Memorial University of Newfoundland

Department of Mathematics and Statistics

# Applied Mathematics 2130

## Technical Writing in Mathematics

Course Outline and Manual

The revised Winter 2009 edition was prepared by Sergey Sadov in collaboration with Danny Dyer and Ivan Booth and with the technical assistance of Karen Williams and Melissa Roberts. Changes in the Fall 2009 edition include Section 2.4 and Index added by Sergey Sadov and numerous small corrections.

Previous contributors to this Manual include the Instructors of Applied Mathematics 2130 from the Department of Mathematics and Statistics at Memorial University of Newfoundland, 1994–2007. As well, there are technical contributions from the systems personnel at the Department of Computer Science, and external contributions (reproduced with permission) from David Goss (Ohio State University), Steven Kleiman (MIT), Gavin Maltby (University of Natal), and Glenn Tesler (MIT).

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# Contents

# Chapter 1

# Introduction

## 1.1 The course and this Manual

The purpose of this course is to teach technical writing. You will learn about typical requirements for research and technical papers and about some computer typesetting and graphical tools used to produce technical reports of professional quality.

You will be offered four projects (laboratories) to work on. In those projects you will have to carry out a mathematical investigation of the given problem or situation, to perform computer simulations, to produce illustrations, and to write a report.

A schedule for the four projects will be handed out on the first day of classes along with a description of the first laboratory. Laboratories 2 to 4 will be made available as the course progresses. All documents related to the course (including the most recent version of this Manual) will be available on the course web page

`http://www.math.mun.ca/~m2130`

Three major topics that you will be mastering in this course and the corresponding chapters in this Manual are:

- Composition of technical, mathematics-intense papers — Ch. 2;
- LATEX typesetting system (LaTeX) — Ch. 3;
- Computer programming and computer-generated graphics — Ch. 4.

An attempt has been made in this Manual to isolate the discussion in Chapters 2–4 from particulars of the computing environment. Chapter 5 provides details about computer facilities on campus available to Math 2130 students and about software pertaining to this course.

## 1.2 Submissions

Students are required to submit a neatly stapled **printed copy** of the report and also to submit all reports **electronically**.

Section 5.1 explains the purpose and procedure of electronic submission. The electronic submission must contain a master LaTeX file and all files that the master file refers to (in most cases, these will be `eps` graphics files). In addition, the electronic submission must include computer code(s) written to produce the reported results.

Two sections in this Manual specifically deal with **report format**.

Section 2.3 contains recommendations about a **logical structure** and size of the reports.

Section 3.2 describes the standard report **layout** and LaTeX commands used to produce it.

## 1.3 Policies

### 1.3.1 Evaluation

Grades in the course are based on four projects each requiring a written report submitted in printed form and electronically. There is no final examination. The first three reports will be returned to the student, while the final one will be retained by the instructor.

The typical weights of the reports are 15 marks, 25 marks, 30 marks and 30 marks respectively. However, your instructor's first day handout takes priority in regard to the method of evaluation.

The following two paragraphs apply to Labs 1 to 3.

Late submissions are subject to penalty. A submission within a week past the due date will result in a deduction of 5 marks. Thus a second lab worth 22 out of 25 marks will receive a final grade of 17 out of 25 if it is just one day late. Further delays result in a deduction of 5 marks per week of lateness.

Within a week or two following the submission date you will be asked to **meet with your instructor** to go over your paper. At the meeting, the instructor will suggest possible improvements in the paper, while you must be prepared to explain mathematical details, the workings of a computer program, sources of information, collaboration, etc. The results of such interviews can affect your grade on the project.

The **evaluation criteria** for the projects address quality of contents and presentation. But before anything else the instructor will check whether you have **completed the assigned task**. In a laboratory that asks to write a computer program that does so and so, neither an amusing narration and fancy graphics nor five pages of mathematical definitions and theorems will help if your program doesn't work or doesn't solve the problem as required.

As long as that principal condition is met, further criteria pertaining to **contents** typically include the following:

- Usefulness of the paper (relevance, informativeness, mathematical and factual correctness);
- Research quality (understanding of underlying mathematics, appropriateness and effectiveness of tools used, scope and depth of analysis);
- Quality of computer programs supporting the research (validity of code, efficiency of algorithm, readability — structure, comments, self-explanatory identifiers, etc.) and explanation of the program's workings.

Depending on the nature of a problem at hand, the relative importance of the listed elements may vary and other elements may be emphasized. If in doubt, ask your instructor what to pay attention to.

The criteria pertaining to **presentation** are very similar to those used in non-technical writing:

- Quality of exposition (structure, style, level appropriate to the assumed readership, clarity with which technical ideas are explained, consistent use of terminology and notation);
- Conformance to language standards (grammar, spelling);
- Conformance to typographical standards (LaTeX typesetting, quality of graphics);
- Proper citations and quotations.

Chapters 2–4 elaborate on many of these points.

This course gives you an opportunity to put the skills you acquired in other courses to work. Some students would try to excuse themselves for spelling and grammar errors saying that this is not a course in English; others with poor knowledge of programming would complain that creating a correctly working program carries so much weight. Such excuses and complaints will be rightfully dismissed by the instructor. Also it is very normal in this course to learn chunks of mathematics on the fly. Thus, if a project asks you to simulate a dynamics described by differential equations and you have not taken Math 3260, just look up a few relevant facts!

### 1.3.2 Academic integrity and academic misconduct

Academic integrity means honesty and courtesy in your course work and research. The opposite is academic misconduct. In our experience, situations occur in this course on a regular basis where students are at risk of violating academic integrity in the following ways:

- forging research results;
- plagiarizing.

### A. Forging research results

A graph downloaded from the Internet and presented as the output of a student's own program is an example of a forged research result. But forging does not necessarily involve someone else's results; it can also occur as entirely one's own activity. If a student's program does not solve an equation as expected and the student decides to "correct" the program's output by hand, hoping to fool the instructor, that's a forge.

Sometimes the borderline between an involuntary mistake and a deliberate forge is shaky. An argument that pretends to be a mathematical proof but fails to be such due to a logical error can be treated as a forge if there is an evidence that the author has been aware of the error and has chosen to disregard it.

If you think that something goes wrong in your project, you should consult with your professor or laboratory assistant at the earliest opportunity. Their advice will likely get you on track. Yet quite a few students find themselves in a situation where the assignment is due the next day and things do not work their way. What should they do?

Desperately filling up pages with material that is not supported by your actual findings is a bad idea. One solution is to **to buy additional time** at the expense of losing 5 marks as allowed by the evaluation policy (Sect. 1.3.1). Another possibility is to **frankly admit** a problem and describe your approach in **as much detail as possible**. If you feel that your method/program is sound but perhaps some detail escaping your view prevents it from yielding satisfactory results, report the research as is. Do not beg for an excuse; instead, try to present an educated guess as to where a weak link could be.

### B. Plagiarism

We urge all students to familiarize themselves with Section 4.11 (Academic Misconduct) in the Memorial University Calendar:

`http://www.mun.ca/regoff/calendar/`

In particular, read carefully Section 4.11.4 (Academic Offences) which defines what plagiarism is and details the range of consequences that result from an act of plagiarism.

This section is not intended to frighten you and to discourage sharing ideas with fellow students or using available information resources. The Calendar points out that "the properly acknowledged use of sources is an accepted and important part of scholarship." Just know the limits. They are sometimes subtle. The next two sections should help you develop a better understanding of situations routinely occurring in practice.

### 1.3.3 Collaborative work

During the course, students are encouraged to work together. Feel free to trade ideas about how to approach a given project, how to write programs, how to use Maple and LaTeX, etc.

All help received should be acknowledged (see Section 2.3.8) and all sources consulted must be referenced.

However, when the time comes to prepare a report, each student will see this activity as entirely his or hers. **Each student is completely responsible for the intellectual content of his or her report** and later may be asked to explain any material contained in the report. All reports must be written by students on their own. They cannot be based on any report previously submitted for this course (say, by a sister, a friend, or a tutor) or a report being submitted concurrently by a classmate. Also, if a student is repeating the course, reports are not permitted to be on the same topics as those submitted previously.

If a student is not able to explain and/or defend the contents of a report, the grade on that report may be adjusted. If there is evidence that written material in the report has been shared, the students concerned may receive a grade of 0 on that report. In the case of a last report, the student(s) may be given an incomplete grade and be required to return to campus for a follow-up interview. Finally, if there is a repetition of this sort of activity, a grade of zero in the course will be given.

### 1.3.4 Use of online materials

The Internet as a source of information can be great if used diligently.

Proper acknowledgements must be made to all resources cited.

Another thing to keep in mind is that for the purposes of this course "research" does not mean googling whatever (hopefully relevant) "stuff" is out there and copying it to your paper. Not all information on the Internet is credible and correct. Also the meaning of being correct is not absolute. A definition acceptable for a Ph.D. level research monograph may be inappropriate in your paper even if it refers to the same concept. On the other hand, a definition suitable for a common language dictionary may lack significant technical details and also be inappropriate for your purposes even if it comes from a reliable source.

Do not yet discard printed books, in particular, textbooks used in courses. They are generally more reliable and definitive sources of information; unlike many Internet sites, they have gone through a strict review process and multiple proofreadings.

## 1.4 MUN Writing Centre

Look up the Writing Centre webpage

`http://www.mun.ca/writingcentre/about/`

and consider dropping in there some day.

Students who experience problems with their writing may find their marks dramatically different depending on whether or not they show their work to a knowledgeable writing advisor before final submission.

If you consider yourself to be a good writer, the mark improvement through the use of the Writing Centre may not be a big issue. But it's a misconception that only weaker students should seek help there. Not quite so! In fact, the better you write, the more efficient the help can be; you and your advisor can concentrate on how to make your paper really enjoyable — and perfection has no end.

Remember however that people in the Writing Centre are not supposed to understand the technical content of your paper and they may not be familiar with specific requirements, traditions and habits of mathematical exposition. Those who help you ought to get due credit, but the remaining deficiencies are yours. No one but you is ultimately responsible for everything in your paper, including style, spelling and punctuation. And your instructor will have the last say in evaluating your writing.

# Chapter 2

# Technical writing

## 2.1 Technical versus non-technical writing

Most things about writing that you learn in English courses apply equally to technical writing. This chapter does not pretend to teach you the rules of grammar, basic principles of composition and style in general. We rather focus on the elements of writing specific to this course. (Yet some common spelling errors etc. will be mentioned — see Sect. 2.4.1.)

Writing requires you to organize your flow of thought into a coherent sequence of units carrying sense. The smallest such unit is a sentence. Then comes a paragraph. A short story may contain no further structural units, while more sizeable pieces of fiction are often organized into Parts and Chapters.

Scholarly writing, as compared to fiction, is characterized by a more sophisticated **hierarchy** of logical units. Some of them, such as Sections, Subsections, References, determine the **plan** of the paper. Others, such as Definitions, Remarks, Tables, help the readers to **pause and digest** one relatively small piece of information at a time. Good graphics can be truly informative and replace a hundred words. Particular to mathematical writing are such units as Theorems and their Proofs. Common in technical reports are fragments of computer code or whole program listings. In Section 2.3 we make recommendations concerning the global structure of your Math 2130 reports. Some useful advice can be found in Appendix B-1, Section 2.

The use of language in technical writing is strongly biased towards **precision and clarity** as opposed to figurative, metaphoric language common in non-technical narration. Technical writers should, as a rule, keep a neutral tone and abstain from emotional bursts. There are also specific problems: whether to use “I” or “We” or write in the third person; what tense to use, etc. Questions of this kind are discussed in Sect. 2.4 and B-1.3.

Writing mathematics properly is a special art, which does not come easily. Section 4 in Appendix B-1 should help you get started.

## 2.2 Writing process

Before setting out to write a report, one must have *something* to report. This means, in our case, *results*: a solution to the assigned problem. Obtaining the results usually takes up most of the time students spend on their Math-2130 assignments. Remember, however: properly presenting your work in writing is not a simple task, either.

With the results obtained and the pertinent information collected, do you need anything else before starting to write? Perhaps. There are questions to answer and decisions to make:

- What is the main **purpose** of your paper? Is it to illustrate a certain mathematical theory with examples you have produced (by hand and with the help of a computer)? Or is it to report your algorithm of solution of the assigned problem and the results of computations? Is it to decipher a cryptic message? Or is it, possibly, to describe a series of computer-assisted experiments with a logical game? Knowing the purpose will prevent you from going off on tangents when writing.
- Decide about scope. **Narrow down** the subject so as to avoid excessive generality. Decide what theory, which results, how many graphs and tables you want to include. Keep focused when writing. It is better to present a small circle of ideas accurately and precisely than to attempt to embrace a larger area in vague terms.
- Decide who is your assumed **readership** is. In most cases the best assumption is that the reader is a fellow student with a background like yours. In some projects, especially those of an entertaining nature, younger math students can be the target audience. Stick with the interests and level of your readers. Do not go over their head, but do not use baby talk.

  Aiming your paper at a graduate level audience or professors will only work in exceptional cases. Never address the paper directly to your professor as if he/she were the only reader.

In summary:

*Identify the purpose. Limit the scope. Speak to your audience.*

Now we come to the main point — how to actually put things down. Writing is a creative process. One's writing strategy reflects one's personality. There is no such thing as a magic writing "algorithm" suitable for everybody. Some people write easily; most struggle. Find what works best for you, making writing less painful and more enjoyable. As food for thought, here are several possible approaches that different people use. Your writing strategy will likely be a mix of these, or possibly even entirely different.

- *Top-down approach*: Start with a plan, set the goal or goals, and proceed section by section. This approach, generally speaking, requires good self-discipline and ability to comprehend the material of the paper in its entirety; otherwise it is easy to miss important points at the outset. The suggested format of a Math-2130 paper (Section 2.3) will hopefully help. You will still need to return to sections already written and to make changes as your work progresses.

- *Bottom-up approach*: Begin by stating the results, then trace back. A method by which the results were obtained must be fully explained. Terminology involved must be defined. Elaborate, fill in details. Make sure the final order of parts makes logical sense: concepts should be introduced before they are referred to.
- *Free flow approach*: Start writing up your thoughts as they come. Write an introduction. Put down relevant definitions, facts, considerations. Describe the work done and the results obtained. State a conclusion. Then edit the obtained loose draft: organize the material into structural units, improve explanations (expand where needed), remove redundancy. In the end, rewrite the introduction anew.

No matter what approach, the chance of writing a good paper from scratch in one attempt is small. Editing will be necessary. Read what you have written. Note what sounds ugly, awkward, cumbersome. Strive for clarity. Move material around to achieve the best logical order. Replace vague phrases and words with clear-cut ones. Check your writing against this Manual; cross-read the papers with a friend; consult with the Writing Centre.

## 2.3 Organization of report

### 2.3.1 General requirements

A typical report in this course contains or may contain the following components, in this order:

- • Title page
- ○ Table of contents
- • Abstract
- • Body of report:
  - Introduction
  - Technical Details
    - Mathematical details
    - Program details
  - Results and Analysis
  - Conclusion
- ○ Acknowledgements
- ○ References
- ○ Appendix

The items marked with solid bullets are mandatory; the presence of others depends on the circumstances. We'll comment on each of them, one by one.

The body of your report (from Introduction to Conclusion), excluding in-text graphs and illustrations, should not exceed **six printed pages**. This means that you must clarify your ideas and arguments and write them in a very precise and concise way.

### 2.3.2 Title page

The title page should give the following information:

- The title and number of the lab.
- The course name and number (Applied Mathematics 2130).
- Your name and student number.
- Your professor's name.
- The date of submission.

A typical title page is shown in Figure 3.2 in Section 3.2.1, where the LaTeX code used to produce it is also given.

About the **title**: try not to repeat the title of the assignment. Devise your own way to describe the topic. The title should not be too general. For example, *Solving a Mathematical Model by Means of Computer Programming* isn't good (although it can be a title of choice for an introductory lecture in a Math Modeling course). While being short, the title should reflect particulars of the work. For example, *Two methods for evaluating the volume of a pyramid* is much better than just *Pyramids.* Word play, subtle humor, puns — these may work, but make sure you show a good taste. Some variants are totally inappropriate: like *Mathematics Supporting Global Warming.* Put *Behind* in place of *Supporting* — and the title becomes acceptable. A perfectly normal, if not at all fancy, version is *Mathematical Modeling of* . . . .

### 2.3.3 Table of contents

The Table of contents is generated automatically by LaTeX from the headings of your sections. Some extra commands are needed in order to include the References and Appendix sections. The details are given in Sect. 3.2.2.

A safe practice, in the first assignment at least, is to adhere to the suggested standard plan and headings. Then, as your experience grows, you can vary the paper structure and the headers of sections and subsections. For instance, the title *Mathematical details* can be changed into *Geometry of tangent circles*, if that's the mathematical subject dealt with. It can be further divided, if appropriate, into subsections like *Tangency condition in terms of coordinates*, *Relation between the radii of four tangent circles*, etc. The table of contents that exhibits such subject-specific and self-explanatory headers allows the reader to grasp the developments in the paper at a glance.

In short reports, the table of contents may not be needed at all. You may still be asked to include it as a typesetting exercise. Check with your instructor.

### 2.3.4 Abstract

From a reader's perspective, the abstract serves the purpose of classification: where does the paper belong? Should I take a closer look? The abstract should be short yet informative. Refer to the real-world problem or the mathematical question that gives rise to the project. Define the area of mathematics involved. Briefly characterize an algorithm and/or a program. Squeeze in the essence of results or mention a particularly striking result, perhaps schematically, in an easy-to-grasp form. Avoid extensive details.

In practice, abstracts are often restricted to a prescribed numbers words. Try to limit yours to 100 words. Learn word-saving tricks. Cross out epithets, excessive verbiage, and the obvious. There is absolutely no room for duplication, elaboration, and emotions.

**Example**. This is a bad abstract:

> **Abstract** In this paper, we discuss how to create a program that is useful for modeling global warming. Unlike in the real world, however, where water vapor and carbon dioxide both play an important role, our program makes simplifying assumption that there is only one gas responsible for the greenhouse effect, whose concentration is proportional to re-radiation of heat. Finally, results of simulations are presented.

Cut out deadwood and unnecessary elaboration, preserve the useful particulars, add some specifics on the method and results, — and a much better version is obtained:

> **Abstract** We discuss a computer simulation of global warming based on a simple mathematical model, in which one gas is responsible for the greenhouse effect and its concentration is proportional to re-radiation of heat. The program iterates over time steps, one per season. Instability of temperature is observed if the intensity of gas emission exceeds a certain critical value.

### 2.3.5 Introduction and Conclusion

If after glancing at the Abstract the readers feel the paper is not out of touch with their interests, they browse through the Introduction and Conclusion. The central part of the paper, with all the technicalities, is the last place the readers will go.

The **Introduction**, as the name suggests, should introduce the reader to the problem being investigated. Motivation and historical background can be included (although some of it can be scattered over later sections, too). The Introduction should also indicate what the reader will find in the remainder of the report. The context and language of the Introduction often makes it clear who the target audience is. Otherwise, state any special assumptions about the readers' background explicitly, e.g. "We assume the reader is familiar with eigenvalue theory for matrices."

When writing the Introduction, assume that the original assignment is not available to the reader. Your paper must be self-contained. Do not copy the assignment's language; use your own words. Some assignments might introduce a little story and characters, like Alice and Bob. In this case, again, your own Introduction must independently describe the situation — so that the reader who didn't see the assignment sheet would know what you are talking about.

The introduction can be viewed as an extended abstract, but it has a broader mission. Hook the readers; make a promise that makes them want to stay with your paper. A potential reader may never get to appreciate the rich and interesting contents if the introduction fails in its mission.

For a typical paper in this course, the Introduction should be from 1/4 of a page to a full page long. It should not tire the reader. It should be rather easy reading, not so technically dense as the subsequent sections. In many cases it is not a place to put precise definitions, but rather a place to motivate and anticipate them by describing the major concepts in a less formal way.

**Example 1**.

> The notion of two graphs having *identical shape* may be important. What exactly does it mean for two graphs to have an identical shape? There are geometric definitions and analytic definitions. They will be discussed in Section 2.

**Example 2**.

> When we say that one geometric figure is an *expansion* of another, there is an intuitive understanding of the two being alike and differing only in size. For the purposes of this project, a precise geometric definition is required. It refers, in turn, to the notion of *isometry*, or distance-preserving transformation of a plane, and to the notion of *homothety*, which is stretching or squeezing in the same proportion along all directions going from a fixed center. [Then the introduction proceeds informally. In a later section, the technical definitions, say, in a coordinate form, are given.] ,

In some cases a precise technical definition properly belongs in the introduction. Suppose the assignment asks you to write a program that counts in how many ways a given positive integer $N$ can be broken into a sum of positive integers. It is rather pointless in this case to keep an informal tone. Get straight to the point:

> The purpose of this laboratory is to design a method and to write a computer program for counting partitions of integers.
>
> **Definition**. A *partition* of a positive integer $N$ is a set of positive integers arranged in non-increasing order $n_1 \geq n_2 \geq \ldots \geq n_k$ such that $n_1 + n_2 + \ldots + n_k = N$.

You can help the reader to understand the definition without breaking away from the formal style by adding a comment or remark explaining important particular cases. For example: "A partition with the least value $k = 1$ consists of one element $n_1 = N$. A partition with maximum number of elements $k = N$ is $n_1 = n_2 = \ldots = n_N = 1$". The project may later deal with special classes of partitions, say, those with non-equal members. It is not necessary (and hardly appropriate) to put all definitions in the Introduction.

The last section — **Conclusion**, or Concluding Remarks — contains a brief **summary** of the findings of your report. By reading only the summary, a reader should be able to ascertain the most important facts resulting from your work. Do not overload the conclusion with details of the results.

It is tempting to create the Conclusion from the Introduction by a simple "copy and paste" method. However such an approach misses the point. Observe the difference. If your introduction sets up a goal or makes a promise (as we suggest it should), the conclusion serves as a "checklist": has the goal been reached? is the promise fulfilled? Sometimes the answer will be — not quite; in that case, you should admit it and explain.

**Example**.

> In this paper, a method for counting all partitions of a given positive integer $N$ is described. A FORTRAN program has been written and the number of partitions has been computed for all $N \leq 30$. It is apparent that the number of partitions $P(N)$ exhibits a rather rapid growth as $N$ increases. We observed and proved that $P(N) > N^2$ for $N > 9$ and that $P(N) < 2^N$ for all $N$, but we did not come up with a definite conclusion about the precise law describing the growth.
>
> The program presented here uses direct enumeration of partitions. A Wikipedia article [2] suggests another, supposedly more economical method for counting partitions, based on recurrence relations. We have also attempted to implement that method but have not been able to complete the programming in a timely manner.

It is too late in the conclusion to bring in new material not found earlier in the paper. Instead, you may discuss possible extensions of the work done or point out some connections between your subject and other applications or techniques, which you might have come across in the course of the work but which have not been been worked out in detail in the paper body.

The Introduction and Conclusion may contain other material that the author considers relevant, for example, a personal remark or opinion which cannot be conveniently expressed in the central, more formal part of the paper.

The size of the Conclusion should not exceed 1/2 of a page; in many cases, one or two paragraphs will suffice. Avoid trivial, non-informative phrases, like "Upon completion of this laboratory, certain conclusions can be drawn". An Introduction or Conclusion longer than one of the central sections is a sign that the material should be re-balanced.

### 2.3.6 Technical details

Other commonly used titles are “Method” or “Methodology”. Feel free to devise a subject-specific, explanatory title. If the original problem has several parts, subdivide this section accordingly. A subdivision may be needed simply for a better balance of section sizes or it can be demanded by the logic of exposition: if, say, different aspects of the method employ different techniques.

This section should describe all the fine (or heavy) details of the problem or model and the details of the mathematical method or algorithm used to solve it, as well as the structure and particulars of your computer code.

The requirements, in brief, are:

*Attention to details and particulars. Relevance to the topic.*

The subsection **Mathematical details** (or whatever your subject-specific title) generally includes notation used, definitions, mathematical formulation (set-up) of a model, relevant theoretical facts, and the mathematical essence of the algorithm used to solve the problem.

Do not attempt to reach far and wide; apply judgement. Suppose, for instance, that the problem is to find the distance between two skew lines in space. A student googles for *distance*, finds a Wiki article on *metric spaces* and blindly copies a definition to her paper. Big math, the prof must be pleased!? — Wrong! Irrelevant! (Plus, the level is inappropriate for the target audience, which is not the “prof”.)

Another student, faced with the equation $x^2 - 5x + 6 = 0$, engages in a lengthy step by step calculation using the quadratic formula. Five lines down, he finds the roots to be 2 and 3. It isn't such a serious crime, but it wastes space on a triviality. In such simple cases, the answer ought to be given next to the equation. (In more involved cases, a detailed solution of a quadratic equation would make sense. Your reader may not immediately see that the roots of the equation $x^2 - 2tx + (t^2 - 1) = 0$ are $x_1 = t + 1$ and $x_2 = t - 1$.)

Neither speculating about things you don't understand nor reiterating banalities is good. Focus on a content that's not over your head and that really matters. Any symbol that appears in your calculations, arguments, or later in tables and graphs, should be defined. Definitions and terminology should be accommodated to the **concrete situation**. In an anecdotal case, a student writing a paper on graphs of polynomial functions started with a definition of a graph from Graph Theory!

Let us elaborate on **definitions** a bit further. They can be stated in two ways.

(1) As stand-alone structure units, in a separate paragraph, with the title word **Definition** in a distinguished font style. This format should be used when you introduce a major concept or when a definition is lengthy. (Do not hesitate to make definitions lengthy and detailed, even boring: precision and disambiguation are the priorities. Think of a legal code.)

(2) Inline definitions (as a part of the flow) can be used if the definition is very short, simple or natural, or if the concept is supposed to be generally familiar to the reader. Example: “To describe the shape of a rectangle with side lengths $a$ and $b$, we introduce the parameter $\mu = b/a$ called the *aspect ratio*.”

The definitions, the notation, and the method should be described in such detail that a motivated reader could **reproduce** your work on his/her own and obtain identical results. How about yourself a few months later? Keep adding details and clarifying your writing until you are able to answer in the affirmative. We refer to Appendix B-1, Section 4, for further tips on mathematical writing. Let us just make one more suggestion regarding the mathematical method in general and contents of your *Mathematical details* section in particular.

Think about **simple particular cases**, where the situation is either obvious or the answer can be obtained easily. Do this before you write a program and before you explore the "real" data, for which you cannot predict the results. While creating your program, you will have convenient simple tests. For example, if you have to write a program to compute the area of a triangle with sides 14.23, 12.497 and 9.72, test your program on the Pythagorean triangle with sides $3, 4, 5$ first; also test it in the case where a triangle degenerates to a segment (say, for the sides 1, 2, and 3).

Also, think what happens when a certain parameter or a ratio of parameters becomes extremely large or extremely small. Quite often, intuition will suggest an answer and you'll be in a better position to make sense of the computed results.

**Example 1.** Suppose, as a part of the assignment, you have to construct a common tangent to two given circles. A circle collapses to a point when its radius tends to zero. Consequently, a common tangent to the two given circles becomes a line passing through the two given points when both radii shrink to zero. This limiting case provides a convenient test for your calculations and your computer program.

**Example 2.** Suppose the assignment asks you to find the number of grid points (whose both coordinates are integers) inside the circle of radius $R$ centered at the origin. Think what happens as $R \to \infty$. Every grid point corresponds to the unit square of which it is the center, so the number of the grid points is approximately equal to the area of the circle, that is, $\pi R^2$. The fraction of the area contributed by incomplete unit squares overlapping with the circle is vanishingly small.

Considerations of this sort can make a valuable part of the *Mathematical details* section or of the *Results and Analysis* section.

The subsection **Program details** should provide a detailed breakdown of your program so that the reader can see how the mathematical ideas of the solution method are coded.

Please learn to differentiate between a mathematical method, or algorithm, and its programming implementation. Sometimes the description of the method and of the program, which implements it, can be intertwined, especially if the method is very straightforward. In other cases you are better to explain the method or its more subtle elements within *Mathematical details*, using conventional mathematical notation (dot or void for the multiplication sign, one-letter variables, subscripted if needed, etc.). The explanation of a program involves the actual syntax of the programming language used, with its own conventions ($*$ for multiplication, multi-letter names of variables, etc.). If necessary, explain the correspondence between mathematical variables and their counterparts in the program.

What is the best way to explain a computer program? On the one hand, from the end-user perspective the program is a black box that takes a specified input and produces an output, which the user should be in position to interpret. On the other hand, you must explain the internals, the workings of the code, and — primarily — the part pertaining to **mathematical operations**. It is the latter that we want you to emphasize in this section. You are writing a research report, not a user manual (a technical text, too, but of a different kind).

If your program validates initial data (reports an error on input of a negative distance, say) — good, but do not get overexcited about the user interface. It is better to make an effort to explain the overall logic of the program, flow control (loops, if/else operators), and the organization of data unless it is very obvious.

**Example 1.** Suppose your program counts the number of partitions $P(N)$ for $N$ from 1 to 30. Your description of the program can begin as follows:

> Each run of the outermost loop of the program corresponds to computation of $P(N)$ for a particular value of $N$:
>
> ```
> DO N=1,MAXN
>   Computation of P(N)
> END DO
> ```
>
> The upper bound, `MAXN`, of the loop is set to 30 by default, but it can be changed through user's input.

**Example 2.** A line like this

```
DISTANCE=TIME*SPEED
```

is self-explanatory and doesn't need comments. A loop like this

```
DO I=TMIN+1,TMAX
 DISTANCE=DISTANCE+DT*SPEED(I)
END DO
```

can be commented on at the author's discretion, for example:

> In this loop, the distance traveled over time interval from `TMIN+1` to `TMAX` is computed. The array `SPEED` is initialized at the beginning of the program according to formula (2.3). [referring to the *Mathematical details* section] The variable `DT` in the program is time step, denoted by $\tau$ in the description of the method. Note that the value `SPEED(TMIN)` is not included, since it has been included in the previous summation.

The better your programming style, the easier it is to explain the program's overall design and logic. Look at Example 1 again. Perhaps, replacing the whole body of the loop (dozens of lines of code) by a framed summary was a good writing trick. But it becomes altogether unnecessary if a subroutine is used instead:

```
DO N=1,MAXN
   CALL NUM_PARTITIONS(N)
END DO
```

Self-explanatory names of variables, short functions, transparent if/else conditions, avoiding fancy syntax constructions like `cond[--i]=(i>=1)` in C, — all this helps.

Do not teach the reader the basics of programming. A general definition of a `for` loop copied from the Internet or a general discussion of the organization of computer memory or a definition of common data types (`int`, `double`) is not what is needed. If you really feel a need to remind the readers about some syntaxic details or other particulars of the programming language used — for instance, if you are writing for your own future reference — then, ok, find a place, but don't make it a big story. Even if you think (possibly correctly) that your instructor is not a Java or Python guru, it is not a reason to incorporate a section-long language tutorial.

When it comes to small details, give priority to those that require special care. Why does the value of `J` change from `0` to `N-1` and not from `1` to `N`? Why is the variable `numpoints` defined as `double`, while it naturally represents a positive integer quantity? (Conceivably, you want to allow it to assume very large values, beyond the range of the type `int`.) Issues like these can be addressed.

For the reader's convenience, include only short, critical fragments of the actual code in the body of the report. The complete listing can be included as an Appendix. In any case, the program code must be submitted electronically as a part of the assignment bundle.

The above is not dogma. For example, it can be important to explain to the reader how exactly your FORTRAN program produces the data file with coordinates — in which case the details of the `WRITE` operator should be discussed, although it is not a computational issue.

### 2.3.7 Results and Analysis

As in the case with the *Technical details* section, this one can be subdivided if the research has several parts.

In different projects, the meaning of "solution" or "results" is different. There have been few labs in this course where the answer can be stated in a really short form, like projects asking to decipher a coded message. In most labs, results come from a series of runs of a program that a student creates. Each single outcome can be just a number or it can be an array of numbers, a table or a graph.

As a bare minimum, your presentation of results should include:

- evidence that your mathematical method and your program are correct. Run sample cases that can be checked by hand calculation. Or demonstrate the workings of your program in cases where the solution is intuitively obvious.
- the solution(s) corresponding to those data provided in the assignment (if such data are indeed provided).

If correctness of the method/program cannot be demonstrated because the program doesn't work correctly, read Sect. 1.3.2.

Many projects in this course are to some degree open-ended. They ask you to go beyond the prescribed sets of data and to explore the problem further on your own. The assignment sheet may or may not give a hint on how to choose data for such experimentation. Some outcomes of your experiments will end up in a trash bin and some will make it into the paper. In the end, we want your Results to be more than just plural for a single 'result'. Interpret them, present them as a manifestation of a certain idea or phenomenon. We want you to spot a trend, to observe a pattern, to discover some sort of "law."

- Tell the reader why or how each example presented is relevant to the conjectured "law."
- Explain the observed pattern/law, at least partially.

The quality of your analysis of results and your mathematical explanations determine the research value of your paper more than anything else. Remember that this is a **mathematics** course and a large component of your grade will be based upon the paper's mathematical content. Don't just state observations. Analyse them and justify them! If the analysis and/or explanation of the results requires a piece of theory that has not been discussed in the *Mathematical details* section, include the necessary definitions and facts here, along with your own calculations and arguments.

In this section you can also explore the efficiency of your code: how fast or slow it is as the size of data fed to the program increases.

### 2.3.8 Acknowledgements and References

Any help that you have received from another person must be acknowledged. An acknowledgement should be expressed in the form of a grammatically complete sentence. If possible, specify the kind of help obtained. It also helps to characterize the status of the person so that those who come across your paper in a few years will know. Don't just say:

- *John Smith for his help with this assignment.*

Say instead:

> John Smith, a Computer Science major, has provided advice about the input/output functions in Java.

Or:

> I acknowledge help from Mr. John Smith, a tutor, in justifying the pattern as described in the Results section.

If you quote any printed material in your report, for example a calculus book, it must be listed as a reference. You should provide specific page number to enable the reader to easily find the place (definition, theorem, historical fact) you are referring to.

**Example.** In the body of the paper, put the reference number and the page number:

If we substitute the elliptic arc equation (2) into the arc length formula [3, p. 548]

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx,$$

we obtain the expression ...

In the References section, describe the source:

[3] J. Stewart, *Calculus: Early Transcendentals*, 5th ed., Thomson-Brook/Cole, 2003.

(In this case, the publisher is a worldwide company and the place of publication is not indicated.)

There are different reference styles. Follow one style consistently. Check with your instructor as to whether a particular style is preferred.

Quoted online resources must also be given proper attribution. For instance, there is a webpage at `http://mathworld.wolfram.com/ContinuityPrinciple.html`. It has a title and, unlike, say, many pages in Wikipedia, it is not anonymous: we can name an author. For this particular page, a bibliographic entry would look like this:

[1] Eric W. Weisstein, Continuity Principle, `http://mathworld.wolfram.com/ContinuityPrinciple.html`. (Accessed Dec. 5, 2008).

Alternatively, *"web sites may be cited in running text instead of in an in-text citation"* (The Chicago Manual of Style Online, section Website). This very line is an example.

The sections **Acknowledgments** and **References**, as well as **Abstract**, are not numbered. See Sect. 3.2 regarding LaTeX typesetting conventions for these sections.

### 2.3.9 Appendix

Material that does not naturally fit in the flow of your paper yet is important for your project's completeness should be put in an **Appendix**. Many papers will not have an appendix. Where an appendix is present, the following kinds of material are found in it:

- Computer code
- Graphs and illustrations
- Particularly long mathematical calculations or proofs

A listing of your program is the most common thing to put in the Appendix. Sect. 3.1.9 suggests how to get a nice printout.

A Math-2130 paper having more than one appendix should be an exception, but if that happens, identify the appendices alphabetically: **Appendix A**, **Appendix B**.

We urge you to learn quickly how to include graphs and illustrations **in the body of your report**. You may use `gnuplot`, `xfig`, the LaTeX picture environment, `Maple`, or PostScript. Graphs and illustrations which are not part of the text can comprise Appendix A of your report, while the computer programs can be presented in Appendix B.

## 2.4 Suggestions about style

### 2.4.1 A note on spelling

There are many spelling checkers available. Use them! On Linux, you can use `ispell`. The *`Kile`* editor has a built-in spell function which uses the `ispell` program. There is no excuse for spelling errors, and they will be **penalized** heavily. However, be warned, the spell checker will not find every error in spelling, nor can it pass judgement on a sentence like

A program too gene rate asset off inter resting numb hers ...

**Some common spelling errors**

- their (whose?) — there (where?)
- its (whose?) — it's (it is)
- sep**a**rate (not sep**e**rate)
- occur**e**nce (not occur**a**nce)
- one's — once
- two — to — too
- then (if ..., then) — than (more than)
- lab$\boxed{\textbf{o}}$ratory (not labratory)

### 2.4.2 Squeeze water out (Eliminate unnecessary words)

Compare:
(1) It can be shown by the implementation of the Cosine theorem that the distance $AB$ is equal to 5.
(2) Applying the Cosine theorem we see that $AB = 5$.

### 2.4.3 A note on "strong words"

Students often write: "it is necessary", "one must" etc. Such strong expressions may justly raise objections.

**Example**. Suppose we are considering the equation $x^2 - 6x + 5 = 0$.
**Bad description**: *"It is necessary to use the Quadratic Formula. Thus we obtain..."*
Is it necessary? Absolutely not. Any one competent in quadratic equations will factor this one on the spot. If you want to emphasize the method, better say:
*"The roots, as given by the Quadratic Formula, are* $\frac{1}{2}(6 \pm \sqrt{D})$*, where* $D = 6^2 - 4 \times 5 = 16$*. Thus* $x_1 = (6+4)/2 = 5$*,* $x_2 = (6-4)/2 = 1$*. "*
Better even (if the method is of no special importance) is just to say
*"The roots are* $x_1 = 5$ *and* $x_2 = 1$*".*
Of course, the style and level of details that you should or should not provide depend on who your readers are. In any case, saying that to use the Quadratic Formula is *necessary* here is unprofessional.

### 2.4.4 Common words in mathematical writing

Learn to use basic mathematical terminology precisely and avoid common misuses. For example, watch the following as you write.

• An *equation* must have two parts (sides) connected by the = sign. A thing like $\sin^2 x + \cos^2 x$ is not an equation; it can be described as an *expression* or, more precisely, as a *trigonometric polynomial.* Also don't call $x + y \geq 2\sqrt{xy}$ an equation; it is an *inequality.*

• At the beginning of a mathematical argument you often make an *assumption*, while at the end you arrive at a *conclusion.* Expressions like *Assume (suppose) that something is ...* or, equivalently, *Let something be* are very standard. Steps of your argument or formulas that you display or refer to should be verbally connected using words like *imply*, *follow*, *yield*, etc. to make the flow of the argument smooth and its logic transparent.

• Here are some common, frequently used, safe verb-noun collocations.
— An *equation* can be *solved* (or *solved for x*). In contrast, a *polynomial* (like $x^2 - 5x + 6$, without any right-hand sign) cannot be solved (there is no equation to solve) but it may *have roots.* Also, a polynomial can be *evaluated* at a particular point; equivalently, a particular *substitution* can be made into it.
— A *computer program* can *implement* a method or an algorithm; it is *created* or *written* by you; to obtain (produce) results you *run* (or *execute*) it; the results are *presented* or *displayed* in tables and graphs.
— A *value* can be *assigned to* a variable (or a variable can be assigned a value). Also, a value can be given *in a closed form* (as a mathematical expression that does not involve an approximation, e.g. $\sqrt{2}$ or $\arctan \pi/3$) or *approximately* (as a decimal fraction), e.g. 1.414 for $\sqrt{2}$.

— An *expression* (a *sum*, an *integral*, but not an equation) can be *evaluated* (calculated, computed); also it can be *transformed* (into an equivalent form), *expanded*, *factored*, *simplified*, *reduced* to a simpler form in a special (particular) case, *broken* into two or more parts (usually, to reveal an essential structure). But do not "solve a sum"!
— A new concept or notation can be *introduced*, *defined*, or described in a manner of this sort: *let us say that X is ...* [an explanation in terms of known or previously defined terms follows].

### 2.4.5 *We* versus *I*

Whether you should write in the 1st person (*we* or *I*) or in the 3rd person is to a large extent a matter of choice and personal style. It is more common to use *I* in the Introduction and Conclusion only, if at all. If you are bold enough to use *I* in the main part of your paper, do so consistently; do not hide behind *we* in "inconvenient" places.

That said, modern professional scientific writing overwhelmingly prefers *we*. Think about it this way: as a reader goes through your paper, he/she feels your company and gentle support: **"We = I, the author, + you, the reader"**.

You should use *I* when expressing personal opinions, e.g. "I found the results obtained in this laboratory truly surprising." There are other situations where the subject *I* sounds more appropriate than *we*. Consider this example: "I have encountered difficulty implementing this algorithm." Substituting *we* in place of *I* would imply that your innocent reader shares your responsibility for the trouble. Do not switch lightly between *we* and *I*. If the subject *I* is indeed necessary, its use should be confined to a particular section — better the Introduction or Conclusion — and there *I*, not *we*, should be used throughout.

While working on the project, you perform concrete actions, like searching a database, writing a program, etc. If you feel uncomfortable to write *we* when describing such actions, use sentences stated in the 3rd person. For example, "A program has been written" instead of "I have written a program". However, papers written in the 3rd person from the beginning to end usually leave an impression of an indirect, cumbersome style.

### 2.4.6 Verb forms: tense, mood, modal verbs

Use **present tense** predominantly when discussing mathematical theories and other abstract facts: they last indefinitely. Naturally, in talking about background and history of the problem you would use the past tense.

You may choose to use either past or present to describe how you have created a computer program and produced the results of numerical simulation. (The story may be the past for you when you submit the report, but it will be current for the reader as he/she reads it.)

Variation of tense can be employed just for a particular purpose. Do not constantly swing between one tense and another, especially within a paragraph or even a section.

Write in the **indicative mood** for the most part; use conditional only when it is unavoidable. Theorems *are* true, not *would be* true.

Watch other **modal verbs**, besides *would*. In many of students' papers the verbs *can* and *must* are unnecessarily frequent.

**Example 1.** Saying that "Equation (1) *can be shown* to yield formula (2)" leaves the reader wonder whether you omit significant steps. If the answer is yes, you ought to show *how* the former yields the latter. Otherwise (if the connection between (1) and (2) is transparent enough), simply say "Equation (1) yields formula (2)."

**Example 2.** Here *must* is used unnecessarily: "Therefore Equation (3) must hold when $t = 0$." It really means that "Equation (3) holds when $t = 0$." Say thus — shorter and better.

Chapter 3

# Typesetting with LaTeX

## 3.1 Elements of LaTeX

### 3.1.1 Preamble

Every LaTeX file has a structure like this:

```
\documentclass[12pt]{article}
\usepackage{2130}
  .........
\begin{document}
  .........
\end{document}
```

The part of the file before the line `\begin{document}` is called *preamble*, while the subsequent part is called *body* of the document.

The first line in the file tells LaTeX to use a file called `article.sty` as the main source of formatting commands. The style file contains certain default parameters that determine layout of the document, in particular:

```
\textwidth
\textheight
\topmargin
\oddsidemargin
\evensidemargin
```

The subsidiary style indicated by the argument "[12pt]" sets up the regular font size for the document to be 12pt. Line spacing and sizes of fonts described in relative terms (Large, large, small, tiny) thus become determined.

The first line may be followed by lines that import additional commands from various *packages*. The package `2130.sty`, which the second line in our document refers to, makes it easy to format a report for this course. For example, it will default to one inch margins all around the standard size of paper. It also makes it easy to create the title page, headers and footers, using the commands

```
\headers{...}{...}{...}                 \footers{...}{...}{...}
\underhead         \nounderhead         \overfoot         \nooverfoot
\underheadoverfoot
```

The package `2130.sty` also contains enhanced graphics command, which significantly extend a graphical facility provided by the standard LaTeX, as described in Section 4.4.3.

Other useful packages that you will likely need to include in your documents are `graphicx` (to enable import of EPS graphics, see Section 4.3.1) and `amssymb` (to enable mathematical symbols like $\mathbb{R}$ and various special characters, like $\varnothing$).

Besides `\usepackage` command(s), the preamble may contain your own definitions. For example, this is a convenient shorthand for an useful but long LaTeX's keyword (see p. 28):

```
  \newcommand{\ds}{\displaystyle}
```

User-defined commands may have parameters, for example: `\newcommand{\pair}[2]{(#1,#2)}`. Now, the command `\pair{A12}{b34}` produces: (A12,b34).

### 3.1.2 Comments

The percent sign % is the commentary symbol in LaTeX. Everything that follows it is ignored till the end of line. There are two exceptions. First, the percent sign itself can be printed with command `\%` and, as a part of this command, it does not begin a commentary. Second, the percent sign within the `verbatim` environment or `\verb` command has no controlling effect.

If there is a need to disable rather long parts of a document from processing, while deleting them is undesirable, the following construction can be used:

```
\iffalse
  No matter how many lines or pages - everything here will be ignored by LaTeX
\fi
```

Finally, you may put the line `\end{document}` earlier in your document and LaTeX will stop exactly there. Everything that follows will be ignored. This is a convenient method to find and fix errors in long documents.

### 3.1.3 Environments

The following construction, very common in LaTeX, is called an *environment*:

```
  \begin{something}
      ..........
  \end{something}
```

In this case, we deal with a (nonexistent) "something"-environment. The whole document is an environment by itself.

Some common environments are: *displaymath, equation, tabular, picture, table, figure, center, thebibliography, itemize, enumerate, verbatim.*

### 3.1.4 Space

LaTeX regards one 'return' or 'enter', one or more 'tabs' and one or more blank spaces, as equivalent to one 'space'. An exception to this rule is presented by the spaces within the `\verb` command and the `verbatim` environment.

#### Paragraphs

There are two ways to start a new paragraph:

1. The usual (simplest) way is to leave a blank line in your text.
2. Use the command `\par`.

#### Line breaks

You may force LaTeX to start a new line within the current paragraph by means of the command `\\` (double backslash). The current line ends immediately, at the same position as if the paragraph had ended. The command `\\` is routinely used as a separator between lines in tables and "arrays" (multi-line formulas in math mode).

Another seemingly similar command is `\linebreak`. However it is rarely used. It causes LaTeX to stretch the current line to page width to compensate for the text that has been forced out to the next line. LaTeX may find the required stretch intolerable and deny the requested line break.

#### Vertical space

LaTeX does a reasonable job of inserting space and turning up new pages. Still, there are times when the user would like to assert his/her own influence.

The regular, preferred method to add vertical space between paragraphs or figures is by using one of the three commands:

`\smallskip` adds a little extra space to the regular space between lines

`\medskip` adds about twice that much

`\bigskip` adds yet about twice that much as `\medskip`.

The command `\vspace{1ex}` instructs LaTeX to push the next line down by the height of a letter “x” in the current font size. These commands ought to be given after a blank line (in particular, they should not be given within a paragraph), otherwise the layout of your page may turn out to be rather odd. The command `\vskip 1ex` has the same effect in most cases; however, its use in LaTeX documents is now deprecated.

There are several allowed measures of length, including inches (in), centimeters (cm) and millimeters (mm), and finally, points (pt), which are the preferred unit to a typesetter. (There are 72 points to an inch.) Besides these *absolute* units, the units relative to the current font size are often used: `\ex` (see above) and `\em` (see below).

The command `\vspace{...}` has no effect at the top of a page or at the bottom. Why would you want space when you are about to move to a new page? If you insist, you must use `\vspace*{...}` to force LaTeX to make space.

In the line break command inside a paragraph of text, as well as inside tables, arrays (in math mode), and parboxes, one can create space by affixing an argument to the command `\\`; for example, `\\[1ex]` leaves 1ex of additional space after the current line.

To make LaTeX to skip more space between paragraphs, as is done throughout this Manual, you may add the following command in the preamble of your document:

```
\setlength{\parskip}{\smallskipamount}
```

Finally, the commands `\pagebreak` and `\newpage` end the current page and start putting text on the new one.

### Horizontal space

The command `\hspace{10pt}` skips 10 points (approx. 3.5 mm) of horizontal space. (The command `\hskip 10pt` has the same effect in most cases, but its use is discouraged.)

At the beginning of a line `\hspace{..}` will have no effect, so you must use the command `\hspace*{..}` instead.

It is recommended that you use the units ‘em’ (the width of a letter ‘m’ in the current font size) for horizontal space.

By default, paragraphs in LaTeX begin with indentation, except those immediately after the headings made with commands like `\section`. To suppress indentation, you can use the command `\noindent`.

### Math space

Spacing commands like `\hspace` and `\vspace` only work in “paragraph mode” (outside math). The horizontal spacing commands in math mode are: `\; \: \, \! \quad \qquad`, which can be found in Maltby.

That’s how much they measure: `\,`$=|\,|$ and `\:`$=|\:|$ and `\;`$=|\;|$ and `\quad`$=|\quad|$ and `\qquad` $=|\qquad|$. The remaining distance `\!`$=|\!|$ is a tiny step *backward*!

Vertical spacing in math mode is best handled with `\\[..]`.

### Centering

To center a block of text, use the *center* environment:

```
\begin{center}
 ....
\end{center}
```

Note the American spelling of "center". LaTeX is **very unforgiving** if you make a spelling error in a command.

## 3.1.5 Math mode

### Inline math and displayed math

LaTeX can typeset in a paragraph mode or in math mode, which, in its turn can be of two kinds: *inline math* or *displayed math.* To begin and end the inline math mode, use the dollar signs: `$...$`. This places the mathematical formula as the next word in the line of text, like here: $(x + y)(x - y) = x^2 - y^2$. However, if you wish to display the formula, use

```
\begin{displaymath}
  ............
\end{displaymath}
```

A shorter delimiter for displayed math (both to begin and to end) is double dollar sign:
`$$....$$`.
The displayed formula is automatically centered on its own line. (This can be artificially changed, but rarely needed.)

A displayed formula is still considered to be a part of a paragraph unless there is a blank line separating it from the paragraph, before or after. In particular, the text following the displayed math (without blank line in between) will not be indented.

A modification of the `displaymath` environment is the `equation` environment, which displays a single **numbered equation**, like this:

$$2 + 2 = 4. \tag{1}$$

LaTeX automatically numbers equations enclosed in the `equation` environment consecutively starting from (1). It does not number inline equations and equations within the `displaymath` environment, but it does number equations within the `eqnarray` environment described below.

There is a difference in style of math formulas depending whether they are printed in the inline or displayed math mode. In the displayed mode, for instance, fractions' numerators and denominators are printed in the regular font size, while in the inline mode they are printed in a reduced font size. There is a way to force LaTeX to type math in a prescribed mode: this is done by the commands `\displaystyle` and `\textstyle`. There is also a similar command `\scriptstyle` (to print in small size, like subscripts or superscripts).

### 3.1.6 Lists

#### Text (paragraph mode)

Use the 'enumerate' environment or the 'itemize' environment

```
\begin{enumerate}                       \begin{itemize}
  \item ......                           \item ......
  \item ......                           \item ......
     .........                              .........
  \item ......                           \item ......
\end{enumerate}                         \end{itemize}
```

to generate a numbered /bulletted list of items, respectively.

#### Math mode — `\eqnarray`

Here, we usually mean a set of equations. Use

```
\begin{eqnarray*}
<formula> & <connective> & <formula> \\
...
...
...
<formula> & <connective> & <formula> \\
\end{eqnarray*}
```

It is possible that some of the '< . . . >' are empty fields. Also, the `\\` can be replaced by `\\[..]` as indicated in the section on vertical space above.

If the `*` is omitted, the equations will automatically be numbered. If a line is not to be numbered, then the command `\nonumber` must be entered somewhere in that line.

#### Math — `\array`

The `\array` environment is used within displayed math in the cases when an additional flexibility as compared to `\eqnarray` is required. Details can be found in Maltby.

In both, `\array` and `\eqnarray` environments, LaTeX is by default in the **inline** math mode, so you have to use the `\displaystyle` command to display fractions, sums, etc. properly. Moreover, this command only has effect between two ampersands or between an ampersand and the endline command `\\`. So you may have to issue the `\displaystyle` command repeatedly. That's why the abbreviation shown on p. 25 makes a lot of sense.

### 3.1.7 Advanced math typesetting

The formula

$$\frac{x^{\sin(x)}}{(\cos(x))^x} \;=\; \int_{\sqrt{x}}^\infty\,f(x)\,dx\,.$$

is obtained from

```
$$
 \frac{x^{\sin(x)}}{(\cos(x))^x} \;=\; \int_{\sqrt{x}}^\infty\,f(x)\,dx\,.
$$
```

To the mathematically literate, many math commands are intuitive. If you want a symbol and cannot remember it, try the obvious, and most times you will be correct!

You now should read Section 3.3, *An Introduction to TEX and friends* by Gavin Maltby. The definitive book is the "LaTeX User's Guide and Reference Manual", by Leslie Lamport.

### 3.1.8 Processing and viewing LaTeX files

Every LaTeX file must be saved with the `.tex` extension.[1] You process the file "mylab.tex" with the command

```
latex mylab
```

Note that there is no need for the extension `.tex`.

At this point, a lot of information will come across your screen. With time, much of it will even make some sense. Everything you see on the screen, and more, gets written to a "log" file. If you processed "mylab.tex", LaTeX will create "mylab.log" for you. If you are lucky and have made no errors, LaTeX will eventually stop and report that the output was written to "mylab.dvi". If you are less fortunate, LaTeX will stop at the first error and leave you hanging at a question mark on the screen. At this point, if you answer `r` (for run), LaTeX will finish processing to the best of its ability, writing all errors to "mylab.log" which one can then review in one window while correcting "mylab.tex" in another.

To view "mylab.dvi", you use a UNIX program called `xdvi` which is run like this:

```
xdvi mylab
```

Again, there is no need for the extension `.dvi`.

A `.dvi` can be converted to a `.pdf` file by the command like this:

```
dvipdf mylab
```

[1]The description below has been in this Manual since its first edition and is still valid if you are working in the UNIX command line. Many integrated environments like Kile, WinEdt, Texnic Center or Scientific Word nowadays make processing a LaTeX document more comfortable.

### 3.1.9 Including source code in LaTeX documents

There are a couple of ways in which you can include the source code from your programs in your LaTeX documents. One way is to use the `verbatim` environment:

```
\begin{verbatim}
      Paste a copy of your code here
\end{verbatim}
```

Remember to **break long lines by hand** so that they fit the page width; otherwise some details of your program will not be visible.

The other way to do so is to use `lgrind`, which formats program sources in a nice style. Comments are placed in Roman font, keywords in bold face, variables in italics, and strings in typewriter font. Source file line numbers appear in the right margin every 10 lines. Suppose that you have a C program in the file `sample.c`. The first step towards including the file in the document is to run it through `lgrind` to produce a file, say `sample.tex`:

```
lgrind -i -lc sample.c > sample.tex
```

This generates a file `sample.tex`, which has the pretty-printed version of `sample.c` with a lot of LaTeX commands. (Note that your program and main document should have different names!) Now, in the declarations at the start of your main LaTeX document file, you have to be sure to include the `lgrind` package:

```
\usepackage{lgrind}
```

At the point in the document where you want to include the source code, give the command:

```
\lgrindfile{sample.tex}
```

Figure 3.1(A) presents an example of how the file would show in your document.

An **alternative method** of including source code, which does not require any intermediate processing, would be to declare a few commands near the start of your LaTeX document:

```
\newcommand{\beginverb}{\begin{verbatim}}
\newcommand{\inputfile}[1]{\input{#1}}
\newcommand{\verbatimfile}[1]{\expandafter\beginverb\inputfile{#1}}
```

and later on make use of the `verbatimfile` command:

```
{ \small
\verbatimfile{sample.c}
\end{verbatim} }
```

(Don't forget to close the brace after `\end{verbatim}`; otherwise the rest of your document will be printed in a small font size.)

The program listing printed with this method is displayed on Figure 3.1(B).

**A.**

```
/* This is a short C program to compute the sum of the integers
 * from 1 to 1000 and print the result.
 */
#include <stdio.h>
#define N 1000

int Sum (int maxnum);

int main ()
{
   printf ("The sum of the integers from 1 to %d is %d\n", N, Sum(N) );
   return (0);
}

int Sum (int maxnum)
{
   int i, total=0;
   for (i=1 ; i<=maxnum ; i++)
      total += i;
   return (total);
}
```

**B.**

```
/* This is a short C program to compute the sum of the integers
 * from 1 to 1000 and print the result.
 */
#include <stdio.h>
#define N 1000

int Sum (int maxnum);

int main ()
{
   printf ("The sum of the integers from 1 to %d is %d\n", N, Sum(N) );
   return (0);
}

int Sum (int maxnum)
{
   int i;
   int total=0;
   for (i=1 ; i<= maxnum ; i++)
      total += i;
   return (total);
}
```

Figure 3.1: Source code printed (A) using `\lgrind` and (B) using `\verbatimfile`

### 3.1.10 Some commands defined in `2130.sty`

#### Text commands

`\TODAY` produces dates in the form 22 December 2008.

`\TODAYAT` produces dates and time in the form 22 December 2008 at 14:57.

`\Cents` produces ¢.

`\INDENT` forces an indented line when `\indent` fails.

`\tildechar` produces ˜

`\hatchar` produces ˆ

`\boxit{object}` produces

$$\boxed{object}$$

`\lbk`, short for `\linebreak`, produces a line break with horizontal justification.

`\pbk`, short for `\pagebreak`, produces a page break with vertical justification.

`\fsize{number}` is an alternative way of changing font size. This was recommended by Lamport. `\normalsize` is `\fsize{0}`. Positive integers increase and negative integers decrease from here.

For example `\fsize{1}` is equivalent to `\large`, and `\fsize{-4}` is equivalent to `\tiny`. Integers too large default to the smallest or largest fonts available.

#### Math commands

`\di` is short for `\displaystyle`

`\toi` produces $\to\infty$

`\dist` produces dist

`\slope` produces slope

`\LOngleftrightarrow` produces $\Longleftarrow\!\!\!\Longrightarrow$ — compare `\Longleftrightarrow`, which produces $\Longleftrightarrow$.

`\LOngleftarrow` produces $\Longleftarrow\!\!=$

`\LOngrightarrow` produces $=\!\!\Longrightarrow$

`\IFF` produces $\Longleftrightarrow$ — compare `\iff`, which produces $\iff$

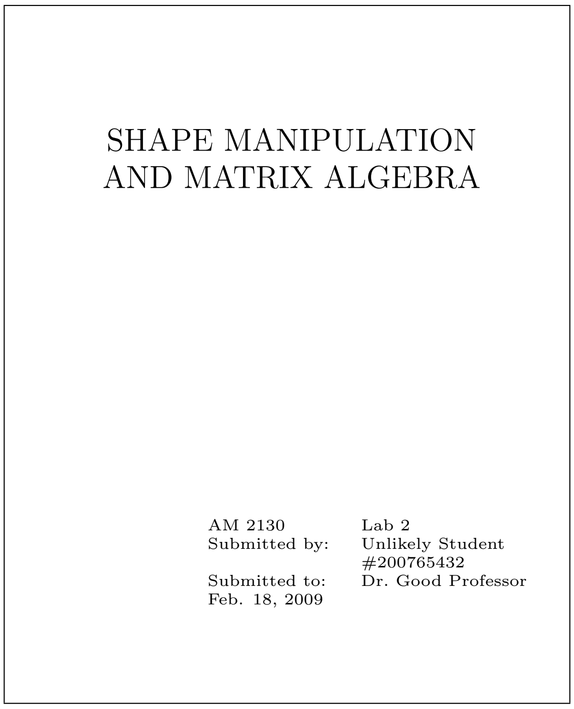
## 3.2 Formatting your Math-2130 report in LaTeX

### 3.2.1 Title page, footers and headers

A typical title page is shown on Figure 3.2. A LaTeX code used to produce it is as follows.

```
\begin{titlepage}
\begin{center}
\large SHAPE MANIPULATION \\
   AND MATRIX ALGEBRA
\end{center}
\vspace{6cm}

\hfill\begin{tabular}{ll}
    AM 2130 & Lab 2 \\
    Submitted by: & Unlikely Student \\
                  & \#200765432 \\
    Submitted to: & Dr.~Good Professor \\
    Feb. 18, 2009
\end{tabular}
\end{titlepage}
```



SHAPE MANIPULATION
AND MATRIX ALGEBRA

| | |
|---|---|
| AM 2130 | Lab 2 |
| Submitted by: | Unlikely Student |
| | #200765432 |
| Submitted to: | Dr. Good Professor |
| Feb. 18, 2009 | |

Figure 3.2: A sample title page for an AM2130 report.

Your document will look nicer with **running footers and headers**. The following few lines can be placed in the preamble or after the title page. The effect is explained in the comments.

```
\lhead{AM2130}       % appears in the header on the left
\rhead{Lab 1: Title} % appears in the header on the right
\lfoot{Your Name}    % appears in the footer on the left
\rfoot{\thepage}     % page number, in the footer on the right
\underheadoverfoot   % dividing lines: under header and below footer
```

### 3.2.2 Table of contents

Assuming you use the standard LaTeX commands to structure your report (“section, “subsection), all that remains to produce the table of contents (TOC) is to insert the command

```
\tableofcontents
```

after the title page code.

You have to run LaTeX compiler **two times** to obtain a correct TOC, as the first run just creates an auxiliary file where the information about sections and subsections found is collected, but LaTeX is not able to incorporate that information into its dvi output immediately.

Some complication occurs with References and Appendix (assuming they are formatted as suggested below). The command `\thebibliography`, which generates the list of references, does not automatically yield an entry in TOC. You have to do some work by hand.

Somewhere soon after your `\thebibliography` command (perhaps, immediately after) insert the following line:

```
\addcontentsline{toc}{section}{References}
```

The heading **References** will then appear in your TOC and it will be printed in the same style as section headers. If you want to somewhat de-emphasize References in TOC, modify the above line:

```
\addcontentsline{toc}{subsection}{References}
```

Similarly, if you formatted your Appendix A using `\section*` command, you should put the corresponding TOC line immediately after it, for example:

```
\addcontentsline{toc}{section}{Appendix A: Program source code}
```

### 3.2.3 Abstract

> **Abstract.** The `quote` environment provides a nice format for an abstract. Type the word *Abstract* in boldface and the rest in a regular font style. Abstract should not be included in the TOC.

### 3.2.4 The body of report

Use the `\section{...}` command for section headers, like Introduction, Technical Details, etc. Use the `\subsection{...}` command for parts of the main sections.

Having third-tier headers (subsubsections) can hardly be justified in a typical Math-2130 report.

Acknowledgements and Appendix should not be numbered as regular sections. Use the `\section*` command, for example,

```
section*{Acknowledgments}
```

The so formatted sections are not automatically referenced in the TOC. You should include a reference to Appendix or Appendices, but probably not to Acknowledgements.

### 3.2.5 References

The list of references is printed using `thebibliography` environment. For example,

```
\begin{thebibliography}{4}
\bibitem{hagin} Frank G.~Hagin, A first course in differntial equations.
             Prentice-Hall, Inc.:New Jersey. 1975.
%
\bibitem{m2130} Math-2130 Course Manual. MUN, 2008.
%
\bibitem{fourier-bio} ``Jean Baptiste Joseph Fourier'' (biography).\\
\verb+http://www-history.mcs.st-andrews.ac.uk/history/Mathematicians/+
\verb+Fourier.html+ \\ (Accessed December 2, 2008.)
%
\end{thebibliography}
```

Don't forget to provide a numerical argument in `\thebibliography` line. It does not necessarily be equal to the actual number of references, it must just have the same decimal length — like in our example, 3 references and the number is 4, both are single-digit numbers. In the absence of the numerical argument LaTeX often gives a misleading information as to where the error is.

### 3.2.6 Appendix and program source

Use the `\section*` command for appendices. Section 3.1.9 explains how to include the source code of your program in the report. (It is not always necessary.)

A little note for those who needs to include a **Maple code** in the report: in our experience, the quickest way that does not compromise on typographical quality is simply to copy and paste the lines from Maple worksheet into the LaTeX file line by line. Maple programs (at least those in Math-2130 projects) are usually rather compact. Maple's numerical outputs can also be copied and pasted, unless they contain exponents. In the latter case please take a trouble to type the answer properly in the LaTeX math mode, for example, $6.67 \times 10^{-11}$. If there are too many such numbers to type, think about summarizing them in a table rather than just copying.

Maple's symbolic answers, when copied into a text file, are difficult to read. Again, if the answer is important (so you are not just filling up a space), it is worth to neatly re-type in a proper mathematical format. *Important* results can hardly be too many...

A copy of your Maple worksheet submitted electronically is a definitive document and will be used by the instructor should any discrepancy between your reported input and Maple's output be revealed.

### 3.2.7 Floating environments: figures and tables

LaTeX has its own ideas as to where it is best to place your figures and tables made by means of the corresponding environments. In the processed document, it usually does not put them where they ought to be according to their position in the source `.tex` file. Many students, and not only, often find this frustrating.

However, LaTeX's behaviour is in most cases backed by well established rules and conventions of typography. In general, try not to fight LaTeX very hard. If it puts your figure on top of the next page rather than dropping it just on spot, think whether you can agree with that decision. Only if the displacement of the figure leads to a mess in the logic of your presentation or if a large unfilled area remains on the page, you should insist.

The way to make LaTeX to put the picture **exactly here** is to use the `figure` environment with argument `[H]`:

```
\begin{figure}[H]
```

The option `H` is not provided by standard LaTeX, you must `\usepackage{float}` in order to be able to use it. All this equally applies to the `table` environment.

### 3.2.8 Automatic numbering, cross-references, and citations

The `\caption` command within a `figure` or `table` environment assigns a number to the figure or table. This number can be captured by the `\label` command put right after. For example, the command `\label{xyz}` allows you to refer to your figure elsewhere in your document in the following way:

"`See Figure~\ref{xyz}` on `page~\pageref{xyz}`."

Similarly, labels can be assigned to sections, subsections (Sect. 3.2.4), equations (Sect. 3.1.5) and theorem-like environments described in Section 3.4.6.

References to sources listed under `\thebibliography` command can be done as follows:

```
Fourier's memoir ``On the Propagation of Heat in Solid Bodies'' was
read to the Paris Institute on 21 December 1807 \cite{fourier-bio}.
```

## 3.3 An introduction to TeX and friends

Original text by Gavin Maltby (1992); adapted by Math-2130 Instructors (1995,1998,2008)

### 3.3.1 An Introduction to TeX

TeX is well known to be *the* typesetting package, and a vast cult of TeX lovers has evolved. But to the beginning TeX user, or to someone wondering if they should bother changing to TeX, it is often not clear what all the fuss is about. After all, are not both WordPerfect and Microsoft Word capable of high quality output? Newcomers have often already seen what TeX is capable of (many books, journals, letters are now prepared with TeX) and so expect to find a tremendously powerful and friendly package. In fact they *do*, but that fact is well hidden in one's initial TeX experiences. In this section we describe a little of what makes TeX great, and why other packages cannot even begin to compete. Be warned that a little patience is required—TeX's virtues are rather subtle to begin with. But when the penny drops, you will wonder how you ever put up with anything different.

#### TeX is a typesetter, not a word-processor

TeX was designed with no limiting application in mind. It was intended to be able to prepare practically any document—from a single page all-text letter to a full blown book with huge numbers of formulae, tables, figures etc.

Conventional word processors have a fundamental limitation in that they try to "keep up" with you and "typeset" your document as you type. This means that they can only make decisions at a local level (eg, it decides where to break a line just as you type the end of the line). TeX's secret is that it waits until you have typed the *whole* document before it typesets a single thing! This means that TeX can make decisions of a global nature in order to optimise the aesthetic appeal of your document. It has been taught what looks good and what looks bad (having been given a measure of the "badness" of various possibilities) and makes choices for your document that are designed to make it "minimally bad".

But TeX's virtues run much deeper than that, which is just as well because it is possible to get satisfactory, though imperfect, results from some word processors. One of TeX's strongest points is its ability to typeset complicated formulae with ease. Not only does TeX make hundreds of special symbols easily accessible, it will lay them out for you in your formulae. It has been taught all the spacing, size, font, . . . conventions that printers have decided look best in typeset formulae. Although, of course, it does not understand any mathematics it knows the grammar of mathematics—it recognises binary relations, binary operators, unary operators, etc. and has been taught how these parts should be set. It is consequently rather difficult to get an equation to look bad in TeX.

Another advantage of compiling a document after it is typed is that cross-referencing can be done. You can label and refer back to chapters, sections, tables etc. by *name* rather than absolute number, and TeX will number and cross-reference these for you. Similarly, it will compile a table of contents, glossary, index and bibliography for you.

Essential to the spirit of TeX is that *it formats the document whilst you just take care of the content*, making for increased productivity. The cross-referencing just mentioned is just part of this. Many more labour-saving mechanisms are provided for through *class* and *style files*. These are generic descriptions of classes of documents, teaching TeX just how each class likes to be formatted. This is taught in terms of font preferences, default page sizes, placement of title, author, date, etc. For instance, a `paper` style file could teach TeX that when typesetting a theorem it should embolden the part that states the theorem number and typeset the text of the theorem statement in slanted Roman typeface (as in many journals). The typist simply provides and indication that a theorem is being stated, and then types the text of the theorem *without* bothering to choose any fonts or do any formatting—all that is done by the style file. Style files exist for all manner of document—letters, articles, papers, books, proceedings, review articles, and so on.

There are many other motivations one could cite for the superiority of TeX. But it is time that we started to get our hands dirty. The novice reader will still have no idea of what a TeX source file looks like. Indeed, why do we keep referring to it as a *source file*? The fact of the matter is that TeX is essentially a *programming language*. Just as in any compiled language (e.g., Fortran, C) one prepares a source file and submits it to the compiler which attempts to produce an object file (`.dvi` file in the TeX case). To learn TeX is to learn the command syntax of the commands that can be used in the source file.

### Typical TeX interfaces

By the nature of TeX most time is spent editing the source document (before submitting it for compilation). No special interface is necessary here, you just use your favourite text editor (perhaps customising it to enhance TeXnical typing). Thus TeX user interfaces are usually small and simple, often even missing. One frequently uses TeX at command line level, just running the editor, compiler etc. as you need them. Sometimes a TeXshell program is present, which runs these for you when you choose various menu options.

Whatever the interface, there are just a few basic steps to preparing a document:

1. Choose a document style to base your document on (e.g., letter, article).

2. Glance through the material you have to type, and decide what definitions might be made to save you a lot of time. Also, decide on the overall structure of the prospective document (e.g., will the largest sectional unit be a chapter or a part?). If you are going to compose as you type, then pause a moment to think ahead and plan the structure of your document. The importance of this step cannot be overstressed, for it makes clear in *your* mind what you want from TeX.

3. Prepare your input file, specifying only the content and the logical structure (parts, sections, theorems,...) thereof and forgetting about formatting details.

4. Submit your input, or source, file to the TeX compiler for compilation of a `.dvi` file.

5. If the compiler finds anything in your source file strongly objectionable, say incorrect command syntax, then return to editing.

6. Run a *previewer* to preview your compiled document on the screen.

7. Go back to editing your document until glaring errors have been taken care of.

8. Make a printout of your compiled document, and check for those errors that you failed to notice on the screen.

Performing these steps may be effected through typing at the system prompt (bare-bones technique) or through choosing menu options in a TeXshell program. The latter will probably provide some conveniences to make your life easier.

### 3.3.2 A review of LaTeX

The TeX typesetting system was designed by the eminent Stanford computer scientist Donald Knuth, on commission from the American Mathematical Society. It was designed with enormous care, to be ultimately powerful and maximally flexible. The enormous success of Knuth's design is apparent from the vast number of diverse applications TeX has found. In reading the following you must keep one thing clearly in mind: *there is only TeX language, and all the other packages whose names end in the suffix -TeX simply harness its power via a whole lot of complicated macro definitions.*

TeX proper is a collection of around 300 so called *primitive* typesetting commands. These work at the very lowest level, affording enormous power. But to make this raw power manageable, some macros must be defined to tame raw TeX somewhat.

In the few years after the initial TeX release (1982), the macro package LaTeX was born. LaTeX was written for general usage. LaTeX scores high points for its enhanced command syntax. By far the majority of LaTeX users will never have to learn "raw" TeX, for they will be shielded from direct exposure by the numerous powerful macro packages.

#### Pre-amble

Every LaTeX document begins with a *pre-amble*. This consists of a set of commands that tells TeX how to process the document. We will explain the important parts of this in the next two sections. The first (*documentclass*) is mandatory, whereas the others are all optional.

#### Document classes

We have explained the concept of a document class. It remains to name a few, and indicate where they would be used. One *always* has to choose a document class when preparing a document with LaTeX.

The basic document classes in LaTeX are `article`, `letter`, `report`, and `book`. Many more are available, but these few cover the majority of straightforward applications. This is because classes are not rigid—you can impose your own parameter choices if you want. So one chooses the class that most closely approximates the document you have in mind, and performs some minor tweaks here and there. The `article` class is used for documents that are to have the appearance of a journal or magazine article, and is the class that should be used for your reports in this course. The `report` class is usually used for larger documents than the `article` class. These classes really only differ in their choice of default page size, font, placement of title and author, sectional units, etc. and on how they format certain LaTeX constructs. You use the same LaTeX commands in each. Since the examples here will be small, we will choose to use the `article` document class.

There are a number of possible options with each document class. The syntax for choosing a document class follows. Do not worry if this leaves you with no idea of how to choose a document style, for we will soon be seeing some examples. Also, remember that an argument in square brackets is optional, and can omitted altogether (including the brackets).

`\documentclass [`*`options`*`]{`*`class`*`}`

where *class* is the main document class (eg. `report`) and the optional argument *options* is a list of document style options chosen from, for example, the following list:

| | |
|---|---|
| `11pt` | chooses 11-point as the default font size for the document, instead of the default 10-point. |
| `12pt` | chooses 12-point as the default font size. |
| `twoside` | formats output as left and right pages, as in a book. |
| `twocolumn` | produces two-column magazine like output. |
| `titlepage` | applies to the `article` style only, causing the title and abstract to appear on a page each. |

In fact there are many, many more document class options but we will not mention any more here.

### Packages

To modify further the main document class, we make use of style files. They are used as in the following example: `\usepackage [`*`options`*`]{`*`package`*`}` where *package* is the modifying style file and *options* is a list of style files modifying the style file.

For example, here is a possible preamble:

```
\documentclass{article}
\usepackage{amsmath}
\usepackage{2130}
```

## 3.3.3 Special symbols

In the coming sections we will see that the the ten characters

`# $ % & ~ _ ^ \ { }`

are reserved for special use.

But what if we need one of these special symbols to appear in our document? The answer for seven of the symbols is to precede them by a \ character, so forming another *control symbol* (remember that \ followed by a space was also a control symbol) .

```
It is not 100\% straightforward to typeset the
characters \$ \& \% \_ \{ \}, but given the
enormous convenience of the use they are normally
reserved for this is a small price to pay.
```

produces

It is not 100% straightforward to typeset the characters $ & % _ { }, but given the enormous convenience of the use they are normally reserved for this is a small price to pay.

### So what are control symbols and words?

In typing a document, we can think of ourselves as being in one of two distinct modes. We are either typing *literal text* (which will just be set into neat paragraphs for us) or we are typing text that will be *interpreted* by LaTeX as an instruction to insert a special symbol or to perform some action. Thus we are either typing material that will go straight into the document (with some beautification), or we are giving commands to LaTeX.

Some commands are implicit, in that we do not have to do anything much extra. For instance, we command LaTeX to end the present sentence by typing a period (that does not follow a capital letter). These are not so much commands as part of having to describe the logical structure of a document.

A *control word* is something of the form `\commandname`, where the command name is a word made up only of the letters a to z and A to Z. A *control symbol* consists of a \ followed by single symbol that is not a letter.

Here are some examples:

- we have met the control space symbol \␣ before,
- the commands to set symbols like % and $ are control symbols
- `\@` is a control symbol that told LaTeX that the very next period did really end the sentence,
- `\LaTeX` is a control word that tell LaTeX to insert its own name at the current point,
- `\pm` instructs that a $\pm$ be inserted,
- `\div` inserts a $\div$ symbol,
- `\infty` inserts a $\infty$ symbol,
- `\em` makes the ensuing text *be emphasized.*

These examples show that control sequences can be used to access symbols not available from the keyboard, do some typesetting tricks like setting the word LaTeX the way it does, and change the appearance of whole chunks of text as with `\em`. We will be meeting many more of these type of control sequences.

Incidentally, underlining was the traditional way in which writers, working only with typewriters, were able to provide *emphasis*. Nowadays, underlining is poor form because it so easy to italize. With TeX, use `{\em something pretty long}` or `\emph{a word or two}` to produce *something pretty long* or *a word or two*.

Another enormously powerful class of control sequences is those that accept *arguments*. They tell LaTeX to take the parts of text you supply and do something with them—like make a fraction by setting the first argument over the second and drawing a line of the appropriate length between them. These are part of what makes LaTeX so powerful, and here are some examples.

- `\overline{words}` causes $\overline{\text{words}}$ to be overlined,
- `\frac{a+b}{c+d}` sets the given two argument as a fraction, doing most of the dirty work for us: $\frac{a+b}{c+d}$,
- `\sqrt[5]{a+b}` typesets the fifth-root of $a + b$, like this: $\sqrt[5]{a+b}$. The `5` is in square brackets instead of braces because it is an optional argument and could be ommited all together (giving the default of square root),

Mandatory arguments are given enclosed by braces, and optional arguments enclosed by square brackets. Each command knows how many arguments to expect, so you do not have to provide any indication of that.

We have actually jumped the gun a little. The above examples include examples of *mathematical* typesetting, and we have not yet seen how to tell LaTeX that it is typesetting math as opposed to some other random string of symbols that it does not understand either. We will come to mathematical typesetting in good time.

We need to dwell on a TeXnicality for a moment. How does LaTeX know where the name of a control sequence ends? Will it accept both `\pm3` and `\pm 3` in order to set $\pm3$, and will `\emWalrus` and `\em Walrus` both be acceptable in order to get *Walrus*? The answer is easy when you remember that a control word consists only of alphabetic characters, and a control symbol consists of exactly one nonalphabetic character.

So to determine which control sequence you typed, LaTeX does the following:

1. when a \ character is encountered, LaTeX knows that either a control symbol or a control word will follow.
2. If the \ is followed by a nonalphabetic character, then LaTeX knows that it is a control *symbol* that you have typed. It then recognises which one it was, typesets it, and goes on to read the character which follows the symbol you typed.

3. If the \ is followed by an alphabetic character, then LaTeX knows that it is a control word that you have typed. But it has to work out where the name of the control word ends and where the ensuing text takes over again. Since only alphabetic characters are allowed, LaTeX reads everything up to just before that first nonalphabetic character as the control sequence name. Since it is common to delimit the end of a control word by a space, LaTeX will *ignore* any space that follows a control word, since you want that space treated as end-of-control-word space rather than interword space.

This has one important consequence: The character in the input file immediately after a control symbol will be "seen" by LaTeX, but *any space following a control word will be discarded and never processed.* This does not affect one much if you adopt the convention of always typing a space after a control sequence name.

There is a rare circumstance in which this necessitates a little extra work and thought, which we illustrate by example:

```
If we type a control word like \LaTeX in the running text
then we must be cautious, because the string of spaces that
come after it will be discarded by the \LaTeX\ system.
```

which produces the output

> If we type a control word like LaTeXin the running text then we must be cautious, because the string of spaces that come after it will be discarded by the LaTeX system.

### Accents

LaTeX provides accents for just about all occasions. They are accessed through a variety of control symbols and single-letter control worlds which accept a single argument—the letter to be accented. These control sequences are detailed in table 3.1.

| | | | | | |
|---|---|---|---|---|---|
| `\`{o}` | ò | (grave accent) | `\u{o}` | ŏ | (breve accent) |
| `\'{o}` | ó | (acute accent) | `\v{o}` | ǒ | (háček or "check") |
| `\^{o}` | ô | (circumflex or "hat") | `\H{o}` | ő | (long Hungarian umlaut) |
| `\"{o}` | ö | (umlaut or dieresis) | `\t{oo}` | o͡o | (tie-after accent) |
| `\~{o}` | õ | (tilde or "squiggle") | `\c{o}` | o̧ | (cedilla accent) |
| `\={o}` | ō | (macron or "bar") | `\d{o}` | ọ | (dot-under accent) |
| `\.{o}` | ȯ | (dot accent) | `\b{o}` | o̲ | (bar-under accent) |

Table 3.1: Control sequences for accents

Thus we can produce ó by typing `\'{o}`, ǎ by typing `\v{a}`, and Pál Erdös by typing `P\'{a}l Erd\"{o}s`. Take special care when accenting an *i* or a *j*, for they should lose their dots when accented. Use the control words `\i` and `\j` to produce dotless versions of these letters. Thus the best way to type to type ĕxĭgent is `\u{e}x\u{\i}gent`.

### 3.3.4 Formatting

#### Font commands

We have seen a little of how to access various symbols using control sequences and we mentioned the `\em` command to emphasize text, but we did not see how to use them. We look here at commands that change the appearance of the text.

Each of the control words here is a directive rather than a control sequence that accepts an argument. This is because potential arguments consisting of text that wants to be emboldened or emphasized are very large, and it would be a nuisance to have to enclose such an argument in argument-enclosing braces.

To delimit the area of text over which one of these commands has effect (its *scope*) we make that text into what is called a *group*. Groups are used extensively in LaTeX to keep effects local to an area, rather than affecting the whole document. Apart from enhancing usability, this also in a sense protects distinct parts of a document from each other.

The LaTeX commands for changing type style are given in table 3.2, and those for changing type size are given in table 3.3. Commands for selecting fonts other than these are not discussed here.

| | | | | | |
|---|---|---|---|---|---|
| `\rm` | Roman | `\it` | *italic* | `\sc` | CAPITALS |
| `\em` | *Emphasized* | `\sl` | *slanted* | `\tt` | `typewriter` |
| `\bf` | **boldface** | `\sf` | sans serif | | |

Table 3.2: Commands for selecting type styles

Each of the type style selection commands selects the specified style but does not change the size of font being used. The default type style is roman (you are reading a roman style font now). To change type size you issue one of the type size changing commands in table 3.3, which will select the indicated size in the currently active style.

| size | default (10pt) | 11pt option | 12pt option |
|---|---|---|---|
| `\tiny` | 5pt | 6pt | 6pt |
| `\scriptsize` | 7pt | 8pt | 8pt |
| `\footnotesize` | 8pt | 9pt | 10pt |
| `\small` | 9pt | 10pt | 11pt |
| `\normalsize` | 10pt | 11pt | 12pt |
| `\large` | 12pt | 12pt | 14pt |
| `\Large` | 14pt | 14pt | 17pt |
| `\LARGE` | 17pt | 17pt | 20pt |
| `\huge` | 20pt | 20pt | 25pt |
| `\Huge` | 25pt | 25pt | 25pt |

Table 3.3: LaTeX size-changing commands.

The point-size option referred to in table 3.3 is that specified in the `\documentclass` command issued at the beginning of the input file. Through it you select that base (or default) font for your document to be 10, 11, or 12 point Roman. If no options are specified, the default is 10-point Roman. The table shows, for instance, that if I issue a `\large` in this document for which I chose the `12pt` document class option the result will be a 14-point Roman typeface.

We mentioned that to restrict the scope of a type-changing command we will set the text to be affected off in a group. Let us look at an example of this.

```
When we want to {\em emphasize\/} some text we
use the {\tt em} command, and use grouping to
restrict the scope.  We can change font {\large sizes}
in much the same way.  We can also obtain {\it italicized},
{\bf emboldened}, {\sc Capitals} and {\sf sans serif} styles.
```

When we want to *emphasize* some text we use the `em` command, and use grouping to restrict the scope. We can change font sizes in much the same way. We can also obtain *italicized*, **emboldened**, CAPITALS and sans serif styles.

Notice how clever grouping allows us to do all that without once having to use `\rm` or `\normalsize`.

One more thing slipped into that example—an italic correction `\/`. This is a very small amount of additional space that we asked to be inserted to allow for the change from sloping *emphasized* text to upright text, because the interword space has been made to look less substantial from the terminal sloping character. One has to keep an eye open for circumstances where this is necessary. See the effect of omitting an italic correction after the emphasized text earlier in this paragraph.

One might expect, by now, that LaTeX would insert an italic correction for us. But there are enough occasions when it is not wanted, and there is no good rule for LaTeX to use to decide just when to do it for us. So the italic correction is always left up to the typist.

### Sentences and paragraphs

Let us create our very first LaTeX document, which will consist of just a few paragraphs.

As mentioned above, paragraph input is free-form. You type the words and separate them by spaces so that LaTeX can distinguish between words. For these purposes, pressing Return is equivalent to inserting a space—it does not indicate the end of a line, but the end of a word. You tell LaTeX that a sentence has ended by typing a period followed by a space. LaTeX ignores extra spaces; typing three or three thousand will get you no more space between the words that these spaces separate than typing just one space. Finally, you tell LaTeX that a paragraph has ended by leaving one or more blank lines. In summary: LaTeX concerns itself only with the logical concepts end-of-word, end-of-sentence, and end-of-paragraph. Sounds complicated? The example in Figure 3.3 should clear things up. Try running LaTeX on this input.

We have learned more than just how LaTeX recognises words, sentences and paragraphs. We have also seen how to specify our choice of document class and how to tell LaTeX where our document begins and ends. Any material that is to be printed must lie somewhere between the

```
\documentclass{article}
\begin{document}
Words within a sentence are ended by spaces.  One space
between words  is     equivalent      to any number. We are only
interested in separating      one    word      from        the
next, not in formatting       the space between them.
For these purposes, pressing Return
at the end of a line
and starting a new word on the next line
just serves to separate
words, not to cut a line short.
The end of a sentence is indicated by a period
followed by one or more spaces.

The end of a paragraph is indicated by leaving a blank line.
All this
means that we can type without too much regard for layout, and
the typesetter will sort things out for us.
\end{document}
```

produces the result

Words within a sentence are ended by spaces. One space between words is equivalent to any number. We are only interested in separating one word from the next, not in formatting the space between them. For these purposes, pressing Return at the end of a line and starting a new word on the next line just serves to separate words, not to cut a line short. The end of a sentence is indicated by a period followed by one or more spaces. The end of a paragraph is indicated by leaving a blank line. All this means that we can type without too much regard for layout, and the typesetter will sort things out for us.

Figure 3.3: Some TeX input and the corresponding output

declaration of `\begin{document}` and that of `\end{document}`. Definitions that are to apply to the entire document can be made before the declaration of the document beginning. The specification of document class must precede all other material.

In future examples we will not explicitly display the commands that select document class and delimit the start and end of the document. But if you wish to try any of the examples, do not forget to include those commands. The `article` document class will do for most of our examples. Of course, the preceding example looks not at all like an article because it is so short and because we specified no title or author information.

Most of what you need to know to type regular text is contained in the example above. When you consider that by far the majority of any document consists of straight text, it is obvious that LaTeX makes this fabulously straightforward. LaTeX will do all the routine work of formatting, and we simply get on with the business of composing.

LaTeX does more than simply choose pleasing line breaks and provide natural spacing when setting a paragraph. Remember we said that TeX has inherited the knowledge of generations of professional printers—well part of that knowledge includes being on the look-out for *ligatures*.

These are combinations of letters within words which should be typeset as a single special symbol because they will “clash” with each if this is not done. Have a look at these words

flight, flagstaff, chaff, fixation

and compare them with these

flight, flagstaff, chaff, fixation

See the difference? In the first set I let LaTeX run as it usually does. In the second I overruled it somewhat, and stopped it from creating ligatures. Notice how the ‘fl’, ‘ff’, and ‘fi’ combinations are different in the two sets—in the former they form a single symbol (a ligature) and in the latter they are comprised of two disjoint symbols. There are other combinations that yield ligatures, but we do not have to bother remembering any of them because LaTeX will take care of these, too.

Notice, too, that LaTeX has been taught how to hyphenate the majority of words. It will hyphenate a word if it feels that the overall quality of the paragraph will be improved. For long words it has been taught several potential hyphenation positions.

LaTeX also goes to a lot of trouble to try to choose pleasing page breaks. It avoids “widows”, which are single lines of a paragraph occurring by themselves at either the bottom of a page (where it would have to be the first line of a paragraph) or at the top of a page (where it would have to be the last). It also “vertically justifies” your page so that all pages have exactly the same height, no matter what appears on them. As testimony to the success of the pagebreaking algorithm, I have (to this point) not once chosen a page break in this document.

### Punctuation

Typists have a convention whereby a single space is left after a mid-sentence comma, and two spaces are left after a sentence-ending period. How do we enforce this if LaTeX treats a string of spaces just like a single one? The answer, unsurprisingly, is that we *do not.*

```
To have a comma followed by the appropriate space,  we simply
type a comma follows by at least one space. To end a sentence
we type a period with at least one following space.     No space will
be inserted if we type a comma or period followed straight away
by something other than a space, because there are times when
we will not require any space, i.e., we do what comes naturally.
```

will produce

> To have a comma followed by the appropriate space, we simply type a comma follows by at least one space. To end a sentence we type a period with at least one following space. No space will be inserted if we type a comma or period followed straight away by something other than a space, because there are times when we will not require any space, i.e., we do what comes naturally.

LaTeX will produce suitable space after commas, periods, semi-colons and colons, exclamation marks, question marks etc. if they are followed by a space. In stretching a line to justify to the right margin, it also knows that space after a punctuation character should be more "stretchable" than normal inter-word space and that space after a sentence-ending period should be stretched more than space after a mid-sentence comma. TeX knows the nature of punctuation if you stick to the simple rules outlined here. As we have already said, those rules tell LaTeX how to distinguish consecutive words, sentences, phrases, etc.

Actually, there is more to ending sentences than mentioned above. Since LaTeX cannot speak English, it works on the assumption that *a period followed by a space ends a sentence unless the period follows a capital letter.* This works most of the time, but can fail. To get a normal inter-word space after a period that does not end a sentence, follow the period by a *control space*—a \␣ (a \ character followed by a space or return). Very rarely, you will have to force a sentence to end after a period that follows a capital letter (remember that LaTeX assumes this does not end a sentence). This is done by preceding the period with a `\@` command (you can guess from the odd syntax that this is rarely needed).

It is time we saw some examples of this. After all, this is our first experience of *control symbols* (do not worry, there are many more to come).

```
We must be careful not to confuse intra-sentence periods
with periods that end a sentence, i.e.\ we must remember
that our task is to describe the sentence structure.  Periods
that the typesetter requires a little help with typically result
from abbreviations, as in etc.\ and  others.  We have to work
somewhat harder to break a sentence after a capital letter,
but that should not bother us to much if we keep up our intake
of vitamin E\@.  All this goes for other sentence-ending
punctuation characters, so I could have said vitamin E\@!
Fortunately, these are rare occurrences.
```

results in

> We must be careful not to confuse intra-sentence periods with periods that end a sentence, i.e. we must remember that our task is to describe the sentence structure. Periods that the typesetter requires a little help with typically result from abbreviations, as in etc. and others. We have to work somewhat harder to break a sentence after a capital letter, but that should not bother us to much if we keep up our intake of vitamin E. All this goes for other sentence-ending punctuation characters, so I could have said vitamin E! Fortunately, these are rare occurrences.

Quotation marks is another area where LaTeX will do some work for us. Keyboards have the characters `‘`, `’`, and `"` but we want to have access to each of ‘, ’, “, and ”. So we proceed like this:

```
‘\LaTeX’ is no conventional word-processor, and
to to get quotes, like ‘‘this’’, we type repeated
{\tt ‘} and {\tt ’} characters.  Note that modern
convention is that ‘‘punctuation comes after
the closing quote character’’.
```

which gives just what we want

> ‘LaTeX’ is no conventional word-processor, and to to get quotes, like “this”, we type repeated `‘` and `’` characters. Note that modern convention is that “punctuation comes after the closing quote character”.

Very rarely, you have three quote characters together. Merely typing those three quote characters one-after-the-other is ambiguous—how should they be grouped? You tell LaTeX how you want them grouped by inserting a very small space called a *thin space*, and invoked with `\,`.

```
``\,`Green ham' or `Eggs?'\,'' is the question.
```

gives the desired result

“ ‘Green ham’ or ‘Eggs?’ ” is the question.

Since we have a typesetter at our disposal, we might as well use the correct dashes where we need them. There are three types of dash: the hyphen , the en-dash, and the em-dash. A minus sign is not a dash.

Hyphens are typed as you would hope, just by typing a `-` at the point in the word that you want a hyphen. Do not forget that LaTeX takes care of hyphenation that is required to produce pretty linebreaks. You only type a hyphen when you explicitly want one to appear, as in a combination like “inter-college”.

An en-dash is the correct dash to use in indicating number ranges, as in “questions 1–3”. To specify an en-dash you type two consecutive dashes on the keyboard, as in `1--3`.

An em-dash is a punctuation dash, used at the end of a sentence—I tend to use them too much. To specify an em-dash you type three consecutive dashes on the keyboard, as in “`...a sentence---I tend to...`”.

```
Theorems 1--3 concern the semi-completeness
of our new construct---in the case that it
satisfies the first axiom.
```

yields

Theorems 1–3 concern the semi-completeness of our new construct—in the case that it satisfies the first axiom.

## Ties

When you always remember to use *ties*, you know that you are becoming TeXnically inclined. Ties are used to prevent LaTeX from breaking lines at certain places. LaTeX will always choose line breaks that result in the most aesthetically pleasing paragraph as judged by its stringent rules. But because LaTeX does not actually understand the material it is setting so beautifully, it can make some poor choices.

A *tie* is the character `~`. It behaves as a normal interword space in all respects *except* that the line-breaking algorithm will never break a line at that point. Thus

```
Dr. Seuss should be typed as Dr.~Seuss
```

for this makes sure that LaTeX will never leave the ‘Dr’ at the end of one line and put the ‘Seuss’ at the beginning of the next.

One should try to get in to the habit of typing ties first-time, not after waiting to see if LaTeX will make a poor choice. This will allow you to make all sorts of changes to your text without ever having to go back and insert a tie at a point that has migrated to the end of a line from the middle of a line as a result of those changes.

Figure 3.4 shows some more examples of places where you should remember to place ties.

```
Lemmas 3 and~4          Chapter~10
Donald~E. Knuth         Appendix~C
width~2                 Figure~1
function~f              Theorem~2
1,~2, or~3              Lemmas 3 and~4
equals~5
```

Figure 3.4: Some places where ties are useful.

### Over-ruling some of TeX's choices

We have seen that ties can be used to stop linebreaks occurring between words. But how can we stop LaTeX from hyphenating a particular word? More generally, how can we stop it from splitting any given group of characters across two lines. The answer is to make that group of characters appears as one solid *box*, through use of an `\mbox` command.

```
For instance, if we wanted to be sure that the word
{\em currentitem\/} is not split across lines
then we should type it as \mbox{\em currentitem}.
```

If for some reason we wish to break a line
in the middle of nowhere, preventing LaTeX from justifying that line to the prevailing right margin, then we use the `\newline` command. One can also use the abbreviated form `\\`.

```
We start with a short line.\newline
And  now we continue with the normal
text, remembering that where we press
Return in the input file probably will not
correspond to a line break in the final
document.  More short lines\\
are easy, too.
```

will produce the line breaks we want

> We start with a short line.
> And now we continue with the normal text, remembering that where we press Return in the input file probably will not correspond to a line break in the final document. More short lines
> are easy, too.

A warning is in order: `\newline` must only end part of a line that is "already set". It cannot be used to add additional space between paragraphs, nor to leave space for a picture you want to paste in. This is not to be awkward, but is just part of LaTeX holding up its end of the deal by making you have to specially request additional vertical space. This prevents unwanted extra space from entering your document.

Later we shall see how to impose our own choice of page size, paragraph indentation, etc. For now we will continue to accept those declared for us in the document class.

### 3.3.5 Document structure

#### Sectioning commands

As part of our task of describing the logical structure of the document, we must indicate to LaTeX where to start sectional units. To do this we make use of the sectioning commands: `chapter`, `section`, `subsection`, `subsubsection`.

Each sectioning command accepts a single argument—the section heading that is to be used. LaTeX will provide the section numbering (and numbering of subsections within sections, etc.) so there is no need to include any number in the argument. LaTeX will also take care of whatever spacing is required to set the new logical unit off from the others, perhaps through a little extra space and using a larger font. It will also start a new page in the case that a new chapter is begun.

It is always a good idea to *plan* the overall sectional structure of a document in advance, or at least give it a little thought. Not that it would be difficult to change your mind later (you could use the global replace feature of an editor, for instance), but so that you have a good idea of the structure that you have to describe to LaTeX.

The sectioning command that began the present sectional unit of this document was

```
\subsection{Document structure}
```

and that was all that was required to get the numbered section name and the table of contents entry.

There are occasions when you want a heading to have all the appearance of a particular sectioning command, but should not be numbered as a section in its own right or produce a table of contents entry. This can be achieved through using the **-form* of the command, as in `\section*{...}`. We will see that many LaTeX commands have such a *-form which modify their behaviour slightly.

Not only will LaTeX number your sectional units for you, it will compile a table of contents too. Just include the command `\tableofcontents` after the
`\begin{document}` command and after the topmatter that should precede it.

#### LaTeX environments

Perhaps the most powerful and convenient concept in the LaTeX syntax is that of an *environment*. We will see most of the “heavy” typesetting problems we will encounter can be best tackled by one or other of the LaTeX environments.

Some environments are used to *display* a portion of text, i.e. to set it off from the surrounding text by indenting it. The `center` environment (note the American spelling!) allows us to centre portions of text, while the `flushright` environment sets small portions of text flush against the right margin.

But the true power of LaTeX begins to show itself when we look at environments such as those that provide facilities for itemized or enumerated lists, complex tabular arrangements, and for taking care of figure and table positioning and captioning. What we learn here will also be applicable in typesetting some complicated mathematical arrangements in the next section.

All the environments are begun by a `\begin{`*`name`*`}` command and ended by an `\end{`*`name`*`}`, where *name* is the environment name. These commands also serve as begin-group and end-group markers (see Sect. 3.3.4), so that all commands are local to the present environment—they cannot affect text outside the environment.

We can also have environment embedded within environment within environment and so on, limited only by memory available on the computer. We must, however, be careful to check that each of these *nested* environments is indeed contained within the one just outside of it.

### `quote` environment

This environment can be used to display a part of a sentence or paragraph in such a manner that the material stands out from the rest of the text. This can be used to enhance readability, or to simply emphasize something. Its syntax is simple:

```
Horace smiled and retaliated:
\begin{quote}
\em You can mock the non-WYSIWYG nature of \TeX\
all you like.  You do not understand that that is
precisely what makes \TeX\ enormously more powerful
than that lame excuse for a typesetter you use.
And I will bet that from start to finish of preparing
a document I am quicker than you are, even if you
do type at twice the speed and have the so-called
advantage of WYSIWYG.  In your case, what you see
is {\em all\/} you get!
\end{quote}
and then continued with composing his masterpiece of the
typesetting art.
```

produces the following typeset material:

Horace smiled and retaliated:

> *You can mock the non-WYSIWYG nature of TeX all you like. You do not understand that that is precisely what makes TeX enormously more powerful than that lame excuse for a typesetter you use. And I will bet that from start to finish of preparing a document I am quicker than you are, even if you do type at twice the speed and have the so-called advantage of WYSIWYG. In your case, what you see is* all *you get!*

and then continued with composing his masterpiece of the typesetting art.

That is a much more readable manner of presenting Horace's piece of mind than embedding it within a regular paragraph. The `quote` environment was responsible for the margins being indented on both sides during the quote. This example has also been used to show how the commands that begin and end an environment restrict the scope of commands issued within that environment: The `\em` at the beginning of the quote did not affect the text following the quote. We have also learned here that if we use `\em` within already emphasized text, the result is roman type—and we do not require an italic correction here because the final letter of 'all' was not sloping to the right.

Although LaTeX does not care too much for how we format our source file, it is obviously a good idea to lay it out logically and readably anyway. This helps minimize errors as well as aids in finding them. For this reason I have adopted the convention of always placing the environment `\begin` and `\end` commands on lines by themselves.

### `center` environment

This environment allows the centering of consecutive lines of text, new lines being indicated by a `\\`. If you do not separate lines with the `\\` command then you will get a centred paragraph the width of the page, which will not look any different to normal. If only one line is to be centred, then no `\\` is necessary.

```
The {\tt center} environment takes care of the vertical
spacing before and after it, so we do not need to leave any.
\begin{center}
If we leave no blank line after the\\
{\tt center} environment\\
then the ensuing text will not\\
be regarded as part of a new\\
paragraph, and so will not be indented.\\
\end{center}

In this case we left a blank line after the environment,
so the new text was regarded as starting a new paragraph.
```

gives the following text

The `center` environment takes care of the vertical spacing before and after it, so we do not need to leave any.

If we leave no blank line after the
`center` environment
then the ensuing text will not
be regarded as part of a new
paragraph, and so will not be indented.

In this case we left a blank line after the environment, so the new text was regarded as starting a new paragraph.

### `verbatim` environment

We can simulate typed text using the `verbatim` environment. The `\tt` (typewriter text) type style can be used for simulating typed words, but runs into trouble if one of the characters in the simulated typed text is a specially reserved LaTeX character. For instance, `{\tt type \newline}` would not have the desired effect because LaTeX would interpret the `\newline` as an instruction to start a new line.

The `verbatim` environment allows the simulation of multiple typed lines. *Everything* within the environment is typeset in typewriter font exactly as it appears in our source file—obeying spaces and line breaks as in the source file and not recognising the existence of any special symbols.

```
\begin{verbatim}
In the verbatim environment we can type anything
we like.
So we do not need to look out for uses of %, $, & etc,
nor will control sequences like \newline have any
effect.
\end{verbatim}
```

will produce the simulated input text

```
In the verbatim environment we can type anything
we like.
So we do not  need to look out for uses of %, $, & etc,
nor will control sequences like \newline have any
effect.
```

The only thing that cannot be typed in the `verbatim` environment is the sequence `\end{verbatim}`. You might notice that I still managed to simulate that control sequence above. One can always get what you want in TeX, perhaps with a little creativity.

If we want only to simulate a few typed words, such as when I say to use `\newline` to start a new line, then the `\verb` command is used. This command has a slightly odd syntax, pressed upon it by the use for which it was intended. It cannot accept an argument, because we may want to simulate typed text that is enclosed by `{braces}`. What one does is to choose any character that is *not* in the text to be simulated, and use a pair of these characters as "argument delimiters". I usually use the `@` or `"` characters, as I rarely have any other uses for them. Thus

use `\%` to obtain a % sign

is typed as

```
use \verb"\%" to obtain a \% sign
```

or

```
use \verb@\%@ to obtain a \% sign
```

### `itemize`, `enumerate`, `description` environments

LaTeX provides three predefined list-making environment, and a "primitive" list environment for designing new list environments of your own. We shall just describe the predefined ones here.

There is delightfully little to learn in order to be able to create lists. The only new command is `\item` which indicates the beginning of a new list item (and the end of the last one if this is not the first item). This command accepts an optional argument (which means you would enclose it in square brackets) that can be used to provide an item label. If no optional argument is given, then LaTeX will provide the item label for you; in an `itemize` list it will bullet the items, in an `enumerate` list it will number the items, and in a list of `description`s the default is to have no label (which would look a bit odd, so you are expected to use the optional argument there).

Remember that `\item` is used to separate list items; it does not accept the list item as an argument.

```
\begin{itemize}
\item an item is begun with \verb@\item@
\item if we do not specify labels, then
      \LaTeX\ will bullet the items for us
\item I indent lines after the first in the
      input file, but that is just to keep things
      readable.  As always, \LaTeX\ ignores additional
      spaces.

\item a blank line between items is ignored, for
      \LaTeX\ is responsible for spacing items.
\item \LaTeX\ is in paragraph-setting mode when
      it reads the text of an item, and so will
      perform all the usual functions
\end{itemize}
```

produces the following itemized list:

- an item is begun with `\item`
- if we do not specify labels, then LaTeX will bullet the items for us
- I indent lines after the first in the input file, but that is just to keep things readable. As always, LaTeX ignores additional spaces.
- a blank line between items is ignored, for LaTeX is responsible for spacing items.
- LaTeX is in paragraph-setting mode when it reads the text of an item, and so will perform all the usual functions

Lists can also be embedded within one another, for they are just environments and we said that environments have this property. Remember that we must nest them in the correct order. We demonstrate with the following list, which also shows how to use the `enumerate` environment.

```
\noindent I still have to do the following things:
\begin{enumerate}
\item Sort out LAN accounts for people on the course
   \begin{itemize}
   \item Have new accounts created for those not already
         registered on the LAN
   \item Make sure all users have a personal directory
        on the data drive
   \item Add users to the appropriate LAN print queues
   \end{itemize}
\item Have a \TeX\ batch file added to a directory that
      is on a public search path
\item Finish typing these course notes and proof-read them
\item Photocopy and bind the finished notes
\end{enumerate}
```

will give the following list

I still have to do the following things:

1. Sort out LAN accounts for people on the course
   - Have new accounts created for those not already registered on the LAN
   - Make sure all users have a personal directory on the data drive
   - Add users to the appropriate LAN print queues
2. Have a TeX batch file added to a directory that is on a public search path
3. Finish typing these course notes and proof-read them
4. Photocopy and bind the finished notes

See how I lay the source file out in a readable fashion. This is to assist myself, not LaTeX. The `description` environment is, unsurprisingly, for making lists of descriptions.

```
\begin{description}
\item[itemize] an environment for setting itemized lists.
\item[enumerate] an environment for setting numbered lists.
\item[description] an environment for listing descriptions
(like for words in a dictionary with boldface and a nice little indentation
after the first line).
\end{description}
```

will typeset the descriptions shown in Figure 3.5. Note that the scope of the `\tt` commands used in the item labels was restricted to the labels.

**itemize** an environment for setting itemized lists.

**enumerate** an environment for setting numbered lists.

**description** an environment for listing descriptions (like for words in a dictionary with boldface and a nice little indendation after the first line).

Figure 3.5: The description environment.

### `tabular` environment

The `tabular` environment is used to produce tables of items, particularly when the table is predominantly rectangular and when line drawing is required. LaTeX will make most decisions for us; for instance it will align everything for us without having to be told which are the longest entries in each column.

This environment is the first of many that use the TeX "tabbing character" `&`. This character is used to separate consecutive entries in a row of a table, array, etc. The end of a row is indicated in the usual manner, by using `\\`. In this way the individual cells of the table, or array, are clearly described to LaTeX, and it can analyse them to make typesetting decisions. Commands issued within a cell so defined are, again, local to that cell.

The `tabular` environment is also our first example of an *environment with arguments*. The arguments are given, in braces as usual, just after the closing brace after the environments name. In the case of `tabular` there is a single mandatory argument giving the justification of the entries in each column: `l` for left justified, `r` for right justified, and `c` for centred. There must be an entry for each column of the table, and there is no default. Let us start with a simple table.

```
\begin{tabular}{llrrl}
\bfseries{Student name} & \bfseries{Number}
      & \bfseries{Test 1} & \bfseries{Test 2} & \bfseries{Comment}\\
F. Basset    & 865432 & 78     & 85     & Pleasing\\
H. Hosepipe  & 829134 & 5      & 10     & Improving\\
I.N. Middle  & 853931 & 48     & 47     & Can make it
\end{tabular}
```

will produce the following no-frills table

| **Student name** | **Number** | **Test 1** | **Test 2** | **Comment** |
|---|---|---:|---:|---|
| F. Basset | 865432 | 78 | 85 | Pleasing |
| H. Hosepipe | 829134 | 5 | 10 | Improving |
| I.N. Middle | 853931 | 48 | 47 | Can make it |

Note that a `\\` was not necessary at the end of the last row. Also note that, once again, the alignment of the `&` characters was for human readability. It is conventional to set columns of numbers with right justification. The `\bf` directives apply only to the entries in which they are given.

A `|` typed in the `tabular` environment's argument causes a vertical line to be drawn at the indicated position and extending for the height of the entire table. An `\hline` given in the environment draws a horizontal line extending the width of the table to be drawn at the vertical position at which the command is given. A `\cline`{$i$-$j$} draws a line spanning columns $i$ to $j$, at the vertical position at which the command is given. A repeated line-drawing command causes a double line to be drawn. We illustrate line drawing in tables by putting some lines into our first table. We will type this example in a somewhat expanded form, trying to make it clear why the lines appear where they do.

```
\begin{tabular}{|l|l|r|r|l|}
\hline
\bfseries{Student name} & \bfseries{Number} & \bfseries{Test 1} & \bfseries{Test 2}
      & \bfseries{Comment}\\
\hline
F. Basset        & 865432     & 78          & 85          & Pleasing\\
\hline
H. Hosepipe      & 829134     & 5           & 10          & Improving\\
\hline
I.N. Middle      & 853931     & 48          & 47          & Can make it\\
\hline
\end{tabular}
```

which will give

| **Student name** | **Number** | **Test 1** | **Test 2** | **Comment** |
|---|---|--:|--:|---|
| F. Basset | 865432 | 78 | 85 | Pleasing |
| H. Hosepipe | 829134 | 5 | 10 | Improving |
| I.N. Middle | 853931 | 48 | 47 | Can make it |

That way of laying out the source file makes it clear where the lines will go. As we (by now) well know, the returns that we pressed after the `\\`s in typing this table might as well have been spaces as far as LaTeX is concerned. Thus it is common to have the `\hline` commands following the `\\`s on the input lines. We will do this in future examples.

The `\multicolumn` column can be used to overrule the overall format of the table for a few columns. The syntax of this command is

`\multicolumn` {*n*}{*pos*}{*item*}

where $n$ is the number of columns of the original format that *item* is to span, and *pos* specifies the justification of the new argument.

```
\begin{tabular}{||l|c|c|c||} \hline
\multicolumn{4}{|c|}{\LaTeX\ size changing commands}\\ \hline
Style option        & 10pt (default) & \ttfamily 11pt & \ttfamily 12pt\\ \hline
\tt footnotesize  & 8pt            & 9pt      & 10pt\\ \hline
\tt small         & 9pt            & 10pt     & 11pt\\ \hline
\tt large         & 12pt           & 12pt     & 14pt\\ \hline
\end{tabular}
```

produces the following table:

| LaTeX size changing commands | | | |
|---|:-:|:-:|:-:|
| Style option | 10pt (default) | `11pt` | `12pt` |
| `footnotesize` | 8pt | 9pt | 10pt |
| `small` | 9pt | 10pt | 11pt |
| `large` | 12pt | 12pt | 14pt |

### `figure` and `table` environments

Figures (diagrams, pictures, etc.) and tables (perhaps created with the `tabular` environment) cannot be split across pages. So LaTeX provides a mechanism for "floating" them to a nearby place where there is room for them. This may mean that your figure or table may appear a little later in the document than its declaration in the source file might suggest. You can suggest to LaTeX that it try to place the figure or table at the present position if there is room or, failing that, at the top or bottom of the present or following page. You can also ask for it to be presented by itself on a "page of floats".

You suggest these options to LaTeX through an optional argument to the environment. One lists a combination of the letters `h`, `t`, `b` and `p` with the following meaning:

**h** the object should be placed *here* if there is room, so that things will appear in the same order as in the source file,

**t** the object can be placed at the *top* of the of a text page, but no earlier than the present page.

**b** the object can be placed at the *bottom* of a text page, but no earlier than the present page.

**p** the object should be set on a *page of floats* that consists only of tables and figures.

A combination of these indicates decreasing order of preference. The default is `tbp`.

You may also force LaTeX to place the figure or table in a desired spot by using capital `H`. To do this you must include the float package by using `\usepackage{float}` in your preamble.

LaTeX will also number a figure or table for you, supply a caption and compile a list of tables and a list of figures. Just include `\listoffigures` and `\listoftables` next to your `\tableofcontents` command at the beginning of the document. To caption a table of figure, include `\caption{`*caption text*`}` just before the `\end{table}` or `\end{figure}` command. Here is a sample source file.

```
\begin{table}[htbp]
   \begin{tabular}{lrll}
   ...
   \end{tabular}
\caption{Mark analysis}
\end{table}
```

To leave space for a figure that will inserted by some other means at a later date, we can use the `\vspace` command:

```
\begin{figure}[htbp]
\vspace{9.5cm}
\caption{An artists impression}
\end{figure}
```

## 3.4 Mathematical typesetting with LaTeX

Original text by Gavin Maltby (1992); adapted by Math-2130 Instructors (1995,1998)

---

The last section taught us a good deal of what we need to know in order to prepare quite complicated non-mathematical documents. There are still a number of useful topics that we have not covered (such as cross-referencing), but we will defer discussion of those until a later section. In the present chapter, we will learn how LaTeX typesets mathematics. It should come as no surprise that LaTeX does most of the work for us.

### 3.4.1 Introduction

In text-only documents we saw that our task was to describe the logical components of each sentence, paragraph, section, table, etc. When we tell LaTeX to go into *mathematical mode*, we have to describe the logical parts of a formula, matrix, operator, special symbol, etc. TeX has been taught to recognize a binary operation, a binary relation, a variable, an operator that expects limits, and so on. We just need to supply the parts that make up each of these, and TeX will take care of the rest. It will leave appropriate space around operators, italicize variables, set an operator name in roman type, leave the correct space after colons, place sub- and superscripts in the correct positions (based on what it is you are working with), choose the correct typesizes, ... the list of things it has been taught is enormous. When you want to revert to setting normal text again, you tell LaTeX to leave math mode and go back into the mode it was in (paragraphing mode).

LaTeX cannot be expected to perform these mode shifts itself, for it is not always clear just when it is mathematics that has been typed. For example, should an isolated letter `a` in the input file be regarded as a word (as in the definite article) or a mathematical variable (as in the variable $a$). There are no reliable rules for LaTeX to make such decisions by, so the begin-math and end-math mode switching is left entirely to you.

The symbol `$` is specially reserved (See Sect. 3.3.3) by LaTeX as the "math shift" symbol. When LaTeX starts setting a document it is in paragraphing mode, ready to set lines of the input file into paragraphs. It remains in this mode until it encounters a `$` symbol, which shifts LaTeX into mathematical mode. It now knows to be on the look-out for the components of a mathematical expression, rather than for words and paragraphs. It reads everything up to the next `$` sign in this mathematical mode, and then shifts back to paragraphing mode (i.e. the mode it was in before we took it in to math mode).

You must be careful to balance your begin-math and end-math symbols. It is often a good idea to type two `$` symbols and then move back between them and type the mathematical expression. If the math-shift symbols in a document are not matched, then LaTeX will become confused because it will be trying to set non-mathematical material as mathematics.

For those who find having the same symbol for both math-begin and math-end confusing or dangerous, there are two control symbols that perform the same operations: the control symbol `\(` is a begin-math instruction, and the control symbol `\)` is an end-math instruction. Since it is easy to "lose" a `$` sign when typing a long formula, a math environment is provided for such occasions: you can use `\begin{math}` and `\end{math}` as the math-shift instructions. Of course, you could just decide to use `$` and take your chances.

Let us have a look at some mathematics.

```
\LaTeX\ is normally in paragraphing mode, where
it expects to find the usual paragraph material.  Including
a mathematical expression, like $2x+3y - 4= -1$, in the
paragraph text is easy.  \TeX\ has been taught to recognize
the basic elements of an expression, and typeset them appropriately,
choosing spacing, positioning, fonts, and so on.
Typing the above expression without entering math
mode produces the incorrect result: 2x+3y - 4= -1
```

will produce the following paragraph

> LaTeX is normally in paragraphing mode, where it expects to find the usual paragraph material. Including a mathematical expression, like $2x + 3y - 4 = -1$, in the paragraph text is easy. TeX has been taught to recognize the basic elements of an expression, and typeset them appropriately, choosing spacing, positioning, fonts, and so on. Typing the above expression without entering math mode produces the incorrect result: 2x+3y - 4= -1

Notice that LaTeX sets space around the binary relation $=$ and space around the binary operators $+$ and $-$ on the left hand side of the equation, ignoring the spacing we typed in the input. It was also able to recognize that the $-1$ on the right hand side of the equation was a unary minus—negating the 1 rather than being used to indicate subtraction—and so did not put space around it. It also italicized the variables $x$, $y$, and $z$. However, it did not italicize the number 1.

In typing a mathematical expression we must remember to keep the following in mind:

1. All letters that are not part of an argument to some control sequence will be italicized. Arguments to control sequences will be set according to the definition of the command used. So typing `$f(x)>0 for x > 1$` will produce

   $$f(x) > 0 for x > 1$$

   instead of the expression

   $$f(x) > 0 \quad \text{for} \quad x > 1$$

   that we intended. Numerals and punctuation marks are set in normal roman type but LaTeX will take care of the spacing around punctuation symbols, as in

   ```
   $f(x,y) \geq 0$
   ```

   which produces

   $$f(x,y) \geq 0 \quad .$$

2. Even a single letter can constitute a formula, as in "the constant $a$". To type this you enter `$a$` in your source file. If you do not go in to math mode to type the symbol, you will get things like "the constant a".

3. Some symbols have a different meaning when typed in math mode. Not only do ordinary letters become variables, but symbols such as `-` and `+` are now interpreted as mathematical symbols. Thus in math mode `-` is no longer considered a hyphen, but as a minus sign.

4. LaTeX ignores all spaces and carriage returns when in math mode, without exception. So typing something like `the constant$ a$` will produce "the constant$a$". You should have typed `the constant $a$`. LaTeX is responsible for all spacing when in math mode, and (as in paragraphing mode) you have to specially ask to have spacing changed. Even if LaTeX does ignore all spaces when in math mode you should (as always in TeX) still employ spaces to keep your source file readable.

The above means that, at least for most material, a typist need not understand the mathematics in order to typeset it correctly. And even if one does understand the mathematics, LaTeX is there to make sure that you adhere to accepted typesetting conventions (whether you were aware of their existence or not). So one could type either

```
$f(x, y)  =  2 (x+ y)y/(5xy - 3  )$
```

or

```
$f(x,y) = 2(x+y)y / (5xy-3)$
```

and you would still get the correct result

$$f(x, y) = 2(x + y)y/(5xy - 3) \quad .$$

There are some places where this can go wrong. For instance, if we wish to speak of the $x$-$y$ plane then one has to know that it is an *en-dash* that is supposed to be placed between the $x$ and the $y$, not a minus sign (as `$x-y$` would produce). But typing `$x--y$` will produce $x - -y$ since both dashes are interpreted as minus signs. To avoid speaking of the $x - y$ plane or the $x - -y$ plane, we should type it as `the $x$--$y$ plane`. We are fortunate that LaTeX can recognise and cope with by far the majority of our mathematical typesetting needs.

Another thing to look out for is the use of braces in an expression. Typing

```
${x : f(x)>0}$
```

will not produce any braces. This is because, as we well know, braces are reserved for delimiting groups in the input file. Looking back to section 3.3.3, we see how it should be done:

```
$ \{ x: f(x)>0 \} $
```

Math shift commands also behave as scope delimiters, so that commands issued in an expression cannot affect anything else in a document.

### 3.4.2 Displaying a formula

LaTeX considers an expression `$ ... $` to be word-like in the sense that it considers it to be eligible for splitting across lines of a paragraph (but without hyphenation, of course). LaTeX assigns quite a high penalty to doing this, thus trying to avoid it (remember that LaTeX tries to minimize the "badness" of a paragraph). When there is no other way, it will split the expression at a suitable place. But there are some expressions which are just too long to fit into the running text without looking awkward. These are best "displayed" on a line by themselves. Also, some expressions are sufficiently important that they should be made to stand out. These, too, should be displayed on a line of their own.

The mechanism for displaying an expression is the *display math* mode, which is begun by typing `$$` and ended by typing the same sequence (which again means that we had better be sure to type them in pairs). Corresponding to the alternatives `\(` and `\)` that we had for the math shift character `$`, we may use `\[` and `\]` as the display-math shift sequences. One can also use the environment

```
\begin{displaymath} ... \end{displaymath}
```

which is equivalent to `$$ ... $$` and is suitable for use with long displayed expressions. If you wish LaTeX to number your equations for you you can use the environment

```
\begin{equation} ... \end{equation}
```

which is the same as the `displaymath` environment, except that an equation number will be generated.

It is poor style to have a displayed expression either begin a paragraph or be a paragraph by itself. This can be avoided if you agree to *never leave a blank line in your input file before a math display.*

We will see later how to typeset an expression that is to span multiple lines. For now, let us look at an example of displaying an expression:

```
For each $a$ for which the Lebesgue-set $L_a(f) \neq \emptyset$ we define
$$ % We could have used \begin{displaymath} here
   {\cal B}_a(f) = \{ L_{a+r}(f) : r > 0  \},
$$ % and \end{displaymath} here and these are easily seen to be completely regular.
```

which produces

For each $a$ for which the Lebesgue-set $L_a(f) \neq \emptyset$ we define

$$\mathcal{B}_a(f) = \{L_{a+r}(f) : r > 0\},$$

and these are easily seen to be completely regular.

That illustrates how to display an expression, but also shows that we have got a lot more to learn about mathematical typesetting. Before we have a look at how to arrange symbols all over the show (e.g. the subscripting above) we must learn how to access the multitude of symbols that are used in mathematical texts.

### 3.4.3 Using mathematical symbols

LaTeX puts all the esoteric symbols of mathematics at our fingertips. They are all referenced by name, with the naming system being perfectly logical and systematic. None of the control words that access these symbols accepts an argument, but we will soon see that some of them prepare LaTeX for something that might follow. For instance, when you ask for the symbol '$\sum$' LaTeX is warned that any sub- or superscripts that follow should be positioned appropriately as limits to a summation. In keeping with the TeX spirit, none of this requires any additional work on your part.

We will also see that some of the symbols behave differently depending on where they are used. For instance, when I ask for $\sum_{i=1}^n a_i$ within the running text, the limits are places differently to when I ask for that expression to be displayed:

$$\sum_{i=1}^n a_i \quad .$$

Again, I typed nothing different here—just asked for display math mode.

It is important to note that *almost all of the special math symbols are unavailable in ordinary paragraphing mode.* If you need to use one there, then use an in-line math expression `$...$`.

#### Symbols available from the keyboard

A small percentage of the available symbols can be obtained from just a single key press. They are $+ - = < > | / ( ) [ ]$ and $*$. Note that these must be typed *within math mode* to be interpreted as math symbols.

Of course, all of $a$–$z$, $A$–$Z$, the numerals $0, 1, 2, \ldots, 9$ and the punctuation characters , ; and : are available directly from the keyboard. Alphabetic letters will be assumed to be variables that are to be italicized, unless told otherwise (see Sect. 3.4.4). The numerals receive no special attention, appearing precisely as in normal paragraphing mode. The punctuation symbols are still set in standard roman type when read in math mode, but a little space is left after them so that expressions like $\{x_i : i = 1, 2, \ldots, 10\}$ look like they should. Note that this means that normal sentence punctuation should not migrate into an expression.

#### Greek letters

Tables 3.4 and 3.5 show the control sequences that produce the letters of the Greek alphabet. We see that a lowercase Greek letter is simply is accessed by typing the control word of the same name as the symbol, using all lowercase letters. To obtain an uppercase Greek letter, simply capitalise the *first* letter of its name.

Just as `$mistake$` produces $mistake$ because the letters are interpreted as variables, so too will `$\tau \epsilon \chi$` produce the incorrectly spaced $\tau\epsilon\chi$ if you try to type greek words like this. TeX can be taught to set Greek, but this is not the way. τεχ, incidentally, is the Greek word for "art" and it is from the initials of the Greek letters constituting this word that the name TeX was derived. TeX is "the art of typesetting".

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\alpha$ | `\alpha` | $\beta$ | `\beta` | $\gamma$ | `\gamma` | $\delta$ | `\delta` |
| $\epsilon$ | `\epsilon` | $\varepsilon$ | `\varepsilon` | $\zeta$ | `\zeta` | $\eta$ | `\eta` |
| $\theta$ | `\theta` | $\vartheta$ | `\vartheta` | $\iota$ | `\iota` | $\kappa$ | `\kappa` |
| $\lambda$ | `\lambda` | $\mu$ | `\mu` | $\nu$ | `\nu` | $\xi$ | `\xi` |
| $\pi$ | `\pi` | $\varpi$ | `\varpi` | $\rho$ | `\rho` | $\varrho$ | `\varrho` |
| $\sigma$ | `\sigma` | $\varsigma$ | `\varsigma` | $\tau$ | `\tau` | $\upsilon$ | `\upsilon` |
| $\phi$ | `\phi` | $\varphi$ | `\varphi` | $\chi$ | `\chi` | $\psi$ | `\psi` |
| $\omega$ | `\omega` | | | | | | |

Table 3.4: Lowercase Greek letters

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\Gamma$ | `\Gamma` | $\Delta$ | `\Delta` | $\Theta$ | `\Theta` | $\Lambda$ | `\Lambda` |
| $\Xi$ | `\Xi` | $\Pi$ | `\Pi` | $\Sigma$ | `\Sigma` | $\Upsilon$ | `\Upsilon` |
| $\Phi$ | `\Phi` | $\Psi$ | `\Psi` | $\Omega$ | `\Omega` | | |

Table 3.5: Uppercase Greek letters

### Calligraphic uppercase letters

The letters $\mathcal{A}, \ldots, \mathcal{Z}$ are available through use of the style changing command `\cal`. This command behaves like the other style changing commands `\em`, `\it`, etc. so its scope must be delimited as with them. Thus we can type

```
... for the filter $\cal F$ we have $\varphi({\cal F}) = \cal G$.
```

to obtain

for the filter $\mathcal{F}$ we have $\varphi(\mathcal{F}) = \mathcal{G}$.

There is no need to tabulate all the calligraphic letters, since they are all obtained by just a type style changing command. We will just list them so that we can see, for reference purposes, what they all look like. Here they are:

$$\mathcal{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$$

### Binary operators

LaTeX has been taught to recognise binary operators and set the appropriate space either side of one—i.e., it sets the first argument followed by a little space, then the operator followed by the same little space and finally the second argument. Table 3.6 shows the binary operators that are available via LaTeX control words (recall that the binary operators $+$, $-$, and $*$ can be typed from the keyboard). Here are some examples of their use:

| *Type* | *To produce* |
|---|---|
| `$a+b$` | $a+b$ |
| `$(a+b) \otimes c$` | $(a+b) \otimes c$ |
| `$(a \vee b) \wedge c$` | $(a \vee b) \wedge c$ |
| `$X - (A \cap B) = (X-A) \cup (X-B)$` | $X - (A \cap B) = (X-A) \cup (X-B)$ |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\pm$ | `\pm` | $\cap$ | `\cap` | $\diamond$ | `\diamond` | $\oplus$ | `\oplus` |
| $\mp$ | `\mp` | $\cup$ | `\cup` | $\bigtriangleup$ | `\bigtriangleup` | $\ominus$ | `\ominus` |
| $\times$ | `\times` | $\uplus$ | `\uplus` | $\bigtriangledown$ | `\bigtriangledown` | $\otimes$ | `\otimes` |
| $\div$ | `\div` | $\sqcap$ | `\sqcap` | $\triangleleft$ | `\triangleleft` | $\oslash$ | `\oslash` |
| $\ast$ | `\ast` | $\sqcup$ | `\sqcup` | $\triangleright$ | `\triangleright` | $\odot$ | `\odot` |
| $\star$ | `\star` | $\vee$ | `\vee` | $\wedge$ | `\wedge` | $\bigcirc$ | `\bigcirc` |
| $\dagger$ | `\dagger` | $\setminus$ | `\setminus` | $\amalg$ | `\amalg` | $\circ$ | `\circ` |
| $\ddagger$ | `\ddagger` | $\cdot$ | `\cdot` | $\wr$ | `\wr` | $\bullet$ | `\bullet` |

Table 3.6: Binary Operation Symbols

### Binary relations

LaTeX has been taught to recognize the use of binary relations, too. Table 3.7 shows those available via LaTeX control words. There are a few that you can obtain directly from the keyboard: $<$, $>$, $=$, and $|$.

To negate a symbol you can precede the control word that gives the symbol by a `\not`. Some symbols come with ready-made negations, which should be used instead of the `\not`'ing method because the slope of the negating line is just slightly changed to look more pleasing. Thus `\notin` should be used instead of `\not\in` and `\ne` should be used instead of `\not =`.

If negating a symbol produces a slash whose horizontal positioning is not to your liking, then use the math spacing characters described in section 3.4.4 to adjust it.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\leq$ | `\leq` | $\geq$ | `\geq` | $\equiv$ | `\equiv` | $\models$ | `\models` |
| $\prec$ | `\prec` | $\succ$ | `\succ` | $\sim$ | `\sim` | $\perp$ | `\perp` |
| $\preceq$ | `\preceq` | $\succeq$ | `\succeq` | $\simeq$ | `\simeq` | $\mid$ | `\mid` |
| $\ll$ | `\ll` | $\gg$ | `\gg` | $\asymp$ | `\asymp` | $\parallel$ | `\parallel` |
| $\subset$ | `\subset` | $\supset$ | `\supset` | $\approx$ | `\approx` | $\bowtie$ | `\bowtie` |
| $\subseteq$ | `\subseteq` | $\supseteq$ | `\supseteq` | $\cong$ | `\cong` | $\Join$ | `\Join` |
| $\sqsubset$ | `\sqsubset` | $\sqsupset$ | `\sqsupset` | $\neq$ | `\neq` | $\smile$ | `\smile` |
| $\sqsubseteq$ | `\sqsubseteq` | $\sqsupseteq$ | `\sqsupseteq` | $\doteq$ | `\doteq` | $\frown$ | `\frown` |
| $\in$ | `\in` | $\ni$ | `\ni` | $\propto$ | `\propto` | | |
| $\vdash$ | `\vdash` | $\dashv$ | `\dashv` | | | | |

Table 3.7: Binary relations

### Miscellaneous symbols

Table 3.8 shows a number of general-purpose symbols. Remember that these are only available in math mode. Note that `\imath` and `\jmath` should be used when you need to accent an $i$ or a $j$ in math mode (see Sect. 3.4.3) —you cannot use `\i` or `\j` that were available in paragraphing mode. To get a prime symbol, you can use `\prime` or you can just type `'` when in math mode, as in `$f''(x)=x$` which produces $f''(x) = x$.

The symbols $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, commonly used to denote number domains (natural, integer, rational, real, complex, and quaternion, respectively) are obtained by the commands like `\mathbb{N}`. You need to `\includepackage{amssymb}` in the preamble of your document.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\aleph$ | `\aleph` | $\prime$ | `\prime` | $\forall$ | `\forall` | $\infty$ | `\infty` |
| $\hbar$ | `\hbar` | $\emptyset$ | `\emptyset` | $\exists$ | `\exists` | $\Box$ | `\Box` |
| $\imath$ | `\imath` | $\nabla$ | `\nabla` | $\neg$ | `\neg` | $\triangle$ | `\triangle` |
| $\jmath$ | `\jmath` | $\surd$ | `\surd` | $\flat$ | `\flat` | $\triangle$ | `\triangle` |
| $\ell$ | `\ell` | $\top$ | `\top` | $\natural$ | `\natural` | $\clubsuit$ | `\clubsuit` |
| $\wp$ | `\wp` | $\bot$ | `\bot` | $\sharp$ | `\sharp` | $\diamondsuit$ | `\diamondsuit` |
| $\Re$ | `\Re` | $\Vert$ | `\\|` | $\backslash$ | `\backslash` | $\heartsuit$ | `\heartsuit` |
| $\Im$ | `\Im` | $\angle$ | `\angle` | $\partial$ | `\partial` | $\spadesuit$ | `\spadesuit` |
| $\mho$ | `\mho` | | | | | | |

Table 3.8: Miscellaneous symbols

## Arrow symbols

LaTeX has a multitude of arrow symbols, which it will set the correct space around. Note that vertical arrows can all be used as delimiters—see section 3.4.3. The available symbols are listed in table 3.9.

| | | | | | |
|---|---|---|---|---|---|
| $\leftarrow$ | `\leftarrow` | $\longleftarrow$ | `\longleftarrow` | $\uparrow$ | `\uparrow` |
| $\Leftarrow$ | `\Leftarrow` | $\Longleftarrow$ | `\Longleftarrow` | $\Uparrow$ | `\Uparrow` |
| $\rightarrow$ | `\rightarrow` | $\longrightarrow$ | `\longrightarrow` | $\downarrow$ | `\downarrow` |
| $\Rightarrow$ | `\Rightarrow` | $\Longrightarrow$ | `\Longrightarrow` | $\Downarrow$ | `\Downarrow` |
| $\leftrightarrow$ | `\leftrightarrow` | $\longleftrightarrow$ | `\longleftrightarrow` | $\updownarrow$ | `\updownarrow` |
| $\Leftrightarrow$ | `\Leftrightarrow` | $\Longleftrightarrow$ | `\Longleftrightarrow` | $\Updownarrow$ | `\Updownarrow` |
| $\mapsto$ | `\mapsto` | $\longmapsto$ | `\longmapsto` | $\nearrow$ | `\nearrow` |
| $\hookleftarrow$ | `\hookleftarrow` | $\hookrightarrow$ | `\hookrightarrow` | $\searrow$ | `\searrow` |
| $\leftharpoonup$ | `\leftharpoonup` | $\rightharpoonup$ | `\rightharpoonup` | $\swarrow$ | `\swarrow` |
| $\leftharpoondown$ | `\leftharpoondown` | $\rightharpoondown$ | `\rightharpoondown` | $\nwarrow$ | `\nwarrow` |
| $\rightleftharpoons$ | `\rightleftharpoons` | $\leadsto$ | `\leadsto` | | |

Table 3.9: Arrow symbols

## Expression delimiters

A pair of delimiters often enclose an expression, as in

$$\left[ \begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right] \quad \text{and} \quad f(x) = \left\{ \begin{array}{ll} x & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{array} \right. \qquad .$$

LaTeX will scale delimiters to the correct size (determined by what they enclose) for you, if you ask it to. There are times when you do not want a delimiter to be scaled, so it is left up to you to ask for scaling.

To ask that a delimiter be scaleable, you precede it by `\left` or `\right` according as it is the left or right member of the pair. Scaled delimiters must be balanced correctly. It sometimes occurs, as in the right-hand example above, that only one member of a delimiting pair is to be visible. For this purpose, use the commands `\left.` and `\right.` which will produce no visible delimiter but can be used to correctly balance the delimiters in an expression. For examples of the use of delimiters, see section 3.4.4 where we learn about arrays.

Table 3.10 shows the symbols that LaTeX will recognize as delimiters, i.e. symbols that may follow a `\left` or a `\right`. Note that you have to use `\left\{` and `\right\}` in order to get scaled braces.

| | | | | | |
|---|---|---|---|---|---|
| $($ | `(` | $)$ | `)` | $\uparrow$ | `\uparrow` |
| $[$ | `]` | $]$ | `]` | $\downarrow$ | `\downarrow` |
| $\{$ | `\{` | $\}$ | `\}` | $\updownarrow$ | `\updownarrow` |
| $\lfloor$ | `\lfloor` | $\rfloor$ | `\rfloor` | $\Uparrow$ | `\Uparrow` |
| $\lceil$ | `\lceil` | $\rceil$ | `\rceil` | $\Downarrow$ | `\Downarrow` |
| $\langle$ | `\langle` | $\rangle$ | `\rangle` | $\Updownarrow$ | `\Updownarrow` |
| $/$ | `/` | $\backslash$ | `\backslash` | | |
| $\vert$ | `|` | $\Vert$ | `\|` | | |

Table 3.10: Delimiters

### Operators like $\int$ and $\sum$

These behave differently when used in display-math mode as compared with in-text math mode. When used in text, they will appear in their small form and any limits provided will be set so as to reduce the overall height of the operator, as in $\sum_{i=1}^{N} f_i$. When used in display-math mode, LaTeX will choose to use the larger form and will not try to reduce the height of the operator, as in

$$\sum_{i=1}^{N} f_i \quad .$$

Table 3.11 describes what variable-size symbols are available, showing both the small (in text) and the large (displayed) form of each. In section 3.4.4 we will learn how to place limits on these operators.

| | | | | | |
|---|---|---|---|---|---|
| $\textstyle\sum \displaystyle\sum$ | `\sum` | $\textstyle\bigcap \displaystyle\bigcap$ | `\bigcap` | $\textstyle\bigodot \displaystyle\bigodot$ | `\bigodot` |
| $\textstyle\prod \displaystyle\prod$ | `\prod` | $\textstyle\bigcup \displaystyle\bigcup$ | `\bigcup` | $\textstyle\bigotimes \displaystyle\bigotimes$ | `\bigotimes` |
| $\textstyle\coprod \displaystyle\coprod$ | `\coprod` | $\textstyle\bigsqcup \displaystyle\bigsqcup$ | `\bigsqcup` | $\textstyle\bigoplus \displaystyle\bigoplus$ | `\bigoplus` |
| $\textstyle\int \displaystyle\int$ | `\int` | $\textstyle\bigvee \displaystyle\bigvee$ | `\bigvee` | $\textstyle\biguplus \displaystyle\biguplus$ | `\biguplus` |
| $\textstyle\oint \displaystyle\oint$ | `\oint` | $\textstyle\bigwedge \displaystyle\bigwedge$ | `\bigwedge` | | |

Table 3.11: Variable-sized symbols

### Accents

The accenting commands that we learned for paragraphing mode do not apply in math mode. Consult table 3.12 to see how to accent a symbol in math mode (all the examples there accent the symbol $u$, but they work with any letter). Remember that $i$ and $j$ should lose their dots when accented, so `\imath` and `\jmath` should be used.

There also exist commands that give a "wide hat" or a "wide tilde" to their argument, `\widehat` and `\widetilde`.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\hat{u}$ | `\hat{u}` | $\acute{u}$ | `\acute{u}` | $\bar{u}$ | `\bar{u}` | $\dot{u}$ | `\dot{u}` |
| $\check{u}$ | `\check{u}` | $\grave{u}$ | `\grave{u}` | $\vec{u}$ | `\vec{u}` | $\ddot{u}$ | `\ddot{u}` |
| $\breve{u}$ | `\breve{u}` | $\tilde{u}$ | `\tilde{u}` | | | | |

Table 3.12: Math accents

### 3.4.4 Some common mathematical structures

In this section we shall begin to learn how to manipulate all the symbols listed in section 3.4.3. Indeed, by the end of this section we will be able to typeset some quite large expressions. In the section following this we will learn how use various alignment environments that allow us to prepare material like multi-line expressions and arrays.

#### Subscripts and superscripts

Specifying a sub- or superscript is as easy as you would hope—you just give an indication that you want a sub- or superscript to the last expression and provide the material to be placed there, and LaTeX will position things correctly. So sub- and superscripting a single symbol, an operator, or a big array all involve the same input, and LaTeX places the material according to what the expression is that is being sub- or superscripted:

$$x^2 \quad , \quad \prod_{i=1}^{N} X_i \quad , \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^2 \quad .$$

To tell LaTeX that you want a single character set as a superscript to the last expression, you just type a ^ before it. The "last expression" is the preceding group or, if there is no preceding group, the single character or symbol that the ^ follows:

| *Type* | *To produce* |
|---|---|
| `$x^2$` | $x^2$ |
| `$a^b$` | $a^b$ |
| `$Y^X$` | $Y^X$ |
| `$\gamma^2$` | $\gamma^2$ |
| `$(A+B)^2$` | $(A+B)^2$ |
| `$\left[ \frac{x^2+1}{x^2 + y^2} \right]^n$` | $\left[ \frac{x^2+1}{x^2 + y^2} \right]^n$ |

Subscripts of a single character are equally easy—you just use the underscore character _ where you used ^ for superscripting:

| *Type* | *To produce* |
|---|---|
| `$x_2$` | $x_2$ |
| `$x_i$` | $x_i$ |
| `$\Gamma_1(x)$` | $\Gamma_1(x)$ |

Now let us see how to set a sub- or superscript that consists of more than just one character. This is no more difficult than before if we remember the following rule: `_` *and* `^` *set the group that follows them as a sub- and superscripts to the group that precedes the sub- and superscript symbols.* We see now now that our initial examples worked by considering a single character to be a group by itself. Here are some examples:

| *Type* | *To produce* |
|---|---|
| `$a^2b^3$` | $a^2b^3$ |
| `$2^{21}$` | $2^{21}$ |
| `$2^21$` | $2^21$ |
| `$a^{x+1}$` | $a^{x+1}$ |
| `$a^{x^2+1}$` | $a^{x^2+1}$ |
| `$(x+1)^3$` | $(x+1)^3$ |
| `$\Gamma_{\alpha\beta\gamma}$` | $\Gamma_{\alpha\beta\gamma}$ |
| `${}_1A_2$` | ${}_1A_2$ |

In the very last example we see a case of setting a subscript to an empty group, which resulted in a kind of "pre-subscript". With some imagination this can be put to all sorts of uses.

In all of the above examples the sub- and superscripts were set to single-character groups. Nowhere did we group an expression before sub- or superscripting it. Even in setting the expression $(x+1)^3$, the superscript $^3$ was really only set to the character ). If we had wanted to group the $(x+1)$ before setting the superscript, we would have typed `${(x+1)}^3$` which gives ${(x+1)}^3$, with the superscript slightly raised. One has to go to this trouble because, to most people, something like $(x^a)^b$ is just as acceptable and as readable as ${(x^a)}^b$. It also has the advantage of aligning the base lines in expressions such as

$$(ab)^{-2} = [(ab)^{-1}]^2 = [b^{-1}a^{-1}]^2 = b^{-1}a^{-1}b^{-1}a^{-1}$$

which looks more pleasing than if we use additional grouping to force

$${(ab)}^{-2} = {[{(ab)}^{-1}]}^2 = {[b^{-1}a^{-1}]}^2 = b^{-1}a^{-1}b^{-1}a^{-1} \quad ,$$

and the latter has rather more braces in it that require balancing.

Here are some more examples, showing how LaTeX will set things just as we want without any further work on our part:

| *Type* | *To produce* |
|---|---|
| `$x^{y^z}$` | $x^{y^z}$ |
| `$2^{(2^2)}$` | $2^{(2^2)}$ |
| `$2^{2^{2^{\aleph_0}}}$` | $2^{2^{2^{\aleph_0}}}$ |
| `$\Gamma^{z_c^d}$` | $\Gamma^{z_c^d}$ |

We can also make use of empty groups in order to stagger sub- and superscripts to an expression, as in

```
$\Gamma_{\alpha\beta}{}^{\gamma}{}_\delta$
```

which will yield

$$\Gamma_{\alpha\beta}{}^{\gamma}{}_\delta$$

One can specify the sub- and superscripts to a group in any order, but it is best to be consistent. The most natural order seems to be to have subscripts first, but you may think otherwise. It is also a good idea to always include your sub- and superscripts in braces (i.e. make them a group), whether they consist of just a single character or not. This enhances readability and also helps avoid the unfortunate case where you believe that a particular control word gives a single symbol yet it really is defined in terms of several.

## Primes

LaTeX provides the control word `\prime` ($\prime$) for priming symbols. Note that it is not automatically at the superscript height, so that to get $f'$ you would have to type

```
$f^\prime$ .
```

To make lighter work of this, LaTeX will interpret a right-quote character as a prime if used in math mode. Thus we can type

```
$f'(g(x)) g'(x) h''(x)$
```

in order to get

$$f'(g(x))g'(x)h''(x) \quad .$$

## Fractions

LaTeX provides the `\frac` command that accepts two arguments: the numerator and the denominator (in that order). Before we look at examples of its use, let us just note that many simple in-text fractions are often better written in the form *num/den*, as with 3/8 which can be typed as `$3/8$`. This is also often the better form for a fraction that occurs *within* some expression.

| *Type* | *To produce* |
|---|---|
| `$\frac{x+1}{x+2}$` | $\frac{x+1}{x+2}$ |
| `$\frac{1}{x^2+1}$` | $\frac{1}{x^2+1}$ |
| `$\frac{1+x^2}{x^2+y^2} + x^2 y$` | $x^2y+\frac{1+x^2}{x^2+y^2}$ |
| `$\frac{1}{1 + \frac{x}{2}}$` | $\frac{1}{1+\frac{x}{2}}$ |
| `$\frac{1}{1+x/2}$` | $\frac{1}{1+x/2}$ |

### Roots

The `\sqrt` command accepts two arguments. The first, and optional, argument specifies what order of root you desire if it is anything other than the square root. The second, and mandatory, argument specifies the expression that the root sign should enclose:

| *Type* | *To produce* |
|---|---|
| `$\sqrt{a+b}$` | $\sqrt{a+b}$ |
| `$\sqrt[5]{a+b}$` | $\sqrt[5]{a+b}$ |
| `$\sqrt[n]{\frac{1+x}{1+x^2}}$` | $\sqrt[n]{\frac{1+x}{1+x^2}}$ |
| `$\frac{\sqrt{x+1}} {\sqrt[3]{x^3+1}}$` | $\frac{\sqrt{x+1}}{\sqrt[3]{x^3+1}}$ |

### Ellipsis

Simply typing three periods in a row will not give the correct spacing of the periods if it is an ellipsis that is desired. So LaTeX provides the commands `\ldots` and `\cdots`. Centered ellipsis should be used between symbols like $+$, $-$, $*$, $\times$, and $=$. Here are some examples:

| *Type* | *To produce* |
|---|---|
| `$a_1+ \cdots + a_n$` | $a_1+\cdots+a_n$ |
| `$x_1 \times x_2 \times \cdots \times x_n$` | $x_1\times x_2\times\cdots\times x_n$ |
| `$v_1 = v_2 = \cdots = v_n = 0$` | $v_1=v_2=\cdots=v_n=0$ |
| `$f(x_1,\ldots,x_n) = 0$` | $f(x_1,\ldots,x_n)=0$ |

### Text within an expression

One can use the `\mbox` command to insert normal text into an expression. This command forces LaTeX temporarily out of math mode, so that its argument will be treated as normal text. Its use is simple, but we must be wary on one count: remember that LaTeX ignores all space characters when in math mode; so to surround words in an expression that were placed by an `\mbox` command by space you must include the space in the `\mbox` argument.

| *Type* | *To produce* |
|---|---|
| `$f_i(x) \leq 0 \mbox{ for } x \in I$` | $f_i(x) \leq 0 \mbox{ for } x \in I$ |
| `$\Gamma(n)=(n-1)! \mbox{ when $n$ is an integer}$` | $\Gamma(n)=(n-1)! \mbox{ when $n$ is an integer}$ |

In section 3.4.4 we will learn of some special spacing commands that can be used in math mode. These are often very useful in positioning text within an expression, enhancing readability and logical layout.

### Log-like functions

There are a number of function names and operation symbols that should be set in normal (roman) type in an expression, such as in

$$f(\theta) = \sin\theta + \log(\theta + 1) - \sinh(\theta^2 + 1)$$

and

$$\lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \quad .$$

We know that simply typing `$log\theta$` would produce the incorrect result $log\theta$ and that using `$\mbox{log}\theta$` would leave us having to insert a little extra space between the log and the $\theta$ $\mbox{log}\theta$. So LaTeX provides a collection of "log-like functions" defined as control sequences. Thus `$\log\theta$` produces the perfect $\log\theta$. Table 3.13 shows various log-like functions that are available and some examples of their use. Notice how LaTeX does more than just set an operation like sup in roman type. It also knew where a subscript to that operator should go.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| `\arccos` | `\cos` | `\csc` | `\exp` | `\ker` | `\limsup` | `\min` | `\sinh` |
| `\arcsin` | `\cosh` | `\deg` | `\gcd` | `\lg` | `\ln` | `\Pr` | `\sup` |
| `\arctan` | `\cot` | `\det` | `\hom` | `\lim` | `\log` | `\sec` | `\tan` |
| `\arg` | `\coth` | `\dim` | `\inf` | `\liminf` | `\max` | `\sin` | `\tanh` |

Table 3.13: Log-like functions

| *Type* | *To produce* |
|---|---|
| `$f(x)=\sin x + \log(x^2)$` | $f(x)=\sin x + \log(x^2)$ |
| `$\delta = \min \{ \delta_1, \delta_2 \}$` | $\delta = \min \{ \delta_1, \delta_2 \}$ |
| `$\chi(X) = \sup_{x\in X} \chi(x)$` | $\chi(X) = \sup_{x\in X} \chi(x)$ |
| `$\lim_{n \rightarrow \infty} S_n = \gamma$` | $\lim_{n \rightarrow \infty} S_n = \gamma$ |

### Over- and Underlining and bracing

The `\underline` command will place an unbroken line under its argument, and the `\overline` command will place an unbroken line over its argument. These two commands can also be used in normal paragraphing mode (but be careful: LaTeX will not break the line within an under- or overlined phrase, so do not go operating on large phrases).

You can place horizontal braces above or below an expression by making that expression the argument of `\overbrace` or `\underbrace`. You can place a label on an overbrace (resp. underbrace) by superscripting (resp. subscripting the group defined by the bracing command.

| *Type* | *To produce* |
|---|---|
| `$\overline{a+bi} = a- bi$` | $\overline{a+bi} = a-bi$ |
| `$\overline{\overline{a+bi}} = a+bi$` | $\overline{\overline{a+bi}} = a+bi$ |

And some examples of horizontal bracing:

```
$A^n=\overbrace{A \times A \times \ldots \times A}^{\mbox{$n$ terms}}$

$\forall x \underbrace{\exists y (y \succ x)}_{\mbox{scope of $\forall$}}$
```

will produce

$$A^n=\overbrace{A \times A \times \ldots \times A}^{\mbox{$n$ terms}}$$

and

$$\forall x \underbrace{\exists y (y \succ x)}_{\mbox{scope of $\forall$}}$$

### Stacking symbols

LaTeX allows you to set one symbol above another through the `\stackrel` command. This command accepts two arguments, and sets the first centrally above the second.

| *Type* | *To produce* |
|---|---|
| `$X \stackrel{f^*}{\rightarrow}Y$` | $X \stackrel{f^*}{\rightarrow}Y$ |
| `$f(x) \stackrel{\triangle}{=} x^2 + 1$` | $f(x) \stackrel{\triangle}{=} x^2 + 1$ |

### Operators; Sums, Integrals, etc.

Each of the operation symbols in table 3.11 can occur with limits. They are specified as sub- and superscripts to the operator, and LaTeX will position them appropriately. In an in-text formula they will appear in more-or-less the usual scripting positions; but in a displayed formula they will be set below and above the symbol (which will also be a little larger). The following should give you an idea of how to use them:

| *Type* | *To produce* |
|---|---|
| `$\sum_{i=1}^{N} a_i$` | $\sum_{i=1}^{N} a_i$ |
| `$\int_a^b f$` | $\int_a^b f$ |
| `$\oint_{\cal C}f(x)\,dx$` | $\oint_{\cal C}f(x)\,dx$ |
| `$\prod_{\alpha \in A} X_\alpha$` | $\prod_{\alpha \in A} X_\alpha$ |
| `$\lim_{N\rightarrow\infty}\sum_{i=1}^{N}f(x_i)\Delta x_i$` | $\lim_{N\rightarrow\infty}\sum_{i=1}^{N}f(x_i)\Delta x_i$ |

We will have more to say about the use of `\,` in section 3.4.4. Let us have a look at each of those expressions when displayed:

$$\sum_{i=1}^{N} a_i\,, \qquad \int_a^b f\,, \qquad \oint_{\cal C} f(x)\,dx\,, \qquad \prod_{\alpha\in A} X_\alpha\,, \qquad \lim_{N\rightarrow\infty}\sum_{i=1}^{N} f(x_i)\Delta x_i$$

## Arrays

The `array` environment is provided for typesetting arrays and array-like material. It accepts two arguments, one optional and one mandatory. The optional argument specifies the vertical alignment of the array—use `t`, `b`, or `c` to align the top, bottom, or centre of the array with the centreline of the line it occurs on (the default being `c`). The second argument is as for the `tabular` environment: a series of `l`, `r`, and `c`'s that specify the number of columns and the justification of these columns. The body of the `array` environment uses the same syntax as the `tabular` environment to specify the individual entries of the array.

For instance the input

```
$A =\left[ \begin{array}{rrr}
12 & 3 & 4\\
-2 & 1 & 0\\
3 & 7 & 9
\end{array}\right]$ ...
```

will produce the output

$$A = \left[ \begin{array}{rrr} 12 & 3 & 4\\ -2 & 1 & 0\\ 3 & 7 & 9 \end{array}\right]$$

Note that we had to choose and supply the enclosing brackets ourselves (they are not placed for us so that we can use the `array` environment for array-like material; also, we get to choose what type of brackets we want this way). As in the `tabular` environment, the scope of a command given inside a matrix entry is restricted to that entry.

We can use ellipsis within arrays as in the following example:

$$\det A = \begin{array}{cccc} a_{11} & a_{12} & \cdots & a_{1n}\\ a_{21} & a_{22} & \cdots & a_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{array} .$$

It has been produced by

```
$\det A = \left| \begin{array}{cccc}
a_{11} & a_{12} & \cdots & a_{1n}\\
a_{21} & a_{22} & \cdots & a_{2n}\\
\vdots & \vdots & \ddots & \vdots\\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{array} \right|.$
```

The `array` environment is often used to typeset material that is not, strictly speaking, an array:

```
$f(x) = \left\{ \begin{array}{ll}
   x & \mbox{for $x<1$}\\
 x^2 & \mbox{for $x \geq 1$}
\end{array} \right.$.
```

which will yield

$$f(x) = \begin{array}{ll} x & \mbox{for } x<1 \\ x^2 & \mbox{for } x \geq 1 \end{array}.$$

### Changes to spacing

Sometimes LaTeX needs a little help in spacing an expression, or perhaps you think that the default spacing needs adjusting. For these purposes we have the following spacing commands:

| | | | |
|---|---|---|---|
| `\,` | thin space | `\:` | medium space |
| `\!` | negative thin space | `\;` | thick space |
| `\quad` | a quad of space | `\qquad` | two quads of space |

The spacing commands `\,`, `\quad`, and `\qquad` can be used in paragraphing mode, too. Here are some examples of these spacing commands used to make subtle modifications to some expressions.

| *Type* | *To produce* |
|---|---|
| `$\sqrt{2} \, x$` | $\sqrt{2} \, x$ |
| `$\int_a^b f(x)\,dx$` | $\int_a^b f(x)\,dx$ |
| `$\Gamma_{\!2}$` | $\Gamma_{\!2}$ |
| `$\int_a^b \! \int_c^d f(x,y)\,dx\,dy$` | $\int_a^b \! \int_c^d f(x,y)\,dx\,dy$ |
| `$x / \! \sin x$` | $x / \! \sin x$ |
| `$\sqrt{\,\sin x}$` | $\sqrt{\,\sin x}$ |

## 3.4.5 Alignment

Recall that the `equation` environment can be used to display and automatically number a single-line equation (see Sect. 3.4.2). The `eqnarray` environment is used for displaying and automatically numbering either a single expression that spreads over several lines or multiple expressions, while taking care of alignment for us. The syntax is similar to that of the `tabular` and `array` environments, except that no argument is necessary to declare the number and justification of columns. The `eqnarray*` environment does this without numbering any equations. Thus

```
\begin{eqnarray}
(a+b)(a+b) & = & a^2 + 2ab + b^2\\
(a+b)(a-b) & = & a^2 - b^2
\end{eqnarray}
```

will give

$$(a+b)(a+b) \quad = \quad a^2+2ab+b^2 \tag{3.1}$$
$$(a+b)(a-b) \quad = \quad a^2-b^2 \tag{3.2}$$

See how we identify the columns so as to align the = signs. We can also leave entries empty. The following output, for instance,

$$\begin{aligned}
\frac{d}{dx}\sin x &= \lim_{h\rightarrow 0}\frac{\sin(x+h)-\sin x}{h}\\
&= \lim_{h\rightarrow 0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}\\
&= \lim_{h\rightarrow 0}\left\{\frac{\sin x(\cos h-1)}{h}+\cos x\frac{\sin h}{h}\right\}\\
&= \cos x
\end{aligned}$$

has been produced from the input below:

```
\begin{eqnarray*}
\frac{d}{dx} \sin x & = & \lim_{h\rightarrow0}\frac{\sin(x+h)-\sin x}{h}\\
 & = & \lim_{h\rightarrow0}\frac{\sin x\cos h + \cos x\sin h - \sin x}{h}\\
 & = & \lim_{h\rightarrow0}\frac{\sin x(\cos h-1)}{h} + \cos x\frac{\sin h}{h}\\[8pt]
 & = & \cos x\,.
\end{eqnarray*}
```

No first column entry is required. The \\[8pt] gives extra space. Note also that `\displaystyle` command is not required in `eqnarray` environment, while it would be required if we were to produce the same result using `array` environment.

### 3.4.6 Theorems, Propositions, Lemmas, ...

Suppose you document contains four kinds of theorem-like structures: "theorems", "propositions", "conjectures", and "wild guesses". Then near the beginning of the document you should have something like the following:

```
\newtheorem{thm}{Theorem}
\newtheorem{prop}{Proposition}
\newtheorem{conjec}{Conjecture}
\newtheorem{wildshot}{Hypothesis} % make it sound good!
```

The first argument to `\newtheorem` defines a new theorem-like environment name of your own choosing. The second argument contains the text that you want inserted when your theorem is proclaimed:

```
\begin{thm} {\slshape $X$ is normal if, and only if, each pair of disjoint
closed sets in $X$ is completely separated.}
\end{thm}

\begin{wildshot} % remember, we chose the name 'wildshot'
The property of Moore extends to all objects of the class $\Sigma$.
\end{wildshot}
```

which will produce the following:

**Theorem 1** *$X$ is normal if, and only if, each pair of disjoint closed sets in $X$ is completely separated.*

**Hypothesis 1** *The property of Moore extends to all objects of the class $\Sigma$.*

Notice that LaTeX italicizes the theorem statement, and that you still have to shift in to math mode when you want to set symbols and expression. Typically, it is the class or style file that determines what a theorem will appear like—so do not go changing this if you are preparing for submission for publication (because the journal staff want to substitute their production style for your document class choice, and not be over-ridden by other commands).

# Chapter 4

# Computer-assisted research: programming and graphing

## 4.1 Programming

### 4.1.1 Development process

A *program development cycle* is the incremental process of building up your code, testing and debugging. It involves four steps:

- Write and modify source code of your program in a text editor.
- Create an executable file from the program source file with a compiler.
- Run your program.
- Observe the effect and decide if further changes in the code are needed.

A computer cannot directly run a program source file, which is a human-readable text file. The source file must be translated into an executable (binary) file, which the computer understands. A *compiler* is a program that does this translation. For each programming language there is its own compiler, sometimes more than one. See information about the compilers available on local machines in Section 5.3.3.

Often during the development cycle you have to repeat certain commands over and over again, for example,

```
gcc project2.c
a.out
```

To minimize typing, you can use a "history" keystroke: Esc-K or ↑ (see Sect. A.4).

A more fundamental time-saving suggestion concerns **testing and debugging**. Do not bother to create a friendly user interface or to add various features to your program until you achieve **basic functionality**. Suppose your program is to compute the area of a polygon given the coordinates of its vertices. Eventually you want the program to prompt the user to enter the number of vertices, $N$, and their coordinates, one by one, like this:

```
Please enter the number of vertices: N=  3
Vertex #1: x=  1.1
           y=  0.4
Vertex #2: x=  3.4
           y=  -4.56
Vertex #3: x=  3.12
           y=  -9.4
```

However, do not begin programming by creating the input interface. Instead, put a temporary initialization block with fixed, **hardcoded** data at the beginning of the program; use simple data to enable an easy check by hand calculation.

```
N=3
x[1]= 0
y[1]= 0
x[2]= 2
y[2]= 0
x[3]= 0
y[3]= 3
```

This set of data corresponds to a right triangle with two sides running along the coordinate axes. Then, as the mathematical part of your program matures, you should change the data: test the program on a triangle that is acute or obtuse; shift it; then test the program on a quadrilateral, etc. As you will be catching mistakes in the program, you will have to run it more than once on each sample data set, while modifying the data infrequently. Eventually you will spend much less time on data input than you would spend via the input prompt.

Syntax errors that depend on a programming language and a compiler used cannot be discussed here in any depth. We will just mention some **common errors** that appear frequently.

- Unmatched bracket in a mathematical expression (() or unmatched opening of a structure in a program, like `DO` loop without closing "`END DO`" in FORTRAN. Most notably, when you find such an error and try to fix it, there is a danger that you *misplace* the closing bracket or the closing keyword and make the problem worse, more difficult to detect.
- All variables in the program must be assigned values before their values are used for the first time. A randomly looking output is the most common consequence of the *failure to initialize*. However, a compiler may initialize your variables to 0 by default — not to the values that ought to be there; the results may look OK at first sight but still be wrong.

- Initializations should be placed *outside* loops that they are supposed to initialize. If the variable `SUM` is an accumulator for a sum computed by a loop, you should put the initialization `SUM=0` outside the loop. This problem is especially common with *nested* loops. You must carefully identify "fast" variables, which must change in the inner loop, and "slow" variables, which change only once per the outer loop.

- "Off by one" error is common even with seasoned programmers. Differentiate between *strict* and *non-strict* inequalities (`i>0` vs. `i>=0`). Check whether it is possible that your loop will skip immediately (say, the loop condition is `while(i>j)` and the initial value of `i` is equal to the initial value of `j`). If this is possible in the program, was it meant?

- Quite often, special arrangements are to be made on the first and/or last pass of a loop. For example, in a loop that produces $n$ pairs of coordinates separated by commas, the comma should be printed only $n-1$ times, — see example on p. 86.

- Array indexing. In FORTRAN and Maple, if the array has $N$ elements, the index runs from 1 to $N$, while in C and Java it runs from 0 to $(N-1)$. Also, when you update the array and the new values depend on the old values, watch the order of the update closely. Consider, for example, the cyclic permutation `x(1)`$\rightarrow$ `x(2)`$\rightarrow \ldots \rightarrow$ `x(n)`$\rightarrow$ `x(1)`:

  **CORRECT**

  ```
  tmp=x(n)
  FOR i=1, n-1
     x(n-i+1)=x(n-i)
  END DO
  x(1)=tmp
  ```

  **WRONG**

  ```
  tmp=x(n)
  FOR i=2, n
     x(i)=x(i-1)
  END DO
  x(1)=tmp
  ```

  In the wrong code, the old value of `x(1)` will propagate through the array, while the old values of `x(2), x(3), ..., x(n)` will be lost.

- When you use output to files in C or FORTRAN, make sure to *close* the file; otherwise a part of the output may be lost. Close the file *outside* the outermost loop: otherwise, a multiple closure will cause the program to crash.

- A confusion between the assignment operator and the equality condition ('`=`' vs. '`==`' in C and Java, '`:=`' vs. '`=`' in Maple and Pascal). Make sure you understand the difference! Other symbols to watch carefully are comma vs. semicolon, and various kinds of brackets.

Students often get lost when a program does not behave the way it is supposed to. How is it possible to find errors that affect functionality and are not easy to catch? A regular approach to find and fix a mistake is to insert **temporary output operators** and, using simple test data (cf. p. 2.3.6), to trace the intermediate results comparing them with those calculated by hand. If the program's functioning disagrees with your mathematical algorithm (most often, due to a typo), you will detect a discrepancy at some point. Debugging is more complicated if the mathematical method is flawed in itself. Dissecting the problem (and the program) into smaller steps is still a dependable approach.

### 4.1.2 Programming style

Your program code should be reasonably self-contained and documented. Put the following at the top of a program:

- Author's name
- Date
- Course and project number
- A brief description of the program (what it does)
- Additional information if several programs have been written for this project

Note that the readability of a program is improved and debugging is helped immensely by the generous insertion of **comments**. Ask yourself: will you be able to understand your program in a half a year period of time?

Use **indentation** to improve readability of loops, especially long ones, and if/else clauses.

**GOOD**

```
for (i=1; i<=n; i++)
{
   if (a[i]>0)
      sum=sum+a[i];
   else
      sum=sum-a[i];
}
```

**BAD**

```
for (i=1; i<=n; i++)
{
if (a[i]>0)
sum=sum+a[i];
else  sum=sum-a[i];
}
```

Use **meaningful names** of variables and functions, but don't make them too long. A variable name like `AreaOfTriangle` is hardly better than just `Area`. If practical, use very short names that match the notation you use in the description of the mathematical method. Strive for **consistency** in your programming style, as in everything else.

Good programmers tend to use **modular approach** (subroutines in FORTRAN, functions in C, classes in C++ and Java) to make the structure of a program more transparent and to confine those few cumbersome mathematical lines of code. Modularity also makes debugging easier. However, in programs with simple "linear" structure or in short programs modularity can be a burden rather than a benefit.

Appropriate **generalization** is another feature of a solid style. Make your program flexible, make it easy to play with parameters. An example of a coordinate-generating code on the next page will help you to grasp the idea.

### 4.1.3 Generating graphics data with your own program

We will discuss two-dimensional graphs only. Essentially, every graph you generate is determined by points, each point being a pair of coordinates $(x, y)$. Continuous mathematical curves, consisting of infinitely many points, are most commonly approximated by polygonal lines consisting of segments whose endpoints are to be computed by the program.

When writing a program, you have to make a decision as to what the **bounds for $x$ and $y$** should be (if the mathematical curve is infinite), and **by how many points** you want to define the curve. To get a rough estimate of a reasonable number, let us take 5 *in* as a largest dimension of a picture and note that human eye, even sharp, can hardly resolve distances less than 1/200 of an inch. Thus $5 \times 200 = 1000$ data points across a sheet is perhaps enough in most cases. The smoother a curve, the smaller number of points will suffice: often 50 or even 10.

A good idea is to have **variables** in your program for the $x$ and $y$ limits and for the number of points, rather than to use specific numbers throughout the code. Compare the two fragments of FORTRAN code:

**GOOD**

```
xmin=-5
xmax=5
numpoints=20
xstep=(xmax-xmin)/numpoints
DO i=0,numpoints
  x=xmin+i*xstep
  y=SIN(x)
  PRINT *,"(",x,",",y,")"
END DO
```

**BAD**

```
DO i=0,20
  x=-5+i*0.5

  PRINT *,"(",x,",",SIN(x),")"
END DO
```

The "bad" code does not look bad at all: it is concise, easy to understands, and correct. But it has two drawbacks:
(1) the code conceals the meaning of the numbers: what are those "20" and "$-5$", and "0.5"?;
(2) if the values that are denoted `xmin`, `xmax`, `numpoints` in the "good" code occur somewhere else in the program and you wish to change all or some of the values, it is easy to do: you just need to change the values assigned to the symbolic names, in one place only.

In simple cases, like in the example above, the points are generated independently of each other; as soon as a point has been computed, it can be printed immediately. In more complicated cases, when dependency of some sort is present, you may have to create an array and to complete computation of all the points before you can print them out.

Another suggestion: use **scaling** parameters and **translation** parameters. The "good" code above is good, in particular, because it is easy to implement this suggestion. Modify the output operator as follows:

```
PRINT *, "(",xscale*(x-xorigin),",",yscale*(y-yorigin),")"
```

This stretches (or shrinks) the distances by a factor `xscale` in the $x$-direction and by a factor `yscale` in the $y$-direction. In addition, the point where (`x,y`)=(`xorigin,yorigin`) will be printed as (0,0), that is, it will become the origin of the coordinate system on the plotting device. The trivial default values can be initialized: `xscale=1`, `yscale=1`, `xorigin=0`, `yorigin=0`. If you are not satisfied with size or position of your graph, it will be very easy to change.

## Formatting coordinates

You must format the coordinates in accordance with the method you intend to use to render your graph. Every pair of numbers must be "framed" with *opening* symbol or keyword preceding the $x$ value and *closing* symbol or keyword following the $y$ value, and a *separator* must be inserted between the $x$ and $y$ values. The list of coordinates as a whole has its own opening (before the first point), closing (after the last point) and a separator (between the successive points). Some of these can be void or blank space.

| Graphing facility | LaTeX picture | Postscript | Maple | Gnuplot |
|---|---|---|---|---|
| Before $x$ | `(` | none | `[` | none |
| Between $x$ and $y$ | `,` | space | `,` | space |
| After $y$ | `)` | `moveto` or `lineto` | `]` | none |
| List opening | `\join` | `newpath` | `[` | none |
| Between points | none | line break | `,` | line break |
| List closing | none | `stroke` or `fill` | `]` | none |

Table 4.1: Openings, closings, and separators for coordinates

A summary of the various coordinate formats mentioned in this Manual is given in Table 4.1. Consider for example a line defined by three points $(-1,1)$, $(0,0)$ and $(1,1)$ (a very rough approximation to a graph of $y = x^2$). Below we show what the data file produced by your program should look like in different cases. In LaTeX and Postscript, a change of scale may be necessary to actually see the picture.

- LaTeX: you must include package `2130.sty` or `curvesb.sty` to enable the `join` command. Import the data file by the `input` command, see p. 120.

```
\join (-1,1)(0,0)(1,1)
```

- Postscript: you must add the heading `%!PS-Adobe-2.0` by hand or have your program to print it automatically.

```
newpath
-1 1 moveto
 0 0 lineto
 1 1 lineto
stroke
```

- Maple: you must cut and paste this array to Maple's `plot` command.

  ```
  [[-1,1],[0,0],[1,1]]
  ```

- Gnuplot: feed the file to Gnuplot's `plot` command; use option `with lines`.

  ```
  -1 1
   0 0
   1 1
  ```

Since the list opening and list closing are to be printed only once, this can be done outside of the coordinate-generating loop. On the other hand, the separator between the pairs must be printed after each pair save the last one. So it must be done within the loop; the last pass of the loop must be a little different. We present short FORTRAN and C codes that produce the data in **Maple's style**. For simplicity of presentation, we sacrificed any flexibility, in violation of a good programming style we promote.

**FORTRAN**

```
OPEN (UNIT=1, FILE ="line1.dat")
WRITE(1,*) "[" !List opening
DO i=0,2
   x=i-1
   y=x**2
   WRITE(1,*),"[",x,",",y,"]"
   IF (i<2) THEN
      WRITE(1,*),"," !separator
   END IF
END DO
WRITE(1,*) "]" !List closing
CLOSE(1)
```

**C**

```
FILE *f =fopen("line1.dat","w");
fprintf(f,"[");//List opening
for (int i=0; i<=2; i++)
{
   x=i-1;
   y=pow(x,2);
   fprintf(f,"[%f,%f]",x,y);
   if (i<2)
      fprintf(f,","); //separator
}
fprintf(f,"]");//List closing
fclose(f);
```

If, instead of connecting the points by lines, you need to render them differently, your program can be written accordingly. For example, the following line in a C program will print a LaTeX command that puts a small solid circle in a specified position.

```
printf("\put(%f,%f){\circle*{0.1}}",x,y);
```

An equivalent FORTRAN code is

```
PRINT *, "\put(", x, "," ,y, "){\circle*{0.1}}"
```

## 4.2 An introduction to Maple

Maple is a computer algebra system (CAS) created around 1980 by a team of researchers based at the University of Waterloo. Currently it is commercial software supported by Maplesoft, Inc. and it is available to MUN students through the LabNET-wide licence. It may be necessary for you to set up your account so that you can use Maple — see Section 5.3.4.

Maple's most vigorous competitor is CAS *Mathematica*, a product of Wolfram Research, Inc. (USA). Another software that has many similar capabilities but focuses on matrix computations at the expense of sophisticated symbolic manipulations is *Matlab*. Users familiar with one of these systems will have little difficulty with another as soon as they understand the basic syntax and work out a few examples. In Math 2130, we focus on Maple.

Maple's functionality and interface have evolved over about 30 years. As of now, there exist three types of user interface in Maple. Two of them are graphical user interfaces: *standard* (modern) and *classic worksheet*, and the third is non-graphical *text-based* interface, which can be used in a command-line mode. Of the two graphical interfaces, the classic worksheet is the one which is easier to transform to a printed document. Our presentation will be based on the classic worksheet. The examples below were tested on Maple version 11 in November 2008. Maple graphics is dealt with in Sect. 4.3.2.

Maple can, in principle, save a worksheet in LaTeX format. However, this feature doesn't seem to be implemented carefully. We suggest that you paste fragments of your Maple code into your reports by hand (cf. Sect. 3.2.6).

### 4.2.1 Basic Arithmetic and Algebra

To start Maple, open a terminal window and at the prompt, simply type

```
xmaple
```

Maple also has a classic worksheet option, which can be accessed by typing

```
xmaple -cw
```

at the prompt. Maple has a very useful help area, where you can find instructions on the many operations it can perform. In the classic worksheet, the *Help* command is located in the top right-hand corner. In the standard Maple, it is located under *Tools*. Also, in all interfaces of Maple, help can be accessed by typing

```
?help
```

Maple can be used as a calulator. Hit *Enter* to execute the command. The keystroke *Shift-Enter* makes carriage return without an immediate execution. Our first examples are:

```
> 4+3;
                                 7
> 2*5;
  6^2;
                                10
```

Note the difference between exact and floating point operations:

```
> 2^64;
```

$$18446744073709551616$$

```
> 2.0^64;
```

$$1.844674407 \cdot 10^{19}$$

Let's see how to do slightly more interesting operations. Symbolic names can be used:

```
> y1:=x^3/2-9/2*x^2-2*x+6;
  y2:=(x^4-x^3-15*x^2+9*x+54)/(2*x^3-2*x^2-8*x+8);
```

$$\mathrm{y1} := \frac{1}{2}x^3 - \frac{9}{2}x^2 - 2x + 6$$

$$\mathrm{y2} := \frac{x^4 - x^3 - 15x^2 + 9x + 54}{2x^3 - 2x^2 - 8x + 8}$$

Expressions can be symbolically factored:

```
> y1f:=factor(y1);
```

$$\mathrm{y1f} := \frac{(x-1)(x^2-8x-12)}{2}.$$

Or expanded:

```
> expand(y1f);
```

$$\mathrm{y1} := \frac{1}{2}x^3 - \frac{9}{2}x^2 - 2x + 6$$

**A remark on symbolic names**. Some names in our examples (`x`, `y1`, `y2`, `y1f`) denote *user-defined variables* — these names are arbitrary, you can change them any way you like. Other names are *keywords* known to Maple (like `factor`, `expand`). These names are protected; an attempt to assign a value to them will prompt an error message. Maple knows a few special constants, which are also protected. The most famous one is `PI` ($\pi = 3.14159\ldots$); not so many students are familiar with Euler's constant `gamma` ($\gamma = 0.57721\ldots$). You may be surprised that the symbols `e` and `E` are not protected; the natural logarithm base $e = 2.71828\ldots$ can be accessed in Maple as `exp(1)` (in symbolic calculations) or as `exp(1.0)` (numerically). The availability of the symbol `e` is convenient for astronomers who use $e$ to denote eccentricities of planetary orbits. The fact that `gamma` is reserved is unfortunate for geometers who like to denote the angles of a triangle as $\alpha$, $\beta$, $\gamma$.

We continue a tour of basic Maple commands. The `simplify` command applies a bunch of algorithms to transform expressions to a simpler form. For example, it will identify common factors in the numerator and denominator and remove them. In our fraction `y2` defined above, Maple finds that the common factor $(x + 2)$ can be canceled:

```
> simplify(y2);
```

$$\frac{x^3 - 3x^2 - 9x + 27}{2(x^2 - 3x + 2)}$$

The `simplify` command also knows trigonometric identities:

```
> simplify(sin(theta)^2+cos(theta)^2);
                                 1
```

Maple's simplification algorithms are powerful but not perfect. For example, Maple fails to notice that $(x+1)^{2n} - (x^2 + 2x + 1)^n = ((x + 1)^2)^n - (x^2 + 2x + 1)^n = 0$:

```
> simplify((x+1)^(2*n)-(x^2+2*x+1)^n);
```

$$(x + 1)^{(2n)} - (x^2 + 2x + 1)^n$$

For any particular $n$, Maple will simplify correctly, but it can take a long time.

```
> simplify((x+1)^(2*700)-(x^2+2*x+1)^700);
                                0
```

Another useful command is substitution, `subs`:

```
> subs(x=Pi/4, sin(x)):    simplify(%);
```

$$\frac{1}{2}\sqrt{2}$$

The colon suppresses printout of a result (try to put a semicolon instead to see the effect). The percent sign refers to the most recent result.

### 4.2.2 Equations

Maple has built-in commands to solve equations automatically.

```
> solve(x^2+x-12=0,x);
                               3, -4
> XX:=solve(x^2+6*x+3,x);
```

$$-3 + \sqrt{6}, \quad -3 - \sqrt{6}$$

The command `evalf` takes exact answers like those above and spits them out in decimal form:

```
> evalf(XX);
         -.550510257, -5.449489743
```

Higher accuracy is available through an optional argument of the `evalf` command.

```
> evalf(XX,20);
         -.55051025721682190180, -5.4494897427831780982
```

Maple knows complex numbers, too. Note that symbol "I" in Maple is the imaginary number $\sqrt{-1}$, usually denoted as $i$.

```
> solve(x^2+x+1=0,x);
```

$$-\frac{1}{2}+\frac{1}{2}\mathrm{I}\sqrt{3},\ -\frac{1}{2}-\frac{1}{2}\mathrm{I}\sqrt{3}$$

Maple's exact answers to equations of degrees 3 and 4 can be impractical. For roots of polynomials of degree 5 and higher no general formulas exist and, unless the equation can be factored, Maple will return gibberish. In all such cases, `evalf` can be used to get an approximation of the roots. The command `fsolve` returns approximations of real roots only.

```
> evalf(solve(x^3+x+1=0,x));
     -.6823278040, 0.3411639019-1.161541400 I, 0.3411639019+1.161541400 I
> fsolve(x^3+x+1=0,x);
                                 -.6823278038
```

Maple can solve not only algebraic equations but many others, too. Beware, however, of its simplistic approach. Every math student knows that the equation $\cos x = 0$ has infinitely many solutions — but not Maple!

```
> solve(cos(x)=0);
```

$$-\frac{1}{2}\pi$$

Nor can Maple find all approximate solutions in a given interval containing many roots:

```
> fsolve(cos(x)=0, x=-100..200);
                               48.69468613
```

### 4.2.3 Calculus

Maple can do calculus both symbolically and numerically. Recall the expression `y1` from our example on page 88; we can use Maple for differentiation, indefinite and definite integration:

```
> y1;
```

$$\mathrm{y1} := \frac{1}{2}x^3-\frac{9}{2}x^2-2x+6$$

```
> diff(y1,x);
```

$$\frac{3}{2}x^2-9x-2$$

```
> Iy1:=int(y1,x);
```

$$\mathrm{Iy1} := \frac{1}{8}x^4-\frac{3}{2}x^3-x^2+6x$$

```
> int(y1,x=-1..1);
```

$$9$$

Maple has commands that find extreme values of functions.

```
> maximize(Iy1);
```

$$\infty$$

```
> minimize(Iy1);
```

$$\frac{(4+2\sqrt{7})64}{8} - \frac{3(4+2\sqrt{7})^3}{2} - (4+2\sqrt{7})^2 + 24 + 12\sqrt{7}$$

The percent symbol can be used as a substitute for the result of the last executed command:

```
> evalf(%);
                              -302.1620735
```

The symbols %%, %%%, etc. refer to the results obtained so many steps back. By Maple's design, you can execute commands that are typed in your worksheet in any order (moving back and forth across the worksheet) simply by hitting *Enter* on a command. This practice should be avoided in worksheets that are to be saved and later read by you or another person, otherwise the results can mislead the reader. In particular, instead of using the percent sign, it is preferable to assign symbolic names to the results you want to re-use.

The `maximize` and `minimize` commands work only on certain functions — namely where no critical points exist or the equation for critical points can be solved exactly. Not the case here:

```
> maximize(x*cos(x), x=0..2);
 RootOf(tan(_Z) _Z -1, 0.8603335890) cos(RootOf(tan(_Z) _Z - 1, 0.8603335890))
```

The command `numapprox[infnorm]` can find the *maximum abslolute value* of more general functions. (The reader can rightly be curious about the command's name; look it up!)

```
> numapprox[infnorm](x*cos(x), x=0..2);
                          0.8322936731
```

Here we encounter for the first time an example of a function from a Maple *package*. The whole package whose name is `numapprox` can be uploaded by the command `with (numapprox);` and then you can use the `infnorm` command without prefix `numapprox`.

Maple is quite knowledgeable in Calculus. It knows limits and Taylor series.

```
> limit(sin(2*x)/ln(1-x), x=0);
                              -2
```

```
> taylor(tan(t),t, 6);
```

$$t + \frac{1}{3}t^3 + \frac{2}{15}t^5 + O(t^7)$$

We leave it to the reader to find out the meaning of the "big Oh" symbol `O(..)`.

### 4.2.4 Arrays

Data in Maple can be grouped to form an array. An array is bounded by square brackets and elements are separated by commas. The elements of an array can be objects of like or different nature; they can themselves be arrays. For example,

```
> A:=[1, 2, [red,blue], x^2-5*x+6, plot1];
```

The elements can be referenced using forward or backward indexing:

```
> A[1];
       1
> A[3];
      [red,blue]
> A[3][2];
       blue
> A[-1];
       plot1
```

The command `nops` returns the number of elements in an array:

```
> nops(A);
       5
```

A sub-array can be selected:

```
> A[3..4];
      [[red,blue], x^2-5*x+6]
```

It is straightforward to change the values of elements of an existing array by assignments like `A[1]:=...`, but adding new elements is tricky. We need the command '`op`' whose effect is just to remove the bounding brackets around the whole array:

```
> op(A);
      1, 2, [red,blue], x^2-5*x+6, plot1
```

To add a new element, say, `elem6`, the following command can be used:

```
> A:=[op(A), elem6];
      A:=[1, 2, [red,blue], x^2-5*x+6, plot1, elem6]
```

The command `seq` provides a convenient way to initialize an array with elements generated according to a given rule:

```
> B:=[seq(i^2, i=3..9)];
      B:=[9,16,25,36,49,64,81]
```

The command `map` performs the specified action on all elements of the array at once:

```
> map(sqrt, B);
      [3,4,5,6,7,8,9]
```

In the commands `seq` and `map` it is possible to use your own function. For example, the following will increment all elements of the array `B` by one:

```
> map(x->x+1, B);
      [10,17,25,37,50,65,82]
```

### 4.2.5 Linear Algebra

Linear algebra is available in Maple via either of two packages, `linalg` or `LinearAlgebra`. The former is not being updated anymore and will be eventually phased out. We will work with the latter package. First, load the library.

```
> with(LinearAlgebra):
```

Here are some basic operations: to create a matrix, to find the inverse, the determinant, the transpose, the characteristic polynomial, the eigenvalues and eigenvectors.

```
> A:=Matrix([[2, 4],[6,8]]);
```

$$\mathtt{A} := \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

```
>  A^(-1);
```

$$\begin{bmatrix} -1 & \frac{1}{2} \\ \frac{3}{4} & -\frac{1}{4} \end{bmatrix}$$

```
> Determinant(A);
```

$$-8$$

```
> Transpose(A);
```

$$\begin{bmatrix} 2 & 6 \\ 4 & 8 \end{bmatrix}$$

```
> CharacteristicPolynomial(A,lambda);
```

$$\lambda^2 - 10\lambda - 8$$

```
> Eigenvalues(A);
```

$$\begin{bmatrix} 5+\sqrt{33} \\ 5-\sqrt{33} \end{bmatrix}$$

```
> EValVec:=Eigenvectors(A);
```

$$\mathtt{EValVec} := \begin{bmatrix} 5+\sqrt{33} \\ 5-\sqrt{33} \end{bmatrix} \quad \begin{bmatrix} \frac{4}{3+\sqrt{33}} & \frac{4}{3+\sqrt{33}} \\ 1 & 1 \end{bmatrix}$$

Note that the `Eigenvectors` command return the eigenvalues as well as the eigenvectors. We can select the matrix comprised of the eigenvectors:

```
> EVec:=EValVec[2];
```

$$\texttt{EVec} := \begin{bmatrix} \dfrac{4}{3+\sqrt{33}} & \dfrac{4}{3+\sqrt{33}} \\ 1 & 1 \end{bmatrix}$$

Now, if we want to separate the eigenvectors from one another and to make an array of them, the following command will do it:

```
> EVecArray:=[seq(Column(EVec,i), i=1..2)];
```

$$\texttt{EVecArray} := \left[ \begin{bmatrix} \dfrac{4}{3+\sqrt{33}} \\ 1 \end{bmatrix}, \begin{bmatrix} \dfrac{4}{3+\sqrt{33}} \\ 1 \end{bmatrix} \right]$$

Matrix multiplication (and dot product of vectors) are denoted by a single dot (period).

```
> B:=Matrix([[2,0,-3],[1,2,1]]);
```

$$\texttt{B} := \begin{bmatrix} 2 & 0 & -3 \\ 1 & 2 & 1 \end{bmatrix}$$

```
> A.B;
```

$$\begin{bmatrix} 8 & 8 & -2 \\ 20 & 16 & -10 \end{bmatrix}$$

An attempt to multiply matrices when dimensions don't match leads to an error message.

```
> B.A;
```

```
    Error, (in MatrixMatrixMultiply) first matrix column dimension (3) <> second
    matrix row dimension (2)
```

### 4.2.6 Programming

Maple can be used for programming. It allows conditionals, loops, and user-defined functions.

Compared to languages such as FORTRAN or C, programming in Maple is more convenient: there is no need to bother about input/output operations and data types, commands can be executed immediately, and you have access to all of the built-in commands and available packages. The price to pay is efficiency. Maple runs loops a lot slower than compiled programs; also it runs out of memory on much smaller sizes of data. Yet, for many applications and many Math-2130 projects these issues are not critical.

Our first example is a summation loop, which computes the arithmetic sum $11 + 32 + 53 + 74 + 95$. Remember to use *Shift-Enter* to type multi-line commands into Maple (cf. page 87).

```
> tot := 0;
  for i from 11 by 21 while (i < 100) do
     tot := tot + i
  end do;
                                  tot :=   0
                                  tot :=  11
                                  tot :=  43
                                  tot :=  96
                                  tot := 170
                                  tot := 265
```

If you are not interested to see the intermediate results, simply replace the semicolons by colons in the above program and add the line $\boxed{\texttt{> tot;}}$ to print the final result.

There are usual conditional commands `if-then-else`. For example, the following command finds the maximum of two numbers.

```
> a:=4: b:=2:
  if (a > b) then  a  else  b  end if;
                                      4
> a:=1: b:=7:
  if (a > b) then a else b end if;
                                      7
```

The closing commands `end do` and `end if` can be replaced by less traditional `od` and `fi`, respectively.

Consider an example of a very simple user-defined function, or procedure. It just adds one to a given number.

```
> increment:=proc(x) return (x+1): end proc:
> increment (2008);
                              2009
```

Procedures can be much more involved; they may have many input values, local variables, and return values of any type. Procedures can contain loops, conditionals and calls to other procedures or to Maple's built-in functions.

There are also commands to test whether inputs are real, complex, matrices, or whatever.

```
> type(4,realcons);  type(c,realcons);
                                    true
                                    false
```

These could be used in `if-then` statements. For example, the following procedure returns the square root of the argument if the root is a real number, and prints an error message otherwise.

```
> SafeSqrt:=proc(x) local sqrtx:
    sqrtx:=sqrt(x):
    if  type(sqrtx,realcons) then
       return evalf(sqrtx)
    else
       print (Sqrt - Error)
    end if:
  end proc:

>  SafeSqrt(3);
             1.732050808
>  SafeSqrt(-3);
               Sqrt - Error
>  SafeSqrt(x^2);
               Sqrt - Error
```

## 4.3 Drawing graphs

Knowing how to generate graphs and incorporate them into a paper is a valuable skill for all technical writers. In this chapter we review a number of ways to generate plots with software.

The most common method to import computer-generated graphs into a LaTeX document involves **Encapsulated Postscript** files. We begin this section with basic information about Postscript, Encapulated Postscript, and LaTeX imports.

We then explain how graphs can be generated in a **Maple** worksheet and include brief descriptions of the **Gnuplot** and **Xfig** graphing packages.

LaTeX has its own graphical facility, the **picture environment**. Its drawing functions are very limited, but there are powerful enhancements, in particular, those included with `2130.sty` file. Besides, the *picture* environment can be used to create a desired layout of imported graphs or to superimpose graphs and text.

We do not advocate exclusive usage of any one of these graphing utilities. Try to draw pictures, and draw your conclusions. In all cases, you have to write a program which in part generates graphics data and it is up to you to select the utility that will handle a current graphics task best.

A practice **not allowed** in this course (except with an express permission of the instructor) is to import graphics files that are not your own production (downloaded from the Internet). Also, do not use bitmap graphics and image compression formats (`gif`, `jpg`, `png`), in particular, scanned pictures and digital photos.

### 4.3.1 Postscript Files

Many graphics utilities, Maple and Gnuplot for example, generate *Postscript* files, which are easily recognized by the `.ps` or `.eps` extension. Postscript is a popular graphical format introduced by Adobe, Inc. It belongs to the category of *vector* graphics, as opposed to *bitmap* graphics, a typical representative of which is the `bmp` format. Postscript is a parent (pretty much alive!) of the Portable Document Format (PDF) also designed by Adobe, Inc.

EPS stands for *Encapsulated Postscript*, which is now the best-supported format for the inclusion of graphics into LaTeX documents. To include an `.eps` file into a `.tex` file is easy.

1. First, you must have the line `\usepackage{graphicx}` in the preamble of you document (between the `\documentclass` and `\begin{document}`). Note the peculiar "x" at the end of the package name.

2. At the spot where you want to drop the `.eps` file into your document, use the command `\includegraphics`. For example, if the name of your graph is `fig1.eps`, the following line will include it into the LaTeX file:

   ```
   \includegraphics{fig1}
   ```

   The extension `.eps` in the file name should be omitted. Often the `\includegraphics` command is used within the *picture* environment or *figure* environment. See p. 60 and p. 114 for information about these environments.

3. Then proceed with your LaTeX file in the usual way.

For most students, the above algorithm of integration of EPS with LaTeX is all that is needed.

For all practical purposes, the only difference between `.ps` and `.eps` files is that the latter have a *BoundingBox* line, which is usually the second line in the file (but sometimes it is found at the very end). If your graphing program generates `.ps` file, but not an `.eps`, you need to **convert** it — see instructions on p. 128. Unless you use raw Postscript programming or older versions of Gnuplot, you will likely never need this.

#### Optional arguments of `\includegraphics`

Consider a more sophisticated version of the above example:

```
\includegraphics[height=8cm, angle=90]{fig1}
```

The expressions `height=8cm` and `angle=90` are optional arguments to the `\includegraphics` command and, as with all optional arguments in LaTeX, they are included within square brackets. Any valid TeX unit of length can be used. Counter-clockwise direction of rotation is deemed positive and the angle is measured in degrees. Other possible optional arguments for `\includegraphics` are `totalheight`, `width`, and `origin`. The difference between `height` and `totalheight` is that the former specifies the elevation of the graphics above the baseline, while the latter equals to height plus depth (the part below baseline). The argument `origin` specifies what point to use for a rotation origin (`origin=c` rotates about the centre).

**Postscript programming**

Postscript (`.ps`, `.eps`) files are usually created by specialized graphing or more universal programs — like Gnuplot, XFig, or Maple. However, Postscript by itself is a human-readable language, and one can write a "program" in Postscript describing a picture. Also, you can generate a Postscript file automatically by your own FORTRAN or C program.

As an example, the following short program in raw Postscript (Fig. 4.1) describes two Pythagorean triangles, shown on the right. Type the program in a text editor and save the file as, say, `triangles.ps`. The unit length in Postscript is *point*, which is 1/72 of an inch.

```
%!PS-Adobe-2.0
% First triangle (contour)
newpath
200 125 moveto
300 125 lineto
200 200 lineto
200 125 lineto
stroke
% Second triangle (filled)
0.8 setgray
newpath
200 0   moveto
275 0   lineto
275 100 lineto
200 0   lineto
fill
showpage
```

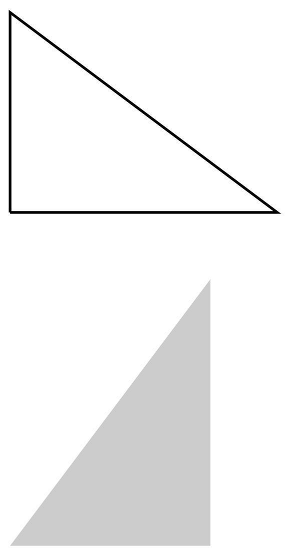


Figure 4.1: A simple Postscript program and its effect

You can immediately open the file `triangles.ps` with Postscript viewer Ghostview:

```
gv triangles.ps
```

You can now insert the picture to a LaTeX document. A conversion to `.eps` is required, but if you want a quick try, just replace the first line `%!PS-Adobe-2.0` in the file by two other lines:

```
%!PS-Adobe-3.0 EPSF-3.0
%%BoundingBox: 198 0 303 202
```

Save the file as `triangles.eps`. You can now type the line `\includegraphics{triangles}` in a LaTeX document and the picture will be inserted. To put the triangles beside the Postscript program on Figure 4.1, we used a superimposition trick described in Section 4.4.4.

### 4.3.2 Maple graphics

Maple has versatile plotting capabilities. They are realized by means of two basic commands: `plot`, `plot3d`, and more functions provided by the `plots` package. We will illustrate the usefulness of these commands with very basic examples. To get a more elaborate description of plots and their options, use Maple's Help and other available sources.

We begin with a graph of the function $y = \frac{\sin x}{x}$ on a specified interval $(-15, 15)$. The formula does not make sense when $x = 0$, but the limit of $y$ as $x \to 0$ exists. Setting $y = 1$ when $x = 0$ makes our function continuous everywhere. Maple knows such tricks as extension by continuity and it automatically determines the $y$-range necessary for the plot.

```
> plot(sin(x)/x,x=-15..15);
```

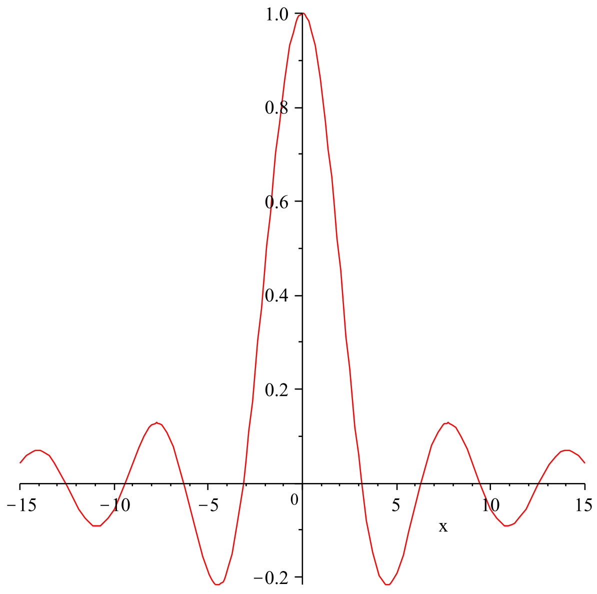


Figure 4.2: Maple's graph of smooth function: $y = \dfrac{\sin x}{x}$

Let's try a graph with vertical asymptotes. Consider the function `y2` on p. 89, which can be factorized as $(x+3)(x-3)^2/(2(x-1)(x-2))$.

```
> plot(y2,x=-10..10);
```

The graph, Figure 4.3, is not looking particularly illuminating. The vertical range can be altered with this graph to enable a clearer picture (Figure 4.4, left):

```
> plot(y2,x=-10..10,y=-40..40);
```

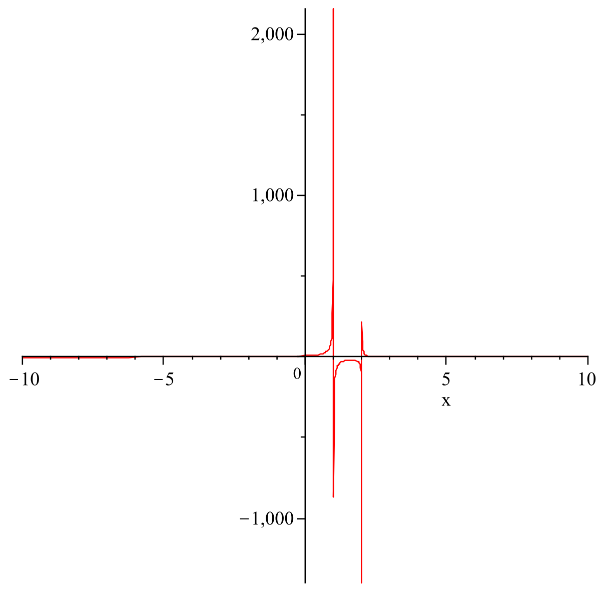


Figure 4.3: Maple's plot of $y = \dfrac{(x+3)(x-3)^2}{2(x-1)(x-2)}$. Default view.

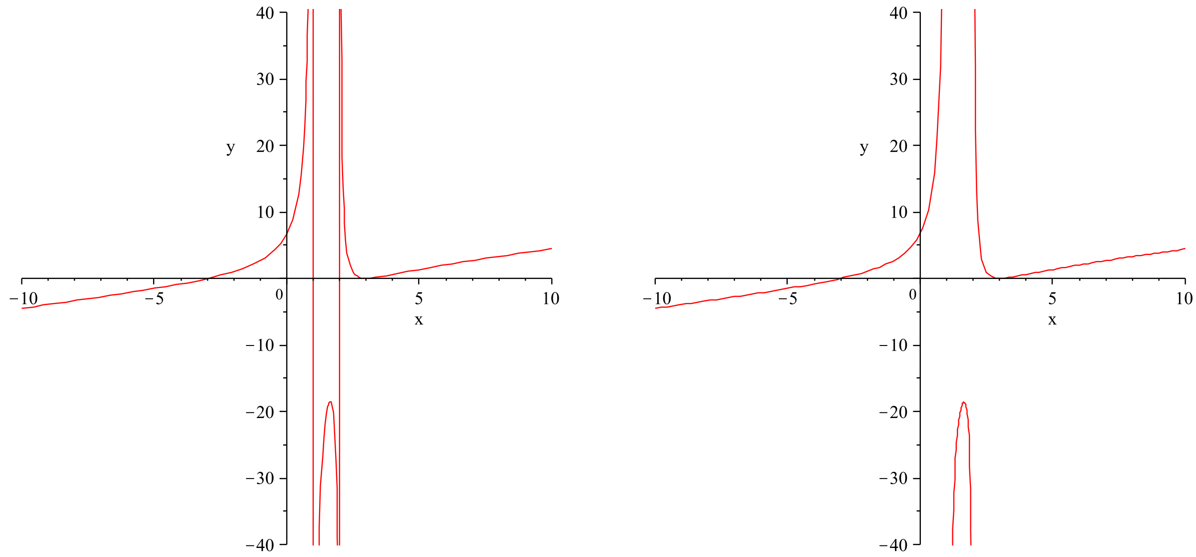


Figure 4.4: Plot with restricted $y$-range: discontinuity exhibited (left) or hidden (right).

A discontinuity detector can be used to remove the unnecessary lines, as on Figure 4.4, right:

```
> plot(y2,x=-10..10,y=-40..40,discont=true);
```

Coordinates can be plotted using the following format, yielding Figure 4.5.

```
> X:=[[-2, 4],[-1,1],[-1/2,1/4],[0,0],[1/2,1/4],[1,1],[2,4]];
```

$$X := \left[[-2,4],[-1,1],\left[-\frac{1}{2},\frac{1}{4}\right],[0,0],\left[\frac{1}{2},\frac{1}{4}\right],[1,1],[2,4]\right]$$

```
> plot(X);
```

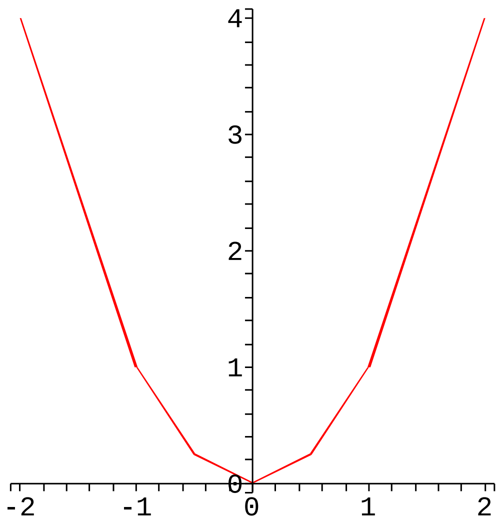


Figure 4.5: Maple's graphs defined by an array of coordinate pairs

Maple can plot parametric curves. In the following example we equalize the $x$ and $y$ scales using the option `scaling=constrained`. Without this option the graph, which is an ellipse (Fig. 4.6), would look like a circle.

```
 > plot([5/2*cos(t), 5/3*sin(t), t=0..2*Pi],scaling=constrained);
```

Attention! A slight syntactic alteration of the command — moving the bracket `']'` — completely changes the picture: instead of a parametric curve we obtain two curves on the same graph (Fig. 4.7), with $t$ being the independent variable (instead of being a parameter). The option `scaling=constrained` is unimportant in this example and omitted.

```
 > plot([5/2*cos(t), 5/3*sin(t)], t=0..2*Pi);
```

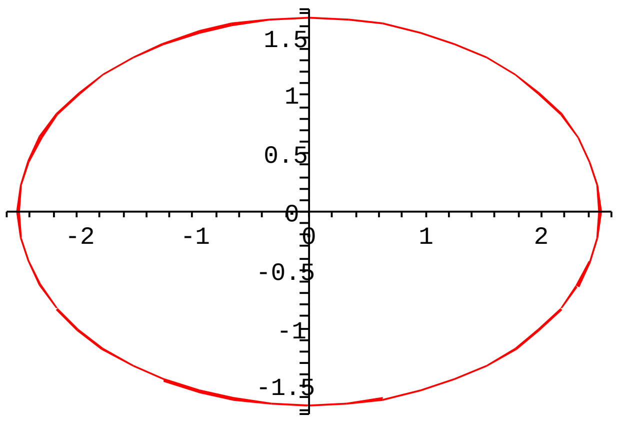


Figure 4.6: An ellipse described parametrically: $x = \dfrac{5}{2}\cos t,\ \ y = \dfrac{5}{3}\sin t.$

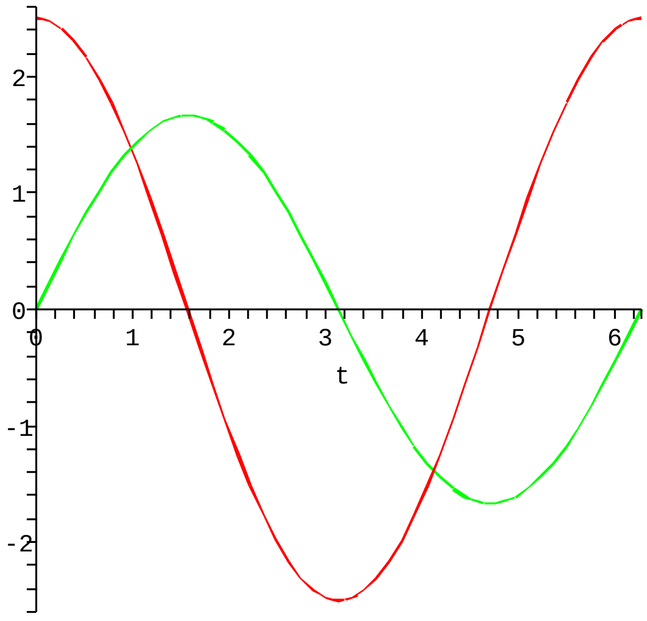


Figure 4.7: Curves $y = \dfrac{5}{2}\cos t$ and $y = \dfrac{5}{3}\sin t$ in the $(t, y)$ axes.

Here is an example of a 3D graph. (Compare Maple's graph with Gnuplot's — Fig. 4.12(D).)

```
> F:=sin(x^2+y^2)/(x^2+y^2);
```

$$F := \frac{\sin(x^2+y^2)}{x^2+y^2}$$

```
> plot3d(F,x=-3..3,y=-3..3);
```

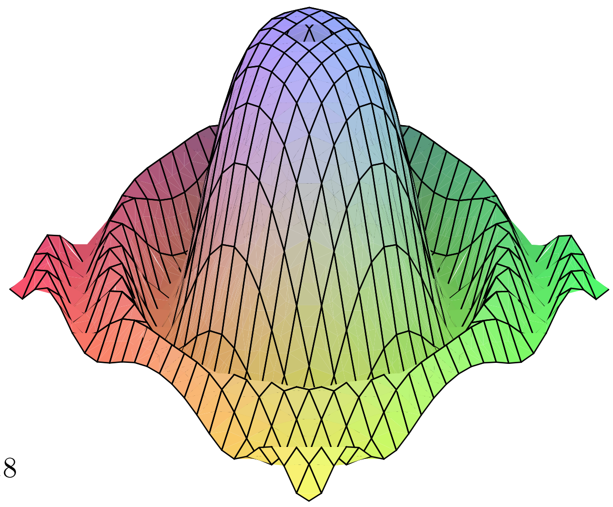


Figure 4.8

The default is not to show the axes. However, they can be added:

```
> plot3d(F, x=-3..3, y=-3..3, axes=boxed);
```

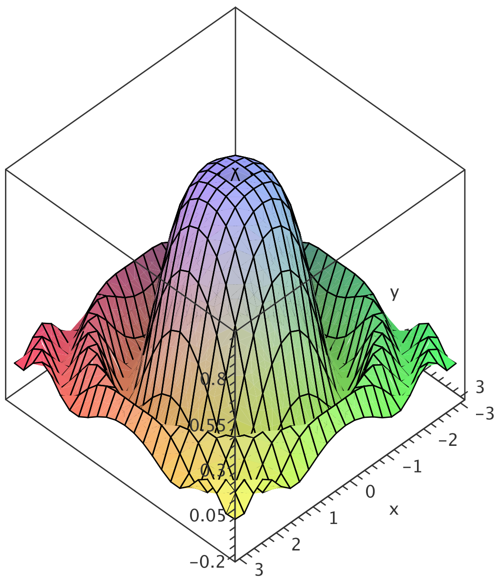


Figure 4.9

For more advanced tasks, we can load the plotting library, `plots`.

```
 >with(plots);
[Interactive, animate, animate3d, animatecurve, arrow, changecoords,
  complexplot, complexplot3d, conformal, conformal3d, contourplot,
  contourplot3d, coordplot, coordplot3d, cylinderplot, densityplot, display,
  display3d, fieldplot, fieldplot3d, gradplot, gradplot3d, graphplot3d,
  implicitplot, implicitplot3d, inequal, interactive, interactiveparams,
  listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d,
  loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare,
  pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d,
  polyhedra_supported, polyhedraplot, replot, rootlocus, semilogplot,
  setoptions, setoptions3d, spacecurve, sparsematrixplot, sphereplot,
  surfdata, textplot, textplot3d, tubeplot]
```

Plotting the graph of an implicit equation $f(x, y) = 0$ now becomes possible. Try

```
> f:=4*x^2+9*y^2-25;
> implicitplot(f, x=-4..4, y=-2..2, scaling=constrained);
```

The result is identical to Figure 4.6. It should not be surprising: the parametrized coordinates $x = \frac{5}{2}\cos t$ and $y = \frac{5}{3}\sin t$ satisfy exactly our present equation $4x^2 + 9y^2 = 25$. Note however that plotting functions implicitly is more difficult for Maple than plotting parametric curves with a command like that on p. 101. The quality of parametric plots will generally be better.

The library `plots` also makes available plotting multiple graphs on the same picture. Each graph (plot) can be created separately; then all of them are submitted at once to the `display` command. When creating the individual plots, use *colon* as a terminator; otherwise Maple will spit out the long and nasty internal representation of a plot. Example: the following commands produce the plot identical to Figure 4.7 save for a label on the horizontal axis.

```
 > sinplot:=plot(5/3*sin(x), x=0..2*PI, color=green):
 > cosplot:=plot(5/2*cos(x), x=0..2*PI, color=red):
 > display([sinplot, cosplot]);
```

While in this case the two curves can be conveniently plotted together using just the regular `plot` command as on p. 101, it is easily conceivable that individual plots can be too many or too complicated, so that computing them separately and then using the `display` command can be the only practical option. The following example, showing an equilateral triangle together with its inscribed and circumscribed circles (Fig. 4.10) would be too cumbersome to describe by a single `plot` command.

```
> triangle:=plot([[1,0],[-1/2, sqrt(3)/2],[-1/2,-sqrt(3)/2],[1,0]], color=red):
> circumcircle:=plot([cos(t),sin(t),t=0..2*Pi],color=blue):
> incircle:=plot([cos(t)/2,sin(t)/2,t=0..2*Pi],color=green):
> display([triangle,circumcircle,incircle],scaling=constrained, axes=none);
```

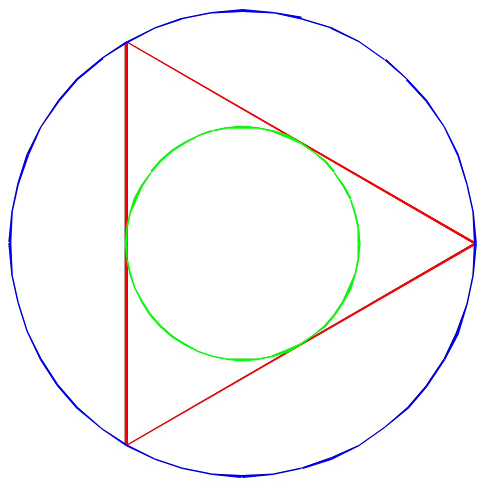


Figure 4.10: Equilateral triangle with inscribed and circumscribed circles

Our last example demonstrates a combination of Maple elements just described and those described in Section 4.2. We construct three similar parametric curves, called cycloids, each given by the parametric equations

$$x = C(0.3t - \sin t), \qquad y = C \cdot 0.6(1 - \cos t). \tag{4.1}$$

The parameter $C$ equals 0.5, 1, and 2 for the three curves, respectively. To make the $x$-span of the curves approximately equal, we choose the $t$-range the bigger the smaller $C$ is. Specifically, we set $t \in [-\pi, 9\pi]$ for $C = 0.5$, $t \in [-\pi, 5\pi]$ for $C = 1$, and $t \in [-\pi, 3\pi]$ for $C = 2$. The pattern is: $t_{\text{min}} = -\pi$ and $t_{\text{max}} = (1 + 4/C)\pi$.

Instead of creating plots of the three curves individually, we take advantage of the clear pattern and create a procedure. The first argument is the value $C$ in Eq. (4.1), and the second argument is the color to strike the curve with.

```
> cycloidC:=proc(C,col) local t,tmax,fx,fy:
      fx:=0.3*t-sin(t):  fy:=0.6*(1-cos(t)):  tmax:=(1+4/C)*Pi:
      return plot([C*fx, C*fy, t=-Pi..tmax], color=col):
  end proc:
```

We then assign the colors:

```
> cycloid_colors:=[red,blue,black]:
```

In the final plotting command, we use the array manipulation methods from Section 4.2.4.

```
> display([seq(cycloidC(2^(n-2), cycloid_colors[n]),n=1..3)] );
```

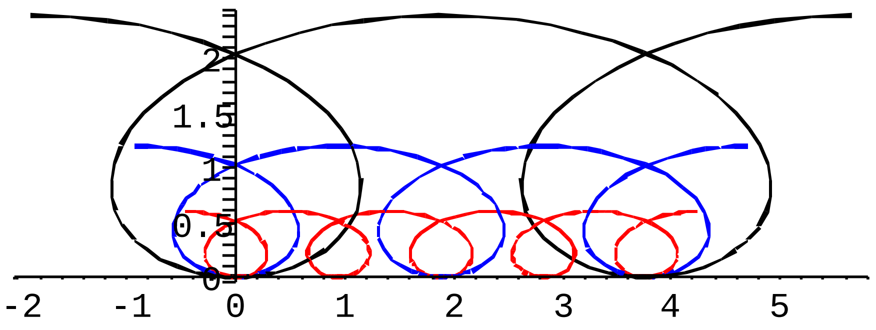


Figure 4.11: Cycloids given by Eq. 4.1 with $C = 0.5$ (red), $C = 1$ (blue), and $C = 2$ (black).

For comparison, here is an equivalent, more explicit Maple code for the same plot.

```
> a:=0.3:   fx:=a*t-sin(t):   fy:=a*(1-cos(t)):
> p0:=plot([0.5*fx,   fy, t=-Pi..9*Pi], scaling=constrained, color=red):
> p1:=plot([    fx, 2*fy, t=-Pi..5*Pi], scaling=constrained, color=blue):
> p2:=plot([  2*fx, 4*fy, t=-Pi..3*Pi], scaling=constrained, color=black):
> plots[display]({p0,p1,p2});
```

**How to insert a Maple graph in a LaTeX document.**

First, you have to save your graph as an Encapsulated Postscript file. The simplest method is to right-click on the graph you want to import and to *save image as EPS file.* Alternatively, especially if you have many pictures in your file, a convenient way to save all of them at once is to click on menu *File/ExportAs* and choose file type LaTeX.

If, say, the name of the Maple worksheet is `cycloids.mw`, then the name of the Maple-created LaTeX file is `cycloids.tex`, and the first plot will be saved as `cycloidsplot1.eps`, the second plot (if exists) — as `cycloidsplot2.eps`, and so on.

As advised on page 87, ignore the LaTeX file that Maple creates, but pick the companion `eps` files.

Once you have extracted an `eps` file or files from a Maple worksheet, import them in your master LaTeX file by means of the `\includegraphics` command.

As a final remark, we ask you **not** to include graphics **modified via the pop-up menu** that appears when you right-click on a picture. While this menu provides convenient manipulations, no record remains of the ultimate parameters. Hence the instructor may not be in position to tell how exactly your graph was generated based on the Maple worksheet you would submit electronically. Of course, if you fully document any manipulations, then these restriction will not apply.

### 4.3.3 Gnuplot

Gnuplot used to be thought of as a UNIX command-line utility, and it still is, but now it is also available for Windows, with a handy clickable menu. It is a freely distributed software.

Gnuplot can display graphs on a computer screen and save them as files of various graphics types. The kind of output display device is described by the parameter `terminal` of Gnuplot's `set` command.

The program is controlled by user's commands typed in the command line; several commands can be put together into a file to form a re-usable script.

Gnuplot can handle two-dimensional and three-dimensional data and graphs. It automatically changes line styles when several curves are plotted. It automatically produces a legend for each curve and permits the user to include such things as arrows and mathematical text anywhere on the graph.

Gnuplot is somewhat clumsy to use even with the simplest of data because of much typing necessary. In exchange, it provides a lot of flexibility in regard to line styles, labeling, and such, — perhaps, significantly more than Maple. It is also a very straightforward tool to plot graphs given as pairs of coordinates. On the other hand, Maple's versatility and symbolic manipulation power is incomparably higher.

The best way to use Gnuplot is to interactively piece together (step-by-step), vary and adjust all the essentials for the desired plot, all the while watching the plot displayed in its own window at every stage. Once the desired plot is generated and fine-tuned, it is easy to redirect the plot into a file.

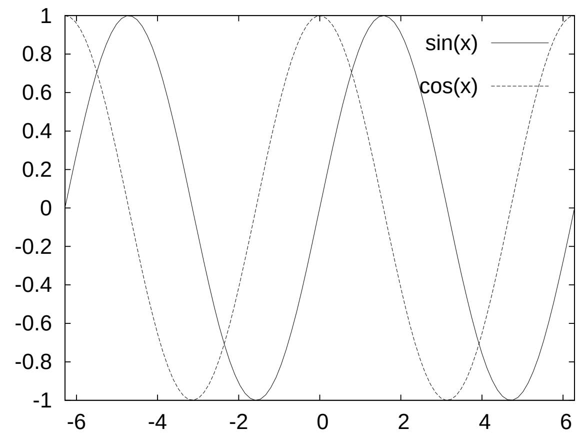
To start a Gnuplot session, simply type `gnuplot` in the command line and the Gnuplot prompt will appear. A Gnuplot session is ended by typing `quit` at the prompt.

The two main plotting commands are `plot` (for 2D data) and `splot` (for 3D data). Their behaviour is controlled by a wide range of options. Most of the options are introduced by the command `set`.

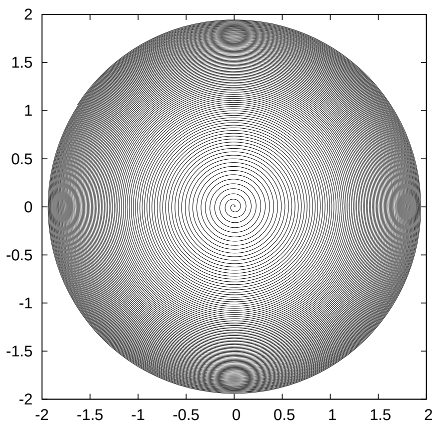
Gnuplot's originally intended and still most popular use is to render plots based on externally prepared data fed to it from a file. Yet modern Gnuplot knows many standard mathematical functions and it is quite capable of plotting on its own, as sample graphs that follow demonstrate. Here we are only able to help you get started with Gnuplot. An interested reader can find out about numerous options and features in Gnuplot's Help and in online tutorials, for example, `http://www.ibm.com/developerworks/library/l-gnuplot`.

#### Plotting functions with Gnuplot

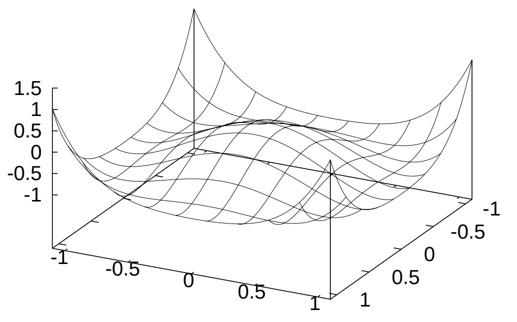
The figures on the next page demonstrate how Gnuplot can be used directly, without data importing. The commands used to generate each plot are given, but we are not attempting to explain the options involved. Look up a reference and play around with them to see, for instance, what effect the absence of the command `unset key` will produce or what will a different numerical value of `spacing`, `samples`, `isosamples`, etc. do.



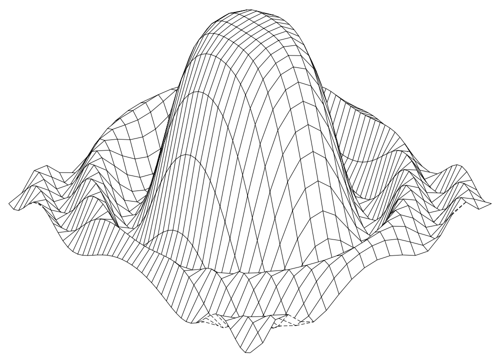
(A)



(B)



(C)



(D)

Figure 4.12: Graphs of functions produced by Gnuplot on its own

```
         (A)
set key spacing 2
plot [0:2*pi] sin(x),cos(x)
```

```
         (B)
unset key;  set parametric;
set trange [1:7]; set samples 10000
plot log(t)*cos(100*t), log(t)*sin(100*t)
```

```
         (C)
unset key; set ztics -0.5,0.5,1.5
set xrange [-1:1]
set yrange [-1:1]
splot (x**2+y**2)*(x**2+y**2-1.5)
```

```
         (D)
unset tics; unset border; unset key
set isosamples 50,20; set view 18,45,1,3
set xrange [-3:3]; set yrange [-3:3];
splot  (sin(x**2+y**2)/(x**2+y**2))
```

**Notes:** (B) represents a parametric curve — logarithmic spiral $\begin{bmatrix} x \\ y \end{bmatrix} = \ln t \begin{bmatrix} \cos(100t) \\ \sin(100t) \end{bmatrix}$.

(D) displays the same function as Figure 4.8 on p. 103.

### Two-dimensional data plots

We will now use Gnuplot to plot data from a file supplied by the user. For two-dimensional plots, the contents of the file, by default, should be a collection of ordered pairs $x$, $y$ separated by white space, one pair per line.

Let's say that you wish to plot two curves on the interval $x \in [-2, 2]$:

$$y = x^3 - 3x \quad \text{and} \quad y = \frac{1}{4}x^3.$$

- Choose the number of points to be used. Let's say, 100 points. Since the length of our interval is 4, the value of $x$ will be incremented by 0.04 from one point to the next.
- Write a program to generate the data sets for both functions. For example, here is a fragment of a C program that does it for the first function.

  ```
  for (x=-2; x<=2; x=x+0.04){
    printf("%f %f\n", x, pow(x,3)-3*x);}
  ```

  (This code prints the data to the terminal, but you can re-direct the output to a file: type `a.out > plot1.dat` — see Section A.7).
- A data file named `plot1.dat` has been created with 100 data pairs. For example, the first and the last lines in the file will be like these:

  ```
  -2.000000 -2.000000
      ......
  2.000000 2.000000
  ```

  Modify the program and create the file `plot2.dat` for the second function similarly.
- Now the command

  ```
  gnuplot> plot 'plot1.dat' with lines, 'plot2.dat' with lines
  ```

  generates the plot displayed in Figure 4.13.

On your terminal, the curves will be drawn in like-style but with different colours. When a hard-copy is generated, `gnuplot` knows that colour is not generally available and thus differentiates between different curves on the basis of different line styles.

### Displaying a surface in 3D space

Gnuplot accepts 3D data in the format similar to 2D: each line in the file represents the coordinates $x$, $y$, $z$ of a single point on the surface. There is an option to remove hidden lines, so that foreground and background can be differentiated.

Figure 4.14 was produced from the file `glass.dat` (author: Gershon Elber, 1990), which is a part of the repository of samples provided with Gnuplot distributions. It can be downloaded separately from `http://www.challenge.nm.org/ctg/graphics/glass.dat`.

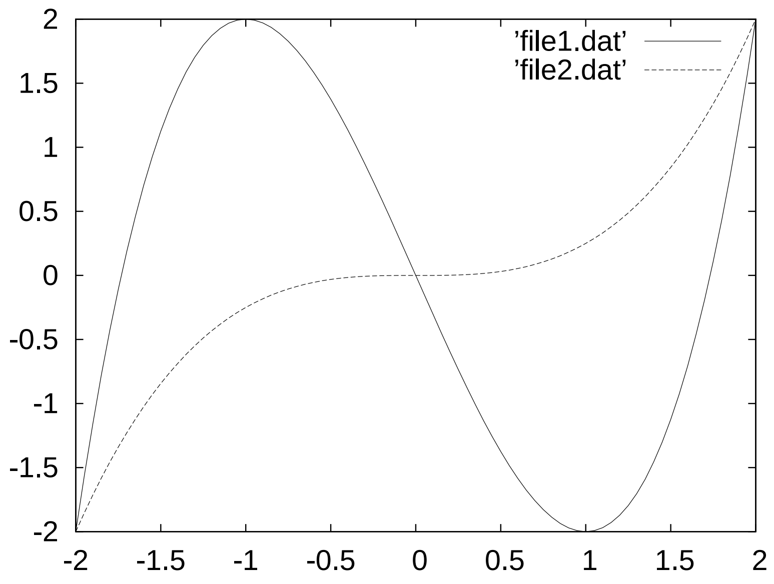


Figure 4.13: Graphs of $y = x^3 - 3x$ and $y = \frac{1}{4}x^3$ produced from data files.

The glass is a surface of revolution, which can be mathematically described by an equation of the form $r = f(z)$ in the cylindrical coordinates in $\mathbb{R}^3$, with $r = \sqrt{x^2 + y^2}$. The function $f(z)$ in this case is not given by a formula, but it is rather a result of an artwork.

Rendering three-dimensional surfaces is no simple business. If you venture to try, we recommend that all slices (level curves) have the same number of points per slice or level curve. The reason for this is that given a set of, say, $x$-slices, Gnuplot attempts to work out the missing $y$-slice data in order to perform the hidden line removal. Gnuplot will not necessarily fail if this advice is not followed, but its behaviour is then somewhat unpredictable.

The data file `glass.dat` contains $16 \times 16 = 256$ coordinate triples $(x, y, z)$. There are 16 data blocks, each describing the level curve $z = z_i$ of the glass (that is, the circle with radius $f(z_i)$), for 16 different (not equally spaced) values $z_i$ in the range from $z_1 = -0.911$ to $z_16 = 1.101$. Each block, in its turn, consists of 16 points, equally distributed along the level circle, with angular separation $360°/16$.

Here are the commands used to obtain figure 4.14, and a brief explanation.

```
unset key                                  Do not show legend
set hidden                                 Do not show hidden lines
set xtics -0.5 0.5 0.5                     Set ticks on x axis starting from −0.5,
set ytics -0.5 0.5 0.5                               with step 0.5, ending at 0.5
set xyplane 0                              Adjust the position of the base xy plane
set border 4095                            Draw a box around the plot
splot 'glass.dat' using 2:1:3 with lines     2:1:3 means interchange x and y data
```

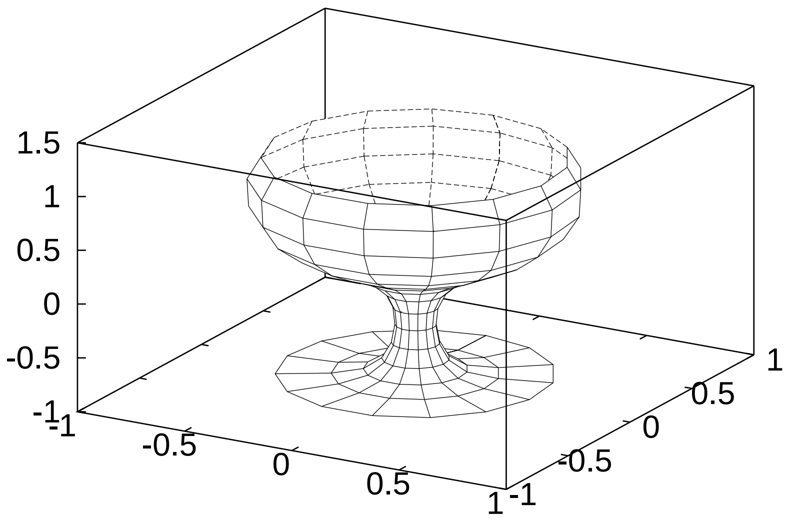


Figure 4.14: Plot produced from file `glass.dat`, with hidden lines removed.

The order in which the columns in the data file are used was changed with the "`using 2:1:3`" option in an effort to get a solid line to represent the forefront portion of the glass. (Otherwise the dashed line appears on the front somehow.) This is a harmless move in this case thanks to the rotational symmetry of the glass.

**Generating Postscript with Gnuplot**

Once, in the course of a Gnuplot session, you have a complete graph exactly as you want it, then simply issue the commands

```
> set terminal postscript eps
> set output 'figure1.eps'
> replot
```

The first of these (abbreviations `term` and `post` can be used) tells Gnuplot you wish to produce data in Encapsulated Postscript format, the second redirects the output into a file (in this case named `figure1.eps`), and the last command just redraws the plot last displayed into the file.

The file `figure1.eps` has now been created in your working directory. You can view it with Ghostview or insert it in your LaTeX document as descrined in Section 4.3.1. A scaling option (`height` or `width` in `\includegraphics`) may be helpful. (If you `set term post` without the `eps` option, then also conversion `ps` to `eps` and rotation by 270° will be required.)

The size of the picture as it will appear in your paper will often be smaller than what you see when previewing the generated file in Ghostview. Have mercy on your instructor's eyes, **use larger font size in labels.** This can be done as follows:

```
> set term post eps "Helvetica" 24
```

### 4.3.4 Using XFig to make diagrams

The program XFig is a handy tool that allows you to quickly draw diagrams consisting of simple shapes, to drag and drop objects, and even to put labels and formulas on your figures in a LaTeX format. It is great for creating **"hand-drawn" diagrams**, but less suitable when it comes to graphs where the positions of objects must be specified by coordinates. In XFig, you build a diagram **by mouse manipulations** and immediately see the result. Thus, as we see, the creation of XFig diagrams is not relied on a digital algorithm (although the resulting file is a digital description of the figure). As such, XFig should not be used in the cases where you must generate definite, unambiguously reproducible graphics. It can be used for auxiliary, artwork-like illustrations or schemes.

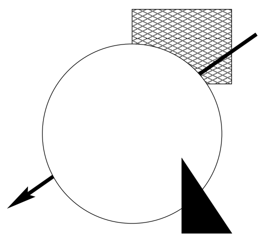
To start XFig, just enter the command

```
xfig
```

The XFig window has several icons along the left, several menu options along the top, including Help, rulers along the top and right, as well as a few setting boxes on the bottom. Playing with the icons on the left and the bottom is perhaps the best way to discover what you can do. The "scull and bones" button allows you to delete previously created objects. Once you get going, a few of the setting boxes on the bottom are worth noting, such as the *Point Posn*, *Depth*, and *Fill Style*.

Notice that the functionality of each of the 3 mouse buttons varies from option to option, and is displayed in the upper right corner of the window.

For example, the following diagram contains 4 objects that were each drawn with a different *depth* setting.

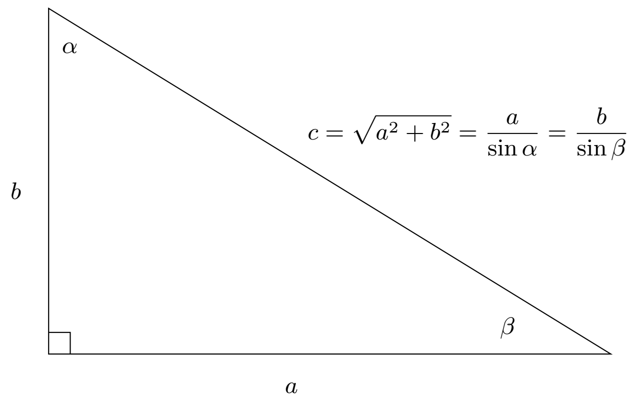


Once you've got a diagram ready for inclusion in a LaTeX document, first use the *File* menu option and save the figure in a file with a `.fig` extension, such as `diagram.fig`. Then use the *Export* menu option to export the figure using the *Encapsulated Postscript* language. Notice the default here will be to create a file named `diagram.eps`, which you can subsequently include in your document by following the instructions in Section 4.3.1.

You will likely want to label your diagrams on occasion, sometimes even using the math mode of LaTeX to display mathematical symbols. For this, you ought to start XFig with the command:

```
xfig -P -specialtext -latexfonts
```

Using the *text* drawing tool in XFig, you can then label your figures as you please, even including math mode text (which you can accomplish by using dollar signs). While XFig will not display your math mode labels very nicely, they will come out just fine in your document, such as in the following figure:


$\alpha$
$c = \sqrt{a^2 + b^2} = \frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$
$b$
$\beta$
$a$

However, when using math mode, you need to select the export option of *Combined PS/LaTeX* rather than *Encapsulated Postscript*. You also should use the magnification option of the XFig *Export* window in order to fine-tune the size of your figure as it appears in your document. This time, to include the diagram in your master LaTeX file, use a different LaTeX command:

```
\input diagram.pstex_t
```

In this import mechanism, only the intermediary file with extention `pstex_t` must be directly referenced from your LaTeX document; yet the actual figure is still an `eps` file (which in this case has extension `pstex`). Thus your **electronic submission** must include both the `.pstex_t` and `.pstex` files besides the main `.tex` file.

## 4.4 The LaTeX `picture` environment and enhancements

### 4.4.1 Introduction

The most straightforward way to draw diagrams in a LaTeXed document is to use the **picture** environment:

```
\begin{picture}(xlen,ylen)(leftcornerx,leftcornery)
...
\end{picture}
```

Here `xlen` and `ylen` are the ranges (from zero) of the $x$ and $y$ coordinates. The **optional** parameters, `leftcornerx` and `leftcornery` change the bottom left hand corner from the origin to the new coordinates. They may be omitted if the bottom left hand corner is the origin.

The picture environment essentially allows you to do only one thing — to `\put`:

```
\put(xcoor,ycoor){object}
```

An *object* can be a LaTeX's graphics element (`\line`,`\circle`), or a text, formula, paragraph (`\parbox`), table, etc. It can also be the `\includegraphics` command referring to an `.eps` file to be imported.

The dimensions of the picture (in the environment's header) and the coordinates in `\put` commands are given just as numbers, without specyfying the unit of length. The default **unitlength** is 1 point ($= 1/72\,in = 0.35\,mm$). It is rather too small for practical purposes and for convenience it can be reset by a command like

```
\setlength{\unitlength}{1cm}
```

printed just before the beginning of the picture environment.

LaTeX has very limited and poor graphical faclities of its own. Its `\line` command cannot even create arbitrary lines. However, with **enhancements** provided by additional packages, LaTeX's picture environment becomes competitive in its ability to generate interesting quality graphics. From now till the last part of this chapter, Section 4.4.4, we will stay entirely within a LaTeX document. No other graphics formats or imports will be involved.

We will describe in some detail the enhancements offered by the `math2130.sty` package. In particular, a necessity to decide which unit length to set for the given picture can be avoided by use of an enhanced picture environment called **scaledpicture**. It is used thus:

```
\begin{scaledpicture}{percent}(xlen,ylen)[(leftcornerx,leftcornery)]
...
\end{scaledpicture}
```

Here, `percent` is a number less than or equal to 100, where 100 gives a diagram that almost fills the entire text width, and any smaller number gives a diagram that spans a percentage of the corresponding 100% diagram.

In most respects, `scaledpicture` behaves like the standard LaTeX `picture` environment, but it has additional features to make it easier to produce diagrams quickly and easily.

### 4.4.2 Lines

A major restriction in the LaTeX picture environment lies with the available slopes of lines that can be drawn. These slopes are restricted to *rationals* of the form $p/q$, where $p$, $q$ are coprime and less than or equal, in size, to 6. The slopes are written as ordered pairs, $(p, q)$ to allow for vertical lines, given by $(0, 1)$ or $(0, -1)$, depending on the direction.

Another complication in the syntax of a line segment, which is given by:

```
\line(p,q){len}
```

is that `len` refers, not to the length on the line, but to its projection in the $x$–direction (unless, of course, it is vertical, when `len` means the vertical length).

Even worse, each line segment must be `put` somewhere with a `\put` command of the form:

```
\put(a,b){\line(p,q){len}}.
```

You can see that the simple process of drawing a straight line is quite complicated.

### 4.4.3 Enhanced Pictures

However, help is at hand with enhanced picture styles. Since you will have started almost every document for this course with

```
\documentclass{article}
\usepackage{2130}
```

it makes sense to learn a few new commands available in the **picture** environment.

#### The `\join` command

The most useful command is:

```
\join(x1,y1)(x2,y2)
```

This draws a straight line from `(x1,y1)` to `(x2,y2)`. Already you can see an advantage over the standard `\line` command. The restrictions on slope no longer apply. Even better, by using

```
\join(x1,y1)(x2,y2)(x3,y3)
```

we get a straight line from `(x1,y1)` to `(x2,y2)` followed by a straight line from `(x2,y2)` to `(x3,y3)`. By doing this successively, with the points sufficiently close together, we can draw curves. So, you must first calculate the appropriate data set for the curve (see Sect. 4.1.3). The computed data set can be either pasted into the `.tex` file or, alternatively, it can be kept in its own file, say, `curve.tex` and the command

```
\input{curve}
```

can be used to import the data into your master LaTeX file.

### Dotted lines

`\dottedline[`*`dotchar`*`]{dotgap}(x1,y1)(x2,y2)...(xn,yn)`

This draws a dotted line joining `(x1,y1)` to `(x2,y2)` and so on, to `(xn,yn)`. The `dotgap` is given in the units equal to the `\unitlength` defined and needs not be an integer. The optional *`dotchar`* may be omitted to give the default of a small dot, but any character may be used.

You can use this, with the appropriate `dotgap`, as a method for putting markers on curves.

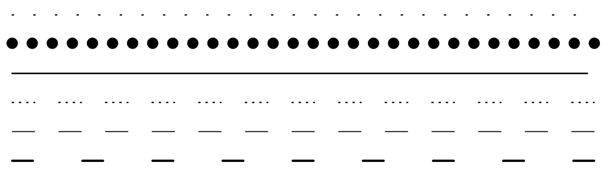
### Dashed lines

`\dashline[`*`stretch`*`{dashlen}[`*`dotgap for dash`*`](x1,y1)(x2,y2)...(xn,yn)`

This draws a dashed line joining `(x1,y1)` to `(x2,y2)` and so on, to `(xn,yn)`. The `dashlen` is the length of the dash. Each dash is in fact a dotted line, and the optional *`dotgap for dash`* is the gap between each dot that is used to construct the dash. Both are in current unitlengths. The optional *`stretch`* is an integer between $-100$ and $+\infty$. You should experiment with these to find the appropriate relationship among the parameters to suit your purpose.

Here are some examples created by:

```
\begin{center}\setlength{\unitlength}{1em}
\begin{picture}(20,7)
   \dottedline{.7}(0,6)(20,6)
   \dottedline[$\bullet$]{.7}(0,5)(20,5)\join(0,4)(20,4)
   \dashline{.8}[0.2](0,3)(20,3)
   \dashline{.8}(0,2)(20,2)
   \thicklines\dashline[-30]{.8}(0,1)(20,1)
\end{picture}\end{center}
```



### Grids

Grids can be created in many ways using:

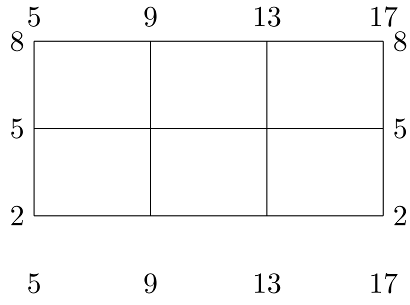
`\grid(xlen,ylen)(`$\Delta$`xlen,`$\Delta$`ylen)[init-x,init-y]`

This creates a grid measuring `xlen`$\times$`ylen` with each `xlen` interval being $\Delta$`xlen`, and each `ylen` interval being $\Delta$`ylen`. The inputs of `init-x` and `init-y` give the coordinates of the bottom left hand corner of the grid.

The next diagram is generated within picture environment by the command

```
\grid(12,6)(4,3)[5,2]
```

(Picture dimensions are (12,6) and `unitlength` is set equal to 1em).



**Circles**

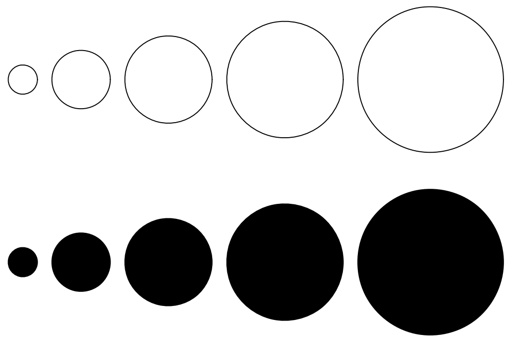
The standard LaTeX picture environment allows you to draw circles using the command:

```
\put(x,y){\circle{diam}}
```

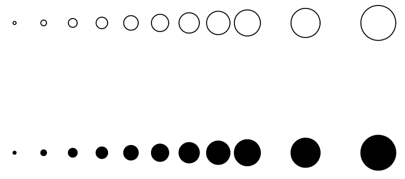
The parameter `diam` is the diameter of the circle (measured in `unitlength`), centered on `(x,y)`, and is an integer in the range $0 \leq$ `diam` $\leq 5$. However, there is also a restriction on the maximum diameter of circle that can be drawn in absolute units: it cannot exceed 15 points (about 0.5 cm). In the enhanced picture enviroment, these restrictions no longer apply, although large circles will turn out to be rectangles with rounded corners.

There is a variation, `\circle*{diam}`, which gives a solid disk instead of a hollow circle. If you try to make a solid circle that is too large, LaTeX will not fill it in!

```
\setlength{\unitlength}{1em}
\begin{center}
\begin{picture}(24,5)
\put(2,3){\circle{0}}    \put(3,3){\circle{1}}   \put(5,3){\circle{2}}
\put(8,3){\circle{3}}  \put(12,3){\circle{4}}  \put(17,3){\circle{5}}
\end{picture}
\end{center}
```



```
\setlength{\unitlength}{0.1em}
\begin{center}
\begin{picture}(140,50)
\put(0,30){\circle{0}}  \put(10,30){\circle{1}}
\put(20,30){\circle{2}} \put(30,30){\circle{3}}
\put(40,30){\circle{4}} \put(50,30){\circle{5}}
\put(60,30){\circle{6}} \put(70,30){\circle{7}}
\put(80,30){\circle{8}} \put(90,30){\circle{9}}
\put(110,30){\circle{10}} \put(135,30){\circle{12}}
\end{picture}
\end{center}
```



In the **scaledpicture** environment, drawing circles is made easy, and consistently correct, with the command `\arc`. This command has three parameters, and is used thus:

`\put(a,b){\arc(p,q){deg}}`

Here, the centre of the arc is at `(a,b)`; the point where the arc begins is `(p,q)`, with this being taken **relative to the centre of the arc**; and the arc is drawn `deg` degrees in the positive (counter-clockwise) sense from the starting point. For example, `\put(0,1){\arc(2,0){360}}` will draw a circle of radius `2`, centered at `(0,1)`.

## Labels

Labels can be placed on a diagram using the command `\put(x,y){label}`. A label is usually a letter or a number typed in math mode, like `$A$`, to produce a slanted $A$ on the picture (for numbers, the math mode ensures that the negative sign will have an appropriate length). While easy in its syntax, this command requires some experience and judgement as to where one should "put" the label if, say, the label is referring to the point at the top right corner of a square. Try it for yourself. Try to label a simple square $ABCD$ and you will need to adjust the values of `x` and `y` several times before the labels finally look nice.

In the **scaledpicture** environment, this is made easy. There is one general command, and several short forms. The great advantage is that they refer to the point that is being labelled, say `(a,b)`.

**The general command**

`\angleput{deg}[scale](a,b){label}` — the default scale number is `1`.

**Short forms**

1. `\cput(a,b){label}` — `deg=0`, `scale=0` — centred on the point.

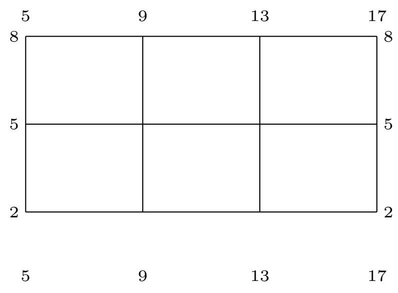
   In the next eight commands, `scale=1`.

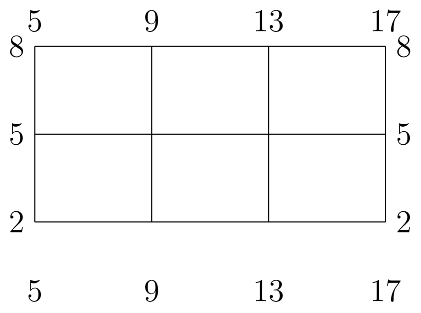
2. `\eput(a,b){label}` — `deg=0` — east of the point
3. `\nput(a,b){label}` — `deg=90` — north of the point
4. `\wput(a,b){label}` — `deg=180` — west of the point
5. `\sput(a,b){label}` — `deg=-90` — south of the point
6. `\neput(a,b){label}` — `deg=45` — northeast of the point
7. `\nwput(a,b){label}` — `deg=135` — northwest of the point
8. `\swput(a,b){label}` — `deg=-135` — southwest of the point
9. `\seput(a,b){label}` — `deg=-45` — southeast of the point

In the **scaledpicture** environment, the font size is chosen accordingly to the `percent` parameter. You may override this by the usual font sizing commands. Compare:

```
\begin{Scaledpicture}{36}(12,6)
\grid(12,6)(4,3)[5,2]
\end{scaledpicture}
```

```
\begin{Scaledpicture}{36}(12,6)
{\large \grid(12,6)(4,3)[5,2]}
\end{scaledpicture}
```





A `scaledpicture` is always centered. In fact, the environment used here was `Scaledpicture`, which produces an uncentred `scaledpicture`. This is useful for putting several diagrams on one line.

To further demostrate the use of labeling commands provided by `scaledpicture`, here is a set of commands for a cubic graph:

```
\begin{scaledpicture}{70}(8,6)(-4,-3)
\xaxis \yaxis \xnums{1} \ynums{1}
\ticks{1}[-0.1] \thicklines
\input{cubic_graph}
\put(-2.07936,0){\circle*{0.1}} \put(0.46295,0){\circle*{0.1}}
\put(3.1164,0){\circle*{0.1}} \put(2,-2.33333){\circle*{0.1}}
\put(-1,2.16667){\circle*{0.1}} \put(0,1){\circle*{0.1}}
\put(-4,-3.7){\large The graph of $f(x)=\frac13 x^3-\frac12 x^2-2x+1$}
\end{scaledpicture}
```

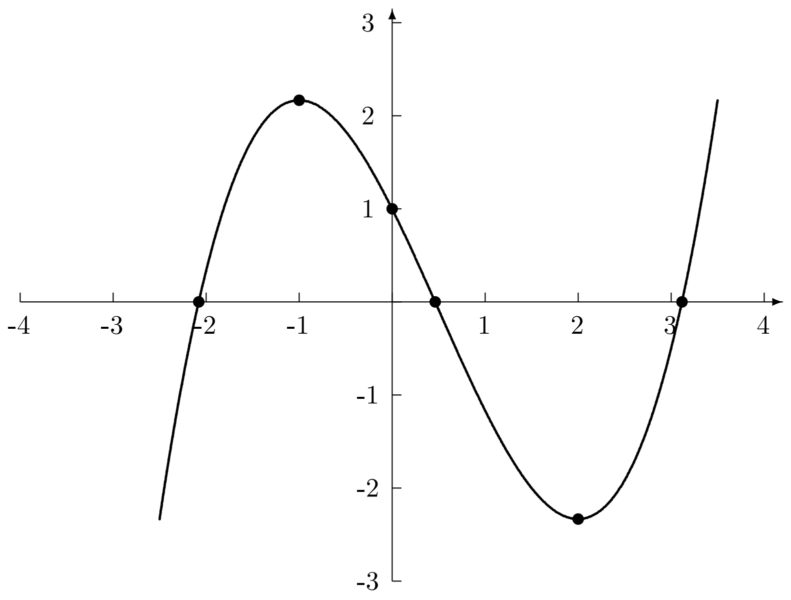


Figure 4.15: The graph of $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$

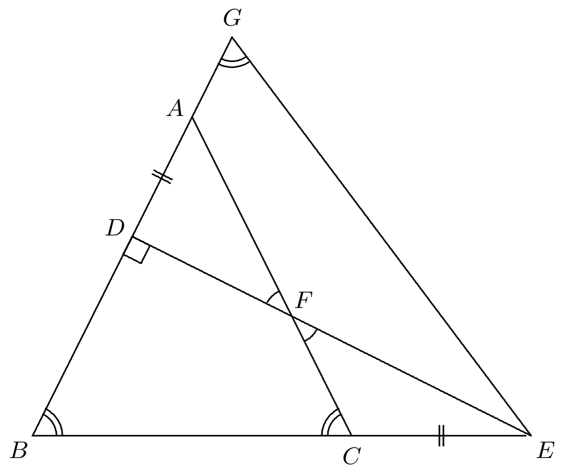
The `\input{cubic_graph}` command imports the contents of the file `cubic_graph.tex` whose first few lines are

```
\join(-2.50,-2.333)(-2.48,-2.200)(-2.46,-2.068)(-2.44,-1.939)(-2.42,-1.812)
(-2.400,-1.688)(-2.380,-1.566)(-2.360,-1.446)(-2.340,-1.329)(-2.320,-1.214)
(-2.300,-1.101)(-2.280,-0.990)(-2.260,-0.882)(-2.240,-0.775)(-2.220,-0.671)
(-2.200,-0.569)(-2.180,-0.470)(-2.160,-0.372)(-2.140,-0.277)(-2.120,-0.183)
(-2.100,-0.092)(-2.080,-0.003)(-2.060,0.084)(-2.040,0.169)(-2.020,0.252)
```

This is a typical example of a file that you can generate by your own program as described in Section 4.1.3.

Finally, here is the set of commands for the diagram that follows:

```
\begin{scaledpicture}{50}(13,12)(0,-1)
\join(0,0)(12.5,0)(5,10)(0,0) \join(12.5,0)(2.5,5) \join(4,8)(8,0)
\swput(0,0){$B$} \seput(12.5,0){$E$} \sput(8,0){$C$} \nput(5,10){$G$}
\angleput{153}[1](2.5,5){$D$} \angleput{153}[1](4,8){$A$}
\angleput{55}[1](6.5,3){$F$} \put(2.5,5){\rtangle{243}{.5}}
\put(10.25,0){\join(.05,-.25)(.05,.25)\join(-.05,-.25)(-.05,.25)}
\put(3.25,6.5){\rotate{63}{\join(.05,-.25)(.05,.25)
\join(-.05,-.25)(-.05,.25)}}
\put(6.5,3){\arc(.5,-.5){15}\arc(.5,-.5){-18} \arc(-.5,.5){15}
\arc(-.5,.5){-18}} \put(0,0){\arc(.6,0){64}\arc(.75,0){64}}
\put(8,0){\arc(-.6,0){-64}\arc(-.75,0){-64}} \put(5,10){\arc(0,-.6){34}
\arc(0,-.6){-27}\arc(0,-.75){34} \arc(0,-.75){-27}}
\end{scaledpicture}
```



Two commands here, `\rotate` and `\rtangle`, are defined in the `2130.sty` file using a bit of Postscript programming and by means of a command `special`. When the `special` command is used, the result may not always be dispalyed correctly. In this case, the `.dvi` picture may have some unrotated elements, but when a `.dvi` file is converted to `.pdf`, the correct rotation is put in place.

### Some other picture commands in `2130.sty`

`\closecurve(a,b,c,d,e,f)` produces a triangle on the vertices $(a, b)$, $(c, d)$, and $(e, f)$.

`\curve(a,b,c,d,e,f)` produces two segments joining $(a, b)$ to $(c, d)$ to $(e, f)$.

`\rotate{deg}{object}` rotates the object through "deg" degrees. This needs Postscript.

`\rtangle{deg}{size}` is used in diagrams to produce a right angle marker. Needs Postscript.

### 4.4.4 Superimposition

The `picture` environment can be used to manipulate position of graphics and text by hand if needed. While this should not be considered as a good practice in general (Do not fight LaTeX! It knows better!), sometimes knowing how to fine-tune your document may help.

As an example, consider the layout of Figure 4.1 on page 98. On the left, we have the text of a program made within the `verbatim` environment, and on the right there is an `eps` picture of the two triangles inserted by means of the `incudegraphics` command. The question is how it is possible to make LaTeX to put the graphics in such a non-standard place. The key trick is the picture environment with a tiny height, which however enables precise positioning of any objects (graphics or text) via the coordinates in the `\put` command.

```
\begin{figure}[H]
\begin{verbatim}
     Text of the Postscript program
\end{verbatim}

\begin{picture}(400,1)
\put(300,30){\includegraphics{triangles}}
\end{picture}

\caption{A simple Postscript program and its effect}
\end{figure}
```

The main difficulty in such cases is determining the coordinates where to `\put` an object. It is, essentially, a trial and error business; the "convergence rate" of the process and precision with which you can drop the thing where you want it to be greatly improves as you gain experience.

Using superimposition within the master LaTeX document, it is possible to insert labels, on graphs created by various software tools. The advantage of this approach is that your labels will always be in the same font style as the rest of your document and their size will not depend on the scaling you apply to the imported graphics.

The pattern is simple:

```
\begin{picture}(...)
\put(0,0){\includegraphics{ your .eps file}}
\put(...){$ label$ or text}
\end{picture}
```

For the dimensions of the picture (in points), you can take the dimensions of the EPS graph, which can be calculated based on `BoundingBox` information. (The `%%BoundingBox` line can be found in most EPS files near the top of the file.) Suppose, for example, that the `BoundingBox` numbers are `50 60 410 302`. Then the horizontal size of the graph is $410 - 50 = 360$ and the vertical size is $302 - 60 = 242$. Thus you can use `\begin{picture}(360,242)`. Setting the picture dimensions precisely is not necessary and you can always make adjustments if you don't like the way the compiled LaTeX document looks.

# Chapter 5

# Local system particulars

The information in this chapter is believed to be mostly valid as of December 5, 2008. As the computer systems undergo updates and upgrades, some of it will become obsolete.

## 5.1 Electronic submissions

The command `submit` is to be used to submit your assignments electronically. Run it from your local shell window prompt in Linux.

You should place all the files you wish to submit for your assignment in a directory with a certain name determined by your instructor: math2130-a1, math2130-a2, etc. We recommend that this directory be different from your working directory for the current assignment (it is convenient to make it a subdirecotry). After you have completed your work, the working directory will contain files with extentions `.log`, `.aux`, `.bak`, which should not be included in the submission. The working directory can also contain drafts that you do not want to submit. Copy only those files to the submission directory that are needed to reproduce your work in a form identical to the printed copy you are submitting. Include the `.tex` file (or files, if there are several) and all graphics files (`.eps`, `.pdf`, `.eepic`, etc.) that your master file refers to. Also include a copy of your computer program (`.f`, `.c`, etc.) or Maple worksheet (`.mw`). If there is a certain data file with "hidden" (not told to the reader) data which your report depends on, it too should be included.

Before issuing the `submit` command, make sure your **current directory is the parent directory of the directory containing the files to be submitted**. For example, if your Assignment 1 files are located in `~/A1/math2130-a1`, use the command `cd ~/A1` to change into the `A1` directory before running the `submit` command.

Actual submission for Assignment 1 for a course called math2130-2 (math2130, section number 2) is done by typing the following in the command line (from the parent directory of the directory `math2130-a1` containing your submission:

```
submit submit math2130-2 a1
```

Assignment 2 will have a submission ID of a2 and so on. Students of section 3 should replace `math2130-2` into `math2130-3`.

You will be asked to enter your password.

(Students who wonder why the word *submit* is repeated twice can compare the above command with the one that an instructor would use to retrieve your submissions:
`submit retrieve math2130-2 a1`.)

If you are not sure of the course or assignment ID to use with the `submit` command, you may type at the shell window prompt

```
submit list
```

to get a list of courses and dates on which there are assignments due.

If you mistakenly submit incorrect file(s) or you make changes to your file(s) after you have submitted them, you may submit again (before the due time). Your most recent submission must exactly match the submitted hard copy of the report.

Be aware that the size of your total submission is limited by 10 MB and the size of each single file being submitted should not exceed 2MB.

Why are the electronic submissions required? They are archived year over year and can be automatically checked to detect any occurrences of textual overlap. An instructor can use electronic submissions to verify whether your reported results are actually obtained through your program. If such a verification is impossible due to insufficient information provided, the instructor has a right to doubt the integrity of your research (see Sect. 1.3.2).

## 5.2 Laboratory computers on campus

### 5.2.1 Where

There are two computer laboratories available to mathematics and statistics majors, HH-3030/3056 and EN-2036. As well students may use computers in various General Access Labs around campus.

For example, students may access their accounts through one of the computers located in Chemistry/Physics General Access Lab (C-2004) or at the Commons located in the QEII Library. They also may access computers in C-2003 and CS-1019 when there are no classes taking place in these labs. Please note that those machines are maintained by the Department of Computing and Communication. Should you have difficulty logging into a computer, please advise the person who is working in the lab.

### 5.2.2 Your computer account

Everyone has been assigned a computer account, which provides access to the desktop workstations, which comprise **MUN LABNeT**. Each student is provided with his/her *home directory* on the student disk, where your personal files reside. You are not able to read or write files in someone else's home directory and no one else can read or write to yours.

Each workstation has a name. Newfoundland communities are used in HH-3030/3056; the workstations in EN-2036 are named after Transformers characters. The machines in HH-3030/3056 will boot to either Linux or Microsoft Windows. They boot to Windows by default. To work with LaTeX and program compilers, you must click on Linux.

To log in, type your user name and hit *Enter*, then type your password and hit *Enter* in the appropriate field in the middle window. Assuming no typing errors, the screen will clear and you will see a desktop with a few icons to the left and a toolbar along the bottom.

The home directory, where all your files reside, does not depend on the particular workstation you are using, nor on the operating system you are booted to. Thus if one day you log onto **lumsden** in HH-3030/3056 under Windows and the next day to **bumblebee** in EN-2036 under Linux, your files will be the same. Whenever you create a file and save it, it does not get saved on the local or remote workstation; rather it is written in your home directory on the student disk and it is then available to you from any other workstation. This is all done in a seamless fashion so you really need not concern yourself.

**Organize your work.** Create a separate subdirectory in your home directory for each course. Within it, create a separate subdirectory for each project. In your Math-2130 directory, you may also create a subdirectory for LaTeX and Maple samples provided by the instructor or for your own experiments with these software systems.

We suggest that you manage your home directory carefully, regularly deleting unnecessary files. Though disk space limitations may not be a big concern these days, regular clean up will keep your directory well organized; otherwise you will feel an increasing discomfort due to accumulating junk files.

**Rules of conduct.** You are strongly encouraged to read and heed the university's policy concerning the appropriate use of campus computers. It can be found at

```
http://www.mun.ca/scac
```

### 5.2.3 Printing

A laser printer is available in each lab. It is important to replenish your paper account in advance of the submission deadline. Inability to get a printed copy of your work due to insufficient funds is not an excuse for missing a deadline. There are several stations across Campus where you can deposit funds onto your Mun One Card. There is one in the library Commons area and one in the Chemistry/Physics (CP2003-4). This money is not locked into a particular lab and can be used on all LABNeT printers. A noted peculiarity is that, once started, a printing job will apparently be finished even if it results in a negative account balance — but the card reader will be showing zero balance until you cover the debt.

## 5.3 Software

### 5.3.1 Processing LaTeX files in the command line

You process the file `mylab1.tex` with the command

```
latex mylab1
```

Note that there is no need for the extension `.tex`.

If there are errors, LaTeX will stop at the first one and leave you hanging at a question mark on the screen. At this point, you may answer `x` to stop processing and to fix the reported error. Or if you answer `r` (for *run*), LaTeX will finish processing to the best of its ability, writing all errors to `mylab.log`, which you can then review in one window while correcting `mylab.tex` in another.

To view `mylab.dvi`, use a UNIX program called `xdvi`:

```
xdvi mylab1
```

Again, there is no need for the extension `.dvi`.

As of now, it is not possible to print your `dvi` document from the viewer's window. For printing, you need to convert it to either to a Postscript file:

```
dvips mylab1.dvi mylab1.ps
```

or to a PDF file:

```
dvipdf mylab1.dvi mylab1.pdf
```

To open a Postscript picture, invoke the GhostView viewer by typing `gv` or `ghostview` on the command line:

```
gv mylab1.ps
```

PDF files are viewed with familiar Acrobat Reader:

```
acroread mylab1.pdf
```

Both GhostView and Acrobat allow you to print the document you are viewing.

LaTeXfiles that do not contain `eps` graphics can be conveniently processed in one step by the `pdflatex` command:

```
pdflatex mylab1
```

Remember to run the LaTeX compiler (whether `latex` or `pdflatex`) **two times** if your file contains either of the following: the table of contents, the bibliography section, automatically numbered equations or figures.

### 5.3.2 Kile — integrated LaTeX environment

Kile (launched by the command `kile`) is a UNIX text editor specifically designed to assist with preparation of LaTeX documents. Besides the many functions common to text editors (including spell checking and text highlighting), Kile allows you to compile LaTeX documents and view them without switching to a shell window.

The icon depicting a little blue wheel invokes `latex` command. The icon where the wheel overlaps with a red curve invokes `pdflatex`. There are buttons that launch `dvi`, `ps`, and `pdf` viewers, and buttons that convert `dvi` to `ps` or `pdf`.

### 5.3.3 Compilers

There are compilers available on the Linux machines for the FORTRAN, C, C++, and Java languages. Some students may use other languages of their choice, like Python, for example, unless the instructor raises objections.

There are naming conventions for source code filenames which must be adhered to with each of the various compilers.

| | |
|---|---|
| FORTRAN | There are two compilers, `f95` and `ifort`, and program source filenames must end in `.f`. |
| C | The compiler is `gcc`, and program source filenames must end in `.c`. |
| C++ | The compiler is `g++`, and program source filenames end in `.c`, `.cc`, or `.cxx`. |
| java | The compiler is `javac`, and the program source file names end in `.java`. |

For example, suppose we have a FORTRAN program named `assign1.f`.

1. Compile the program with the command
   `lumsden $` *f95 assign1.f*
   which creates the executable file `a.out`. If you want to have the executable file named differently, use the `-o` option as in
   `lumsden $` *f95 assign1.f -o assign1*
   which will name the executable file `assign1` instead of the default `a.out`.

   If you just want to get an error report, use
   `lumsden $` *f95 assign1.f -c assign1*
   Any syntactical errors in your program will be reported by the compiler. If any are found you will have to go back to the first step and re-edit your source file.

2. Run your program, which is done simply by typing the name of the executable file. For example, if you created the executable file `assign1` as in Step 1, it is run with the command

```
lumsden $ assign1
```
otherwise, the command would be
```
lumsden $ a.out
```

### 5.3.4 Maple

To enable access to Maple in your account, a licence information must be set. This needs to be done only once, by issuing the command

```
export LM_LICENSE_FILE=28002@noether
```

After that, you should be able to use Maple.

The command *xmaple* invokes Maple 11 (the most recent version currently available in the lab) in the standard mode.

The command *xmaple ␣ -cw* invokes Maple 11 in the Classic Worksheet mode.

The command *xmaple 9* invokes Maple 9.

Maple can be invoked in a non-graphical command line mode by the *maple* command. The session opens quickly. This mode is particularly advantageous if you connect to Maple from your home computer and do not need graphics. To end the session, type **quit**.

### 5.3.5 Miscellaneous

1. Gnuplot is invoked by the *gnuplot* command.
2. XFig is invoked by the *xfig* command.
3. To convert a `ps` file into `eps` file, many UNIX systems have a command `pstoeps` or `ps2eps`. Windows-based versions of GhostView have an EPS convertor under *File* menu. Unfortunately, these options are not available on the LabNET machines. Here is a conversion method, which is a little ugly, but it works. Suppose `fig1.ps` is the name of the original Postscript file.
   (a) Type the command

       *gs ␣ -sDEVICE=bbox fig1.ps*

   (b) Among the output that appears on the screen, find and select by mouse a line that looks like this:
   ```
   %%BoundingBox: 20 118 575 673
   ```
   (c) Open the file `fig1.eps` in a text editor (say, in Kile). The first line will look like this:
   ```
   %!PS-Adobe-2.0
   ```

Change it into

```
%!PS-Adobe-3.0 EPSF-3.0
```

Paste the selected BoundingBox specification immediately after, so the second line in your updated file must be similar to this:

```
%%BoundingBox: 20 118 575 673
```

(d) Finally, save the file as `fig1.eps`.

4. Do not use command *pdflatex* or the corresponding icon in Kile if your LaTeX file contains references to `eps` graphics. Compile your `tex` file into a `dvi` file and then convert `dvi` into `pdf` if you wish. You can successfully use *pdflatex* if the only type of graphics in your document is that provided by the LaTeX's *picture* environment or its extensions as described in Sect. 4.4.

5. The command `lgrind` (Sect. 3.1.9) may not work on your home computer (although you may have successfully installed a LaTeX distribution).

# Appendix A: Quick UNIX reference

UNIX is a common name for a family of operating systems, most popular of which in this epoch is Linux. This document is by no means a UNIX tutorial. Sophisticated UNIX users may find some of the material here grossly simplified. Only a small subset of the shell commands available is given and very few options are mentioned.

## A.1 Files

A file is a collection of data with a unique name. Names are case-sensitive (lowercase and uppercase letter are considered distinct). All permanent storage on UNIX consists of files. In this course you will encounter several different types of files that are recognized by their *extensions*. The extension is the ending of the file name following the (last) dot. Some extensions are expected or enforced by certain programs, while others are merely conventions.

Some file types, most relevant to this course, are:

- *Text files.* They are human-readable and can be edited by means of a text editor. There are many different kinds of text files:

  *Program source files*

  These files are source code written by programmers in a high-level language like FORTRAN or C. The compilers for these languages require the source file names to end in `.f` or `.c` respectively.

  *Data files*

  Input data files contain data that will be fed into a program. Output data files contain output produced by a program that you want to keep in a file.

  *LaTeX files*

  These files are "source" files for documents such as the one you are reading. LaTeX software requires that the file name end in `.tex`.

  *Webpages*

  are files with extensions **htm**, **html**. They stand for **H**ypertext **M**arkup **L**anguage.

- *Executable files*

  These files are programs that can be run directly; that is you can type their names into the shell like the **UNIX** commands. UNIX shell commands, compilers, text editors, Internet browsers belong to this category. Also this type of file is produced by a *compiler* from a *source* file written in FORTRAN, C, or another language. The FORTRAN and C compilers name this file `a.out` by default.

- **dvi** *files*

  These files are created by LaTeX to be printed on various devices, such as the screen or a laser printer. These files' names will end in `.dvi`. This stands for **de**vice **i**ndependent.

- **pdf** *files*

  The extension pdf stands for **P**ortable **D**ata **F**ormat, developed by Adobe, Inc. Files in this format are produced by many word and data processors, including LaTeX. *Acrobat Reader* is the most popular viewer for these files.

- **ps** *and* **eps** *files*

  The extension `ps` denotes **P**ost**s**cript, another page description format developed by Adobe, Inc. In fact, Postscript is pretty much a programming language. Postscript graphics files can even be created by hand, but usually their creation is helped by a software. Encapsulated postscript (eps) format is almost identical. The only difference is that an `eps` file must contain the Bounding Box information (see Sect. 4.3.1). This tiny difference is important when you have to incorporate a Postscript graphics into your LaTeX document.

- **mw** *and* **mws**

  These are the extensions assigned to Maple worksheets.

Depending on context, the term *file name* can refer either to the full name or to the part preceding the dot after which the extension follows. So, we would often say "the LaTeX file `project3`"; here the full file name `project3.tex` is implicit.

Each file must have a unique name which distinguishes it from all other files. If a file is named sensibly, the name will give a clue as to the contents. One important point is to distinguish between various input and output data files, as these can become quite numerous.

## A.2 Directories

A group of files is stored in a *directory*. A directory can in turn contain other directories. The UNIX *file system* contains all files on the computer, and as such is made up of a hierarchy of directories. It can be thought of as an upside-down "tree" of directories (Fig. A.1).

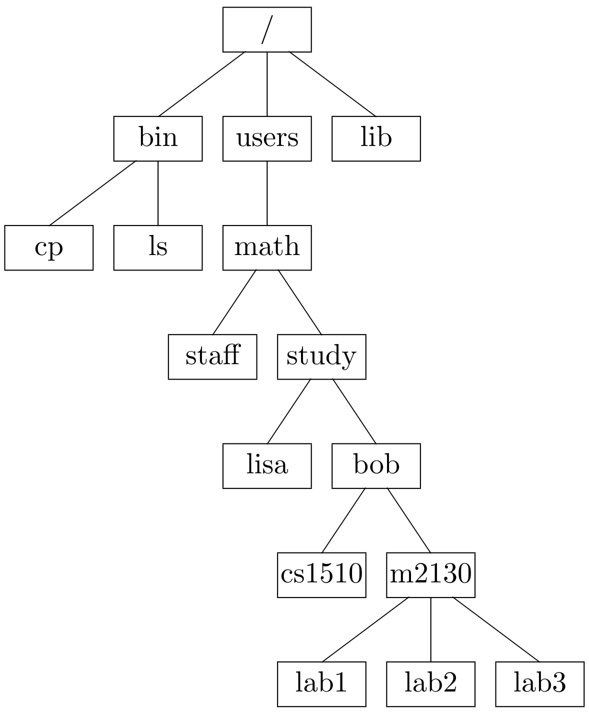


Figure A.1: Sample directory tree

The purpose of directories is to help users organize their files. Each user can create any number of subdirectories within their home directory, to any depth in the tree. If you look at the depicted directory structure, you can see that the directory `bob` contains two first-level subdirectories named `cs1510` and `m2130` and the latter contains three second-level subdirectories named `lab1`, `lab2` and `lab3`.

## A.3 Pathnames

There can be files with the same name in different directories. For example, there are likely dozens of files by the name `lab1.tex` on the system, created by different students. Yet there is a method to identify any file in the file system unambiguously. The way do this is to use a *pathname*. This name specifies the path you must travel through the tree to reach the file. The full name of a file, as understood by the system, is the pathname from the root directory. A full name begins with an initial slash (`/`), with further slashes separating each directory name. The actual name of the file, by which it is known to its owner, is the part of the pathname after the final slash. For example, the full pathname of the directory `bob` in the above example is `/users/math/study/bob`.

Note that UNIX is different from Microsoft Windows, where the backslash (\) is the separator for pathnames and the top-level directory is the drive name (for example, `C:\`).

Along with **absolute pathnames** just described, there are **relative pathnames**. They describe how you reach the required file or directory from the current directory. Sometimes you may have to go up the reversed tree (towards the root). The directory one level higher the current directory is denoted `../` (double dot). Thus, the relative pathname from Bob's directory `lab1` to his directory `cs1510` is `../../cs1510`. In the same situation, the pathname `../lab1` points at the directory we are in. The standard way to refer to a directory from itself is by the relative pathname `./` (the "dot directory", means — stay here). Obviously, in practice, if a file is in the current directory, you just call it by its name.

## A.4 Shell

While a graphical user interface is available for most modern programs, it doesn't hurt to be familiar with an old-fashioned but always reliable command-line mode.

To enter the command-line mode, you need to open a terminal shell window (click on the icon that looks like a display). Now, what you type is interpreted by a special program called the *shell*. It launches and runs other programs for you. The shell tells you it is waiting for input by displaying a prompt (like `lumsden $`) at the beginning of a line.

Two combinations of keys deserve a special mention:

- Almost any program started from the command line can be stopped by typing **Ctrl C**.

- The combination **Esc K** returns the most recent command entered in the shell. Pressing **Esc K** twice will quickly bring back the second most recent command, etc. Knowing this is especially convenient for those who run compilers in the command line. In another type of shell, the single key ↑ scrolls up the commands history.

A command followed by the ampersand character (&) launches a program in a mode detached from shell's interactive session, so that you can enter other commands in the same window while the program is running. For example, you can open a text editor and modify your program in it, while the command line will be available for compilation of the program:

`lumsden $` *kile myprog.f &*

## A.5 Basic UNIX commands

A UNIX command consists of one or more words separated by spaces. The first word is the name of the program you want to run, either a standard system program or one you created yourself. The rest of the words are *arguments* to the program, which usually are either the names of files, or *options*, which tell the program to modify its usual behaviour. Options usually begin with a dash (`-`), for instance `-l`.

**Notation used**. *This typeface* is anything you type in literally. Anything in *italics* is not typed in literally. For example, *file* means that you should not type the word "file", but that you should type a file name instead. Anything followed by the ellipses (. . . ) can be repeated. For example, *file* . . . means you can type in several file names. In the next section examples, `this typeface` will denote what the computer displays, as opposed to what the user types.

**Commands**

*ls*

list the names of all files in the current directory.

*cp original-name new-name*

create a copy of the original file with a different name.

*mv original-name new-name*

rename the file with a different name, or move the file to another place.

*rm file-name* . . .

remove the file(s).

*cat file-name* . . .

display the contents of the file(s).

*p file-name* . . . or *more file-name* . . .

display the contents of the file(s), one screen at a time.

*cd directory-name ...*

changes to the directory named.

*mkdir directory-name*

create a new directory.

*rmdir directory-name*

remove a directory.

*pwd*

display the current directory.

*man subject*

display the UNIX manual for the subject, can be used to find out details about a command's syntax.

*lpq* and *lprm request-ID*

*lpq* can be used to obtain a listing of the jobs in the printer queue waiting to be executed. If by misfortune you have asked to print a document and suddenly wish to cancel the print job, use *lpq* to obtain the request-ID number associated with your printing job and remove it from the printer queue with the command *lprm request-ID.*

## A.6 Working with directories and files

You change your current directory with the *cd* command. *cd* takes one argument, the pathname of the directory you want to change to. For example, the command

```
lumsden $ cd /users/math/study
```

makes `/users/math/study` the current directory. In this case, an absolute pathname was used, which isn't a common practice for ordinary users.

Much more practically useful are relative pathnames. You must, of course, know what your current directory is. If not sure, use the *pwd* command. For instance, if the current directory is `/users/math/study/bob/m2130/lab1`, then typing *pwd* you get system's response:

```
Current directory is /users/math/study/bob/m2130/lab1
```

Now, typing *cd ..* will make `/users/math/study/bob/m2130` the current directory. Note that if the user Bob types *cd /users/math/study/lisa*, the system will reject this command as Bob is not authorized to access Lisa's home directory.

The special variant

*cd ␣~*

makes the user's home directory the current directory. Thus, if Lisa types *cd ~*, the current directory becomes `/users/math/study/lisa`, no matter what it was before.

The command *ls* will list the names of all the files in the current directory. If you give it a directory name, it will list the names of all the files in that directory. If you give it a file name, it will list that name, which can be useful if you just want to check if such a file exists.

For our sample directory tree, if the current directory was `/users/math/study` and you ran *ls*, it would display

```
lumsden $ ls
    bob   lisa
```

The command *ls ␣ -l ␣ name* will give you a detailed information about a file or files.

The command *ls ␣ -a* displays, unlike the bare *ls*, those files and directories whose names begin with a dot, like the name `.www`, which often denotes the directory in user's account accessible from the Internet.

**Wildcards** can be used to specify a pattern that we want a file name or a directory name to match. The most useful wildcard character is the asterisk `*`, which matches any, even empty, combination of characters. The command

*ls ␣ *tex*

will display all files with names ending in `tex`. The command

*ls ␣ *tex**

will, in addition, display all files with names containing `tex` at the beginning or in the middle, like `text1.doc`, `latexnotes`, etc.

The command

*cd ␣ ../*2*

issued from Bob's directory `m2130/lab1` will change directory to Bob's `math2130/lab2`, because only one directory name matches the pattern `../`*2.

The *cp* command creates a new copy of a file. For example, if the current directory contains one file, `lab1.tex`, and you ran *cp lab1.tex newlab.tex*, then *ls* would display

```
lumsden $ cp ␣ lab1.tex ␣ newlab.tex
lumsden $ ls
    lab1.tex   newlab.tex
```

and the contents of `newlab.tex` would be identical to `lab1.tex`. If a file named `newlab.tex` had already existed, that file would have been overwritten.

The command *cp* with wildcards in its arguments is very convenient when you copy your files from the working directory to the submission directory (see Sect. 5.1). The command *cp ␣ * ␣ m2130-a1/* will copy all files from the current directory to its subdirectory named `m2130-a1`. This may not be exactly the desired outcome: for instance, `.log`, `.aux`, `.dvi`, and `a.out` files need not be included. So a better command is

```
ls ␣ *tex ␣ *eps ␣ *.f
```

which will copy all (possibly, single) LaTeX source file(s), any encapsulated Postscript figures, and any Fortran programs.

The *mv* command is similar to the copy command. However the original name of the file will be lost after the command is finished. You can also use *mv* to move a file from one directory to another. The *mv* command has similar behaviour to *cp* if its last argument is a directory. One difference between *mv* and *cp* is that *cp* will not copy a directory, whereas *mv* can rename it or move it into another directory.

The command to remove a file is *rm*. The arguments to *rm* are the files you want to remove. Be careful with this command. Once you remove a file, it is difficult to get it back. All the files on the system are saved every night, so if you do accidentally remove a very important file, the system administrator may be able to help you. Be especially careful with *rm*␣∗ command!

The command to create a directory is *mkdir*. It takes one argument, the pathname of the directory you wish to create. For instance, if Bob ran *mkdir lab4* in his `m2130` directory, he would see

```
lumsden $ mkdir lab4
lumsden $ ls
    lab1   lab2   lab3   lab4
```

The command *rmdir* takes one argument, the pathname of a directory you wish to remove. If the directory is not empty, *rmdir* will not function.

The command *p* is actually a shorter name for the command *less*. This is a program you can use to quickly look at text files. *p* will display a screenful of the file and then stop with a ":" prompt at the bottom of the screen. You can hit the space bar to see the next page, or the `u` key to go upwards in half-screenfuls. When you reach the end of the file, `p` will prompt with the name of the file followed by `(END)`. Type the `q` key to quit from `p`.

## A.7 Redirection of output

Most students have little trouble making their programs to print on the screen, but they experience more difficulties in creating programs that would flush their output to a file. Not that creating such a program in any language is very difficult, but it certainly requires more effort and time. Redirection is a trick that can help to work around this problem.

Every program running under UNIX has a "file" already opened: *standard output*. Normally, it is your shell terminal's window. You can *redirect* it to a text file in your directory. For instance, if you want your program `a.out` to print the output to a file `data-out` instead of the screen, you can run it like this:

```
lumsden $ a.out > data-out
```

Note that this could be a problem: if you have messages printed out prompting for certain kinds of input, those messages will also be redirected into the file instead of the screen. You should in this case write your programs so that they not be prompting for input. Or at least know how many numbers and in what sequence the program wants you to enter; you can then feed the data into your program blindfold.

Note also that if you run the same command again, the old contents of the file `data-out` will be lost.

## A.8 Access privileges

We mentioned previously that the system will deny Bob to access Lisa's directory and vise versa. Technically, the access is governed by certain attributes assigned to each directory and file on the system. Each file has its *owner*, who belongs to a *group*. The owner can open or close access to his/her files independently for *self*, *group*, *and others* (or for *all* at once), and for three purposes: *to read*, *to write*, and *to execute*. The command is *chmode*. Consider a few examples.

The following prohibits writing to file `project1.tex` to all, even to its owner. (A possible purpose is to protect the important work after it is finished from accidental deletion.)

```
lumsden $ chmode a -w project1.tex
```

To prohibit group members and others (i.e. everbody but the file owner) from any kind of access to the file `project1.tex`:

```
lumsden $ chmode go -wrx project1.tex
```

To grant everybody a permission to read all existing `.pdf` files in current directory:

```
lumsden $ chmode a +r *.pdf
```

Programs that you routinely work with (editors, compilers, etc.) properly set the access privileges to the files that they create or modify. If, by any chance, a program says "Access denied", you may need to explore the status of the file concerned. A possible reason is that the file is currently being used by another program, which temporarily closed an access to it.

# Appendix B: Two papers on mathematical writing

## B-1. Writing a Math Phase Two Paper

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*Word-smithing is a much greater percentage of what I am supposed to be doing in life than I would ever have thought.*

Donald Knuth [6], p. 54

**Abstract.** In this paper, we discuss the kind of writing that is appropriate in a paper submitted to the math department to complete Phase Two of MIT's writing requirement. We review the general purpose of the requirement and the specific way of completing it for the math department. Then we consider the writing itself: the organization into sections, the use of language, and the special problems of presenting mathematics. We conclude with a short example of mathematical writing.

### 1 Introduction.

MIT established the writing requirement to ensure that its graduates can write both a good general essay and a good technical report. Correspondingly, the requirement has two phases, which engage students at the beginning and toward the end of their careers. The requirement is governed by an institute committee, the Committee on the Writing Requirement (CWR). The requirement is administered by the Undergraduate Education Office, which works in cooperation with the individual departments on Phase Two. The general information given here about the requirement is taken from the MIT *Bulletin* and the CWR's brochure [3], which are the official sources.

To complete Phase One, students must achieve a suitable score on the College Board Achievement Test or Advanced Placement Examination, pass the Freshman Essay Evaluation, pass an appropriate writing subject in Course 21, or write a satisfactory five page paper for any MIT subject, Wellesley exchange subject, or UROP activity. In level, format, and style, a paper should be like a magazine article for an informed, but general, readership. Papers are judged on their logical structure, language and tone, technical accuracy, and mechanics (grammar, spelling, and punctuation) by the instructor of the subject and by evaluators for the Undergraduate Education Office. A paper judged not acceptable may be revised and resubmitted twice. Students must complete Phase One by the middle of their third semester at the Institute.

To complete Phase Two, students must receive a grade of B or better for the quality of writing in a cooperative subject approved by the student's major department, receive a grade of B or better in one of several advanced subjects in technical writing, or write a satisfactory ten page paper for any MIT subject or UROP activity approved by the major department. A student with two majors needs only to complete the requirement in one department. In level, format, and style, a Phase Two paper should be like a formal professional report. Thus a term paper or laboratory report may have to be reworked substantially before it is acceptable as a Phase Two paper. A paper is judged by its supervisor and by departmental evaluators. Students must complete Phase Two by the end of registration day of their last semester; otherwise, they cannot graduate unless there are exceptional circumstances and they successfully petition their departmental coordinators and the CWR.

In the Department of Mathematics, there is no cooperative subject, and nearly every student writes a paper to satisfy Phase Two. About three quarters of the papers begin as term papers for a single course, 18.310 *Principles of Applied Mathematics*. Every paper must have some technical mathematics in it. When the student and the supervisor feel the paper is ready, the student picks up a cover sheet in the Undergraduate Mathematics Office, Room 2–108. The student fills out the top, and gives it to the supervisor, who must vouch for the paper's technical accuracy, and may comment on the quality of the writing. The student then submits the paper to the undergraduate office. The paper must be submitted by the start of IAP if the student intends to graduate the following June. After a paper is submitted, it is read for organization and language by the departmental coordinator, who determines whether the paper is acceptable as is or needs to be improved. If the paper requires further work, the department's Writing TA contacts the student and sets up an appointment to discuss the areas requiring further work. The student submits further revisions of the paper to the TA, and when the revisions are perfected, the paper is resubmitted to the coordinator. On occasion, the coordinator works directly with the student. The goal is to help students both improve their papers and become better writers. The paper must be approved by registration day of the student's last semester.

The present paper is a primer on mathematical writing, especially the writing of short papers. Indeed, this paper itself is intended to be a model of format and style. Mathematical writing is primarily a craft, which anyone can learn. The aim is to inform efficiently. The basic principles are discussed and illustrated here. Some of these principles are simple matters of common sense; others are conventions that have evolved from experience. None need be followed slavishly, but none should be breached thoughtlessly. When they are breached, the breach may stand out like a sore thumb — just as unconventional spelling does. However, the writing itself should fade into the background, leaving the information to be conveyed out front. Following these principles will not cramp anyone's style; there is plenty of room for individual variation. The various principles are discussed more fully in a number of works, including the following works on which this primer is based: Alley's down-to-earth book [1], Flander's article [4]and Gillman's manual [5]for authors of articles for MAA[1] journals, the notes [6]to Knuth's Stanford course on mathematical writing, and Munkres' brief manual of style [7].

Section 2 of this paper discusses the normal way a short mathematical paper is broken into sections. We consider the purpose and content of the individual sections: the abstract, the introduction, the several sections of the main discussion, the conclusion (which is rare in a mathematical work), the appendix, and the list of references. Section 3 below deals with "language," that is, the choice of words and symbols, and the structuring of sentences and paragraphs. We consider seven goals of language: precision, clarity, familiarity, forthrightness, conciseness, fluidity, and imagery. We discuss the meaning of these goals and how best to meet them. Sections 2 and 3 are based mainly on Alley's book [1]. Section 4 deals with a number of special problems that arise in writing mathematics, such as the treatment of formulas, the presentation of theorems and proofs, and the use of symbols. The material is drawn from all five sources cited above. Section 5 gives an illustrative sample of mathematical writing. We treat the two fundamental theorems of calculus, for the most part paraphrasing the treatment in Apostol's book [2], pp. 202–4; we state and prove the theorems, and explain their significance. Finally, the appendix deals with the use of such terms as lemma, proposition, and definition, which are common in treatments of advanced mathematics and appear every year in a few Phase Two papers.

## 2 Organization.

Most short technical papers are divided up into sections, which are numbered and titled. (The pages too should be numbered for easy reference.) Most papers have an abstract, an introduction, a number of sections of discussion, and a list of references. On occasion, papers have appendices, which give special detailed information, or provide necessary general background to secondary audiences. In mathematics, few papers have a section of conclusions and recommendations. Such a section would discuss the results from an overall perspective, bring together the loose ends, and possibly make recommendations for future research. In mathematical papers, these issues are almost always incorporated into the introduction. Normally, short papers have no formal table of contents.

Sectioning involves more than merely dividing up the material; you have to decide about what to put where, about what to leave out, and about what to emphasize. If you make the wrong decisions, you will lose your readers. There is no simple formula for deciding, because the decisions depend heavily on the subject and the audience. However, you must structure

[1]the Mathematical Association of America

your paper in a way that is easy for your readers to follow, and you must emphasize the key results.

The title is very important. If it is inexact or unclear, it will not attract all the intended readers. A strong title identifies the general area of the subject and its most distinctive features. A strong title contains no distracting secondary details and no formulas. A strong title is concise.

The abstract is the most important section. First it identifies the subject; it repeats words and phrases from the title to corroborate a reader's first impression, and it gives details that did not fit into the title. Then it lays out the central issues, and summarizes the discussion to come. It is drawn completely from the paper. However, it includes no general background material. The abstract is a table of contents in a paragraph of prose. It allows readers to decide quickly about reading on. While many will decide to stop there, the potentially interested will continue. The goal is not to entice all, but to inform the interested efficiently. Remember, readers are busy. They have to decide quickly whether your paper is worth their time. They have to decide whether the subject matter is of interest to them, and whether the presentation will bog them down. A well-written abstract will increase the readership.

The introduction is the place where readers settle into the "story," and often make the final decision about reading the whole paper. Start strong; do not waste words or time. Your readers have just read your title and abstract, and they have gained a general idea of your subject and treatment. However, they are probably still wondering what exactly your subject is and how you will present it. A strong introduction answers these questions with clarity and precision. It identifies the subject precisely and instills interest in it by giving details that did not fit into the title or abstract, such as how the subject arose and where it is headed, how it relates to other subjects and why it is important. A strong introduction touches on all the significant points, and no more. A strong introduction gives enough background material for understanding the paper as a whole, and no more. Put background material pertinent to a particular section in that section, weaving it unobtrusively into the text. A strong introduction discusses the relevant literature, citing a good survey or two. Finally, a strong introduction discusses the organization of the paper; it summarizes the contents again, but in more detail than in the abstract, and it says what can be found in each section. It gives a "road map," which indicates the route to be followed and the prominent features along the way. This road map is placed at the end of the introduction to ease the transition into the next section.

The body discusses the various aspects of the subject individually. In writing the body, your hardest job is developing a strategy for parcelling out the information. Every paper requires its own strategy, which must be worked out by trial and error. There are, however, a few guidelines. First, present the material in small digestible portions. Second, beware of jumping haphazardly from one detail to another, and of illogically making some details specific and others generic. Third, if possible, follow a sequential path through the subject. If such a path simply does not exist, then break the subject down into logical units, and present them in the order most conducive to understanding. If the units are independent, then order them according to their importance to the primary audience.

There are three main reasons for dividing the body into sections: (1) the division indicates the strategy of your presentation; (2) it allows readers to quickly and easily find the information that interests them; and (3) it gives readers restful white space, allowing them to stop and reflect on what was said. Make the introduction and the several sections of the body roughly equal in length. When you title a section, strive for precision and clarity; then readers will have an easier time finding particular information. In a short paper, do not use subsections; they make the flow too choppy.

Each main point should be accented via stylistic repetition, illustration, or language. Stylistic repetition is the selective repetition of something important; for example, you should talk about the important points once in the abstract, a second time in the introduction, and a third time in the body. When appropriate, repeat an important point in a figure or diagram. Finally, accent an important point with a linguistic device: italics, boldface, or quotations marks; a one sentence paragraph, a short sentence at the end of a long paragraph, or a repetition of parallel phrases or sentence structures. In particular, set a technical term in italics or boldface — or enclose it in quotation marks if it is only moderately technical — once, at the time it is being defined. Do not use underlining when italics or boldface is available. Use headings such as **Table 1-1**, **Figure 1-2**, and **Theorem 5-2**, and refer to them as Table 1-1, Figure 1-2, and Theorem 5-2; note that the references are capitalized and set in roman.

The list of references contains bibliographical information about each source cited. The style of the list is different in technical and nontechnical writing; so is the style of citation. In fact, there are several different styles used in technical writing, but they are relatively minor variations of each other. The style used in this paper is commonly used in mathematics. The citation is treated somewhat like a parenthetical remark within a sentence. Footnotes are not used; neither are the abbreviations "loc. cit.," "op. cit.," and "ibid." The reference key, usually a numeral, is enclosed in square brackets. When citing particular material such as pages, sections, or equations, do so at the point of citation; within the brackets, place the page numbers, section numbers, or equation numbers preceded by a comma after the reference key. If the citation comes at the end of a sentence, put the period after the citation, not before the brackets or inside them.

## 3 Language.

In the subject of writing, the word "language" means the choice of words and symbols, and their arrangement in phrases. It means the structuring of sentences and paragraphs, and the use of examples and analogies. When you write, watch your language. When it falters, your readers stumble; if they stumble too often, they will lose their patience and stop reading. Write, rewrite, then rewrite again, improving your language as you go; there is no short cut!

Alley [1], pp. 25–130 identifies seven goals of language: two primary goals — precision and clarity — and five secondary goals — familiarity, forthrightness, conciseness, fluidity, and imagery. These goals often reinforce one another. For example, clarity and forthrightness promote conciseness; precision and familiarity promote clarity. We will now consider these goals individually.

Being precise means using the right word. However, finding the right word can be difficult. Consult a dictionary, not a thesaurus, because the dictionary explains the differences among words. For example, the *American Heritage Dictionary* is a good choice, because it has many notes on usage. Consult a book on usage, such as *Webster's Dictionary of English Usage*. Always consider a word's connotations (associated meanings) along with its denotations (explicit meanings); the wrong connotations can trip up your readers by suggesting unintended ideas. For example, the word "adequate" means enough for what is required, but it gives you the feeling that there is not quite enough; its connotation is the exact opposite of its denotation. Strong writing does not require using synonyms, contrary to popular belief. Indeed, by repeating a word, you often strengthen the bond between two thoughts. Moreover, few words are exact synonyms, and often, using an exact synonym adds nothing to the discussion.

Being precise means giving specific and concrete details. Without the details, readers stop and wonder needlessly. On the other hand, readers remember by means of the details. Being precise does not mean giving all the details, but giving the informative details. Giving the wrong details or giving the right ones at the wrong time makes the writing boring and hard to follow. Being specific does not mean eradicating general statements. General statements are important, particularly in summaries. However, specific examples, illustrations, and analogies add meaning to the general statements.

Being clear means using no wrong words. An ambiguous phrase or sentence will disrupt the continuity and diminish the authority of an entire section. A common mistake is to use overly complex prose. Do not string adjectives before nouns, lest they lose their strength and precision; instead, use prepositional phrases and dependent clauses, or use two sentences. Keep your sentences simple and to the point. It may help to keep most of them short, but you need some longer ones to keep your writing from sounding choppy and to provide variety and emphasis.

A pronoun normally refers to the previous noun. Unfortunately, it is common to abuse pronouns, particularly "it," "this," and "which." Make sure the reference is immediately clear, especially when you refer broadly to a preceding phrase, topic, or idea. It is also common to use a plural pronoun such as "their" to refer back to a singular, but indefinite, antecedent such as a "reader." This usage is still considered unacceptable in formal writing; reformulate your sentence if necessary. The pronouns "that" and "which" are not always interchangeable. Either may be used to introduce a restrictive clause, but use "that" ordinarily. Only "which" may be used to introduce a descriptive clause, and the clause must be set off with commas. Strunk and White in their classic guide to style [8], p. 47 recommend "which-hunting." ,

Punctuation is used to eliminate ambiguities in language, and to smooth the flow of the text. A simple misuse of punctuation can weaken your authority. Learn how to punctuate properly, and use a handbook like *The Chicago Manual of Style.* In optional cases, use the punctuation if it promotes clarity at all, but strive for consistency through out the paper. Here are a few rules.

Use periods only to end sentences. (A complete sentence within parentheses should begin with a capital letter and end with a punctuation mark, unless the sentence is part of another and would end with a period.) Avoid abbreviations that require periods; for example, use "that is" instead of "i.e." Always use commas to separate three or more items in a list and to set off contrasted elements (they often begin with "but" or "not"). Most of the time, use a comma after an introductory phrase. Use colons to introduce lists, definitions, and explanations, but not in continuing statements: if a statement is stopped at the colon, then the words should form a complete sentence. Use a semicolon to join two sentences to indicate that they are closely linked in content; however, if you insert a conjunction (not an adverb), then use a comma. Use a dash as a comma of extra strength — but use it sparingly. Place closing quotation marks (") after commas and periods; it is a matter of appearance, not logic.

To inform, you must use language familiar to your readers. Define unfamiliar words, and familiar words used in unfamiliar ways. If the definition is short, then include it in the same sentence, preceding it by "or" or setting it off by commas or parentheses. If the definition is complex or technical, then expand it in a sentence or two. Do not use words like "capability," "utilize," and "implement"; they offer no precision, clarity, or continuity and smack of pseudo-intellectualism. Beware of words like "interface"; they are precise in some contexts, yet imprecise and pretentious in others.

Jargon is vocabulary particular to a certain group, and it consists of abbreviations and slang terms. Jargon is not inherently bad. Indeed, it is useful in internal memos and reports. However, jargon alienates external readers and may even mislead them. So beware. Cliches are figurative expressions that have been overused and have taken on undesirable connotations. Most are imprecise and unclear. Avoid them, or be laughed at. In addition, avoid numerals because they slow down the reading. Write numbers out if they can be expressed in one or two words and are used as adjectives, unless they are accompanied by units, a percentage sign, or a monetary sign. For instance, write, "The equation has two roots," and "One root is 2." Do not begin a sentence with a numeral or a symbol; reformulate the sentence if necessary.

Be forthright: write in an unhesitating, straightforward, and friendly style, ridding your language of needless and bewildering formality. Beware of awkward and inefficient passive constructions. Often the passive voice is used simply to avoid the first person. However, the pronoun "we" is now generally considered acceptable in contexts where it means the author and reader together, or the author with the reader looking on. (Still, "we" should not be used as a formal equivalent of "I," and "I" should be used rarely, if at all.) For instance, do not write, "By solving the equation, it is found that the roots are real." Instead write, "Solving the equation, we find the roots are real," or "Solving the equation yields *real* roots." Beware of dangling participles. It is wrong to write, "Solving the equation, the roots are real," because "the roots" cannot solve the equation.

Concise writing is vigorous. Conciseness comes from eliminating needless repetition, fat phrases, and empty words, thus reducing sentences to their simplest forms. Conciseness comes from eliminating pretentious diction, thus being clear and forthright. Concise writing is simple and efficient, thus beautiful.

The flow of a paper is disturbed by weak transitions between sentences and paragraphs. To smooth the flow, start a sentence where the preceding one left off. Use connective words and phrases. Avoid gaps in the logic, and give ample details. Do not take needless jumps when deriving equations. Use parallel wording when discussing parallel concepts. Do not raise questions implicitly, and leave them unanswered.

Many papers stagnate because they lack variety. The sentences begin the same way, run the same length, and are of the same type. The paragraphs have the same length and structure. Do not worry about varying your sentences and paragraphs at first; wait until you polish your writing. Remember though, if you have to choose between fluidity and clarity, then you must choose clarity.

The very structure of a sentence conveys meaning. Readers expect the stress to lie at the beginning and end. They take a breath at the beginning, but will run out of breath before the end if the structure is too complex, for instance, if the subject is too far from the verb.

Most people think and remember images, not abstractions, and images are clarified by illustrations. Illustrations also give readers rest stops, so complex ideas can soak in. Moreover, illustrations can make a paper more palatable and less intimidating. However, illustration can be overdone; it must fit the audience and the subject.

Illustrations cannot stand alone; they must be introduced in the text. Assign them titles, like Figure 5-1 or Table 5-1, for reference. Assign them captions that tell, independently of the text, what they are and how they differ from one another, without being overly specific. In addition, clearly label the parts of your illustrations: label the axes of graphs with words, not symbols; identify any unusual symbols of your diagrams in the text. Do not put too much information into one illustration, because papers without white space tire readers. For the same reason, use adequate borders. Smooth the transitions between your words and pictures. First of all, match the information in your text and illustrations. Secondly, place the illustrations closely after — never before — their references in the text.

## 4 Mathematics.

Mathematical writing has some special problems because it tends to involve many abstract symbols and formal arguments. Here are some principles to keep in mind.

Formulas are difficult to read because readers have to stop and work through the meaning of each term. Do not merely list a sequence of formulas with no discernible goal, but give a running commentary. Define all the terms as they are introduced, state any assumptions about their validity, and give examples to provide a feeling for them. Similarly, motivate and explain formal statements. Do not simply call a statement “important,” “interesting,” or “remarkable,” but show why it is so.

Display an important formula by centering it on a line by itself, and give a reference number in the margin if it is especially important or if you need to refer to it. Also display any formula that is more than a quarter of a line long or that would be broken badly between lines. Punctuate the display with commas, a period, or whatever as you would if you had not displayed it.

Be clear about the status of every assertion; indicate whether it is a conjecture, the previous theorem, or the next theorem. If it is not a standard result and you omit its proof, then give a reference, preferably in the text just before the statement (If you give the reference in the statement, then do so after the heading like this: **Theorem 5-1 [2], p. 202**.) Tell whether the omitted proof is hard or easy to help readers decide whether to try to work it out for themselves. If the theorem has a name, use it: say "by the First Fundamental Theorem," not "by Theorem 5-1." State a theorem before proving it. Keep the statement concise; put definitions and discussion elsewhere.

Prefer a conceptual proof to a computational one; ideas are easier to communicate, understand, and remember. Omit the details of purely routine computations and arguments — ones with no unexpected tricks and no new ideas. Beware of any proof by contradiction; often there is a simpler direct argument. Finally, when the proof has ended, say so outright; for instance, say, "The proof is now complete." In addition, surround the proof — and the statement as well — with some extra white space.

Here are some more principles:

1. SEPARATE SYMBOLS IN DIFFERENT FORMULAS WITH WORDS.

   **BAD:** Consider $S_q$, $q = 1, \ldots, n$.

   **GOOD:** Consider $S_q$, for $q = 1, \ldots, n$.

2. DO NOT USE SUCH SYMBOLS AS $\exists$, $\forall$, $\wedge$, $\Rightarrow$, $\approx$, $=$, $>$ IN TEXT; REPLACE THEM BY WORDS. THEY MAY, OF COURSE, BE USED IN FORMULAS PLACED IN TEXT.

   **BAD:** Let $S$ be the set of all numbers of absolute value $< 1$.

   **GOOD:** Let $S$ be the set of all numbers of absolute value less than 1.

   **GOOD:** Let $S$ be the set of all numbers $x$ such that $|x| < 1$.

3. DO NOT START A SENTENCE WITH A SYMBOL.

   **BAD:** $ax^2 + bx + c = 0$ has real roots if $b^2 - 4ac \geq 0$.

   **GOOD:** The quadratic equation $ax^2 + bx + c = 0$ has real roots if $b^2 - 4ac \geq 0$.

4. BEWARE OF USING SYMBOLS TO CONVEY TOO MUCH INFORMATION ALL AT ONCE.

   **VERY BAD:** If $\Delta = b^2 - 4ac \geq 0$, then the roots are real.

   **BAD:** If $\Delta = b^2 - 4ac$ is nonnegative, then the roots are real.

   **GOOD:** Set $\Delta = b^2 - 4ac$. If $\Delta \geq 0$, then the roots are real.

5. IF YOU INTRODUCE A CONDITION WITH "IF," THEN INTRODUCE THE CONCLUSION WITH "THEN."

   **BAD:** If $\Delta \geq 0$, the roots are real.

6. USE CONSISTENT NOTATION. DO NOT SAY "$A_j$ WHERE $1 \leq j \leq n$" ONE PLACE AND "$A_k$ WHERE $1 \leq k \leq n$" ANOTHER.

7. Keep the notation simple. For example, do not write "$x_i$ is an element of $X$" if "$x$ is an element of $X$" will do.

8. Precede a theorem, algorithm, and the like with a complete sentence.

| | |
|---|---|
| **BAD:** | We now have the following<br>**Theorem 4-1.** $H(x)$ is continuous. |
| **GOOD:** | We can now prove the following result.<br>**Theorem 4-1.** The function $H(x)$ defined by Formula (4-1) is continuous. |

## 5 Example.

As an example of mathematical writing, we discuss the two fundamental theorems of calculus. Our discussion is based on that in Apostol's book [2], pp. 202–7. The First Fundamental Theorem says that the process of differentiation reverses that of integration. This statement is remarkable because the two processes appear to be so different: differentiation gives us the slope of a curve; integration, the area under the curve. Here is a precise statement of the theorem.

**Theorem 5-1** (First Fundamental Theorem of Calculus). *Let $f$ be a function defined and continuous on the closed interval $[a, b]$ and let $c$ be in $[a, b]$. Then for each $x$ in the open interval $(a, b)$, we have*

$$\frac{d}{dx}\int_c^x f(t)\,dt = f(x).$$

*Proof.* Take a positive number $h$ such that $x + h \leq b$. Then

$$\int_c^{x+h} f(t)\,dt - \int_c^x f(t)\,dt = \int_x^{x+h} f(t)\,dt.$$

By hypothesis, $f$ is continuous. Hence there is some $z$ in $[x, x + h]$ for which this last integral is equal to $h\,f(z)$ by the Mean Value Theorem for integrals [2], p. 154 (which is not hard to derive from the Intermediate Value Theorem). (The setup is shown in Figure 5-1; the Mean Value Theorem says that the area under the graph of $f$ is equal to the area of the rectangle.) Therefore,

$$\frac{1}{h}\left(\int_c^{x+h} f(t)\,dt - \int_c^x f(t)\,dt\right) = f(z).$$

Now, $x \leq z \leq x + h$. Hence, as $h$ approaches 0, the difference quotient on the left approaches $f(x)$. A similar argument holds for negative $h$. Thus the derivative of the integral exists and is equal to $f(x)$. The proof is now complete.

The First Fundamental Theorem says that, given a continuous function $f$, there exists a function $F$ (namely, $F(x) = \int_c^x f(t)\,dt$) whose derivative is equal to $f$:

$$F'(x) = f(x).$$

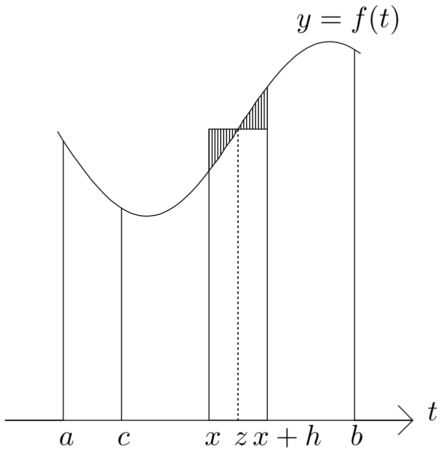

Figure B.1: Geometric setup of the proof of the First Fundamental Theorem

Such a function $F$ is called an *integral* (or a *primitive* or an *antiderivative*) of $f$. Integrals are not unique: if $F$ is an integral of $f$, then obviously so is $F + C$ for any constant $C$. On the other hand, there is no further ambiguity: any two integrals $F$ and $G$ of $f$ differ by a constant, because their difference $F - G$ has vanishing derivative,

$$(F - G)'(x) = F'(x) - G'(x) = f(x) - f(x) = 0 \text{ for every } x,$$

and hence $F - G$ is constant by a simple consequence of the Mean Value Theorem for derivatives (see [2], Theorem 4.7(c), p. 187).

When we combine the First Fundamental Theorem with the fact that an integral is unique up to an additive constant, we obtain the following theorem.

**Theorem 5-2** (Second Fundamental Theorem of Calculus). *Let $f$ be a function defined and continuous on the open interval $I$, and let $F$ be an integral of $f$ on $I$. Then for each $c$ and $x$ in $I$,*

$$\int_c^x f(t)\,dt = F(x) - F(c). \tag{5-1}$$

*Proof.* Set $G(x) = \int_c^x f(t)\,dt$. By the First Fundamental Theorem, $G$ is an integral of $f$. Now, any two integrals differ by a constant. Hence $G(x) - F(x) = C$ for some constant $C$. Taking $x = c$ yields $-F(c) = C$ because $G(c) = 0$. Thus $G(x) - F(x) = -F(c)$, and Equation (5-1) follows. The proof is now complete.

The Second Fundamental Theorem is a powerful statement. It says that we can compute the value of a definite integral merely by subtracting two values of any integral of the integrand. In practice, integrals are often found by reading a differentiation formula in reverse. For example, the integrals in Table 5-1 were found in this way. The notation in the table is standard [9],

**Table 5-1**
**A brief table of integrals**

| | |
|---|---|
| 1. | $\int x^a\,dx = \frac{x^{a+1}}{a+1} + C,\ a \neq -1$ |
| 2. | $\int x^{-1}\,dx = \ln x + C$ |
| 3. | $\int \sin x\,dx = -\cos x + C$ |
| 4. | $\int \cos x\,dx = \sin x + C$ |
| 5. | $\int e^x\,dx = e^x + C$ |

p. 178: the equation

$$\int f(x)\,dx = F(x) + C$$

is read, "The integral of $f(x)\,dx$ is equal to $F(x)$ plus $C$."

A longer table of integrals is found on the endpapers of the calculus textbook [9].

## Appendix. Advanced mathematics

In many treatments of advanced mathematics, the key results are stated formally as theorems, propositions, corollaries, and lemmas. However, these four terms are often used carelessly, robbing them of some useful information they have to convey: the nature of the result.

A theorem is a major result, one of the main goals of the work. Use the term "theorem" sparingly. Call a minor result a "proposition" if it is of independent interest. Call a minor result a "corollary" if it follows with relatively little proof from a theorem, a proposition, or another corollary. Sometimes a result could properly be called either a proposition or a corollary. If so, then call it a proposition if it relatively more important, and call it a corollary if it is relatively less important. Call a subsidiary statement a "lemma" if it is used in the proof of a theorem, a proposition, or another lemma. Thus a lemma never has a corollary, although a lemma may be used, on occasion, in deriving a corollary. Normally, a lemma is stated and proved before it is used.

The terms "definition" and "remark" are also often abused. A formal definition should simply introduce some terminology or notation; there should be no accompanying discussion of the new terms or symbols. A formal remark should be a brief comment made in passing; the main discussion should be logically independent of the content of the remark. Often it is better to weave definitions and remarks into the general discussion rather than setting them apart formally.

Typographically, the statements of theorems, propositions, corollaries, and lemmas are traditionally set in italics, and the headings themselves are set in boldface or in caps and small caps (**Theorem** or THEOREM and so forth). The texts of definitions and remarks are set as ordinary text; so are the texts of proofs, examples, and the like. These headings are traditionally set in italics, boldface, or small caps. (There is also a tradition of treating definitions typographically like theorems, but this tradition is less common today and less desirable.) All these formal statements and texts are usually set off from the rest of the discussion by putting some extra white space before and after them.

Assign sequential reference numbers to these headings, irrespective of their different natures, and use a hierarchical scheme whose first component is the section number. Thus "Corollary 3-6" refers to the sixth prominent statement in Section 3, and indicates that the statement is a corollary. If the statement is the second corollary of the third proposition in the paper, then it may be more logical to name the statement "Corollary 2," but doing so may make the statement considerably more difficult to locate.

## References

[1] M. Alley, *The Craft of Scientific Writing,* Prentice–Hall, 1987.

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[8] W. Strunk, Jr., and E. B. White, *The Elements of Style,* Macmillan Paperbacks Edition, 1962.

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# B-2. Some Hints on Mathematical Style

David Goss

Many years ago, just after my degree, I had the good fortune to be given some hints on mathematical writing by J.-P. Serre. Through the years I have found myself trying to repeat this very sound advice to other mathematicians who are also starting out. Recently, I have been involved in the publishing of a proceedings volume, as well as being an editor of the Journal of Number Theory. Many of the papers coming my way are from young authors; so I have written down these hints in order to speed the process along.

This is a second (and, most probably, final) version of these "hints". I have added comments from a number of mathematicians who read a first version. These hints are presented as a source of ideas on mathematical style. Feel free to use them in any way that you deem useful.

- Two basic rules are: (1). *Have mercy on the reader*,

  and,

  (2). *Have mercy on the editor/publisher.* We will illustrate these as we move along.

- General Flow of the Paper.
  - **Definition**: *All* basic definitions should be given if they are not a standard part of the literature. It is perhaps best to err on the side of making life easier on the reader by including a bit too much as opposed to too little (Rule 1).
    - Some redundancy should be built into the paper so that one or two misprints cannot destroy the understandability. The analogy is with "error-correcting codes" which allow transmission of information through noisy and defective channels.
  - As a very general rule, the definitions should go *before* the results that they are used in (Rule 1).
  - The "quantifiers" should always be clear (Rule 1). Some examples to avoid:
    - "We have $f(x) = g(x)$ $(x \in X)$." What does the parenthesis mean? That $f(x) = g(x)$ for *all* $x \in X$, or, for *some* $x \in X$?
    - What does "$f_{t,u}(x, y) = O\left(g_{t,u}(x, y)\right)$" mean? Very often it means that $t, u, y$ are fixed and $x$ is allowed to vary. Quantifier problems are especially troublesome with "big O" notation.
    - The word "constant" is terribly ambiguous. It is important to make explicit *exactly* which variables the constant depends on.
  - **Theorem/Proposition/Lemma/Corollary**: Give clear and unambiguous statements of results. These are what other people are reading your paper for; so you should ensure that these, at least, can be understood by the reader (Rule 1).

    - The statement of the Theorem/Proposition/Lemma/Corollary should **not** include comments (except for an occasional brief remark in parenthesis) or examples.

  - If you use or quote an important result of another person, you should give a reference. You should avoid giving the impression that such a result is obvious, a generally accepted fact, due to you, and so on.
    - A reference to a book should always give the page!
    - Try to avoid using "by the proof of" when the proof is in the paper and the statements can be rewritten to be *directly* quoted.
    - A "well-known" result that is *not* in the literature should be proved if needed (Rule 1).
  - **Proof**: A proof should give enough information to make the theorem believable **and** leave the reader with the confidence (as well as the ability) to fill in details should it be necessary (Rule 1).

- Other comments:
  - One should, of course, observe the usual conventions in terms of spelling, punctuation, and the other basic elements of style. Use complete sentences, with subject, *verb*, and complement (Rule 1).
    - A verb should **not** be replaced by a symbol. It is bad to write: "... if $x = 2$, $y = 3$, $z = 4$" meaning "... if $x = 2$ and $y = 3$, then $z$ is equal to 4".
    - It is also bad to write: "... we prove $\zeta_{\mathbf{Q}}(2n) \in \pi^{2n}\mathbf{Q}$" instead of: "... we prove *that* $\dfrac{\zeta_{\mathbf{Q}}(2n)}{\pi^{2n}}$ belongs to $\mathbf{Q}$" (or "is rational").
  - Use the present – not the past – form.
    - As an example of bad writing, we have: "We have proved that $f(x)$ was equal to $g(x)$...". This is corrected to: "We have proved that $f(x)$ is equal to $g(x)$...".
  - Long computations that can easily be carried out by the reader should be avoided. The ideas and results should be given with an invitation to the reader to do the calculation should it be desired (Rule 1).
    - The *exception* to this rule is when the long computation is an *essential* part of the argument. In this case, it should be given in full (Rule 1).
  - One should avoid giving the reader the impression that the subject matter can be mastered only with great pain. In fact, this is an *ideal* way to lose readers (or audiences!).
  - One should avoid using abbreviations like "w.r.t." (with respect to), "iff" (if and only if), and "w.l.o.g." (without loss of generality). They simply do not look very nice (and "iff" is offensive! – Rules 1 and 2).
  - You should **not** begin a sentence with a math symbol. This can confuse the printer as well as the reader (Rules 1 and 2).

    * As a example of such bad writing, we have: "... we want to prove the continuity of $f(x) = 2\cos^2(x) \cdot \sin(x)$. $\cos(x)$ being continuous....". This is corrected to: "...$f(x) = 2\cos^2(x) \cdot \sin(x)$. Since $\cos(x)$ is continuous...".
  - If your paper raises a natural question, and you don't know the answer, by all means *say so*! This may turn out to be more interesting than the theorems that you prove.
    * Conversely, refrain from making "conjectures" too hastily. Use instead the words "question" or "problem". Remember that a good "question" should be answerable by "yes" or "no". To ask "under what conditions does A hold" is not a question worth printing.
  - It is often helpful to begin a new section of the paper with a summary of the general setting.
  - After the paper is finished, it should be reread (and, perhaps, rewritten) from the reader's point of view (Rule 1).
  - A good way to begin is to use a standard classic of mathematical exposition (e.g., Bourbaki-Algebra, works by Serre or Milnor) as a basic model.

- Some further sources to look at:
  - P. Halmos: *How to write mathematics*, Enseign. Math., **16**, (1970), 123–152.
  - W. Strunk, Jr., & E. B. White: *The Elements of Style*, Macmillan Paperbacks Edition, (1962).
  - D. Knuth et al.: *Mathematical Writing*, MAA Notes #14, (1989).
  - Some conventions on citations and pronouns may be found in: S. Zucker: *Variation of a mixed Hodge structure II*, Inventiones Math. 80, (1985), p. 545.

- Finally, I quote from a letter Serre wrote commenting on my original version: "It strikes me that mathematical writing is similar to using a language. To be understood you have to follow some grammatical rules. However, in our case, nobody has taken the trouble of writing down the grammar; we get it as a baby does from parents, by imitation of others. Some mathematicians have a good ear; some not (and some prefer the slangy expressions such as "iff"). That's life."

# Index

## LaTeX commands, keywords, packages, and environments

## Personal notes