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User's Manual for the FLORA Equilibrium and Stability Code

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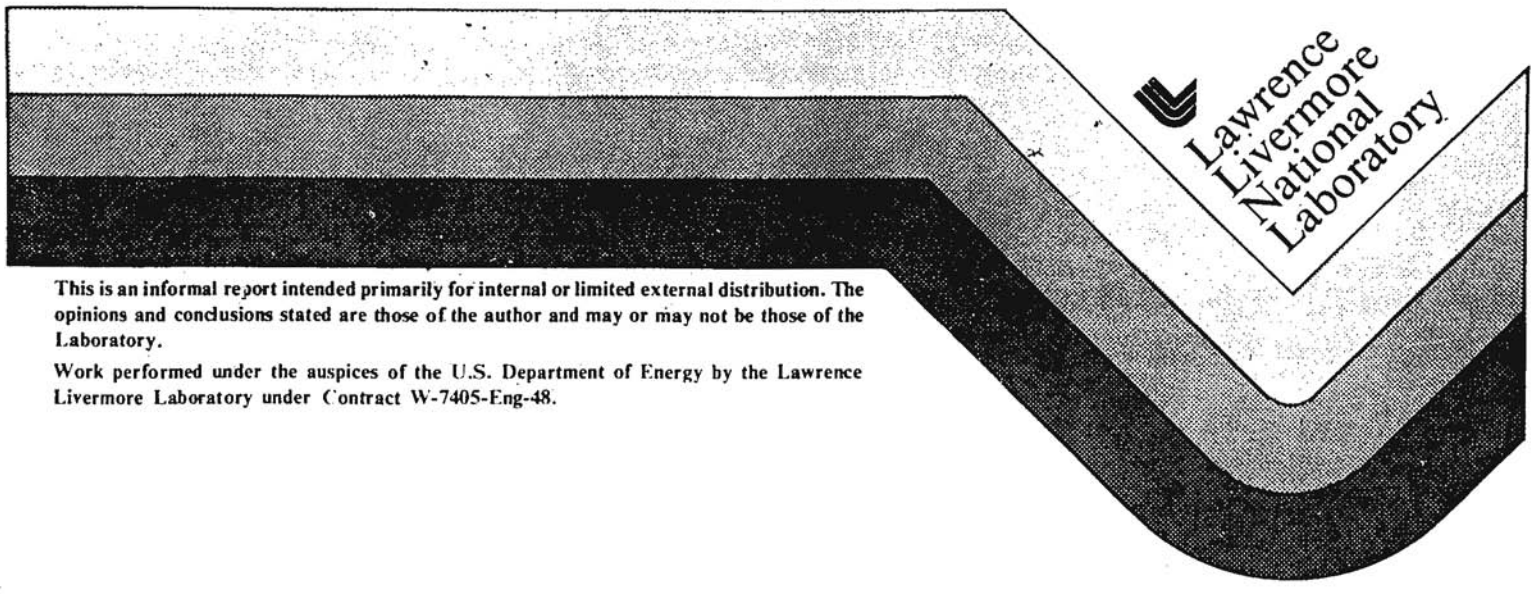
Bruce I. Cohen

April 1, 1985



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User's Manual For The FLORA Equilibrium And Stability Code

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Abstract

This document provides a user's guide to the content and use of the two-dimensional axisymmetric equilibrium and stability code FLORA. FLORA addresses the low-frequency MHD stability of long-thin axisymmetric tandem mirror systems with finite pressure and finite-larmor-radius effects. FLORA solves an initial-value problem for interchange, rotational, and ballooning stability.

1 INTRODUCTION

This user guide is a brief description of the FLORA code and is designed to be used in conjunction with the code lisitng. The theory and general equations which this program solves are described elsewhere[1,2] in detail.

FLORA solves, in a 2-D domain (z,ψ), for the linearized stability of a long thin axisymmetric equilibrium. It uses an initial-value method in which an equilibrium is given an initial perturbation to its magnetic B field, and the time behavior of the perturbation is followed. The perturbation has been Fourier expanded in the azimuthal (θ) direction and each mode (m) must be examined separately. The values of m can be arbitrary with an upper limit around O(10^3) because of accuracy consideration as the modes become more highly localized.

The complex partial differential equation of motion for the perturbed radial displacement of the field lines (Appendix A) is solved as a coupled system of two real p.d.e.'s and the solution consists of two parts, the real part (called *XRO* in FLORA) and the imaginary part (called *XIO*). The system is solved by bringing the coupling terms in each equation to the right hand side and using an iterative technique.

2 FLORA EQUILIBRIUM OVERVIEW

FLORA equilbrium are specified by the following spatial quantities:

pressure ($P_\perp$,$P_\parallel$)

density

vacuum B fields

electric potential ϕ

Tandem mirror systems are simulated by assuming symmetry around the midplane ($z = 0$) and calculating half of the total system. The half system can consist of either two or three cells, referred to as the center cell, the choke cell, and the end-plug cell. The cell boundaries are defined by solenoids which generate the vacuum B fields. Within each cell several pressure components (both perpendicular and parallel) can be specified which will satisfy pressure balance equations and, together with the vacuum B fields, will generate the self-consistent finite-β B fields. Densities and potentials are defined by analytic functions (Appendix A). The potentials are not self-consistent with the equilibria.

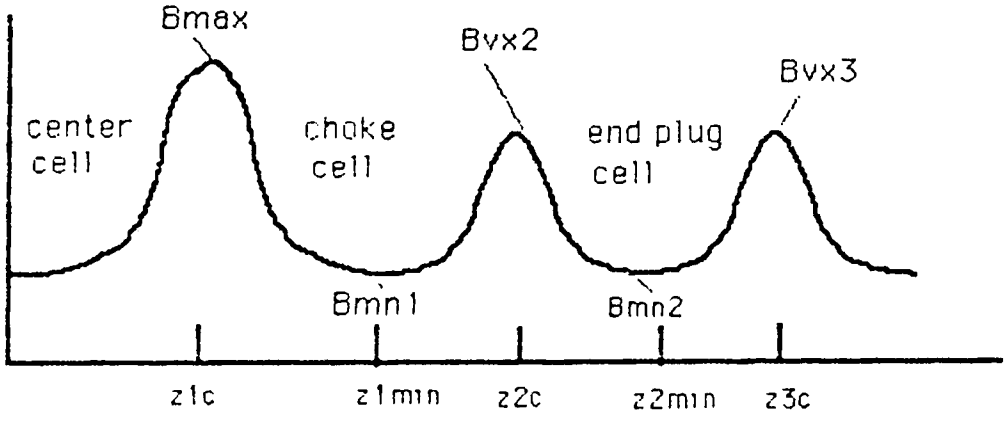
3 VACUUM B FIELDS

The vacuum B at any point in space is

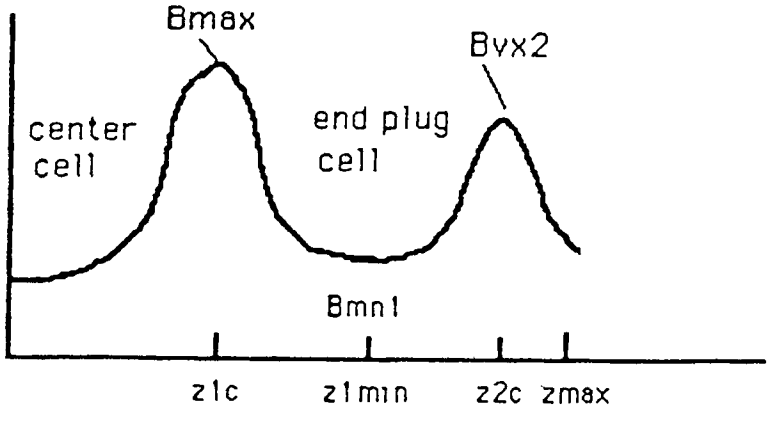
$$BVAC(z) = \sum_{s=1}^{NCOIL} b_s(z_{sc}, z) + BCENTER(z)$$

(independent of ψ because of the paraxial model) Ncoil is 2 or 3. b_s is th on-axis B_z field of a solenoid located at z_{sc} (Appendix A). BCENTER is a constant from $z = 0$ to a specified transition region ($z = ztrans$) beyond which it rapidly falls away (see Appendix A). It is designed to represent the center cell vacuum magnetic field.

For a 3-region case ($NCOIL = 3$)



For a 2-region case ($NCOIL = 2$)



Note:

1. BMAX, BVX2, BVX3, BV0 are the resultant values due to all the sources present.

2. Each solenoid is specified by 4 input parameters:

   B field strength (Gauss)

   axial length (cm)

   radius (cm)

   z location of center (cm) (z_{1c},z_{2c},z_{3c})

The magnetic field strength input is actually the desired total vacuum magnetic field at the center of each solenoid on axis, excluding the center cell field .

4. PRESSURE

The general form of the perpendicular pressure is

$$p_\perp(z,\psi) = \sum_s (a_s\, B^4 + b_s\, B^2 + c_s) p'_{\perp s}(\psi) + p_{\perp e}$$

where $p_{\perp e}$ is due to hot electrons and the sum is over all other species. This is solved together with the perpendicular pressure balance equation,

$$B^2(z,\psi) + 2\, p_\perp(z,\psi) = BVAC^2(z)$$

Then the parallel pressure is obtained from the parallel pressure balance equation,

$$p_\perp = -B^2 \frac{d\,(p_\parallel / B)}{dB}$$

or

$$p_\parallel(\psi, B) = \sum_s p'_{\perp s}(\psi) \left(-\frac{a_s}{3} B^4 - b_s\, B^2 + c_s + d_s\, B \right)$$

The coefficients a_s, b_s c_s and d_s are calculated from the conditions for zero pressure and zero slope at appropriate axial positions.

In addition, in the center cell there can be a z-independent pressure component (ppas1) with $p_\perp = p_\parallel = constant$ with respect to B. In the outer cells the passing component has B dependence.

The passing and trapped groups will be described separately, following a brief description of the hot-electron pressure.

5. HOT ELECTRON PRESSURE

The perpendicular hot electron pressure,

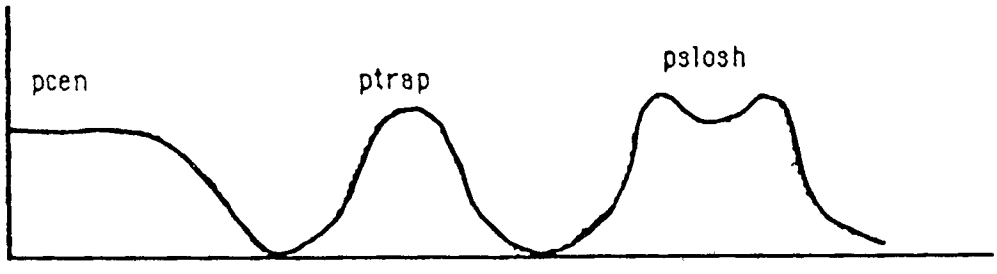
$$p_{\perp e} = \left(a_e\, B^4 + b_e\, B^2 + c_e\right) p'_e(\psi)$$

is separated from the other species in order to properly treat it as a "stiff" component in the manner of the rigid Elmo Bumpy Torus model[3,4]. It is included in the total perpendicular pressure only when calculating the magnetic B fields. It is not included in other pressure-dependent equilibrium quantities, for example $Q(= B^2 + p_\perp - p_\parallel)$. It is therefore not dynamically included as a source of instability. In addition, the hot-electron density is assumed to be negligible, compared to the warm electron and ion densities, in order to satisfy charge neutrality. Note that this does not imply a constraint on the hot electron pressure.

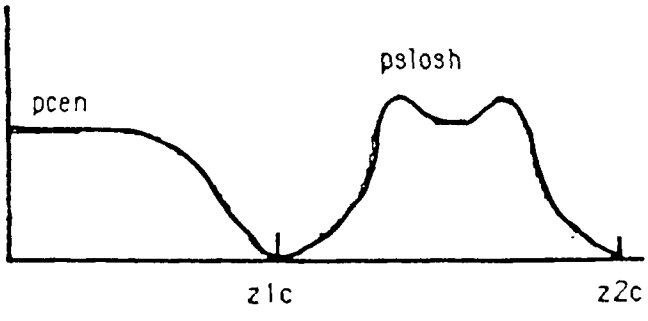
6. TRAPPED PRESSURE

Each cell can contain a trapped species whose $p_\perp$ peaks at the B field minimum and goes to zero with zero slope at the cell limits. In the case of unequal magnetic mirror peaks the smaller magnetic field peak determines the maximum magnetic field beyond which the pressure is zero. In the plug cell a sloshing profile is constructed from the difference of two trapped profiles. A hot electron pressure can exist in the cell adjoining the central cell(i.e. the choke cell in the three region case, the end plug cell in the two region case). A code option permits the axial profile of the hot electrons to be elongated with a region of constant pressure.

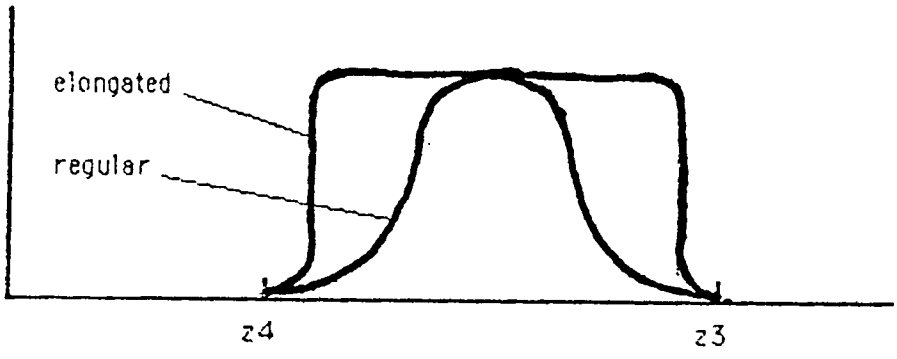
3 region case



2 region case



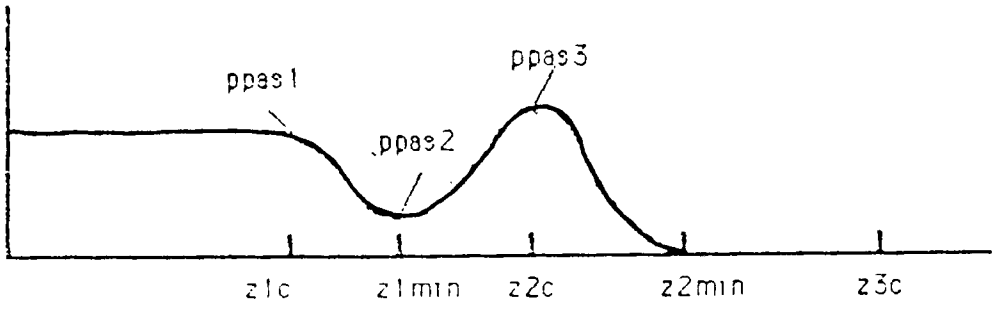
Hot electron pressure



7. PASSING PRESSURE

For the three region case only, a passing pressure can exist which is constant in the center cell, minimizes at the minimum B field of the choke cell, peaks at the entrance to the end plug cell and goes to zero at the minimum B field of the end plug cell. In contrast to the trapped pressures, the passing pressures in all cells are related and the input for the pressure in the choke and end plug cells are expressed as fractions of the center cell passing pressure.

Passing pressure



8. RADIAL (ψ) PRESSURE PROFILES

All ions and warm electrons have the same $p_s'(\psi)$. Hot electrons have a separate $p_e'(\psi)$. For ions the form is

$$p_2(\psi) = p_{2t}(\psi) - p_3(\psi)\ dip$$

$$p_{2t}(\psi) = \frac{1}{2}\left[1 - \tanh\left\{\frac{2\left(\frac{\psi}{\psi_0} - 1\right)}{p2wide}\right\}\right]$$

$$p_3(\psi) = p3a + p3b\ \psi + p3c\ \psi^2 + p3d\ \psi^3$$

This allows hollow profiles The constants p3a,...p3d are calculated such that $p_2(\psi)$ has a maximum at $\psi = \psi_0\ (1 - p2wide)$ and $p_3(\psi)$ goes to zero at $\psi = \psi_0$. Setting the input quantity *dip* to zero removes the hollowness.

Hot electrons have a p_1' of the form

$$p_1(\psi) = \begin{cases} p_{e1}, & \text{if } 0 \le \psi \le \psi_{me} \\ p_{e2}, & \text{if } \psi_{me} \le \psi \le \psi_{max} \end{cases}$$

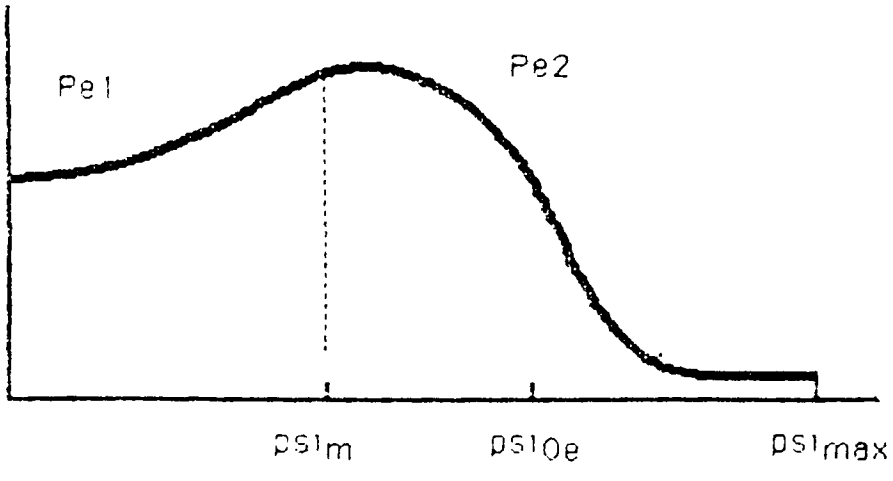
where

$$p_{e1} = p_{e10} + be1\ \frac{\psi}{\psi_{me}} + ce1\ \left(\frac{\psi}{\psi_{me}}\right)^2 + de1\ \left(\frac{\psi}{\psi_{me}}\right)^3$$

$$p_{2e} = \frac{1}{2}\left[1 - \tanh\left\{\frac{2\left(\frac{\psi}{\psi_{0e}} - 1\right)}{p2ewide}\right\}\right]$$

p_{e1} and p_{e2} are matched to give continuous pressure and slope at ψ_{me}. p2ewide is an input and $\psi_{me} = \psi_{0e}(1 - p2ewide)$. p_{e10} (an input) sets the normalized value of p_{e1} at $\psi = 0$ and is designed to adjust the profile from disk-shaped ($p_{e10} = 1$) to ring-shaped ($p_{e10} = 0$)

$p_1(\psi)$ vs ψ



9. MASS DENSITY, ρ

The general form for each componenet s is

$$\rho_s = \left[p'_{\perp s}(\psi)\left(e_s\, B^4 + f_s\, B^2 + g_s\right)\right] a_{mass}$$

The coefficients e_s, f_s, g_s are are based on the assumption that each density component is proportional to its related perpendicular pressure with the proportionality constant determined from input data. The total mass density is

$$\rho = \sum_s \rho_s + \rho_0$$

where ρ_0 is a constant calculated from input data. ($\rho_0 = ncenter \times cold \times a_{mass}$ see Appendix B)

10. GRID STRETCHING

Nonuniformly spaced physical grids in (z,ψ) are analytically mapped onto a uniform computational grid (u,v) to improve numerical accurracy in the finite differencing scheme. The mappings have the form

$$z = zmax^{1-xu}\, u^{xu}$$
$$\psi = \psi_{max}^{1-xv}\, v^{xv}$$

where

$$xu = \frac{\ln fz}{\ln fu}$$
$$xv = \frac{\ln f\psi}{\ln fv}$$

fz, fu, $f\psi$, fv are input data within the range 0 to 1 These relations have the properties;

$$zmax \equiv umax$$
$$\psi_{max} \equiv vmax$$

The z value, $z = fz \times zmax$, maps to the u value, $u = fu \times umax$;and similarly for ψ and v

For example, if $fz = .5$ and $fu = .7$ then the inner 50% of the physical space will be represented by 70% of the computational points, with the concentration of points increasing at smaller values.

11. ENERGY CHECK

The energy constant $H^{(2)}$ (eq. 3, ref. 2) is calculated by a two- dimensional numerical integration over the total (z,ψ) space. In FLORA variables,

$$H^{(2)} = \int \frac{d\psi\, dz}{B}(kinetic + linebending + curvature + flr)$$

where

$$kinetic = \rho\left[\underline{XRO}_t^2 + \underline{XIO}_t^2\right] + \rho\left(\frac{r\,B}{m}\right)^2\left[(r\,\underline{XRO})^2_{\psi\,t} + (r\,\underline{XIO})^2_{\psi\,t}\right]$$

$$linebending = \frac{Q}{r^2B^2}\left[(r\,B\,\underline{XRO})_z^2 + (r\,B\,\underline{XIO})_z^2\right] + \frac{Q\,r^2}{m^2}\left(\left[B\,(r\,\underline{XRO})_\psi\right]_z^2 + \left[B\,(r\,\underline{XIO})_\psi\right]_z^2\right)$$

$$curvature = \left[-\left(p_\perp + p_\parallel\right)_\psi B\,r\,r_{zz} + r^2\,(p_{\perp e})_\psi\,(p_\perp)_\psi\right]\left(\underline{XRO}^2 + \underline{XIO}^2\right)$$

$$flr = \underline{YYY}\left\{(m^2-1)\left(\underline{XRO}^2 + \underline{XIO}^2\right) + \left[\underline{XRO} - r\,B\,(r\,\underline{XRO})_\psi\right]^2 + \left[\underline{XIO} - r\,B\,(r\,\underline{XIO})_\psi\right]^2\right\}$$

$\underline{XRO}$ and $\underline{XIO}$ are the real and imaginary parts of the perturbation, $\underline{YYY}$ is the quasi-elastic term, $Q = B^2 + p_\perp - p_\parallel$, m is the azimuthal mode number and subscripted quantities are derivatives with respect to the subscript (e.g. $\underline{XRO}_{\,t} \equiv \frac{d\underline{XRO}}{dt}$).

The accurracy of this check is limited because the derivatives are calculated by finite differences. If the relative energy is defined as:

$$Rel = \frac{\int kinetic + \int potential}{|\int kinetic| + |\int potential|} \quad ,$$

where $potential = linebending + curvature + flr$, then typical results have $Rel \approx$ a few percent . The relative energy error can be significantly worse in extreme cases of very low beta, for which the line- bending terms involve the products of a relatively large quantity Q and very small quantities, the z derivatives of the flute-mode amplitudes.

12. INPUT DATA

Apendix B is a list of the input data. All input is in format-free Namelist mode, and is echoed to the output one-dimensional plot file. Most input is Data loaded with default values, located mainly in subroutines Input and Inputtm. The input file name should be INFLM8, or else the code execution line must be extended to account for a different name on the data file.

13. CODE EXECUTION

At execution time the user's private file list must contain the controllee XFLM8 and a data input file. To execute XFLM8, simply type its name if the input file is INFLM8. Otherwise type

$$XFLM8 \quad INFLM8 = inputfile$$

where *inputfile* stands for the name of the input file.

After completion of the run there will be two new plot files in the users private file list unless one or both are suppressed by input data (both files have names that begin with F3).

Both the executable file XFLM8 and the fortran source file FLRM8 can be obtained from Filem storage by typing

FILEM READ 326 *.HANDOUT XFLM8 FLRM8*

14. SOME OBSERVATIONS BASED ON EXPERIENCE

The equilibrium calculations are relatively inexpensive in time (a few seconds for meshes of ≈2500 points),so an obvious strategy is to optimize the equilibrium as much as possible by a number of short runs of one or two time steps before doing a full stability run of typically a few hundred time steps. The equilibrium runs can be further economized by turning off the 3-D plots whenever possible (NO3D=1) .

The diagnostic 1-D plot of FLUTE3 is very useful, even for low-m, high-β cases. FLUTE3 is the line-average of the square of the curvature-driven MHD growth rate in the limit of high m and low β, usually referred to as γ^2_{mhd}. Any design changes (e.g. pressure profiles, e-ring positions,etc.) which reduce FLUTE3 move the system toward stability. Note that even with regions of positive FLUTE3, it is possible that the system is stable due to flr and wall effects.

To avoid numerical instabilities or intolerable inaccuracies, the time step dt must be constrained. A conservative first guess is to satisfy the conditions

$$\omega_{flr}\ dt \leq .1$$
$$(FLUTE3)^{\frac{1}{2}}\ dt \leq .1$$

ω_{flr} is the real frequency due to the flr gyroscopic terms in the Lagrangian (ref 1, 2, Appendix A). If the flr term XXX is turned off ($sf6 = 0$) one of the constraints on dt is relaxed. If YYY is also turned off ($sf8 = 0$) the system of P.D.E's is decoupled and the iterations can be dispensed with ($LMAX = 0$). In the general coupled case $LMAX = 4$ has usually been required to insure numerical convergence

15. ACKNOWLEDGMENTS

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APPENDIX A

A.1 PRESSURE NORMALIZATION

Input quantities are beta's which are converted to pressures as follows:

$$pcenter = betcent * \left(\frac{BV0^2}{2}\right) \quad \ldots \text{(ions)}$$

$$pcentee = betcene * \left(\frac{BV0^2}{2}\right) \quad \ldots \text{(warm electrons)}$$

$$ppas1 = betpas1 * \left(\frac{BV0^2}{2}\right) \quad \ldots \text{(sum over species)}$$

$$p1trap = betrap * \left(\frac{BMN1^2}{2}\right) \quad \ldots \text{(sum over species)}$$

$$pslosh = betslsh * \left(\frac{BMN2^2}{2}\right) \quad \ldots \text{(ions)}$$

$$psloshe = betslse * \left(\frac{BMN2^2}{2}\right) \quad \ldots \text{(warm electrons)}$$

ppas2 and ppas3 are inputed as fractions of betpas1 .

A.2 EQUATIONS FOR a, b, c, OF PRESSURE FORMULAS

Define a function $H(z_1, z_2)$as

$$H(z_1, z_2) = \begin{cases} 1, & \text{if } z_1 \le z \le z_2 \ , \\ 0, & \text{otherwise.} \end{cases}$$

Then

$$\begin{aligned} pperp(B) &= pcen(B) + ppas1(B) + ppas2(B) + ppas3(B) + ptrap(B) + pslosh(B) \\ &= abp\ B^4 + bbp\ B^2 + cbp \end{aligned}$$

$$abp = \sum_{i=1}^{6} a_i$$

$$bbp = \sum_{i=1}^{6} b_i$$

$$cbp = \sum_{i=1}^{6} c_i$$

$$a_1 = \frac{pcenter + pcentee}{[1 - (BV0/BMAX)^2]^2\ BMAX^4} H(0, z_{1c})$$

$$a_2 = \frac{ptrap}{\left[1-(BMN1/BVX2)^2\right]^2 BVX2^4} H(z_{1c}, z_{2c})$$

$$a_3 = pslosht \left\{ \frac{H(z_{2c}, z_{3c})}{\left[1-\frac{BMN2}{BVX3}\right)^2\right]^2 BVX3^4} - \alpha \frac{H(z_2, z_3)}{\left[1-\frac{BMN2}{BM1}\right)^2\right]^2 BM1^4} \right\}$$

$$a_4 = -\frac{(ppas1 - ppas2)}{\left(BMAX^2 - BMN1^2\right)^2} H(z_{1c}, z_{mn1})$$

$$a_5 = \frac{b_4\left[1-(BMN1/BVX2)^2\right] - b_5'}{2\,BMN1^2} H(z_{mn1}, z_{2c})$$

$$a_6 = \frac{-ppas3 + \; 2\,a_5'\,BVX2^2 + b_5')\,\left(BVX2^2 - BMN2^2\right)}{\left(BVX2^2 - BMN2^2\right)^2} H(z_{2c}, z_{mn3})$$

$$b_1 = -2\,a_1\,BMAX^2$$

$$b_2 = -2\,a_2\,BVX2^2$$

$$b_3 = -2\,a_3\,BVX3^2$$

$$b_4 = \frac{(ppas1 - ppas2)\,2\,(BMAX)^2}{\left(BMAX^2 - BMN1^2\right)^2} H(z_{1c}, z_{mn1})$$

$$b_5 = \frac{(ppas3 - ppas2) - .5\,\alpha_1\left(BVX2^4 - BMN1^4\right)}{BVX2^2 - BMN1^2 - .5\left(BVX2^4 - BMN1^4\right)/BMN1^2} H(z_{mn1}, z_{2c})$$

$$b_6 = 5\left(\alpha_1 - 4\,BVX2^2\,a_6'\right) H(z_{2c}, z_{mn2})$$

$$c_1 = a_1\,BMAX^4 + ppas1\;H(0, z_{1c})$$

$$c_2 = a_2\,BVX2^4$$

$$c_3 = a_3\,BVX3^4$$

$$c_4 = \left[ppas1 - \frac{(ppas1 - ppas2)\,BMAX^4}{\left(BMAX^2 - BMN1^2\right)^2}\right] H(z_{1c}, z_{mn1})$$

$$c_5 = \left(ppas2 - a_5'\,BMN1^4 - b_5'\,BMN1^2\right) H(z_{mn1}, z_{2c})$$

$$c_6 = \left[-a_6\,BMN2^4 - \frac{BMN2^2}{2}\left(\alpha_1 - 4\,BVX2^2\,a_6'\right)\right] H(z_{2c}, z_{mn2})$$

$$\alpha_1 = 4\,a_5'\,BMX2^2 + 2\,b_5'$$

$$pslosht = pslosh + psloshe$$

Note: a_5'means a_5 without H. i.e. $a_5 = a_5'\;H(z_{mn1}, z_{2c})$. Likewise for b' and c'

A.3 HOT ELECTRON PRESSURE

For $long = 0$

$$pperpe(B) = abf\;B^4 + bbf\;B^2 + cbf$$

$$abf = \frac{pring}{\left[1 - \left(\frac{BMN2}{BV3}\right)^2\right]^2 BV3^4} H(z_3, z_4)$$

$$bbf = -2\ abf\ BV3^2$$

$$cbf = abf\ BV3^4$$

for long=1 and

$$BVAC > fring\ BVX2 = B^*$$

$$abf = \frac{-pring}{(BVX2^2 - B^{*2})^2} H(z_3, z_4)$$

$$bbf = -2\ abf\ B^{*2}$$

$$cbf = \frac{pring\ BVX2^2}{(BVX2^2 - B^{*2})} \left(BVX2^2 - 2\ B^{*2}\right) H(z_3, z_4)$$

for long=1 and

$$BVAC < B^*$$

$$abf = 0$$

$$bbf = 0$$

$$cbf = pring\ H(z_3, z_4)$$

A.4 SOLENOIDAL VACUUM B FIELD

$$b_s(z) = \frac{K_s}{A_s} \left\{ \frac{z_{sc} + \frac{AL_s}{2} - z}{\left[A_s^2 + \left(z_{sc} + \frac{AL_s}{2} - z\right)^2\right]^{\frac{1}{2}}} - \frac{z_{sc} - \frac{AL_s}{2} - z}{\left[A_s^2 + \left(z_{sc} - \frac{AL_s}{2} - z\right)^2\right]^{\frac{1}{2}}} \right\}$$

$$s = 1, NCOIL$$

FLORA determines K_s for each coil by simultaneous solution of this equation at $z = z_{1c}, z_{2c}, z_{3c}$ for $NCOIL = 3$ or $z = z_{1c}, z_{2c}$ for $NCOIL = 2$ given the desired vacuum magnetic field amplitudes as input.

A.5 CENTER CELL VACUUM B FIELD

$$BCENTER(z) = \begin{cases} bceng, & \text{if } z \leq z_{trans}\ ; \\ bceng\ \exp^{\frac{z_{trans} - z}{l_{trans}}}, & \text{otherwise.} \end{cases}$$

A.6 SELF-CONSISTENT B FIELD

Define

$$U1 = P2(\psi)\ abp + P1(\psi)\ abf$$

$$U2 = P2(\psi)\ bbp + P1(\psi)\ bbf + \frac{1}{2}$$

$$U3 = P2(\psi)\ cbp + P1(\psi)\ cbf - \frac{BVAC(z)^2}{2}$$

then

$$B(z,\psi) = \left\{ \frac{-U2}{2\,U1} \pm \left[\left(\frac{U2}{2\,U1} \right)^2 - \frac{U3}{U1} \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}}$$

For very low pressures

$$B(z,\psi) = BVAC(z)^2 - 2\,U3' - 2\,U2'\ BVAC(z)^2 - 2\,U1\ BVAC(z)^4 + f(U1^2)$$

where

$$f(U1^2) = 4\,U3'\,U2' + 8\,U1\,U3'\ BVAC(z)^2 + 4\,U2'^2\ BVAC(z)^2 + 12\,U1\,U2'\ BVAC(z)^4 + 8\,U1^2\ BVAC(z)^6$$

$$U2' = U2 - \frac{1}{2}$$

$$U3' = U3 + \frac{BVAC(z)^2}{2}$$

A.7 ELECTRIC POTENTIAL

$$\phi(z,\psi) = \phi_1(z)\ \phi_2(\psi)$$

where

$$\phi_1(z) = phice\ \exp^{arg1} + \frac{phipl}{\cosh(arg2)}$$

$$\phi_2(\psi) = \left(1 - \frac{\psi}{\psi_3}\right)^{ypot} H(0,\psi_3)$$

$$arg1 = -xpot \left(\frac{z - z_1}{z_1 - z_0} \right)^2 (1 - H(0, z_{1c}))$$

$$arg2 = \left(\frac{z - z_0}{z_0 - z_2} \right)^2 wpot$$

A.8 FINITE LARMOR RADIUS TERMS

$$XXX(z,\psi) = \rho(z,\psi)\,(2\omega_{E\times B} + \omega_{\nabla B} + \omega^*)\;\; sf6$$
$$YYY(z,\psi) = -\rho(z,\psi)\,(\omega_{E\times B} + \omega_{\nabla B})\,(\omega_{E\times B} + \omega^*)\;\; sf8$$

where

$$\omega_{E\times B} = c\frac{\partial\phi}{\partial\psi}$$
$$\omega_{\nabla B} = \frac{P_{\perp i}\,(z,\psi)}{\omega_{ci}\;\rho\,(z,\psi)}\frac{\partial B}{\partial\psi}$$
$$\omega^* = \frac{B\,(z,\psi)}{\omega_{ci}\;\rho\,(z,\psi)}\frac{\partial P_{\perp i}}{\partial\psi}$$

$\rho(z,\psi)$ is the ion mass density,$P_{\perp i}$ is the ion perpendicular pressure, ω_{ci} is the ion larmor radius and c is the speed of light.

A.9 THE EQUATION OF MOTION FOR THE PERTURBED RADIAL DISPLACEMENT, χ

The equation of motion for the perturbed radial displacement is

$$\left(\rho\,T\,r^4 B\chi_\psi\right)_\psi + \left(1-m^2\right)\frac{\rho}{B}T\,\chi$$
$$-r^2\rho_\psi\chi_{tt} - m^2\,r_{zz}r\,(p_\perp + p_\parallel)_\psi\,\chi$$
$$-m^2 r\,B\left[\frac{Q}{r^2B^3}\,(r\,B\,\chi)_z\right]_z + r\left\{B\left[\frac{Q}{B}r^2\left(B\,(r\,\chi)_\psi\right)_z\right]_z\right\}_\psi = 0$$

where $\rho\,T = -\rho\frac{d^2}{dt^2} - \imath\,m\,XXX\frac{d}{dt} - m^2YYY$ subscripts mean derivatives, and all coefficients are presumed to depend on z and ψ.

APPENDIX B

The input data is entered via four namelists. Within each namelist the order is arbitrary. Default values are preset for most data (see the code listing).

Namelist 1

input	description
no1d	1[0] turns off[on] 1-D plots
no3d	1[0] turns off[on] 3-D plots

Namelist 2

input	description
aname	problem identification , up to 5 fields of 8 letters each
bias	Time centering parameter. bias=0. for fully centered bias=1. for fully forward bias.
ex0	initial perturbation coefficient, set = 1 for random initialization.
ex1	initial perturbation coefficient, set = 1 for cosine initialization.
fi1	minimum z boundary condition, set = 1. for 0 slope, set=-1. for 0 value
fizx	maximum z boundary condition, set = 1. for 0 slope, set=-1. for 0 value
fj1	minimum psi boundary condition. set =1. for 0 slope, set=-1. for 0 value For fj1=0., mm=1 results in 0 slope, mm $\geq$ 2 results in 0 value
fjrx	maximum psi boundary condition, set =1. for 0 slope, set =-1. for 0 value
fpsi	grid stretching parameter (see sec. 9). default = .5
fu	grid stretching parameter (see sec. 9). default = .5
fv	grid stretching parameter (see sec. 9). default = .5
fz	grid stretching parameter (see sec. 9). default = .5
jfour	ψ index at which XROis Fourier-analyzed in z
kplotm	index of spatial location of time history plots. If set = 0, center of region automatically chosen.
kzs	flute mode initialization. kzs = 1 ,ex0 =1., ex1=0. sets initial condition, r B XRO= 0.,and r B XIO= 0.
lmax	iteration parameter
mm	azimuthal mode number
ndiag	number of time steps between diagnositc plots
nen	number of time steps between energy checks

Namelist 2 continued

input	description
nfourmax	no. of times the buffer is read to the history file for Fourier analysis
nfourp	Fourier analyze XROevery nfourp'th time step
nmax	total number of time steps for problem
sf6	arbitrary scaling factor on the gyroscopic flr term XXX
sf8	arbitrary scaling factor on the quasi-elastic flr term YYY
swg1	arbitrary scaling factor on the curvature-drive term default=1
swg2	arbitrary scaling factor on some of the line-bending terms default = 1
swg3	arbitrary scaling factor on some of the line-bending terms default = 1
swg4 .	arbitrary scaling factor on some of the line-bending terms default = 1

Namelist 3

input	description
bceng	center cell magnetic field in Gauss
betcene	peak center cell electron $\beta_\perp$
betcent	peak centew cell ion $\beta_\perp$
betpasl	center cell passing β
betrap	peak choke cell $\beta_\perp$
betring	peak hot electron $\beta_\perp$
betslse	peak warm sloshing electron $\beta_\perp$
betslsh	peak sloshing ion $\beta_\perp$
cold	a global density minimum as a fraction of ncenter, the center cell density
dip	parameter in ψ pressure profile (sec. 8)
dpasl	center cell passing density
dpsihrel	ψ width relative to ψ_{max} of transition to halo region
dltrap	peak density in choke cell
echarg	ion charge . Default=4.8e-10
fring	used for elongated hot electrons. See sec A.3
long	switch which sets hot electron z-length as elongated (long=1) or regular (long=0)
ncenter	center cell density (particles/cm^3)
nsloshin	peak plug cell density (particles/cm^3)

Namelist 3 continued

input	description
pe10	coefficient of hot electron radial pressure profile (sec. 8)
ppas2	minimum passing pressure in the choke cell, expressed as a fraction of ppas1 (sec. 7)
ppas3	maximum passing pressure at the inboard mirror of the end plug cell
psi0rel	ψ value relative to ψ_{max} at which ion radial pressure is half the maximum.
psi0erel	ψ value relative to ψ_{max} at which hot electron radial pressure is half the maximum.
psihrrel	ψ value relative to ψ_{max} of halo.
psislp	coefficient of p_{e1} (sec. 8)
psi3rel	ψ value relative to ψ_{max} beyond which electric field=0
p2wide	parameter inversly proportional to ramp width of p_{2t} the ion radial pressure profile (sec. 8)
p2ewide	parameter inversly proportional to ramp width of p_{2e} the hot electron radial pressure profile (sec. 8)
p1floor	value to which p_1 is set if $p_2 \leq p2flag$
p2flag	value of p_2 at which p_1 and p_2 are set constant
p2floor	value to which p_2 is set if $p_2 \leq p2flag$
rp1	mirror ratio of the inner component of the sloshing profile (sec. 6)
rw	wall radius in cm.
rw1	slightly less , within one grid cell, or equal to wall radius rw
wpot	exponent coefficient in end plug cell potential
xpot	exponent coefficient in center cell potential
ypot	power of polynomial in potential ψ dependence
z3rel	outer axial position where hot electrons go to 0

Namelist 4

input	description
als	three element array of z location of each solenoid center
as	three element array of radius of each solenoid
bmx1	magnetic field at the choke coil solenoid (Gauss), (sec. 3)
bmx2	magnetic field at the inboard end plug solenoid, (Gauss),(sec. 3)

Namelist 4 continued

input	description
bmx3	magnetic field at the outer end plug solenoid, (Gauss),(sec. 3)
bceng	magnetic field in center cell (Gauss)
dt	time step (sec.)
dphi	ignore
epsp	$\approx$ minimum pressure (normalized to 1), below which B is calculated by an expansion . see sec A.4
kin	number of points used in Simpson's quadrature for r_{ZZ} .default=23
ltrans	transition length for central cell vacuum
ncoil	number of solenoid coils (also regions)
phicen	maximum electric potential in center cell
phiplg	maximum electric potential in end plug cell
pfudge	parameter on sloshing shape. default=0.
theta0	ignore
zmax	maximum z of the domain
z1c	z location of choke solenoid
z2c	z location of end plug inboard solenoid
z3c	z location of end plug outer solenoid

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