The Millikan oil-drop experiment: a measurement of *e*

Purpose

To demonstrate the quantization of electric charge and to determine the electronic charge *e*.

Key concepts

- Terminal velocity
- Viscosity; Stokes law
- Brownian motion

References

A copy of Millikan's original paper is available in the lab. It is well worth reading: Physical Review, Volume 32, pp 349-397 (1911).

Introduction

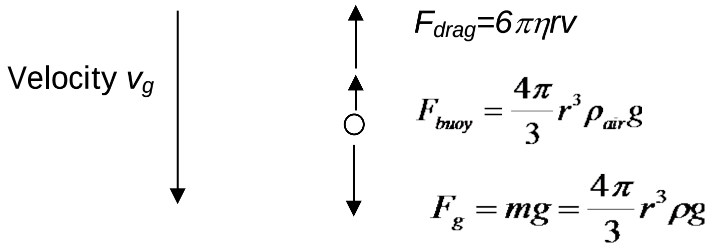
The oil-drop experiment is one of the classic experiments in physics. For the first time, it showed that electric charge is not a continuous quantity but comes only in integer multiples of the basic charge quantum *e*. The first measurement, published by Millikan in 1911, determined the value of *e* with a precision of 0.2%. This measurement also led to the first accurate determination of the mass of the electron (from the previous measurement of *e/m* by Thomson) and Avogadro's number N_A from earlier electrolysis measurements of the Faraday $F=N_Ae$.

The idea of the experiment is very simple. Our experiment uses microspheres of polystyrene instead of oil drops, but is otherwise very similar to Millikan's original experiment.

1. First measure the terminal velocity of a droplet falling under the influence of gravity. Competition between gravity and the viscous drag force exerted by the air enables the size of the sphere to be determined.

2. Then measure the terminal velocity of a charged droplet in the presence of an electric field *E.* This permits a measurement of the additional electric force *qE* acting on the droplet, and thus the charge *q*.

Consider the forces acting on a spherical droplet of radius *r* and density ρ falling with velocity *v* through air with density $\rho_{air.}$



$$F_{drag}=6\pi\eta rv$$

$$F_{buoy}=\frac{4\pi}{3}r^3\rho_{air}g$$

$$F_g=mg=\frac{4\pi}{3}r^3\rho g$$

The total downward force acting on the droplet is

$$F = m\frac{dv}{dt} = mg - F_{buoyant} - F_{drag}. \tag{1}$$

The buoyant force $F_{buoyant}$ is simply the weight of air displaced by the droplet while the drag force on a sphere moving slowly in a viscous fluid is given by Stokes' Law $F_{drag} = 6\pi\eta rv$ where η is the viscosity of the air.

When the acceleration is zero, v becomes the terminal velocity v_g and

$$\frac{4}{3}\pi r^3(\rho - \rho_{air})g = 6\pi\eta r v_g. \tag{2}$$

So, by measuring the velocity v_g we can determine the radius of the droplet:

$$r = \sqrt{\frac{9\eta v_g}{2(\rho - \rho_{air})g}}. \tag{3}$$

You should also be able to show (by integrating equation 1) that, after a droplet is released, its velocity increases as

$$v = v_g\left(1 - e^{-\frac{t}{T}}\right), \tag{4}$$

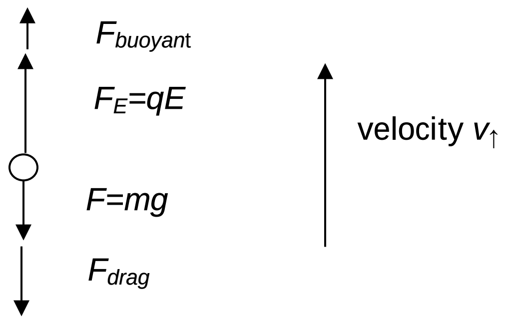
where the time T to reach 63% of the final v_g is $T = \frac{v_g}{g} = \frac{2}{9}\frac{r^2(\rho - \rho_{air})}{\eta}$.

The value of viscosity η at a temperature T (in ^oC) can be found in the CRC Handbook, for example and expressed as

$$\eta_T = 1.824(1 + .005(T - 20^oC))\times 10^{-5} \text{ SI units (kg/m/s).} \tag{5}$$

Before you begin your experiment, calculate the time it will take for a 1 μm diameter sphere to fall a distance of 1 mm and how long it will take for such a sphere to reach terminal velocity. This will be useful in selecting which spheres to observe in your experiment.

Now consider the case where there is an electric charge q on the drop and there is an electric field E present, causing the droplet to rise at terminal velocity $v_\uparrow$.



In this case, the electric force qE can be written as

$$qE = \frac{4}{3}\pi r^3(\rho - \rho_{air})g + 6\pi\eta r v_{\uparrow} = 6\pi\eta r(v_g + v_{\uparrow}). \qquad (6)$$

Finally we have

$$q = \frac{6\pi\eta r(v_g + v_{\uparrow})}{E}. \qquad (7)$$

When the electric field is reversed, so that the electrostatic force and particle velocity point *down*, you can show that

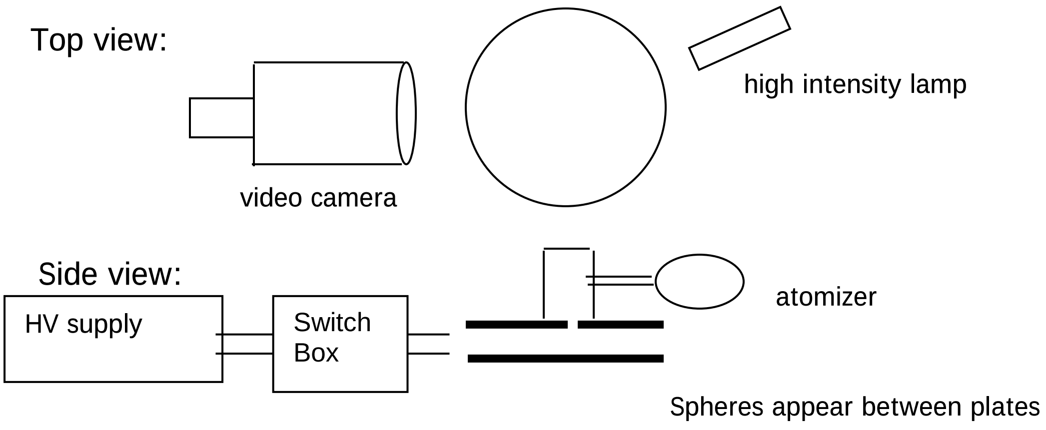
$$q = \frac{6\pi\eta r(v_{\downarrow} - v_g)}{E}, \qquad (8)$$

where $v_{\downarrow}$ is the velocity measured when the electrostatic force is *down.*

The experiment thus consists of measuring terminal velocities of the polystyrene spheres (our “droplets”) as they move under the influence of gravity alone (v_g) and with applied electric fields ($v_{\uparrow}$ or $v_{\downarrow}$, depending on the direction of E).

Apparatus

A sketch of the apparatus is shown below.



This is a modern version of the experiment. You will observe the spheres using a video camera and make velocity measurements by observing their motion on a video monitor.

The main part of the apparatus consists of two aluminum plates separated by a circular lucite ring. This ring separates the plates by a distance 0.750 in. A solution of microspheres is squirted into a container attached to the upper plate, and the spheres fall through a small 0.5 mm diameter hole into the viewing volume. The central region between the plates where the spheres appear is illuminated by a high intensity lamp, and the video camera is focused on this region. The spheres appear as faint light objects against a dark background.

The experiment is strongly affected by vibrations and by convection currents in the air. To minimize these effects, the apparatus is mounted on a heavy metal base-plate and an infra-red (heat) absorbing glass is placed at the end of the lamp to avoid heating the air in the gap. Be careful not to touch the glass - it gets very hot!

A variable high voltage (up to 7.5 kV) is applied between the plates. The switches that are attached to the base plate allow three different configurations: 1) both plates shorted together, 2) upper plate at positive potential with respect to the lower plate , and 3) the upper plate at negative potential with respect to the lower plate. The high voltage is fed through a large series resistor that limits the current if you happen to touch the apparatus. Nonetheless, this is not advised! The potential difference can be varied by means of a large potentiometer knob and its value determined using a DVM (where 1 volt accurately corresponds to 1 kV potential difference). In normal operation, both switches in the right-hand switchbox, which controls the polarity should be moved *together.* The labels “positive” and “negative” refer to the potential of the top plate relative to the bottom plate. (What does this imply about the force on *negatively* charged particles?) The switch in the left-hand switchbox shorts the plates together when you are measuring the free-fall velocity v_g . When you wish to apply high voltage, leave this switch in the “operate” position. Ask your instructor if you are confused about how these switches operate.

Procedure

The video monitor has several equally spaced horizontal lines marked on its surface. You will need to calibrate the distances between these lines using a ruler situated at the center of the plates. A photograph of the monitor taken with the ruler in place is provided at the end of this section of the manual. Note that the 1/64” scale is shown. Use this photograph to calibrate the apparatus.

There are three adjustments on the camera: an adjustable iris to vary the overall light intensity reaching the CCD camera, a zoom ring, and a focusing ring. The zoom ring is set at maximum magnification. The focus ring is locked in place so that the region at the *center* of the plates is in focus. **Do not change the zoom or focus settings of the camera, as these will change the magnification and hence the calibration!** Translating the entire camera using the micrometer on the based will not change the magnification. Use this micrometer to adjust the focus during the experiment. The iris can also be adjusted at any point during the experiment. It is often advantageous to have the iris nearly closed. This increases the depth of focus.

BEFORE proceeding: Make sure that you have correctly calculated the time that you expect for a 1 μm diameter sphere to fall one division on the monitor. The diameter of the spheres that we are using is approximately 1 μm. *Ignore all objects on the monitor with free-fall velocities that deviate more than 20% or so from this value!* Next, calculate the times that you expect for a charged sphere (1 e) to move one division under positive (electrostatic force pointing up, Eq. 7) or negative (electrostatic force pointing down, Eq. 8) potentials. Assume a potential of 1 kV. Make sure that you have read the rest of this write-up and have prepared an Excel spreadsheet to perform your analysis.

The solution of spheres is in a small vial taped to the side of the apparatus. Remove the lid of the vial and place the small tube from the atomizer into the solution. Give the atomizer 2 – 3 puffs. You should see the spheres appear on the monitor. Wait one or two minutes before applying a voltage.

Set the polarity switch so that negatively charged spheres will move upward. Start with 1 kV. Ignore the spheres that move very rapidly when you turn on the voltage. They have a large charge. The spheres receive their electric charge due to *triboelectricity* (friction), as they are squirted into the air. With practice, it is possible to keep a single sphere within the field of view for many minutes by either letting it fall under gravity alone with the plates shorted or by applying positive or negative voltages.

Obviously, for maximum sensitivity, you want to select the spheres with the smallest charge on them, i.e. the ones that move the slowest when the electric field is turned on. Do not waste your time trying to measure spheres with very large (more than 5e) charges. Use a stopwatch to measure the time a sphere takes to pass between the lines on the monitor (e.g. t_E/division), and then to measure the time it takes to fall under gravity alone (t_g/division**)**. Estimate the uncertainty in these time measurements. **It is especially useful if you can continue to observe a single drop for two or three consecutive upward and downward excursions**. **For the downward excursions, you should make some of your measurements with the plates shorted (to measure v_g) and others at "negative" potential (to measure $v_{\downarrow}$).** You will probably notice that the spheres exhibit *Brownian motion*, which is due to thermal agitation. This makes it difficult sometimes to keep a sphere within the field of view for extended periods, so you must choose whether it is best to make more measurements over short distances or to make fewer but more accurate measurements over greater distances. If you find that the spheres rapidly drift out of your field of view, then it is likely that the apparatus is not level.

You will find the voltage of 1 kV convenient for measuring charges of 1e, 2e (use two divisions on the monitor), and (if you use all four divisions on the monitor) 3e. You can then decrease the voltage to 500 V to measure charges of 3e to 5e. Note that you can use either positive or negatively charged spheres. In analyzing the data, it is only important to know whether a sphere is moving up or down. Think carefully about your data collection strategy. In particular, make sure that you cover a range of charges, and that timing errors (probably on the order of 1 sec for this apparatus) do not wash out the effects of quantization. One hint: make sure that your data collection time for a given sphere is at least 20 seconds and use an appropriate number of divisions on the monitor. Data on highly charged spheres tends to be less useful. The idea is to collect a large number of drops over a range of charges 1 – 5e so that you can clearly see the effects of quantization.

You will need measurements on at least 15 – 20 separate spheres with a range of charges to reach a conclusive result. Do not forget to make a note of the air temperature during each lab period.

Data analysis

The density of the spheres is 1.05 g/cm^3.

For each sphere that you observe, determine the velocity v_g and its uncertainty, then determine the radius r of the sphere using Eq. 3. If you have made several measurements on the same drop, you can average them to determine v_g.

Now determine the velocities $v_\uparrow$ and $v_\downarrow$ for each of your measurements with positive and negative potentials respectively. Note that the charge on a sphere can change between measurements and hence these measurements should be tabulated individually (do not average them). It is very unlikely that the charge will change while an electric field is applied.

An important correction to your data

Now it seems that you are ready to calculate the charge q on each of the spheres using Eqs. 7 and 8. However, the expression given for η_T in equation 5 is slightly wrong in the present situation! The expression for the drag force $6\pi\eta rv$ (Stokes' Law) actually applies to streamline motion in a uniform fluid, but in this experiment, the radii of the spheres are typically just a few times the mean free path of air molecules and this "graininess" of the air reduces the effective viscosity. Taking this into account, the relevant value for the air viscosity can be written as

$$\eta = \frac{\eta_T}{\left(1 + \frac{a}{rP}\right)} \quad , \qquad (9)$$

where r is the radius of the sphere in m, $a = 8.1\times10^{-8}$ m, and P is the air pressure in atmospheres. The length a/P is close to the mean free path of the air molecules. (In fact, Millikan used his data to determine the value of *a* directly, by inspecting the small, systematic variations in his derived values of *q* as a function of *r*. For your experiment, however, it will be sufficient to use this corrected value of η).

Substituting Eq. 9 into the expression for the sphere radius *r* (Eq. 3) and squaring we obtain an expression for the correct radius r_{corr}:

$$r_{corr}^2 = \frac{r^2}{1+\frac{a}{r_{corr}P}} \quad .$$

This can be rewritten as a quadratic equation

$$r_{corr}^2 + \frac{ar_{corr}}{P} - r^2 = 0 \ ,$$

which has a solution correct to first order

$$r_{corr} = r - \frac{a}{2P} \quad . \qquad (9)$$

For each sphere, determine its radius r_{corr} using this equation, make the correction to the air viscosity, and then determine the charge *q* on the sphere using Eqs. 7 and 8.

At this stage, you should be able to see the effects of charge quantization in your data. A useful way to look at this is to order your values for *q* in ascending order, and then to Insert _Chart _Column in your Excel sheet).

Finally, assign a value of the integer number of charges on each sphere and determine a best value for the charge *e*. A quick way to do this is to simply assign an integer value n_i to each data point q_i "by eye" and then determine *e* from a weighted average of q_i/n_i.

A better approach uses a chi-squared analysis. Let

$$y_i = q_i/e \qquad (10)$$

be the result of dividing a measured charge q_i by *e.* If your data were "perfect," y_i would be an integer. The difference $y_i - ROUND(y_i,0)$ represents the deviation of y_i from the ideal value. The function ROUND(y_i,0) evaluates the nearest integer to y_i and is available in Excel. The best choice of *e* minimizes

$$\chi^2(e) = \sum_i \frac{1}{\sigma_{y_i}^2}[y_i - ROUND(y_i,0)]^2, \qquad (11)$$

where $\sigma_{y_i} = \sigma_{q_i}/e$. You can implement this process in Excel by choosing 10 or so test values for *e* and finding the minimum in $\chi^2(e)$. If your weighting is correct, you can then use the scheme in Appendix A.10 (the "$\chi^2 + 1$ rule") to estimate the error in *e.* You will probably find that the error estimated in this way is a few percent. There may, however, be systematic errors in the experiment. Discuss the possible systematic errors.

A note on error analysis

A simple means of estimating the uncertainty in *q* due to measurement errors in v_g is to simply to replace the value of v_g in your Excel worksheet by the value $v_g + \Delta v_g$ and to see by how much the final result for *q* varies (remembering that $\Delta v_g / v_g = \Delta t_g / t_g$). Similarly, you can determine the effect of uncertainties in $v_\uparrow$ or $v_\downarrow$ due to timing errors. You can then assume that errors in *q* due to uncertainties in v_g and $v_\uparrow$ (or $v_\downarrow$) add quadratically.

Measurement of *e/m* for electrons

Purpose

The object of the experiment is to obtain a value of *e/m* for electrons from a measurement of their deflection in a uniform magnetic field. The first measurement of this quantity by J. J. Thomson, over 100 years ago, was crucial to acceptance of the idea of electrons as particles. Evaluation of the systematics is key to this experiment. Quantitatively examine the effect of the Earth's field and low anode voltage (< 200 V) on your results.

Key concepts

Lorentz force
Helmholtz coils
Electron gun

Introduction

When a charged particle moving with velocity ***v*** passes through a magnetic field ***B*****,** it experiences a force *e**v** x **B*** where *e* is the particle's charge. Since this force is perpendicular to the direction of motion, it is a centripetal force and, if the magnetic field is uniform, the trajectory of the particle will be a helix, in general. If ***v*** is perpendicular to ***B*****,** the trajectory is circular and we have:

$$F = \frac{mv^2}{r} = evB,$$

where *r* is the radius of the circle.

Electrons originating from a heated filament and accelerated across a potential difference *V* between the filament and a positive plate will have a final kinetic energy given by $\frac{1}{2} mv^2 = eV$ assuming that they are emitted with negligible energy. Thus

$$v = \sqrt{\frac{2eV}{m}}.$$

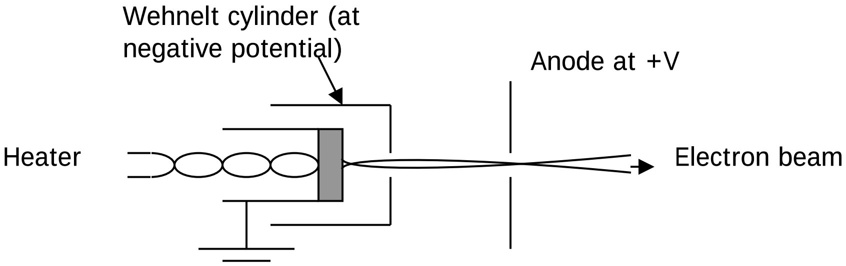
If these electrons are then allowed to enter a uniform magnetic field, the velocity *v* can be eliminated from these equations, giving a relation

$$V = \frac{e}{2m}(rB)^2.$$

The principle of this experiment is to measure *V* as a function of $(rB)^2$.

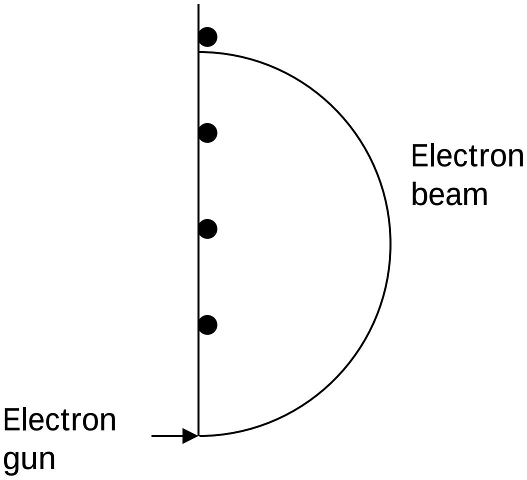
The apparatus

The apparatus consists of an evacuated spherical glass vessel containing an electron gun and acceleration electrodes. The electron gun consists of an indirectly heated barium oxide-coated cathode, a Wehnelt cylinder for focusing the beam and an anode with a hole in it, through which the electrons eventually emerge with energy *eV.*



After being accelerated, electrons emerge from a hole in the anode into a region which is free of electric fields. They travel at constant velocity in this region. To make the electron paths visible, the tube contains a small amount of a noble gas that becomes ionized and glows when electrons pass through it.

The diameters of the electron orbits resulting from application of a magnetic field can be determined by reference to a "ladder" which is placed vertically in the tube. The rungs on the ladder are separated by 20 mm.



The coils providing the deflecting magnetic field are configured as "Helmholtz coils", with coil separation equal to the coil radius, R_C. You should be able to show (using Biot-Savart) that the magnetic field at a distance x along the axis of a coil with radius R_C and carrying a current I is given by

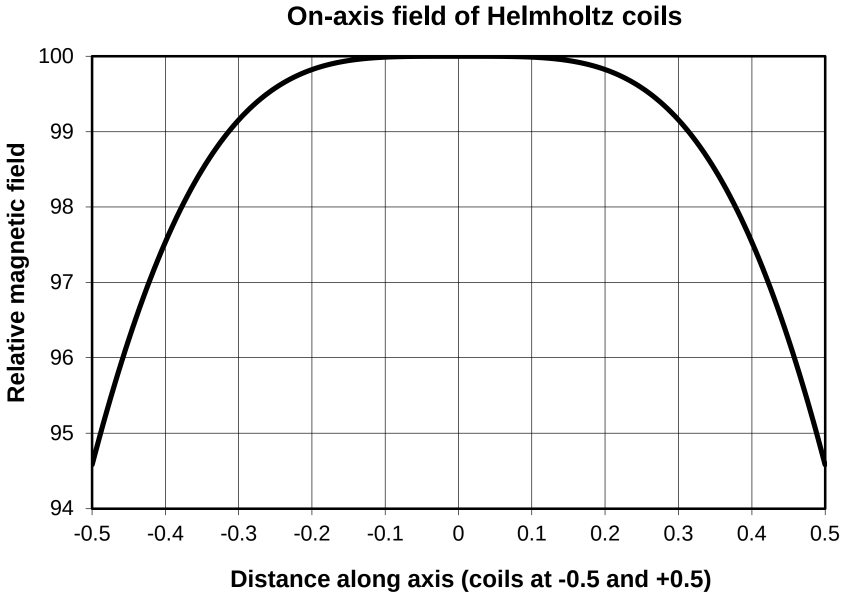
$$B = \frac{\mu_0}{2} \frac{NIR_C^2}{(R_C^2 + x^2)^{3/2}},$$

where N is the number of turns on the coil.

If two coils are separated by a distance *d,* the field at the center is then given by

$$B = \mu_0 NI \frac{R_C^2}{\left(R_C^2 + \left(\frac{d}{2}\right)^2\right)^{3/2}}.$$

When $d = R_C$ (the Helmholtz coil configuration), it is easy to show that $\partial B/\partial x = 0$ (obviously) and, more importantly, that the second derivative $\partial^2 B/\partial x^2 = 0$ at the center. The result is that the field is very nearly uniform over an extensive volume near the center, as shown in the diagram below. (Note the suppressed zero on the vertical axis).



Procedure

First measure the Helmholtz coils so that you can determine the magnetic field for a given current, i.e. determine the constant *k* in the relation *B=kI.* There are 124 turns on each coil. Given that the radius of the coils is equal to the separation, what is the best way to measure the diameter?

Is the earth's magnetic field likely to affect your results? According to NOAA geophysical data the magnitude of the field in Minneapolis is 0.56×10^{-4} Tesla, and points mostly downward with a small component north. Find the field components and write them in your logbook. Good information source is http://www.ngdc.noaa.gov/geomag-web/?id=declinationFormId#igrfwmm
Compare the maximum horizontal component of the earth's field with the field you expect when a current of several amperes is passed through the Helmholtz coils. Orient your apparatus to either minimize the effect or measure the effect.

This could include taking some data with and against the field or calculating the effect of a sideways or downward pointing field. This is an important systematic error.
You might want to do all this for the Pre-Lab and be prepared ahead of time.

The apparatus includes a unit that provides three of the power supplies that you will need: one that supplies the heater potential, one the negative (beam focusing) potential to the Wehnelt cylinder, and one that supplies up to 300 V to the anode. Connect all the grounds at the power supply and at the base of the apparatus (the cathode), then single wires should go to the appropriate connectors on the apparatus. There is a separate current supply for the Helmholtz coils. For maximum accuracy, use separate multimeters to measure the anode potential and coil current, rather than those attached to the supplies.

To begin, turn the anode supply to maximum (~300V) and then turn the heater supply to 6 – 7 V. You should see the heater light up and, after a short time the cathode should be hot enough for you to see an observable electron beam. **The cathode has a finite lifetime, and so the heater potential should always be kept at a minimum, only enough to produce an observable electron beam.**

Turn on the current to the Helmholtz coils (making sure that they are connected correctly) and align the tube so that the beam forms a circle, not a spiral which occurs when the beam and magnetic field are not precisely perpendicular. Vary the Wehnelt voltage to see the effect on the beam. Adjust the Wehnelt voltage in order to produce the narrowest beam possible.

Take a series of measurements of the anode potential V and coil current I for each rung of the ladder. Repeat some measurements to estimate errors. You should use parallax to determine where the beam actually passes a given rung, i.e. there are actually two rungs at each diameter, separated by a small distance. View directly along these rungs to ensure that your measurements have no dependence on viewing angle.

Results and Systematics

Now determine e/m from your data in two different ways: (1) calculate e/m and its uncertainty for all measurements and form a weighted average. (2) Rewrite the formula for e/m so it represents a slope (or inverse slope), then fit using LSQ-fit, making sure that the largest errors correspond to the vertical axis. Look at your results and see if certain groups of points are systematically higher or lower, or seem to have a different slope. If they do, you should try to fit them separately and then quantify the differences. This is how we look for systematic errors, e.g do your individual values of e/m show any dependence on orbit radius r, or on the potential V? If groups of points seem to be different, then give the data points different symbols and try fitting their slopes separately. Reevaluate your assigned error and then combine the slopes in a weighted average. A plot of

$\sqrt{V}$ *vs.* I can be revealing. Use this to determine if there are any background magnetic fields that might be affecting your measurement of e/m.

Photo-electric effect: a determination of *h*.

Purpose

To determine Planck's constant using the photoelectric effect, in which light removes electrons from a metallic electrode.

Key concepts

Photons
Monochromator
Diffraction grating
Photoemission
Stopping potential
Work function

Introduction

The photoelectric effect was first observed a little over 100 years ago; it is the emission of electrons from a surface when exposed to short wavelength light. The effect was not explainable using the wave theory of light, which was prevalent at the time. In 1905, however, Einstein explained the effect, based on Planck's quantum theory of light emission and absorption. It is readily understandable in terms of the complete absorption of incident **photons** each with energy $h\nu$, where ν is the frequency of the light beam. This energy is transferred to electrons in the surface, which results in a linear relationship between the energy of the absorbed light photon and the maximum energy of the emitted electron.

For many years the experiment to check this prediction was carried out by applying a retarding potential between the anode and cathode of a vacuum tube and measuring the voltage needed to bring the current of photo-emitted electrons to zero. This was a difficult measurement because it not only required measurement of a very small current but, more importantly, the current did not go to zero at a well-defined potential.

With the development of very sensitive solid-state amplifiers (integrated circuits), another technique became possible. In the equipment used in the present experiment, electrons knocked out of a cathode (-) strike a wire ring anode (+). As this ring charges up, its potential decreases relative to the cathode and electrons need greater and greater energies to overcome this potential difference. Eventually, when the potential difference equals the maximum energy of the photo-emitted electrons (the stopping potential), no more electrons are collected, causing the potential to stop increasing. A photocell similar to the one used in the experiment is on the lab-bench. The large metallized surface is the cathode, and the ring is the anode.

Measuring the stopping potential is not so simple! A voltmeter typically measures potential by measuring the current developed across a large resistance. However, if a typical digital voltmeter (DVM) input resistance of a few MΩ were placed directly across the anode-cathode system (capacitance C~10-20 pF), the time constant RC of the system would be a small fraction of a millisecond. Unless the photocurrent charging time were very much less than this, charge leakage from the anode would be so large that the anode potential could never reach the stopping potential. Modern field-effect transistors have input resistances up to $\sim 10^{14}\Omega$. Using such a transistor to make a unity gain amplifier or buffer makes it possible to have such a very large input resistance, but with a low resistance at the output of the circuit, i.e. the DVM is buffered or separated from the input circuit. The output voltage is the same as the input voltage, but the voltmeter resistance does not affect the time constant of the circuit. Here, an integrated circuit operational amplifier which uses a field effect transistor is used to provide such a buffer. You will learn more about this type of circuit in Physics 4051.

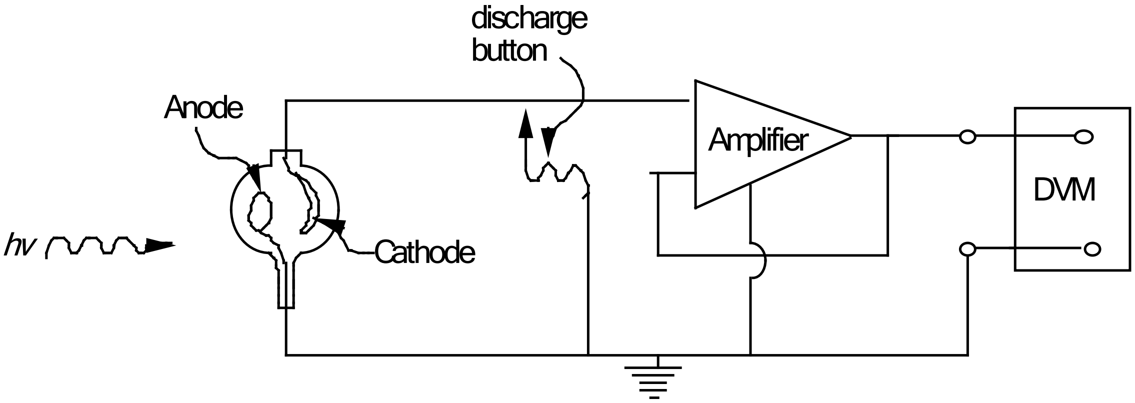


Figure 1. Schematic of a typical circuit diagram for reading out photoelectrons

In the present experiment light photons of various energies $E = h\nu = hc/\lambda$ are provided by a small incandescent bulb. Various wavelengths can be selected by a **monochromator,** which consists of a diffraction grating and some mirrors. The experiment consists of, first, calibrating the monochromator, i.e. determining the relationship between wavelength and monochromator dial reading, and then shining known wavelengths on to the photocathode of the photocell and measuring the corresponding stopping potentials.

Procedure:

A. Calibration of the monochromator

Look inside the monochromator to see how it operates (top swings open). The angle of the diffraction grating to the incident light beam is adjusted by means of a multi-turn dial. Calibration consists of determining the relationship between dial reading and the wavelength of the light emerging from the output slit. You are provided with a mercury lamp, which provides several sharp lines of well-known wavelengths:

yellow	579.05 nm
yellow	576.96
bright green	546.07
blue	491.60
bright blue	435.84
faint blue	434.75
faint blue	433.92
violet	407.78
violet	404.66

1. Find the photodiode and make sure that the front of the enclosure tube is protected with a rubber stopper.

2. We will be using a separate spectroscopic Hg lamp with its own power supply. The supply that powers the fancy DVM is labeled “Photoelectric Effect Apparatus”. Toggle the detector (not Lamp) switch on the power supply to “on” and also the power push button on the DVM. Do it **now** because it takes 20 minutes for the DVM to warm up.

3. Turn the metal knob above the input slit to make a slit width of approximately 0.5 mm.

4. Turn on the Hg light and adjust the light position and the input mirror to focus an intense beam of light on the slit.

5. Remove the photoelectric detector assembly (keep it upright) and insert the eyepiece.

6. Find a bright line. Move the eyepiece to obtain a sharp image and put the cross hairs in the X orientation.

7. Obtain dial readings for as many of the Hg lines listed above as possible. Make a graph of dial reading vs. wavelength λ. The points should lie on a smooth line which is almost straight. Using the Excel workbook *LSQ Fit*, fit these data to a straight line. You should include the errors on your dial reading, but you are assuming that the error on the line’s wavelength is negligible. Since LSQ-fit only handles errors in y, the dial reading should be on the y-axis. Using the results of your fit, you will be able to determine the wavelength that corresponds to a given dial setting.

If you encounter some difficulty in assigning dial settings to the wavelengths given above, you will probably find it helpful to first plot data only for the most

intense lines, then add other data when the overall trend is seen. The relative intensities of the lines can be found either in the *CRC Handbook of Chemistry and Physics* or at http://physics.nist.gov/PhysRefData/ASD/lines_form.html
The spectrum you want is Hg I.

B. Measure electron energy as a function of incident wavelength.

1. Set up the DVM as follows. Voltage range should be set to *Stopping Potential -2 – 0V* and current ranges to 10^{-13}.

2. Set the current amplifier to zero as follows: first *disconnect* the ‘A’, ‘K’ and ‘down arrow’ (GROUND) cables from the back panel of the apparatus. Press the phototube signal button in to *calibrate* setting. Adjust the current calibration knob until the current reads zero. Press the phototube signal button again to get back to the *measure* setting.

3. Replace the Hg lamp with the incandescent lamp. Adjust the lamp position and input mirror so that the light beam is narrow and focused on the slit.

4. Set the monochromator for a wavelength in the green (~ 530 nm). Remove the eyepiece and hold a piece of white paper over the exit port of the monochromator. Verify that you see a bright green spot (square in shape) centered on the exit port. Adjust the focus on the entrance slit so that the spot is as bright as possible. Now put the detector in place on the exit port. Connect the detector to the digital voltmeter.

5. *Reconnect* the ‘A’, ‘K’ and ‘down arrow’ (Ground) cables to the back of the apparatus.

6. Adjust the *voltage adjust* knob until the current on the ammeter is zero, meaning that the photoelectrons are no longer getting to the anode.

7. Make these stopping voltages versus wavelength measurements over as wide a range of wavelengths as possible, with some measurements close to the threshold wavelength. It may be more efficient to first select a new voltage differential and then adjust the wavelength dial until you once again get a zero current, but there are many ways to do it. Record your methodology as well as your data.

8. Widen the slit width to 2 mm and repeat steps 6-7. You have enough time in this lab to do at least two complete stopping potential curves. Two or more curves allow you (a) determine whether the energy of the ejected electrode depends on light intensity and (b) get a handle on the systematics of the experiment.

C. Analysis.

Plot stopping potential vs. 1/λ. Determine the slope of this graph and compare it with the accepted value of Planck's constant (in eV-s). Use LSQ fit and make sure to put the data with the largest error on the y-axis so LSQ-fit can properly evaluate the fit and the χ-squared.

Find the work function of the photocathode by properly evaluating the intercept of your fit. Look online for typical work functions and make a guess as to what the photocathode material must be. We don't actually know what it is for sure, we want you to look it up and give some possible options.

Explore your systematics. Make sure you make a chi plot for your fit and evaluate whether the fit is linear, if you have outliers or problems at the ends of your fit where your calibration may no longer be applicable. Explain any systematic effects that you find. Did you underestimate or overestimate your errors? If so, you should go back and evaluate whether a reasonable change in your input errors really would affect your fit or not. Evaluate the differences between the low intensity and high intensity runs and state whether your experiment depends on intensity within the errors that you have actually assigned (i.e. how many sigma do the two results differ?) Since you already know that it should not, according to Einstein, then finding out that they differ by 20 sigma is a big clue that your errors were underestimated. Go back and fix your analysis.

Excitation and ionization energies of helium

Purpose

To observe and measure the energy level spectrum of the helium atom using an electron beam of variable energy.

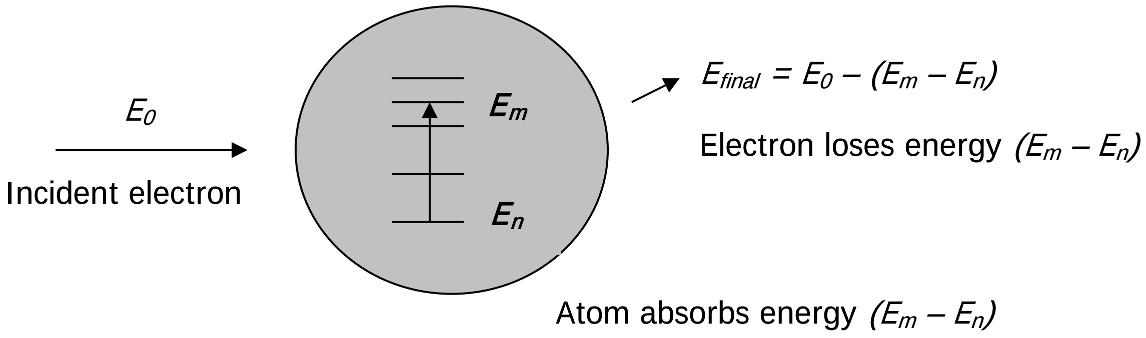
Key Concepts

- Excitation
- Ionization
- Selection rules
- Metastable states
- Singlet and triplet states

Introduction

The early understanding of the quantum nature of atoms came from the study of spectroscopy and the need to explain the line spectra emitted by excited atoms. The Bohr model of electrons moving in planetary orbits around a nucleus gave a quantitative description of the spectra observed in hydrogen or any other (ionized) atom containing only a single electron. The frequency ν of the observed radiation is related to the energy difference ΔE between discrete energy levels through $\Delta E = h\nu$. Although the Bohr model fails completely for even the next simplest atom, helium, the existence of discrete energy levels of atoms was established.

It follows that the transfer of energy to atoms by any means should occur in discrete amounts. One such means is via collisions with an electron beam. At low energies, an electron with energy E_0 can scatter elastically from an atom and, since the electron is so light, transfer negligible energy, $\sim (m_e / M_{atom})E_0$ (depending on scattering angle) to the atom. However, if the incident electron energy is greater than the energy difference ($\Delta E_{mn} = E_m - E_n$) between two discrete energy states of the atom, m and n, then this amount of energy can be absorbed in an inelastic collision. The final electron energy will be $E_0 - \Delta E$.



So, if an incident electron has just enough energy to excite the atom, usually from the ground state, it follows that its final energy will be very small. If the appearance of these very low energy electrons can be detected as the incident energy is gradually increased, it should be possible to identify those incident energies where inelastic collisions occur, and thus to identify the discrete energy levels of the atom. That is the aim of this experiment.

The first observation of atomic energy levels through electron excitation was made by Franck and Hertz in 1914, and led to a Nobel Prize in 1925. Their classic experiment was done with mercury vapor and was able to show multiple excitations of a specific energy level of mercury. Your experiment will use helium gas and is of a form actually used by Hertz in 1923; it should allow you to identify several different levels of the helium atom, which can then be correlated with spectroscopic data.

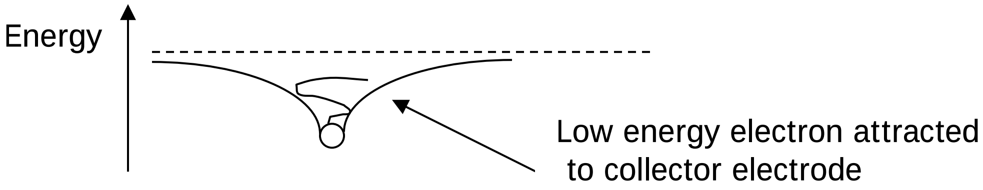
The apparatus

The apparatus consists of a spherical glass bulb that contains helium gas at very low pressure and an “electron gun” to shoot a divergent beam of electrons into the gas. A large diameter wire ring inside the bulb is arranged so that it cannot be hit directly by electrons from the source. This ring is used as a collector electrode to detect low energy electrons or positive ions resulting from inelastic collisions in the gas.

The electron gun consists of a hot filament from which electrons are emitted thermally. They are accelerated through a potential difference V towards a cylindrical anode. Most electrons strike the anode and are collected as part of the anode current, but some pass through the cylinder and emerge, with kinetic energy eV, into the spherical region of the glass bulb. The inner surface of the glass bulb is coated with a thin layer of conducting material and is electrically connected to the anode so that the region is essentially field-free. Non-interacting electrons and those that scatter elastically hit the conducting surface and add to the anode current.

The collector ring is connected through a shielded coaxial cable to a sensitive **picoammeter** to measure the small currents**.** In order to complete the circuit, the shielding of the coaxial cable needs to be connected to the ground of the power supply. This is done by using a BNC Tee connector at the input of the picoammeter and a BNC cable connected to ground. The meter and the anode/inner glass wall are set at a potential of + or -1.5 V relative to ground by means of a battery. The electron gun arrangement therefore “floats” at a negative potential relative to the anode and ground. The diagram below shows how the electrical connections should be made. F3 and F4 are connections to the filament and C5 is an additional electrode, called a Wehnelt cylinder, which helps to focus the beam and limit the region of the filament over which electrons are emitted.

When the battery is connected at a potential of +1.5 V with respect to the anode and walls, the collector ring acts as a shallow potential well and only those electrons with kinetic energy < 1.5 eV will be captured on it. Higher energy electrons will merely be deflected slightly before they hit the outer wall.



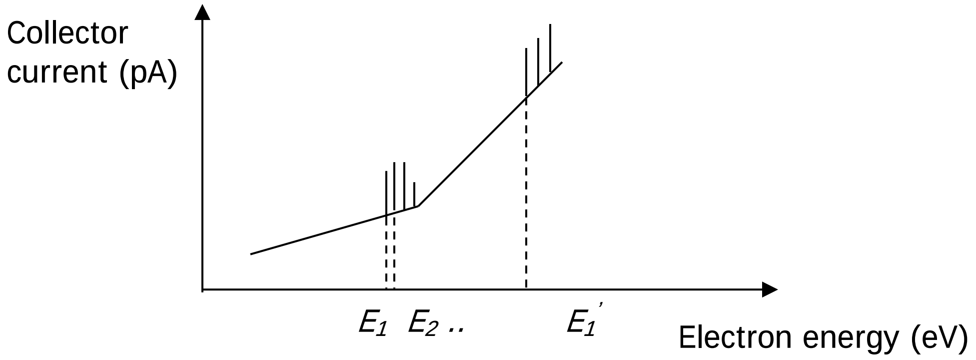
Determination of the excitation potentials of helium

Connect the apparatus as indicated in the diagram. Do not exceed 3V potential difference across the filament terminals F3 and F4; excessive filament current will considerably shorten its lifetime. Set the filament voltage to 2.0 V by adjusting the potentiometer connected to the 3V output of the power supply. Use a voltmeter to measure the filament voltage. You should see the filament glowing; give it a few minutes to warm up and reach equilibrium before taking data seriously.

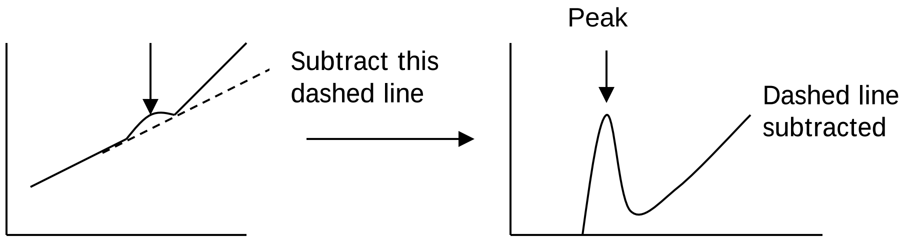
Connect the 10-turn potentiometer across the output terminals of the voltage supply. You will take measurements for voltage ranges from 0 – 50 V (measured with the voltmeter). For the lower voltages, set the power supply to about 25 V and use the potentiometer to vary the voltage. For voltages from 25 V to 50 V, set the power supply to 50 V. Use the potentiometer for all fine adjustments. First do a quick scan of collector current *vs.* anode potential. You should be able to locate two separate regions where peaks in the current occur; these are the signals you're looking for. Make a note of the relevant voltage ranges. It is likely that the collector current fluctuates due to pickup of electromagnetic noise – the ring is an excellent antenna. You can reduce the pickup considerably by carefully covering the glass sphere with some aluminum foil and connecting this to **ground potential on the power supply.**

Take a very careful set of measurements to accurately map out the structure – take fewer points where the data are smoothly varying and many more over the region of the structure. The resolution of your apparatus is demonstrated by the width of the excitation peaks. Decide what is the best spacing between data points in those regions (probably 0.1 V spacing through the peaks and 0.5 or 1 V between the structures). It is best to take data while changing the voltage monotonically.

Your data should look something like the sketch below. Not the change in slope of the current after the first series of excitations. Why does this occur?



Important Note: The peak corresponding to the second excitation energy E_1' is generally small and sits on a rising current which can distort your estimate of E_1'. In practice, it appears as a "shoulder" and not a "peak." To obtain a best value, you will have to subtract a linear background estimated from the current vs energy below the peak. In order to determine this background slope, you will need to take several points *before* the shoulder. ***Make sure that you do this!***



In practice, the excitation peaks are smeared due to two effects: the finite energy spread of thermionically emitted electrons (their average energy is 2kT ~0.2eV at 2000K), and the fact that electrons are emitted from different points along the cathode, across which there is a total potential difference of 3V (although the Wehnelt cylinder which is connected to the negative side of the filament, reduces this effect considerably).

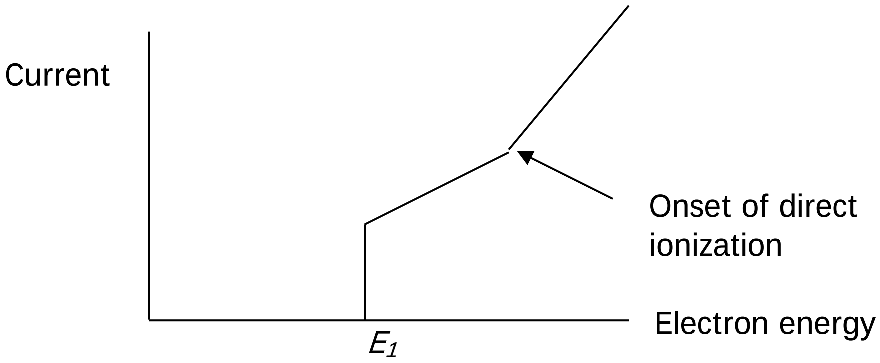
From your data you should be able to obtain accurate values for several excitation energies E_1, E_2,… and also the higher E_1',…*etc*. The higher set of excitations is produced by electrons that undergo two separate inelastic interactions: after exciting one atom to a higher energy level, an electron still has enough energy to excite another atom.

A small complication – contact potential difference

The difference $E_1' - E_1$ represents the actual first excitation energy. You will notice that this is different from E_1. This is because of the **contact potential** that arises whenever different metals are connected. The metal of the cathode is different to that of the anode. Also, the electrons are emitted from a point on the filament where the potential is not zero. Obviously, the contact potential can be found by comparing the value $(E_1'-E_1)$ with E_1 and the appropriate correction made to E_2, E_3, *etc*. Note that, apart from E_1', the upper potentials are not simple repeats of the lower ones, since the first energy loss of the electron can occur at any of the excitation potentials, making the incident electron energy spectrum more complicated in the upper excitation region.

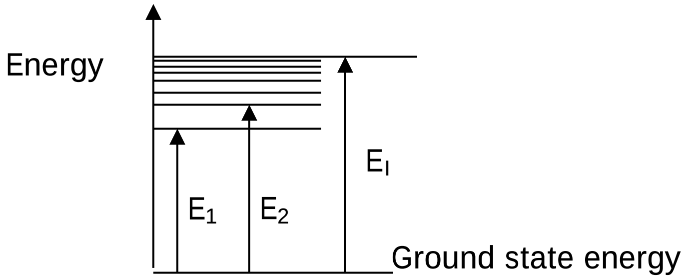
Direct determination of the ionization potential.

Reverse the battery in its holder. The collector should now collect only positive ions and the current should be in the opposite sense. Again, do a preliminary run-through to estimate how the current varies up to a maximum of about 25 V. Your data should exhibit a shoulder before the onset of a significant ionization current, as shown schematically below.



This shoulder is due to the presence of **metastable states** of the helium atom. These are excited states that, because of certain **selection rules,** are unable to decay rapidly back to lower energy states via photon emission, which normally occurs very rapidly in ~10^{-8} s. If a first inelastic electron collision results in such a metastable state, the atom may remain in that state for a very long time, and so a second electron only needs to provide a small additional energy in order to ionize the atom. (If the shoulder begins at energy E_1, what does this say about the lowest excited state of the helium atom?)

Determine a precise value for the ionization energy of helium by an appropriate extrapolation. Construct an energy level spectrum of the helium atom:



Using the relationship

$$h\nu = \frac{hc}{\lambda} = E_m - E_n,$$

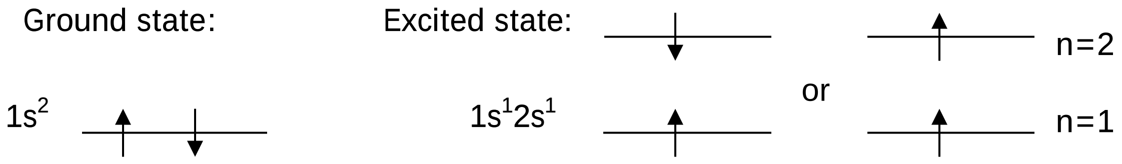
you can also predict the wavelengths of light which should be emitted as the atom de-excites to lower energy states. Compute the expected wavelengths corresponding to the peaks that you observed. One or two of these wavelengths appear in the table below, which compiles all of the optically active transitions for singly ionized helium (i.e. those transitions which result in the emission of light.) The letters W, M, S refer to the strength of the lines: weak, medium, strong).

λ (nm)	strength	λ (nm)	strength	λ (nm)	strength
32.0	W	402.6	M	1031	W
51-52	S	412.1	W	1083	S
52.2	S	438.8	W	1197	M
53.8	S	447.1	S	1253	W
58.4	S	447.2	M	1279	M
59.1	M	471.3	M	1297	W
257.7	M	492.1	M	1508	W
281.8	W	501.6	S	1700	S
294.5	W	504.8	W	1869	S
301.3	M	587.6	S	1909	M
318.8	W	667.8	S	1954	W
382.0	W	706.5	S	2058	S
388.9	S	718.1	M	2112	M
396.5	W	946.3	W		

Some quantum mechanics of the helium atom

The structure of the hydrogen atom and its emission spectra can be predicted exactly by solving the Schroedinger equation. However, the presence of an extra electron in even the next simplest atom, helium, has profound consequences: the problem is now a 3-body problem that must include the repulsive potential between the electrons as well as the electron-nucleus interactions, and sophisticated perturbation theory is required to obtain a solution. In addition, the relative orientation of the electron spins has very important consequences.

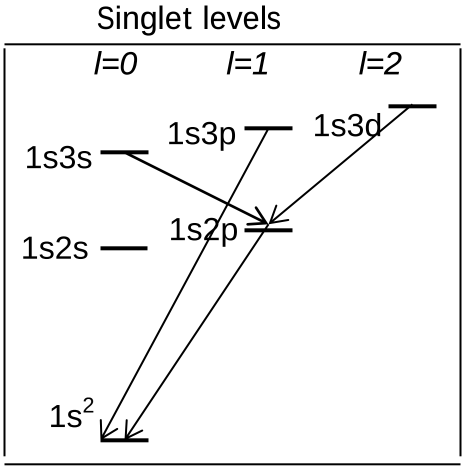
The ground state of helium contains two electrons in the *n=1* shell and the electron spins must be opposed (Pauli principle). The ground state is described by the notation $1s^2$, *i.e.* two electrons with *n=1, l=0.* Where the angular momentum *l = 0, 1, 2, 3…* is represented by symbols *s, p, d, f…*

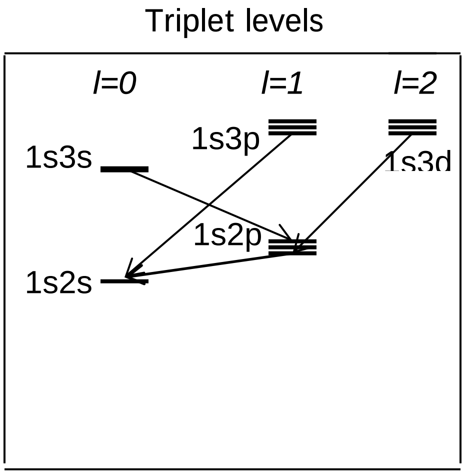


In contrast, the first excited state $1s^12s^1$ has one electron in the *n=2* shell, but this electron can have one of two possible orientations, either spin up or spin down. If the two electron spins are antiparallel, the total spin is S=0, called a spin singlet state of the atom, but if they are parallel the total spin is S=1, called a triplet state because of the three possible values of m_S =+1, 0, -1. The triplet energy levels are slightly lower than the singlet levels. Since the relative orientation of the electron spins generally does not change during a transition (the $\Delta S=0$ rule), transitions between triplet and singlet states do not occur with significant probability. So in helium, there are essentially two independent emission spectra, one corresponding to transitions between triplet states exclusively, and one corresponding to singlet transitions. There are essentially no such selection rules for excitation by transfer of kinetic energy from incident electrons.

Of course, the excited states can also have orbital angular momentum; e.g for *n=2, l = 1* is allowed, in addition to *l = 0.* The corresponding 1s2p states are split slightly. Since the photon has one unit of angular momentum, all transitions involving a photon require the additional selection rule $\Delta l = \pm 1$.

The diagram below shows what we expect of the lowest energy levels of helium, along with the possible radiative transitions. The splitting of the triplet levels has been considerably exaggerated.





Blackbody radiation: a spectroscopic study

Purpose

To investigate the emission of electromagnetic radiation from the filament of an incandescent lamp as a function of the filament temperature. Show that the power goes as the fourth power of temperature and measure *hc*/*k* or find Planck's constant *h*.

Key concepts

Line spectrum
Continuous spectrum
Spectrometer
Stefan-Boltzmann law
Wien's law

Introduction

Planck's derivation in 1900 of an expression for the continuous spectrum of radiation emitted from a small hole in a heated cavity (the blackbody spectrum) motivated the development of quantum physics. He assumed that the radiation was emitted by oscillators located within the walls of the cavity. He found that, by assuming that oscillators with a natural frequency ν are able to emit radiation only with quantized energies $h\nu$, $2h\nu$, $3h\nu$...., he could explain the observed blackbody spectrum, something the earlier, purely classical Rayleigh-Jeans Law was unable to do. The fundamental constant *h* became known as Planck's constant. With this assumption, and using the classical statistics previously developed by Boltzmann, the mean energy of such a quantized harmonic oscillator at temperature *T* is:

$$E_{ave} = \frac{h\nu}{e^{h\nu/kT} - 1},$$

where *k* is Boltzmann's constant, which can be determined from the universal gas law. Then, including a factor for the density of states with wavelengths between λ and $\lambda + d\lambda$ that can exist in the cavity and using the fact that $\nu = c/\lambda$, the intensity of emitted radiation (energy /unit area/unit time) at a given wavelength and temperature is given by:

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5}\left[\frac{1}{e^{hc/\lambda kT} - 1}\right].$$

This blackbody intensity is the maximum that can be emitted by a hot body. In practice, the intensity emitted by a hot body is less than this. It is given by the blackbody intensity multiplied by an emissivity factor $\varepsilon(\lambda, T)$ which is a number less than 1, and which is both temperature and wavelength dependent.

It is easy to show that the spectrum $I\ (\lambda, T)$ has a characteristic peak at a wavelength λ_{max} which varies inversely with absolute temperature.

$$\lambda_{\max} = \left(0.2014\frac{hc}{k}\right)\frac{1}{T} = \frac{2.898 \times 10^{-3}}{T}\, m.$$

This is known as the Wien displacement law; it quantifies the way in which the color of a very hot object varies from red to white to blue as it is heated to higher temperatures. Integrating the blackbody spectrum over all wavelengths yields the total power radiated by an area A at temperature T:

$$P = \varepsilon A \sigma T^4,$$

where the constant $\sigma = 2\pi^5 k^4/15c^2h^3 = 5.67\times10^{-8}\ W/m^2K^4$ is called the Stefan-Boltzmann constant. Such behavior had been established experimentally about ten years before Planck's theoretical explanation of the phenomenon.

In this experiment you will study how the blackbody spectrum emitted by the tungsten filament of an incandescent light bulb varies as a function of temperature. The temperature of the bulb is determined by measuring its resistance, which varies with T in a known manner. In general, measurements of visible and near infrared wavelengths are difficult to perform with a constant or uniform response. The equipment used here is no exception. You will find that the response of the spectrometer is very wavelength dependent, falling quite rapidly at longer wavelengths. As a consequence, you will only be able to verify the temperature dependence implied by Planck's law at fixed wavelengths. You will not be able to verify the exact form of Wien's law, but you can see it qualitatively in action. However, the Stefan-Boltzmann law can be verified by measuring the electrical power dissipated in the bulb as a function of temperature.

The spectrometer

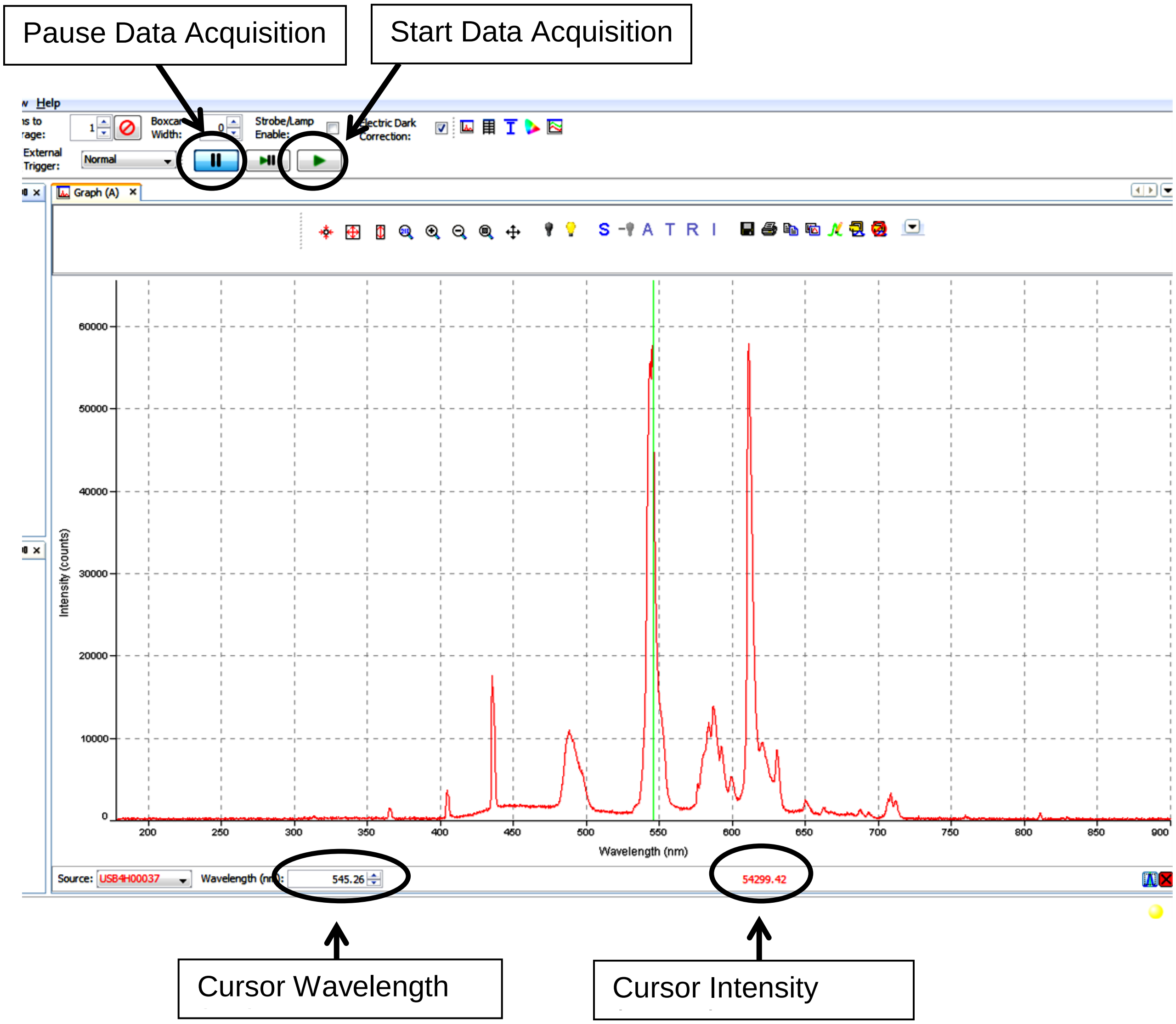
The miniature spectrometer, which measures intensity versus wavelength, (Ocean Optics, USB4000) is actually incorporated into a plug-in card for a PC. A silica fiber optic cable is used to transmit incident radiation to the PC card where it is dispersed by a reflection diffraction grating with 6000 lines/cm, and the resulting spectrum detected in a linear CCD (charge coupled device) array. This is a light-sensitive 2-dimensional array containing many pixels, each of which stores a charge proportional to the number of photons incident on it, i.e. proportional to the product of light intensity and integrating time. After the pre-set integration time, the CCD array is scanned electronically and the output of each channel is passed through an ADC (analog to digital converter) before being displayed on the monitor. The sensitivity depends on wavelength, and this particular device covers a spectral range of 350 nm in the near ultraviolet to 950 nm in the near infra-red, although the efficiency falls significantly at the higher wavelengths. The spectrometer has a resolution of a few nm. The integration time of the CCD array can be easily increased or decreased to accommodate a wide range of intensity levels.

Procedure:

A. Initial check-out of the spectrometer system.

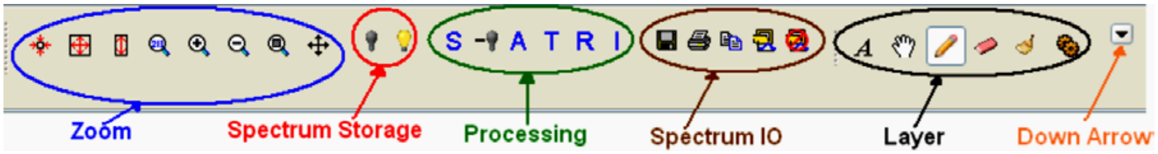
NOTE: THE FIBER OPTIC IS VERY THIN. KEEP IT A STRAIGHT AS POSSIBLE, AVOIDING ANY KINKS.

1. Run the data acquisition program, SpectraSuite by Ocean Optics, , which can be found under Start/All Programs/Ocean Optics or on the Windows desktop. A screen shot of the program with a spectrum of the room lights is shown below.



Source: USB4H00037 Wavelength (nm): 545.26 54299.42

Become familiar with the principal operations that you'll need, shown below. (More detailed instructions can be found in the user manual in the U:\pub\OceanOptics folder.)



Select Scope Mode by clicking on the “S” in the “Processing” options. You will need to remove the background dark current which comes from thermal activation of electrons in the CCD. To do this, check the box labeled Electrical Dark Correction, which is in the line just below the main toolbar. Ask your instructor if you cannot figure out how to do this. DO NOT use the other “dark spectrum” subtraction features!

Find how to operate the cursor by clicking on the spectrum. The current cursor position and the corresponding intensity are displayed at the very bottom in the “Wavelength” window and the (red) number shown to the right of it. (Note: you may also enter a number directly in the “Wavelength” window to move the cursor to a specific position.) Using the Pause and Start Data Acquisition buttons can allow you to “freeze” the acquired data.

Learn how the Integration Time function permits you to effectively change the scale in a controlled way as the radiation intensity changes.

Note that the fluctuating signal that you observe is actually due to the statistics of photon counting; the "average over N spectra" and "boxcar" features may help you to obtain a best estimate of relative intensity. (The latter feature takes an average over several adjacent channels, so be careful if the spectrum is varying non-linearly in the region of concern).

2. Check the calibration of the spectrometer (which should already be satisfactory) by observing the light emitted by the fluorescent lamps in the lab. These work by generating an alternating high voltage across a glass tube containing a small amount of mercury and a low-pressure noble gas (usually argon) which acts to start an electrical discharge. The argon is easily ionized, the gas heats and the mercury vaporizes. Mercury atoms are excited by electron collision in the discharge and emit both UV and visible light when they de-excite. The UV emission at 253.7 nm causes further excitation of the phosphor coating of the tube, which glows as it emits visible light. The overall spectrum thus consists of light emitted by the phosphor superimposed on the characteristic spectrum of mercury. The strongest visible wavelengths emitted by mercury are: 435.8 nm, 546.1 nm, 577.0 nm, 579.1 nm, plus two less intense lines at 365.0 nm and 404.7 nm. Since the yellow doublet is broadened by phosphor, you may not get as good a calibration check as the thin minor lines in the blue/purple.

Draw a sketch of the spectrum in your logbook and clearly label the peaks, noting if they are in the right place or not.

Since the pure Hg spectrum looks blue, manufacturers can choose different phosphors (which are generally complex metal oxides) to enhance different parts of the spectrum, e.g., cool white, warm white, *etc.* Take a look at http://www.gelighting.com/na/business_lighting/education_resources/learn_about_light/distribution_curves.htm and see if you can determine which bulbs are installed in room 62. Write your best guess in the logbook.

3. The light source you will investigate is the filament of a 7.5 W incandescent light bulb. You need a way to measure its temperature as you heat it up. Since its resistance changes as a function of temperature, you need to determine the constant of proportionality that will allow you to measure temperature by measuring resistance. The temperature dependence of the resistivity ρ of the tungsten filament has been determined in a separate experiment. Between room temperature and 4000K, it is given to a precision of better than 1% by:

$$\rho = .0264(T-60)\mu\Omega cm,$$

where T is the absolute temperature in Kelvin**. From this you can find an expression for *T* which depends on the ratio of the filament's resistance at temperature *T* to its resistance at room temperature**. This requires you to measure the resistance of the filament at room temperature, a step which is best carried out using an ordinary ohmmeter and the room thermometer mounted on a wall in the lab. Make sure that the filament is cool when you measure the room-temperature resistance. Note that the process of measuring the resistance will heat the filament somewhat, so be careful.

B. Investigating the Planck spectrum

1. As mentioned earlier, the measured spectral intensity I_{meas} can be written as a product of the blackbody spectrum I_{BB} multiplied by two wavelength dependent factors: $\varepsilon_{det}(\lambda)$ which incorporates the wavelength dependence of the detector, and $\varepsilon_{em}(\lambda,T)$ which is the emissivity of the filament material. Since the temperature dependence of the emissivity is very small, only a few percent over the range of temperatures you will investigate, it can be safely ignored

At the temperatures and wavelengths relevant to your experiment, the factor $e^{hc/\lambda kT}$ is always very much larger than 1 and so the measured spectrum can be written as

$$I_{meas} = \frac{2\pi hc^2}{\lambda^5}\varepsilon_{\det}(\lambda)\varepsilon_{em}(\lambda)e^{-hc/\lambda kT}.$$

Taking logarithms, this becomes

$$\ln(I_{meas}) = f(\lambda) - \frac{hc}{\lambda kT},$$

where *f*(λ) is a function only of wavelength. By plotting $ln\ (I_{meas})$ vs T^{-1} for a known wavelength, the factor *hc/k*, which involves three fundamental constants, can be determined. Thus, your experiment involves measuring the intensity and the temperature of the filament for a series of temperatures and wavelengths.

2. To make the measurements, you will position the fiber in a stable position to observe the light bulb and cover the entire apparatus with a black cloth to shield it from the room light. The bulb will get very hot, so **KEEP THE FIBER AT LEAST 2 CM AWAY FROM THE BULB AT ALL TIMES** or it will melt. Set up two accurate digital meters to determine the *current* through the filament (in series) as you apply a *voltage* (measured across the bulb) from the power supply for each applied potential**.** The power supply for the bulb provides a variable potential up to 400 V. Increase the potential slowly to minimize thermal shock to the filament, and **DO NOT EXCEED 200 V** or you will burn up the filament very rapidly. **At high power, this lamp is very bright; shield it from your eyes**.

3. Decide whether it is better to take a series of measurements with the filament temperature increasing or decreasing (why should this matter?) Decide what range of temperatures you expect to cover (remembering that the intensity will vary over a very large range). It is suggested that you adjust the intensity to vary by a factor of about 2, roughly, between adjacent measurements. This should give you measurements at about 10 or more different temperatures. By varying the Integration Time, you will be able to keep the maximum of the observed spectrum between 1000 and 60,000 counts per channel. Make sure that you do not saturate the CCD, which will occur at about 65,000 counts per channel. Decide how you will average your data with either the Average or Boxcar features. Do not forget to click on the setting Correct for Electrical Dark; this takes an average over several pixels that are shielded from light and then subtracts this average from all pixels. Finally, make sure that the apparatus is completely shielded from room lights. This is particularly important for measurements at the lowest filament temperatures. Photographic cloth is available for this purpose, but make sure that it does not touch the bulb.

4(a) Choose a set of 4 fixed wavelengths that you will be using throughout the entire experiment. For each filament temperature that you choose, you will measure the relative intensity at each of these wavelengths in the spectrum. As described earlier, the response of the spectrometer is a strong function of wavelength. There are two important factors that determine this response. One factor is the **quantum efficiency** of the CCD array that falls rapidly at longer wavelengths, i.e., long wavelength photons have insufficient energy to knock out an electron from a given pixel (the photoelectric effect). The second factor is the **blazing** of the diffraction grating; in order to maximize the amount of light diffracted into the first order of diffraction, and not have the light dispersed through many orders, the grooves are given a special shape. This grating is blazed to maximize the light intensity diffracted from the center of the visible region. It is also responsible for the structure: additional peaks that appear in the measured spectra.

4(b) Sketch (or photograph and paste into logbook) a picture of the spectrum at one of your temperatures and clearly mark where your 4 chosen wavelengths lie on the sketch. If you find that one of your chosen wavelengths sits on the structure, rather than the continuum, pick a different wavelength. For best results choose 4 wavelengths that sit on the leading edge of the blackbody curve, not at the higher wavelengths where the curve falls sharply due to inefficiency.

4(c) At each temperature, produce a table giving voltage *V*, current *I,* the integration time factor, and the relative intensities at the four fixed wavelengths. Do not rush your measurement. It takes some time for the temperature to stabilize, so make sure that the readings are not still drifting when you are taking the data. Look carefully at the spectrum at each temperature and, ignoring the spectral features, see if you can find the broad maximum of the underlying continuum blackbody spectrum and write that number down in the last column. λ_{max} is supposed to be the wavelength of the maximum of the underlying spectrum, not the wavelength of the highest peak, so you need to be aware of the entire spectrum. Go back to your sketch of the spectrum and mark on it where you chose λ_{max} for that temperature.

V(volt)	I(amp)	Int. Time	I(λ_1)	I(λ_2)	I(λ_3)	I(λ_4)	λ_{max}

Then make a summary table giving various quantities that you will need for further analysis, i.e., power, relative intensities corrected for differing integration times. You can choose how to make these tables – maybe you want it all in one table with corrected values right after the uncorrected ones.

V(volt)	I(amp)	Power (watt)	R (Ω)	T (K)	I(λ_1) (corr.)	I(λ_2) (corr.)	I(λ_3) (corr.)	I(λ_4) (corr.)

5. Plot λ_{max} as a function of 1/T by hand, right in your logbook. Since the wavelength dependence of the detector and grating is not given, this cannot be very quantitative, so don't worry about error bars. Is it linear? If so, find the slope and determine the correction factor needed to make it match Wien's Displacement Law. If not linear, which part of the curve has the right slope? Does your analysis show that long wavelengths are inefficiently sampled? Finally, look up the temperature of the sun and mark that on your plot. Is it the right color (λ)?

6. The next part **is** quantitative and requires proper error analysis. Make a log-log plot of temperature vs. power to find how the power dissipated varies with the temperature of the filament. Discuss the source of any possible deviations from the Stefan-Boltzmann law.

7. After correcting relative intensities for the integration time at each temperature, plot *intensity (I) vs. 1/T* for each wavelength on a semi-log plot and determine the factor *hc/k.* Use the least squares fitting program available on the lab computers. Assuming that c and k are well-known from other experiments, this allows you to find Planck's constant. Remember that you must manipulate any equations such that the vertical axis contains the variable with the dominant error, since LSQfit cannot use x-axis errors in its fit.

C. Suggestions for some further investigations, if you have time.

1. Measure the temperature at various points in a candle flame. You are now in a position to measure the temperature of any hot object. From the data you have already taken, choose two wavelengths λ_1 and λ_2 where you have already measured the relative intensities at a known filament temperature, T_0, say. The ratio of these intensities is

$$R_{120} = \frac{I_{10}}{I_{20}} = \left(\frac{\lambda_2}{\lambda_1}\right)^5 .\exp\left(-\frac{hc}{kT_0}\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right]\right).\frac{\varepsilon_1}{\varepsilon_2},$$

where the ε factors represent the emissivity and detector efficiency factors at the two wavelengths.

Now, if you measure the ratio R_{12} of the intensities at the same two wavelengths but at another (unknown) temperature T, and then take a ratio of the ratios, the unknown ε factors cancel out and so the unknown temperature T can be found from

$$\frac{R_{12}}{R_{120}} = \exp\left(-\frac{hc}{k}\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right]\left[\frac{1}{T} - \frac{1}{T_0}\right]\right)$$

e.g., if the wavelengths λ_1 and λ_2 are 600 and 800 nm respectively, then

$$\frac{1}{T} = \frac{1}{T_0} + \frac{1}{5996}\ln\left(\frac{R_{12}}{R_{120}}\right)$$

Use a lens to give you a magnified image of the flame and then use the fiber optic cable to sample the temperature at various points in the image.

2. Look at the spectrum of light from low pressure discharge tubes such as a sodium lamp, observing how the spectrum changes as the tube warms up. Look at the sky, the sun, white light reflected from various colored objects, *etc., etc.*

High-Resolution Spectroscopy of Hydrogen and Sodium

Purpose

To calibrate a diffraction grating spectrometer; to use this for very accurate measurements of visible lines of hydrogen and sodium spectra and to analyze these in terms of elementary atomic theory.

Key concepts

- Spectrometer
- Diffraction grating
- Atomic energy spectra
- Rydberg constant
- Balmer series
- Fine structure

Introduction

One of the most effective ways of investigating the structure of atoms is by studying their emission and absorption of electromagnetic radiation. The absorption of energy by an atom increases the energy in its electron system. When the atom returns to a lower energy state, the energy released is emitted as electromagnetic radiation, i.e. photons. If the atoms are not very close, as in a gas at low pressure, only certain discrete atomic energy levels exist, so that the emitted photons have well-defined energies which correspond to the difference between the initial energy E_i and the final energy E_f of the atom, i.e., $\Delta E = E_I - E_f = h\nu = hc/\lambda$ where ν and λ are the frequency and wavelength of the photon. h is Planck's constant.

Niels Bohr showed that the energy levels in the simplest atom, hydrogen, have values = $-hcR_\infty/n^2$ where n is an integer. R_∞ is known as Rydberg's constant:

$$R_\infty = \frac{2\pi^2 me^4}{ch^3}, \qquad (1)$$

and m and e are the mass and charge of an electron. The energies of the emitted photons are then

$$E = h\nu = chR_\infty\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right), \qquad (2)$$

where n_1 and n_2 are integers representing the order of the energy levels, starting with $n_1 = 1$ for the lowest energy.

Equation 2 is strictly correct only for a hydrogenic atom with an infinitely massive nucleus. For the real hydrogen atom, we must use the *reduced mass* of the electron in Eq. 1 rather than the bare mass. Then Eq. 2 becomes

$$E = h\nu = chR_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right), \qquad (3)$$

where

$$R_H = \frac{M}{M+m} R_\infty = 1.096776 \times 10^7 \text{m}^{-1}, \qquad (4)$$

where M is the mass of the proton.

The spectra of heavier atoms are much more complex and Bohr's theory fails completely to explain them. Nevertheless, the Bohr's theory was seminal in our first understanding of quantum mechanics.

Principle of the spectrometer

In this experiment, a diffraction grating is used to separate light waves of different wavelengths. This is a flat square of glass covered with a thin layer of plastic having a great many fine, parallel grooves on the surface. As shown in Fig. 1, light from a source T is focused by a concave mirror M_1 through a narrow slit S_1. These rays are made parallel by mirror M_2 and reach the grating G at an angle ϕ with respect to a line perpendicular to G. Waves reflected (diffracted) at an angle θ produce constructive interference when the relationship $\boldsymbol{\sin\theta + \sin\phi = m\lambda/d}$ is satisfied. Here d is the distance between grooves on the grating and m is an integer, the order number. This particular instrument is constructed so that m = 1.

The mirror M_3 collects the light leaving G at the angle θ and concentrates it on the slit S_2. Light which passes through S_2 enters a photomultiplier tube P, in which it is converted into an electrical signal which can be observed on an oscilloscope. Just before the light rays reach S_2 a small fraction of them are reflected upward and pass through a magnifying lens so the color of the light can be noted by the observer.

The grating can be rotated about an axis which lies in the plane of its front surface. This rotation changes both angles θ and ϕ, but as can be seen from the diagram, they are related by the equation

$$\theta - \phi = \alpha \text{ (a constant);}$$

i.e., the angle α between the incident and reflected beams stays constant.
Simple manipulation then gives the condition for constructive interference as

$$\sin\phi + \cos\phi\left(\frac{\sin\alpha}{1+\cos\alpha}\right) = \frac{\lambda}{d(1+\cos\alpha)}$$

Since α is a small angle (8.40^0 in this instrument), there is a nearly linear relationship between sin ϕ and the wavelength of the light reaching the detector. The grating rotation mechanism is designed so that the dial reading is proportional to sin ϕ (although there is a zero offset of several turns).

Since, in general, the dimensions of the spectrometer, including the value of d, are not accurately known, it is customary to calibrate the instrument using the well-known wavelengths of several strong lines in the spectrum of light from a mercury arc. From this information one can obtain a relation between dial reading and λ which can then be used in the study of other spectra.

Procedure:

A. Calibration of the spectrometer using the Hg spectrum

Turn on the Hg arc and place it so that the mirror M_1 focuses light from the Hg tube on slit S_1. Make sure that the source is well-focused on the slit. There is no need to adjust the slit width, which should be at zero on the scale. The high voltage supply for the photomultiplier tube should be set at -600 volts. The output signal can be observed with an oscilloscope. (The signal will have a frequency of 120 Hz). Turn up the scope gain for maximum sensitivity, i.e., so that the signal is clearly visible, and you are able to detect the faintest lines.

Start with the grating rotator all the way out (fully counterclockwise). Decide whether to use inches or mm (toggle button) and stick with it! Slowly turn the handle clockwise. After a few turns, you should see a very large signal on the oscilloscope. If the light source is properly focused, the observed waveform should have a maximum amplitude of a few volts, and you should be able to determine the position on the digital readout. Make sure that you have the focus optimized. If you look in the eyepiece, the color of the observed line should match that of the Hg source. This dial reading, which we will call D_0, corresponds to the angle at which the line perpendicular to the grating is centered between M_2 and M_3. In this case, light from the entrance slit is imaged directly onto detector. Record the dial reading corresponding to D_0 (or zero the digital reading, if you want to define D_0 as the origin)

Now rotate the handle clockwise until you reach the first diffracted peak. Record the position for each peak that you find. You should find about 12 lines.

When very strong lines are found (the easiest to see is probably the famous green line at 546.07 nm), look through the eyepiece and record their color next to the dial setting. It will be very helpful to record the approximate signal strength of each line, using the oscilloscope, so that you can first identify the strongest spectral lines with the known spectrum. **Having these benchmarks available will be invaluable if you accidently reset the digital readout or bump the spectrometer. Also, if you leave for the day and return at a later time to finish the experiment, you should check a couple of the strong lines to make sure that the calibration has not changed**

You should find strong Hg spectrum lines at the following wavelengths
(measured in *air* in units of nm):

yellow	579.05 nm
yellow	576.96
bright green	546.07
blue	491.60
bright blue	435.84
faint blue	434.75
faint blue	433.92
violet	407.78
violet	404.66

The list of lines given above is a good start, but not enough to get a good calibration for this lab where high resolution is essential. You MUST go to either http://physics.nist.gov/PhysRefData/ASD/lines_form.html (the spectrum you want is Hg I) or use *The CRC Handbook of Chemistry and Physics.* A total of 25 to 30 lines may be found in the Hg spectrum. (You may find some additional faint lines from argon. Why?) Make sure you also choose some lines in the UV where your eyes won't do much good, but the oscilloscope will. Plot a graph of λ vs. dial setting on millimeter graph paper for as many of the Hg lines that you can observe. An iterative approach is useful. Start with the strong lines and sketch the λ vs. dial setting plot first. A second pass through the range with the sketch to guide you will help you find the fainter lines. If some points do not fall on the straight line, check their dial reading again. Do this promptly so that your data can be checked for errors in measurement or analysis. Start by fitting the data to a line using *LSQ fit.* The line will actually have a slight curvature, which you should be able to observe by looking at the plot of the deviations χ_i. You must therefore fit a second order polynomial to the curve. A fitting program *LSQfit_2D* is available on the lab PCs. Ask your instructor if you cannot figure out how to use it properly. If *D* is your dial setting, you will be fitting to an equation

$$D = a + b\lambda + c\lambda^2,$$

and you will be obtaining values (with uncertainties) for *a, b,* and *c.* **See the hint at the end of this lab write-up for a simpler way to compute the uncertainties**. Note that your data points all lie between 350 and 580 nm. Is there any advantage to shifting your origin by subtracting a constant, such as 450 nm, from each of your wavelengths *before* doing the fit? Think about how your dial reading error is being incorporated in the fit: a half-tic uncertainty is really the same whether the dial reads 10 or 800.

***Caution!** It usually takes an entire lab period to get familiar with the equipment and do the calibration and understand the error propagation. If you are coming back a week later to start part B, you will need to do the calibration once more in case other users have changed settings. It should not be hard just to take a few data points and run it through the same program again; if your data is the same, you can just use the fit from before.*

B. The hydrogen spectrum.

Place the hydrogen tube in place of the Hg source (make every effort to put the thin region of the tube at the same position as the Hg source) and determine the dial readings corresponding to at least four lines from the H spectrum. From the calibration information obtained above, calculate λ for each H line.

Note that these are the wavelengths measured in *air*. The original reference wavelengths for Hg were also obtained in air. Air has an index of refraction n = 1.00029. In an experiment of this precision, we need to account for the fact that the wavelength in air is different from that in vacuum! By using the relation

$$\lambda_{air} = \lambda_{vac} / n,$$

determine the vacuum wavelength λ_{vac} for each of the four transitions. This is the wavelength that you would use to determine the frequency ν of the emitted light. These lines are members of the "Balmer Series," corresponding to the energy levels with n_1 = 2 and n_2 = 3, 4, 5, and 6. To confirm this, calculate the ratio of $\frac{1}{\lambda_{vac}}$ to $\left(\frac{1}{4} - \frac{1}{n_2^2}\right)$ which should be the same for each of your wavelengths. Obtain a best value for the hydrogenic Rydberg constant R_H (with an uncertainty) by calculating a weighted average. Compare with the predicted value from Eq. 4.

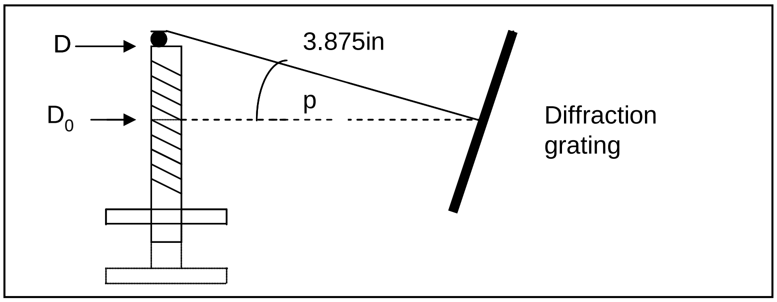
C. The sodium spectrum.

Place the sodium source in position and let it warm up for 10 minutes or so. The light is red at first and will become more orange when the tube gets hot. The reason for this is that sodium has too low a saturated vapor pressure at room temperature and it must be heated before it can maintain a discharge. Rather than simply heating the sodium, this is achieved by adding a "starter" gas, neon, which easily produces a discharge. This heats the initially solid sodium until its vapor pressure increases sufficiently to maintain the discharge. Since the ionization energy of neon is 21.6 eV and that of sodium is 5.1 eV, practically all of the final discharge current is due to sodium ions, and the excitation of the neon becomes negligible. Sodium lights are often used for street lighting, and their initial red color is due to the neon starter gas

Locate all the visible Na lines and determine their wavelengths. Check that the doublets are well-resolved by looking through the eyepiece while changing the width of the input slit (set it back to the nominal setting of zero when you are done). From the average doublet separation, calculate the energy difference between the two levels and compare your result with the expected value of 2.1 meV. Consider the accuracy and reproducibility of your data and include in your calculation of average doublet separation only those line pairs which are clearly doublets. [Note that you can reduce the error in your measurement of the doublet splitting by considering only the change ΔD between the two lines and then evaluating the corresponding separation $\Delta\lambda$. Why is this better than calculating the wavelengths of each of the two lines from the full calibration and then taking their difference? See HINT at end of write-up.]

The sodium spectrum is a relatively simple one, since the atom has eleven electrons, ten of which form a spherically symmetric "closed shell" of charge surrounding the nucleus. The relevant energy levels are due to states of the one remaining electron, which is sometimes called the *valence* electron since it is responsible for chemical binding. The *fine structure* that you measure in this experiment is due to the fact that the electron has *spin* and a corresponding *magnetic moment*. This magnetic moment can have either of two different orientations in a magnetic field (corresponding to “spin up” or “spin down”). The corresponding energy difference between these two states is $\Delta E = 2\ \mu_B B$ where μ_B is the magnetic moment in Bohr magnetons, 9.274×10^{-24} J/T and B is the magnetic field experienced by the electron in the atom. All excited levels of the atom have this doublet fine structure. The splitting is more than an order of magnitude greater in sodium than in hydrogen).

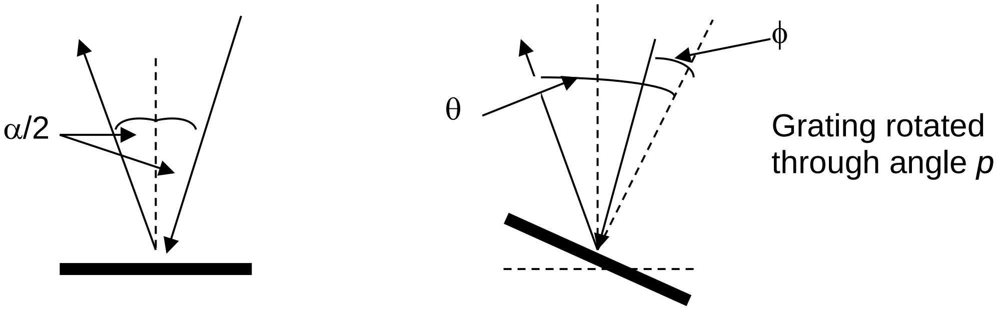
D. Determination of the grating groove spacing, d.



The mechanism which rotates the diffraction grating is shown above: as the handle is turned, it drives a long screw which in turn pushes a lever. This lever is rigidly attached to a point below the center of the grating. At dial setting D_0, determined earlier, the grating angle p is 0^o and the angle of incidence ϕ equals the angle of reflection θ. The length of the lever is 3.875 inches, so that for any other dial setting D, the grating angle p is given by: (check your units!)

$$\sin p = \frac{D - D_0}{3.875 in}.$$

Now consider the two situations illustrated below, with dial settings D_0 and D:



In both cases, corresponding to dial settings D_0 and some other value of D, respectively, the paths of the incident and reflected rays are the same and so the angle $\alpha = 8.40^o$ between them is the same. But the angles of incidence and reflection are changed when the grating is rotated through an angle p. From the second diagram, it follows that the angle $p = \alpha/2 + \phi$ where ϕ is the angle of incidence. So now we can determine ϕ for any dial setting D using this expression and the one above. Inserting the value of ϕ into the diffraction equation gives a value for the number n of grooves per cm:

$$n = \frac{1}{d} = \frac{\sin\phi(1+\cos\alpha) + \cos\phi\sin\alpha}{\lambda}.$$

A Useful Hint for Error Propagation

Consider the calibration equation given above: $D = a + b\lambda + c\lambda^2$

Since our goal is to write a formula for λ as a function of the dial reading D, we will also need a formula for the uncertainty in wavelength σ_λ. Naïvely we could invert the formula above and perform the standard procedure for propagating errors given in the appendix. However since the error σ_λ has four contributions (from σ_D, σ_a, σ_b, and σ_c), each of which requires a separate differentiation, you would quickly find that the standard method of error propagation is very tedious and time consuming. A more clever method is useful in this case.

Instead of inverting quadratic equation above, consider performing the error propagation as if D was the dependent variable, *i.e.* you should do an implicit differentiation: $\sigma_D^2 = \sigma_a^2 + \lambda^2\sigma_b^2 + \sigma_\lambda^2 b^2 + \cdots$ You will find that this formula is trivial to solve for $\boldsymbol{\sigma_\lambda}$. However be careful with the logic of your final formula. Ask yourself if it makes sense; *i.e*. would you expect that having an error on a improves the error from D? Obviously not. You should fix the equation to reflect a more reasonable formula (perhaps some minus signs are really plus signs). You may argue that this treatment is not rigorous, and you would be right. However the procedure here results in the correct formula as you may verify for yourself by consulting any textbook on error analysis.

Now consider the formula for the energy splitting of the doublets in the sodium spectrum:

$$\Delta E = hc\Delta\nu = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

Again if you naively use your error propagation formula for λ you will likely find that your error on ΔE is actually greater than ΔE itself. The reason for this is that the error on D from σ_a should have no effect on a relative splitting in wavelength, since σ_a represents an error in the offset. However, performing the error propagation on ΔE will result in a contribution from σ_a regardless. Let us start by making the approximation:

$$\Delta E \approx hc\frac{\Delta\lambda}{\bar{\lambda}^2}$$

In addition, we may assume that the error on ΔE is completely due to the error on $\Delta\lambda$ and ignore the contribution from $\bar{\lambda}$ (indeed this term is negligible in comparison). Now let us again consider our calibration equation for the dial reading D and write:

$$\Delta D = b\Delta\lambda + 2c\bar{\lambda}\Delta\lambda$$

We may now perform the error propagation for $\Delta\lambda$ considering the contributions from σ_b, σ_c, $\sigma_{\Delta D}$, and $\sigma_{\bar{\lambda}}$. Again you will likely want to do the derivatives implicitly and make the logical corrections to the signs in the final formula.

X-ray crystallography: A determination of Avogadro's number from crystal lattice spacing

Purpose

This experiment will introduce you to the basic techniques of X-ray crystallography. You will make an accurate determination of the lattice spacing of some simple crystals and use your data to determine Avogadro's number.

Key concepts

K, L, M...radiation
Bremsstrahlung
Bragg diffraction
Spectrometer
Unit cell

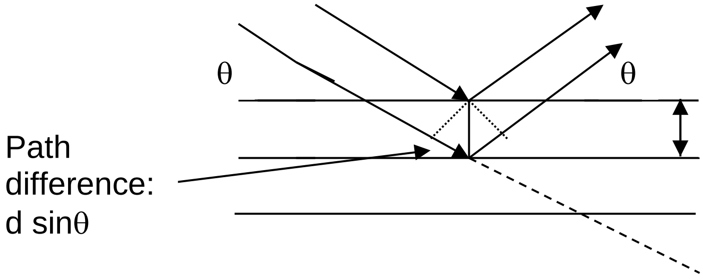
Introduction

X-rays have wavelengths comparable to the interatomic distances in solids, i.e. of the order of 0.1 nm. In 1912, von Laue first suggested that the interference phenomena which occur when a beam of X-rays is incident on the regular structure of crystals could be used to investigate that structure. The technique was further developed by W.H. and W.L. Bragg. Nowadays, the extremely complex structure of biological molecules are investigated using these same basic techniques.

The basic Bragg scattering law can be obtained particularly simply in 2-dimensions. First, consider diffraction of X-rays from a single plane of atoms; clearly, if the angle of incidence is equal to the angle of reflection (the law of reflection for light) then all scattered rays are in phase, corresponding to constructive interference. But then consider scattering from planes of atoms separated by a distance d; in this case, constructive interference occurs only when the path difference between waves scattered from adjacent planes is an integral number of wavelengths. From the diagram below, it should be clear that this condition can be written as

$$2d\sin\theta = n\lambda$$

where λ is the X-ray wavelength and the integer n is called the order of diffraction.



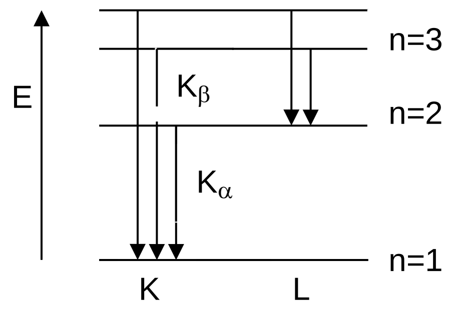
Note that the angle θ is measured from the crystal plane, not from the normal as is customary when considering light. Also note that the X-rays are scattered through an angle **2θ.**

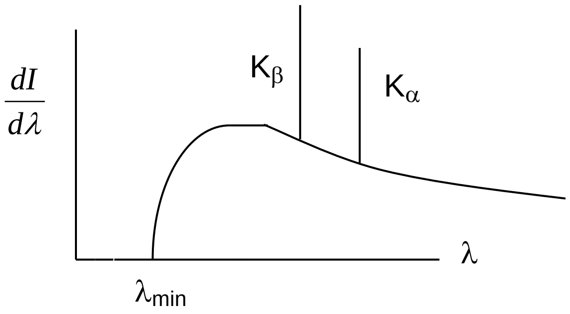
Production of X-rays

X-rays are produced by bombardment of a target by a beam of electrons. (In this experiment the target is molybdenum). The resulting X-ray spectrum has two distinct components: a continuous bremsstrahlung spectrum which is characteristic of the incident beam energy, and several discrete spectral lines which are characteristic of the target material.

Bremsstrahlung is radiation produced by the deceleration of electrons in the very strong electric fields near the target nuclei. Its intensity is dependent upon the target material, but the shape of the spectrum depends only on the primary electron energy E_0. The maximum possible X-ray energy $E_0 = h\nu = hc/\lambda$. If the electrons were accelerated through a potential V so that they have energy $E_0=eV$, then the corresponding minimum X-ray wavelength is given by $\lambda = c/\nu = hc/eV$. The continuous spectrum extends to longer wavelengths corresponding to smaller fractions of the incident energy being radiated away.

In contrast, a discrete spectrum is emitted when electrons from higher shells replace inner-shell electrons which have been knocked out of the target atoms by the incident electrons. The highest energy X-rays, called K X-rays, occur from electron transitions to final n=1 atomic states while L X-rays correspond to transitions to n=2 states. A further classification occurs depending upon which higher shell the occupying electron comes from; in the case of molybdenum the only significant discrete components are called K_α and K_β, corresponding to transitions from n=2 to n=1 and from n=3 to n=1 states, respectively. This is shown schematically below:





The K_α and K_β lines of molybdenum have wavelengths 0.0711 and 0.0631nm. (Actually these lines have fine structure due to spin-orbit interactions but this splitting is not easily resolved). It is these characteristic lines that you'll use in this experiment. The K_β line is about 5 times weaker than the K_α and may be reduced by a much larger factor using selective filtering. In particular, a zirconium absorber placed in the beam has a very strong absorption edge at λ = 0.069 nm with the result that wavelengths below this are selectively absorbed.

Bragg diffraction

There are three principal methods used in X-ray diffraction studies. The method due to **Laue** (1912) uses the continuous component of the spectrum incident upon a carefully aligned single crystal; from the observed interference pattern, which is generally recorded photographically, the overall crystal structure can be inferred. In the **powder method**, the target is in the form of many small crystals oriented in random directions; X-rays are diffracted into cones at well-defined angles depending upon the crystal lattice parameters. In the present experiment, you will use the **rotating crystal technique** along with the equation for Bragg scattering to determine crystal lattice spacing. Photographic recording is again possible, but you will use a Geiger counter to detect individual scattered X-rays. This makes it possible to use very low X-ray intensities (for safety reasons).

The experiment and procedure

There is a sign-in notebook next to X-ray apparatus. If you or your group members are going to be using this machine, you must always sign-in at the beginning of this lab.

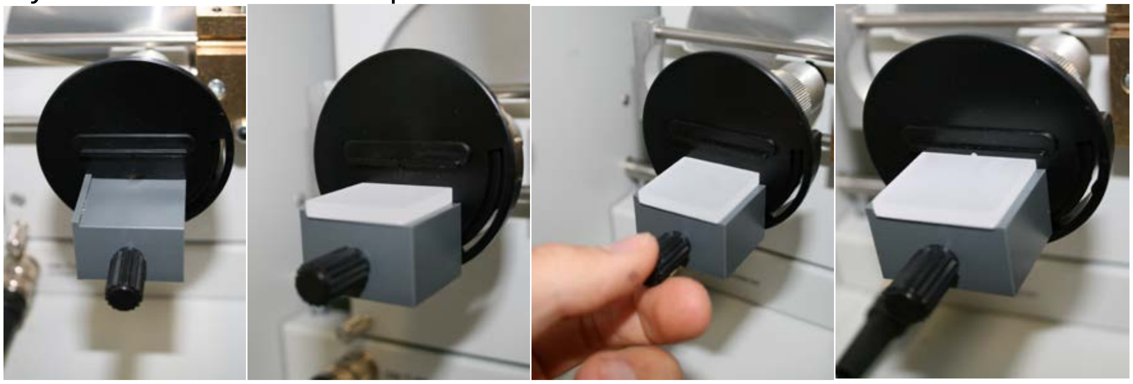
The LEYBOLD X-ray apparatus (#554 800) has many components and comes with software to automate data collection. Take some time to identify the source of the X-rays, the central crystal target stage and the sensor arm, which has the detector (a Geiger tube which counts the number of scattered X-rays).

The LEYBOLD X-Ray apparatus: A) is the display and control panel (Note: For this experiment, the controls will be adjusted using computer software.); B) is the X-ray source (a Molybdinum X-ray tube); C) is the target stage; and D) is the sensor arm.

Locate the crystals available for diffraction. They are NaCl, LiF, and KBr. Pick which one you will use for the first round of data collection.

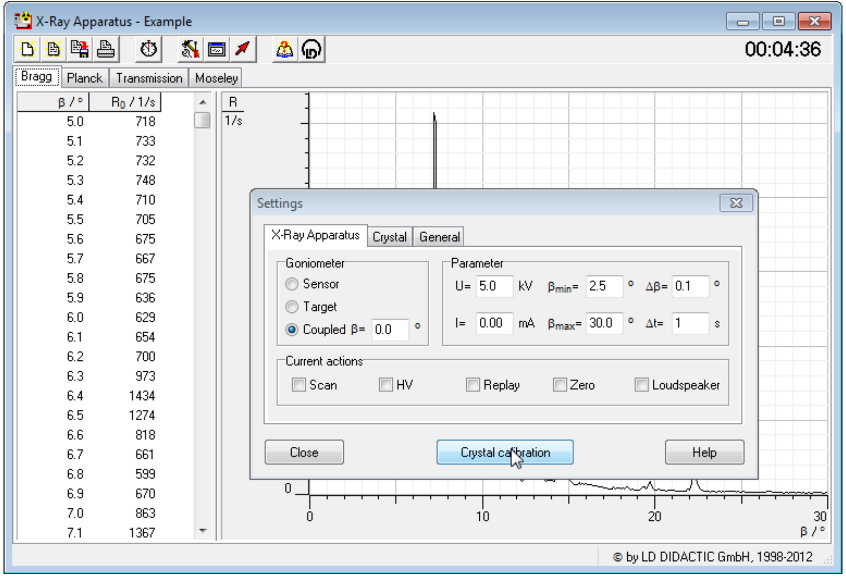
Turn on the X-Ray machine and open the apparatus software, XRay.exe, found on the desktop.

When placing a crystal on the target stage inside the X-Ray apparatus, it is important to secure the crystal so it does not fall during data collection. To secure the crystal, loosen the stage and lower it a little bit, enough so that the crystal can fit under the lip of the black mount that the stage is connected to. Take your chosen crystal and place it on the target stage. (When handling a crystal, grab it only by the **short sides** and make sure to **use gloves**; the moisture of your hands will damage the crystals otherwise.) **Gently** raise the stage to secure the crystal between it and the lip of the mount.



Whenever a crystal is placed or rearranged on the stage, the goniometer will need to be calibrated. This is to ensure that the detector is in the best location to detect x-rays. In order calibrate the apparatus, go to the software and click the "Settings" button (it has a picture of a hammer).

In the Settings menu, you will notice many different inputs. Ignore these for now and click the “Crystal Calibration” button at the bottom of the window.



β / °	R_0 / 1/s
5.0	718
5.1	733
5.2	732
5.3	748
5.4	710
5.5	705
5.6	675
5.7	667
5.8	675
5.9	636
6.0	629
6.1	654
6.2	700
6.3	973
6.4	1434
6.5	1274
6.6	818
6.7	661
6.8	599
6.9	670
7.0	863
7.1	1367

Goniometer
○ Sensor
○ Target
● Coupled β= 0.0 °

Parameter
U= 5.0 kV β_{min}= 2.5 ° Δβ= 0.1 °
I= 0.00 mA β_{max}= 30.0 ° Δt= 1 s

Current actions
☐ Scan ☐ HV ☐ Replay ☐ Zero ☐ Loudspeaker

In the new menu, you will set the parameters for the calibration search. Select your crystal from the first drop down menu and make sure the Mo-tube is selected for the anode. Make sure the doors to the X-ray tube and crystal are closed. Once everything is set, hit the “Start Search” button. Once the calibration is done (and if it is successful), hit the Adopt button to return to the Settings window. (If the calibration is not successful, try rearranging the crystal by either turning it 90 degrees or flipping it over. If this doesn’t work, consult your TA.)

Back in the *Settings* window, notice the different parameters that can be adjusted for data collection. These parameters determine how the X-ray apparatus will collect data. The Goniometer section lists which arms will be adjusted during data collection. The crystal is located on the target arm and the GM tube is located on the sensor arm. This experiment requires a 2:1 coupled motion between the sensor and target so make sure **coupled** is selected.

In the Parameters section, there are six items that can be adjusted. **U** is the Mo-tube Voltage and this should always be set to **35keV**. **I** is the Mo-tube current this should be set to **1mA**. With the goniometer in coupled mode, $\boldsymbol{\beta_{min}}$, $\boldsymbol{\beta_{max}}$, and **Δβ** all refer to the angle of the **target arm**. $\boldsymbol{\beta_{min}}$ and $\boldsymbol{\beta_{max}}$ determine the range of angle measurements. For now, use a range of 0 to 45 degrees. If you want to later change it, go ahead. **Δβ** is the step size of the target arm angle. When looking for peaks, it is important to have a step size as small as possible in order to get the most accurate shape of the peak. **Δt** is the step time; this is the amount of time the sensor will stay at each step. The longer the sensor stays at each step, the more accurate the count rate will be for that step. For our purposes, a step time of 1 or 2 seconds is plenty accurate.

Once you have set the parameters, close both doors to the apparatus and hit the Scan button. The X-ray tube will turn on and the sensor will start collecting data. Close the Settings window to return to the histogram of the data. The histogram and table in the X-ray Apparatus software will update as the data is being taken. The display on the X-ray machine also shows the count rate and angle of one of the arms (°T stands for target arm angle and °S stands for sensor arm angle). When the data collection is done, be sure to save the file for future reference. Copy the data from the table into Excel in order to complete error analysis.

Repeat this process for the two other crystals.

You should be able to locate 3 or 4 orders of diffraction in each case for each of the two X-ray wavelengths**.** Take enough data points so that you can produce a plot showing the detected intensity as a function of angle for each crystal.

Repeat one of the data runs with a zirconium filter in front of the collimator to see its effect.

Some points to consider:

(i) What is the best way to determine the center of a peak? By estimating the angle at which maximum counting rate occurs, or by estimating the positions of the two half-maxima where the counting rate changes most rapidly?

(ii) If your counting rates are low, have you done the obvious thing of checking the alignment of the crystal by eye? Could one face of the crystal be better than another (less damaged)?

(iii) How do you know that zero scattering-angle corresponds to zero on the angle scale? Such an offset would result in your calculations of *d* having a dependence on angle. This is an example of a **systematic error**. Is there evidence in your data for such an error?

Imagine that you are off by 0.3^o, then $\theta_{measured} = \theta_{true}+0.3^0$ Since $2d \sin\theta = n\lambda$, you know the ratios between orders: $\sin\theta_1/\sin\theta_2 = ½$ and $\sin\theta_2/\sin\theta_3=2/3$ and so on. Thus, you can use this knowledge to find a correction angle θ_C, either for one ratio (left), or by adjusting the correction to minimize χ^2 in your fitting program (right). The equations can be reduced to an expression for θ_c by using the trig identity for sin (a+b).

$$\frac{\sin(\theta_{1meas}+\theta_C)}{\sin(\theta_{2meas}+\theta_C)}=\frac{1}{2}$$

$$\sum_{n,xtal}\frac{\left(\frac{\sin(\theta_{nmeas}+\theta_C)}{\sin(\theta_{n+1meas}+\theta_C)}-\frac{n}{n+1}\right)^2}{\sigma^2_{n,n+1}}$$

Data analysis

For each crystal construct the following table:

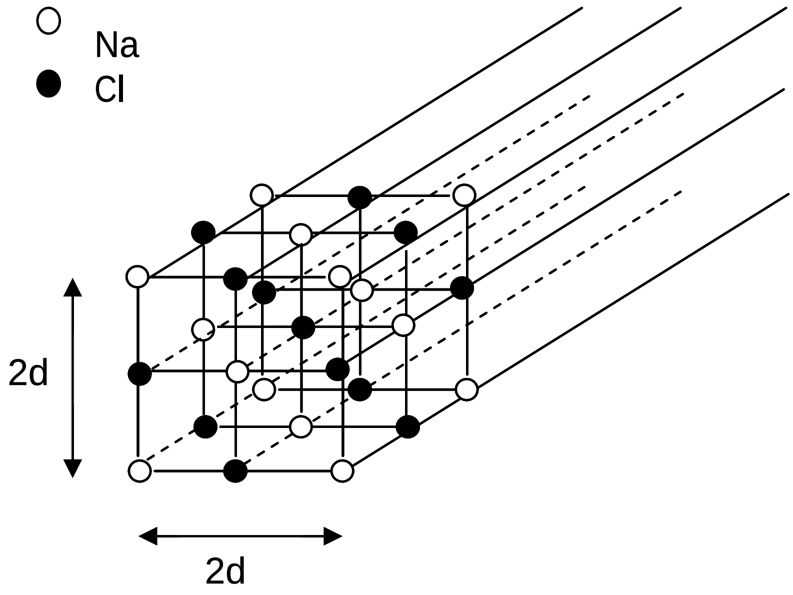
$2\theta_{meas}$	θ_{Bragg}	$\delta\theta_{Bragg}$	N	λ	d_{calc}	δd

$\delta\theta_{Bragg}$ and δd are the uncertainties (standard deviations) in your measurement of the scattering angle and in the value of d derived from this measurement. **You should differentiate the Bragg equation to determine how, in fact, δd depends on $\delta\theta_{Bragg}$.**

What is your best estimate of the lattice spacing of each crystal? This should be derived by taking a weighted mean of all the measurements taken for each crystal.

Determining Avogadro's number

Avogadro's number can be determined by a variety of means. However, X-ray crystallography provides the most accurate determination.



Each of the crystals you have measured is an **alkali halide;** these have cubic structures. The drawing above shows the face-centered cubic crystal structure of sodium chloride. On the left is shown the **conventional unit cell** of the crystal; it represents the periodicity of the crystal lattice. Notice that, if this is considered part of a much larger crystal, the volume of the conventional unit cell contains **four** NaCl molecules. To see this, consider just the Na atoms in the figure and *count* the number of atoms that occupy a cube of volume $8d^3$, accounting for the fact that the atoms are shared with adjacent cubes. For example, the atoms in the center of each face are shared between two cubes.

Now, you can calculate the volume occupied by 1 mole of the crystal (molecular weight/density) and then determine Avogadro's number.

The table below gives some relevant data taken from the *CRC Handbook of Physics and Chemistry*

Atom	Atomic Weight
Li	6.941
F	18.998
Na	22.990
Cl	35.453
K	39.098
Br	79.904

So that you don't have to measure the density of your samples, you can look them up: the *CRC Handbook* gives the specific gravities of NaCl, KBr, and LiF as 2.165, 2.745, and 2.635, respectively.

Semiconductors:
Band Gap of Germanium and the Hall Effect

Purpose

To determine the band gap of pure germanium (Ge) and to measure the carrier concentration and carrier type in doped Ge using the Hall effect.

Key concepts

Semiconductors
Band gap
Doping
Hall Effect

References

Eisberg and Resnick, *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles* (New York: Wiley, 1985); Section 13.9.
Kittel, *Solid State Physics*, *7th ed.* (New York: Wiley, 1995), Chapters 7 and 8.

Introduction

Semiconductors are ubiquitous in everyday life. They form the building blocks of integrated circuits and are the basis for most lasers (such as those in CD players, laser pointer, *etc*.). It is hard to believe that the technological potential of materials such as Si and Ge were essentially ignored until the 1930's. In part this was due to the fact that the physicists had not yet developed the means to understand the physics of semiconductors. This changed with the development of quantum mechanics and solid state physics, but an equally important factor has been the growth of materials science as a research discipline. It is now possible to grow and process semiconductor materials in all sorts of ways.

As the name implies, semiconductors are "poor" conductors. Unlike genuine insulators such as glass, they can carry a current. The typical density of current carriers, however, is orders of magnitude smaller than the electron density in a metal. More importantly, the density of carriers depends strongly on temperature as well as the presence of impurities. The utility of semiconductors derives in large part from our ability to control their properties by introducing impurities, a process known as *doping.*

The unique properties of semiconductors come from the way in which the electronic states of the material are filled. In a metal such as Cu, the highest energy level occupied by electrons sits in the middle of a large number of available states as shown in Fig. 1(a). The presence of these nearby empty states allows the metal to carry a current easily.

In a semiconductor, there is a *gap* E_g that separates the filled states from the empty states of lowest energy. Fig. 1(b) shows the situation for a semiconductor such as pure germanium (Ge). The filled states below the gap are referred to as the *valence band*, and the states above the gap are known as the *conduction band*. It can be shown rigorously that a filled band cannot carry an electrical current. This is a consequence of the Pauli exclusion principle. This means that in order for current to flow in a semiconductor, it is necessary to add electrons to the conduction band OR to remove electrons from the valence band. The simplest means to accomplish this is by increasing the temperature.

At non-zero temperature, some electrons, represented by the black circles in Fig. 1(c), are thermally excited from the valence band to the conduction band. The number of electrons in the conduction band is very small, but they are free to move because of the large number of empty states. Similarly, the *holes* formed in the valence band due to the absence of electrons, which are shown by white circles in Fig. 1(c), can also carry a current. Appealing to elementary statistical mechanics, we would expect the number of mobile carriers to be an exponential function of the ratio $E_g / k_B T$. The actual result for the number of carriers is

$$n = f(T) e^{-\frac{E_g}{2k_B T}}, \qquad (1)$$

where the function *f(T)* varies with temperature as $T^{3/2}$.

In a simple transport experiment, we measure the *conductivity* σ, or *resistivity* $\rho = 1/\sigma$. We expect the conductivity to be proportional to the number of carriers, with an extra factor μ, known as the *mobility*, that accounts for how difficult it is for the electrons to move through the material. It can be shown that

$$\sigma = ne\mu \approx \sigma_0 e^{-\frac{E_g}{2k_B T}}, \qquad (2)$$

where σ_0 is *nearly* independent of temperature at room temperature. There are a lot of details that go into Eq. 2, but the result is very simple: the conductivity of a pure (also known as *intrinsic*) semiconductor depends exponentially on the ratio $E_g/2k_BT$.

The first part of this experiment is devoted to measuring the band gap E_g of germanium (Ge). Although the technological importance of Ge is now minor relative to Si (used in electronics), the gallium arsenide family of semiconductors (used in high-speed electronics, semiconductor lasers, and red photodiodes), and the gallium nitride family (used for blue laser diodes and the high-intensity photodiodes that are now replacing conventional light-bulbs), Ge played an important historical role in the development of semiconductor technology. Germanium was the first semiconductor to be grown in large and very pure crystals, and it was used in the first transistors fabricated in the 1940's and 50's. It is still used today for particle detectors and in some special electronic devices. One of the reasons that silicon is superior to Ge for most electronic applications is its larger band gap. The band gap also plays a major role in determining the optical properties of semiconductors.

Procedure (Band Gap Measurement)

The equipment is very sensitive, so you will see warnings throughout. Please pay attention to them. Since hooking it up the wrong way causes chips and ammeters to blow, please have the TA check your circuit before turning it on.

The apparatus consists of several parts. The pure Ge crystal is mounted on a fiberglass circuit board. Electrical leads are soldered to the ends of the crystal and a heater and thermometer are mounted on the circuit board so that they are in good thermal contact with the crystal. **Make sure that you are using the correct circuit board, which should have the part number 586851.** The circuit board should be plugged into the "Base Unit" (part number 586850), which includes connectors for the current, voltage, heater, and thermometer leads. The principle of the experiment is very simple: a current flows through the sample, and you will measure the current and voltage across the sample as a function of the temperature, which you will change by adjusting the heater power.

THE CIRCUIT BOARD AND CONNECTORS ARE FRAGILE AND MUST BE PLUGGED IN PROPERLY, WITH THE Ge CRYSTAL VISIBLE THROUGH THE HOLE IN THE BASE UNIT. DO NOT USE FORCE! If you have any doubt about how to do this, please ask an instructor! Make sure that the current adjustment knob is set all the way counterclockwise (at 2 mA) before the board is plugged in. The knob should remain at this position throughout the first part of this experiment.

Hook up the leads as shown in Fig 2. Turn on the HP 6234A power supply. Once you have verified that the output of this supply is at +12 V, turn on the power switch on the Base Unit itself. Verify that a current of 1.8 – 2.0 mA is flowing through the crystal. You should measure a voltage drop of about 1 V.

Now turn on the power supply for the heater. The heater coil on the circuit board is protected by a relay. Set the voltage knob on the heater power supply to 10 V. Rotate the current knob on the power supply about 30 degrees from the full counterclockwise position, and then hit the *Heater* button on the base unit. Then, *quickly* set the current to 0.1 A. The red light will fade or go out, but the heater coil and thermistor (a resistor that we use as a thermometer) will still be connected.

The thermistor can be read with a voltmeter and has a calibration:

$$T(K) = 273 + 100 \cdot V, \tag{3}$$

where V is the voltage (in volts). You can assume that this calibration is exact (within 2 K or so), but you will find that there are other systematic errors that affect your experiment. Make sure that the thermistor is reading properly. With 0.1 A going through the heater, you should find a temperature slightly above room temperature. Note that if you reduce the heater current all the way to zero, the thermometer and heater will become disconnected. To reconnect them, repeat the procedure described in the previous paragraph. You will also find that the heater is automatically disconnected when the sample temperature exceeds approximately 420 K. If this happens, wait a few minutes before repeating the procedure of the previous paragraph. **Do not exceed a heater current of 2.5 Amps!**

Decide how you should take data to verify Eq. 2 and measure the band gap E_g of Ge. Is it better to take data while warming or cooling? How long should you wait at each temperature? The temperature will never be completely stable, and so you will always have to take data while it is drifting. However, you should wait *at least* 2 minutes at each setting for it to settle. Estimate the errors in the temperature and the voltage at each point. How should your data points be distributed? Make sure to take at least 15 points over the temperature range of the experiment. Figure out a way to plot your data so that they are linear on a semi-logarithmic scale (while you are doing the experiment!).

Decide how you should analyze your data to determine the band gap of germanium. Compare with values in the literature. To what extent does Eq. 2 describe the temperature dependence of the conductivity? Determine the *resistivity* ρ of your sample at room temperature and at 420 K. Remember that it is the resistivity (**not** the *resistance)* which is a fundamental property of a material. The dimensions of the Ge crystal are: length = 20.0 mm, width = 10.0 mm, thickness = 1.0 mm. How does the resistivity of your Ge sample compare with that of a good metal, such as Cu, at room temperature? Explain.

Doped Semiconductors and the Hall Effect

Controlling the conductivity of a semiconductor by adjusting the temperature is tedious and impractical for making useful devices. In practice, the electronic properties of pure semi-conductors are modified by *doping* with impurities. For example, adding a small amount of phosphorus to Ge has the effect of adding electrons to the conduction band, as shown in Fig. 3(a). In this case, the impurity is called a *donor*, and the resulting semiconductor is called *n-type.*

Boron, however, has the effect of removing electrons from the valence band, and it is called an *acceptor.* In this case, the resulting semiconductor is called *p-type.* The *holes* remaining in the valence band are shown in Fig. 3(b).

The difference between the effect of donors and acceptors on charge transport is illustrated in Fig. 4. The case of donor impurities is easy to understand: the electrons added to the conduction band can now carry a current. The effect of acceptors is slightly more subtle. In this case, electrons are removed from the valence band, and the *holes* that are left behind can conduct a current. The holes*,* which correspond to the absence of electrons, behave like *positive* charge carriers! If a voltage is applied as shown in Fig. 4, a current flows to the right, but this can be thought of as *either* negative charge carriers flowing to the left or positive charge carriers flowing to the right. Can we tell the difference between the two cases, and is it possible to measure the number of charge carriers?

The answer to both of these questions is "yes" and involves the use of a magnetic field to measure the *Hall effect.* Consider the case of an n-type semiconductor, shown in Fig. 5. A current flows to the right, so negatively charged electrons flow towards the left with a velocity *v.* Suppose that a magnetic field is applied

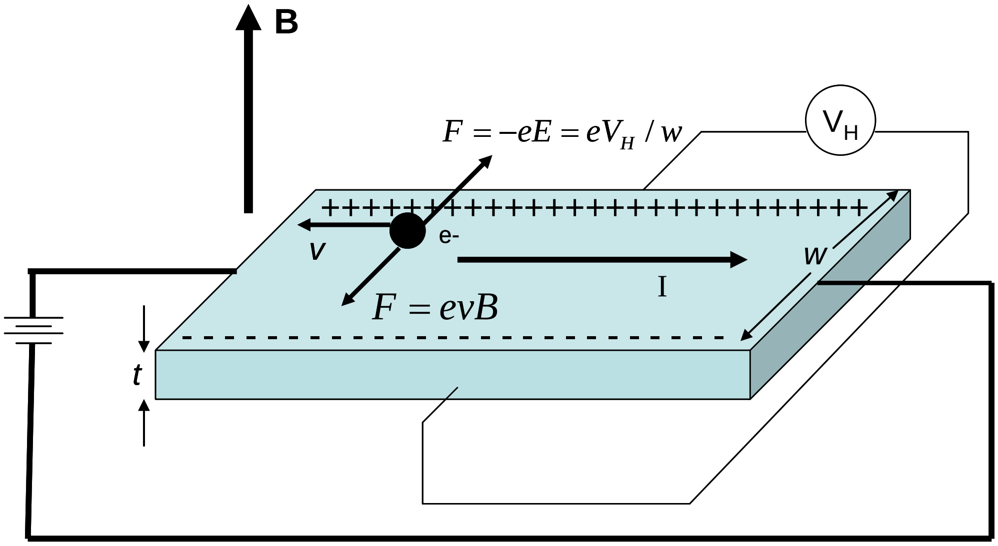


Fig. 5: The geometry of a Hall effect experiment. The magnetic field **B** is pointing up and the current is flowing to the right. The forces shown are for negatively charged carriers, for which V_H is positive when **B** is positive.

in the direction shown. Using the right-hand rule, you should be able to show that there is a force *evB* on the electrons in the direction shown, and that they will therefore move towards the front edge of the rectangular sample, which will become negatively charged with respect to the back edge. Of course, this process cannot go on forever, as eventually there will be an electrostatic force $qE = eV_H / w$ that builds up and exactly cancels the force due to the magnetic field, where V_H is the voltage shown in the figure, and *w* is the width of the sample. (Recall that $V = -e\int_0^w Edx$, and we will assume here that the electric field *E* is constant.) Equating the two forces,

$$v = V_H / Bw, \qquad (4)$$

which would allow us to determine the drift velocity of the carriers. The velocity is related to the current by $I/A = nev$, where $A = wt$ is the cross-sectional area of the sample and *n* is the density (number per unit volume) of carriers. Substituting for *v* in Eq. 4, and rearranging, we find

$$n = \frac{IB}{eV_H t}. \qquad (5)$$

Repeat the analysis of Fig. 5 for the case of *positively* charged carriers flowing to the *right*, for which the direction of the current is the same. You will find that the magnetic Lorentz force points in the same direction as in Fig. 5, but the voltage V_H will be negative, so that

$$p = -\frac{IB}{eV_H t}, \qquad (6)$$

where *p* is the number of *positively charged* carriers per unit volume. Note the minus sign. Eqs. 5 and 6 therefore allow us to measure the density of carriers from a measurement of the *Hall voltage* V_H, provided that the current, magnetic field and thickness of the sample are known. Just as importantly, we can determine the *sign* of the charge carriers! For the geometry of Fig. 5, a positive value of V_H when B is positive indicates that the charge carriers are electrons, while if V_H is negative when B is positive, then the charge carriers are positively charged holes. Note that this distinction is possible only because the Lorentz force depends on the charge *and velocity* of the carriers.

Procedure (Hall Effect)

There are two doped Ge crystals mounted on boards labeled 586852 and 586853. One of these is n-type (negatively charged electrons are carriers) and the other is p-type (positively charged holes are carriers). You must determine the carrier density **and sign** for each crystal. The four terminals on the base unit labeled L, R, T, and B are connected to the crystal. These correspond to left, right, top and bottom in Fig. 5, **provided that the Ge crystal is facing you as shown in Fig. 6.** Insert the card with one of the doped crystals into the base unit. Be very gentle, and make sure that the crystal faces through the hole in the base unit. If you have any problem with either removing the previous card or inserting the new one, please ask an instructor for assistance. Remember to note which card you are measuring in your lab book! Hook up the leads, using the Phllips voltmeter between contacts T and B to measure $V_{H.}$ You must be very careful to ensure that the polarity of your current and Hall voltage leads is correct! For all of the instruments provided, red terminals are positive. Set the current flowing through the crystal to 20 mA as measured with an ammeter. Adjust the knob labeled “COMP” so that $V_H = 0$ in zero magnetic field. Do this before inserting the sample into the magnet. This compensates for small deviations from the ideal geometry of Fig. 5. Leave the heater and thermometer leads disconnected for this part of the experiment.

The post on the base unit fits into a hole underneath the magnet. Make sure that the surface of the crystal is parallel to the pole pieces of the magnet, so that the current path is perpendicular to the magnetic field. A magnetic field sensor can be slid into the magnet next to your crystal, with the flat face of the sensor parallel to the slab of Ge. **Be careful. The magnetic field sensor is very fragile**. *Before* inserting the field sensor, you need to set the “zero.” To do this, place the sensor at least 12 inches from the magnet, turn the dial on the meter to “Zero” and then hit the button labeled “Hold Reset.” The meter will flash “Auto” while it is zeroing sensor. This will take several seconds. When it is done, set the dial back to “Measure,” and then the sensor is ready to use. There is a piece of tape on the sensor, and you should use this to make sure that the sensor always faces the same direction. Unfortunately, you do not know which orientation of the sensor corresponds to a positive magnetic field. The meter attached to the field sensor will read either positive or negative, but this may not correspond to the direction of the magnetic field defined as positive in Fig. 5. A coil of hook-up wire is provided, which can be connected to the power supply. Since you will know which way current flows in this coil, your right hand can then be used to determine the direction of the magnetic field which corresponds to a positive reading on the sensor. The direction of the field should be switched by reversing the leads at the magnet power supply. Do this only when the supply is turned off. Do not rotate the magnet or change any connections on the magnet. Finally, the default setting for the field sensor is in units of Gauss, and there are 10^4 Gauss per Tesla.

Now determine the carrier density for each of the two crystals, labeled 586852 and 586853. To change the crystals, first slide the field sensor out of the magnet and then remove the base unit from the magnet. Reverse this procedure after inserting the new card. The thickness of each crystal is 1.0±0.1 mm. Approximately 10 data points evenly distributed between -1.5 kilo-Gauss and +1.5 kilo-Gauss will suffice. Does V_H depend linearly on magnetic field? Does the slope of V_H *vs.* B reverse when you change the crystal? Which crystal is n-type and which is p-type? Make a small table in your lab book indicating the serial number of each crystal, the carrier density, and whether it is n-type or p-type.

How do the carrier densities you found for the two Ge crystals compare with that for copper, in which there is approximately 1 free electron per Cu atom?

Can you guess how the magnetic field sensor used in this experiment works?

Counting statistics and radioactive decay

Purpose

To use a Geiger counter to investigate the random statistics of radioactive decay and to study the Poisson and Gaussian distribution functions which describe them. The half-life of the nucleus ^{80}Br will be measured after its production by neutron activation.

Key Concepts

- Geiger counter
- Statistical fluctuations
- Poisson distribution
- Gaussian (normal) distribution
- Radioactive decay
- Half-life

Introduction

The topic of statistics will be covered in lectures and is also presented in an Appendix to this lab manual. However, in case you are not yet familiar with that material, we will give an abbreviated discussion here. You will also need to refer to the appendix for further details.

The observation of a random process, such as the decay of radioactive atoms, will produce results that have a well-defined distribution about the average rate of decay if a large number of observations are made. In this experiment it is instructive to compare the observed data with two distribution equations: the Poisson distribution and the Gaussian (or Normal) distribution. For the observations to be described by either equation, a fundamental requirement is that the events being observed be both independent and random. This is certainly true of the decay of atomic nuclei.

Poisson Distribution

For data to be described by the Poisson equation it is necessary that a large number of events may occur but that, on average, only a small fraction of the possible total occur during any one observation. In this experiment, where you will count the number of decays in a given time interval, this condition is satisfied because the radioactive material contains many millions of active nuclei but the time of one observation is short compared with the half-life. In addition, the detector intercepts only a small fraction of the radioactive decay products.

The Poisson distribution gives the probability *P(x)* of observing *x* events when the average (mean) number of events per observation is $\bar{x}$.

$$P(x) \equiv P_P(x, \bar{x}) = \frac{\bar{x}^x}{x!} e^{-\bar{x}}$$

The probability *P(x)* should be multiplied by the total number of observations to obtain the number of times that *x* events is expected to be observed.

The calculation of *P(x)* is not difficult if $\bar{x}$ is a fairly small number. Since the factorial of zero is one, then $P(0) = e^{-\bar{x}}$. Thus, $P(1) = P(0)\bar{x}$ and $P(2) = ½ P(1)\bar{x}$ and $P(3) = 1/3\ P(2)\bar{x}$, etc. Using this step-by-step process avoids the need for very large or very small numbers. Alternatively, you can find the function in the Excel library of functions, and allow the computer to do the calculations for you.

Gaussian Distribution

If the average number of counts observed in each run is more than about 10, the calculation of *P(x)* gets rather tiresome. For averages greater than this value, the Poisson distribution can be replaced by the Gaussian or "Normal" distribution, which has the equation

$$G(x) \equiv P_G(x, \bar{x}, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left[-\frac{(\bar{x}-x)^2}{2\sigma^2}\right]}$$

The evaluation of this expression requires the specification of two numbers: the average number of events per observation (counts per run) which is $\bar{x}$, and also the standard deviation σ, which describes the spread of the distribution on both sides of the average. The Gaussian distribution is symmetric about the average rate. This is not true of the Poisson distribution, which is very asymmetric for small values of $\bar{x}$.

The standard deviation σ, is the square root of the average of the square of the deviations from the average of all data points. That is, it is the R.M.S. deviation from the average. For randomly occurring events the ideal value of σ is $\sqrt{\bar{x}}$.

To make a comparison of a set of data with the theoretical Gaussian distribution, it is usually worthwhile to group the data into intervals before plotting a graph of the distribution. For example, one can plot the number of runs resulting in 90 to 94 counts per run, the number of runs resulting in 95 to 99 counts, etc. In this case the interval is 5 units. On the same graph, one then plots the theoretical Gaussian curve with an adjustment or **normalization** so that the area under the curve will be the same as the area under the graph of actual data. This normalization is obtained by multiplying the values of *G(x)* by $N\Delta$, where *N* is the total number of runs and Δ is the width of the interval described above.

It is interesting to compare the spread in data with the ideal distribution, which predicts that 31.7% of the data should deviate more than 1σ from the mean.

Plateau Curve of Geiger Counter

Before using a Geiger counter, it is necessary to determine the proper operating voltage. If the voltage is too low the counter will be inefficient, but if it is too high, it will go into breakdown mode. The Geiger counter is connected to a unit

“Model 500 Nuclear Scaler” which contains both a high voltage supply and a scaler. We no longer use the scaler function, so just treat this like a fancy power supply. The counts from the Geiger counter are recorded using the NI PCI6259 DAQ card connected to the computer. To avoid damaging the tube, follow these steps in the order listed (*and also be careful of damaging the thin window of the counter*).

1. Before pressing the power switch on the Model 500, set the high voltage knob to 200 volts or lower. Then turn on the electric power by pushing the POWER switch.

2. Open the RadCounterInterface program on the computer desktop. This is your scaler. You can choose a time interval over which you will accumulate counts. When you hit START, it will accumulate counts for that time interval. Choose a filename and location for your data to be saved. STOP stops the run.

3. The radioactive sources are kept in a safe to which only the instructors have access. Use an appropriate source such as ^{137}Cs (which emits both βs and γs) to check out the operation of the Geiger counter. Place the source below the counter, choose an infinite time interval and push START. Raise the high voltage until the read-out indicates that βs are being detected. The voltage at which counts are first observed is the **“threshold voltage”.**

4. Gradually increase the voltage at about 10 or 20-volt intervals. At each voltage take a half-minute run. Plot a graph of counting rate vs. voltage; this should show a **“plateau”** where the counting rate stays fairly constant with increasing voltage. This plateau is not perfectly flat; the counting rate actually increases slightly with increasing voltage. Set the voltage ~20 V above the beginning of the plateau, but not more than 60 V.

5. Record your plateau curve and label your operating voltage in your logbook. This is the voltage you will use for the rest of the lab.

6. With no source present, take a few one-minute runs to determine the “background” counting rate caused by cosmic-rays and other stray radiation.

When raising the high voltage be sure to monitor the count rate. If the rate starts to rise very quickly, back off the voltage. Never exceed the high voltage value that causes the counting rate to become more than 10 per cent larger than the rate on the plateau. Excessive voltage applied to a counter tube will ruin it immediately.

Procedure:

Part A. Statistical Fluctuations in Counting Rates

Since the decay of any individual atom is unpredictable, the number of counts recorded in a given time will fluctuate about the mean or average value. To obtain information about such fluctuations, perform the following steps.

1. Using a radiation source, determine a source position that will give a counting rate of 1 - 2 counts per second. Check that you have the right counting rate before proceeding with the experiment!

2. Set the time interval to one second and take data for 300 time intervals. Many of these one-second intervals will have zero counts. Determine how many counts occur during the first 30 one-second intervals, the first 100 and all 300 intervals.

3. For each of the sets of intervals, plot a graph having the number of counts per interval, x, along the x-axis and the number of times each x was observed along the y-axis. On each graph also plot the predictions of the Poisson distribution *P(x)* multiplied by the total number of intervals (30,100 or 300).

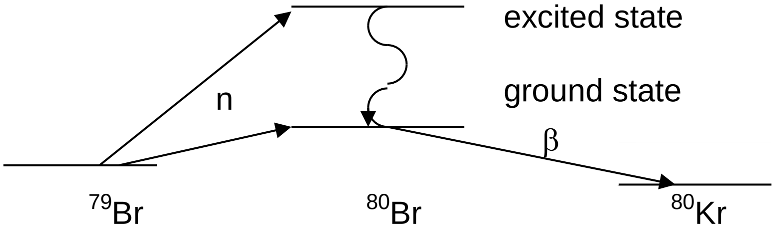
4. For the three sets of data, how well do they fit with expectations? Find the reduced chi-squared to see how well your data matches the Poisson distribution. What is the confidence level of your fit? The definition of chi can be found in the appendix: $\chi_i = \frac{y_{i\_measured} - y_{i\_fit}}{\sigma_i}$ and where y_{i_fit} = *P(x)* in this case. How many degrees of freedom are there in *P(x)*? In carrying out this analysis, make sure you understand the distinction between the uncertainty in a given bin (which is the square root of the number of events in that bin) and the standard deviation of the distribution (which should be the square root of the mean).

5. Find a source location that will produce 20 to 30 counts per second and repeat the above procedure to compare your results with those expected of a Gaussian distribution. With these higher counting rate data a better graph results if you group the data along the number of counts axis into groups of three or four and plot the total number of observations in that group. For each Gaussian use $\sqrt{x}$ as the standard deviation. Again, use a chi-squared test to determine how well your data fit to the underlying probability distribution, this time *G(x)*. Draw some conclusions about the shape of the distribution as a function of the length of the run.

Part B. Study of the Decay of ^{80}Br

Two good references on this subject are "Introductory Nuclear Physics" by Halliday, pp 95-96 and "Nuclear Physics", Kaplan, p 445

When ^{79}Br is irradiated with slow neutrons some of the neutrons are absorbed to form ^{80}Br. 96% of the ^{80}Br nuclei decay to ^{80}Kr by the emission of 2 MeV β-rays. (The other 4% emit 0.87 MeV positrons, which are not of interest in this experiment.) In the neutron absorption process some of the ^{80}Br nuclei are left in an excited state which decays more slowly than does the ground state because of the need for a large change in angular momentum. A ^{80}Br nucleus that was formed in the excited state first emits low energy γ-rays as it drops to its ground state and then decays to ^{80}Kr by the emission of a 2 MeV β-ray which is indistinguishable from the β-ray emitted from a ^{80}Br that was originally formed in its ground state. However, we can determine what fraction of the ^{80}Br nuclei were formed in the excited state by observing the two decay rates in the emission of these β-rays. A simplified schematic of these processes is shown below.



1. First read the whole set of instructions, decide what data are needed, and how it should be recorded. This part of the experiment will take you 3 hours and cannot be stopped and restarted during that time. The usual procedure is to do the first part of the lab week 1, including planning for steps 2-8 below. Then be ready to start Bromine data at the beginning of lab on week 2.

2. Obtain the background counting rate of your counter by making two or three 2-minute runs with no source near the counter. Then place a freshly irradiated sample of NaBr close to the counter tube and take a series of 2-minute runs lasting at least 3 hours. Finally check the background rate again.

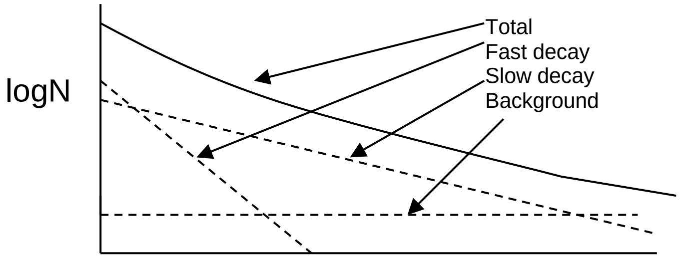
The irradiation must be carried out by the lab instructor and is accomplished by placing the sample inside a Pu-Be neutron source. This is a common source of neutrons for the laboratory, in which α-particles emitted in Pu decay interact strongly with ^{9}Be in the reaction:

$$^{9}\mathrm{Be} + {}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C} + n + 5.76\ \mathrm{MeV}$$

The TA then removes the sample, now containing some fraction of ^{80}Br, and brings it to you. Make sure you know the time that the TA removed the sample from irradiation, in order to be able to complete step 7. ***It is advisable to get an instructor to begin irradiation of the source several hours before your lab***

session begins so that it is available near the beginning of the regularly scheduled lab period.

3. Your data will consist of three separate components as illustrated in the diagram below.



Time t after end of irradiation

4. Subtract the background counts from the results of each run and make a semi-log of the net counting rate as a function of time. The decay rate should be rapid at first, when most of the β-rays come from the ^{80}Br originally in the ground state, and slower later, when the rate is determined by the decay from the high angular momentum excited state of ^{80}Br.

5. Do a least squares fit to the exponential decay for times between 1.5 to 3 hours, where the fast decay has become negligible. Then subtract this slow decay data from the total to leave only the fast component. Do a least squares fit to this fast component.

6. Compare your results for the two half-lives to the expected values, which can be found in the *CRC Handbook* or on the Web at *http://nucleardata.nuclear.lu.se*

7. What fraction of the ^{80}Br atoms were in the excited state at the end of the irradiation period? Make sure that you read the notes below carefully.

8. Your final plot should include all the components shown in the sketch above.

Some notes on radioactive decay

All individual atom of a radioactive isotope have the same probability to decay in a given time. This implies that the total activity, or rate of decay *dN/dt* is proportional to the number of radioactive nuclei *N* present in the sample at time *t*. Thus

$$\frac{dN}{dt} = -\frac{N}{\tau},$$

where τ is s a constant. When integrated, this gives the number of atoms as a function of time:

$$N(t) = N_0 \exp(-t/\tau),$$

where N_0 is the number of atoms present in the sample at time $t=0$. It should be simple to show that τ is the *mean life* of the isotope; this is related to the half-life $t_{1/2}$ by $t_{1/2} = \tau.\ln 2$.

What you actually observe in this experiment is a decay rate dN/dt.
By differentiation of the expression for $N(t)$ above

$$\frac{dN}{dt} = -\frac{N_0}{\tau} e^{-t/\tau},$$

and so

$$\frac{dN}{dt}\Big|_{t=0} = -\frac{N_0}{\tau}; N_0 = -\tau \frac{dN}{dt}\Big|_{t=0},$$

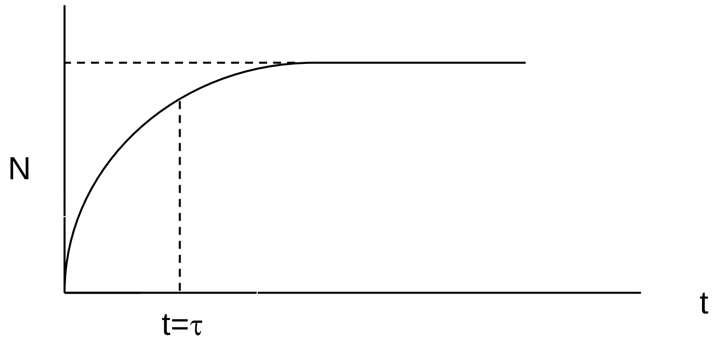
giving the number of radioactive nuclei present initially.
How does the sample of ^{80}Br atoms increase during neutron irradiation? These atoms are produced at a rate proportional to the neutron flux ϕ and decay at the rate shown by the equation above. So at any time t the net rate of production is given by

$$\frac{dN}{dt} = k\phi - \frac{N}{\tau},$$

where k is a constant. Integrating this gives

$$N(t) = k\phi\tau(1 - \exp(-t/\tau)),$$

which reaches about 2/3 of its final rate after about one mean-life.



Compton scattering using a scintillation counter

Purpose

To introduce the scintillation counter as a tool of γ ray spectroscopy.
To calibrate a scintillation counter using standard γ sources; to study Compton scattering and to determine the mass of the electron.

Key concepts

Scintillation
Photomultiplier
Compton effect
Photoelectric effect
Multichannel analyzer

Introduction

1. Scintillation counter and the multichannel analyzer

The scintillation counter is a widely used detector of charged particles. It consists of a scintillating medium coupled to a **photomultiplier tube (PMT).** The passage of a charged particle through the scintillator excites and ionizes the material which then emits small flashes of visible light, or scintillations, as it de-excites. This visible light is, in turn, detected in the photomultiplier tube where the flash of light is turned into a burst of electrons and amplified by a very large factor. In this experiment the charged particles which are detected are actually secondary electrons produced in the scintillator by the interaction of γ rays through Compton scattering, the photoelectric effect, or pair production.

Several materials scintillate. The first use of a scintillation counter was by Rutherford in his classic experiments which demonstrated the existence of the atomic nucleus: α particles scattered from gold foils were detected by their scintillations produced in a zinc sulphide screen. In those days, the scintillations were actually detected by eye by looking at the screen with a microscope. Nowadays, scintillators come in many forms, from inorganic crystals to plastics, and also as liquids and gases. They are used in many fields, from medicine to oil-prospecting to particle physics. The scintillation process is relatively inefficient: in the most efficient scintillators, only one visible photon (~3 eV) is emitted for every 100 eV of primary energy deposited, approximately.

A schematic diagram of a scintillation counter is shown in figure 1. The scintillator in our experiment is a 2 in. diameter by 2 in. long crystal of sodium iodide (NaI) containing about 0.1% Thallium (Tl). One side of the crystal is placed against the glass face of the photomultiplier tube while the other sides of the crystal are surrounded by a light-tight seal to keep out room light (and also to keep out moisture, since NaI is highly hygroscopic).

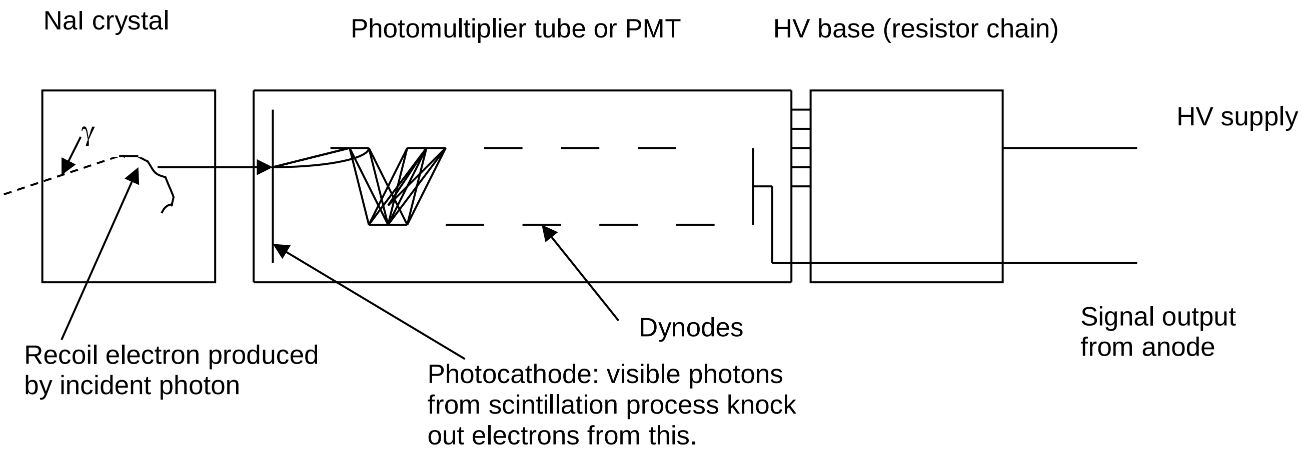


Figure 1. Schematic diagram of a scintillation counter

On the inner surface of the phototube's glass face is a thin photocathode where the visible scintillation photons knock out electrons via the photoelectric effect. About 10-20% of the incident photons are able to knock electrons into the vacuum of the photomultiplier tube. These electrons are focused and accelerated across a potential of 100 V, typically, to an electrode called a **dynode**. Each incident electron has enough energy to knock out several additional electrons from the dynode surface. The number of these secondary electrons is proportional to the incident electron energy, i.e. to the accelerating voltage. This amplification process is then repeated at 12 successive dynodes in this particular photomultiplier tube so that, e.g. if the secondary emission factor at each dynode is 3 to 5 secondaries/incident electron, then the initial burst of photoelectrons will be amplified by a factor 3^{12} to 5^{12}, or about 10^6 to 10^8, depending upon the high voltage supplied to the tube. This signal may be further amplified or shaped for analysis by the subsequent electronic circuitry.

The amount of visible light produced in the scintillator is proportional to the energy deposited by the electron, and the amplification process in the photomultiplier tube is also linear so that the output signals are proportional to the electron energy. Each signal is passed to a **multichannel analyzer** in which it is sorted according to its amplitude and assigned a channel number proportional to this amplitude. The number of signals collected in each channel is stored in the analyzer memory and displayed on the computer screen. The horizontal scale on the display shows channel numbers and the vertical scale shows the number of counts in each channel, so that, over time, the screen builds up a histogram corresponding to the spectrum of energies deposited in the scintillator. The input to the analyzer is automatically shut off during the analysis process, typically about 1 ms per signal, resulting in "dead time". This dead time is subtracted from the real clock time to give the actual "live time". Both live time and real time are displayed on the analyzer screen.

2. γ-ray interactions in matter

At the energies typical of nuclear γ-decay, up to a few MeV, the principal processes through which γ-rays interact with matter are Compton scattering and the photoelectric effect. In the latter, all of the primary γ-ray energy is passed on to the knock-on electron and so mono-energetic γ-rays produce only mono-energetic secondary electrons, ideal for scintillation spectroscopy. (Actually, not all of the primary γ-ray energy is transferred to the knock-on electron, because some of the primary energy is needed to overcome the electron binding energy in the atom. However, the ionized atom is promptly neutralized by an electron from the crystal and releases this extra binding energy in the form of X-rays which are rapidly absorbed by nearby atoms which then eject electrons in turn, so that no energy is lost).

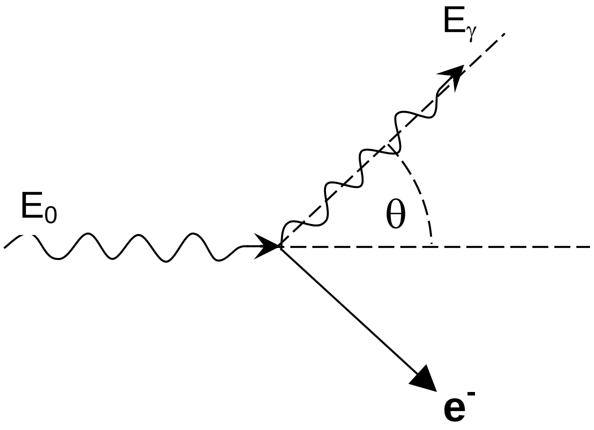


Figure 2. Compton scattering

The Compton process, illustrated in Figure 2, is more complicated. In such an interaction between a photon of energy E_0 and an electron, the photon is scattered with an energy E_γ which depends on its scattering angle θ

$$E_\gamma = \frac{E_0}{1 + \frac{E_0}{m_e c^2}(1 - \cos\theta)}$$

and the scattered electron has energy

$$E_{el} = E_0 - E_\gamma$$

You should be able to derive these relationships! (Note that the original classic experiment of Compton, which provided one of the earliest verifications of the particle-like behavior of the photon, was done in 1920 using X-rays. That experiment showed a characteristic increase in the wavelength of X-rays scattered at various angles from the incident beam. In the case of the much shorter wavelength γ-rays, it is more appropriate to use energy rather than wavelength as the relevant photon property.)

Why have we chosen to use a NaI scintillation counter for this experiment? This is the most commonly used scintillator for γ-spectroscopy because of iodine's high Z! The probability for the photoelectric effect to occur is roughly proportional to Z^5 while that for Compton scattering, where only a fraction of the initial γ energy is transferred to an electron, is proportional to Z (i.e. to the number of electron targets in the material). The result is that the photoelectric effect is strongly enhanced in a high Z material.

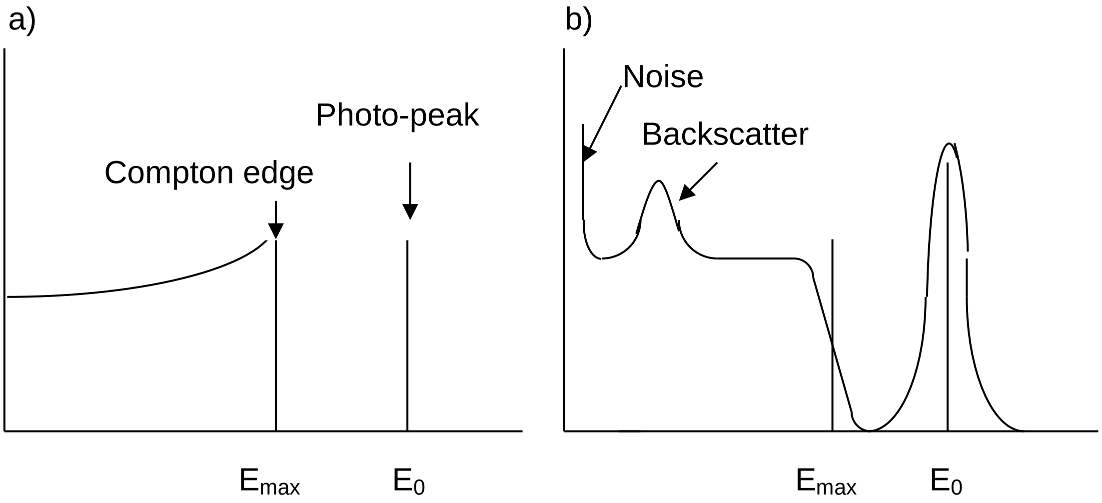


Figure 3 a) Spectrum of recoil electron energies from interaction of mono-energetic photons in matter; b) as observed in a detector with finite energy resolution.

Of course, in general, a beam of mono-energetic γ-rays incident on the scintillation crystal will interact via both Compton and photoelectric effects, so that the resulting spectrum of knock on electrons will contain both components. (At the γ energies used in this experiment, pair production is not important because of its threshold at $2m_ec^2 = 1.02$ MeV, and, in any case, its probability is very small compared to the other processes). Figure 3a shows the electron energy spectrum produced by mono-energetic γ-rays in an idealized scintillation counter with perfect resolution. The photo-peak obviously corresponds to electrons produced with the primary γ-ray energy E_0 while the continuous energy spectrum is due to Compton-scattered electrons which have a maximum energy

$$E_{\max} = \frac{E_0}{1 + \frac{mc^2}{2E_0}}$$

(when the photon is scattered at 180^0). In figure 3b, the effect of finite detector resolution becomes apparent: the photo-peak is smeared with a σ of 5%, typically, and the Compton edge becomes less distinct. The finite resolution is a consequence of photon statistics: a 1 MeV electron produces about 10^4 visible photons in the scintillator, about half of which will reach the photocathode of the phototube, and about 10% of which will actually produce an electron at the photocathode. So there are about 500 electrons available for amplification in the tube. These are random processes and so the standard deviation σ on the number N of these photoelectrons is given by $\sqrt{N}$ and the relative standard deviation is then $\frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{500}}$ which is about 5%.

In addition to an electronic noise signal which is apparent in Figure 3b, actual spectra often show a "backscatter" peak which is due to photons which do not come directly from the source. If there is a significant amount of material surrounding the source or if the source itself contains any significant amount of backing, a small fraction of the photons may Compton-scatter from this material and lose most of their energy before reaching the scintillator. A broad peak occurs in the energy of these scattered photons because most will be scattered over a range of very large angles, near 180^0. (At what energy does this backscatter peak occur?)

From the discussion so far, it is clear that the spectrum produced by γ-rays can be quite complicated, especially if several different mono-energetic γ-rays are present. To minimize backscattering, the amount of material surrounding source and detector should be minimized. It is also important to remember that a γ-ray can scatter several times inside the scintillator. A Compton scatter followed by a photoelectric conversion in the scintillator will also lead to a full-energy peak. The size of the detector is an important consideration. Obviously a very large scintillator will absorb all of the energy of an incident γ-ray, even if it makes many Compton scatters before disappearing in photo-electric conversion; in that case the output spectrum would contain only a full-energy peak without the Compton continuum.

Procedure:

A. Turn on your equipment

Our PMT is provided with an Ortec Scinitpack 296 base with a built-in High Voltage supply and pre-amplifier. It is powered via an external Ortec 114 power supply. Make sure the tiny on/off button on the base is pushed in and the Ortec 114 is on. We also boost and shape the signal with an additional preamp. The preamp (Ortec 451 Spectroscopy Amplifier) is located in the NIM crate. Turn on the NIM crate and the preamp. Typical settings to start with for the preamp are:

Fine Gain = 1.0
Coarse Gain = 20
Left Slider (output) = POS
Right Slider (input) = POS

Connect the "UNIPOLAR" output from the preamp to the input of the “Ortec EASY MCA” and connect the MCA to the computer using the USB cable. The Ortec MCA has 2048 channels and input range of 0 to 10 V.

B. Calibration of the scintillation counter

First you must relate channel number on the analyzer display to photon energy, using standard sources of known energies. This process is known as "calibration". A plot of the positions (channel numbers) of the full energy peaks vs. photon energies should be linear.

0. Start the Ortec MAESTRO software which is found under:
   All Programs / MAESTRO / MAESTRO for Windows.
   Select the detector in the pull down menu on the top right; it should read something like: "0001 SPA-MXP-010 MCB 130." Hit the "Go" button to acquire data. You should see counts accumulating on the display. This is due to the ubiquitous γ-ray background arising from the small amounts of radioactive uranium, thorium and potassium present in the earth. These have half-lives longer than the age of the earth and still exist in measurable quantities, typically a few parts per million.

1. Eventually, you will be investigating Compton scattering of γ-rays from a strong ^{137}Cs source. These have an energy of 0.6616 MeV and represent the maximum energy you will be concerned with. Place a **small** (few μCi) ^{137}Cs source a few cm from the scintillator and adjust the preamp gain (amplification) of the system so that the full energy peak occurs near the top of the scale. Use the cursor to estimate the position of the center of the 0.6616 MeV peak. Record the gain settings. Make sure the dead-time stays below a few percent during your calibration; if the data rate is too high and

pulses pile up on top of one another, there may be significant distortion in the recorded spectra. (It is also interesting to observe the pulses from the scintillation counter directly on an oscilloscope before they are analyzed. If you don't know how to do this, ask an instructor to demonstrate).

2. Select sources of lower energy than the ^{137}Cs and again find the channel numbers corresponding to the full energy peak. Some useful sources in addition to ^{137}Cs(0.662 MeV), are ^{22}Na(0.511, MeV), ^{133}Ba(.081, .276, .303, .356, .384 MeV, and ^{241}Am(.060MeV). In practice, the higher energy ^{133}Ba peaks cannot be used reliably with this detector because they all merge together. (The 0.081 MeV peak is o.k.)

3. Make a plot of channel number vs energy. This completes the calibration of the detector. **Make sure that you have recorded all the analyzer settings that you have used**. When you come back for the second week of lab, you will need to make sure that the settings are the same. You can always check your calibration using one of the reference sources.

A. A study of Compton scattering

The aim will be to record the spectra of ^{137}Cs photons scattered from a target at various scattering angles. From these spectra, you can find the energy of the scattered photons and also the number of scattered photons.

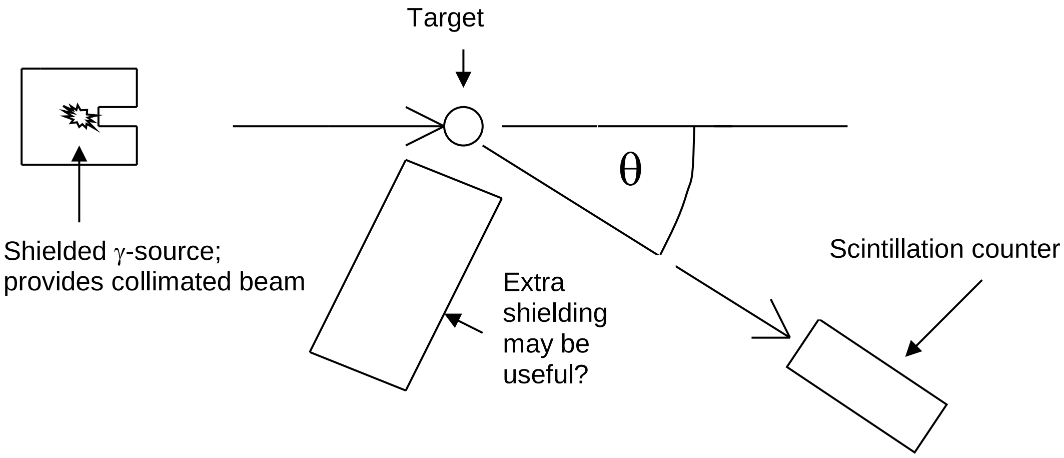


Figure 4. Set-up for Compton scattering experiment.

1. Install the **strong** ^{137}Cs in the lead housing**. This must be done by an instructor who has access to the key.** The source currently has a strength of about 0.3 milli-Curie and should be treated with care. It is safe within its housing, but it is still preferable to stay at least a couple of feet away from it.

2. Use a cylindrical target of aluminum and record spectra for a wide range of scattering angles. You must record data with the target in place and with the target removed. Subtract the background (obtained with the target removed) to find the effect of the target alone. Note that you must measure the background at each scattering angle and with the same lifetime (i.e. length of run) as used for the run with the target in place. Make sure that you record the lifetime at each angle. Store your data for each scattering angle on the adjacent computer. It is easiest to do the background subtraction in Excel, although instructions are also provided for doing it on the MCA.

   You want to minimize the number of photons that travel directly from the source to the detector (without scattering). These contribute to the background, which can be much larger than the scattering signal. Think about how you can place lead shielding to minimize this background. You may need to rearrange the shielding at each angle.

   Make sure that you always place the target at the same position and that the distance from the target to the detector is always the same! Give some thought as to how you should determine the energy of the photo-peak and its uncertainty.

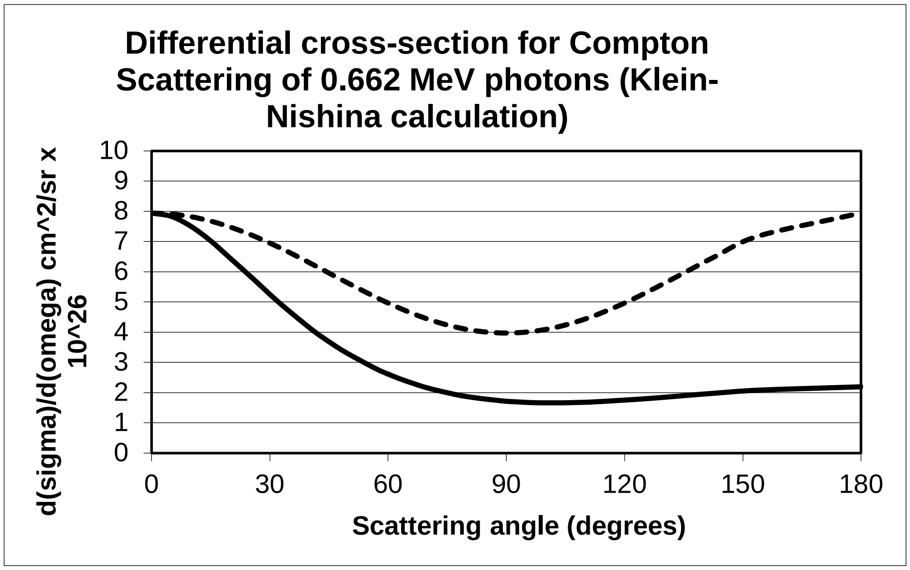
3. The expression given earlier for the energy of the recoil photon in Compton scattering can be inverted and written as

   $$\frac{1}{E_\gamma}=\frac{1}{E_0}+\frac{(1-\cos\Theta)}{mc^2}.$$

   In your data, the position of the photo-peak gives the scattered photon energy. By plotting $1/E_\gamma$ against $\cos\theta$, determine the rest energy of the electron.

4. Determine whether your data support a quantum-mechanical calculation of the process, versus a classical calculation by measuring the angular distribution of the scattered photons - *i.e.* the probability of scattering versus scattering angle by following the procedure below.

In any scattering process, the angular distribution of the scattered particles gives very important information about the nature of the interaction. The solid line in the figure below shows the predicted shape of the angular distribution (or *differential cross-section*) of Compton scattered photons at 0.662 MeV, as given by Klein and Nishina who performed the original quantum mechanical calculation of this fundamental process in 1929. In contrast, the dashed line is the classical calculation of electromagnetic radiation scattered by electrons (Thomson scattering) – it goes like $(1+\cos^2\theta)$.



Determine the relative scattering probability from the numbers of scattered photons that you have recorded at each scattering angle. The efficiency of the NaI crystal varies as a function of energy.

E_γ (MeV)	Rel. eff.(%)	Photo-fraction (%)
0.10	100	97
0.20	96	84
0.30	80	68
0.40	69	55
0.50	62	45
0.60	56	38
0.66	54	35

The relative efficiency is the probability that a γ-ray will interact inside the 2” diameter crystal and the photo-fraction is the fraction of gammas with deposited energies in the full energy peak (the rest are in the Compton continuum). Thus only 54% of the 0.662 MeV gammas interact at all and only 35% of those that do will deposit all of their energy. The product of efficiency and photo-fraction gives the probability that an incident photon of a given energy will actually produce an electron with energy in the photo-peak.

Determine the actual number of photons scattered into your detector by first finding the number of gammas in the photo-peak at each scattering angle and using the efficiency data. This is proportional to the differential cross-section. (Remember to normalize your data to the same lifetimes!). Come up with a way to compare your data with the theoretical differential cross section curves shown above to show whether you agree with Thomson or Klein-Nishina scattering cross sections. Either plot theory and experiment on the same graph or try finding the residuals (difference between theory and experiment as a function of angle).

Zeeman Effect

Read about the Zeeman effect in a standard textbook (such as Eisberg and Resnick, *Quantum Physics of Atoms, Molecules, Solids, Nuclei, and Particles*). The following discussion is a summary, but it is not meant to serve as a self-contained explanation. You MUST complete the reading for his lab, including all of the background sections of this write-up, before your first lab period.

Suppose that we wish to measure the magnetic moment of an atom. We consider only the *z*-component of the magnetic moment, where *z* is the same axis used to define the total angular momentum *J*; i.e. J_z assumes one of the quantized values $J_z = m_J \hbar$, where $m_J = -J, -J+1, \ldots, 0, \ldots . J-1, J$. The *z*-component of the magnetic moment of the atom in any of these states is

$$\mu_z = g_J m_J \frac{e\hbar}{2m} = g_J m_J \mu_B,$$

where

$$\mu_B = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24} \text{ J/T} = 5.79 \times 10^{-5} \text{ eV/T}$$

is the *Bohr magneton* and g_J is the *Landé g-factor*. The evaluation of the Landé g-factor requires a full understanding of the addition of angular momentum in quantum mechanics, which we will not review here. In general,

$$g_J = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)},$$

where *J, L,* and *S* are the total, orbital, and spin angular momentum of the electronic state of the atom. Thus, given *J, L*, and *S*, we can predict the allowed values of the magnetic moment of the atom.

We now place the atom in a magnetic field $\boldsymbol{B} = B_0 \hat{z}$, and so the magnetic energy of the atom is

$$E = -\boldsymbol{\mu} \cdot \boldsymbol{B} = -\mu_z B_0 = -g_J m_J \mu_B B_0.$$

Each atomic level is therefore split into $2J+1$ components, corresponding to the $2J+1$ possible values of m_J. The energy separation between adjacent levels is $\Delta E_Z = g_J \mu_B B_0$, where ΔE_Z is known as the *Zeeman splitting*. This is the Zeeman effect.

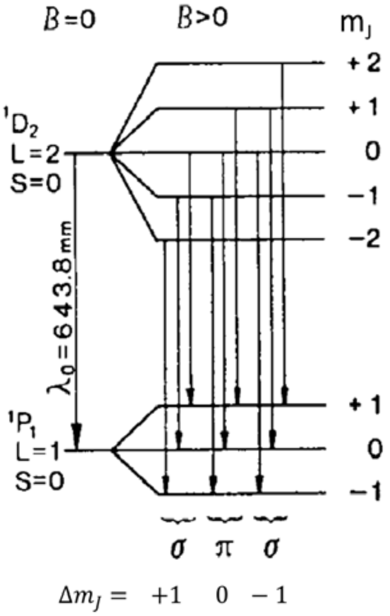
We now consider one case in which the Zeeman effect is easy to understand from a semi-classical perspective. Suppose that the total spin S of the atomic state is zero. This occurs for atoms with two valence electrons in a spin singlet state, and in this situation, $J = L$ and $g_J = 1$. The magnetic moment of each state is then $\mu_z = m_J \mu_B$, which is exactly the magnetic moment that one would derive for a Bohr atom, in which the electrons travel in classical circular orbits about the nucleus with quantized angular momentum $m\hbar$. In this picture, the electron orbit can be treated as a current loop, so that $\mu_z = I\pi r^2 = \frac{e}{T}\pi r^2$, where $T = 2\pi r/v$ is the period of circular orbit of radius *r*. Using the fact that $L = m_e vr = m_J\hbar$, for the Bohr atom, you can show that $\mu_z = m_J\mu_B$ and that the Zeeman splitting between adjacent levels is $\Delta E_Z = \mu_B B_0$. This case (the Zeeman effect for atoms with a total electron spin of zero) is known for historical reasons as the *Normal Zeeman Effect.*

Detection of the Normal Zeeman Effect

The original observation of the Zeeman effect in 1897 was a triumph of high resolution optical spectroscopy. The actual splittings between the levels at a relatively large magnetic field (~ 0.2 T) are only of the order of 10^{-5}eV, which is in the microwave region of the spectrum. Before the 1940's, there were no sources of microwave radiation powerful enough to observe the Zeeman effect directly. Zeeman's approach was to observe the optical radiation emitted by the transition between two *different* atomic states. In our version of the experiment, the atom in question is cadmium, which has two valence electrons. In the subsequent discussion, the multi-electron states of the atom are denoted in spectroscopic notation:

$$^{2S+1}L_J,$$

where we shall use the convention of referring to *L* by letter (S, P, D, etc. for *L* = 0, 1. 2, …)



The two valence electrons in Cd may be either in a state of total spin 0 (a singlet) or total spin 1 (a triplet). The Normal Zeeman Effect is observed in transitions between two singlet states, and we consider that between the $^1D_2$ (*S*= 0, *L*=2, *J*=2) and $^1P_1$ (*S*=0, *L*=1, *J*=1) states. The $^1D_2$ state is higher in energy by 1.926 eV. Because the selection rule $\Delta L = \pm 1$ is satisfied, this transition is optically allowed and leads to emission at a wavelength of 643.8 nm, which is in the red part of the visible spectrum.

In the presence of a magnetic field, the $^1D_2$ state splits into five separate levels, and the $^1P_1$ state splits into three, as shown in Fig. 1, in which each of the levels is labeled by its m_J value. The arrows show the optical transitions that satisfy the selection rule $\Delta m_J = 0, \pm 1$. There are nine such transitions. The energy E_γ of the photon emitted in each allowed transition is obtained from the difference of the total energies: $E_\gamma = E_0 +$

$E_{Z,i} - E_{Z,fl}$, where $E_0 = 1.926$ eV is the difference in the energies of the $^1D_2$ and $^1P_1$ states in zero magnetic field, and $E_{Z,i}$and $E_{Z,f}$ are the Zeeman energies of the initial (upper) and final (lower) states. We see that only three spectral energies appear: $E_0 - \mu_B B_0, E_0,\ and\ E_0 + \mu_B B_0$, corresponding to $\Delta m_J = +1, 0,$ and -1. In other words, the 643.8 nm line splits into three lines when a magnetic field is applied. The splitting is extremely small relative to the optical transition energy E_0. In a field of 0.1T, $\mu_B B_0 / E_0 = 3 \times 10^{-6}$. In order to resolve the Zeeman effect, we therefore need to resolve a splitting that is only a few parts per million. We do this using a unique and powerful tool, which is the Fabry-Perot etalon.

Fabry-Perot Etalon

The Fabry-Perot etalon is discussed extensively in Hecht's *Optics* and Melessinos' *Experimental Modern Physics.* You should read the relevant sections of those books or an equivalent introduction to this useful device. The etalon in our experiment is a single quartz plate of thickness 3.00 mm and index of refraction *n* = 1.4519 at 509 nm and n = 1.4560 at 644 nm. The two surfaces of the etalon are polished so that they are exactly parallel. A lens with a relatively long focal length (in our case 100 mm) is placed in front of the etalon so that light rays converge at a very small angle θ relative to the optical axis. The two surfaces of the etalon are coated so that the internal reflectivity is of order 90%. This means that each ray incident on the etalon will undergo many internal reflections. At each reflection, a small amount of light is transmitted, and the interference pattern that we observe is due to the superposition of these transmitted rays, which are focused to a point by a second lens

You should show that all of the transmitted rays generated by the same incident ray in Fig. 2 will interfere constructively when the condition

$$m\lambda = 2nt\cos\theta \tag{1}$$

is satisfied, where *m* is an integer. If the incoming beam has cylindrical symmetry, then each incident ray in Fig. 2 is focused to a point on a circle of radius $r = f_{eff}\theta$, where f_{eff} is the effective focal length of the imaging system (see Fig. 3). As we will see below, we do not need to know f_{eff}. The important point is that the interference pattern generated by the etalon is a series of circles.

Using the small angle expansion for $\cos\theta$, we can rewrite Eq. 1 as

$$m\lambda = 2nt\left(1 - \frac{\theta^2}{2}\right) \tag{2}$$

Note that the center of the circular pattern ($\theta = 0$), will in general correspond to a large ($\sim 2 \times 10^4$) but *non-integer* value of *m*, which we will call m_0:

$$m_0 = \frac{2nt}{\lambda}. \tag{3}$$

As θ increases, *m* decreases by 1 for each fringe. We will index each visible fringe starting from the center of the pattern with an integer *p*, with *p=1* corresponding to the first visible fringe. (In other words *p* increases by 1 while *m* decreases by 1 for each fringe.) You should now show that

$$r_{p+1}^2 - r_p^2 = 2f_{eff}^2/m_0, \tag{4}$$

where r_p and r_{p+1} are the radii of consecutive fringes. In other words, the difference in the *squares* of the radii of adjacent fringes is a constant for a given wavelength.

As a tool for measuring an absolute wavelength, the Fabry-Perot etalon is almost useless, because we have no idea what m_0 is. The etalon, however, is a very powerful tool for measuring extremely small *changes* in wavelength, and this is exactly what we need in order to measure the Zeeman effect. Because m_0 is so large, a very small change in wavelength leads to a measurable shift in the radius of a fringe. A change in λ of λ/m_0, or approximately 1 part in 20,000, will cause the circular fringe pattern to grow by one complete fringe. In addition, the width of each fringe is reduced by the multiple reflections which occur inside the etalon. The reduction factor, which is known as the finesse (see Hecht for an extensive discussion), is approximately 30 for the reflectivity of the coatings used in our etalon. This means that it is possible to resolve changes in wavelength of the order of $\lambda/\Im m_0$, where $\Im$ is the finesse. The resolution for our etalon is thus of the order of a few parts per million, a precision that is much better than can be achieved by an ordinary grating spectrometer of moderate length.

We now consider the interference fringes that are created by two waves of nearly identical wavelengths λ_a and λ_b. Each wavelength produces its own set of concentric fringes. We will assume that the difference $\Delta\lambda = \lambda_b - \lambda_a$ in the two wavelengths is so small that the spacing between the *a* and *b* fringes of the same index *m* in Eq. (2) is less than the spacing between two fringes of different orders. By writing out Eq. 2 for each of the two wavelengths, making judicious use of the extremely good approximations $m \approx m_0$ and $\lambda_a \approx \lambda_b$, and then using Eq. 4, you can (and should) show that

$$\frac{1}{\lambda_b} - \frac{1}{\lambda_a} = \frac{1}{2nt}\frac{r_{p,b}^2 - r_{p,a}^2}{r_{p'+1,a}^2 - r_{p',a}^2} = \frac{1}{2nt}\frac{r_{p,b}^2 - r_{p,a}^2}{r_{p'+1,b}^2 - r_{p',b}^2} = \frac{1}{2nt}\frac{\delta}{\psi}, \tag{5}$$

or

$$\Delta E = -hc\frac{\Delta\lambda}{\lambda^2} = \frac{hc}{2nt}\frac{\delta}{\psi}, \tag{6}$$

where δ is the difference in the squares of the radii for any pair of *a* and *b* fringes of the *same* order *p*, ψ is the difference in the squares of the radii of any two *a* fringes or any two *b* fringes of two *consecutive* orders p' and $p'+1$, and $E = hc/\lambda$ is the energy corresponding to the photon wavelength of interest. ΔE is the energy splitting of interest (the Zeeman splitting in this experiment). Note that Eq. 4 implies that you can pick any values of *p* and *p'* that you wish. In practice, one usually picks $p' = p$ or $p' = p - 1$. In the case of our etalon, we know *n* and *t*, and we can measure the radius of each fringe. The units of measurement do not matter, because the radii appear only in the dimensionless ratio δ/ψ. In this experiment, the fringe patterns will be recorded with a CCD camera and you can measure the radii using any technique that you wish. To keep all algebra manageable, measure energies in units of eV, and remember that $hc = 1.24 \times 10^{-4}\ \mathrm{eV \cdot cm}$.

We should pause at this point to consider one "real life" limitation that impacts the application of the above derivation. The assumption that all rays are imaged to a radius $r = f_{eff}\theta$, where f_{eff} is independent of *r*, applies only to an ideal imaging system. This assumption breaks down in the presence of spherical aberrations, which cause the effective focal length to decrease for rays further from the optical axis. As a result, deviations from Eqs. 4 and 5 will be observed as the fringe index *p* increases. This is the reason why one should use $p' = p$ or $p' = p - 1$ in applying Eq. 5. **The analysis works best for the fringes nearest the center of the pattern.**

The analysis described above is unambiguous provided that the separation between *a* and *b* fringes is always smaller than the separation between fringes of different orders. In the case of the anomalous Zeeman Effect discussed below, it is possible to violate this condition at the highest magnetic fields available with our magnet.

Selection Rules and Polarization

For the case of the normal Zeeman Effect described above, we expect to see each circular fringe of the Fabry-Perot etalon split into three fringes as the magnetic field increases from zero. We have not yet considered, however, the direction (relative to the magnetic field) along which light from each transition is emitted, nor the polarization state of the emitted light. These are somewhat more advanced topics, relying on knowledge of both the quantum mechanical wave functions associated with each state and the corresponding electric dipole moments associated with each transition.

For an electric dipole transition with $\Delta m = \pm 1$ the photon associated with the transition carries away exactly one unit of angular momentum oriented either parallel or anti-parallel to the magnetic field, which defines the *z*-axis. When viewed along the direction of the magnetic field ($\hat{z}$ in Fig. 4) the electric field of the classical electromagnetic wave associated with the photon will rotate either clockwise or counter-clockwise relative to the observer. This is just circularly polarized light, with the polarization vector rotating in the *y*-*z* plane.

What about the $\Delta m = 0$ transition? In this case, the dipole moment producing the emitted light oscillates parallel to the magnetic field, and so any electric field of the associated wave must be parallel to the magnetic field. From Maxwell's equations, however, we know that the polarization of the wave must be perpendicular to the direction of the propagation. This means that the $\Delta m = 0$ transition cannot be observed along the direction of the magnetic field! In other words, only two lines will appear when the emission is observed along the direction of the magnetic field. One line is right circularly polarized and the other is left circularly polarized. The splitting between the two lines is $\Delta E = 2\mu_B B_0$. The Zeeman effect observed in this geometry is known as the *longitudinal Zeeman effect*.

Now suppose that we observe the light emitted in a direction perpendicular to the magnetic field (*i.e.,* we are viewing the system along the *x*-axis in Fig. 4 when the magnetic field is along the z-axis). This is the *transverse Zeeman effect.* Now the $\Delta m = 0$ transition, for which the light is linearly polarized along the *z*-direction, can be observed. The two $\Delta m = \pm 1$ transitions are also observable, but in this case the *x*-component of the oscillating electric field cannot propagate in the *x*-direction, and so the light emitted is linearly polarized in the $\hat{y}$ direction. There are now three visible lines, and the central ($\Delta m = 0$) line is twice as bright as the other two.

Fig. 4 also introduces some spectroscopy jargon. Transitions with $|\Delta m| = 1$ are denoted σ transitions, while those with $\Delta m = 0$ are called π transitions. Only the two σ transitions are observed when the emission is viewed along the magnetic field axis, and one line will be right circularly polarized (we call it σ^+) and the other (denoted σ^-) left circularly polarized. When viewed along an axis perpendicular to the field, all three transitions are observable. For the geometry of Fig. 4, the π transition is horizontally polarized while the two σ transitions are vertically polarized.

In the experiment, you will use a polarizer and quarter-wave plate to verify the polarization states of each of the lines for the cases of the transverse and longitudinal Zeeman effects.

Anomalous Zeeman Effect

In many modern physics textbooks, the Zeeman effect is introduced in the context of two electron spin states, and hence it may be frustrating that we have not yet introduced spin into the discussion.

Shortly after Zeeman's original discovery in 1897, it was observed that for certain allowed transitions, more lines appeared in the presence of a magnetic field than expected based on the argument presented above. Moreover, the smallest observed splittings were found to be ½ of the expected value. Because no classical description described these observations, they were denoted the *Anomalous Zeeman Effect.* The anomalies were resolved over twenty years later when spin was discovered and a full quantum mechanical treatment of angular momentum was developed. In modern language, the anomalous Zeeman Effect occurs when the total spin is not zero.

In Cd, the anomalous Zeeman effect appears in the spectrum of the $^3S_1$ ($2S + 1 = 3$, $L = 0$, $J = 1$) to $^3P_2$ ($2S+1 = 3$, $L = 1$, $J=2$) transition. Remember that the letter "S" in spectroscopic notation refers to the $L=0$ angular momentum state, and not the total spin, which is 1 in each of these states. The energy level diagrams for these transitions are shown in Fig. 5.

Exercise: Compute the Landé g-factor for each of the two levels in Fig. 5, and then determine the energy levels in the presence of a magnetic field (relative to the zero-field energy of the $^3S_1 \rightarrow {}^3P_2$ transition). You should find 9 distinct spectral lines in the presence of a magnetic field, with a splitting between adjacent lines that is $\Delta E = \frac{\mu_B B_0}{2}$, exactly half of the value you found for the normal Zeeman effect.

In practice, the anomalous Zeeman effect is harder to analyze than the normal effect. First the splitting between adjacent levels (the one we really care about) is only ½ as large for a given value of the magnetic field. On the other hand, the splitting between the highest and lowest levels (labeled 1 and 9 in Fig. 5) is twice as large as the maximum splitting in the case of the normal effect. This causes the different orders of interference of the Fabry-Perot to "cross," resulting in a very messy spectrum.

There are two quantitative measurements that are possible with our apparatus. In the longitudinal geometry, only the $\Delta m_j \pm 1$ transitions are observable. These form two bands, just as in the longitudinal normal Zeeman effect, but each band contains three lines. The $\Delta m_j = +1$ band contains the lines 1, 2, and 3 in Fig. 5, while the $\Delta m_j = -1$ band contains lines 7,8, and 9. Although the individual lines cannot be resolved at low field, one can measure the splitting between the centers of the two bands (lines 2 and 8). Show that this splitting should be $\Delta E = 3\mu_B B_0$.

With our apparatus, which is limited to about 0.55 T in magnetic field, and our optics, there is one practical way to resolve the smallest splitting in Fig. 5, which is $\Delta E = \mu_B B_0/2$ between adjacent levels. In the **transverse geometry**, with the polarizer set to detect only the $\Delta m = 0$ transitions (lines 4, 5, and 6) in Fig. 5, one can set the field to the maximum value and just barely resolve the three individual lines that make up this band.

Equipment

The magnet: The two coils of the electromagnet are wired in parallel. The leads are color-coded and should not be reversed. Please do not touch any of the wiring for the magnet. (Doing so will change the calibration of the magnet.) The leads are connected to a power supply through an aluminum box that contains a protection relay. To turn on the magnet, simply plug in the 12V supply to the box (the green power light will come on) and then press the red button labeled "latch." You will see an amber light turn on, after which the magnet current is controlled by the dial on the large power supply, which has a power switch on the back. The current going through the magnet is monitored by the voltage across a 0.050 Ω series resistor that is also inside the aluminum box. This shunt voltage is read out by a voltmeter. The magnetic field *B* (in Tesla) is related to the shunt voltage *V* (in Volts) by

$$B = 1.004V + 0.006 \rightarrow \text{ for V } < \text{ 0.5 V}$$

$$B = 0.531V + 0.235 \rightarrow \text{ for } 0.5 < \text{V} < 0.65 \text{ V.}$$

With one exception discussed below, you will always use the magnet at voltages less than 0.5 V, and hence you will use the first calibration equation. The uncertainty in the magnetic field is of the order of 5% of the actual field. Note that the magnet saturates at very high fields, and this is why the second calibration equation is provided. You will also notice that if the magnet is left on at high current for an extended period of time, it will become extremely hot. **Avoid keeping the current above 8 A (0.4 V on the shunt resistor) for more than a few minutes or so at a time**. If the magnet gets too hot, a thermostat will open the relay coil in the aluminum box, and you will not be able to start the magnet again until it cools down.

The magnet is on a rotary table, allowing you to rotate between the longitudinal and transverse configurations. **Make sure you rotate the bearing on which the magnet sits. DO NOT MOVE THE LEGS supporting the table. Also, make sure you do not twist any wires when rotating the table, which only needs to move through an angle of 90 degrees and back.**

The lamp: A Cd discharge lamp is mounted in the center of the magnet. Note that light from the lamp is emitted both along the axis of the magnetic field (through holes bored in the pole pieces) and perpendicular to the magnetic field.

The optics: We first consider the optics that are in the light-tight box on the table. Lift the curtain on one side of the box to view them. Light from the cadmium lamp is collected by two lenses, one of which is built into the housing (the long black cylinder) that contains the Fabry-Perot etalon. The light then passes through the etalon plate (which is fixed inside the housing). The lenses after the etalon, including the focusing ring on the camera, image the interference pattern onto the CCD chip inside the camera, where the image is collected and sent to the computer.

All of the optics discussed so far are mounted to an optical rail**.** You will be able to focus the interference pattern using the focusing ring on the camera. **Otherwise, do not move any of the optics inside the light-tight box** (except for the red color glass filter discussed in the following paragraph). If any of the lenses are moved, the apparatus will need to be realigned. A polarizer is in a mount outside the box. There is a lever on the mount to rotate the polarizer. The mount is calibrated from -90 degrees to + 90 degrees.

Two optical band-pass filters are provided in order to select one of the two different atomic transitions used in the experiment. The red filter, which slides into a slot in the etalon housing, passes light at 644 nm and should be in place when studying the normal Zeeman effect. The 508 nm interference filter slides into the rail at the position shown in the drawing, and it should be in place when studying the anomalous Zeeman effect. **Note that only one of these filters should be in place at a time**. Otherwise, no light will be transmitted! As with any optic, please take care not to touch the surfaces of these filters with your fingers.

Finally, you are provided with a quarter waveplate (labeled $\lambda/4$), that you will use when studying the selection rules discussed above. Please read up on the use of a quarter-wave plate and polarizer to detect circularly polarized light before coming to the lab. The optical axes of the waveplate are in the horizontal and vertical directions. You can select either right or left circularly polarized light by adjusting the polarizer to ± 45 degrees.

Software

Instructions for using software to record and save images from the camera will be provided next to the experiment. On the desktop of your computer you will find a program called **ZeemanLab.vi**. When this program is started it opens a video capture window. The live video feed from the camera is on the left side of the screen. Figure 8 shows a screenshot of what you can expect to see for a sample lab. You can adjust the gain and integration time to obtain a viewable image. By default, these controls are set at a high level, and so you should see the ring pattern expected upon opening the program. If the circular fringes are too bright, they will be artificially broadened, in which case you should reduce the gain so that none of the fringes appear to be "saturated." Finally, the program gives you the option of integrating many frames, which will help to increase the signal to noise ratio when the images are dim. **Keep in mind that the goal is to get fringes that are sharp and well-resolved. Do not waste time trying to get the brightest image possible.**

When you hit the "Capture" button, a negative grayscale image will appear in the right-hand window of the program. The fringes are now black on a white background. Use the "Pan" and "Zoom" features to select an area containing the three or four innermost fringes in zero field. A zoom setting of 1.8 is probably suitable for viewing the first three or four fringes. Outside of this region, spherical aberrations will influence your results (in a bad way) as discussed above. There are also two sliders that allow you to set the maximum (black) and minimum (white) levels for the image, enhancing the contrast. Once you have a centered image with well-resolved fringes, you can hit the "accept image" button.

The program will now return to a main window, where you will be able to print and save the interference pattern as well as to center the fringes on a circular coordinate system and record their diameters. This provides one way (but by no means the only one) by which you can measure the diameter of each circle. **Make sure you have saved and printed each image that you intend to analyze and that you have recorded in your lab book the shunt voltage (i.e. magnetic field), wavelength, geometry (longitudinal or transverse to field), and polarizer setting that goes with each saved image.**

Procedure

To get started, put the magnet in the proper orientation, with the magnetic field perpendicular to the optical rail. This also allows for the most light to reach the detector. Make sure that the red color glass filter is in place (it slides into a slot in the etalon housing). Drop the curtain on the side of the light-tight box. **Make sure that the 508 nm filter and the quarter-wave plate are NOT in the set-up. Set the polarizer at either positive or negative 45 degrees. Turn on the Cd**

lamp and let it warm up for a minute or so. Start up the camera program as described above. In the video capture window, you should see a series of concentric rings, which are the interference fringes of the Fabry Perot etalon.

If you do not see anything on the display:

1) Make sure that the gain and intensity controls in the program are turned all the way up.
2) Check that the camera is operating correctly. For example, if you open the curtain on the light-tight box, you should see the display on your screen turn bright. If not, check all connections and the software. If you are still stumped, ask a TA for assistance.
3) If the camera is working abut you still do not see fringes, it is almost certain that either something is blocking the optical path between the cadmium lamp and the camera OR that both filters are in place.

If the image is slightly blurry, try adjusting the focusing ring on the front of the camera. Otherwise do not touch anything on the lenses. **If you see a pattern that is either significantly off-center or blurry**, then the apparatus needs to be realigned. Consult a TA.

You are more likely to see that your fringes are “too bright,” and under these conditions they will appear to be broader than they actually are due to saturation of the pixels on the CCD. If this happens, reduce the gain in the software so that the fringes are no longer saturated.

Once you see a normal fringe pattern:

Turn on the magnet power supply and increase the current so that the voltage across the series resistor reads 0.5 V. This corresponds to a magnetic field of approximately 0.5 T. As you increase the field, you should see each fringe “split” into three concentric fringes. If you do not see this: 1) check that the polarizer is at 45 degrees or 2) go back to the magnet instructions and make sure it is powered up properly.

Once you have well-resolved fringes and can see the splitting, you are ready to begin taking data. Turn the magnet current back to zero.

Figure 7. Screenshot of the data collection window in ZeemanLab.vi, showing a Fabry-Perot interference pattern obtained with the red color-glass filter in place. The magnetic field is set to zero. The magnet is perpendicular to the optical rail (as in Fig. 6). Note that the image on the right is inverted (fringes are black on white),

Data Collection and Analysis

Your instructor will tell you which of the following parts to complete. Ask!

I. Testing the Fabry-Perot Etalon: Keep the red color glass filter in place. At zero magnetic field, capture and save an image of the fringe pattern following the instructions above. Use these data to test whether or not Eq. 4 is valid. Use the four rings closest to the center.

II. Transverse Normal Zeeman Effect

A. Keep the magnet in the transverse orientation. With the polarizer at 45 degrees and the red color glass filter in place, record and save images of the interference pattern for four or five magnetic fields between 0 and 0.5 T. For each field, record the diameters of the fringes for the first three orders of interference. As discussed above, you will usually carry out the analysis using the first two orders (i.e. the first two rings in zero field). If the first fringe is too close to the center, however, you will not be able to see all three Zeeman-split lines at high field. In that case, use the second and third orders for your analysis.

B. Plot d^2 (the square of the diameter of each fringe) as a function of the magnetic field. Determine the uncertainty in d^2.

C. Using Eq. 6, determine the normal Zeeman splitting as a function of magnetic field.

D. Determine (with an uncertainty) the Bohr magneton.

E. With the magnet at high field (shunt voltage of 0.50 V), record the Fabry-Perot fringe pattern for polarizer settings of 0 degrees (vertical), 45 degrees, and 90 degrees (horizontal). Do not adjust the optics between images. Save these images, making absolutely sure to keep track of which image goes with each polarizer setting. Given that there are three lines total for each order of interference, which ones appear and which ones are missing for each polarizer setting? In your lab book, compare the patterns and explain the differences.

III. Longitudinal Normal Zeeman Effect.

Rotate the magnet 90 degrees so that the magnetic field is parallel to the optical rail. Light from the Cd lamp will now go through the holes in the pole pieces. View the interference pattern on the screen. Note that you can look at the screen while rotating the magnet in order to optimize the alignment. If the fringes are dim, try increasing the gain. Make sure the number of frames to average is set to the maximum value.

Put the quarter-wave plate in place between the magnet and the polarizer and set the polarizer at 0 degrees (vertical).

A. Into how many components does each fringe split as the magnetic field increases?

B. Repeat IIA – D, recognizing that you have a different number of Zeeman split lines for each order of interference.

C. With the magnet at high field (shunt voltage of 0.50 V), record and save the interference patterns for polarizer settings of -45 degrees, 0, and +45 degrees. How many fringes appear in each case? Discuss in the context of the selection rules discussed above.

IV. Longitudinal Anomalous Zeeman Effect

Remove the quarter-wave plate and red color glass filter. Insert the 508 nm interference filter between the polarizer and the light-tight box.

A. You are still in the longitudinal geometry, and so you can only observe the $\Delta m_j = \pm 1$ transitions in Fig. 5. As the field increases, you will see each fringe split into two bands, as for the normal Zeeman effect. Each of these bands, however, now consists of three individual lines as discussed above, but you will not be able to resolve these at low field. Note also that as you increase the field, the upper band from the first order fringe will start to overlap with the lower band from the second order fringe. Make sure you take data at fields below the value at which this starts to happen.

B. Pick an appropriate magnetic field and determine the splitting between the two bands using Eqs. 5 and 6. How does this compare with the same splitting measured for the normal Zeeman effect?

V. Transverse Anomalous Zeeman Effect (somewhat tricky)

Rotate the magnet back to the transverse configuration, with the magnetic field perpendicular to the optical rail. The interference pattern will be much brighter. Turn down the gain if necessary.

In this geometry, it is possible to resolve the smallest splitting between the levels in the anomalous Zeeman effect. This is $\Delta E = \frac{\mu_B B_0}{2}$, and can be considered the most direct evidence for spin.

A. Set the polarizer at 90 degrees (horizontal). For this polarizer setting, only the three central lines (the $\Delta m_j = 0$ transitions) out of each group of nine are visible (see Fig. 5). Set the magnetic field to the highest possible value. This is the ONLY time you should do this, and make sure to use the “high-field” calibration to convert shunt voltage to magnetic field. Record and save an image, making sure that gain setting on the camera is appropriate and that you average sixty frames. You should now resolve (barely) three lines within each of the first few fringes on the screen. Save the image and set the magnetic field back to zero.

B. Analyze the data using Eqs. 5 & 6. Determine the splitting between adjacent fringes and compare with the expected value of $\Delta E = \frac{\mu_B B_0}{2}$.

Assuming $\Delta E = s\mu_B B_0$, what do you find for the constant *s?*

Photon counting statistics

Purpose

To measure photons by a SiPM detector and explore different distribution functions

Key Concepts

Photons
Oscilloscope
Analog and Digital Signals
Poisson distribution
Gaussian (normal) distribution

Introduction

In this lab, the quantized nature of single photons along with the phenomena of single photon events will be studied. Afterward, different distribution functions will be investigated. Two different ways will be employed to observe the events of single photons. One way is plotting the behavior on an oscilloscope and the other is digitizing the signal and evaluating a histogram of events. In the first part of this lab, you should learn all the basic functions of an oscilloscope and then use the oscilloscope to display the signal from the detector. In the second part, control software will be employed to examine histograms of the photon events. Finally, analysis of the data from the histograms and error analyses will be carried out.

The topic of statistics will be covered in lectures and is also presented in an Appendix to this lab manual. However, in case you are not yet familiar with that material, we will give an abbreviated discussion here.

The observation of a random process, such as the detection of single photons, will produce results that have a well defined distribution about the average detection rate if a large number of observations are made. In this experiment it is instructive to compare the observed data with two distribution equations: the Poisson distribution and the Gaussian (or Normal) distribution. For the observations to be described by either equation, a fundamental requirement is that the events being observed be both independent and random. This is true for the pulsed light emitted by the LED.

Poisson Distribution

For data to be described by the Poisson equation it is necessary that a large number of events may occur but that, on average, only a small fraction of the possible total occur during any one observation. In this experiment, where you will count the number of photons in a given time interval, this condition is satisfied because the LED creates many pulses of photons, but the detector detects only a small fraction of the photons emitted.

The Poisson distribution gives the probability $P(x)$ of observing x events when the average (mean) number of events per observation is M

$$P(x) = \frac{M^x e^{-M}}{x!}.$$

The probability $P(x)$ should be multiplied by the total number of observations to obtain the number of times that x events is expected to be observed.

The calculation of $P(x)$ is not difficult if M is a fairly small number. Since the factorial of zero is one, $P(0)=e^{-M}$. Then $P(1) =P(0)M$, $P(2)=P(1)M/2$, $P(3)=P(2)M/3$, etc. Using this step-by-step process avoids the need for very large or very small numbers. Alternatively, find the function in the Excel library of functions, and allow the computer to do the calculations for you.

Gaussian Distribution

If the average number of counts observed in each run is more than about 10, the calculation of $P(x)$ gets rather tiresome. For averages greater than this, the Poisson distribution can be replaced by the Gaussian or "Normal" distribution, which has the equation

$$G(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-M)^2}{2\sigma^2} \right].$$

The evaluation of this expression requires the specification of two numbers: the average number of events per observation (counts per run) which is M, and also the standard deviation, σ, which describes the spread of the distribution on both sides of the average. The Gaussian distribution is symmetric about the average rate. This is not true of the Poisson distribution, which is very asymmetric for small values of M.

The standard deviation, σ, is the square root of the average of the square of the deviations from the average of all data points. That is, it is the R.M.S. deviation from the average. For randomly occurring events the ideal value of σ is $\sqrt{M}$.

To make a comparison of a set of data with the theoretical Gaussian distribution it is usually worthwhile to group the data into intervals before plotting a graph of the distribution. For example, one can plot the number of runs resulting in 90 to 94 counts per run, the number of runs resulting in 95 to 99 counts, etc. In this case the interval is 5 units. On the same graph, one then plots the theoretical Gaussian curve with an adjustment or **normalization** so that the area under the curve will be the same as the area under the graph of actual data. This normalization is obtained by multiplying the values of $G(x)$ by $N\Delta$ *,* where N is the total number of runs and Δ is the width of the interval described above.

It is interesting to compare the spread in data with the ideal distribution which predicts that 31.7% of the data should deviate more than 1σ from the mean.

The Equipment:

This experiment consists of 6 major pieces of equipment: the LED source, the Silicon Photo Multiplexer (SiPM), the SP5600 Power Source and Amplification Unit (PSAU), the Digitizer, an oscilloscope and a computer. A brief and proper description of their functions follows.

The LED source produces pulses of light. This light is transmitted through a **fragile** fiber optic cable, where it is detected by the SiPM detector, which consists of 100 Avalanche PhotoDiodes. Be very careful with the fiber optic cable. The SiPM detector will translate the detected photons into an analog signal; specifically, when a photodiode of the detector detects a photon, it will emit an electron as an electric signal. Sometimes, the detector may detect more than one photon at once, in which case, it will emit the corresponding number of electrons. The results from all photodiodes of the detector are combined to form the output analog signal.

The detector is connected to the PSAU which provides two main functions: supply power to the detector and amplify the signal of the detector. From here, the signal can go to two places: the oscilloscope or the Digitizer. You will first be examining the signal with the oscilloscope. In the second part of the lab, the signal will be sent to the Digitizer, which will digitize the detector signal and then send it to the computer.

The Control Software is another key component of this experiment. The Control Software is a Graphic User Interface (GUI) that controls the PSAU and the Digitizer. It is also here that signals from the Digitizer are displayed.

Part One

As mentioned previously, in this part, you will be working with an oscilloscope to display the detector signal of photon events. Oscilloscopes are powerful tools that are commonly used in most lab settings. A wealth of knowledge about the behavior of a signal can be displayed on the monitor. Two main types of oscilloscopes are analog and digital oscilloscopes. This section will give a brief introduction to digital and analog signals along with the functions of the oscilloscope that you will be using.

In this lab, you are investigating the signal produced by a detector struck by photons. For this SiPM detector, the produced signal is an analog signal, which means it can take on a variety of values. Analog signals contain a lot of information. In contrast, a digital signal is a stream of 1's and 0's, usually encoded as a high voltage value (1) or a low voltage value (0). As a result, there is less information in a digital signal. That being said, digital signals tend to be much faster and easier to work with as compared to analog ones.

The main function of an oscilloscope is to give a visual display of signals. How this happens depends on what type of oscilloscope you are working with. For the analog oscilloscope in this lab, a CRT is used to shoot electrons at screen made of phosphorous. When an electron strikes this screen, the phosphorous will glow for a second or two. This is a (relatively) long amount of time and allows for many signal cycles to be displayed at once, creating the display of the signal.

Explore the capabilities of the oscilloscope by connecting the oscilloscope to the function generator. Connect the output terminal of the function generator to the “CH1” input of the oscilloscope. Turn on the function generator and create your signal. For now, use a sine wave. Select any frequency you want. Next, turn on the oscilloscope and try to display the signal on the graph. In order to optimize your display, you will need to use the adjustment knobs. The section labelled “Vertical” allows you to control which signals are displayed and it allows you to control the vertical parameters of the displayed signal (e.g. the scaling of the axis (“**VOLTS/DIV**”) and the position.) Similarly, the horizontal axis scale can be adjusted with the “**A and B**” dial. What variable is represented by the horizontal axis? What is the range of scale factors for the horizontal axis and for the vertical axis? Sketch the graph of your signal in your lab notebook. Try other frequencies and signal types (e.g. Square or Triangle).

Now that you know some of the capabilities of the oscilloscope, you can now investigate the behavior of the SiPM detector with it. This will be mostly the same as before, with one major change. Unfortunately, when the oscilloscope tries to display and analog signal, some of the information of a signal can be lost. Here’s why. You may have noticed the “Trigger” section of the oscilloscope. In order for the oscilloscope to produce a continuous graph of the signal, it needs a reference point from which to start displaying the signal. This is usually done by setting a certain voltage threshold. Once the signal reaches that threshold, the oscilloscope will start to display the signal. This is the trigger. The problem with this is that an analog signal can take on different voltage amplitudes between different signal cycles (for example, the amplitude could be 2.00 Volts a certain part of one cycle and 1.8 Volts for the same part during the next cycle.) This behavior can confuse the oscilloscope. Here, it is useful to have a digital signal, which, for a certain cycle section, will have the same voltage value for different cycles.

Procedure

Display the detector signal on the oscilloscope. First, make sure that the CH0-signal from the PSAU is connected to CH 1 of the oscilloscope and that the CH0 TRIG-signal is connected to CH2.Turn on all equipment.

Before opening the SiPM Control Software, you will need to copy a version of it to your desktop. This will allow you to save data later. Locate the "SiPM Stuff" folder on the desktop. Create a copy of this folder onto your desktop and rename that folder. Inside this new folder, you will find the SiPMkit_ControlSoftware.exe file. Open this file. **Remember**, whenever you open the control software, open it from this location.

The Control Software oversees everything except the LED intensity amplitude. We won't use most of the features of this software, but take a minute to find the controls that will be used (e.g. bias voltage, gain.)

The bias voltage is the operating voltage for the SiPM detector. In order to get the best data, the operating voltage needs to be optimized. (Too high of a voltage will create a lot of noise within the detector.)

The initial signal from the detector is actually very weak and hard to measure. This is where the Gain comes in. The Gain is a multiplication factor that is applied to the signal in order to be measured more easily. **Be careful.** There is a limit to how much gain a detector can handle. For this lab, this won't be an issue because the gain can only be set as high as 50, which is more than enough for our purposes.

The LED amplitude dial controls the intensity of the light pulses emitted by the LED Driver. As you will demonstrate, the LED intensity plays a major role in the photon detection.

Note: You cannot adjust parameters for a particular device unless that device has been "Started" within the program. For now, set the CommPort to 6(or whichever one has the PSAU USB connected to it) and run the software by clicking the button with the single white arrow near the top of the screen (if you see a black arrow instead, then the program is already running.)

Important: When you are finished using the program, press the "Stop Program" button in the bottom right corner of the interface. **Do not** press the red octagon button near the top of the window. This will cause connection issues between the computer and PSAU.

Now, Start the PSAU. If you get a connection error, restart the program and try again. If the problem persists, consult your TA. Once the PSAU has started, press the green ch0 button in order to initialize the detector. Now the bias voltage and the gain of the detector can be adjusted. For now, set the bias voltage to 70.3 and set the gain to 35.

You should see two signals on the oscilloscope display. If you don't see a certain signal, press that channel's respective button inside the section labeled "**MODE.**" If the button is illuminated, that means the signal is being displayed. If you still don't see the signal, try adjusting the vertical position of that channel using the knobs at the top of the "Vertical" section. If you still don't see the signal, consult your TA.

Now you will adjust the settings on the oscilloscope in order to see the display of the signal. Adjust the horizontal scale to its lowest value.

In the section labeled "Trigger," set CH2 as the trigger. You may want to readjust to voltage scale of CH2 so the entire pulse can be seen on the display.

Next, adjust the voltage scale for CH1. A display similar to **Figure 1** should appear on the monitor.

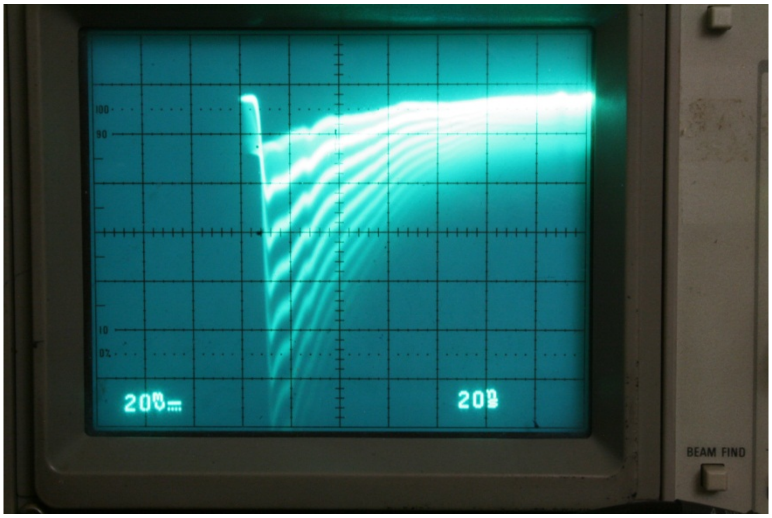


Figure 1: Example display from the oscilloscope

If you experience that the signal for CH1 is a bit dim, the signal can be brightened by turning off the display of CH2.

You should now notice that there are bands between peaks for the detector signal. What do these bands represent? They represent distinct photon events. For instance, the first band represents a single-photon event. This means the detector measured one single photon. The second band corresponds to the detector measuring two photons at once. The third band means three photons simultaneously, etc. You may notice that some of the peaks appear brighter on the display than others. A brighter peak means there were more events of that kind (i.e. the detector registered that event more frequently), a dimmer peak means there were less events of that kind.

Now, adjust the bias voltage, the gain, and the LED amplitude and record how the display changes. First, investigate the effects of LED intensity. What happens if the LED intensity is 0? Try it and record what the oscilloscope displays. Sketch the display in your lab notebook. Then, investigate the effects of LED intensity for a range of values between 3 and 5.4 on the LED knob. **Record these effects**. Also, using your phone, take a picture of the display. Paste these pictures into your lab notebook. You may paste however many you want, but have at least one from the low end of the range, one from the middle part, and one from the high end.

Set the LED intensity so you see 6 or 7 strong bands. Now, adjust the bias voltage and gain and then record their effects. For the bias voltage, use a range 69.5 - 72 volts. For the gain, use a range 25 - 40. Also, take photos of these new displays and paste them into your lab notebook. You don't need all of them for instance, just a few of the more interesting ones. In total, you should have at least 10 photos in your lab notebook.

Part Two

In this section, you will use the Digitizer to evaluate the signal from the SiPM detector. Make sure the CH0 port on the PSAU is connected to the CH0 port of the Digitizer. Also, double check that there is a cable connecting the TRIG IN port of the Digitizer to the TRIGGER OUT port of the LED driver. Now, start the Control Software and run the program.

Like before, start the PSAU and press the green ch0 button in that section. Set bias voltage to 70.4, the gain to 35, and the LED amplitude to about 3. Now, start the Digitizer and press the green ch0 button in that section as well. Before adjusting any parameters, let's examine the Digitizer section.

As mentioned previously, the Digitizer will transform the analog signal from the SiPM detector into a digital one. In the Digitizer section of the Control Software, you can control how the Digitizer does this by adjusting the Trigger, Gate, and Baseline parameters. For now, make sure the Trigger Mode switch is set to "external."

The converted SiPM signal is displayed on a graph on the right side of the interface. You will notice that there are tabs at the top of this section. These tabs allow you to view different types of graphs of the signal. For now, select the "wave: tab". Does this graph look familiar? It is the same type of graph as one appears in the oscilloscope. The difference is that you only see one or two cycles of the signal at a time where as with the analog oscilloscope, you saw many cycles at once. Record what you see and sketch this display in your lab notebook. Before moving on, set the "pregate" value in the Digitizer section to some value over 50. Now, select the "Histogram" tab.

As you may know, a histogram is a recording of how many times a certain event happens. The display you see now records the number of events that have a certain ADC charge. There are many peaks in this graph. These peaks are Poissonian and each peak represents a number of photon events, except for the 0th peak. The 0th peak represents instances where the detector fired, even though there was no photon detected. The 1st peak represents single-photon events, the second peak represents double-photon events, etc. You can save an image of the histogram by pressing the "Print Display" button. You can name the file using the text directly above that button. The .bmp will be saved to the folder named "pictures" inside the folder you created on the desktop.If you plan to save more than one display, **make sure** to change the file name each time. Otherwise, the old file will be overwritten by the new one. In your lab notebook, paste a copy of the histogram. Similarly, you can save the histogram data by using the Save button located next to the text field labelled "Histo File Name." These files will be added to the "data" folder.

Note: For histograms, it is usually helpful to collect data for an extended period of time in order to get good statistics. For our purposes, 30 seconds represent a good amount of time.

If it helps, you can remove the shaded area of the graph by right clicking the small box labelled “plot one” (the black one with the 3 white triangles.) Go to Fill Base Line and select None.

Now, adjust the bias voltage, gain, and LED intensity in the same manner as you did while exploring the detector behavior using the oscilloscope. Start with LED amplitude set to 0 and record what you see. Either Sketch or Print this image into your lab notebook. Compare this to the oscilloscope display you had for the same settings and record this in your lab notebook. Then, record about 10 graphs from for the range of LED amplitude values from 2 to 5. Make sure to note the number of peaks, the relative strength of each peak, and any underlying trends. If it helps, you can change the scale of the x or y axis by right clicking on that axis, deselecting the autoscale property, and then setting your own upper or lower limit.

Compare three of your graphs (one from the low end of the range, one from the middle, and one from the high end.) What differences do you notice between them? Is the higher intensity graph still Poissonian? If not, what other type of graph does it look like? Make sure to paste the images of these graphs into your lab notebook.

Now that you have a general feeling of how the LED intensity affects the SiPM signal, investigate how the gain and the bias voltage affect the histograms. For each of the three intensity values you compared, investigate effects of the bias voltage (range 69.5-72) and gain (25-40.) Record these effects and paste a few of the graphs into your lab notebook. Record the displays of what you feel are the optimal settings for each LED-intensity. Paste these results in your lab notebook. Make sure to have all of these displays in your lab notebook before you start the analysis section. **Note:** The histogram will not record events with ADC counts greater than 23000.

Analyses

Analyze the histograms of the three LED intensities you compared previously. The low intensity graph should be described by a Poisson distribution. Quantitatively confirm this by plotting the data into Excel and fitting a Poisson distribution to it. What is your value for *M*, the average number of counts per detection? Paste a graph of your fit into your lab notebook.

The high intensity graph should be described by a Gaussian distribution. Quantitatively confirm this by fitting a Gaussian function to your data. What is your value for *σ*? Is the mid-intensity graph described by a Poissonian or a Gaussian function? Fit your data to both a Poisson function and a Gaussian function and compare your fits. Which one yields a better fit? Paste both fits into your lab notebook. Record your values for *M* and *σ*.

Something to consider: When trying to fit to a Poisson distribution, are all of the data points from your histogram necessary?