

Rocket Lab

1 Introduction

A Severe Strain on the Credulity

As a method of sending a missile to the higher, and even to the highest parts of the earth's atmospheric envelope, Professor Goddard's rocket is a practicable and therefore promising device. It is when one considers the multiple-charge rocket as a traveler to the moon that one begins to doubt . . . for after the rocket quits our air and really starts on its journey, its flight would be neither accelerated nor maintained by the explosion of the charges it then might have left. Professor Goddard, with his "chair" in Clark College and countenancing of the Smithsonian Institution, does not know the relation of action to re-action, and of the need to have something better than a vacuum against which to react . . . Of course he only seems to lack the knowledge ladled out daily in high schools.

New York Times Editorial, 1920

The problem of the flight of a rocket is an interesting problem for both practical and more theoretical reasons. The flight of a rocket can be pursued at several different levels theoretically. The simplest level is a basic constant force problem using Newton's Laws:

$$\sum \vec{F} = m\vec{a}. \quad (1)$$

At this simple level we would only deal with two forces – gravity and rocket thrust. In the case of vertical flight this leads to:

$$T - mg = ma = m \frac{dv}{dt} = m \frac{d^2y}{dt^2} \quad (2)$$

where T is the thrust. Assuming constant thrust, solution of this differential equation is trivial:

$$a = \frac{T}{m} - g \quad (3)$$

$$v = v_0 + at = v_0 + \left(\frac{T}{m} - g\right)t \quad (4)$$

$$y = y_0 + \frac{1}{2}at^2 = y_0 + \frac{1}{2}\left(\frac{T}{m} - g\right)t^2. \quad (5)$$

This solution is appropriate in cases such as in when the rocket's mass is almost constant and air resistance is negligible, but neither of these conditions will hold for this lab.

In order to analyze our model rockets we will have to deal with at least three complicating factors. In our rocket engines, thrust as a function of time will be much closer

parabolic than constant, so we will have to deal with varying the thrust. Next, the mass of the rocket will change as it burns off its propellant, so we will not be able to use a constant mass in our equations. Finally, the air resistance will add another force to our equation of motion.

1.1 Time-Varying Thrust

The simplest time-varying thrust that we can consider is essentially a square wave – the rocket engine is on at some constant value until it burns out. While this is not a good approximation to our rocket, it is a useful case to consider because it shows some features that will also appear in our more realistic thrust curves. The on/off character of this type of thrust curve leads to the technique of splitting the problem in two. First, solve for the rocket’s motion for the case of constant thrust, using the equations 3–5. Then after the rocket engine quits, start with initial conditions based on when the engine stopped and solve using the same equations with a thrust of 0.

In order to deal with thrust curves that are more realistic than square waves, we need to come up with expressions for thrust as a function of time: $T \rightarrow T(t)$. We will find the thrust curves in two ways: by using the manufacturer’s published curves (Section 2.2) and experimentally using a force meter (Section 2.3).

1.2 Time-Varying Mass

Since we will not be able to directly measure how the mass of the rocket engine changes during the rocket’s flight, we need a proxy that will allow us to deal with mass as a function of time ($m \rightarrow m(t)$). The obvious choice is to assume that for each unit of mass that is lost a constant amount of thrust is derived:

$$-\frac{dm_e}{dt} \propto T. \quad (6)$$

or

$$\frac{dm_e}{dt} = -\alpha T(t), \quad (7)$$

where α is a proportionality constant. With this idea we can use the initial (m_{ei}) and final (m_{ef}) mass of the rocket engine along with our thrust curve to get $m_e(t)$. Solving this differential equation we get:

$$m_e(t) = m_{ei} - \alpha \int T(t)dt \quad (8)$$

or after the engine is exhausted:

$$\alpha = \frac{m_{ei} - m_{ef}}{\int T(t)dt} \quad (9)$$

The quantity $\int T(t)dt$ is known as the impulse, and it is provided by the engine manufacturer, though it also be calculated directly from the thrust function. So we can find α using the equation 9 and then use it in the equation 8 for the time varying mass, $m_e(t)$.

1.3 Air resistance

Air resistance is a complicated subject and for most real problems it cannot be dealt with analytically. Typically one of a variety of approximations is applied depending upon which fluid regime the problem falls under [Marion and Thorton, 1995]. For this lab we will use :

$$f(t) = -bv(t)^2 \quad (10)$$

where here “-” means that the drag force is in the opposite direction from the direction that the rocket moves and b is the drag constant. Other common forms for the drag force depend on the speed to the first power. The constant b depends on the fluid the rocket is moving through and the shape of the rocket:

$$b = \frac{1}{2}\rho c_d A \quad (11)$$

where ρ is the density of air, c_d is the drag coefficient, and A is the surface area of the cross-section of the rocket. We will measure the cross-sectional area, and the density of air can be determined from the ideal gas law and the weather conditions at the time of the flight of the rocket. The drag coefficient depends on the shape of the rocket and the smoothness of its surface. The drag coefficient is one of the major unknown quantities that we will determine in this lab. Values for this quantity should end up being somewhere between 0.2 and 2.

Since the form for the drag force law changes as the speed of an object changes, the drag coefficient is not constant over wide speed ranges. In this lab we will first use one type of rocket engine to determine the drag coefficient. With another type of rocket engine we will then use the drag coefficient as a known value, and predict the maximum height of the rocket.

1.4 Combined Equation

Finally, we get to an equation including time-varying thrust, time-varying mass, and air resistance

$$a(t) = \frac{T(t)}{m(t)} - g - \frac{bv(t)^2}{m(t)} \quad (12)$$

This is the equation that we will solve to find the drag coefficient for the rocket.

2 Experimental Procedure

2.1 Building your Rocket

Follow the instructions included with the rockets for putting your rocket together. Sand and paint your rockets in order to have a smooth surface that minimizes drag.

1. Does painting your rocket cause have any positive or negative effects on the range of the rocket? If so, try to estimate these effects.

2.2 Finding Thrust Function from Manufacturer’s Thrust Curves

In order to use the manufacturer’s thrust curves in data analysis, the curves must be converted into a numerical form. To do this we will convert the continuous curve into

discrete thrust versus time values using a computer program available in room 118. The program allows you to calibrate and then trace out a graph in order to convert the plot into data points. Use this program to get thrust function data for both types of engines that will be used in this lab.

To use this program log into the Unix dummy terminal next to the digitizing board. After logging in, you may have to enter the terminal type which is “vt100”. Tape your thrust plot to the board so that it won’t move while you are taking data. The program to use is called “digitizer.” Start the program by typing its name. Hit the “0” button on the cursor/mouse whenever you need to mark a location. For each axis in turn, choose a linear scale and then calibrate the digitizing board by choosing minimum and maximum locations and enter the numerical values for those locations. Then the program will have enter you a name and comment for your data file. You will then trace out the thrust curve with the cursor, taking data points at intervals that you find appropriate. More points should be taken when the graph is curved, fewer when its straight. You should include x- and y-error bars for your data based on the thickness of the thrust curve line. This will require you to mark three locations for each data point: the data point itself, a high or low y-value that will set the y-error, and a high or low x-value which will set the x-error. When you are done collecting data, press “Control-D” to stop the program. You should then plot your data using a program of your choosing to check that your curves match the manufacturers curves. Include these plots in your lab notebook.

1. If we could assume that the manufacturer’s thrust curves were free of error, we would still have uncertainty in our thrust versus time values because of the method that we are using to obtain our data from the curves. What factors limit our ability to convert the curves into numerical data?
2. Estimate the uncertainty due to these limitations. How does this uncertainty compare to the variations between engines due to the manufacturing process? You may be better able to answer these questions after firing some of the engines.

2.3 Experimental Determination of Thrust Curves

Some groups will determine the rocket engines thrust curves experimentally. Check with your instructor to see if your group should do this section and if so which engines you should use.

Thrust curves will be determined using a Pasco force meter connected to a computer. See the pictures on the web for one possible way to set up this equipment. The equipment should be setup under a hood to minimize the smoke dispersal through the building. Calibrate the force meter with known masses. Have your instructor check your setup before firing the rocket engines.

Verify the thrust curve by firing three engines of a given type. Plot your three data curves. Include these plots in your lab manual.

1. How much variation is there between the curves from the three trials? How much of this variation do you think is due to the instruments used to measure the thrust curves and how much is due to intrinsic variations in the rocket engines?
2. How well do your thrust curves compare to the manufacturer’s curves? Are the differences between them systematic or random?

Based on your results in this section and the previous section, decide whether you are going to use the data from the manufacturer’s thrust curves or your experimental

data to model the thrust of the rockets. Justify your decision. If you decide to use your experimental data, then you should come up with an experimental thrust curve that combines your trials and an estimate of the uncertainty for it.

2.4 Rocket Launches

At least four people will be needed to complete the rocket launches, so you will need to coordinate with other groups to schedule your launches. If no one in your groups has done a launch previously, then your instructor will want to go with to help you set up.

Rocket engines are described by names such as A8-3. The letter in the name stands for the total impulse of the rocket, with A engines having total impulse of up to $2.5 \text{ N}\cdot\text{s}$, with each subsequent letter having up to twice the total impulse (B has $5 \text{ N}\cdot\text{s}$, etc.). The first number in the name is the rockets average thrust in N. Finally, the last number is the delay time in seconds between the burnout of the rocket and the ejection of the parachute. In practice, this delay time seems to vary quite a bit from the published values.

Do three rocket launches with A engines and use this data to determine the drag coefficient of your rocket. Also, do three launches with B engines and compare this data to predictions based on the drag coefficient from the A engine launches.

Several measurements will be needed for each rocket launch including the apogee (maximum altitude) of the rocket, the flight time till apogee and the initial and final mass of the rocket engine. Note that you will also need the total mass of the loaded rocket for the calculations — the engine masses are just used to find how the rocket mass changes. As described above, you will also need to determine the cross-sectional area of the rocket. Finally, you will need some weather conditions for each time that you launch, in order to determine the density of air. The name of individual measuring each piece of data should be recorded as well as the data itself in order to aid in the isolation of any systematic error.

2.4.1 Apogee Time

While doing rocket launches, anyone (including the rocket launcher) not making angle measurements for the geometrical apogee determination should measure the time to apogee. Time to apogee can be difficult to determine, so having several people measure it and taking a mean is advantageous.

These rockets have an explosive charge which will deploy their parachutes. The second number on the rocket engine name is the number of seconds until the charge is supposed to detonate, but for real rockets the time till the charge varies wildly from this number. During a flight, depending on the engine and the mass of the rocket and the performance of the engine, the apogee will happen two different ways. Either the rocket's motion will turn over before the parachute pops out, or the apogee will occur when the parachute pops out. If the motion turns over, determine the apogee based on when the rocket is at its highest height. If the parachute comes out first, mark the apogee time as when the parachute comes out since the rocket's upward motion will be halted by the parachute. You should keep this behavior in mind when comparing your experimental results to the theoretical calculations, since the theoretical calculations do not account for the possibility of the parachute coming out early.

2.4.2 Apogee Determination

Two methods will be used to determine the maximum altitude of the rocket: geometrical and electronic. The geometrical method relies on multiple observers determining the angle the rocket makes with respect to the horizontal at its maximum height. The electronic method relies on an altimeter that measures pressure changes that occur as the rocket rises in order to find its altitude.

Electronic Apogee Determination

For the electronic method we will be using the PerfectFlite Alt15K altimeter. Note that the altimeter will likely not function properly when the temperature is below 0 °C. After each flight the data for that flight will have to be uploaded to a computer from the altimeter since the altimeter can only store one set of data at a time. The altimeter also reports the apogee altitude as a series of beeps after the flight. Detailed instructions for using the data retrieval program are included in the manual [*Perfect Flite*, 2003].

In order to use the altimeter you will have to install the payload section on your rocket and include the altimeter inside the payload section. Make sure to secure the payload and altimeter section to the rest of your rocket, so that it all descends as one piece. After the flight connect the altimeter to the computer using the Serial/USB cable and extract the data. Be sure to save the data with a name that you will be able to remember later on.

Geometrical Apogee Determination

The simple method of geometrically determining the apogee of the rocket only requires the determination of one angle and one distance. An observer measures their distance from the launch site and then measures the angle the rocket is above the horizontal at its apogee. In this case the altitude is

$$h = x \tan \theta. \tag{13}$$

This method's accuracy is limited by the need for the rocket's launch to be perfectly vertical, but it is still a good first approximation to finding the altitude. For each flight you should use this method, as well as the one described below to determine the altitude. Note that this method gives the most accurate results when θ is near 45°.

Figure 1 shows a more reliable method of determining altitude. In this method only the distance d , which is the distance between the two observers making the measurements, is required. As shown in the figure only the three angles a , b , and c are required. These angles are the horizontal angles (b and c) between the line connecting the two observers and the line of the rocket's flight. The other angle (a) is the vertical angle between the ground and the rocket's location. With this method one observer could measure one angle while the other measures two angles. To make things more robust we will have both observers measure both the horizontal and vertical angles. This will allow us to make two altitude calculations with the same set of data – one set where both angles are used from one observer and the other set where both angles are used from the other observer. Using these measurements the height of the rocket is

$$h = \frac{d \tan a \sin c}{\sin (b + c)} \tag{14}$$

[*Benson*, 2003]. Beware that this formula assumes that all angles are less than 90 degrees. If your angles are larger, then you will have to make appropriate transformations. When

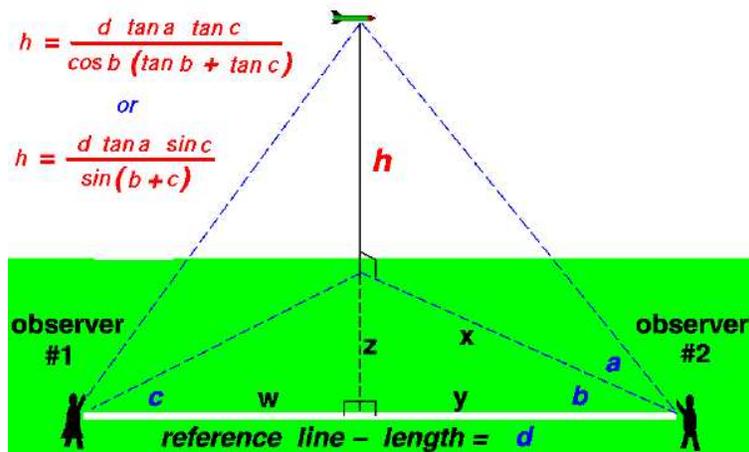


Figure 1: The figure above shows how to determine the altitude of the rocket using measurements from two observers. This figure is adapted from <http://www.grc.nasa.gov/>.

deciding where to locate the observers for your launches, note that this method also gives the best results when the angles are near 45° . For this reason, you probably want to set up your observers so that their locations make a equilateral right triangle, with the launching pad at the right angle. In order to get angles near 45° for the vertical angles, you will also want the distances from the observers to the launch pad to be roughly the same as the expected maximum altitude of the rocket.

Note that you should end up with four geometrical estimates of the apogee height for each launch — two from the simple one-angle method and two from the multi-angle method.

3 Data Analysis

3.1 Using Mathematica

We will use Mathematica to solve our version of the rocket equation (equation 12). While performing this analysis you can refer to the sample file performing this calculation in a simplified case.

The first step in this calculation is to enter your thrust data. Those groups that determined the thrust curve experimentally should use evaluate their two types of thrust data, and use whichever type they find appropriate here. Be sure to justify your choice.

In order for Mathematica to be able to use this data you must enter the data into a table. To do this you will likely want to use the `ReadList` function. Something along the lines of

```
table1 = ReadList["/home/s03/abstudent/A8.thrust_data", {Number, Number, Number, Number}]
```

should work. The first parameter is the path to the data file, while the other parameters correspond to the columns in your data file. You may need to edit your file to get it into a form that Mathematica can deal with. Once you have your data table, you will

probably want to use the `Interpolation` function to make the data into a continuous function. Be sure to graph your thrust curve over the full time range that you want do calculations over to be sure that it looks reasonable. Mathematica often puts spikes and other unphysical features into Interpolating Functions. If you have problems of this sort, you may have to alter some of the options to `Interpolation`. For each engine, calculate the total impulse of the engine and compare it to the expected value.

For the A engine data, consider the drag coefficient, c_d to be an unknown and solve for it by trial and error for each of the three launches that you did with the A engines. Find the mean and standard deviation for the drag coefficient.

For the B engines, use the drag coefficient that you determined above to find a predicted apogee (including uncertainty) for the B engine flights. Also, re-calculate the drag coefficient using your B engine apogee data as the known quantities.

1. Compare the prediction of the B engine apogee with your experimental determinations of the apogee. Do they agree? Why or why not?
2. Compare the estimates of the drag coefficient that got for the A engines with the estimate from the B engines. Do they agree? If not, how could you explain the discrepancy?

3.2 Using a Spreadsheet

This section will repeat the calculations done for one rocket launch with Mathematica above using a spreadsheet. Only some groups will do this section, check with your instructor to see if you should. For those of who know how to program, you could also write a Fortran (or other programming language) program to do this part. Check with your instructor if you want to write a program to do this part.

Here we will use a spreadsheet (e.g. Gnumeric or Excel) to solve equation 12 using Euler's Method [*Boyce and DiPrima*, 1992, or almost any differential equations textbook]. This method basically involves stepping through the motion of the rocket a small time step at a time. At each time step the value for the acceleration, velocity, and displacement is determined from values for the previous time step.

To use this method you will first need to transfer all of the relevant constants, such as gravitational acceleration, to the spreadsheet. Also, include the value for the drag coefficient that you determined with Mathematica for this run since for the purposes of this calculation we will be considering the drag coefficient to be a known quantity. Then move have to transfer columns (or rows) of data for the thrust as a function of time and the mass as a function of time. You will probably want to use a Mathematica command like `Table[Cos[i], i, 0, 8, .1] // TableForm`, substituting whatever you called your functions for `Cos`. These calculations will be performed with time steps of 0.1 s and 0.01 s, so you should prepare data for each case. Also include a column for time with the difference between consecutive cells following the time step sized listed above.

Next you will make the calculations of acceleration, velocity, and displacement. When setting up these calculations, for the first cell for each you will want to put the initial value, which will likely be 0. In the next cell you will set up a formula to calculate the value for that cell based on previously calculated values. When you are setting up these calculations that when you place a \$ in a cell reference, that reference will remain unchanged when you copy and paste your formula. So if you are storing your drag coefficient in the cell B4, you will probably want to refer to it as `$$B$4`.

For the acceleration, you should use a formula like

$$a_{i+1} = \frac{T_{i+1}}{m_{i+1}} - g - \frac{bv_i^2}{m_{i+1}} \quad (15)$$

where the indices refer to current and previous time steps. Similarly for velocity and position we get

$$v_{i+1} = v_i + a_i \Delta t \quad (16)$$

and

$$x_{i+1} = x_i + v_{i+1} \Delta t + \frac{1}{2} a_{i+1} (\Delta t)^2 \quad (17)$$

Plot the position, speed, and acceleration versus time for both time steps sizes.

1. How well do these results match your Mathematica results?
2. How do the results compare with different time step sizes?

4 Conclusion

As well as answering the questions listed above, comment on your results.

1. Do your results support the idea of the drag coefficient being a true constant for the speed regime your rockets were tested in? Why or why not? Make sure to use the uncertainties that you obtained for the drag coefficients in defending your answer.
2. Describe in more detail than is presented here how the electronic altimeter works.
3. Explain how the geometrical method of determining altitude using two observers improves accuracy. Hint: there is more to it than simply having better statistics to average over.

References

- Benson, T., A beginner's guide to model rockets, 2003, <http://www.grc.nasa.gov/WWW/K-12/airplane/rkthowhi.html>.
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- Marion, J. B., and S. T. Thornton, *Classical Dynamics of Particle and Systems*, Thomson, Brooks/Cole, Pacific Grove, California, 1995.
- Perfect Flite, *Alt15/WD User's Manual*, 2003, <http://www.perfectflite.com/>.