Laboratory 1:

Analog Systems of 1. and 2. Order

Prof. Dr. Martin J. W. Schubert

Electronics Laboratory

Regensburg University of Applied Sciences

Regensburg

Abstract. Electronic control loop circuits as typical e.g. for ΔΣ A/D converters are compared to generalized first and second order models and verified by both simulation and measurement.

1 Introduction

Any system that can oscillate is of at least second order, i.e. it has at least two memories. The order of a model is the maximum of poles or zeros in its transfer function. As higher order systems are difficult to treat by analytical calculus, second order system considerations are popular. They apply to higher order system models also, when the first two poles are significantly lower than the rest of poles and zeros.

How to work through this laboratory [1]:

- There is no need to fully understand the theory in section 2 to benefit from this tutorial. For theory it is enough to study the model summary in Fig. subsection 2.3. An in-depth mathematical derivation of the models is given in [2].
- The Spice [3] simulations of section 3 are useful but no precondition for the hands-on training. They can be made with a personal computer running the free LTspice simulator. Fill the “simulated” fields of the tables in chapter 5. Respective LTspice [4] input files [5] are given on the web together with this documentation.
- During the hands-on training concentrate on section 4 working with the *Bode 100* network analyzer [6],[7], [8]. Fill the "measured" fields of the tables in chapter 5.

The organization of this laboratory is as follows: Section 2 presents theoretical background according to [2], sections 3 and 4 offer simulation and experimental verification, respectively. Section 5 contains tables common to sections 3 and 4. In section 6 you can check your understanding of the fundamental goals of this laboratory. Section 7 draws relevant conclusion.

2 Theory

Typically the biasing voltage U_B is ≠0V. In this case we use a mathematical trick :

Calculate with: $\mathbf{U' = U - U_B}$ (2.1)
After Solution: $\mathbf{U = U' + U_B}$. (2.2)

2.1 General and Particular First Order System Models

2.1.1 Signal Transfer Function of a First Order Electronic Circuit

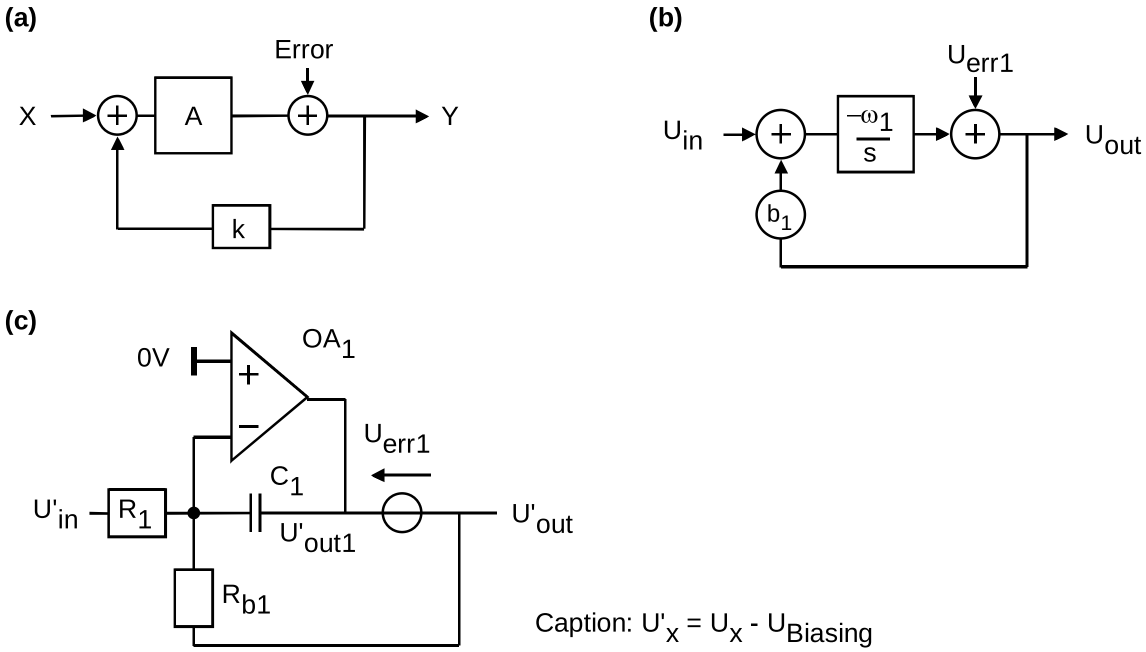


Fig. 2.1.1-1: (a) High-level schematics, **(b)** depicted for modeling, **(c)** OpAmp realization with $\omega_1=1/R_1C_1$, $b_1=R_1/R_{b1}$ and error source U_{err1}.

The signal transfer function (STF) is generally defined as $$STF = \frac{U'_{out}}{U'_{in}}$$ (2.3)

Table 2.1.1: General and particular 1st order model taken from [2] (2.4)

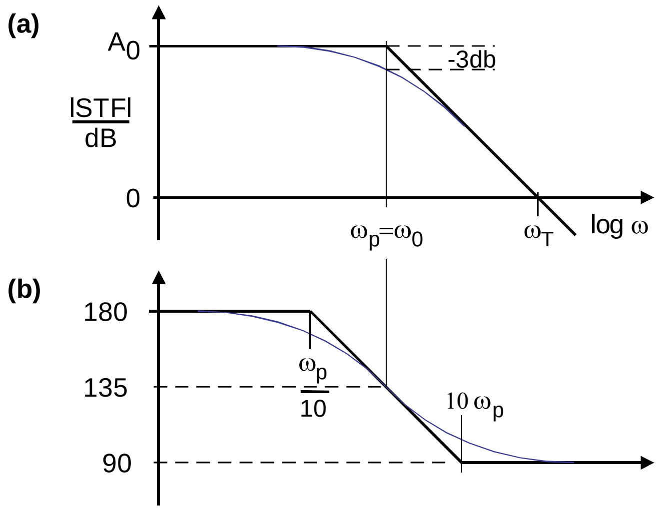
(a) General 1st Order Model	**(b) Particular Model of Fig. 2.1.1-1(b)**
$STF_1 = \frac{U'_{out}}{U'_{in}} = \frac{A_0\omega_0}{s+\omega_0} = \frac{A_0}{s'+1}$ with $s' = \frac{s}{\omega_0}$	$STF_1 = -\frac{\omega_1}{s+b_1\omega_1} \xrightarrow{\lvert s\rvert\to 0} -\frac{1}{b_1} = -\frac{R_{b1}}{R_1}$, with $b_1 = \frac{R_1}{R_{b1}}$, $\omega_1 = \frac{1}{R_1C_1}$
	(c) Mapping general to particular param.
A_1: amplification at ω=0, ω_0: cutoff frequency, used in $s'=s/\omega_0$	$A_0 = -\frac{R_{b1}}{R_1}$, $\omega_0 = \frac{1}{R_{b1}C_1}$

Table 2.2.1(a) with parameters A_0 and ω_0. They are chosen such, that both parameter affect one single aspect of the circuit only: A_0 is the DC amplification and ω_0 the cutoff frequency as shown in Fig. 2.1.1-2 below. Any model – electrical, mechanical, fluid, etc. – with a single pole only (to be considered) can be described with this general model.

Table 2.1.1(b) shows the transfer function of the circuit in Fig. 2.1.1-1(b).

Table 2.1.1(c) maps the particular parameters of (b) to the general model parameters of (a).

Fig. 2.1.1-2: First Order System:
(a) Magnitude and
(b) Phase diagram



The pole can be identified by its -45° phase shift

Goal: Find two particular parameters that effect one of the two general parameters only:

1. Which device parameter in Fig. 2.1.1-1(c) affects DC amplification A_0 and not pole ω_0 ?

....R_1....

2. Which device parameter in Fig. 2.1.1-1(c) affects pole ω_0 and not DC amplification A_0 ?

....C_1....

3. What is the DC amplification A_0 as $f(R_x, C_1)$? A_0 = $-R_{b1}/R_1$

4. What is the cutoff frequency ω_0 as $f(R_x, C_1)$? f_0 = $1/(2\pi R_{b1} C_1)$

2.1.2 Noise Transfer Function

The noise transfer function (NTF) is generally defined as

$$NTF = \frac{U'_{out}}{U'_{err}} \tag{2.5}$$

From noise source U_{err1} we measure at the output $U'_{out}=NTF \cdot U_{err1}$. Take the *NTF* from [2] or compute it from STF by translating U_{err1} into an equivalent input signal: Divide it by (ω_1/s) and multiply with STF:

$$NTF_1 = \frac{s}{s + b_1\omega_1} = \frac{sR_{b1}C_1}{1 + sR_{b1}C_1} \xrightarrow{|s|\to 0} 0 \tag{2.6}$$

Important (e.g. for ΔΣ modulators [9], [10], [11]): Low frequencies are suppressed proportional to s and therefore particularly for low frequencies.

2.1.3 Stability

There is a single pole in the negative s-plane. It is $s_p = -b_1 \cdot \omega_1$

Consequence: (Check the correct statement):

The system is **X** always stable **O** stability depends on device parameters.

2.2 General and Particular Second Order System Models

2.2.1 Signal Transfer Function of a Second Order Electronic Circuit

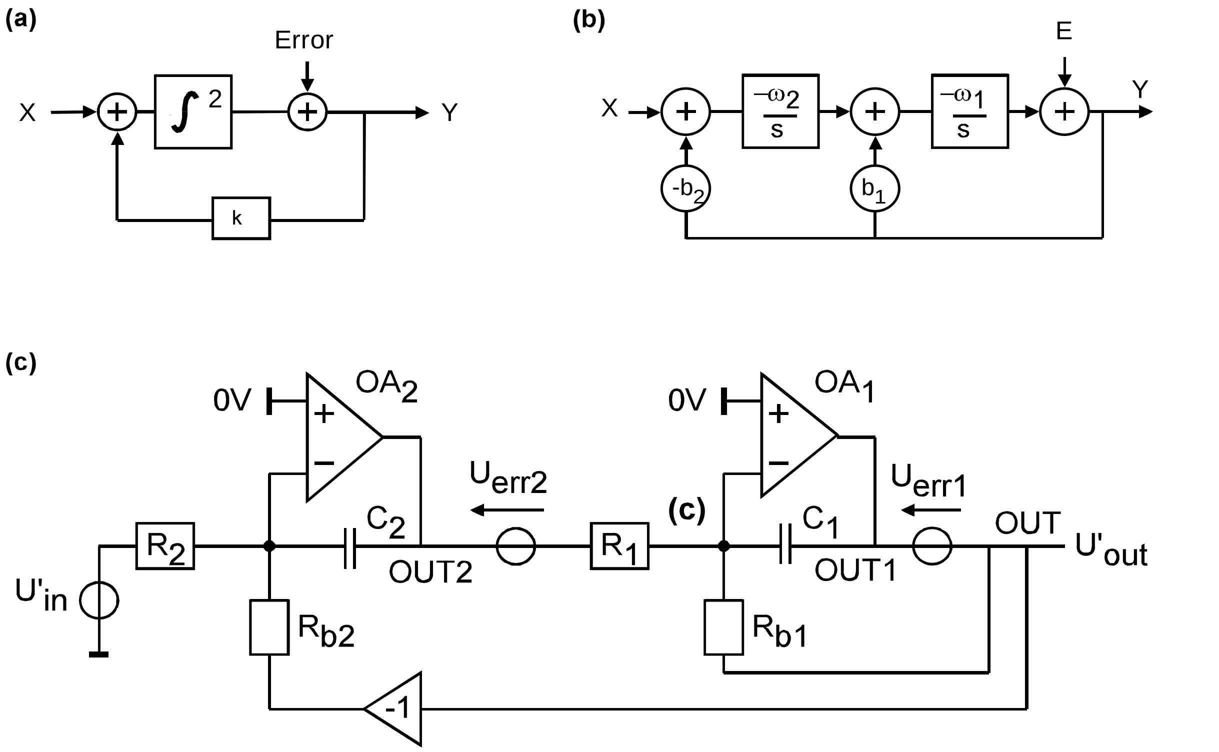


Fig. 2.2.1: (a) High-level schematics, **(b)** depicted for modeling, **(c)** Realization with $\omega_1=1/R_1C_1$, $\omega_2=1/R_2C_2$, $b_1=R_1/R_{b1}$, $b_2=R_2/R_{b2}$ and error source U_{err1}.

A_0, ω_0, D are a set of parameters describing the general 2nd order model. They are chosen such, that any of these parameters affects one aspect of the circuit only: A_0 is the DC amplification, ω_0 the cutoff frequency and D adjusts the stability.

Goal: Find three particular parameters that effect one of the three general parameters only:

1. Which device parameter in Fig. 2.2.1(c) affects DC amplification. A_0 only? ....R_2....

2. Which device parameter in Fig. 2.2.1(c) affects stability Parameter D only? ....R_{b1}....

3. Which two parameter Fig. 2.2.1(c) affects the cutoff frequency ω_0 only?

C_1 C_2 while C_2/C_1 keeps constant: $C_1 = C_{10}\cdot c_r$, $C_2 = C_{20}\cdot c_r$
....

Further questions important to understand the circuit:

4. What is the DC amplification A_1 of the amplifier stage with index 1? ..$-R_{b1}/R_1$..

5. What is the DC amplification A_2 of the amplifier stage with index 2? ...A_0/A_1..

6. Compute phase and magnitude of $STF_2(s=j\omega_0)$. (Hint: It is easiest to compute $STF_{2,general}(s'=j)$.)
. (2.7)

$$STF_{2,general}(s'=j)=\frac{A_0}{s'^2+2Ds'+1}=\frac{A_0}{j^2+2Dj+1}=\frac{A_0}{-1+2Dj+1}=-j\frac{A_0}{2D}=\frac{A_0}{2D}e^{-j90°}$$

Table 2.2.1: General and particular 2nd order model taken from [2] (2.8)

(a) General 2nd Order Model	**(b) Particular Model of Fig. 2.1**
$STF_{2,general}=\frac{A_0}{s'^2+2Ds'+1}=\frac{A_0\omega_0^2}{s^2+2D\omega_0 s+\omega_0^2}$	$STF_{2,particular}=\frac{U_{out1}(s)}{U_{in}(s)}=\frac{\omega_1\omega_2}{s^2+b_1\omega_1 s+b_2\omega_1\omega_2}$
$s'=\frac{s}{\omega_0}$: frequency normalized to ω_0 A_0 : amplification at ω=0, d : attenuation / time, dimension: 1/time $D=\frac{d}{\omega_0}$: attenuation / wave, dimensionless ω_0 : cutoff frequency, used in $s'=s/\omega_0$ Poles of the 2. order system: $s_{p1,2}=\begin{cases}\omega_0\left(-D\pm\sqrt{D^2-1}\right) & if \quad D\geq 1\\ \omega_0\left(-D\pm j\sqrt{1-D^2}\right) & if \quad D<1\end{cases}$	$A_0=\frac{1}{b_2}=\frac{R_{b2}}{R_2}$ with $\omega_x=\frac{1}{R_x C_x}$, $b_x=\frac{R_x}{R_{bx}}$, (x=1,2) $\omega_0=\sqrt{b_2\omega_1\omega_2}=\frac{1}{\sqrt{R_1C_1R_{b2}C_2}}$ $D=\frac{b_1\omega_1}{2\omega_0}=\frac{\sqrt{R_1R_{b2}}}{2R_{b1}}\sqrt{\frac{C_2}{C_1}}$

2.2.2 Noise Transfer Function of the 2nd-Order System

From noise source U_{err1} we measure at the output $U'_{out}=NTF_1 \cdot U_{err1}$. To compute NTF_2 from STF we translate U_{err1} into an equivalent input signal by dividing it by $\omega_1\omega_2/s^2$ and then multiply with the STF:

$$NTF_2(U_{err1})=\frac{STF_2}{\frac{\omega_1\omega_2}{s^2}}=\frac{s^2}{\omega_1\omega_2}\frac{\omega_1\omega_2}{s^2+\omega_{k1}s+\omega_1\omega_{k2}}=-\frac{s^2}{s^2+\omega_{k1}s+\omega_1\omega_{k2}}\xrightarrow{|s\to 0|}0 \quad (2.9)$$

From noise source U_{err2} we measure at the output $U'_{out}=NTF_3 \cdot U_{err2}$. To compute NTF_3 from STF we translate U_{err2} into an equivalent input signal by dividing it by ω_2/s and then multiply with the STF:

$$NTF_3(U_{err2})=\frac{STF_2}{\frac{\omega_2}{s}}=-\frac{s}{\omega_2}\frac{\omega_1\omega_2}{s^2+\omega_{k1}s+\omega_1\omega_{k2}}=-\frac{s\cdot\omega_1}{s^2+\omega_{k1}s+\omega_1\omega_{k2}}\xrightarrow{|s\to 0|}0 \quad (2.10)$$

Important (e.g. for ΔΣ modulators): Low frequencies are suppressed proportional to s^2.

2.2.3 Stability Investigation Considering the System's Poles

From (2.8) we know: $D = \frac{b_1 \omega_1}{2\omega_0} \quad \Leftrightarrow \quad b_1 = 2D\frac{\omega_0}{\omega_1}$ (2.11)

Knowing $\omega_0 = \frac{1}{\sqrt{R_1 C_1 R_{b2} C_2}}$ and $R_{b1} = \frac{\sqrt{R_1 R_{b2}}}{2D}\sqrt{\frac{C_2}{C_1}}$ from (2.8) we can compute the aperiodic (dead-beat) limit case using D=1 and derive the other cases from it (see table below).

Table 2.2.3: Controlling stability with parameter D for R_1=R_2=R_{b2}=100KΩ and C_1=C_2. (2.12)

Case	D	=> b_1 (ω_0/ω_1)	=> R_{b1} (DevParam)	R_{b1} / KΩ
creep	$D > 1$	$b_{1,creep} > 2\omega_0/\omega_1$	$R_{b1,aper} < R_{b1,db\lim}$	< 50
dead-beat (=aperiodic) limit:	$D = 1$	$b_{1,db\lim} = 2\frac{\omega_0}{\omega_1}$	$R_{b1,db\lim} = \frac{\sqrt{R_1 R_{b2}}}{2}\sqrt{\frac{C_2}{C_1}}$	50
Butterworth:	$D = \sqrt{1/2}$	$b_{1,BW} = \sqrt{2}\frac{\omega_0}{\omega_1}$	$R_{b1,BW} = R_{b1,db\lim} \cdot \sqrt{2}$	70.71
Phase margin 45°	1/2	$b_{1,PM45°} = \omega_0/\omega_1$	$R_{b1,PM45°} = R_{b1,db\lim} \cdot 2$	100
Ideal oscillator:	$D = 0$	$b_{1,osc} = 0$	$R_{b1} \to \infty$	∞

Fill the last column in the table above such that it computes the value of R_{b1} for R_1=R_2=R_{b2}=100KΩ and C_1=C_2=220pF.

Consequence: The system is o always stable **x** stability depends on device parameters.

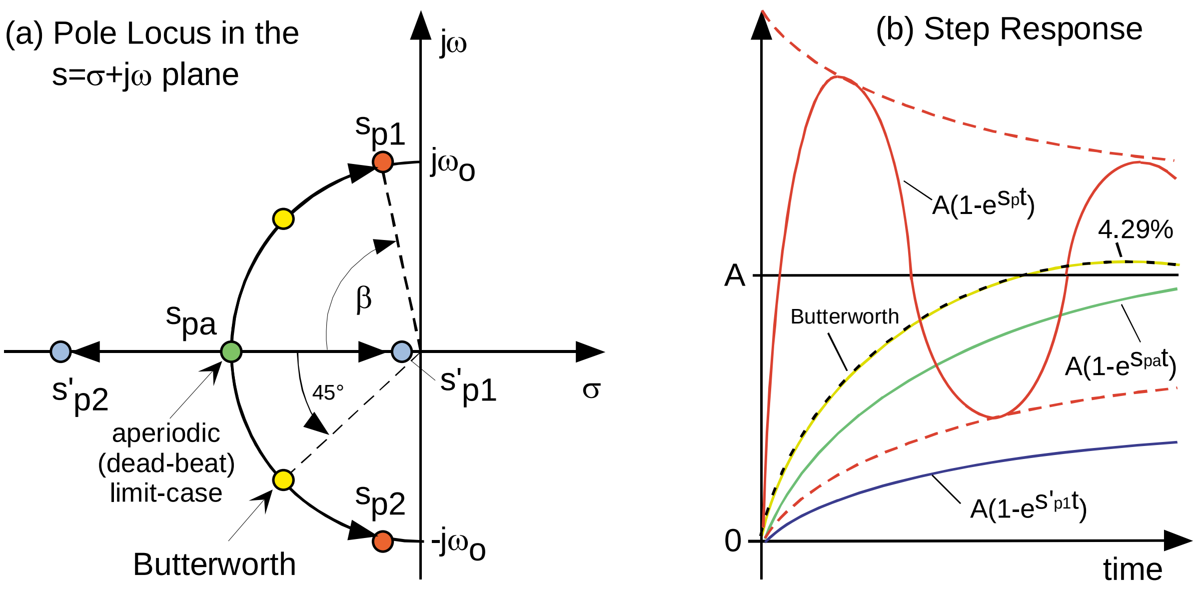


Fig. 2.2.3: (a) Locus of poles in the s-plane and **(b)** respective step responses

2.3 Model Summary

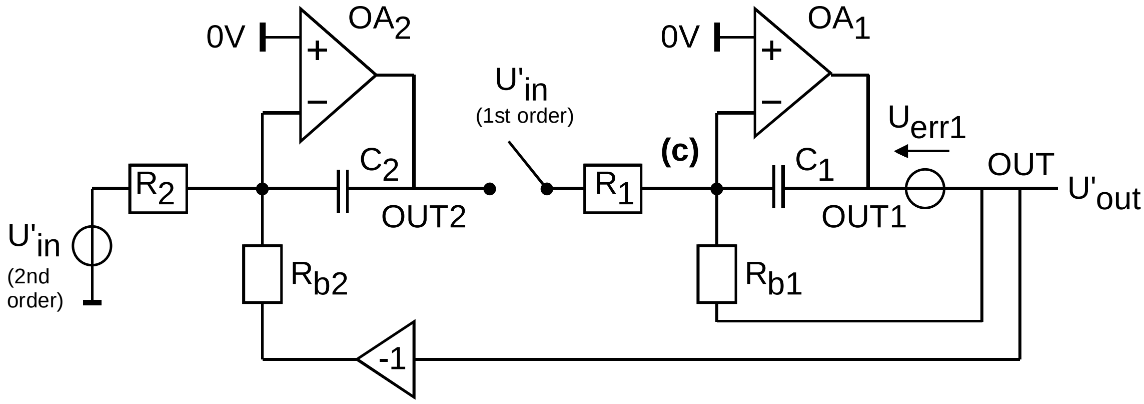


Fig. 2.3: 1st and 2nd order system, depending on switch S.

Table 2.3-1: Summary of 1st and 2nd order models and parameters (2.14)

Property	**Parameter**	**1st Order Model**	**2nd Order Model**
Signal Transfer Function, $s'=s/\omega_0$	$STF(s')$	$\frac{A_0}{s'+1}$	$\frac{A_0}{s'^2+2Ds'+1}$
Signal Transfer Function using	$STF(s)$	$\frac{A_0\omega_0}{s+\omega_0}$	$\frac{A_0\omega_0^2}{s^2+2D\omega_0 s+\omega_0^2}$
Noise Transfer Function	$NTF(s)$	$\frac{s}{R_1C_1}STF = \xrightarrow{s\to 0} 0$	$\frac{s^2}{R_1C_2R_1C_2}STF \xrightarrow{s\to 0} 0$
DC amplification	A_0	$\frac{R_{b1}}{R_1}$	$\frac{R_{b2}}{R_2}$
System bandwidth / cutoff frequency	f_0	$\frac{1}{2\pi R_{b1}C_1}$	$\frac{1}{2\pi\sqrt{R_1C_1R_{b2}C_2}}$
Stability control parameter	D		$\frac{\sqrt{R_1R_{b2}}}{2R_{b1}}\sqrt{\frac{C_2}{C_1}}$

Table 2.3-2: Controlling 2nd order system stability for R_1=R_2=R_{b2}=100KΩ and C_1=C_2. (2.15)

Case	***D***	**=> R_{b1} =f(*DevParam*)**	**R_{b1} / KΩ**
creep	$D>1$	$R_{k1,aper} < R_{b1,db\lim}$	< 50
dead-beat (=aperiodic) limit:	$D=1$	$R_{b1,db\lim} = \frac{\sqrt{R_1R_{b2}}}{2}\sqrt{\frac{C_2}{C_1}}$	50
Butterworth:	$\sqrt{1/2}$	$R_{b1,BW} = R_{b1,db\lim}\cdot\sqrt{2}$	70.71
Phase margin 45°	1/2	$R_{b1,PM\,45°} = R_{b1,db\lim}\cdot 2$	100
Ideal oscillator:	$D=0$	$R_{b1}\to\infty$	∞

3 Model Verification by Spice Simulation

3.1 Verifying the First-Order Model with Spice

Simulation is a valuable tool to proof and better understand the analytical calculus in the previous chapter. Nowadays many derivatives of the UC Berkeley's Spice [3] simulator exist. LTspice [4] is available free of charge and without simulation limitations.

The input file for the circuit in the figure above is available from the author. Use it to proof the analytical results. First of all summarize them:

Table 3.1: Impact of device parameters R_{b1} R_1 and C_1 on Amplification A_1 and pole f_p.

	DC ampl. A_0	cutoff frequ. f_0	adjust A_0 only by	adjust f_0 only by
Device parameter:	$-R_{b1}/R_1$	$1/(2\pi R_{b1}C_1)$	R_1	C_1

3.1.1 Variation of R_1

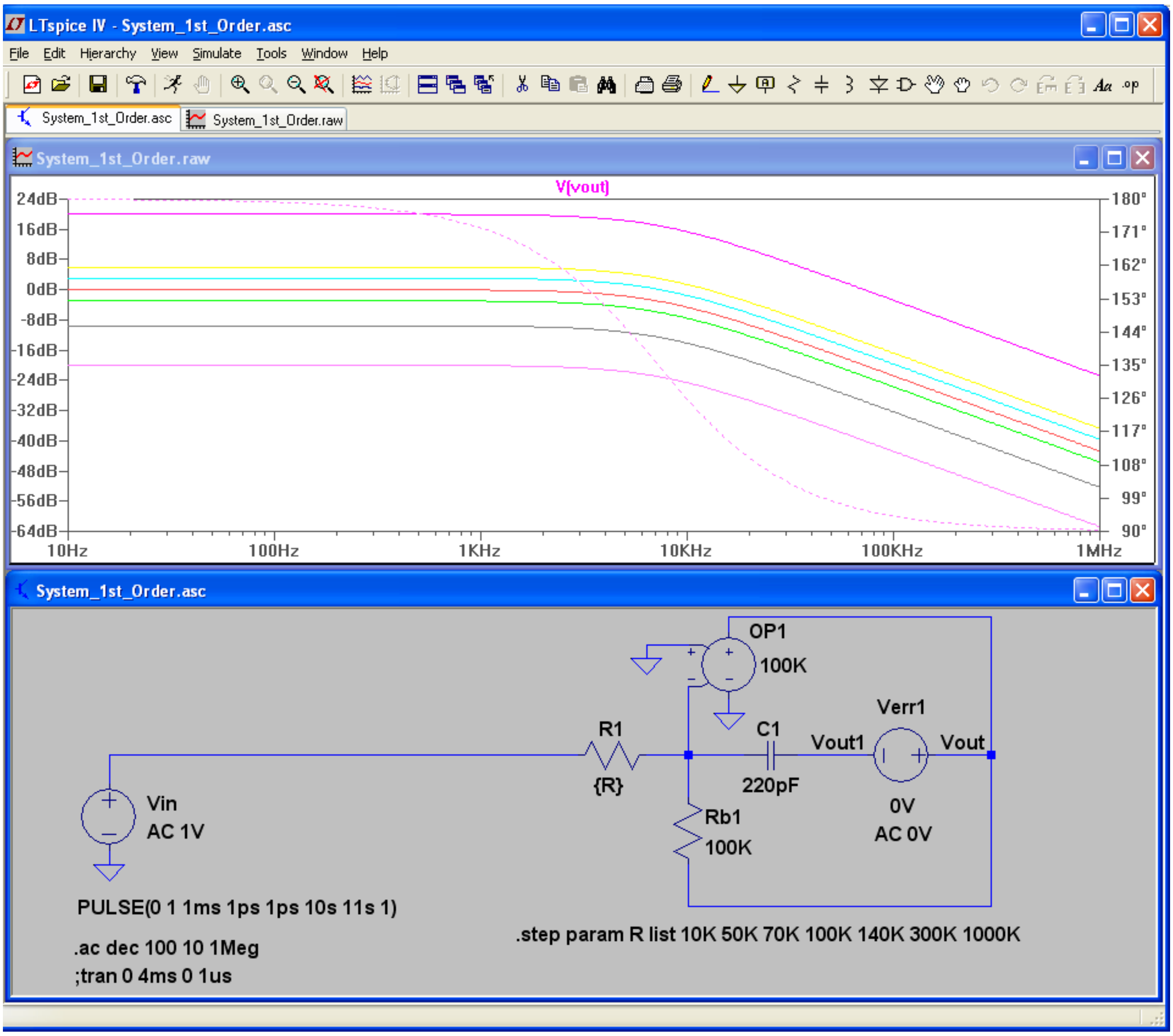


Fig. 3.1.1: LTspice simulation of the 1st-order model: Variation of R_1 for the *STF*.

3.1.2 Variation of R_{b1}

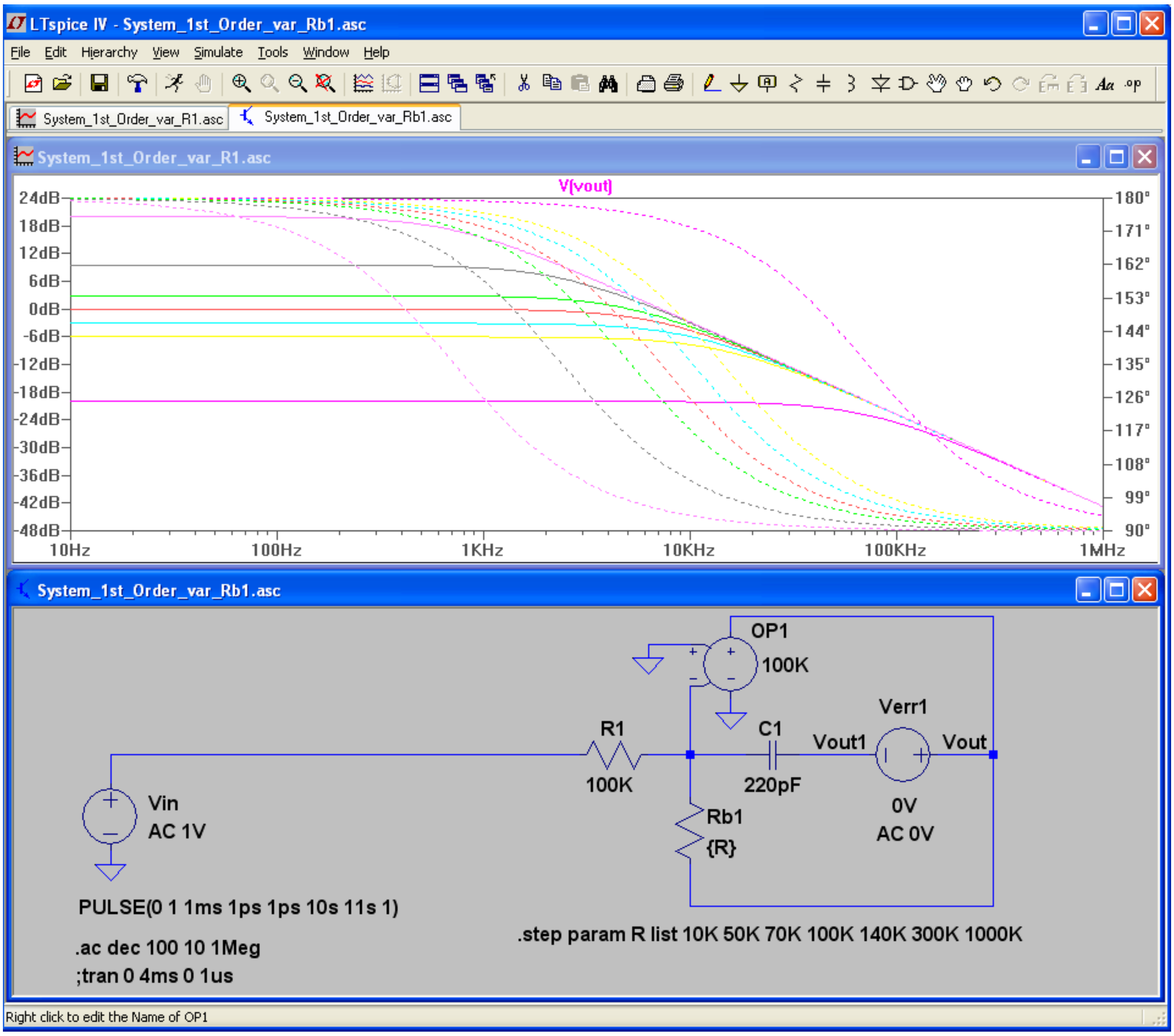


Fig. 3.1.2: LTspice simulation of the 1st-order model: Variation of R_{b1} for the *STF*.

3.1.3 Homework: Fill Tables 5.1-1 and 5.1-2.

To quantitatively proof table 3.1 fill the lines labeled with "simulated" in tables 5.1-1 and 5.1-2. Use AC mode for simulation. Note that LTspice-measurements are typically better done with constant device parameters (i.e. without parameter list). Use a cursor to measure A_1 and f_p at –45° phase drop compared to phase at f=0.

***NTF*$_1$:** In the AC mode: vary R_1 by a list at U_{in}=0V, U_{err1}=1V. Agreement with Eq. (2.6)? **yes**

Stability: AC mode: are there oscillations in the step response? **no**

3.2 Verifying the Second-Order Model with Spice

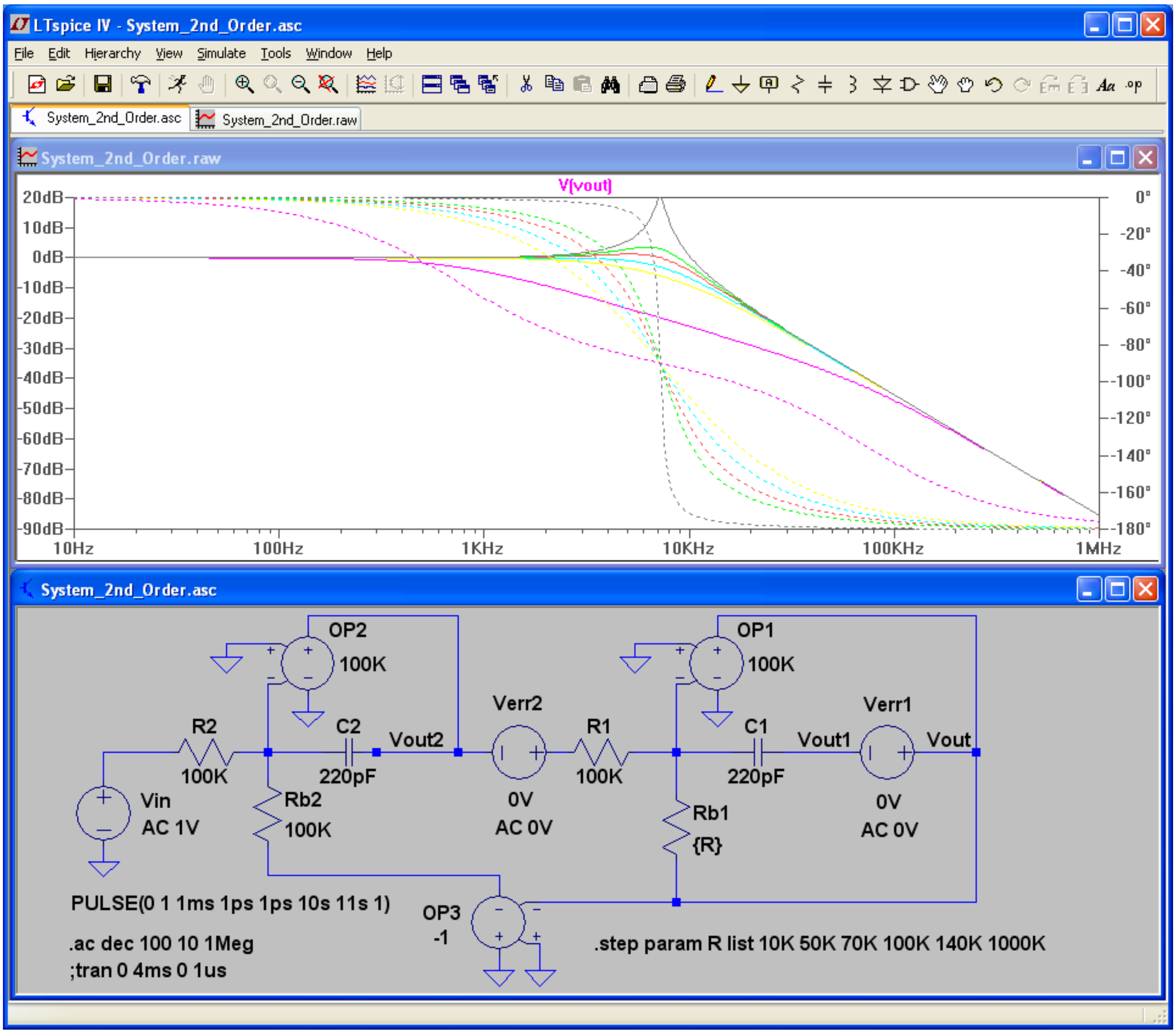


Fig. 3.2: LTspice simulation of the second-order model, here: variation of R_{b1}.

The input file for the circuit in the figure above is available from the author. Use it to proof the analytical results. First of all summarize them:

Table 3.2: Impact of device parameters R_{b1} , R_2 , c_r on DC-ampl. A_0 , D and cutoff frequ. f_0.

	DC ampl. model A_0	Damping factor model D	cutoff frequ. model f_0	adjust A_0 only by	adjust D only by	adjust f_0 only by
Device Parameter:	$\frac{R_{k2}}{R_2}$	$D=\frac{\sqrt{R_1 R_{k2}}}{2R_{k1}}\sqrt{\frac{C_2}{C_1}}$	$\frac{1}{\sqrt{R_1 C_1 R_{k2} C_2}}$	R_2	R_{b1}	c_r

Observe the AC-plot in the top part of Fig. 3.2 (colors inverted to save ink). It is the system $A(s)=1/(s'^2+2Ds'+1)$ with $s'=\omega/\omega_0$ for varying values of D. How can we measure $f_0=\omega_0/2\pi$ for different values of D? (Hint: look at the dashed curves, the phase information of the plot.)

Measure f_0 at phase = -90°

. . . .

Vary one after the other the parameters R_2, R_{b1}, c_r (with $C_1=c_r\cdot C_{10}$, $C_2=c_r\cdot C_{20}$) in the spice model to proof table 3.2 qualitatively like in Fig. 3.2.

To quantitatively proof table 3.2 fill the lines labeled with "simulated" in tables 5.2-1 and 5.2-2. Use AC mode for simulation. Note that LTspice-measurements are typically better done with constant device parameters (i.e. without parameter list). Use a cursor to measure **A_0**, **f_0**, **$A(f_0)$** at –90° phase drop compared to phase at **f**=0.

***NTF_2*:** Settings: AC mode, U_{in}=0V, U_{err2}=0V, U_{err1}=1V.

Does the result agree with Eq. (2.9)? **yes** What is the slope of $U_{out}(f<f_0)$? **40** dB/dec

***NTF_3*:** Settings: AC mode, U_{in}=0V, U_{err2}=1V, U_{err1}=0V.

Does the result agree with Eq. (2.10)? **yes** What is the slope of $U_{out}(f<f_0)$? **20** dB/dec

Stability: Oscillations in the step response oscillate at $\omega_p=\mathrm{Im}\{s_p\}$. What is its model?

$f_p.=\omega_p/2\pi$ with $\omega_p = \omega_0\cdot\sqrt{1-D^2}$ for D≤1 (creep case if D>1)

. . . .

Proof, that $A(f_0)=-j\dfrac{A_0}{2D}$. (Hint: s'=j when $\omega=\omega_0$.)

Important consequence: At $f=f_0$ here will always be a –90° phase shift.

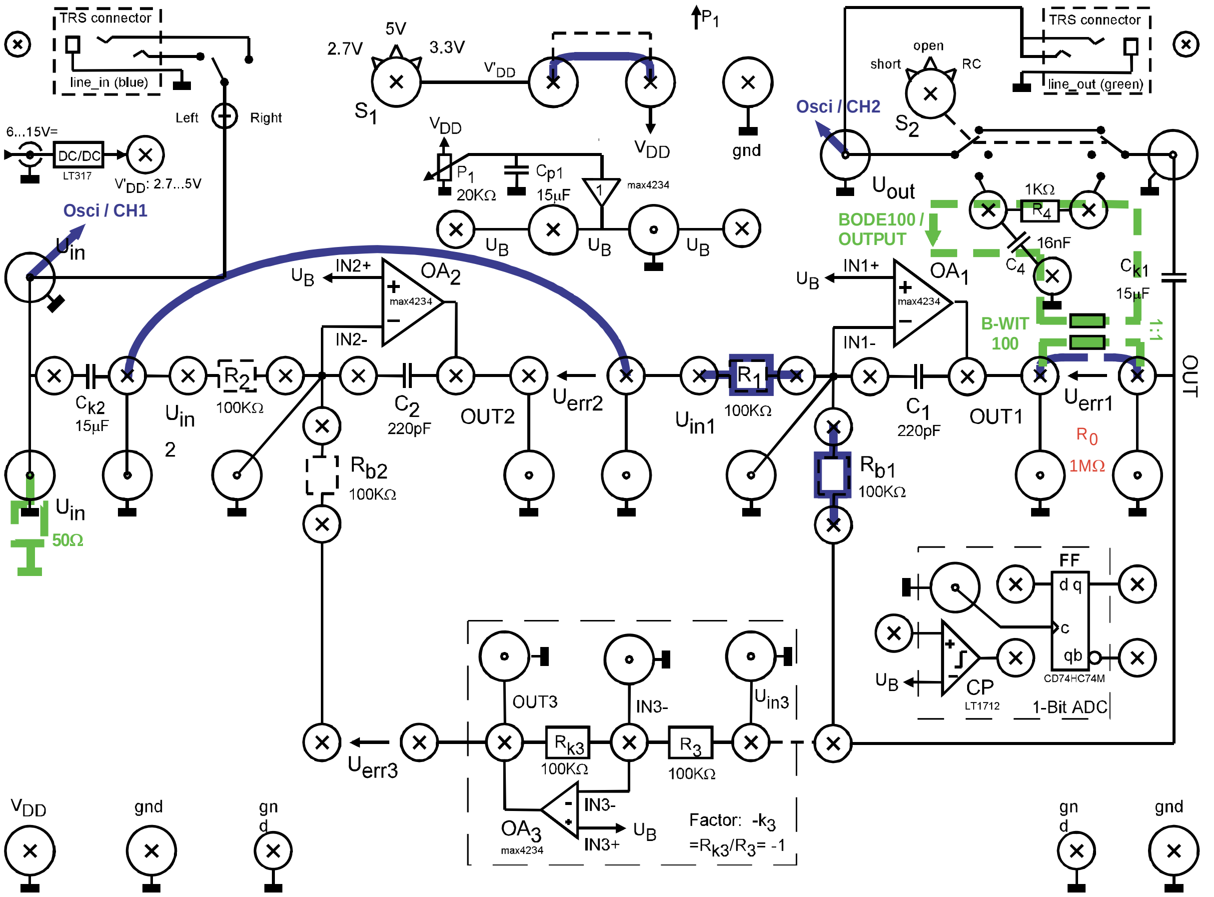
$STF_{2,general}(s')$ = $\dfrac{A_0}{s'^2+2Ds'+1}=\dfrac{A_0}{j^2+2Dj+1}=\dfrac{A_0}{2Dj}=-j\dfrac{A_0}{2D}$

. . . .

4 Experimental Verification of 1st and 2nd Order Model

To get started with the *BODE100 Vector Network Analyzer* for this laboratory refer to [8].

4.1 First-Order System Characterization



Caption: 2 mm banana, 4 mm banana, BNC

Fig. 4.1: First Order System Configuration

Preparing the Board: Setup & Calibration:

Disconnect TRS*) connector from the computer's sound card. (Speakers may be connected.)

- Verify V_{DD}=3.3V,
- Adjust the *IN1+* inputs of the OpAmp OA_1 to V_B=V_{DD}/2.
- Set U_{err1} = 0V using short circuit
- Disconnect the output of OA_2 from U_{in1} (i.e. 'U_{err2}' is a break) and connect U_{in1} to U_{in2}.
- Set R_{b1} = R_1 = 100KΩ, C_1 = 220pF

Preparing the Oscilloscope: Let's Use Following Default Settings

- Oscilloscope's *CH1* shows the boards input voltage U_{in}.
- Oscilloscope's *CH2* shows the boards input voltage U_{out}.
- Trigger channel *CH1*.

Preparing *Bode 100* for Gain/Phase mode mesurements:

- *Bode Analyzer Suite* Toolbar: *Configuration* → *Device Configuration* → set switch right, connecting *Receiver 1* to *CH1*. Then shorten *CH1* to *OUTPUT* externally with a BNC cable.
- Click the *Gain/Phase* toolbar button .
- *Source Frequency*: 100Hz, *Level*: 0dB, *Attenuators*: 20dB, *Receiver Bandwidth*: 100Hz.
- Connect (a) *Bode 100's OUTPUT* and *CH1* and the board's U_{in}, (b) *CH2* to board's U_{out}.
- Click the *Continuous Measurement* or *Single Measurement*, button to measure.

Train in the *Gain/Phase* mode the settings of *OUPUT Level* and *Attenuators*. In the *Frequency Sweep* mode things may happen too fast to observe.
Constant parameters below: C_1=220pF, R_{b1}=100KΩ

***R_1 = R_{b1}* = 100KΩ:** (Results in integral multiples of 10dB.)

1. Set *Bode 100*'s *OUTPUT* such, that *OA_1's* output is max. & sinusoidal *Level* = 0 dB
2. Adjust *Attenuator CH1* such, that *CH1* is well loaded. *Attenuator* *CH1* = 20 dB
3. Adjust *Attenuator CH2* such, that *CH2* is well loaded. *Attenuator* *CH2* = 20 dB

Note your measurements: *Mag (dB)*: ...38 mdB... *Phase* (°): ....179...

***R_1* = 1MΩ:** (Results in integral multiples of 10dB.)

4. Set *Bode 100*'s *OUTPUT* such, that *OA_1's* output is max. & sinusoidal *Level* = 13 dB
5. Adjust *Attenuator CH1* such, that *CH1* is well loaded. *Attenuator* *CH1* = 30 dB
6. Adjust *Attenuator CH2* such, that *CH2* is well loaded. *Attenuator* *CH2* = 10 dB

Note your measurements: *Mag (dB)*: ...-20 dB.... *Phase* (°): ....-180....

***R_1* = 10KΩ:** (Results in integral multiples of 10dB.)

7. Set *Bode 100*'s *OUTPUT* such, that *OA_1's* output is max. & sinusoidal *Level* = -20 dB
8. Adjust *Attenuator CH1* such, that *CH1* is well loaded. *Attenuator* *CH1* = 0 dB
9. Adjust *Attenuator CH2* such, that *CH2* is well loaded. *Attenuator* *CH2* = 20 dB

Note your measurements: *Mag (dB)*: ....19.8.... *Phase* (°): ....172....

Keep in mind for the following measurements, that *Bode 100's OUTPUT Level* must be adjusted proportional $1/A_0.=R_1/R_{b1}$! Eventually peaking around f_0 must be respected, too.

***Bode 100*, *Frequency Sweep* mode:**

Click toolbar button to switch to the *Frequency Sweep* mode.
Freq.: 100Hz-1MHz, Log. 201 Points, Receiver Bandwidth 100Hz, Level+Attn. optimized.
Activate: Trace *2, Measurement: Gain, Display: Data, Format: Phase (°).*
Check → *Configuration* and → *Calibration*. Everything ok? Let's start the measurements!

STF: Proof the theory of tables 2.2.1 and 2.4 regarding A_0 and/or f_0 dependencies:
Connect the Line-out TRS*) plug to the speakers, listen to one or two measurements to get a feeling for measurement duration and speed.
Perform the 8 measurements required to fill tables 5.1-1 and 5.1-2. Observe the *Bode100*'s overload indicator and oscilloscope channel *CH2*. Avoid overloads by setting *OUTPUT Level* and *Attenuator CH2* properly.

- A_0 and f_0 for $\boldsymbol{R_1}$**=10KΩ,** R_{b1}=100KΩ, C_1=220pF, 2.2nF → note results in table 5.1-1
- A_0 and f_0 for $\boldsymbol{R_1}$**=100KΩ,** R_{b1}=100KΩ, C_1=220pF, 2.2nF → note results in table 5.1-1
- A_0 and f_0 for $\boldsymbol{R_1}$**=1000KΩ**, R_{b1}=100KΩ, C_1=220pF, 2.2nF → note results in table 5.1-1

- A_0 and f_0 for R_1=100KΩ, $\boldsymbol{R_{b1}}$**=10KΩ,** C_1=220pF → note results in table 5.1-2
- A_0 and f_0 for R_1=100KΩ, $\boldsymbol{R_{b1}}$**=1MΩ,** C_1=220pF → note results in table 5.1-2

Conclusion: R_{b1} has impact on X A_0 X f_0, R_1 on X A_0 O f_0, C_1 on O A_0 X f_0

NTF: Measuring the Noise Transfer Function
- U_{in}=0V (attach BNC short circuit or 50Ω to U_{in}).
- R_{b1}=100KΩ, R_1=10, 100 or 1000KΩ (has no impact)
- Let *CH2* connected to U_{out} as is.
- Feed *Bode100's OUTPUT* to U_{err1} using injection transformer *B-WIT100* of *Omicron Lab.*

Perform a frequency sweep, **observe**: Low frequencies are suppressed with 20 dB/dec**.**

Measuring the Errors in the Feedback Path
Use the same measurement setup as above but measure U_{out1} instead of U_{out}. Then U_{err1} is within the feedback path.
Observe: Low frequencies are suppressed with 0 dB/dec

<u>Conclusions of the frequency-domain measurements:</u>
Errors at the system's input are amplified by the X STF O NTF
Errors at the forward network's output are amplified by the O STF X NTF
Errors at the feedback network's output are amplified by the X STF O NTF

Time-Domain Measurements
Observe stability in the time domain
- $R_1 = R_{b1}$ = 100KΩ,
- U_{in} = 1$V_{peak\text{-}peak}$, rectangular, 4 KHz from the waveform generator
- Observe U_{in1} and U_{out} at oscilloscope channels *CH1*, *CH2* for C_1=220pF and 2.2nF.

Can you observe any voltage overshoot? No

Analog Audio Signal Processing

Remove waveform generator
Connect the board's line-in TRS*) plug to line-out of the computer's sound card (green).
Connect the board's line-out TRS plug to the speakers.
Play some music, switch amplification (A_0) & bandwidth (f_0) independently**).
Perceive the effects on the sound.

*) TRS = Tip, Ring, Sleeve, connected to left channel, right channel and ground, respectively.
**) Care about other students: low volume. Short test times please!

4.2 Second-Order System Characterization

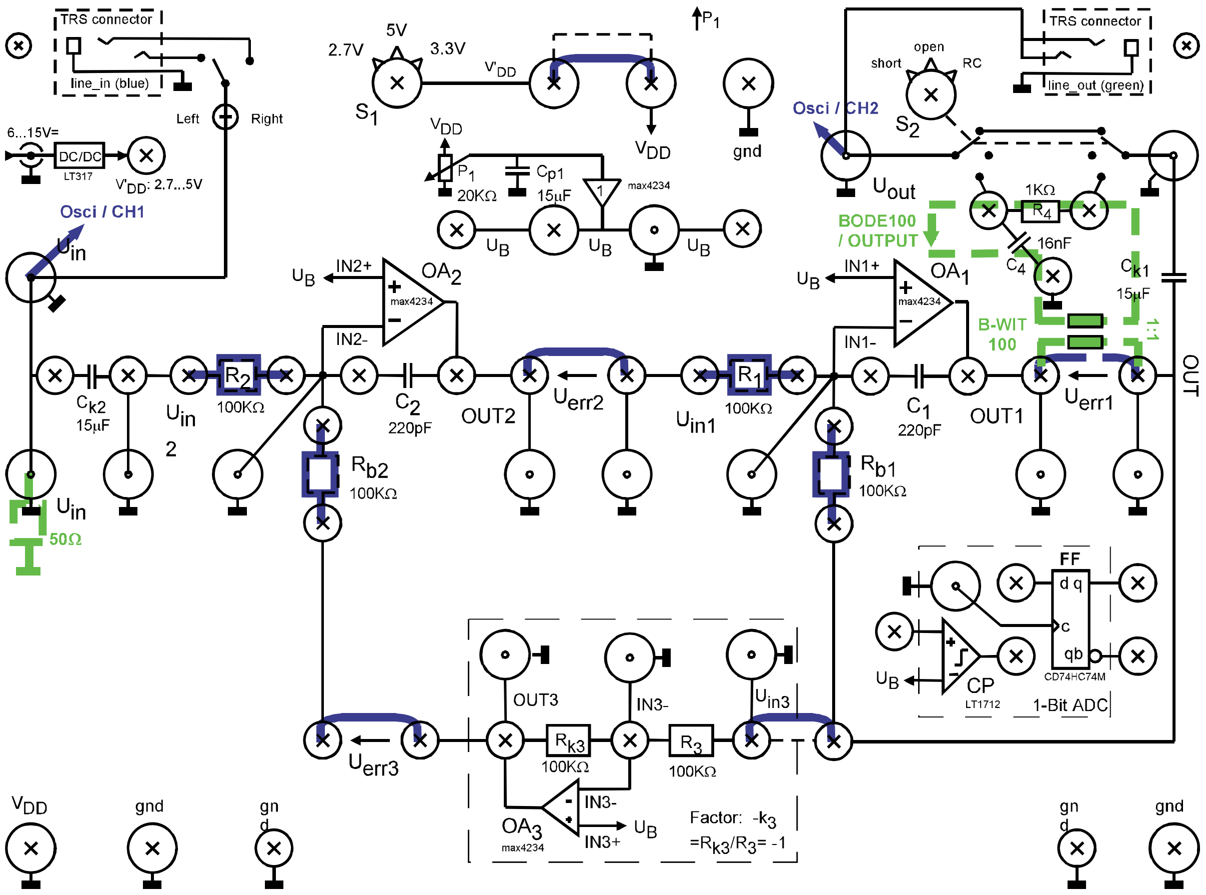


Fig. 4.2: Second Order System Configuration

Caption: 2 mm banana, 4 mm banana, BNC

Board: Setup & Calibration:

Remove TRS connector coming from the computer's sound card.

- Verify that V_{DD}=3.3V.
- Adjust the *IN1+* inputs of the OpAmps OA_1, OA_2, OA_3 to ca. V_B=V_{DD}/2.
- Remove bypass of OA_2.
- Set $U_{err2} = 0V$ and $U_{err1} = 0V$ using short circuits
- Set $R_{b1} = R_1 = R_{b2} = R_2 = 100K\Omega$, $C_1 = C_2 = 200pF$

Oscilloscope: Default Settings

- Oscilloscope's *CH1* shows U_{in}, *CH2* shows U_{out}, trigger channel *CH1*.

Bode100, Gain/Phase mode: *Source Frequency*: 100Hz, optimize *Level* and *Attenuations*.

***Bode100, Frequency Sweep* mode:**

Use settings as described in subsection 4.1.5.
Activate: Trace *2, Measurement: Gain, Display: Data, Format: Phase (°).*
Check for *Configuration*
Check for *Calibration* when *CH1* is internally connected to *OUTPUT*.

STF: Proof the theory of tables 2.4-1 and 2.4-2 regarding A_0, f_0, D dependencies:
Perform the 6 measurements to be noted in table 5.3-1. Consider proper settings of *OUTPUT Level* and *Attenuator CH2* to avoid overloads. Use a cursor to measure f_0 at –90° phase drop. Try also R_{b1} > 100KΩ and free oscillation: $R_{b1} \to \infty \Leftrightarrow D=0$.

Conclusions:
R_2 has impact on X A_0 O f_0 O D, R_{b1} on O A_0 O f_0 X D, $C_{1,2}$ on O A_0 X f_0 O D

NTF: Measuring the Noise Transfer Function
- U_{in}=0V (attach BNC short circuit or 50Ω to U_{in}).
- R_2=10, 100 or 1000KΩ (has no impact)
- Let *CH2* connected to U_{out} as is.
- Feed *Bode100's OUTPUT* to U_{err1} using injection transformer *B-WIT100* of *Omicron Lab.*
- Perform a frequency sweep.

Observe: Low frequencies are suppressed with 40 dB/dec.

Measuring the Errors in the Feedback Path
- Feed *Bode100's OUTPUT* to U_{err3} using injection transformer *B-WIT100* of *Omicron Lab.*

Observe: Amplification at low freq. of the error U_{err3} injected into the feedback path: .0. dB

<u>Conclusions of the frequency-domain measurements:</u>

Errors at the system's input are amplified by the	X *STF*	O *NTF*
Errors at the forward network's output are amplified by the	O *STF*	X *NTF*
Errors at the feedback network's output are amplified by the	X *STF*	O *NTF*

Time-Domain Measurements

(a) Observe stability in the time domain
Set $R_1=R_2=R_{b2}$=100KΩ, R_{b1} variable. Remove *Bode 100's OUTPUT* from the board's U_{in}. Feed a rectangular waveform with frequency $f_0/10$ (ca. 700Hz) and $1V_{peak\text{-}peak}$ to U_{in}. Perform the 3 measurements to be noted in table 5.2-2. **(1)** Dead-beat limit case: R_{b1}=50KΩ ⇔ $D=1/2$, **(2)** Butterworth case: R_{b1}=70KΩ ⇔ $D=1/\sqrt{2}$, **(3)** Phase-Margin=45° case: R_{b1}=100KΩ ⇔ $D=1$. **(4)** Try also R_{b1} > 100KΩ and free oscillation: $R_{b1} \to \infty \Leftrightarrow D=0$.

(b) Observe stage amplifications in the time domain
Observe the amplification A_{V2} of the input stage (around OA_2) as a function of R_{b1}! How can A_{V2} be modeled as function of A_0 and A_{V1}, the amplification of the output stage (around OA_1)? (Hint: The total amplification must be $A_0 = A_{V2}\,A_{V1}$ while $A_0=R_{b2}/R_2$ and $A_{V1}=-R_{b1}/R_1$.)

A_{V0} and A_{V1} given => $A_{V2}=A_{V0}/A_{V1}$.
....

Analog Audio Signal Processing
- Remove any sources connected to U_{in}.
- Connect the board's line-in TRS plug to line-out of the computer's sound card (green).
- Connect the board's line-out TRS plug to the speakers.
- Play music, switch stability (D by R_{b1}) amplification (A_0) & bandwidth (f_0) independently.
- Perceive the effect on the sound, particularly for $R_{b1} \to \infty$ for the different capacitances.

5 Simulated and Experimental Results Summary

5.1 First Order System

Table 5.1-1: Impact of C_1 and R_1 on Amplification A_0 and f_0 (= pole f_p).

[1)] Use a simple resistor for the 1MΩ measurements to avoid parasitic capacitive effects.

	R_1 →	10 KΩ		100 KΩ		1 MΩ[1)]	
C_1 ↓		A_0/dB	f_0/Hz	A_0/dB	f_0/Hz	A_0/dB	f_0/Hz
220 pF	simulated:	20	7210	0	7210	-20	7210
	measured:	20	7010	0	7500	-20	7496
2200 pF	simulated:	20	721	0	721	-20	721
	measured:	20	737	0	737	-20	737

Table 5.1-2: Impact of R_{b1} on Amplification A_0 and f_0 (= pole f_p), constant: R_1=100KΩ

C_1 ↓	R_{b1} →	10 KΩ		100 KΩ take val. from above		1000 KΩ	
check CH_2	Overload?	A_0/dB	f_0/Hz	A_0/dB	f_0/Hz	A_0/dB	f_0/Hz
220pF	simulated:	-20	72100	0	7210	20	7210
	measured:	-20	77254	0	7200	20	717

5.2 Second Order System

Table 5.2-1: Parameter's impact of on Amplification A_0, cutoff frequency f_0 and stability D.

$D(R_{b1})$ ↓	$A_0(R_2)$	10 KΩ Output level = -20dB	100 KΩ Output level = 0dB	1000 KΩ Output level = 10dB
		A_0/dB, f_0/KHz, $A(f_0)$/dB	A_0/dB, f_0/KHz, $A(f_0)$/dB	A_0/dB, f_0/KHz, $A(f_0)$/dB
R_{b1}=50KΩ, $C_{1,2}$=220pF	simulated:	X	0 , 7.23, -6.01	X
	measured:		0, 7.20, -5.88	
R_{b1}=70KΩ, $C_{1,2}$=220pF	simulated:	X	0, 7.23, -3.09	X
	measured:		0, 7.20, -3.07	
R_{b1}=100K, $C_{1,2}$=220pF	simulated:	20, 7.23, 20.0	0, 7.23, 0	-20, 7.23, -20.0
	measured:	20, 7.20, 20	0, 7.20, -0.118	-20, 7.30, -20
R_{b1}=100K, $C_{1,2}$=2200pF	simulated:	X	0, .724, -0.006	X
	measured:		0, 747, -0.104	

Table 5.2-2: Stability observations in the time domain: Voltage overshoot and oscill. frequ.

Case		Aperiodic (dead-beat) limit-case		Butterworth		Phase Margin = 45°	
U_{in} = 1V		mV	%	mV	%	mV	%
U_{out} voltage Overshoot:	simulated:	0	0	40.5	4.05	163	16.3
	measured:	0	0	50	5	180	18
Oscillation frequency:	as f(f_0)	–	–	≤ f_0, exact: $\mathrm{Im}\{s_{p1,2}/2\pi\} = f_0\sqrt{1-D^2}$			

6 Check Your Knowledge

Can you resolve acronyms STF and NFT? What is their general definition as function of U_{in}, U_{err}, U_{out}? Can you apply these voltages if they are not given? What are the STF_1, STF_2 and NTF_1, NTF_2 for the particular 1st and 2nd order circuits of this laboratory?

Typically circuits are driven with a biasing voltage U_B to avoid a second power supply. This DC-offset is contradictory to the rules of LTI systems [2]. What do we do to get mathematically rid of U_B?

1st order system: What two parameters characterize a very general 1st order *STF* with DC-amplification and a single pole? Which device parameters in our particular circuit have impact on only one of these general parameters? Tendency of this impact (e.g. lower $x \rightarrow$ higher y)? Can this system become instable by parameter variations?

2nd order system: What three parameters characterize a very general 2nd order *STF* with DC-amplification and two poles? Which device parameters in our particular circuit have impact on only one of these general parameters? Tendency of this impact (e.g. lower $x \rightarrow$ higher y)? What stability case is given for general parameter D=1? What case is $D=1/\sqrt{2}$? How do you identify these cases in a Bode diagram? Is D=0 more or less stable than D=1 or $D\rightarrow\infty$?

7 Conclusions

Draw your personal conclusions from this laboratory.

The general control loop model combined with the general 1st- and 2nd- order system models in the Laplace domain apply well to 1st- and 2nd- order circuit behavior for both simulation and measurement. The comparison of simulated and experimental results is respectable in face of the fact that very simlpy spice models were used. Particularly the operational amplifiers were assumed to be near-ideal having neither poles nor zeros, infinite input and zero output impedance. A better OpAmp macro should introduce at least two additional poles for every OpAmp.

8 References

[1] M. Schubert, Courses at Regensburg University of Applied Sciences. Available: http://homepages.fh-regensburg.de/~scm39115/ → Offered Education → Courses and Laboratories

[2] M. Schubert, [1] → Document "Linear Control Loop Theory".

[3] The Spice Page, EECS Department of the University of California at Berkeley, available: http://bwrc.eecs.berkeley.edu/Classes/icbook/SPICE/.

[4] LTspice, available: http://www.linear.com/designtools/software/

[5] M. Schubert, [1] → LTSpice Input Files.

[6] Bode 100 User Manual, available http://www.omicron-lab.com/manuals/pdf.html.

[7] Bode 100 Network Analyzer Suite, available http://www.omicron-lab.com/downloads.html.

[8] M. Schubert, [1] → Document "*BODE100* Quickstart for Spectrum Analysis".

[9] J. C. Candy, G. C. Temes, 1st paper in "Oversampling Delta-Sigma Data Converters, Theory, Design and Simulation", IEEE Press, IEEE Order #: PC0274-1, ISBN 0-87942-285-8, 1991.

[10] S. R. Norsworthy, R. Schreier, G. C. Temes, „Delta-Sigma Data Converters", IEEE Press, 1996, IEEE Order Number PC3954, ISBN 0-7803-1045-4.

[11] M. Schubert, [1] → Document "Delta-Sigma Modulation"".