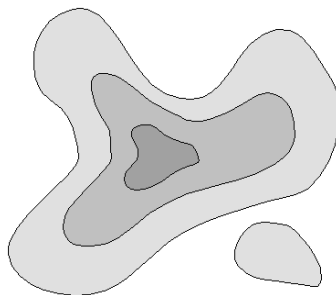


# **FOURPOT**

Fourier transform based processing of 2D potential field data

Version 1.1

User's guide



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## 1. Introduction

The FOURPOT program is designed for frequency domain processing and analysis of two-dimensional (2D) potential field arising, in particular, from geophysical gravity and magnetic field measurements. The data can be irregularly or regularly sampled. The frequency domain operations include high pass, low pass and directional filtering, upward and downward continuation, pole reduction of magnetic data, 1.st and 2.nd degree vertical and horizontal ( $x$  and  $y$  directed) gradients, total horizontal gradient, total gradient (analytical signal) and tilt gradient. Pseudo-gravimetric and pseudo-magnetic fields can be computed also.

Considering a continuous function  $f(x)$  with continuous first derivatives the Fourier transform  $F(k_x)$  and its inverse transform can be written as (e.g., Blakely, 1995):

$$F(k_x) = \int_{-\infty}^{\infty} e^{-ik_x x} f(x) dx \quad \text{and} \quad f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ik_x x} F(k_x) dk_x$$

Considering discrete and 2D data the governing equations are:

$$F_{nm} = \sum_{k=1}^N \sum_{l=1}^M e^{-2\pi i \left( \frac{nk}{N} + \frac{ml}{M} \right)} f_{kl} \quad \text{and} \quad f_{kl} = \frac{1}{NM} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} e^{+2\pi i \left( \frac{nk}{N} + \frac{ml}{M} \right)} F_{nm} ,$$

where  $N$  and  $M$  are the number of data values in  $x$  and  $y$  directions. The transform is complex which means that it has both amplitude and phase spectra.

FOURPOT computes the discrete 2D Fourier transform using the fast Fourier transform (FFT) algorithm. The Fourier transform represents a sum of sine and cosine terms with different spatial frequencies ( $k_x$  and  $k_y$ ) that are defined by data sampling ( $d_x$  and  $d_y$ ) in  $x$  and  $y$  directions. The highest spatial frequency is the so-called Nyquist frequency (e.g.,  $\max(k_x) = 0.5/d_x$ ). The lowest frequency is based on the data coverage (e.g.,  $\min(k_x) = 0.5/(\max(x)-\min(x))$ ). Considering that the inverse of the spatial frequency represents wave length ( $\lambda_x = 1/k_x$ ), zero frequency means infinite wave length, i.e., constant level of data. Because of the properties of the Fourier transform (symmetry, linearity, shift and derivate properties) several computational operations can be performed in Fourier transformed frequency ( $k_x, k_y$ ) domain more efficiently than in the spatial ( $x, y$ ) domain. For more detailed information about Fourier transform methods in potential field analysis, please, see Blakely (1995), for example.

## 2. Installing the program

This version of FOURPOT is a 32-bit application that can be run on Windows XP/Vista system with a graphics display of at least 1024×768 pixel resolution. Memory requirements and processor speed are not critical, unless very large data sets are used. The program has a simple graphical user interface (GUI) that is used to handle file input and output, to perform the computational operations, and to visualize the data and the results. The user interface and the data visualization are based on DISLIN graphics library.

The program requires these two files:

FOURPOT.EXE	the executable file and
DISDLL.DLL	dynamic link library for the DISLIN graphics.

The distribution file (FOURPOT\*.ZIP) also contains a description file (\_README.TXT), this user's manual (FOURPOT\_MANU.PDF), and some example data files (\*.DAT). To install the program, uncompress (e.g., Winzip or 7Zip) the distribution file and move the resulting FOURPOT folder to a preferred place (e.g., C:\TOOLS). To be able to start the program from a shortcut that locates in a different directory or from the desktop, you should move or copy the DISDLL.DLL file into the WINDOWS\SYSTEM (or SYSTEM32) folder or somewhere along the system path.

### 3. Menu items

#### 3.1. File menu

The *File* menu contains following options:

<i>Read data</i>	read in (irregularly spaced) data in default file format.
<i>Read matrix</i>	read in (regularly spaced) data in matrix format.
<i>Save data...mat</i>	save the data of current view in a matrix format.
<i>Save data...dat</i>	save the data of current view in general xyz column format.
<i>Read disp.</i>	read in new graph parameters from a *.DIS file.
<i>Save Graph as PS</i>	save the current graph in Adobe's Postscript format.
<i>Save Graph as EPS</i>	save the current graph in Adobe's Encapsulated PS format.
<i>Save Graph as PDF</i>	save the current graph in Adobe's PDF format.
<i>Save Graph as WMF</i>	save the current graph in Windows metafile format.
<i>Save Graph as GIF</i>	save the current graph in GIF file format.

These menu items bring up a standard (Windows) file selection dialog that is used to provide the file name for I/O operations. All data files are in text format (see chapter 5). The graphs are saved A4 landscape mode (see Appendix A). The preferred output format is PDF or PS, because their resolution is much better than that of GIF and WMF files. The PDF format is particularly useful because Adobe's Acrobat Reader can be used to take bitmap snapshots at desired resolution.

#### 3.2. Edit menu

The *Edit* menu contains following items:

<i>Inverse→data</i>	set current results (of the inverse FFT) as new initial data.
<i>Subtract inverse</i>	subtract the inverse results from the original data.
<i>Add inverse</i>	add the inverse results to the original data.
<i>Color scale</i>	change the color scale (rainbow, gray scale, etc.).
<i>Padding mode</i>	change the method used to pad and taper data
<i>Data type</i>	define potential field data (generic, gravity, magnetic).

<i>Miscellaneous</i>	define some additional settings.
<i>Low pass filter</i>	perform (GUI directed or manual) low pass filtering.
<i>High pass filter</i>	perform (GUI directed or manual) high pass filtering.
<i>Direction filter</i>	perform (GUI directed or manual) direction filtering.

The use of *Inverse* → *data* menu item can be used to put the current inverse results as new input data for successive frequency domain operations. For example, tilt gradient can be computed from the upward continued data. Likewise, the *Subtract inverse* menu item allows computing the residual field directly as the difference between the original (interpolated) data and the upward continued or low pass filtered data. On the contrary, the *Add inverse* menu item is used to add the inverse results to the original data. Although this item is not very useful, it can be used to enhance high frequency content of the data, for example.

The *Padding mode* menu item is used to demonstrate the way padding and tapering is made. Padding is an operation where null values are added around the original data area (see Appendix B). The purpose of padding is to extend the data (N and M) to even power of two (e.g. 64, 128, 256, 512 etc.) as the FFT algorithm requires this. Tapering is an operation where the "padded" values are made such that they prevent rapid amplitude changes at the border of the data area. This means that the data and their derivative are made more or less continuous in padding. Together padding and tapering effectively remove so-called Gibbs phenomenon as well as ringing and other artifacts in the inverse transformed data. Padding will be discussed more in the next chapter.

FOURPOT does not consider what kind of data that are passed to it. Strictly speaking pole reduction and pseudo-gravity, for example, are applicable to magnetic data only. For teaching and testing purposes the *Data type* menu item allows defining the actual data type. In practice, this has effect only on the information text next to the graph and the scaling of the radial amplitude spectrum (see chapter 4.7).

The *Miscellaneous* menu item contains following items:

<i>Contour/Image</i>	change between contour map and image (pixel) map modes.
<i>Show/Hide points</i>	show/hide the actual data points in original data view.
<i>Meters ↔ Kilometers</i>	swap units between meters and kilometers.

<i>Reverse sign</i>	reverse the sign of the original data.
<i>Autom. low pass filter</i>	automatic low pass filtering when activated.

Note: The information text next to the graph will show the dimensions incorrectly unless the *Meters↔Kilometers* menu item is utilized. Although knowledge about the spatial dimensions is usually not required, the pseudo-gravity and pseudo-magnetic field need to know the real dimensions for the amplitude of the transformed field to be more or less "truthful". The *Reverse sign* item can be useful when working on the potential of the original field, since often potential fields are defined as the negative gradient of the scalar potential.

The quality of some frequency domain filtering operations is strongly affected by the "smoothness" and continuity of the data. To circumvent computational artifacts in the inverse transformed data it is often necessary to perform low pass filtering before any other processing operations. The *Automatic low pass filter* menu item activates automatic filtering every time the FFT is computed. The inner and outer radii of the filter can be changed manually using the dialog that appears after using the menu item. Note that the radii are not defined as distances but as a ratio (a value between 0 and 1) of the minimum Nyquist frequency of the data. Automatic values are used if the radii are set to zero. If the values are omitted totally (i.e., a blank line is given) the automatic mode is deactivated.

Low pass and high pass filtering are the two most typical frequency domain processing operations. Considering 2D data, low pass filtering removes (nullifies) values outside certain radius from the origin of the frequency (amplitude) spectrum (see Appendices C, E and F). On the contrary, high pass filtering removes low frequency data around the origin of the spectrum. Together low and high pass filtering can be used as a band pass or notch filtering. The direction filtering, which is applicable to 2D frequency data only, can be used to remove or to enhance linear features in the original data (see Appendices G and H). The frequency filtering can be accomplished either graphically with the mouse or manually using the given numeric values. Frequency filtering will be discussed more in the next chapter.

### 3.3. Process menu

The *Process* menu contains the items for data processing operations:

<i>Upward continuation</i>	performs upward continuation.
<i>Downward continuat.</i>	performs upward continuation.
<i>Pole reduction</i>	performs reduction to magnetic pole.
<i>dF/dz gradient</i>	computes vertical gradient of the data.
<i>dF/dx gradient</i>	computes horizontal x gradient of the data.
<i>dF/dy gradient</i>	computes horizontal y gradient of the data.
<i>d<sup>2</sup>F/dz<sup>2</sup> gradient</i>	computes second vertical gradient of the data.
<i>d<sup>2</sup>F/dz<sup>2</sup> gradient</i>	computes second horizontal x gradient of the data.
<i>d<sup>2</sup>F/dz<sup>2</sup> gradient</i>	computes second horizontal y gradient of the data.
<i>d<sup>2</sup>F/dxdy gradient</i>	computes xy gradient of the data.
<i>d<sup>2</sup>F/dxdz gradient</i>	computes xz gradient of the data.
<i>d<sup>2</sup>F/dydz gradient</i>	computes yz gradient of the data.
<i>Hori. gradient</i>	computes horizontal gradient.
<i>Total gradient</i>	computes total gradient (analytic signal).
<i>Tilt gradient</i>	computes tilt gradient.
<i>Pseudo gravity</i>	computes pseudo-gravity field from magnetic data.
<i>Pseudo magnetic</i>	computes pseudo-magnetic field from gravity data.
<i>Gravity potential</i>	computes gravity potential from gravity field (Bouguer data).
<i>Magnetic potential</i>	computes magnetic potential from total field (TMI) data.
<i>Undo process</i>	reverts back to unprocessed (but frequency filtered) data

Upward continuation makes the data appear to have been measured higher above the surface of the earth (or the plane of measurements). It is used to estimate the (low frequency) regional trend of the data. Likewise, downward continuation makes the data appear to have been measured below the plane of measurements, i.e., inside the earth. It is used to enhance the high frequency content of the data and to estimate the depth to the top of the targets. Unlike upward continuation, downward continuation is not a stable procedure. If the plane of continuation is located below the actual potential field sources the results become erratic. Examples of upward and downward continuation are shown in Appendix I.

Important: When performing upward or downward computation the height difference or elevation is read from the *Height* text field in the control pane on the left side of the graph area (see Appendix A). The height value is always positive, that is to say, the difference is



computed downwards or upwards depending on the selected task. The dimension of height is the same as for the data itself (e.g., kilometers or meters depending on sampling).

Unlike gravity field the static magnetic field of a symmetric body (e.g vertical prism) exhibits non-symmetric anomaly shape because the inclined direction of the inducing magnetic field. The behavior of Earth's magnetic field can be estimated using the IGRF model (international geomagnetic reference field). The *Pole reduction* item is used to transform magnetic field data as if it were measured on a magnetic pole. This helps estimating the true dip and strike directions of the targets.

Important: When performing pole reduction the inclination and the declination of the magnetic field are read from the *Incl.* and *Decl.* text fields. The values are provided as angles in degrees. Inclination ( $I = \pm 90^\circ$ ) is taken from horizontal plane and it is positive downwards (northern hemisphere) and negative upwards (southern hemisphere). Declination ( $D = \pm 90^\circ$ ) is positive in clock-wise orientation from the true north direction. The pole reduction is unstable operation if the inclination is very small (near the equator).

The vertical and horizontal x and y gradients are used for the visual enhancement of some features in the data. Both first and second degree gradients of the data  $f = f(x,y)$  can be computed. Here x direction represents the horizontal axis and y direction the vertical axis of the mapped data. The vertical gradients are particularly useful in enhancing the lateral dimensions of anomalous sources ( $df/dz$  for magnetic data and  $d^2f/dz^2$  for gravity data). The less usual gradients  $d^2f/dxdy$ ,  $d^2f/dxdz$ ,  $d^2f/dydz$  can be used to compute the off-diagonal components of the gravity tensor provided that the gravity potential is first computed. Examples of gradient operations are shown in Appendix J and K.

The horizontal gradient,  $h(x,y) = [d^2f/dx^2 + d^2f/dy^2]^{1/2}$ , total gradient  $a(x,y) = [d^2f/dx^2 + d^2f/dy^2 + d^2f/dz^2]^{1/2}$  (also known as analytical signal), tilt gradient  $t(x,y) = \arctan[df/dz / h(x,y)]$  and the horizontal gradient of tilt gradient  $s(x,y) = [d^2t/dx^2 + d^2t/dy^2]^{1/2}$  are useful operations that can be used in visual interpretation of magnetic and gravity maps. They are used especially to produce the location of lateral contacts of different geological units. Examples of these gradient operations are shown in Appendix L and M.

Note: The horizontal gradient of tilt gradient  $s(x,y)$  can be computed manually as follows. First the user needs to compute the tilt gradient  $t(x,y)$  and use *Inverse→data* menu item to replace the original data. Then one computes the 2.nd horizontal  $x$  and  $y$  gradients  $d^2t/dx^2$  and  $d^2t/dy^2$  and saves their results into two separate files. Finally, using a spreadsheet program one computes the amplitude of the horizontal tilt gradient  $s(x,y) = [d^2t/dx^2 + d^2t/dy^2]^{1/2}$ .

The gravity and magnetic potentials (and hence the fields) of an anomalous target with the same dimensions but different petrophysical properties (density and magnetic susceptibility) are interrelated via Poisson's relation. The so-called pseudo-gravity field can be computed from the measured magnetic data. Likewise, pseudo-magnetic field can be computed from measured gravity data. For these operations some assumed values of density, susceptibility and the intensity of the inducing magnetic field must be provided in the dialog that appears after using the *Pseudo gravity* or *Pseudo magnetic* menu items. Also, as discussed in previous chapter, the actual units of the spatial dimensions must be defined in order to compute the amplitude of gravity effects. An example of pseudo-magnetic field is shown in Appendix M. Note that the validity of the pseudo-magnetic and pseudo-gravity field computations has not been verified in this version of FOURPOT. The potential of the gravity and magnetic field data can be computed also. In the latter case the current values of field inclination and declination are used to account for the direction of the inducing magnetic field.

The "*Exit*" menu has two items. The first one can be used restart the whole program in a mode that is more suitable to widescreen or normal 4:3 displays. The second menu item is used to confirm the exit operation. Before exit the user should save processed data, because they are not saved automatically. Errors that are encountered before the GUI starts up can be found from the FOURPOT.ERR file. When operating in GUI mode, run-time errors arising from improper parameter values, for example, are shown on the display screen.

## 4. Using the program

When the program is started it reads graph parameters and some additional settings from the FOURPOT.DIS file (see chapter 6 for more information). If this file does not exist, default parameters will be assigned and the file is created automatically. The program then builds up the GUI shown in Appendix A. None of the program controls (widgets), however, are active before data has been read in.

The data processing and analysis is performed in few successive steps that are discussed next. The preliminary step is always to check that the input data file is in correct format so that it can be read in (see chapter 5 on file formats). After reading in the data one should apply *Plot matrix*, *Padding* and *Plot FFT* buttons to perform interpolation and padding and to compute the 2D frequency spectrum. Only after the FFT has been computed one can perform frequency filtering (*Edit* menu) and frequency domain data processing (*Process* menu).

### 4.1. Reading in the data

The first thing to do after starting up the program is to read in the data using the *Read data* (\*.DAT) or *Read matrix* (\*.MAT) menu items. Note that Geosoft XYZ formatted files can be read as normal DAT files provided that the file header is added by hand. See chapter 5 for more information about file formats.

After successful data input the program automatically interpolates the data on a regular grid and plots a contour map as seen in Appendix A. The initial grid sampling is based on the number of data values and the spacing between the first two data points in the file. The grid dimensions, i.e, the number of grid nodes in  $x$  and  $y$  directions ( $N$  and  $M$ ) will appear in *Dim x* and *Dim y* text fields in the control pane. At any time the original data can be plotted again with the automatic grid spacing using the *Plot data* button.

### 4.2. Interpolation

All input data are assumed to be irregularly sampled. Even if the original data were already evenly discretized it will be re-interpolated before it's passed to FFT. Therefore, after reading in the data the user must always apply the *Plot matrix* button, which is used to perform the

required interpolation using the grid dimensions ( $N$  and  $M$ ) provided on the *Dim x* and *Dim y* text fields. Note that the grid spacing ( $d_x$  and  $d_y$ ) does not need to be equal in  $x$  and  $y$  directions.

The *Plot data* function is used merely to visualize the original data and it does not affect the current grid dimensions. If the grid dimension changes from the default or initial values (because of interpolation or padding), one can revert back to original (automatically computed) sampling using the *Def size* button. The *Aspect ratio* button, on the other hand, is used to reset either the *Dim x* or *Dim y* dimension so that the original shape of the data area is preserved. This means that when a new value of *Dim x* or *Dim y* is given, the other value is set so that the original aspect ratio remains when *Aspect ratio* button is applied.

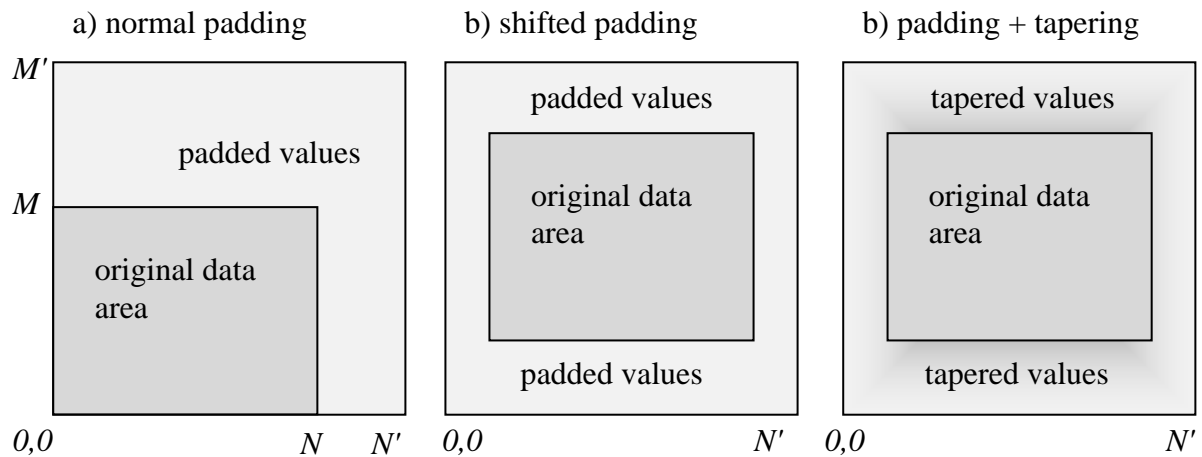
Important! The horizontal and vertical dimensions ( $N$  and  $M$ ) of the data matrix passed to the FFT algorithm must be powers of two (e.g., 64, 128, 256, 512, etc.). The interpolation discussed above is performed using the GETMAT subroutine included in the DISLIN graphics library (see DISLIN user manual). This subroutine works best if the data are already regularly sampled and the dimensions coincide with the original grid. Therefore, it is recommended that highly irregular data are interpolated on a regular grid using a more advanced third party software (e.g., Golden Software Surfer) and more suitable algorithm (e.g., minimum curvature). Naturally, if evenly discretized data are passed to FOURPOT one should take care that the interpolation (*Plot matrix*) is made using the same grid sampling as that of the original data so that as minimal distortion is made by GETMAT as possible.

#### 4.3. Padding and tapering

Padding and tapering are essential parts of successful Fourier transform processing. The *Padding* button will automatically add extra columns and rows around the interpolated data matrix so that the grid dimensions ( $N$  and  $M$ ) will increase to the next even power of two (see Figure 4.1 and Appendix B). The padding can be performed with or without tapering. Tapering means that the padded data are such that the level and the derivative of the data are preserved.

Sometimes the original grid dimension may already be so close to some power of two that the tapering will not have enough time to suppress data discontinuity. To further increase the

padded dimension from 64 to 128, for example, one simply needs to change the current value of *Dim X* or *Dim Y* to some number between 65 and 128. This will then be adjusted to 128 when *Padding* button is pressed. Note that the dimension of the grid can be decreased only using re-interpolation by *Plot matrix* button.



**Figure 4.1.** Schematic view of a) padding without shift, b) padding with shift and c) shifted padding with tapering. The new dimensions  $N'$  and  $M'$  are powers of two.

Padding and tapering depend on the selection made with the *Padding mode* item in *Edit* menu. The padding modes are: a) *Zeros* which actually use the median of the outmost data points instead of zeros for padding, b) *Linear extrapolation* will extend with the outmost data point, c) *Mean value based* will use the nearest points inside some increasing search radius and d) *Gradient based*, which uses the mean value and the derivative of the field to extend the data to the padding zone. The *Shift yes/no* option is used to define if padding is added only to the top and to the right of original data area or if padding is made all around the data area (which looks like the data were shifted).

**Important!** Best results from Fourier transform processing are obtained using gradient based padding with shift. The other padding options are kept primarily for teaching purposes so that the users can see the effect and importance of padding and tapering.

#### 4.4. Fourier transform

After sampling, interpolation and padding one can use the *Plot FFT* button to compute the FFT and to see the 2D frequency spectrum (see Appendix C).

At this point one should remember that the frequency spectrum  $F$  is complex, i.e., it has both real and imaginary parts,  $F = \text{Re}(F) + i \cdot \text{Im}(F)$ . Thus, the 2D Fourier graph actually represents the 10-base logarithm ( $\log_{10}$ ) of the *amplitude spectrum*  $A = (\text{Re}(F)^2 + \text{Im}(F)^2)^{1/2}$ . The origin ( $k_x=0, k_y=0$ ) of the graph is located at the middle of the graph and the horizontal and vertical axis range between the Nyquist frequencies, the value of which are shown in the information text next to the graph. Lowest frequencies or largest wave length (long variations of data) are located in the middle of the graph and highest frequencies or shortest wave lengths (short variations of data) are located far from the origin. Continuous linear features in the data appear as linear features in the FFT spectrum as well (the angle of slope is inversed, though).

Only after the FFT has been computed one can perform the frequency filtering and other frequency domain processing. The inverse FFT and the difference between the original (gridded) data can be plotted using *Inverse FFT* and *Plot*.

Note that each Fourier operation is made on the interpolated and padded data matrix (*Plot matrix/Padding*). Multiple Fourier operations (e.g. pole reduction and total gradient) can be performed in a row by using the *Inverse→Data* menu item after the first operation and performing new FFT and the second Fourier operation. Alternatively, one can save the current inverse results and restart the program using the saved results as new input data.

#### 4.5. Inverse Fourier transform and the difference

After the Fourier transform and the 2D frequency spectrum has been computed one can perform frequency filtering tasks in the *Edit* menu and/or any of the frequency domain processing tasks in the *Process* menu. The *Plot inverse* button can be used to display the current inverse results, that is to say the either the inverse FFT of the current frequency (filtered and/or processed) spectrum. When tasks are selected from the *Process* menu the inverse results will be shown automatically.

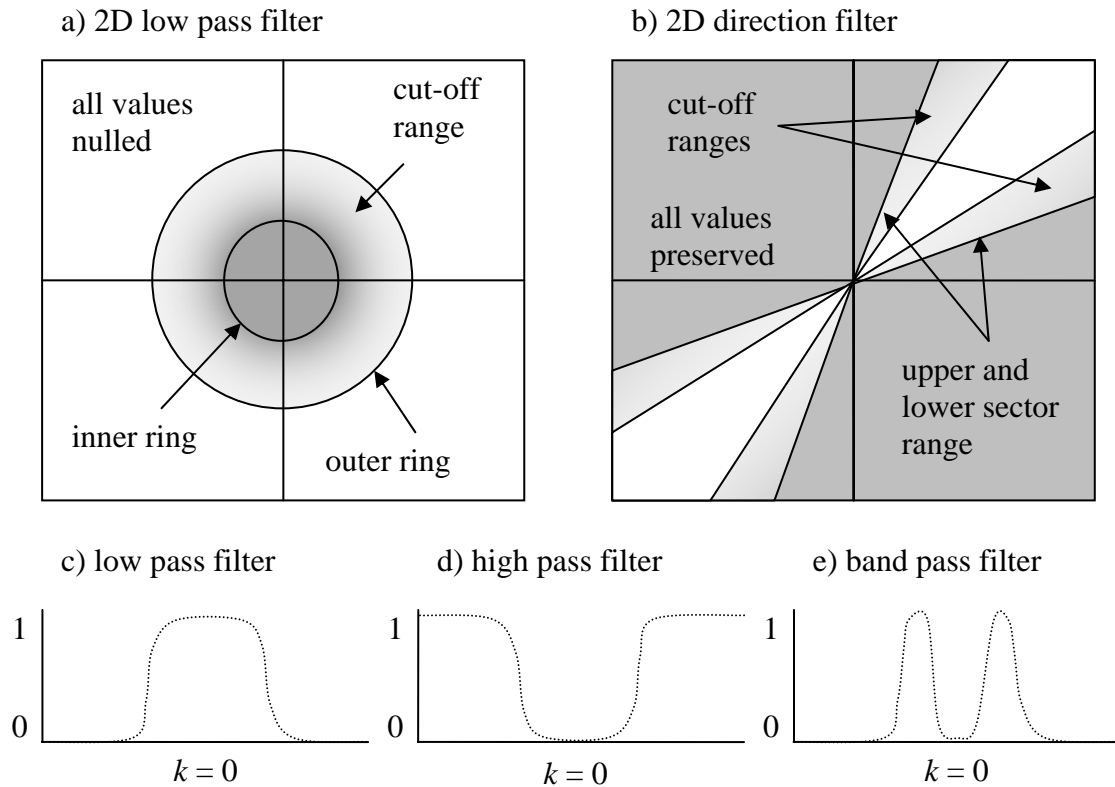
Successive utilization of the *Plot inverse* and *Plot data* buttons can be used to visualize the changes that are made to the data due to the frequency domain operations. As an alternative, the *Plot diff* button can be used to visualize the difference between the original (interpolated) data and the data obtained from the inverse FFT after the most recent Fourier operation. If no operations have been made the inverse will be and should be (almost) equal to the original data. Note that pressing the *Def. size* or *Padding* buttons will invalidate the current frequency spectrum and inverse results and the user needs to perform the frequency filtering operations again. Note also that after *Padding* the *Plot matrix* function becomes inactive, because the N and M values are affected by the padding to the next power of two. To be able to re-grid the original data one needs to apply the *Def. size* button.

#### 4.6. Frequency filtering

Important: Unlike the other processing operations, frequency filtering affects the current amplitude spectrum on which other processing functions are performed and from which the inverse transform is computed. This means that frequency filtered data are always passed to further processing operations without the use of *Inverse* → *data* menu item. As a matter of fact, low pass filtering is often an essential preliminary step for successful operation of the gradient filters.

Like the rapid changes in the data can cause problems when its FFT is processed, rapid changes in FFT spectrum can cause oscillations in the inverse transformed data. This is known as the Gibbs phenomenon. Therefore, instead of using an abrupt box-car shaped filter function a smooth bell shaped (sine and cosine) functions are applied over certain cut-off range. In low pass and high pass filtering this means in practice that instead of a single radius of a single filter ring one needs to provide the radii for the inner and outer ring of the cut-off

range separately. Between the two rings the filtering is made using a quarter sine or cosine functions. Likewise, in directional filtering one needs to provide three values that define the upper and lower sector range and the width (as an angular difference) of the cut-off range. The filter rings and sectors and the cut-off ranges are illustrated in Figure 4.2.



**Figure 4.2.** Schematic view of a) cut-off rings and b) sectors of 2D filters and the smooth (bell shape) filter function of c) low-pass, d) high pass and e) band pass filters.

#### 4.6.1. Low pass and high pass filtering

In low pass filtering all data outside the outer ring (high frequencies) are nullified and all data inside the inner ring are preserved (see Figure 4.2a and c). On the contrary, in high pass filtering all data inside the inner ring (low frequencies) are nullified and all data outside the outer ring are preserved (see Figure 4.2d). The radius of the rings are defined as wave length ( $\lambda$ ), which is the inverse of radial frequency  $k_r = [k_x^2 + k_y^2]^{1/2}$ , where  $k_x$  and  $k_y$  are the axes of the 2D frequency graph. The unit of the wave length is either meter or kilometer depending on the selection made using the *Meters ↔ Kilometers* item in the *Edit-Miscellaneous* menu.



Note: If the sampling intervals are different in the x and y directions the spatial frequencies ( $k_x$  and  $k_y$ ) will be different also. Because, a single value of wave length is used as filter dimension both in x and y directions, the filter rings will appear as an ellipses instead of circles in the 2D frequency graph.

#### 4.6.2. Directional filtering

Directional filtering is equal to the so-called  $fk$ -filtering commonly used in seismic and GPR data processing. Directional filtering removes (or enhances) linear features with certain angle in the data. In directional filtering the data are removed inside two sectors that are symmetric around the origin (see Figure 4.2b). The filter removes the frequency data inside a sector defined by two angles that correspond to the upper and lower limit of the sector. The angles are defined in degrees in counter-clockwise direction taken from the positive  $k_x$  axis. In addition, the cut-off range is defined by providing a third parameter as an angular difference either from the upper or the lower sector (the width of the cut-off range will be the same on both sides). Because of the symmetry of the frequency spectrum the directional filter will be mirrored around the origin automatically. Thus, one needs to define only single sector either above the  $k_x$  axis or left to the  $k_y$  axis.

#### 4.6.3. Interactive and manual filtering

Filtering can be done either a) interactively using the mouse or b) manually by providing the numerical values of the filter cut-off ranges. The selection is made when the corresponding menu item is chosen in the *Edit* menu.

When interactive mode is chosen most of the other program widgets become inactive and the mouse cursor changes from normal arrow into a cross-hair above the graph area (note: only above the graph area). The 2D frequency spectrum is shown together with auxiliary polar coordinate system grid. The user is given instructions to provide first the inner and then the outer ring by pressing the left mouse button above the graph. To validate the last value and to end the editing mode one must press the right mouse button. The distance from the origin determines the radius of the filter rings. Likewise in directional filtering the user is instructed to provide the upper and lower angle of the filter sector and the width of the cut-off range.

The width can be defined as a angular difference from the lower or upper sector and the filter cut-off range will be adjusted symmetrically at both sides automatically.

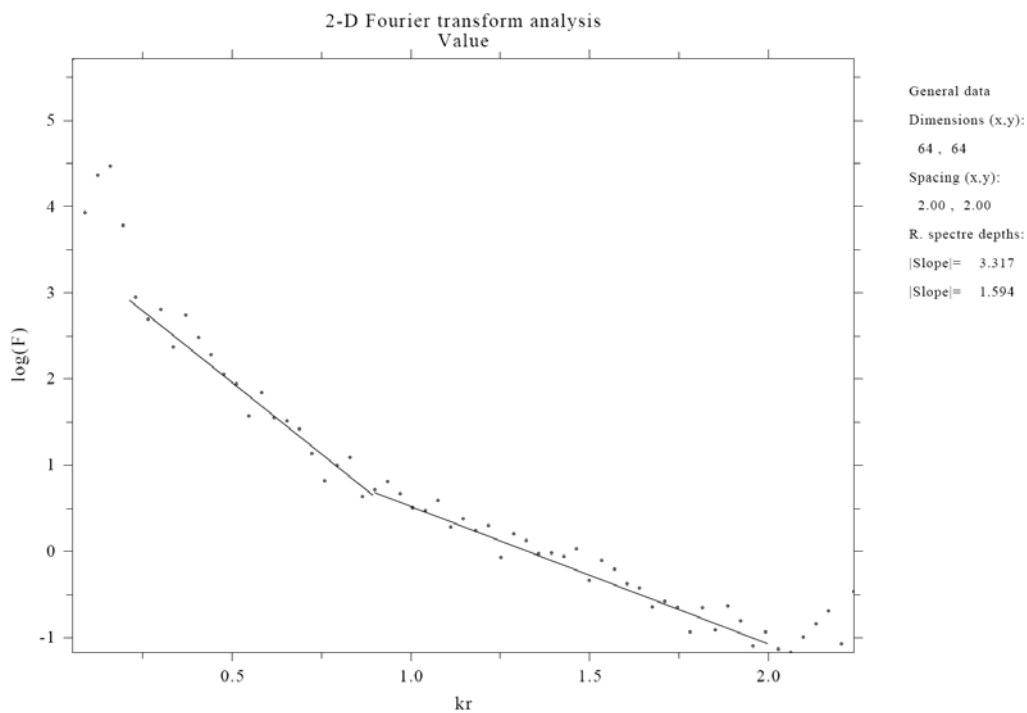
Note that the ring and sector values can be provided in reverse order in which case they will be automatically arranged so that the inner ring is smaller than outer ring or the upper angle is larger than lower angle. Values can be omitted by pressing the right mouse button instead of the left one. In low pass and high pass filtering omitting the first value cancels the whole interactive editing mode. Omitting the second value will make the two rings the same which means that the sine/cosine bell function is replaced by a box-car filter. In directional filtering the upper and lower angle must always be provided but the cut-off range can be omitted if the right mouse button is pressed when its value is asked for.

Important: After interactive editing the given filter rings and sectors will be shown by dotted lines on the frequency spectrum. The wave lengths and the angles will appear in the *Ring 1*, *Ring 2*, *Sect 1*, *Sect 2* and *Taper* text fields in the left control pane. Their values, however, are not stored if multiple filtering are made in this version of FOURPOT. Also note that together low and high pass filtering can be used to accomplish band pass and notch filtering. The *Ring*, *Sect* and *Width* text fields, however, will show the parameters of the last filtering only. Therefore, the user should remember to memorize or save the parameters of the frequency filters because they may be needed to reproduce the processing results afterwards.

After the user gets accustomed with interactive filtering one should learn manual filtering which is more accurate than interactive filtering. Manual filtering is made by providing the wave lengths (in units of dimension) of the filter rings (low pass and high pass filtering) or the angles of (in degrees) the filter sectors and width of the cut-off range (directional filtering) in directly the corresponding *Ring 1*, *Ring 2*, *Sect 1*, *Sect 2* and *Width* text fields in the left control pane. Then by selecting the desired menu item from the *Edit* menu one can perform low pass, high pass or direction filtering. Some default parameters are used if the text fields are equal to zero.

#### 4.7. Radial amplitude spectrum

The *Radial spectrum* button, which is active only when the 2D frequency spectrum is active, will compute the amplitude spectrum as a function of the radial frequency. The radial amplitude is the mean of the 2D Fourier amplitude spectrum  $A = |F| = [\text{Re}(F)^2 + \text{Im}(F)^2]^{1/2}$  along rings with radius  $k_r = [k_x^2 + k_y^2]^{1/2}$ . The amplitude spectrum has traditionally been used to estimate the depth to the bottom of potential field sources. This is established by fitting linear lines to the decaying amplitude curve on a semi-logarithmic scale. An example is shown in Figure 4.3.



**Figure 4.3.** Radial amplitude spectrum with two manually fitted lines. The slopes give estimates for the depth to the bottom of the potential field sources.

In frequency domain the Fourier transform of a potential field can be formulated as  $F \approx Ce^{-hk}$  giving  $\log(F/C) = -h \cdot k_r$ . Thus, the depth to the top of an anomaly source ( $h$ ) is equal to the tangent or the slope of the linear parts of the amplitude spectrum. For general data type the coefficient  $C = 1$  and the vertical axis is the logarithm of the amplitude spectrum,  $\log|F|$ . For gravity data the coefficient  $C = -1/(k_x k_y k_r)$ , and in case of magnetic data  $C = -1/(k_x k_y)$  (selected by the *Data type* item in *Edit* menu). Because of the data type the slopes and, therefore also

the depth estimates are quite different. Depending on the data the normalization might not be useful at all, so one should prefer to general data type. Please, refer to Bhattacharyya (1967) and Ruotoistenmäki (1987) for more information about the use of radial amplitude spectrum for depth determination. Note that the depth interpretation option has not been fully tested in this version of FOURPOT.

Interactive line fitting is accomplished by pressing the *Define line* button when radial amplitude spectrum is displayed. The mouse pointer changes into a cross-hair cursor over the graph area. The user should select the start and end points of the line over the scattered data plot of the radial amplitude spectrum by pressing the left mouse button. The editing mode is ended by pressing the right mouse button. The graph will be redrawn with the newly edited line on it and the slope of the line will be displayed on the information text on the right side of the graph. Multiple lines can be added to the same graph pressing the *Define line* button again. The most recent line can be deleted from the graph pressing the *Delete line* button.

Note: Low pass and high pass filtering affect the computation of the radial spectrum. Because of the cut-off range of the filters the radial spectrum may look quite different from the one shown in Fig. 4.3, for example. Also note that the vertical axis of the radial spectrum (and 2D spectrum as well) is logarithmic and that the nulled values are plotted as zeros on the log-scale. Naturally this means that nulled values are plotted incorrectly (they are really zeros).

#### 4.8. Color mapping (range, center and levels)

The *Range* and *Center* scale widgets can be used to change the color mapping of the data maps and 2D frequency spectrum. The *Levels* widget is used to define the number of contour levels (the default is 21 levels).

By default the color scale (eg. rainbow or reverse rainbow) is evenly distributed between the minimum and the maximum data values of the current map. The *Range* widget allows changing the range of the color scale by between 5 % and 125 % of the original minimum and maximum values. Decreasing the value is useful if the maximum data value of the map is larger than the mean data level due to outlier, for example. Increasing its value is useful if the maps get saturated at their minimum or maximum limit values.

The *Center* scale widget changes the middle point of the color scale between -85 % and +85 % of the current data range. Increasing the center value will emphasize small data values and increases the amount of colors at the beginning part of the color scale. Decreasing the center value will emphasize large data values and increases the amount of colors at the end of the color scale.

Important: After changing any of the abovementioned scales one needs to press the *Update plot* button to make the changes visible. Because the scale widgets give different results for different graphs one can reset the color mapping parameters to their defaults using the *Reset scale* button.

Note that FOURPOT was not designed to be used as a plotting program. To prepare the final results the user is advised to use plotting programs like Golden Software's Surfer.

## 5. File formats

Potential field data can be read into FOURPOT in two file formats: a) generic column formatted data file (\*.DAT) and b) regularly gridded data matrix (\*.MAT).

### 5.1. Column formatted data files

The format of a \*.DAT file is illustrated below:

```
4096  1  2  4
      0.00      0.00 -0.8665106E+01  0.33489
      10.00     0.00 -0.8558651E+01  0.44134
      20.00     0.00 -0.8365253E+01  0.63474
      ...
```

The data file can contain multiple columns and the header line at the first line is used to define the number of lines (NOP) and the indices of the columns that contain the x and y coordinates (ICO1, ICO2) and data (ICO3). FOURPOT ignores empty lines and lines starting with ", /, l, L or # characters. Thus, provided that NOP is large enough ICO\* refer to correct column indices, FOURPOT can also read Geosoft XYZ files (\*.XYZ) illustrated below:

```
5000  1  2  4
/ Test data
LINE 1
      0.00      0.00 -0.8665106E+01  0.33489
      10.00     0.00 -0.8558651E+01  0.44134
      20.00     0.00 -0.8365253E+01  0.63474
      ...
LINE 2
      0.00     10.00 -0.8165106E+01  0.23489
      10.00     10.00 -0.8258651E+01  0.34134
      20.00     10.00 -0.8465253E+01  0.53474
      ...
```

Note: The header line can be omitted in which case the program shows the first line of the data file and asks the parameter (NOP, ICO1, ICO2, ICO3) values from the user.

### 5.2. Regularly gridded matrix files

The matrix format (\*.MAT) expects that the data are stored in a regular grid *without* x and y coordinates. The matrix format is provided for easier functionality with programs such as Matlab and Maple. The format of a matrix file (\*.DAT) is illustrated in the example below:

```

    64    64
-0.43160E+09
-0.43320E+09
-0.43270E+09
...

```

The header line defines the number of data points in x and y directions (NP and MP). The following (NOP = NP × MP) lines contain the actual data values. Note that because the matrix format does not contain coordinates the sampling frequencies and corresponding wave lengths shown by FOURPOT are not correct if they are not provided correctly in the dialog that appears after the matrix data are read in. The dialog window is used to define the x and y coordinates of the origin and the x and y step between the matrix elements. Note that if the x and/or y steps are negative, the axis gets reversed. Moreover, the data are mirrored in x or y direction if the last two parameters from zero to one in the auxiliary dialog.

Since the matrix file does not contain coordinates it is important to know the order in which the data are stored. The origin is always located in the bottom left corner. By default the data is stored in column-wise fashion from bottom to top and from left to right. Using a generic programming language notation we would have:

```

do i= 1,np
  do j= 1,mp
    read or write f(i,j)
  end do
end do

```

When matrix data are read in, the order can be changed so that for each column the rows will be read by giving a negative value for the number of points in x direction (NP= -NP).

Furthermore, if MP < 0 the matrix file is considered to have multiple data columns which will be read row by row (the file really looks like matrix). If NP > 0 and MP < 0 the rows are considered to be the y columns of the mapped data as in the default case. If NP < 0 and MP < 0 the rows are considered to be data rows (but the origin is still in lower left corner). Using a generic programming language notation we would have in this case:

```

do j= 1,mp
  read (f(i,j), i=1,np)
end do

```

## 6. Graph options

Several graph parameters and text strings can be changed by editing the FOURPOT.DIS file. This allows one to localize the graphs into another language, for example. Note that if the format of the FOURPOT.DIS file is important. If the format of the file becomes invalid, one should delete the file and a new one with default parameter values will be generated automatically next time the program is started. The file format is illustrated below:

```
# Fourpot ver. 1.10 parameter file

  36   32   24   0   0
  0    0    2   1   0   1   0   0   0
350   600 0.90 0.80 0.91 0.00 0.00 0.00
  75.0   10.0 52000.0 1.00000 0.01000 0.0 0.0
```

2-D Fourier transform analysis

Original data

Interpolated data

Transformed data

Inverse data

Difference

Radial ave. spectre

X

Y

Field

Kx

Ky

log<sub>10</sub>(F)

Kr

log(F)

Low pass filtered

High pass filtered

Direction filtered

Upward continued

Downward continued

Pole reduced

dF/dz gradient

dF/dx gradient

dF/dy gradient

d<sup>2</sup>F/dz<sup>2</sup> gradient

d<sup>2</sup>F/dx<sup>2</sup> gradient

d<sup>2</sup>F/dy<sup>2</sup> gradient

d<sup>2</sup>F/dxdy gradient

d<sup>2</sup>F/dydz gradient

d<sup>2</sup>F/dydz gradient

Horizontal gradient

Total gradient

Tilt gradient

Pseudo gravity

Pseudo magnetic

Gravity potential

Magnetic potential

Unprocessed data



- The two first lines are used as a header to identify the file and version number.
- The 1.st actual data line defines three character height values. The first one is used for the main title, the second one is for the axes labels and the third one is used for the plot legend.
- The 2.nd line defines some integer valued options. The first one sets the screen mode between screen normal 3:4 aspect ratio and wide screen mode. The second one defines the color scale. The third and the fourth parameter define padding and tapering modes. The fifth parameter sets either contour or image map mode. The sixth parameter defines the use of automatic low pass filtering mode. The extra parameters are reserved for future use.
- The 3.rd line defines the  $x$ - (horizontal) and  $y$ - (vertical) distance (integer values) of the origin of the main graph (in pixels) from the bottom-left corner of the page, and the length of the  $x$ - and  $y$ -axis relative to the size of the remaining (origin shifted) width and height of the plot area. The total size of the plot area is always 2970×2100 pixels (landscape A4). The fifth parameter defines the aspect ratio of the graph area when widescreen mode is used. The remaining parameters are reserved for future use.
- The 4.th line defines the initial values of the magnetic and gravity field components used in pole reduction and pseudo-field computations. These parameters include: the inclination and declination (in degrees from horizontal plane and true north) and intensity (in nano-Teslas) of the magnetic field, the density contrast (in grams per cubic centimeter) and susceptibility (dimensionless in SI units).
- The 5.th line should be left empty.
- The following 15 lines define various text items of the graphs. These include: the main title of the graph, the 6 possible sub-titles of the graph, the axis labels and color scale label of the data in spatial domain, the axis labels and color scale label of the data in frequency domain, the axis labels of the graph of the radial spectra. The maximum length of each text string is 70 characters.
- The 16.th line between the two text blocks should be left empty.
- The last 23 lines define various text items (max. 70 characters) that are shown in the information text next to the graph. These include the (possible) filtering operations made to the FFT data and the processing operation made to the inverse FFT data.

Note that the format of the FOURPOT parameter file is likely to be changed in the future. Also note that the ^ character can be used to define superscripts (exponent), \_ character is used to generate subscripts, and the \$ character is used to move the text back to the baseline.

## **7. References**

- Bhattacharyya, B.K., 1967. Some general properties of potential fields in space and frequency domain; a review. *Geoexploration*, 5, 127-143.
- Blakely, R.J., 1995. *Potential theory in gravity and magnetic applications*. Cambridge Univ. Press.
- Claerbout, J., 1976. *Fundamentals of geophysical data processing. With applications to petroleum prospecting*: McGraw-Hill Book Co.
- Ruotoistenmäki, T., 1987. Estimation of depth to potential field sources using the Fourier amplitude spectrum. Bulletin 340, Geological Survey of Finland. (PhD thesis).

## **8. Additional information**

I made the first version of FOURPOT in 2003 when I worked at the Geological Survey of Finland for the 3-D crustal model project funded by the Academy of Finland. The original idea was to utilize the depth analysis methods of Ruotoistenmäki (1987). In 2005, when I started as a lecturer of applied geophysics in Oulu the objective became more educational and I have used FOURPOT in the teaching of gravity and magnetic data processing.

The Fourier transform is based on the FFT algorithm FORK by Jon Claerbout (1976). Look for his webpage via Stanford Exploration Project (<http://sep.stanford.edu>) for more details. Thanks to Richard Stuart for the comments on smooth frequency filtering. More information about the geophysical use of Fourier transform methods in potential field processing can be found in Blakely (1995), for example.

The FOURPOT program is written in Fortran 90 style using Intel Visual Fortran version 11.1. The graphical user interface is based on the DISLIN graphics library version 9.4 by Helmut Michels. For more information, please, visit <http://www.dislin.de>. Because the DISLIN graphics library is independent from the operating system, FOURPOT could be compiled on other operating systems (Solaris, Linux, Mac OS X) without large modifications. At the

moment, however, the source code is not made available and I do not provide any support for the software. However, if you find the results erroneous or if you have suggestions for improvements, please, inform me.

## **9. Terms of use and disclaimer**

The FOURPOT software is totally free for personal and scientific use. If you find it useful, please, send me a postcard. If you decide to use results computed with FOURPOT in publications, please, use this user manual and the web page at <http://www.cc.oulu.fi/~mpi/softat> as a reference, because I have not (yet) published this work in any journal.

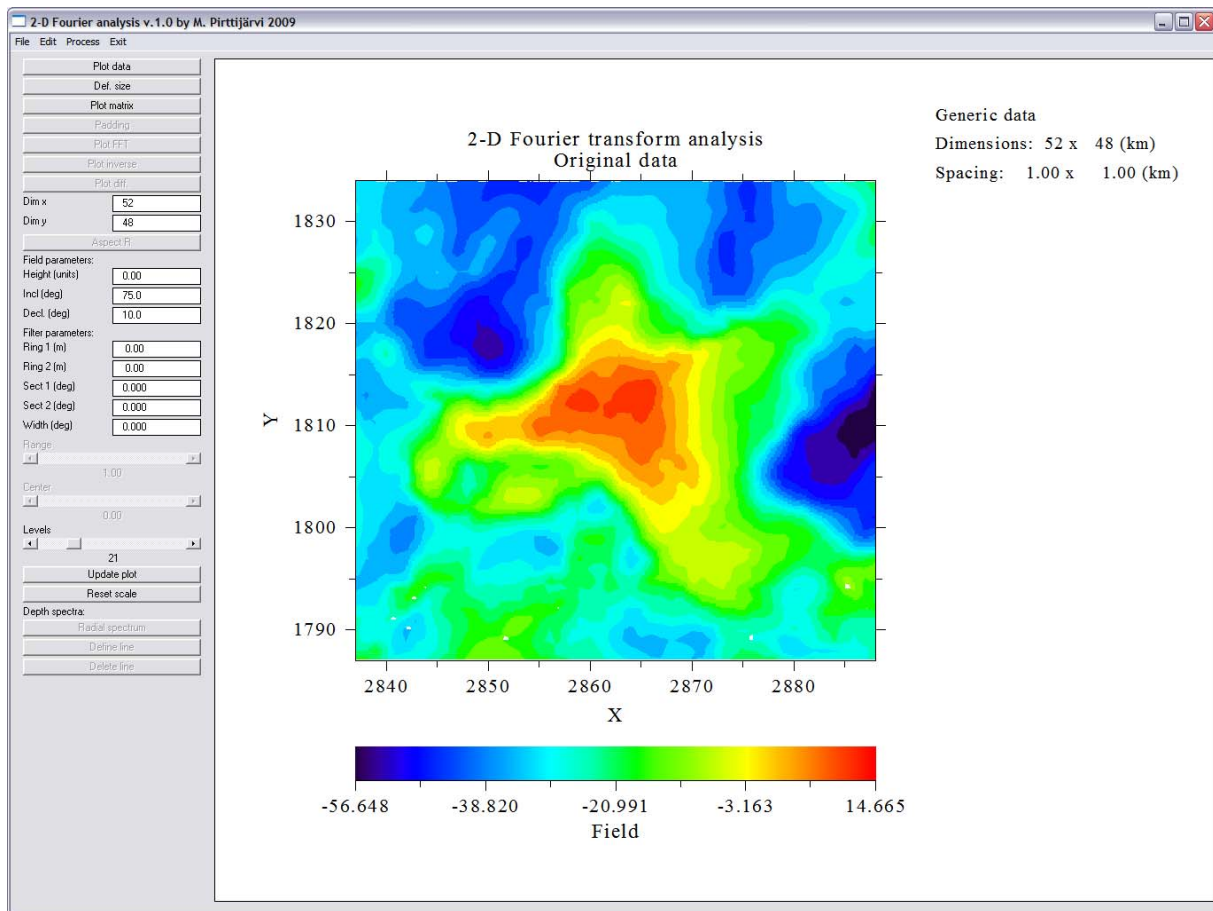
The FOURPOT program is provided as is. The author (MP) and the University of Oulu disclaim all warranties, expressed or implied, with regard to this software. In no event shall the author or the University of Oulu be liable for any indirect or consequential damages or any damages whatsoever resulting from loss of use, data or profits, arising out of or in connection with the use or performance of this software.

## **10. Contact information**

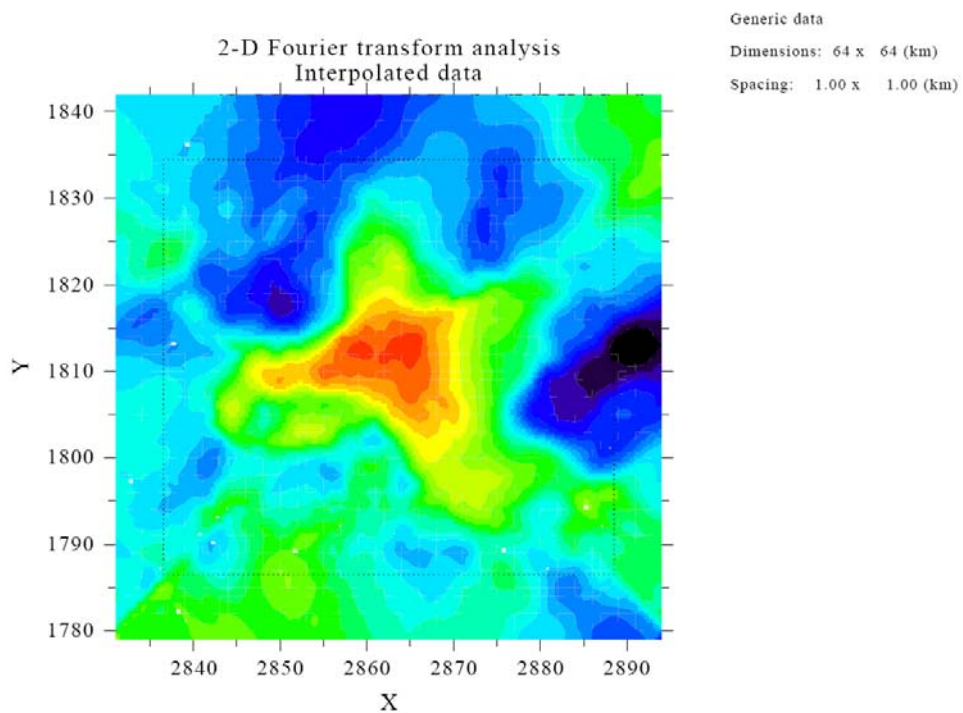
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URL: <http://www.gf.oulu.fi/~mpi>

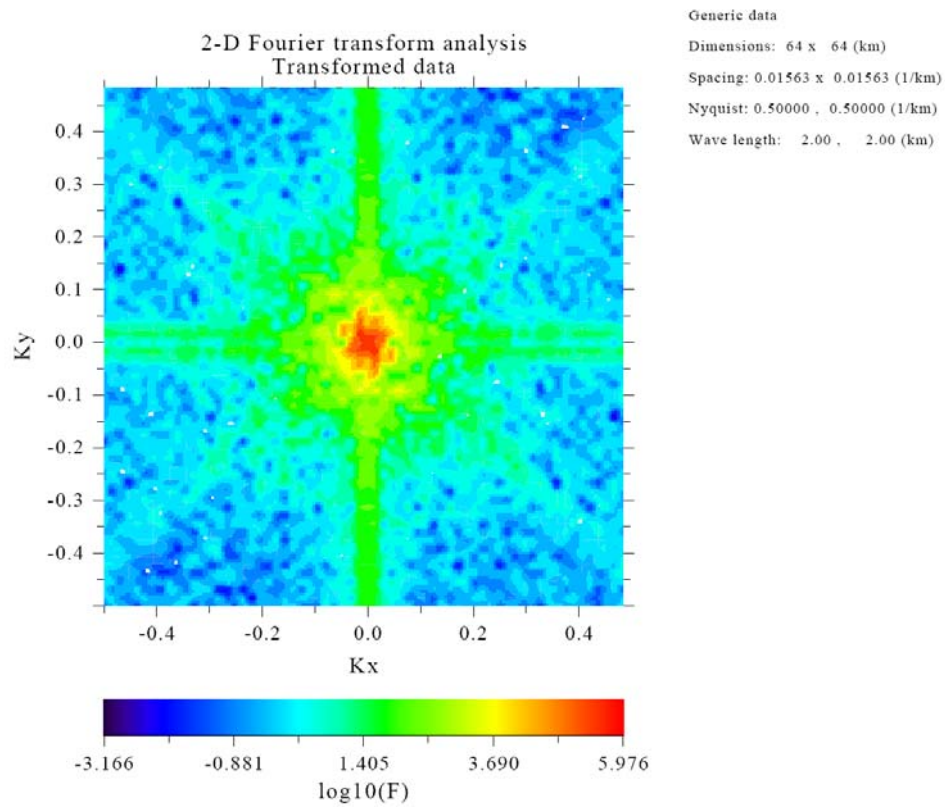
## Appendix A. FOURPOT GUI after data input



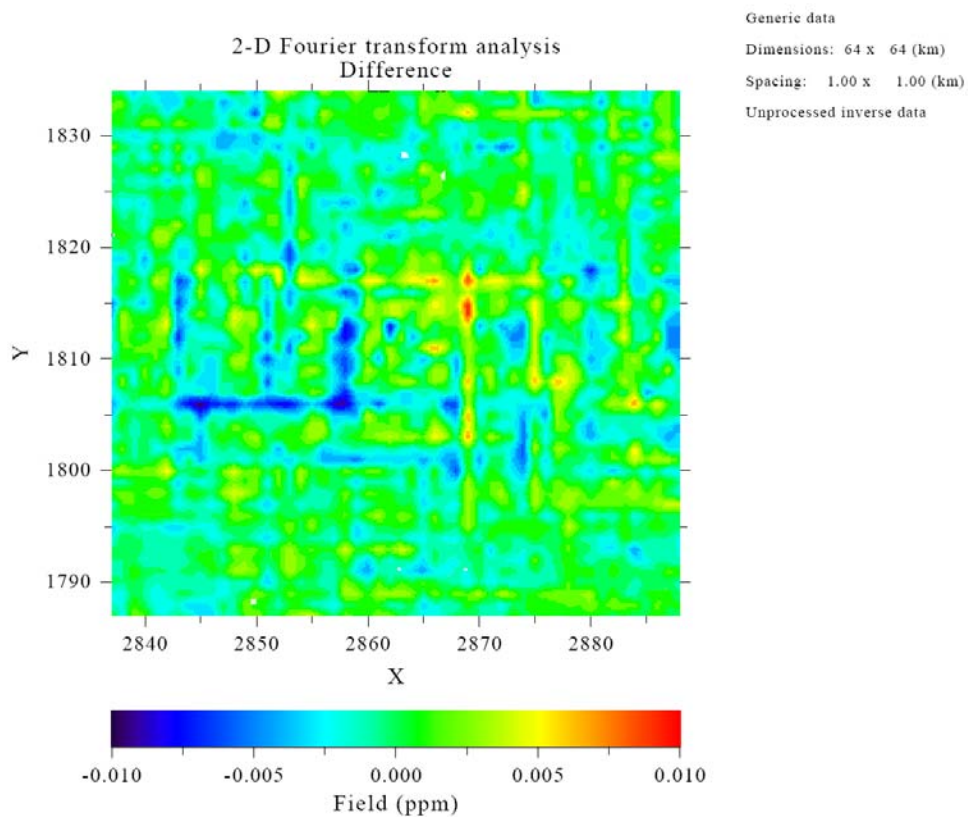
## Appendix B: Data map after padding (with shift) and tapering



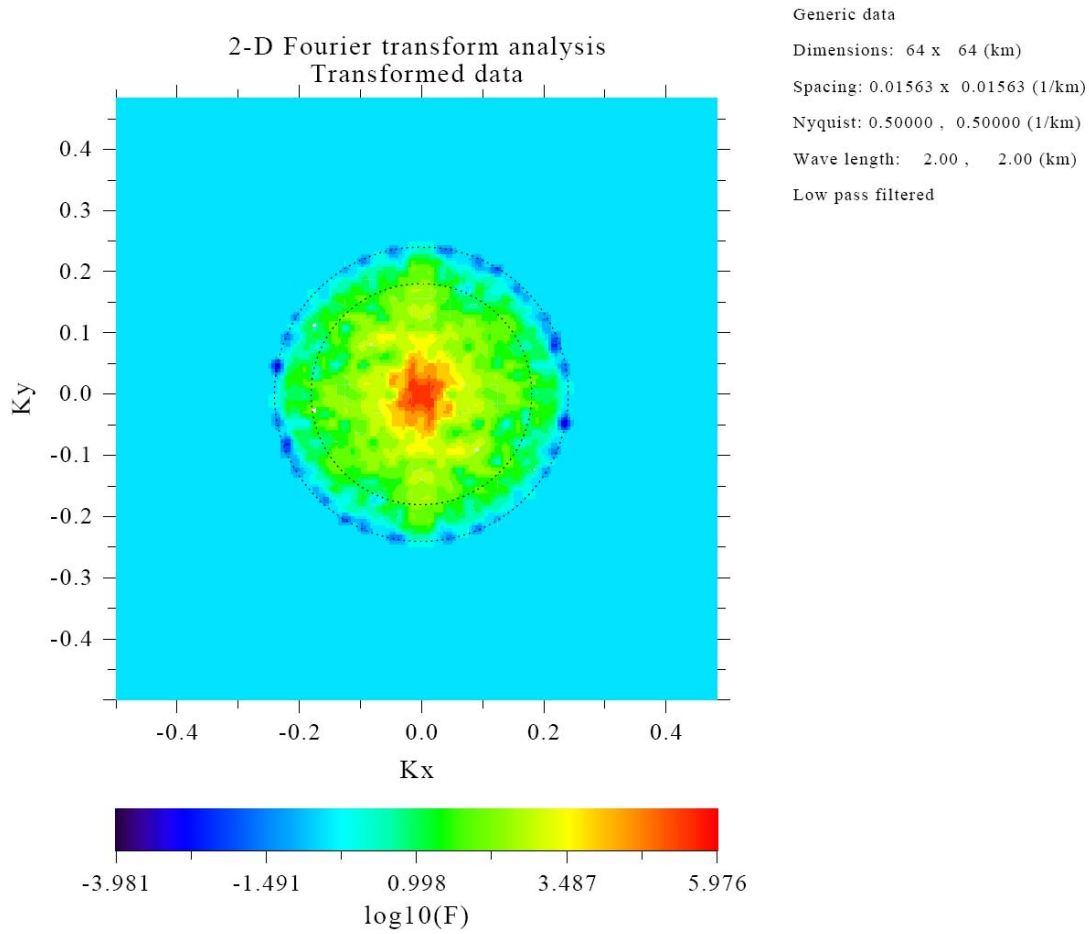
## Appendix C: 2D frequency (amplitude) spectrum



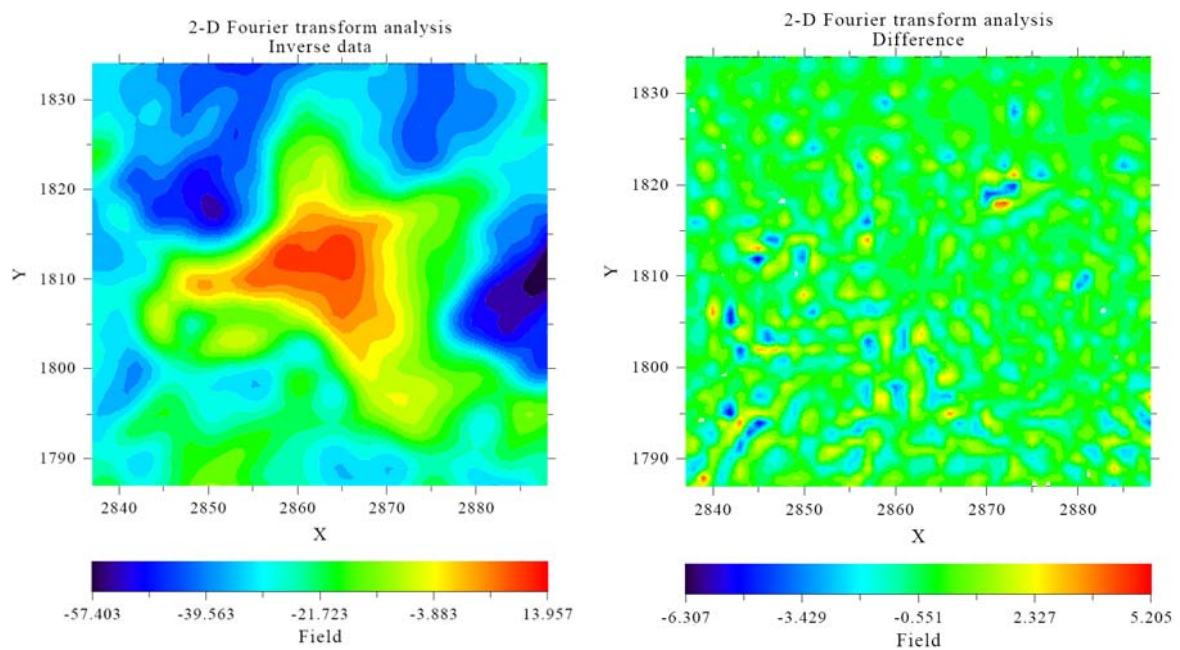
## Appendix D: Difference between original data and inverse transformed data



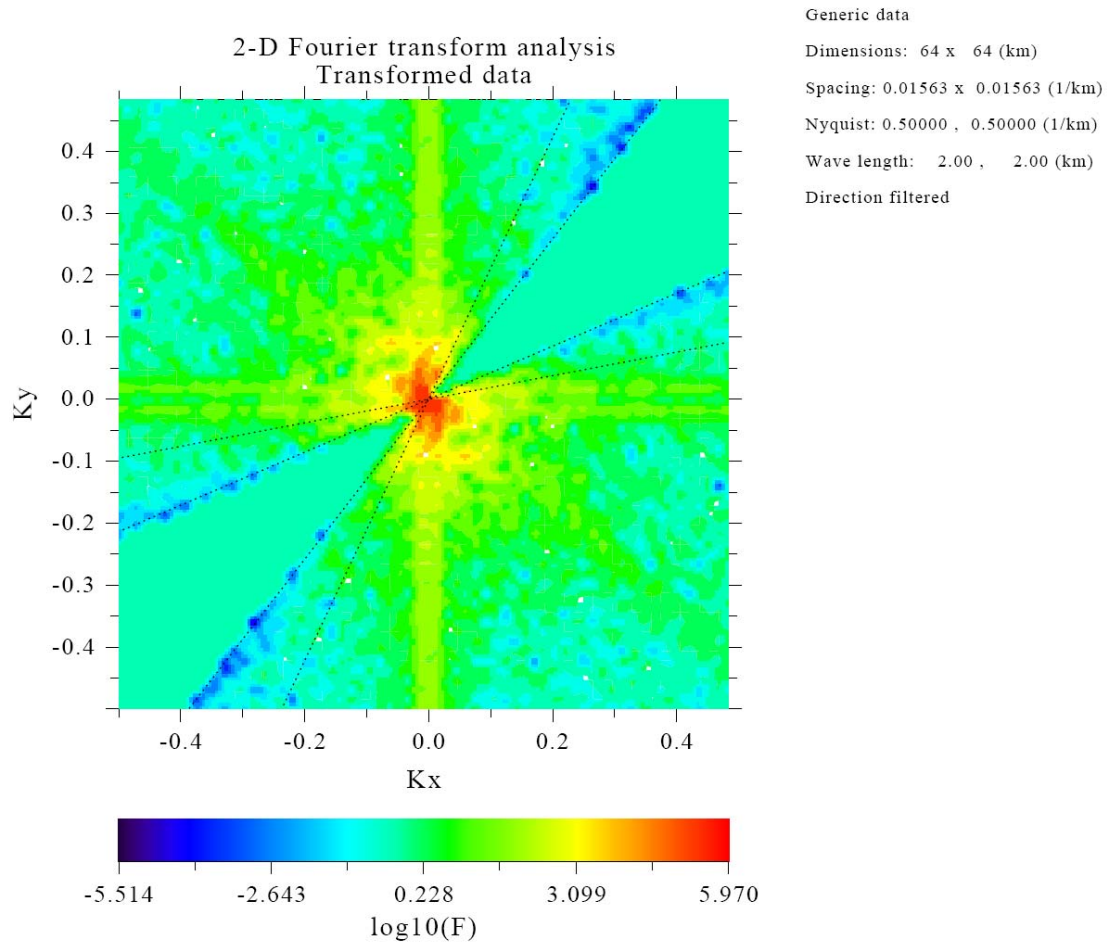
## Appendix E: Low pass filtered frequency spectrum



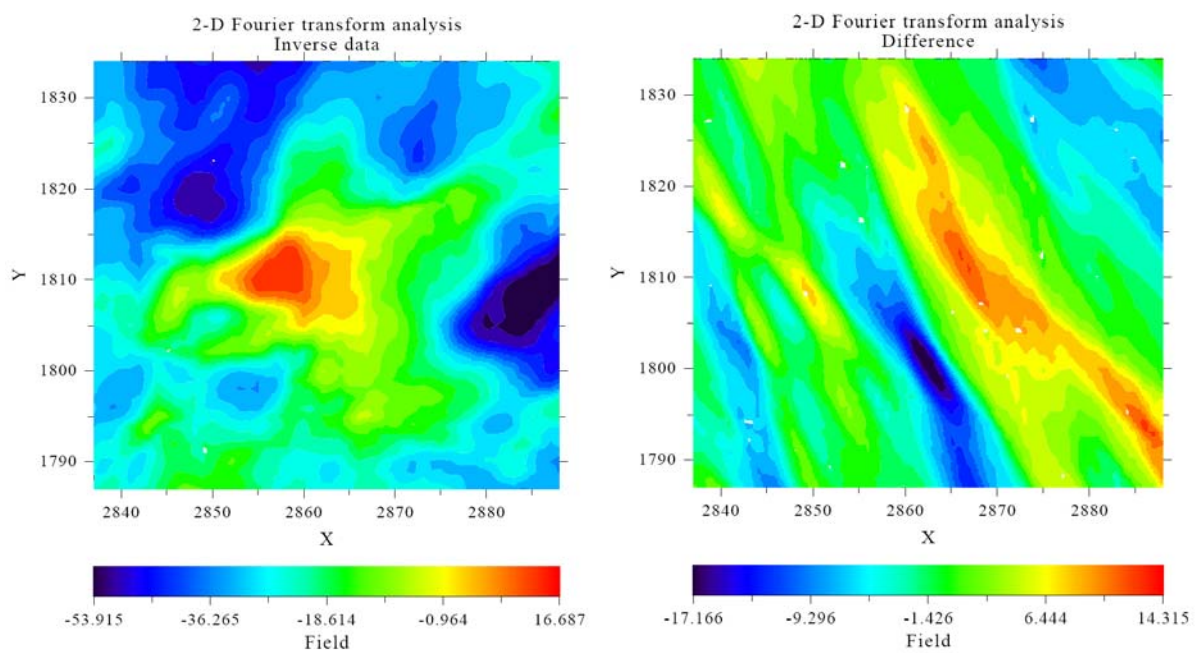
## Appendix F: Low pass filtered data and its difference to original data



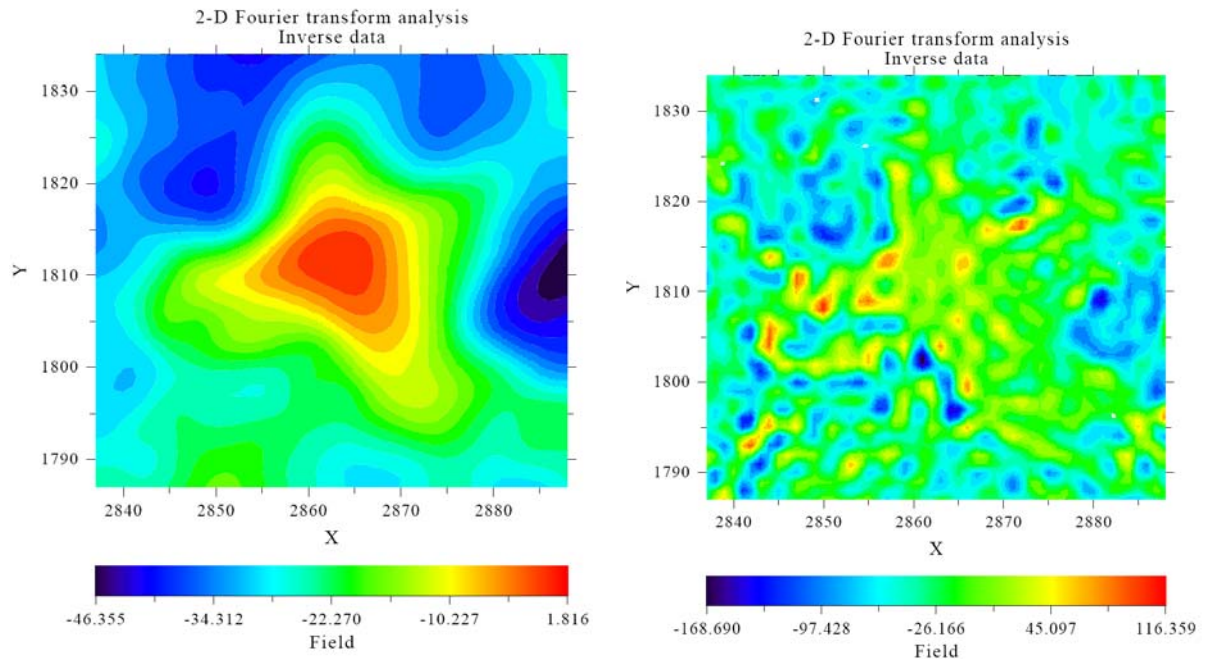
## Appendix G: Direction filtered frequency spectrum



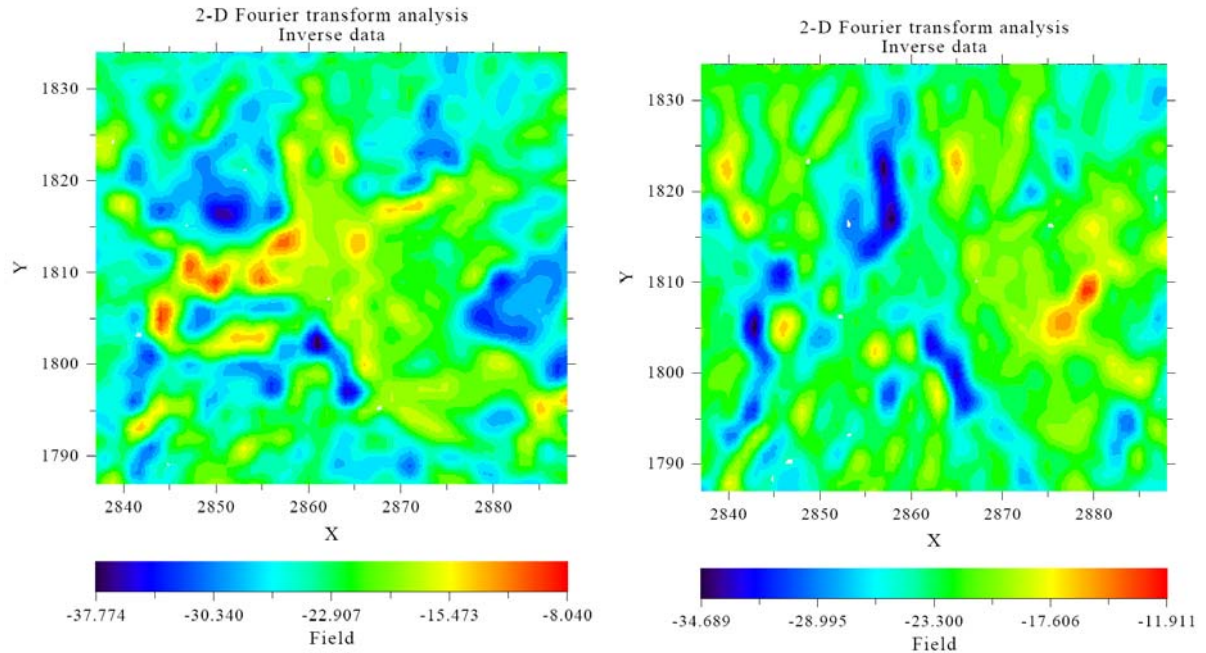
## Appendix H: Direction filtered data and its difference to original data



## Appendix I: Upward and downward continued data (h= 2 km)

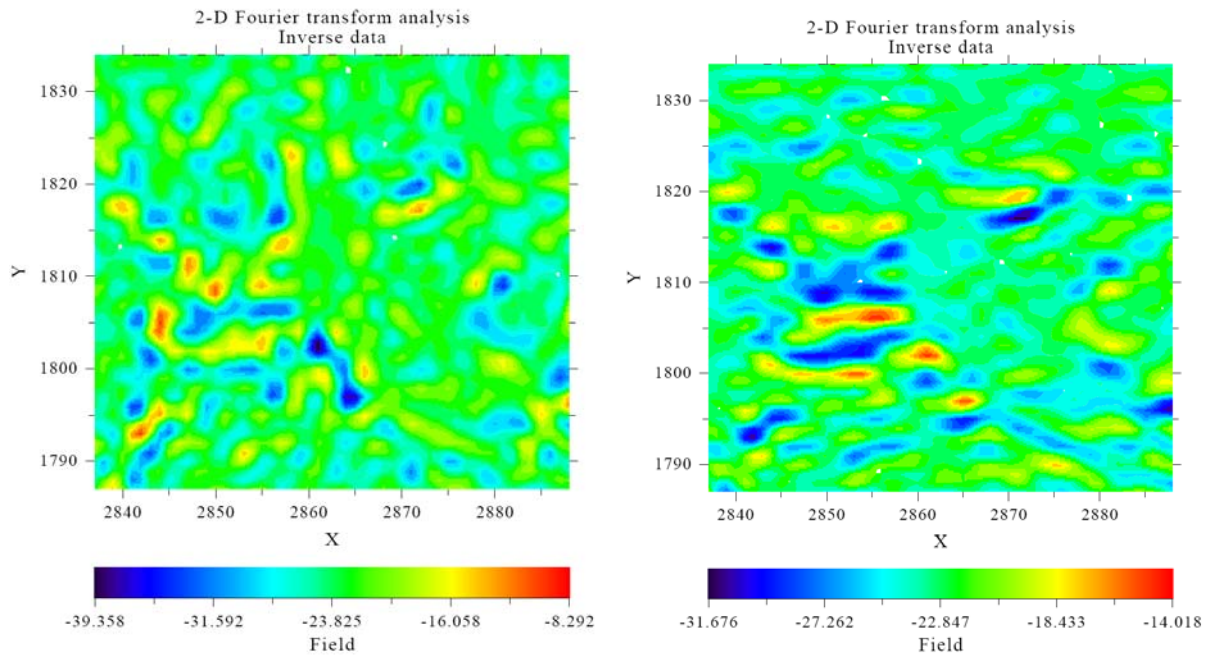


## Appendix J: First vertical and horizontal x gradient

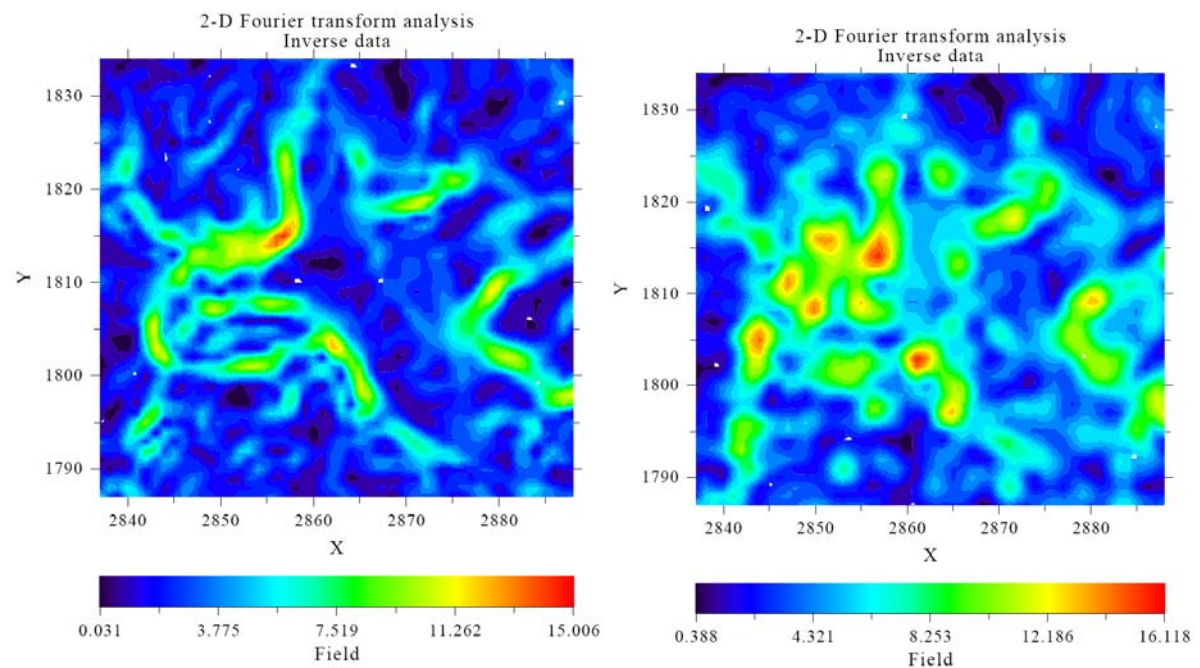




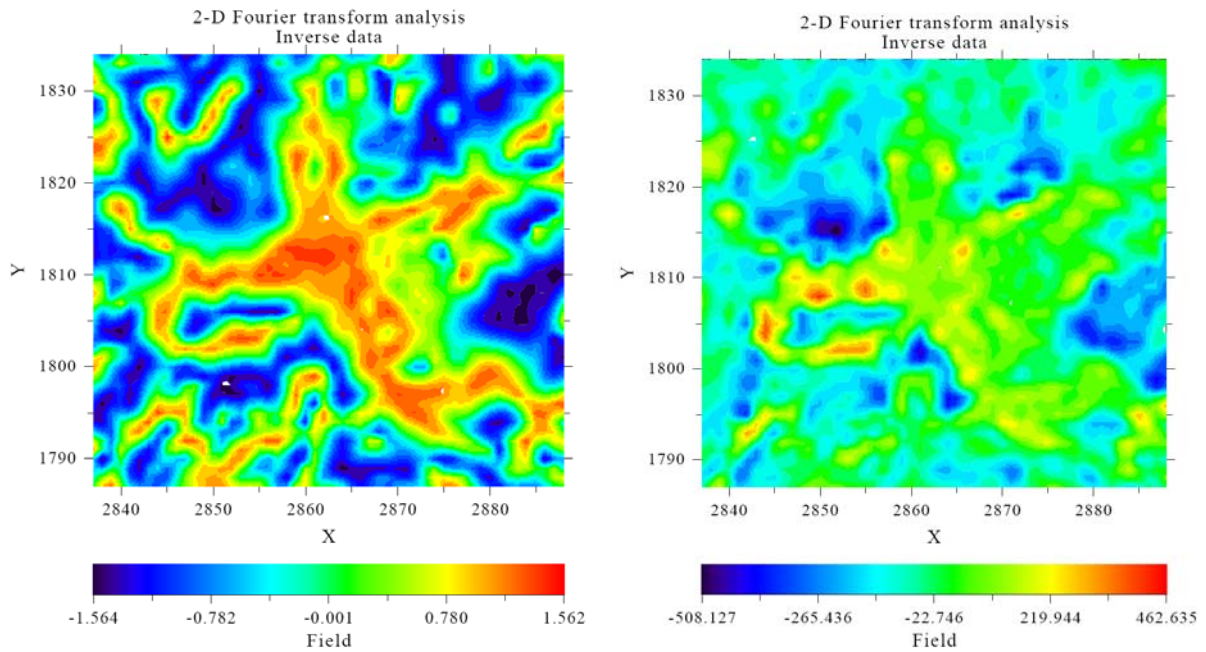
## Appendix K: Second vertical and horizontal y gradient



## Appendix L: Horizontal gradient and total gradient (analytical signal)



**Appendix M: Tilt derivative and pseudo-magnetic field ( $k= 0,01$  SI,  $\Delta\rho= 0,2$  g/cm<sup>3</sup>, and  $T_0= 52000$  nT)**



**Appendix N: Program GUI with 2.nd vertical gradient after low pass filtering and modification of the color scale**

