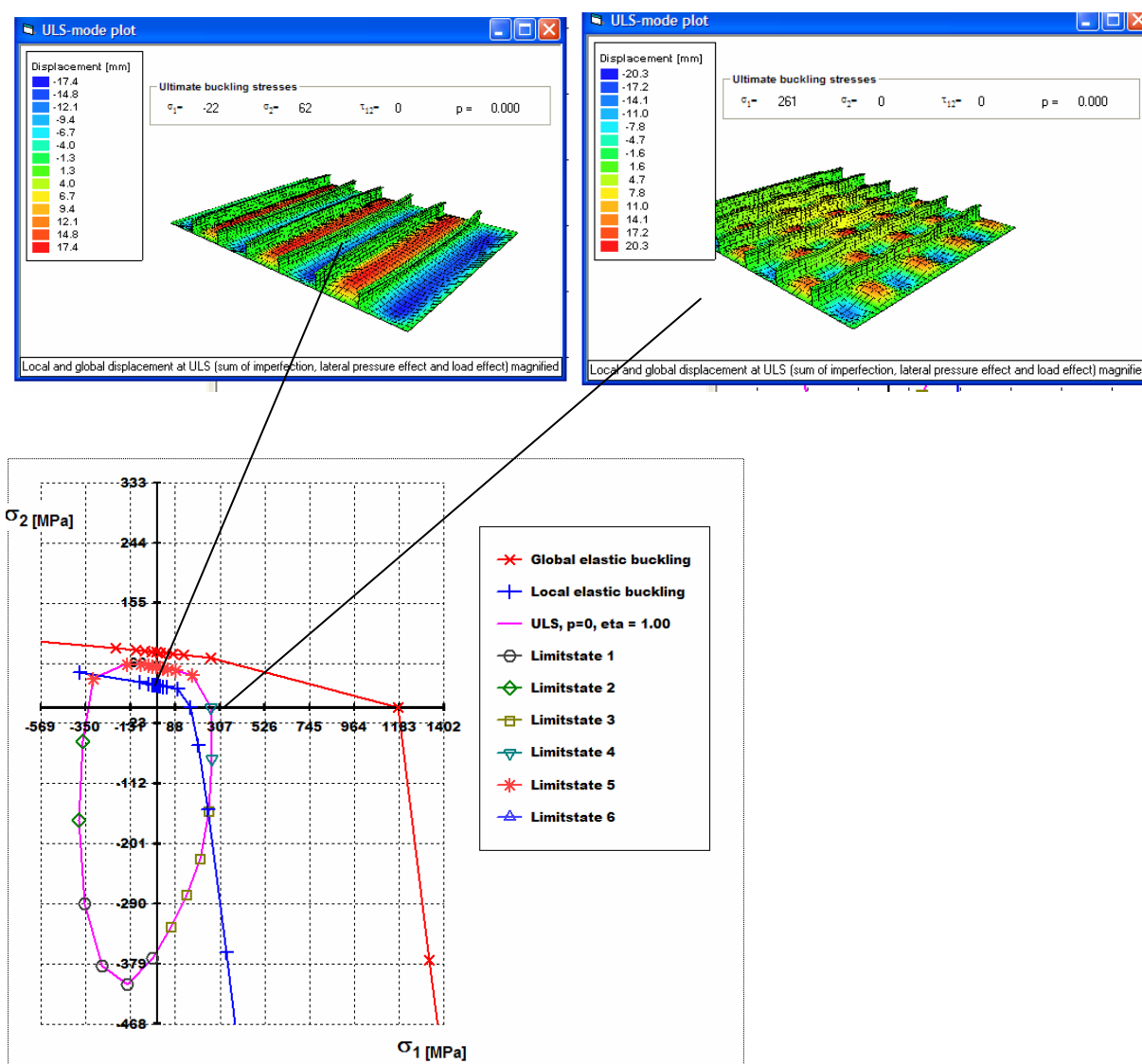


Fig. 18 Capacity Curves in 2D load space with associated 3D visualization of ULS modes for two different points in load space

## 6.10 Output window- Detailed result S3 element (stiffened panel)

Detail result summary sheet: In the **standard** mode a detailed results window is available by selecting Detailed results from the **Result** menu. The output parameters are commented in figure below.

**Applied loads, mapped from input window**

**Ultimate loads with GEB cut-off = do not accept global stiffener buckling**

**Buckling loads, GEB and LEB cut-off**

**Minimum local eigenvalue; LEB**

**Minimum global eigenvalue; GEB**

**Real ULS; accepts global stiffener buckling**

**Calculated usage factor**

**Tree view control menu**

**Ultimate stress and usage factor for each limit state, minimum ultimate stress given in detail result summary above.**

**Table showing the maximum displacement selected locations in panel.**

**Detailed results summary**

	Applied loads	Ultimate loads*	Buckling loads	Local eigenvalue	Global eigenvalue	Ultimate loads**	Allowable usage factor
$\sigma_1$ [MPa]	100.00	170	126	126	2577	170	1.00
$\sigma_{2,1}$ [MPa]	0.00	0	0	0	0	0	Ultimate capacity Status
$\sigma_{2,2}$ [MPa]	0.00	0	0	0	0	0	0.59 OK
$\tau_{12}$ [MPa]	0.00	0	0	0	0	0	Buckling strength Status
$p$ [MPa]	0.000	0.000	0.000	N/A	N/A	0.000	0.79 OK

Ultimate loads\*\* - panel capacity (accepting local and global buckling).  
 Buckling loads - minimum of local eigenvalue, global eigenvalue and ultimate loads\*\* (no buckling accepted).  
 Ultimate loads\* - minimum of global eigenvalue and ultimate loads\*\* (global buckling not accepted).

**Stress control limit states**

Stress control limit state	$\Delta_u$	$P_u$	$\sigma_{1u}$	$\sigma_{2,1u}$	$\sigma_{2,2u}$	$\tau_{12u}$	Usage
1	5.15	0.000	515	0	0	0	0.19
2	1.84	0.000	184	0	0	0	0.54
3	1.93	0.000	193	0	0	0	0.52
4	1.77	0.000	170	0	0	0	0.59
5	0.99	0.000	199	0	0	0	0.50
6	1.77	0.000	177	0	0	0	0.57

**Weakest link displacements at ultimate capacity**

Maximum state displacement between stiffeners at ultimate capacity (local mode) [mm]\* : 9.24  
 Maximum lateral stiffener displacement (global mode) [mm]\* : 0.22  
 Maximum sideways displacement in top of stiffener (local mode) [mm]\* : 1.04  
 Maximum sideways displacement across stiffener web height (local mode) [mm]\* : 2.04  
 Failing stress control limit state: 4

\*The imperfection amplitude is not included.

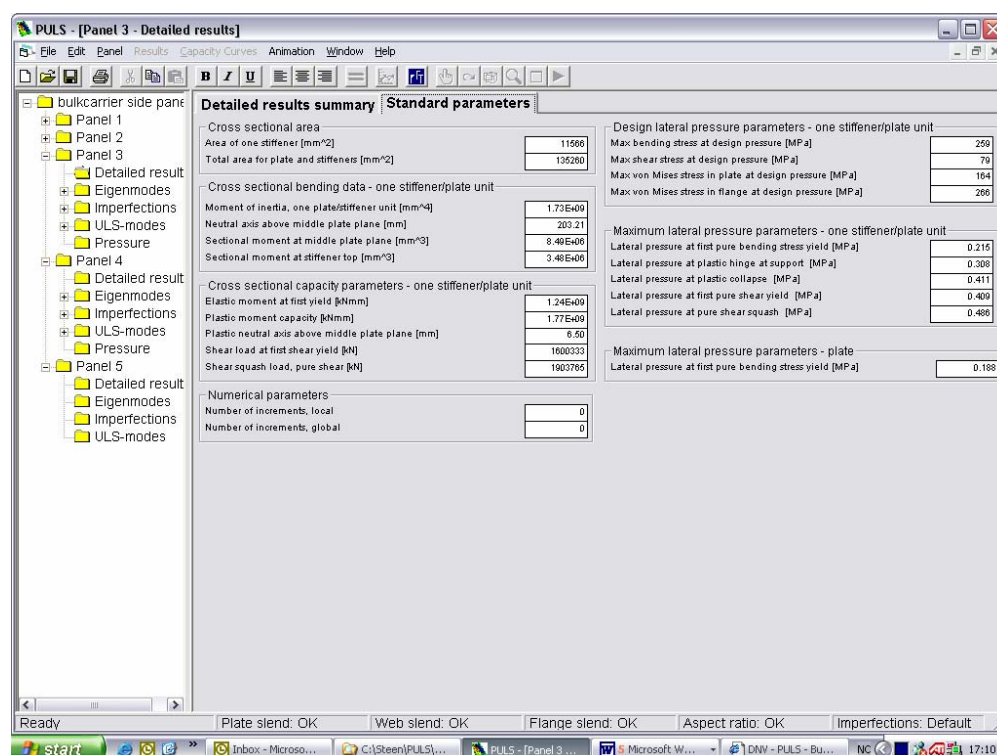
Stiffness coefficients sheet: Orthotropic macro stiffness coefficients, L-linear, I-initial, U-ultimate secant.

Standard Parameters sheet: The stiffened panel element has also a Standard parameter list. Most of these parameters are not a part of the PULS buckling models and they are only meant as general user info.

They are categorised into

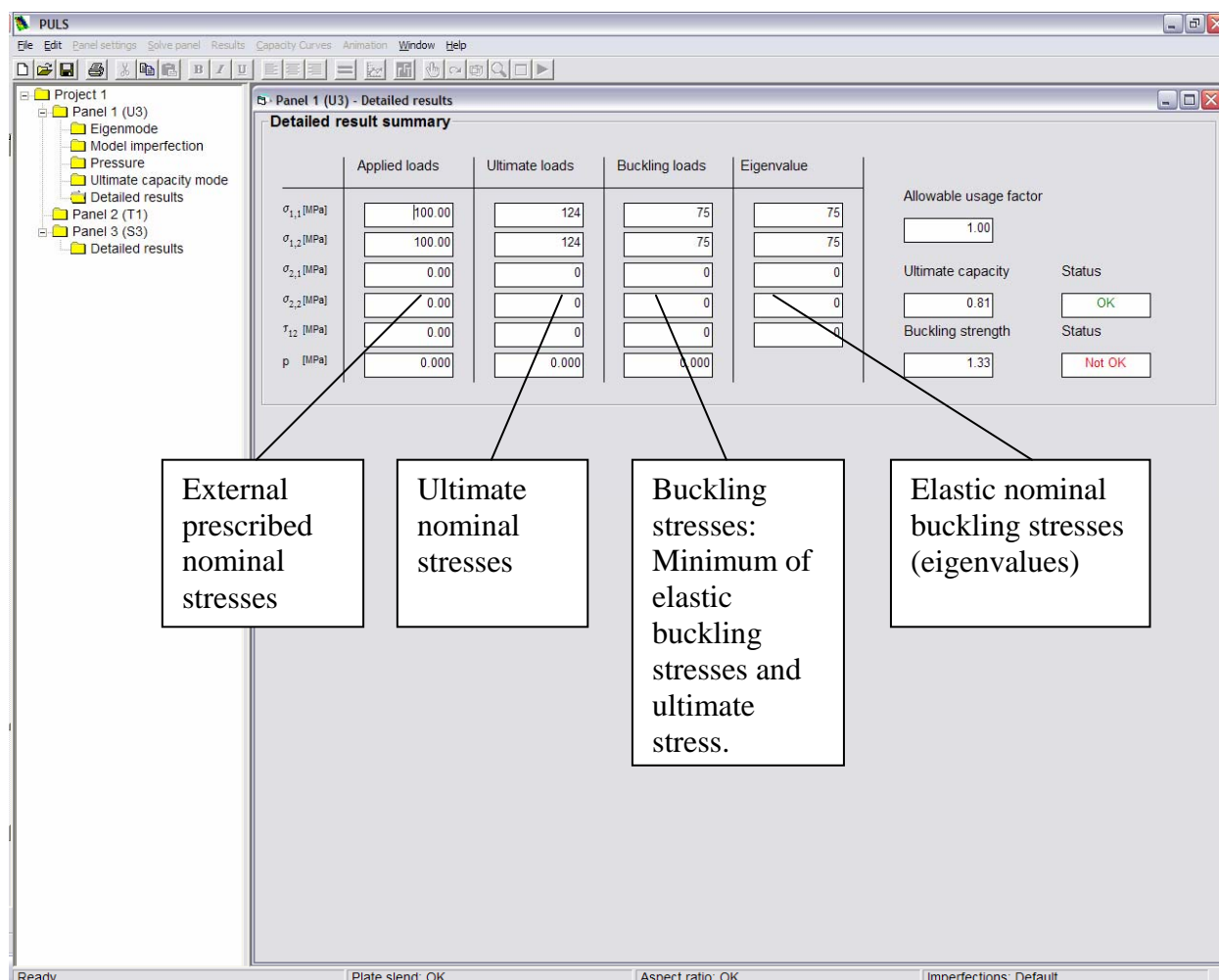
- Cross-sectional data; cross sectional data for plate and stiffener,  $x_1 = \text{const.}$
- Cross-sectional bending data; valid for one stiffener/plate unit, full plate width  $s$  is included
- Cross-sectional capacity parameters; standard moment and shear load capacities
- Numerical parameters; number of increments in PULS procedure
- Design lateral pressure parameters; Stresses in single stiffener/plate unit according to linear beam theory for the prescribed pressure
- Maximum lateral pressure parameters - one stiffener/plate unit; pressure limits according to linear beam theory for a single plate/stiffener unit
- Maximum lateral pressure parameters - plate; reference pressure limit  $p_F$  based on first edge bending stress limit for a clamped plate unit with length  $s$ .

JTP requirements sheet: Not implemented yet.



## 6.11 Output window- Detailed result U3 element (unstiffened plate)

In the **standard** mode a Detail results Window is available by selecting Detailed results from the **Result** menu. The output parameters are commented on figure below.



## 6.12 Status bar

The status tool bar is located at the bottom of the Window. Below is given examples for the stiffened panel S3.



Ready -

program ready.

Press Tab key -

a new number has entered a cell, tab to next field to register the number.

Recalculation necessary

A new set of data has been registered, recalculation is necessary for updated buckling and ULS strength.

## **6.13 Program hints**

The usage hints are given here for overview. It refers both to program operation and how to address special problems and applications cases.

### **General:**

- i) *User specified model tolerances can be specified for the plate and stiffener (U3 and S3), see under Panel menu on tool bar. In current version only model imperfection amplitudes not imperfection forms can be specified. Default settings correspond to normal fabrication standards of steel structures.*
- ii) *Excel Report can be printed, see under File menu. .*

Always remember to press the solve button before printing in order to avoid mismatch between input data and results. This is controlled also by the status bar information at the bottom of the PULS Windows.

### **General: Eigenmode, Imperfection, Pressure and ULS mode windows:**

- i) *The windows, figures and plots can be copied and pasted into other programs such as Word etc.*
- ii) *By clicking the right mouse bottom a property menu for the graphic is available*
- iii) *By holding the pointer over the panel a tool tip box appears and shows the displacement or stress (depending on plot) in the selected point.*

### **General: Capacity Curve Windows:**

- i) *The capacity curves (elastic buckling and ULS) can be copied and pasted as numbers into e.g. Microsoft Excel for comparison purposes.*

#### **Procedure:**

Click once with the left mouse button on the curve to be copied. This marks all points on the curve. Then follow the standard copy and paste procedure for transferring the numbers into Excel for recreating the capacity curve. NB! When clicking the first time on a capacity curve the marked points (black) may seemed moved somewhat relative to the coloured continuous curve. This "error" is eliminated by e.g. a slight drag on the outer curve frame before clicking on the curve or e.g. by a stretch or move operation on the axes described above.

- ii) *The axes can be stretched/compressed and the origin can be moved by a drag option using the mouse.*

Stretching/compressing axes: Press the shift key on the keyboard and drag the mouse with left button pressed.

Moving origin: Press the shift key down and drag the mouse with the right button pressed.

iii) *The text box in the capacity curve figures can be resized and moved in the window*

iv) *Graphical visualisation of buckling (or ULS mode) mode from any point in load space laying on the capacity curves.*

Procedure: A point in load space is selected by first select the whole curve (all points visualised by coloured marks) and then click once more on the point of interest (only this point is to be marked by colour symbol). Then double click on the selected point and the buckling mode window pops up. The corresponding co-ordinates in load space are given in the inserted frame above (always given in MPa). Several points may be “called up” and moved around inside the window for comparison purposes, illustrating different type of failure modes as function of the load combination.

### **U3 element:**

i) *Detailed results summary*

The applied nominal stresses are listed together with the ultimate stresses, buckling loads, the eigenvalues and the usage factor.

The results are based on a proportional load path for the in-plane stresses meaning the same ratio between each nominal stress component; applied, at the point of elastic buckling (eigenvalues) and ultimate state. The lateral pressure is as specified in the input, i.e. it is kept fixed.

ii) *Eigenmode*

The eigenmode shows the minimum ideal elastic buckling mode according to linear elastic buckling theory. The deflections are normalised to unity.

iii) *Imperfection mode*

Shows form of geometrical imperfections used in the non-linear analysis. Deflection  $w_{imp}$  in [mm]

iv) *Pressure*

The pressure plot shows the displacements in the plate caused by the applied pressure. The plot generated by a regular solution is non-linear while the plot generated by the only pressure mode is linear. The shown stresses are bending stresses or membrane stresses depending on the users choice in the load tab strip.

v) *UC mode*

Shows the deflected form in the UC state.

The UC deflection shown in the figure is the total deflection ( $w_{tot}$ ) including the geometrical imperfection and the deflections due to pressure;  $w_{tot} = w_{imp} + w_{load} + w_{pressure}$ .



By clicking on the UC figure a yellow frame pops up showing the co-ordinates of the point and the corresponding UC out-of-plane deflection in [mm].

*vi) UC membrane stresses*

Shows membrane stresses in the UC state.

By clicking on the figure a yellow frame pops up showing the co-ordinates of the point and the corresponding UC membrane stresses stress in [MPa].

The membrane stresses for any required load level can be visualised by adjusting a “fictive” material yield stress in the input cell such that the applied nominal stresses becomes equal to ultimate stresses. A bit tricky approach, but it is necessary to limit the storage of data.

Only membrane stresses are given. However, bending stresses are easily calculated from the method, but not included in the current PULS versions due to storage limitations.

*vii) Capacity curves*

Only uniform nominal stresses  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$  are possible for capacity curve calculations.

Capacity curves can be generated in load space  $\sigma_1$ -  $\sigma_2$  with fixed  $\sigma_3$ ,  $\sigma_1$ -  $\sigma_3$  with fixed  $\sigma_2$ ,  $\sigma_2$ -  $\sigma_3$  with fixed  $\sigma_2$ . Five levels of the pre-stress may be specified using the add button in the dialog box. Lateral pressure can also be specified covering a special application of local pressure on a plate.

NB! Capacity curves involves a sequence of non-linear calculations for different load paths and may require some computer time (number of load paths = number of points in load space \* number of fixed stress values)

**S3 element:**

*i) Detailed results summary*

The applied nominal stresses are listed together with the ultimate stresses, the eigenvalues in local and global mode and the usage factor.

The results are based on a proportional load path for the in-plane stresses meaning the same ratio between each nominal stress component; applied, at the point of elastic buckling (eigenvalues) and ultimate state. The lateral pressure is as specified in the input, i.e. it is kept fixed.

The results for each of the five limit states are listed. The critical limit state is picked out and given in the detailed summary list.

A set of orthotropic stiffness coefficients are given indicating the degree of reduced stiffness (increased flexibility) of the panel due to local buckling from the plate between stiffener and sideways buckling of stiffeners. The stiffness coefficients given in PULS 1.5 represent linear values (sub/superscript L) and tangent values for zero load (sub/superscript I) and at the secant values as representative for the ultimate state (sub/superscript U). A superscript N symbolises neutral relevant for the bending stiffness about the instantaneous neutral axis of plate and stiffener combination.

*ii) Eigenmodes*

Shows the minimum ideal elastic buckling mode according to linear elastic buckling theory in the local and global mode separately.

The global eigenvalue is calculated based on an orthotropic plate theory with the reduced bending stiffness coefficients in the ultimate state.

The eigenmodes are normalised to unity.

*iii) Imperfection mode*

Shows form of geometrical imperfections used in the non-linear analysis. Deflection  $w_{imp}$  in [mm]

*iv) Pressure*

The pressure plot shows the displacements in the plate caused by the applied pressure. Both the solution for regular and only pressure mode is linear.

*iv) UC mode*

Shows the deflected form in the UC state separated into local and global modes.

The ULS deflection shown in the figure is the total deflection ( $w_{tot}$ ) including the geometrical imperfection and the displacements due to pressure;  $w_{tot} = w_{imp} + w_{load} + w_{pressure}$ .

*v) UC membrane stresses*

The membrane stresses is shown at the local UC state. This is not exactly as the same as the real UC state, but for qualitative illustration of non-linearly redistributed stresses the 3D graphics may be useful. The graphical presentation is only an added feature, it has no influence on the UC results which consider interaction between local and global buckling effects.

*vi) Capacity curves*

Capacity curves can be generated in load space  $\sigma_1$ -  $\sigma_2$  with  $\sigma_3$  = fixed,  $\sigma_1$ -  $\sigma_3$  with  $\sigma_2$  = fixed,  $\sigma_2$  -  $\sigma_3$  with  $\sigma_1$  = fixed.

NB! Capacity curves involves a sequence of non-linear calculations for different load paths and may require some computer time (number of load paths = number of points in load space \* number of fixed stress values)

*vii) Secondary stiffeners*

The standard UC and buckling check in present 2.0 version control the strength of the secondary stiffeners by using a simplified theory. The secondary stiffeners are modelled as lateral springs and their strength are controlled by a max deflection criterion. Thus their bending stiffness is considered, but not their axial nor their rotational/torsional stiffness.

## 7 PULS EXCEL SPREADSHEET

### 7.1 Introduction

The PULS Excel spreadsheet and the PULS GUI use the same computational routines. The spreadsheet offers easy input of a large number of panels, and therefore makes parameter studies easy to perform. The spreadsheet is organized in input sheets and output sheets for the S3 and U3 elements. The input and output parameters are the same as in the Puls GUI. The Puls spreadsheet is able to read and write pbp-files which are compatible with the Puls GUI.

### 7.2 Input columns

Most of input columns are self explanatory. The input parameters that are found necessary to explain are tabulated below.

Input parameter	Relevant element	Description	Input options
Stiffener type	S3	Choose stiffener profile by entering either L, T or F. User defined stiffener profile is not an option.	L – Angle profile T – Tee-bar F – Flatbar
Stiffener boundary	S3	Stiffener end support.	C – Continuous stiffener S – Sniped stiffener
Tilt angle	S3	Angle between plate normal and the undeformed stiffener. Default is 0 degrees.	Any angle between -45 and 45.
In-plane support	S3, U3	In-plane membrane support. In girder mode: Loads can not be specified for free edges.	Int – Integrated panel GL - Left/right edges free GT - Upper/lower edges free
Rotational support	U3	Specifying rotational support of plate. Both numerical values and strings can be specified.	SS - Simply supported CL - Clamped Corr - Corrugated (left-right or upper-lower must be specified)

			as corrugated) Numerical values can also be specified.
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### 7.3 Buttons in input sheet

A summary of the buttons in the input sheets are tabulated below.

Button	Relevant element	Description
Clear input sheet	S3, U3	Removes all input data in sheet
Panel illustration	S3, U3	Opens a window displaying a general panel.
Import panels from file	S3, U3	Opens a pbp-file generated of either the Puls GUI or the Puls spreadsheet. Only steel panels are opened.
Calculate and export panels to file	S3, U3	All panels are calculated and written to a pbp-file.
Define corrugated panel	U3	The spreadsheet sets the necessary options to generate a corrugated panel successfully.
Calculate panel	S3, U3	Calculate all panels in the input file and writes all input and output into the output sheet. If an error occurs during the computation, an error message is written in the output sheet.
Profile table	S3	Lets the user pick profile types from a table.
Hide/Show secondary stiffener properties	S3	Hides/shows the columns for the secondary stiffener properties
Program info	S3	Displays message box containing release date and PulsComClasses-version.

### 7.4 Option buttons

There are two groups of option buttons in the input sheet:

- 1) **Row by row/ combinations of input:** These options let the user choose how the panels are generated from the input sheet. The row by row option makes a panel out of each row (one row = one panel, two rows = two panels...). The combination of input combines input from every cell in each column with every other multiple cell columns. The figure below shows an example of a “combination of input”-input. There are specified two lengths, three thicknesses and two load sets which results in 12 panels. All parameters can be combined this way. When combining loads, all loads in each row must be specified as shown in the example.





## **8 REFERENCES**

- 1 Brush and Almroth. "Buckling of Bars, Plates and Shells", McGraw-Hill 1975
- 2 Washizu, K. (1975). "Variational methods in elasticity and plasticity", Pergamon Press, Second Edition, Bath, Great Britain.
- 3 DNV RP-C201, "Buckling Strength of Plated Structures", October 2002
- 4 DNV, Rules for Classification of Ship, Part 3, Chapter 1, Sec.14, January 2000
- 5 DNV, Classification Note No.30.1, July 95
- 6 Steen, E., Østvold, T.K and Valsgård, S. "A new design model for ultimate and buckling strength assessment of stiffened plates", PRADS 2001, Shanghai, China.
- 7 Byklum, E. and Amdahl, J. "Nonlinear Buckling Analysis and Ultimate Strength Prediction of Stiffened Steel and Aluminium Panels", The Second International Conference on Advances in Structural Engineering and Mechanics, Pusan, 21-23 August 2002.
- 8 Steen, E. "Application of the perturbation method to plate buckling problems", Research Report in Mechanics, No. 98-1, ISBN 82-553-1149-1, University of Oslo, Department of Mathematics, Mechanics Division, 1998.
- 9 Steen, E. "Buckling of Stiffened Plates using a Shanley Model Approach", Research Report in Mechanics, No. 99-1, ISBN 82-553-1182-3, University of Oslo, Department of Mathematics, Mechanics Division, 1999.
- 10 Steen, E and Østvold, T. K. "Basis for a new buckling model for strength assessment of stiffened panels", DNV Seminar: Buckling and Ultimate Strength of Ship structures, 20 September 2000, Høvik, Norway.
- 11 Østvold, T. K and Steen, E. "Windows program implementation of new buckling code", DNV Seminar: Buckling and Ultimate Strength of Ship structures, 20 September 2000, Høvik, Norway.
- 12 Steen, E and Østvold, T. K. "Simplified treatment of buckling effects in global linear FE ship models using an orthotropic macro material modelling technique", DNV Seminar: Buckling and Ultimate Strength of Ship structures, 20 September 2000. Høvik, Norway.
- 13 Byklum, E. and Amdahl, J. "A simplified method for elastic large deflection analysis

- of plates and stiffened panels due to local buckling”, Thin Walled Structures, vol. 40, no. 11, pp. 923-951, 2002.
- 14 Byklum, E. “Ultimate strength analysis of stiffened steel and aluminium panels using semi-analytical methods”, Dr.ing. thesis, Department of Marine Technology, Norwegian University of Science and Technology, 2002.
  - 15 Byklum, E., Steen, E. and Amdahl, J. “A semi-analytical for global buckling and postbuckling analysis of stiffened plates”, Thin Walled Structures, vol. 42, no. 5, pp. 701-717, 2004.
  - 16 Steen, E. , Byklum, E. and Vilming, Kjetil G. “Computer efficient non-linear buckling models for capacity assessments of stiffened panels subjected to combined loads”, ICTWS 2204, Fourth International Conference on Thin-Walled structures, 22-24 June, Loughborough University, UK.
  - 17 Steen, E. , Byklum, E., Vilming, K. G. and Østvold, T. K. “Computerized Buckling Models for Ultimate Strength Assessment of Stiffened Ship Hull Panels”, PRADS 2204, Lubeck/Travemunde, Germany
  - 18 Brubak, L., Hellesland, J., Steen, E., Byklum, E. “Approximate buckling strength analysis of plates with arbitrarily oriented stiffeners”, NSCM 2004, Nordic Seminar on Computational Mechanics, KTH, Stockholm, Sweden

## NOTATION

- $\sigma_1$  - Nominal uniform stress in stiffener direction, i.e. along  $x_1$  – axis, compression positive – tension negative, unit N/mm<sup>2</sup>
- $\sigma_2$  - Nominal uniform stress in perpendicular to stiffener direction, i.e. along  $x_2$  – axis, compression positive – tension negative, unit N/mm<sup>2</sup>
- $\sigma_3$  - Nominal uniform in-plane shear stress, positive/negative, unit N/mm<sup>2</sup>
- $\tau_{12}$  Shear stress; same as  $\sigma_3$
- $\sigma_F$  - Characteristic yield stress, unit N/mm<sup>2</sup>
- $\sigma_{Fp}$  - Characteristic yield stress of plate, unit N/mm<sup>2</sup>
- $\sigma_{Fs}$  - Characteristic yield stress of stiffener, unit N/mm<sup>2</sup>
- E - Young’s modulus N/mm<sup>2</sup>
- $\nu$  - Poisson’s ratio
- G Shear module (not input),  $G = E/(2(1+\nu))$
- $\eta$  - Calculated usage factor for actual applied load combination



$\eta_{\text{allow}}$	Allowable usage factor defined in rules
$L_1$	Plate length in $x_1$ direction, stiffener span between rigid lateral supports, distance between transverse frames
$L_2$	Plate length in $x_2$ direction, total pane width for S3 (not input), $L_2 = (N_s + 1) s$
$N_s$	Number of stiffeners for S3
$N_{\text{sec}}$	Number of sniped stiffeners perpendicular to continuous stiffeners (for S3)
$s$	Stiffener spacing (for S3)
$h$	Stiffener height including flange thickness (for S3)
$t_w$	Stiffener web thickness (for S3)
$b_f$	Total flange width (for S3)
$t_f$	Flange thickness (for S3)
$t_p$	Plate thickness
$e_f$	Flange eccentricity, distance from centroid of flange to web plate middle-plane (for S3)
$D_{ij}$	Bending stiffness coefficients, macro material coefficients ref. plate middle-plane, <b>D</b>
$C_{ij}$	Membrane stiffness coefficients, macro material coefficients ref. plate middle-plane, <b>C</b>
$Q_{ij}$	Coupling bend.-memb. stiffness coeff., macro material coeff. ref. plate middle-plane, <b>Q</b>
$\tilde{D}_{ij}$	Neutral bending stiffness coefficients, macro material coefficients ref. centroid, <b><math>\tilde{D}</math></b>
$N_1$	Line load in $x_1$ -direction, unit N/mm
$N_2$	Line load in $x_2$ -direction, unit N/mm
$N_3$	In-plane shear load ( $x_1$ - $x_2$ plane), unit N/mm
$M_1$	Line moment about $x_2$ -axis, ref. plate middle-plane, unit N
$M_2$	Line moment about $x_1$ -axis, ref. plate middle-plane, unit N
$M_3$	Twisting line moment, ref. plate middle-plane, unit N
$x_1$	Cartesian in-plane co-ordinate in continuous stiffener direction, plate middle-plane
$x_2$	Cartesian in-plane co-ordinate, perp. continuous stiffener direction, plate middle-plane
$x_3$	Cartesian co-ordinate perpendicular to plate plane
$\varepsilon_1$	Normal strain in plate middle-plan in $x_1$ -direction
$\varepsilon_2$	Normal strain in plate middle-plan in $x_2$ -direction
$\varepsilon_3$	Engineering shear strain in plate middle-plan in $x_1$ - $x_2$ plane ( $\varepsilon_3 = 2 \varepsilon_{12}$ )
$\varepsilon_{12}$	Shear strain tensor in plate middle-plan in $x_1$ - $x_2$ plane
$\kappa_1$	Curvature in plate middle-plan in $x_1$ -direction, $\kappa_1 = -w_{,11}$
$\kappa_2$	Curvature in plate middle-plan in $x_2$ -direction, $\kappa_2 = -w_{,22}$
$\kappa_3$	Twisting curvature in plate middle-plan in $x_1$ - $x_2$ plane, $\kappa_3 = -w_{,12}$

$\Lambda$	Load proportionality factor, unit load factor ( $\Lambda=1$ ) corresponds to applied loads $\sigma_{10}, \sigma_{20}, \sigma_{30}$
$\Lambda_E$	Load proportionality factor at ideal elastic buckling for U3, eigenvalue
$\Lambda_{GE}$	Load prop. factor at ideal elastic buckling in global mode, global eigenvalue GEB
$\Lambda_{LE}$	Load prop. factor at ideal elastic buckling in local mode, local eigenvalue LEB
$\Lambda_u$	Load proportionality factor at ideal ultimate load
$L_0$	Load effect (= $S_d$ )
$L_u$	Characteristic resistance (= $R_k$ )
$S_d$	Load effect (= $L_0$ )
$R_k$	Characteristic resistance (= $L_u$ )
$\gamma_m$	Material factor, offshore rule notation

#### Superscripts or subscripts

L	Linear properties
N	Non-linear properties
U	Ultimate limit state
I	Initial properties for zero load
T	Transposed
0	Applied nominal stresses/loads, reference nominal stresses/loads

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