

# 1 Network Theory

This article presents the theory that is employed to perform pipe network calculations – essentially it is a look under the hood of EPA-NET. The important point is that one could make computations by-hand if needed; the quasi-linearization approach is general and the computation method can be extended to other kinds of systems (electric circuits, non-linear structures, logistics).

Pipe networks, like single path pipelines, are analyzed for head losses in order to size pumps, determine demand management strategies, and ensure minimum pressures in the system. Conceptually the same principles are used for steady flow systems: conservation of mass and energy; with momentum used to determine head losses. The following brief description of the arithmetic behind such analyses and the algorithm common to most, if not all network analysis programs is accomplished using a simple example.

Keep in mind that the network is simply an extension of the branched pipe example.

## Illustrative Example

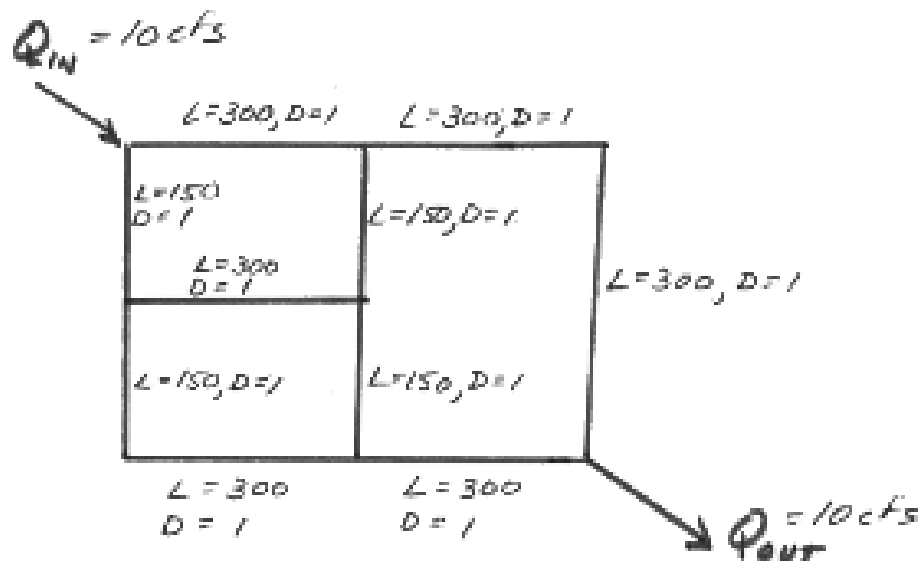


Figure 1: Pipe network for illustrative example with supply and demands identified. Pipe dimensions and diameters are also depicted.

Figure 1 is a sketch of the example problem that will be used. The network supply and

demand node<sup>1</sup> are annotated with the discharge, in this case  $10 \text{ ft}^3/\text{s}$ . **Sketch the network**  
The first step in analysis is to sketch the network

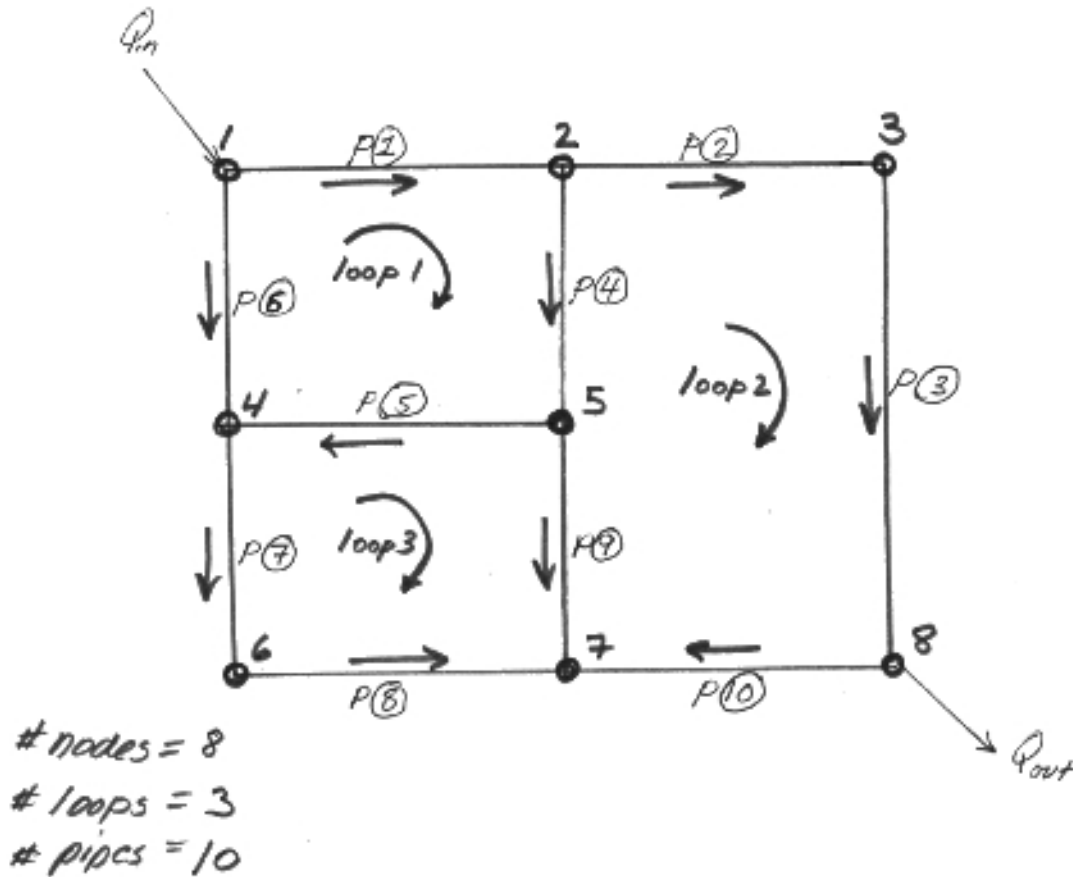


Figure 2: Pipe network for illustrative example with loops, pipes, and nodes labeled.

### Check geometry

A unique solution exists only if the sum of the node count and loop count is equal to the pipe count. In the current example this sum is 11, but the drawing shows only 10 pipes. A modeling trick is to add a fictitious pipe at either a supply or demand point to satisfy this geometric requirement. The particulars of this imaginary pipe are irrelevant as flow in this pipe should vanish at the solution.

### Prepare $f, K, Re$ Tables

The next step is to prepare tables for use in the head loss equations. In these notes, the

<sup>1</sup>A single demand is unusual, but helps with clarity in the example.

Darcy-Weisbach formula is used for head loss, thus the relevant equations for any particular pipe are:

1. The head loss coefficient (just the constant part) to be multiplied by  $|Q|Q$  to obtain loss for the pipe;

$$K = \frac{4\rho}{\pi^2 g D^5} \quad (1)$$

2. The Reynolds number coefficient, to be multiplied by  $Q$  to obtain the pipe Reynolds number for determination of friction factors.

$$\frac{Re}{Q} = \frac{8L}{\mu\pi D} \quad (2)$$

3. An the friction factor table (if variable factors are to be used); typically the Colebrook-White formula is used, but table look-up is also valid and fast. In this example fixed values will be used, so the Reynolds number component is superfluous.

The head loss in any pipe is  $H_{loss} = fK|Q|Q$

### Write the mass balance for each node and head loss for each loop

This step builds the equation system, the matrix below has two partitions, the upper partition corresponds to the nodal equations, and the lower partition to the loop equations. Notice that the lower partition will change value for any change in the discharges.

-1	0	0	0	0	-1	0	0	0	0	0
1	-1	0	-1	0	0	0	0	0	0	0
0	1	-1	0	0	0	0	0	0	0	0
0	0	0	0	1	1	-1	0	0	0	0
0	0	0	1	-1	0	0	0	-1	0	0
0	0	0	0	0	0	1	-1	0	0	0
0	0	0	0	0	0	0	1	1	1	0
0	0	1	0	0	0	0	0	0	-1	-1
$fK Q_1 $	0	0	$fK Q_4 $	$fK Q_5 $	$-fK Q_6 $	0	0	0	0	0
0	$fK Q_2 $	$fK Q_3 $	$-fK Q_4 $	0	0	0	0	$-fK Q_9 $	$fK Q_{10} $	0
0	0	0	0	$-fK Q_5 $	0	$-fK Q_7 $	$-fK Q_8 $	$fK Q_{10} $	0	0

The convention used here is that flow into a node is algebraically positive, while flow away from a node is negative. The sign convention in the loop partition is that if the loop traverse is the same direction as the assumed flow, then the sign convention is a positive loss, while a negative loss is computed for the opposite situation. The array above is a coefficient matrix dependent on discharge. This array is represented here as  $\mathbf{A}(\mathbf{Q})$ .

The discharges are written as the vector  $\mathbf{Q}$

$$\begin{matrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 = \mathbf{Q} \\ Q_7 \\ Q_8 \\ Q_9 \\ Q_{10} \\ Q_{11} \end{matrix}$$

The demand is written as the vector  $\mathbf{D}$

$$\begin{matrix} -10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 = \mathbf{D} \\ 0 \\ 10 \\ 0 \\ 0 \\ 0 \end{matrix}$$

So the resulting system of equations<sup>2</sup> are

$$[\mathbf{A}(\mathbf{Q})] \cdot \mathbf{Q} = \mathbf{D} \quad (3)$$

### Finding a Solution to the Network Equations

The network equations include the node equations<sup>3</sup> and the loop equations<sup>4</sup>. The set of equations are solved simultaneously for pipe discharge, and then these results are used to determine system pressures or other nodal quantities. The system for a pipeline network happens to be a quadratic system of equations, therefore non-linear, and therefore some adaptation of linear solvers is used.

At the “correct” solution the following matrix-vector system is true.

$$[\mathbf{A}(\mathbf{Q})] \cdot \mathbf{Q} = \mathbf{D} \quad (4)$$

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<sup>2</sup>Non-linear because of the  $|Q|Q$  term.

<sup>3</sup>These equations are statements of conservation of mass.

<sup>4</sup>These equations are statements of conservation of energy and independence of path.

In this expression,  $\mathbf{A}$  is a function of  $\mathbf{Q}$ ; therefore the coefficient matrix is a function of the solution vector — a non-linear system. If the system were a univariate non-linear equation, we would proceed by subtracting the right-hand side from the left hand side and re-expressing the entire relationship as a function of the solution variable that is supposed to equal 0.

### Newton-Raphson Method Theory

The Newton-Raphson method extends the classical Newton's method to vector valued functions of vector arguments. The first derivative of Newton's method is replaced by the Jacobian of the function, but otherwise the method is for all purposes identical to the univariate Newton's method.

Starting with the network equations, they are rewritten into a functional form suitable for a Newton's-type approach.

$$[\mathbf{A}(\mathbf{Q})] \cdot \mathbf{Q} - \mathbf{D} = \mathbf{f}(\mathbf{Q}) = \mathbf{0} \quad (5)$$

Recall from Newton's method that

$$x_{k+1} = x_k - \left(\frac{df}{dx} \Big|_{x_k}\right)^{-1} f(x_k) \quad (6)$$

thus the extension to the pipeline case is

$$\mathbf{Q}_{k+1} = \mathbf{Q}_k - [\mathbf{J}(\mathbf{Q}_k)]^{-1} \mathbf{f}(\mathbf{Q}_k) \quad (7)$$

where  $\mathbf{J}(\mathbf{Q}_k)$  is the Jacobian of the coefficient matrix  $\mathbf{A}$  evaluated at  $\mathbf{Q}_k$ . Although a bit cluttered, here is the formula for a single update step, with the matrix, demand vector, and the solution vector in their proper places.

$$\mathbf{Q}_{k+1} = \mathbf{Q}_k - [\mathbf{J}(\mathbf{Q}_k)]^{-1} \{[\mathbf{A}(\mathbf{Q}_k)] \cdot \mathbf{Q}_k - \mathbf{D}\} \quad (8)$$

The Jacobian of the pipeline model is a matrix with the following properties:

1. The partition of the matrix that corresponds to the node formulas is identical to the original coefficient matrix — it will be comprised of 0 or  $\pm 1$  in the same pattern at the equivalent partition of the  $\mathbf{A}$  matrix.
2. The partition of the matrix that corresponds to the loop formulas, will consist of values that are twice the values of the coefficients in the original coefficient matrix (at any supplied value of  $\mathbf{Q}_k$ ).

In the current example the Jacobian would look like the following array (columns and rows are abbreviated to fit the page) :

$$\begin{array}{cccccc}
 -1 & 0 & 0 & \dots & 0 & 0 \\
 1 & -1 & 0 & \dots & 0 & 0 \\
 \dots & & & & & \\
 \dots & & & & & \\
 0 & 0 & 1 & \dots & -1 & -1 \\
 2fK|Q_1| & 0 & 0 & \dots & 0 & 0 \\
 0 & 2fK|Q_2| & 2fK|Q_3| & \dots & 2fK|Q_{10}| & 0 \\
 0 & 0 & 0 & \dots & 0 & 0
 \end{array}$$

In this document, the pipeline solution is a true “Newton’s” method because analytical Jacobian values are used. If a numerical method to approximate the derivatives is used it would be called a quasi-Newton method.

As an algorithm, the engineer would supply a guess for  $\mathbf{Q}_k$ , compute the update value, use this just computed value as the new guess, and repeat the computation until the computed vector is relatively unchanging. Typically, even with a poor first guess the solution can be found in  $\approx 2 \times \text{rank}(\mathbf{A})$

This method is the basis of nearly all network models (it is even used in groundwater hydraulics and surface water networks). The method with some effort can be extended to transient systems<sup>5</sup>.

### Spreadsheet Example

Figure 3 is an image of a spreadsheet model to implement such calculations. The purpose here is to illustrate a bit of the layout, the spreadsheet itself required manual recalculation, and iteration counter, and reasonably intricate updating and linear algebra operations. All the pieces needed are part of a spreadsheet system and an actual spreadsheet will be illustrated in class.

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<sup>5</sup>In a transient solver, there would be a set of iterations per time-step, hence the method is used over-and-over to evolve forward in time. In a transient case, analytical derivatives would be extremely desirable, but if geometry changes as in open channel cases, the programs usually sacrifice speed and use numerical approximation of the Jacobian at each time step.



## 1.1 Readings

1. [http://en.wikipedia.org/wiki/Newton's\\_method](http://en.wikipedia.org/wiki/Newton's_method).
2. [http://en.wikipedia.org/wiki/Pipe\\_network\\_analysis](http://en.wikipedia.org/wiki/Pipe_network_analysis).

## 1.2 Forces and Stresses on Pipes

Pipes are structural elements that are subjected to internal and external forces. The internal forces are pressure and momentum transfer (when direction changes), and cavitation when the liquid pressure is so low that the liquid phase changes (switches between gas and liquid rapidly). The external forces are the restraining forces to counteract momentum changes, crushing forces of loads outside the pipe (including air pressure for negative pressure pipelines), and thermal forces as temperature of the surroundings change.

### 1.2.1 Forces on Bends and Transitions

Momentum is used to calculate the forces in bends and transitions. These forces are computed to design thrust blocks and connections to keep a pipeline fixed in space (instead of flopping around like a garden hose with a nozzle).

For a bend, the momentum equation has at least two components if the plane of the bend is perpendicular or parallel to the gravitational acceleration direction, three otherwise. The momentum equations are

$$\begin{aligned}\sum F_x &= \rho Q(V_{2x} - V_{1x}) \\ \sum F_y &= \rho Q(V_{2y} - V_{1y}) \\ \sum F_z &= \rho Q(V_{2z} - V_{1z})\end{aligned}\tag{9}$$

The subscripts 2 and 1 refer to the downstream and upstream locations (flow from 1 to 2). Usually the third equation includes a body force (gravity) in the force summation.

### Example

Figure 4 depicts a section of pipe with a direction change. The horizontal bend is  $30^\circ$ . The 1-meter diameter pipe carries water at  $3\text{m}^3$  per second. The pressure in the bend is approximately 75 kPa (gage), and the volume of the bend is  $1.8\text{m}^3$ . The bend itself (metal) weighs 4kN. What forces are applied to the bend by the anchor (thrust block) to hold the bend in place? Assume the pipe walls carry no force along their axis (i.e. expansion joints that cannot transmit substantial longitudinal load).



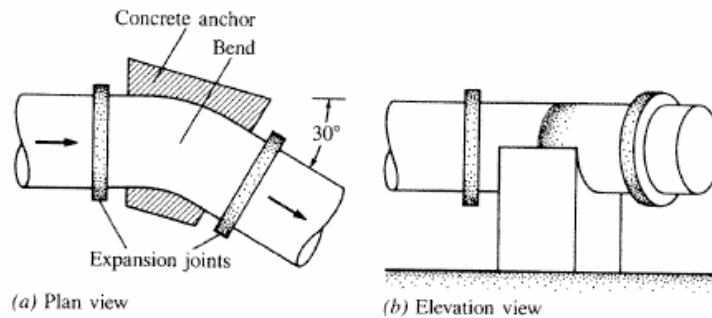


Figure 4: Pipe with direction change. The physical restraint that holds the pipe is called a thrust block.

*Solution*

Apply the momentum equations to the control volume between the expansion joints, as depicted in Figure 5.

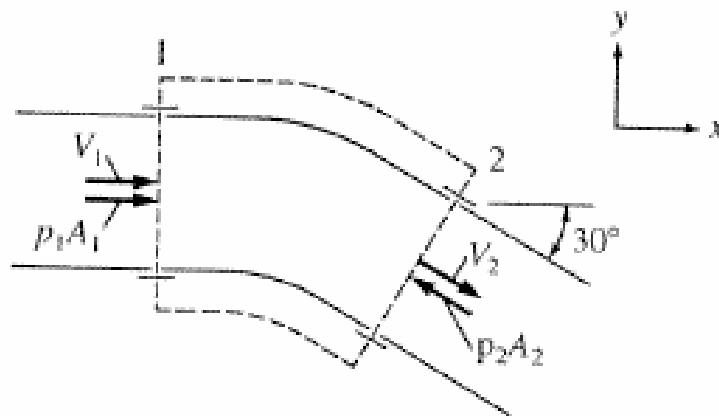


Figure 5: Control volume (FBD) of pipe bend. z-axis is perpendicular with plane of the figure.

First the x-component of the momentum equation is

$$p_1A_1 - p_2A_2\cos(30^\circ) + F_{thrust,x} = (1000\text{kg/m}^3)(3\text{m}^3/\text{s})((Q/A_2)\cos(30^\circ) - (Q/A_1)) \quad (10)$$

The two pressures are equal as is are the cross sectional areas so the equation reduces o

$$75,000\text{Pa} A(1 - \cos(30^\circ)) + F_{thrust,x} = (1000\text{kg/m}^3)(3\text{m}^3/\text{s})((Q/A)(\cos(30^\circ) - 1)) \quad (11)$$

The cross sectional area is  $A = (\pi D^2/4) = 0.785m^2$ , so solving for the force of the thrust block gives

$$F_{thrust,x} = -75,000Pa \cdot 0.785m^2(1-0.866) + (1000kg/m^3)(3m^3/s)((3.31-3.82)m/s) = -9,420N \quad (12)$$

Thus the thrust block pushes to the left on the pipe.

Repeating a similar analysis for the y-component of momentum is

$$F_{thrust,y} = -75,000Pa \cdot 0.785m^2(\sin(30^\circ)) + (1000kg/m^3)(3m^3/s)(-3.82\sin(30^\circ)m/s) = -35,170N \quad (13)$$

Finally the z-component is considered to find the vertical force the thrust block must provide to hold the system in place against its weight.

$$\sum F_z = W_{bend} + W_{water} + F_{anchor,z} = (1000kg/m^3)(3m^3/s)(0) \quad (14)$$

Solving for the anchor force produces

$$F_{anchor,z} = -W_{bend} - W_{water} = -(-4000N) - (1.8m^3)(-9800N/m^3) = 21,600N \quad (15)$$

Recall these are all vector components, so direction (up/down; left/right) matters. So the resultant force the block must be able to transmit to the bend is  $\mathbf{F}_{thrust} = -9,420\mathbf{i} - 35,170\mathbf{j} + 21,600\mathbf{j}N$ .

Once the forces are understood, then we could design the block foundation (recall we need to include the dead weight of the block itself!).

Transitions are analyzed in nearly identical fashion, again an example will illustrate the analysis.

### Example

Water flows through the pipe diameter reducer shown in Figure 6 at a rate of 25 cubic-feet-per-second. The minor loss coefficient of this fitting is 0.20, based on velocity in the smaller pipe. What longitudinal force (from a thrust block) is required to hold the reducer in place? Upstream pressure is 30 psi (gage).

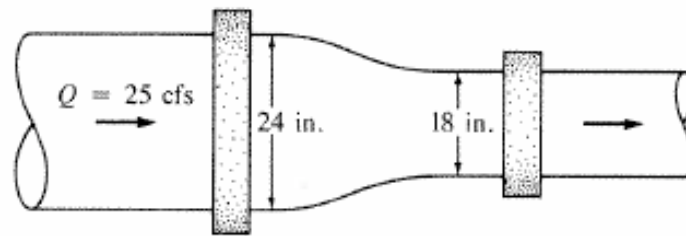


Figure 6: Pipe reducer (transition) diagram.

*Solution*

First sketch a control volume (FBD) to identify momentum terms and such, as in Figure 7. Again we assume the pipes are connected by zero-force joints, so the external force is applied to only the reducer.

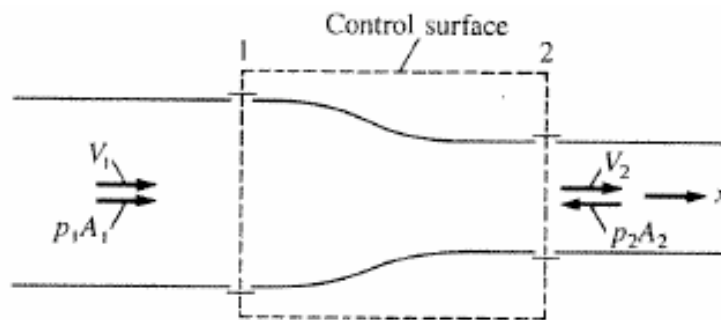


Figure 7: Control volume (FBD) of transition.

Next use the energy equation to find the downstream pressure

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + 0.20 \frac{V_2^2}{2g} \quad (16)$$

Substituting in numerical values, and solving for  $p_2$  we obtain  $p_2 = 4147 \text{ lb}/\text{ft}^2$  (note the units!).

Now we apply the momentum equation to find the required restraining force

$$\sum F_x = \rho Q (V_{2x} - V_{1x}) \quad (17)$$

Substitute in values

$$(4320lb/ft^2)(\pi(2)^2/4) - (4147lb/ft^2)(\pi(1.5)^2/4) + F_{anchor,x} = (1.94slug/ft^3)(25ft^3/s)(14.15 - 7.96ft/s) \quad (18)$$

Solving for the anchor force one obtains  $F_{anchor,x} = -5943lb$

To conclude, the forces on fittings are analyzed by momentum. Remember that the pressures matter as does gravity in the vertical direction. Generally one solves for the forces of the anchor on the control volume, and then uses these forces in later parts of the design to design foundations to resist the hydraulic forces.

### 1.2.2 Cavitation

Cavitation occurs in flowing liquids when the flow passes through a zone of low pressure near the vapor pressure of the liquid. Small bubbles of vapor are formed, then when returned to a liquid, release substantial energy that damages the pipe and equipment.

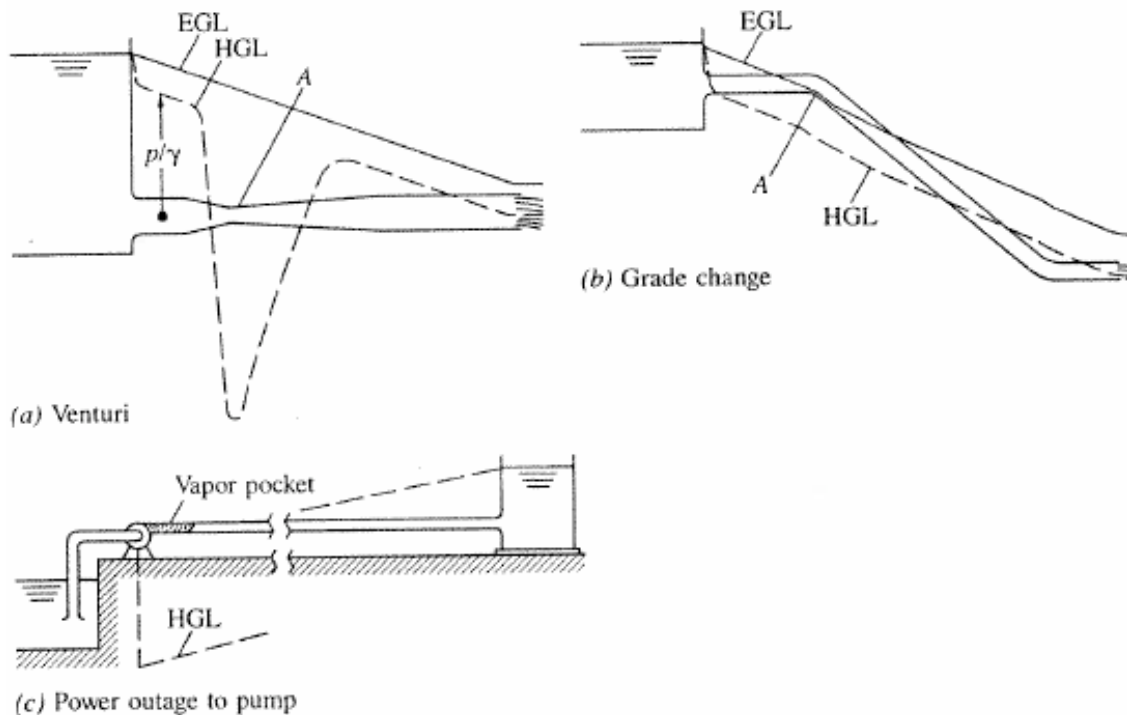


Figure 8: Typical locations of cavitation in hydraulic systems. The cavitation in pumps is problematic and is the reason the engineer makes the Net-Positive-Suction-Head available ( $NPSH_a$ ) computations.

Good hydraulic design practice is to avoid negative (gage) pressures in a system, and where necessary, keep these negative pressures greater than -10 feet of head. If the pressure head gets at all close to -33 feet of head, cavitation is almost assured<sup>6</sup>. The choice of -10 feet is somewhat arbitrary, but is intended to serve as a guideline (with a reasonable factor of safety). The conditions need to be checked at all operating conditions (changes in flow, start-up, and shut-down).

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<sup>6</sup>i.e. If the hydraulic grade line is at an elevation 33 feet below the pipe system cavitation will occur

### 1.2.3 Internal Pressure

Thin walled pipes are examined using a hoop-tension analysis.

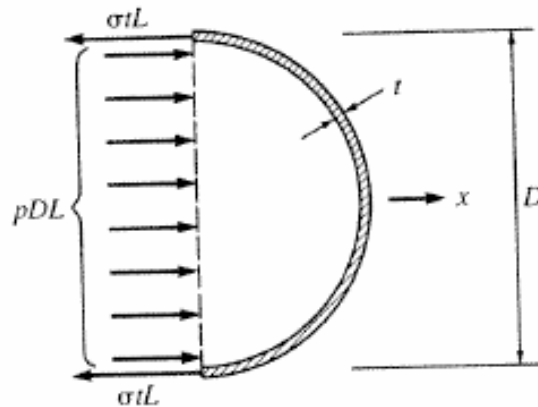


Figure 9: Thin wall pipe, free-body-diagram.

A force balance on the ring section in Figure 9 produces the following formula for internal (pipe) stress

$$\sigma = \frac{pD}{2t} \quad (19)$$

where  $t$  is the pipe thickness.

Thick walled pipes consider that the internal stress is non-uniform and the formula becomes

$$\sigma = \frac{pr_i^2}{r_o^2 - r_i^2} \left(1 - \frac{r_o^2}{r_i^2}\right) \quad (20)$$

Once the internal stress is known (based on geometry and pressure) then materials that do not yield under such stresses can be selected.

### 1.2.4 Temperature Stress and Strain

Temperature stresses develop when temperature changes occur in pipes, often between installation, and service, but also in service. If a pipe is restrained and subjected to a temperature change, then the pipe is subjected to an equivalent longitudinal deflection of

$$\Delta L = L\alpha\Delta T \quad (21)$$

The term  $\alpha$  is the coefficient of thermal expansion and is tabulated for different materials.

If the equivalent deflection is converted into a stress (i.e. via a stress-strain relation) the result is

$$\sigma = E\alpha\Delta T \quad (22)$$

where  $E$  is the elastic modulus of the material. Most real systems use expansion fittings (joints) that move slightly to accommodate thermal changes, although long plastic sewer pipes are often welded (solvent or ultrasonic) and do not have expansion fittings.

Thermal issues would be important in above-ground large-diameter pipe systems and in systems that carry heated liquids (e.g. crude oil pipelines).

### 1.2.5 External Loading

Most pipes are placed in a trench and buried. These pipes need to be able to resist internal loads (pressure), possible thermal loads (temperature), and the external load of the backfill and any additional overburden anticipated.

Trench and backfill design is largely empirical because the systems are complex to analyze.

Figure 10 shows a pipe with three different bedding conditions (these are tabulated in CITE). The load on the pipe is equal to the weight of the soil above the pipe, plus any added weight above the soil, less the shear force (like a skin friction) between the trench wall and the backfill.

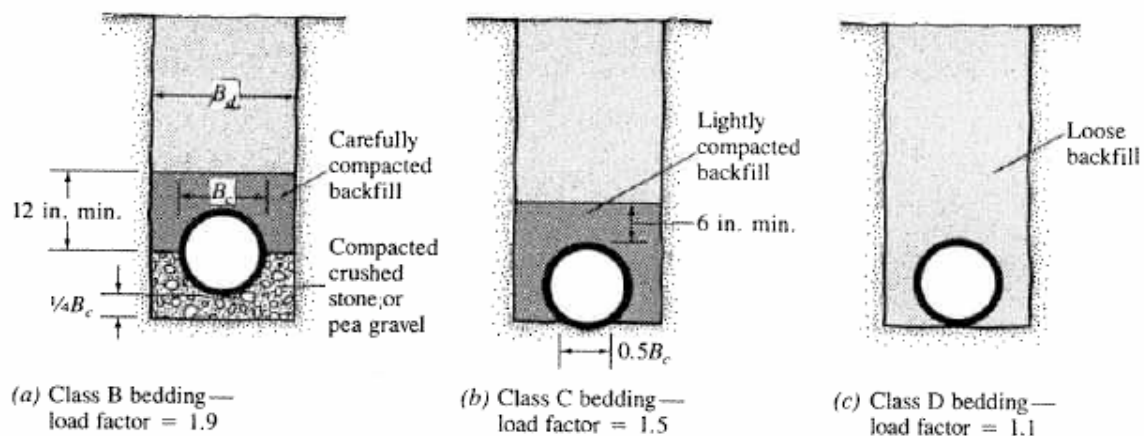


Figure 10: Different trench bedding conditions (from CITE)

The load on the pipe per unit length is  $W = C\gamma_{soil}B^2$  where  $\gamma_{soil}$  is the unit weight of the soil,  $B$  is the trench width, and  $C$  is a coefficient that is a function of fill height and soil type.

Figure 11 is a chart that relates the  $C$  factor to different soil types, and depth to width ratios. Once the load is computed, then the engineer can select a pipe material and thickness to resist the anticipated load. Observe that changing the backfill can have an effect of performance by lowering the  $C$  factor (i.e. backfill in a saturated clay with cement stabilized pea-gravel in a deep trench could reduce the load factor by nearly one — allowing a thinner wall pipe to serve).

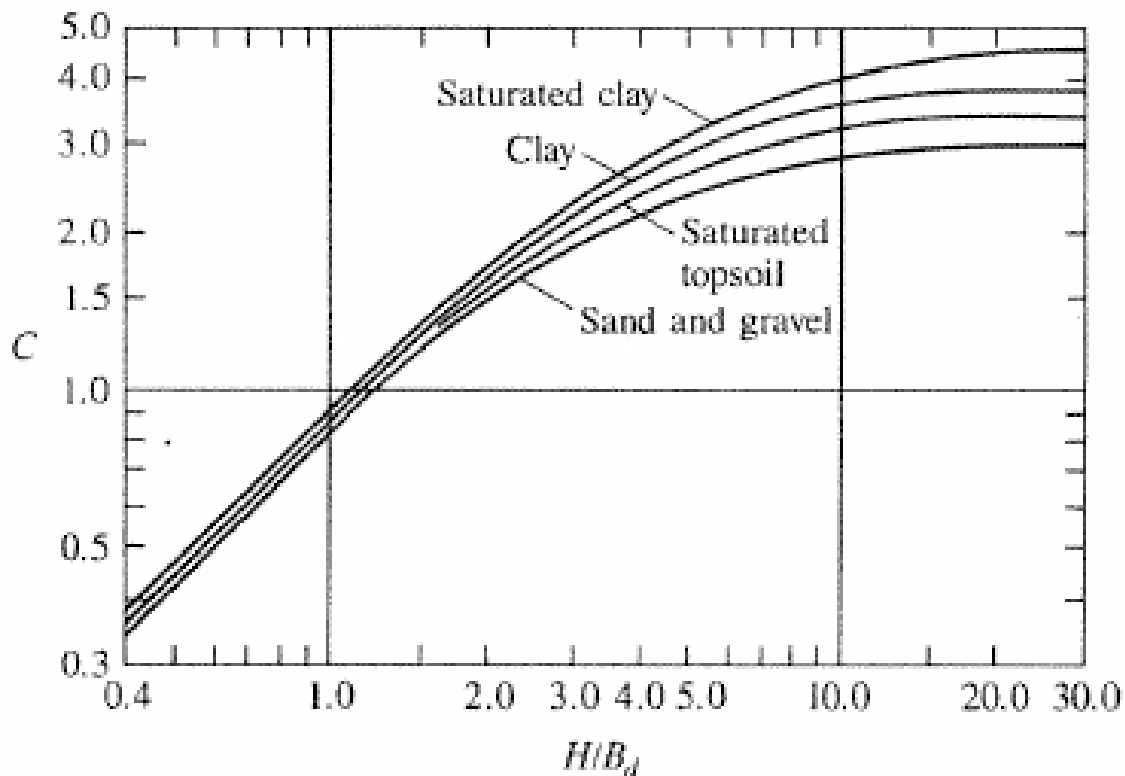


Figure 11: Load factors for trench design (from CITE)

The typical formula is called Marston's general formula for loads on buried objects (CITE). If the pipe is rigid, the formula is

$$W = C \gamma_{soil} B_d^2 \quad (23)$$

where  $B_d$  is the trench width. The coefficient  $C$  is taken from a soil description and either a tabulation or a chart such as Figure 11.

If the pipe is flexible the formula is modified

$$W = C \gamma_{soil} B_c B_d \quad (24)$$



where  $B_c$  is the diameter of the conduit. If the trench is wide compared to the pipe (say two pipe diameters) then the formula will over estimate the applied load and more extensive (than here) geotechnical considerations are applied. These formulas estimate only the soil load, added live loads (trucks, railroads, etc.) are added to these applied loads to examine the anticipated load on the buried pipe.

### 1.2.6 Rigid Pipe Strength

Rigid pipe strength is determined using some variation of a three-point bearing test as depicted in Figure 12, possibly to failure of the testing machine has the capacity. Like a concrete compression test the instrument records loading rate and deformations.

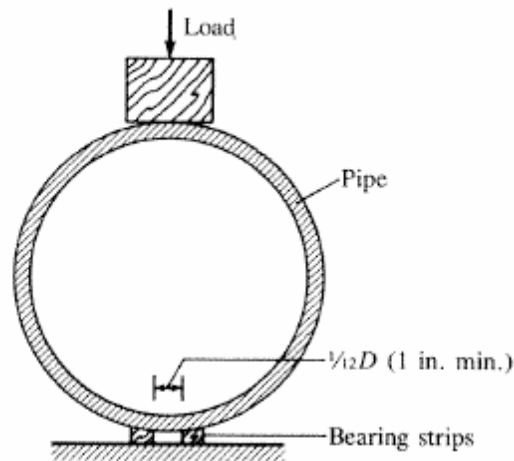


Figure 12: Three-point bearing test. The test is identical to tests you performed in the materials laboratory.

The results of such tests are tabulated by manufacturers and used by the design engineer to select materials.

Such strength values are used with a factor of safety to support the external loads as expressed in Equation 25

$$L_{safe} = \frac{L_{3-edge} F_{load}}{F_{safety}} \quad (25)$$

where  $L_{safe}$  is the safe design load (per unit length),  $L_{3-edge}$  is the material strength from a 3-edge test (manufacturer supplied).  $F_{load}$  is the load bedding factor (from Figure 10) and  $F_{safety}$  is the factor of safety (an expression of uncertainty).

Flexible pipes are treated in a similar fashion and design guidance is provided in Manuals of Practice and from manufacturer tests.

## 1.3 Pressure Pipe Materials

The common pressure pipe materials are steel, concrete, ductile iron, plastic (ABS and PVC). Less common, but still available are vitrified clay, and asbestos-cement. Lead, wood, stone and brick pipes are obsolete and except for a historical installation would not be used. Lead would be unavailable in the US.

### 1.3.1 Steel

Steel pipe in civil engineering use is medium carbon content. Stainless would be unusual in a pressure-pipe system (but is available). Typical recommendations are the use of a tensile stress for design of steel pipe to be equal to 1/2 the yield stress (factor of safety of at least 2).

Corrosion is an issue and coatings are used to protect the pipe as is cathodic protection. Steel would probably be considered a quasi-flexible pipe and backfill would be critical.

### 1.3.2 Ductile Iron

Ductile (grey) iron is common in civil engineering. It is used for force mains in sewage as well as water supply lines. Joints are important, and the material can be coated with epoxies for corrosion control. It is a rigid material and can tolerate poor backfill conditions somewhat better than flexible materials.

Figure 13 lists some important material properties. Even though tolerant of poor backfill, good backfill design should be used if at all possible.

### 1.3.3 Concrete

Concrete pipe is common in civil engineering both as a pressure pipe and as unpressurized drainage pipes (culverts, sewers, etc.). When used unpressurized, corrosion is an issue, especially with sewage where microbial induced concrete corrosion (MICC) can destroy the pipe crown in as few as two years.

Concrete is a rigid material; Figure 14 is a typical table of strengths for different bedding conditions.

**Table 3: Mechanical properties of pipes, fittings and accessories**

Type of casting and material <sup>1)2)</sup>	Minimum tensile strength MPa	Minimum ring crush strength MPa	Maximum Brinell hardness HB
Pipes			
- grey cast iron	200	350 <sup>3)</sup>	260
- spheroidal graphite cast iron	420	--	230
Fittings and accessories			
- grey cast iron	150	--	260
- spheroidal graphite cast iron	420	--	250
1) Other types of cast iron shall satisfy the criteria laid down for grey cast iron. 2) Tensile and ring crush strength for other products see annex A. 3) 332 MPa for nominal sizes equal to or greater than DN 250.			

Figure 13: Ductile iron properties (from CITE)

#### 1.3.4 Plastic

Plastic (PVC) pipes are flexible and the concept of “crush” is somewhat exaggerated. Surely PVC can be crushed, but the failure is considerably more ductile than iron or concrete. Figure 15 is a photograph of PVC pipe in a compression test frame. Observe the large deformation of the material before failure. If the material is buried and backfilled correctly, the deformation would be opposed by the compression of backfill at the springline of the pipe.

CONCRETE PIPE CULVERT CRUSHING STRENGTH (LBS. PER LIN. FT. ULTIMATE STRENGTH, OR CLASS)							PC - 1
DIAMETER (IN.)	AREA (SQ. FT.)	METHOD A BEDDING MAXIMUM HEIGHT OF COVER IN FEET				DIAMETER (IN.)	
		STRENGTH OR CLASS					
		NON REINF.	III	IV	V		
12	0.8	1800 (14')	14'	19'	29'	12	
15	1.2	2125 (14')	14'	19'	29'	15	
18	1.8	2400 (14')	14'	20'	29'	18	
21	2.4	2700 (13')	14'	20'	29'	21	
24	3.1	3000 (13')	14'	20'	29'	24	
27	4.0		14'	20'	29'	27	
30	4.9		14'	20'	29'	30	
33	5.9		14'	20'	29'	33	
36	7.1		14'	20'	30'	36	
42	9.6		14'	21'	30'	42	
48	12.6		14'	21'	30'	48	
54	15.9		14'	21'	30'	54	
60	19.6		14'	21'	30'	60	
66	23.8		14'	21'	30'	66	
72	28.3		14'	21'	30'	72	
78	33.2		14'	21'	30'	78	
84	38.5		14'	21'	30'	84	
90	44.4		14'	21'	30'	90	
96	50.3		14'	21'	30'	96	
102	56.7		14'	21'	30'	102	
108	63.6		14'	21'	30'	108	

Heights of cover shown in table are for finished construction.  
 To protect pipe during construction, minimum heights of cover prior to allowing construction traffic to cross installation are to be 2' or 3.0' whichever is greater. This cover shall extend the full length of the pipe culvert. The approach fill ramp is to extend a minimum of 10(Dia.+3') on each side of the culvert, or to the intersection with a cut.  
 Minimum finished height of cover to be  $\frac{D_{ia}}{2}$  or 2.0' whichever is greater, except pipe under entrances and median crossovers where a 9' min. will be permitted.

Sheet 1 of 17

SPECIFICATION REFERENCE	<b>CONCRETE PIPE</b> <b>CLASS TABLE FOR H-20 LIVE LOAD</b> VIRGINIA DEPARTMENT OF TRANSPORTATION	107.05
302 232		

Figure 14: Concrete pipe material properties.

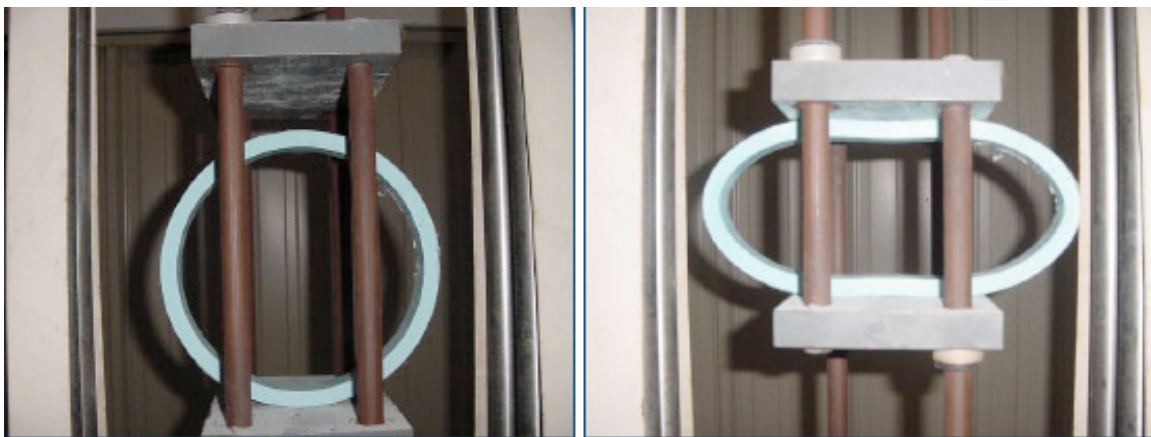


Figure 15: Three-point bearing test on flexible pipe.

<b>General</b>		
Physical Properties of PVC Pipe	Value	Test Method
<b>GENERAL</b>		
Cell Classification	12454	ASTM D1784
Maximum Service Temperature	140°F	
Color	white, dark gray	
Water Absorption % increase 24hrs @ 25°C	.05	ASTM D570
Hardness, Rockwell	110-120	ASTM D785
Poisson's Ratio @ 73°F	.410	
Hazen-Williams Factor	C=150	

<b>Mechanical</b>		
Physical Properties of PVC Pipe	Value	Test Method
<b>MECHANICAL</b>		
Specific Gravity (g/cu,cm)	1.40 ± .02	ASTM D792
Tensile Strength, psi @ 73°F	7,450	ASTM D638
Modulus of Elasticity, psi @ 73°F (Tensile Modulus)	420,000	ASTM D638
Flexural Strength, psi @ 73°F	14,450	ASTM D790
Compressive Strength, psi @ 73°F	9,600	ASTM D695
Izod Impact, ft-lb./in. @ 73°F	.75	ASTM D256

Figure 16: Plastic (PVC) properties (from CITE)

## References

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