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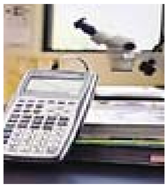


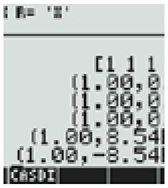
» **HP Mobile Calculating Lab Kits**

Learn how the HP Mobile Calculating Lab (MCL) helps educators teach math and science in a dynamic way.

» **Customization**

Richard J. Nelson

One of HP's greatest contributions to calculators is the feature of customization. Read through a list of features found on HP calculators past and present.



» **Using PROOT with Duplicate Roots**

Namir Shammas

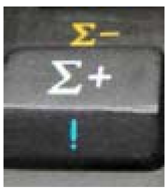
In this article, Namir focuses on how the function PROOT handles polynomials that have duplicate roots in addition to studying and comparing errors within the function PROOT.

» **Repurposing the HP 20b/30b Calculator Platform**

Jake Schwartz

As mentioned in the Customization article, repurposing is a form of customizing your calculator. Jake provides an updated overview of an interesting user community repurposing project.

» **HP's Calculator Manuals**



» **Fundamentals of Applied**

Issue 24
July 2011

Welcome to the twenty-forth edition of the HP Solve newsletter. Learn calculation concepts, get advice to help you succeed in the office or the classroom, and be the first to find out about new HP calculating solutions and special offers.

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From the Editor

Learn more about current articles and feedback from the latest Solve newsletter including One Minute Marvels and other community news.

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» **How Fast is Fast?**

Richard J. Nelson

Calculator speed is a calculator technical

Richard J. Nelson

This overview of HP's calculator manuals covers the entire history of what HP has done to provide the user with what he or she needs to use an HP calculator.

Math Series #7

Richard J. Nelson

What is the difference between average, mean, median and mode? Learn the basics behind these mathematical concepts through data sets and statistics.

attribute that is often discussed by users. This article discusses calculator speed examining “how fast is fast” and “how fast is fast enough.”

Back-To-School Summer Savings

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This school year, HP is offering something for everybody!
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How does the calculator you are now using stack up? To find out use it to calculate the number of cubic inches in a "Cosmic Cube" a million light years on each edge. You should be able to easily get the answer to at least three digits of precision. For the 12 digit answer see below.

ANSWER: Cosmic cube volume: $5.16707817405 \times 10^{70}$ cubic inches.

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HP Mobile Calculating Lab Kits

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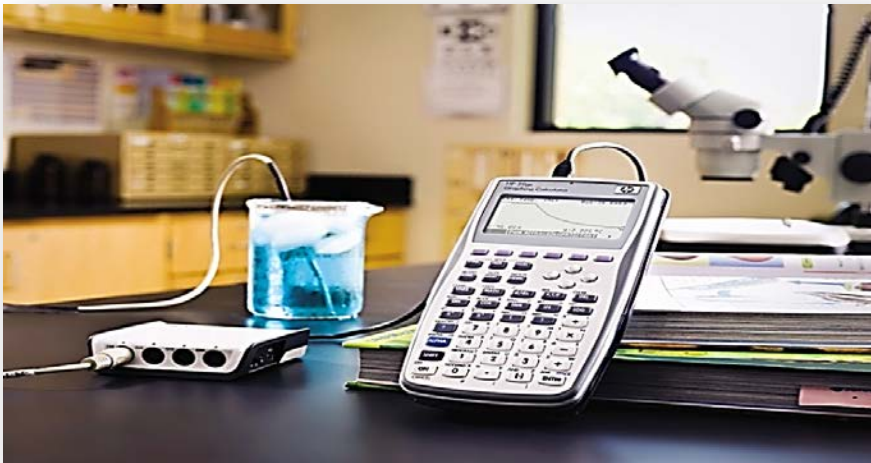
HP Mobile Calculating Lab: A Dynamic way to Teach Math and Science



As technology advances, the ability to challenge today's tech-savvy students in the fields of math and science has become increasingly difficult. Educators have the need for a more challenging curriculum that offers students real-life hands-on experiments to keep them fully engaged. HP provides Educators the solution with the HP Mobile Calculating Lab (MCL).

The HP MCL consists of the HP 39gs graphing calculator, the HP StreamSmart 410 data streamer, and a set of Fourier™ sensors. For science teachers, the HP MCL gives students direct experiences with the laws of nature. For math teachers, these same laws of nature can be explored to develop the concepts of variable and function, and to connect mathematics to the students' everyday experiences through mathematical modeling.

The HP MCL kit is unlike any other lab kit as it allows students to stream data continuously instead of logging data at certain time intervals. The benefit is that there is very little students have to set up. They just start monitoring the experimental setup. When they see something of interest, they can stop and zoom in on the data they want and make conjectures or start analyzing the data. The following experiment illustrates the benefits of data streaming.

An accelerometer is connected to the HP MCL and the StreamSmart application is launched. An accelerometer is strapped to a ruler with a rubber band. One end of the ruler is clamped to a table top and the other end is flexed and released to set the ruler in motion. This is done several times over a 5-second period. The result is shown in Figure 1, where 2 or 3 of the events can be seen as small peaks.

Figure 1

Zooming in (horizontally in time and vertically) on one of the peaks reveals the detail shown in Figure 2. What looked like just a bump or two is actually a clearly-defined set of dampened oscillations. The simple ruler can behave in a way that is both complex and beautiful!

Figure 2

Using the StreamSmart Application, the student can trace to each peak and add the point to a data set manually. Figure 3 shows the final data set graphically. Students can see that the oscillations are fairly regular; that is, the "peaks" are evenly spaced even though the amplitude decreases over time. Connecting phenomena of growth and decay to exponential models is natural here. Figure 4 shows an exponential fit for the data set.

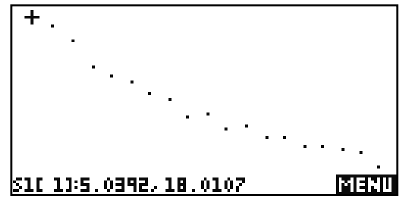


Figure 3

In this experiment, there was very little for students to set up, simply the ruler and accelerometer. The HP MCL automatically identified the sensor, selected a graphing window for the data stream, and began streaming the data. The students saw something interesting to examine(interesting to them!) and zoomed in to find the data they wanted from the data stream. They went on to analyze, create a model, make predications, etc. Using the HP MCL was fast, easy, and –best of all- brought exploration to the forefront of classroom activities.

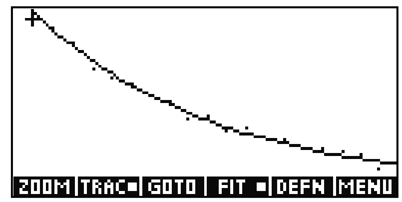


Figure 4

The HP Mobile Calculating Lab is a comprehensive STEM solution that is available in 2 kits. The Starter Kit has just the sensors you need to get started in either math or science. The Advanced Kit has a more extensive collection of sensors which lets you complete all the experiments in the enclosed HP MCL Experiment Guide.

To access more information about the HP MCL Kit and Fourier Systems sensors, visit, www.hp.com/go/MCL and www.fourier-sys.com

Customization

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Customization

Richard J. Nelson

Introduction

I have attended many meetings[1] with HP managers and one of the most common topics of conversation may be summarized by the question, “How may HP distinguish its products from others in the market place?” In other words, what makes HP calculators special and unique? Obviously these managers, from the GM on down, were asking a classical marketing question. My marketing background is much less than my engineering background so I resonate best with the technical features and benefits of HP’s machines.

One vital distinguishing feature, at least to me, is described in the title of this article – Customization.

Before we explore what this means let’s agree, or at least read what I think, are the two most important requirements for a calculator. We have all seen many handheld “boxes” with a display, microprocessor, and an input device - usually a keyboard. PDA’s, notebook computers, and cell phones are examples, especially the latter in the smart phone category. Each of you has at least one of these devices and an integrated calculator is usually one of its features. What makes an HP handheld calculator important enough to also buy and use?

To me the reason a handheld scientific, business, or graphing calculator is essential, and sustains a billion dollar a year business, is due to two basic reasons with a third characteristic, that it is a calculator.

1. It is convenient - easy to use. Convenience means that it is consistent and always ready to solve your problem(s). It instantly turns on and its battery life is very long so it usually works when you pick it up.
2. It is low cost. The three devices listed above usually start in price above the high end calculator models.
3. The last “requirement” is in its name – calculator. Have you noticed that most computer – the three devices listed above are actually computers – calculator emulations are those of well-known calculator models? Basically a calculator is optimized with a completely different design vision than that which is used for a computer - handheld or otherwise. The basic idea is that a calculator has a key per function, i.e. a calculator has to have as efficient a user interface as possible. What is more efficient than pressing a single key?

Customization defined

HP was the first calculator manufacturer to add customization features to its machines and to this day HP has created more methods and has put more types of customization features into its machines than any other manufacturer. Basically a customization feature is one that helps make a calculator what is has to be: convenient and low cost. Let me repeat the last sentence of the paragraph above. What is more efficient than pressing a single key? Let’s go through the list of customizing features found on HP calculators past and present.

1. Programming

This feature was first seen on the HP-65A calculator in 1974. Running a program allows the user to perform hundreds, and even thousands of keystroke pressings by running a single program. The user customizes the program to perform the computation task exactly as he or she needs.

Fig. 1 – The HP-65 had customization features.

The requirement for having a calculator programmable has always been a debatable issue for just about every level of machine. Will the user make the effort to write a program? Research shows that the number of users who program is very small, less than 1% of the total machines sold are user programmed.

Adding programming to a model requires more ROM memory, more RAM memory, a programming "language" of sorts, and documentation. The issue of what the user expects is also a factor. Most users are familiar with electronically transferring information[(2)] to their machines. They expect to be able to down load the more intricate programs into their machines without thinking or effort. This adds some form of input/output to the calculator which further increases the programmable calculator cost.

I argue that even the low end scientific calculator should have some very simple keystroke remembering basic programming capability for quick simple keystroke repeatable calculations.

2. Dedicated program Keys

Being programmable does not completely meet the requirement of being truly customizable as defined above. The HP-65A also added dedicated program keys to start a program previously keyed into memory. Press one of five top row keys and you run five of your programs.

3. Off line storage

The HP-65A[(3)] had a built-in card reader to load and store programs – see Fig. 1. The stick-of-gum sized magnetic cards were used to store, share, and sell/buy programs. You could fill program memory with large programs that could completely customize your machine and make it into a dedicated surveyors, medical, engineering, or financial calculator.

4. Key assignments

The HP-41 is the prime example of a machine with a full set of customization features and it had everything. Programming allows the features to be customized, but it was the User Mode key that converted the meaning of every key to be customized so the user could run any operation, function, or program by pressing a single key. A running program could also redefine the complete keyboard so customization was also dynamic. The 41 also had a feature not seen on other calculators. Since any key could have any function assigned to it the user may not know what to expect. If, however, you held down the key the assigned (or internal native) function would be displayed. If you continued to hold down the key the display would switch to NULL and you could release it and no action was performed.

The HP-41C manual concludes with: "You have just completed the HP-41C Owners Handbook and Programming Guide. You have certainly noticed that programming the HP-41C is simple, and even fun.

Yet the capability of the system is astounding. Your programming expertise will increase as you continue to use your HP-41C. And you will find it an easy matter to completely customize your HP-41C."

The 41C has 130 functions. The HP 48/49/50 series of machines has 2,000+ functions. The key assignment ability for these machines extends to six “planes” of key assignments with the most powerful calculator key assignment capability ever.

5. Overlay capability

Obviously anyone may cut a piece of paper with holes for the keys and place an overlay on any calculator to re-label the keys. Overlays may be made of paper, plastic or electrostatic cling material. The HP-41 included the use of full keyboard overlays[(4)] in the case design[(5)] so that the overlays could be easily managed as part of the system. See ***HP Solve*** *issue 16*[(6)]. Fig. 2 is reproduced from that article. Overlay “D” was called the rubber duck because it completely covered the keyboard to protect it from splashing water.

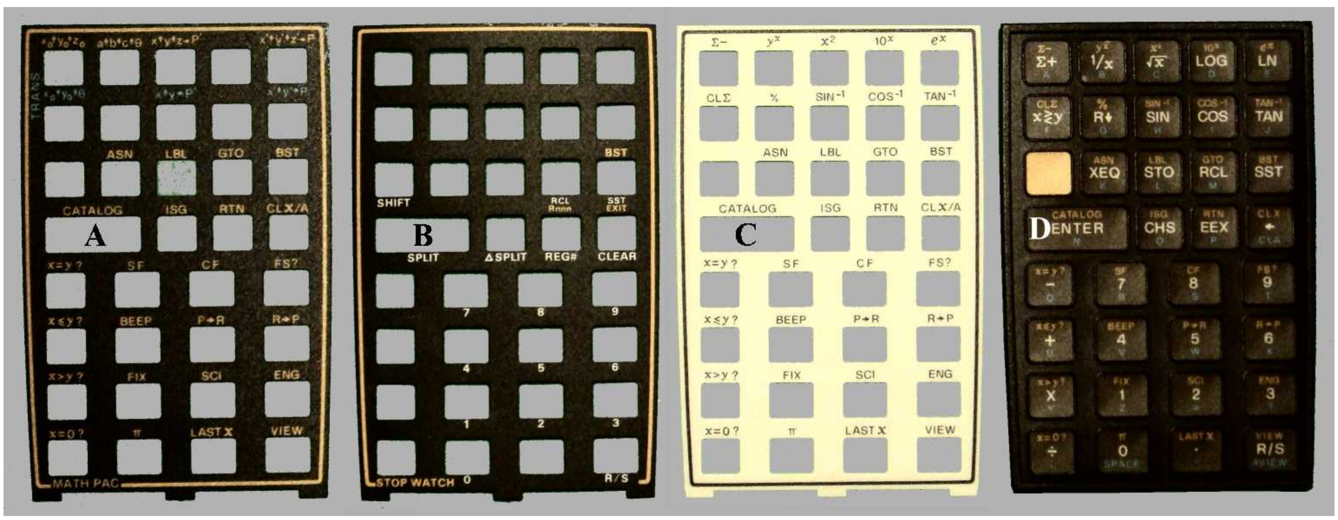

Fig. 2 – Typical keyboard overlays provided by HP for HP-41 PACs (A & B) and general use (C & D).

Fig. 3 – The first HP repurposeable calculator.

HP provided overlays with their HP-41software sold as plug-in modules. A few other machines also included a hardware overlay capability – the HP-71B and HP-75C are examples.

6. Machine repurposing

The customizing examples given above are features found on older HP machines. Has HP exhausted its customizing ideas? No, the most recent HP customization feature is exhibited in the 2008 HP20b. A special calculator connector and HP supplied cable allows the complete calculator to be customized[(5)]. This capability may also be found on the 2010 HP30b. Several Rom images have been converted/developed that may be loaded into these machines. The HP User Community has finally caught up with HP in this regard and the most ambitious project is the WP-34s. This high end scientific calculator is only possible because of HPs vision of customization. See additional information elsewhere in this issue.

7. Expandability

High end calculators will often have some form of expandability. The best example of this is the HP-41 which is best described as a complete HP calculator system because of its four expansion ports. You could customize your HP-41 by simply plugging in ROM and RAM modules. You could customize your HP-41 by adding an HP-IL module to enable connecting to the rest of the world. You could customize your HP-41 by plugging in a bar code reader, printer, disc drive, cassette drive, magnetic card reader or HP instrument. You could also plug in a host of third party accessories as well.

Fig. 4 – Pocket customization example.

Observations and Conclusions

The vital character of a calculator is defined in terms of the current market environment, and the power of a convenient and simple user interface of a single key press is emphasized with customization. HP excels when it comes to providing customization features in its calculators. The best example of the full gamut of customizing features is represented by the legacy HP-41C/CV/CX series of calculators. The seven customizing features of programming, dedicated program keys, off line program and data storage, key assignments, overlays, repurposing, and expandability are discussed. Reference links are included to further explore the subject.

---

(1) Examples are HHC Conferences, user Group meetings, CES Show meetings, and HP facility visits.

(2) Camera manufacturers provide a cable to connect to the camera to the USB port of a computer to down load photos. High end graphing calculators connect to computers to upload and download programs and even operating system upgrades.

(3) The next calculator, the 1976 HP-67A also had a built-in card reader. The most popularly programmed calculator ever was the 1979 HP-41C/CV/CX. There were several off line I/O storage options available for the HP-41 including an HP-IL interface, a card reader, and a barcode wand.

*(4) See **HP Solve** issue 16 for an article The HP-41 system – 30 Years Old that explains the requirements for (HP-41) overlay use (must allow at least 3 reversible overlays on the machine)with 12 examples (photos). A pdf file of the article is at: http://h71028.www7.hp.com/enterprise/downloads/The%20HP-41%20System%20V3.pdf*

(5) Three Slots at the bottom of the keyboard provided space for the tabs at the bottom of the overlau to be inserted. A sliding lever "tab" at the top of the keyboard held the overlay in place at the top. This design allowed the overlay to be easily replaced as needed. the thickness of the slots allowed three reversible overlays to be stacked on the machine at one time. One overlay has the standard notations printed on it so that the machine looked normal. You then have five additional overlays that you could easily swap around as needed.

(6) Details of this capability with extensive photos may be found at: http://hhuc.us/2008/Interfacing%20to%20the%20HP20b%20V3.pdf

Using PROOT with Duplicate Roots

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Using PROOT with Duplicate Roots

Namir Shammas

Introduction

The PROOT function in the HP-50G returns the real and complex roots for polynomials with real and complex coefficients. This function appeared in the HP-48G/GX/G+ graphing models. HP has implemented a nice algorithm, based on the Laguerre method, to handle a wide variety of polynomial coefficients. As mentioned in an earlier article, the PROOT function has the following features:

1. Works with polynomials that have real and complex coefficients (and of course a combination of the two).
2. Solves for all real and complex roots.
3. There is no need to supply guesses for the roots. The function PROOT internally determines the initial guesses.

Article Goals

In this article I focus on the following aspects of function PROOT:

1. How the function PROOT handles polynomials that have duplicate roots. In this article I am using the term *duplicate roots* to be clear and concise. Many articles on the Internet use the term *multiple roots* which I find a bit ambiguous. Typically, you must read these articles' text to determine if they are referring to *several* polynomial roots that have *the same value*, or possess *unequal values*.
2. Study the errors in the results of PROOT as how they relate to the values of the duplicated roots and the number of duplicated roots.
3. Compare the errors of function PROOT with similar errors in the similar Matlab function roots.

This article considers univariate polynomials that have a single value for the duplicate roots. This is a simple case, compared to univariate or mutltivariate polynomials that have several sets of duplicate roots. An example of the polynomials I will cover is:

P(x) = (x – 1)^10

Here is an example of a univariate polynomial with several sets of duplicate roots:

P(x) = (x – 1)^10 * (x + 1)^15 * (2*x – 7)^20

Another example of a mutltivariate polynomial with several sets of duplicate roots appears next:

P(x,y,z) = (x – 2)^12 * (y + 1)^14 * (4*z – 7)^22

The last two examples refer to polynomials whose root error analysis is very complicated.

Why Bother with Duplicate Roots?

If you apply math in practical calculations, you may ask why you should bother with worrying about duplicate roots, since they may rarely occur in your field. To be honest, I have not found a practical case that yields polynomials with duplicate roots (at least with 5 or more duplicates). I have stumbled on the topic of duplicate polynomial roots while researching general solutions for polynomial roots. The cases of polynomials with duplicate roots that I have encountered on the Internet are purely mathematical in nature. It may be one of these cases where we have the knowledge and the algorithms in case a real-world application requires a solution.

Demonstrating PROOT Errors

Duplicate polynomial roots are surprisingly difficult to solve for, causing many popular algorithms to stumble or even fail. To give you a quick idea about errors in calculating duplicate roots, enter the following program object in the stack:

```
<< -> X N
<< 1 N START X NXT N ->ARRY DUP PCOEFF PROOT ARRY-> DROP >>
>>
```

Store the above program object in the variable TDR (short for Test Duplicate Roots, or you can use any other name you prefer). To use the TDR program, enter the value for the root to duplicate and the number of duplicates in the stack. Then run the TDR program. This program performs the following tasks:

1. Builds an array of duplicate roots.
2. Creates a copy of the array.
3. Calculates the corresponding polynomial coefficients.
4. Solves for the polynomial roots.
5. Converts the array of roots into single stack values. This step allows you to examine the calculated roots without using a matrix editor.

Let’s use the program object TDR in a simple case. Enter 1 and then 5 in the stack to build the array [1 1 1 1 1]. Next, run the TDR program. Figure 1 shows the output (generated using Fix 2 display mode).

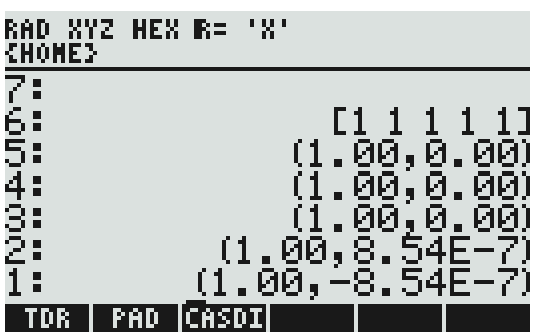

Fig. 1- Solving for a polynomial with duplicate roots.

The stack level 6 shows a copy of the array of roots that the program built. The stack levels 1 to 5 show the roots obtained by PROOT. The values in levels 3 to 5 are exact (albeit in complex form) values. By contast, the values in levels 1 and 2 show minor deteriorations in the form of small imaginary parts of the complex numbers.

Now clear the entire stack and rerun program TDR, This time enter 1 and 10 in the stack, to build an array of ten duplicate roots. Run program TDR to obtain the output in Figure 2.

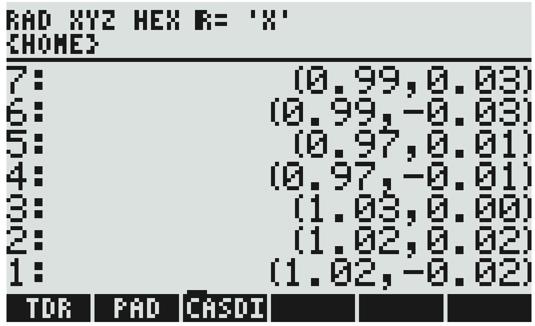

Fig. 2 - The last seven roots.

Levels 1 to 7 in Figure 2 show the seven last roots, out of ten roots. It's easy to notice the deterioration of the results as the real component values fall above and below 1. In addition, the results include imaginary parts that are not so small as in the simple test you did earlier.

If you clear the first seven stack levels, you get the output shown in Figure 3.

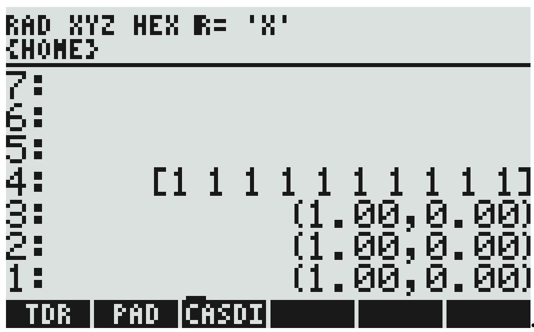

Fig. 3 - The first three roots.

Levels 1 to 3 show the first three roots which are exact. You can conclude that the algorithm used by PROOT may experience deterioration of results due to deflation of the targeted polynomial.

Analyzing PROOT Errors

How are the errors in calculating the duplicate roots affected by their values and their count? I will attempt to answer this question in this section.

First I will define a domain of values for the duplicate roots and the number of duplicates. I studied the roots of 1, 2, 3, 4, 5, 10, 20, 30, 40, 50, and 100. As far as the number of duplicates, I considered the values in the range of 5 to 40, in increments of 5. As the number of duplicates exceeds 40, the PROOT function experiences runtime errors.

I used the following program object to calculate a single value that summarizes the errors in the obtained roots:

```
<< -> X N
<< 1 N START X NXT N ->ARRY DUP PCOEFF PROOT – ABS N SQRT /
>>
```

Store the above program object in the variable PC. This program takes the value of the duplicate root and the number of duplicates from levels 2 and 1, respectively. The program performs the following tasks:

1. Creates the array of roots.
2. Duplicates the array of roots in the stack.
3. Invokes the PCOEF function to create the array of polynomial coefficients.
4. Calls the function PROOT to calculate the roots of the polynomial.
5. Subtracts the array of calculated roots from the array of supplied roots.
6. Calculates the norm of the array of root differences (or errors in the calculated roots, if you prefer) by using the ABS function.
7. Divides the norm by the square root of the number of duplicated roots. The result is a measure for the square root of the mean squared errors. I will call this result NormErr.

Table 1 shows the results-- the square root of the mean squared errors in a two dimensional table of duplicate root values and number of duplicate roots.

Table. 1- The resulting square roots of the mean squared errors (NormErr).

		Number	of	Duplicate	Roots			
Root	5	10	15	20	25	30	35	40
1	7.57356E-07	0.023712099	0.111591213	0.311436851	0.511658272	0.631873293	0.796489399	0.917001403
2	1.11504E-06	0.019690194	0.313862912	0.691337317	1.046742843	1.462345524	1.975058665	Error
3	2.03981E-06	0.009003057	0.357305746	0.982268009	1.799450896	2.536997271	3.025519804	3.419951916
4	3.09378E-06	0.015701459	0.640147421	1.833327277	2.659316327	3.259642713	4.057980913	4.48239371
5	4.01014E-06	0.118694337	0.714660864	2.353802393	3.319326705	4.181961208	5.054055422	5.910306454
10	6.03879E-06	0.236864513	1.761367587	3.112862947	4.833480825	6.318296467	8.025091633	9.178041139
20	1.10905E-05	0.204998177	3.020160338	6.325574864	10.46742843	14.68723352	19.75058665	Error
30	2.52447E-05	0.641002908	4.241309839	9.73553898	17.99616639	25.36997271	30.2545014	34.20109414
40	3.02134E-05	0.149300066	6.408848421	18.33327277	26.59316327	32.59867889	40.58167795	44.82187293
50	3.36434E-05	1.187462764	5.521664308	23.53802393	33.19556004	41.82057503	50.53939049	59.10306454
100	7.57356E-05	2.371209903	11.16477563	31.12862947	48.54449639	63.1786003	80.34521181	91.96021967

Figure 4 shows a linear plot for the NormErr results. The graph includes a legend. Series 1 refers to 5 duplicate roots, series 2 refers to 10 duplicate roots, and so on. The lines are not perfectly linear because of the rounding errors.

Figure 5 plots the same data using log-log scales. The series labeling is the same as in Figure 4.

Applying simple linear and multiple linearized regressions to the data, I found the several empirical models. I will share three models that I found most interesting. The first model is (where N is the number of duplicated roots):

Number of observations = 88
Adjusted R-Square = 0.96926
F statistic = 915.27
NormErr = –0.596421714900844 – 0.246024709841173 * Root + 0.0706708871132657 * N + 0.0302490769565316 * Root * N

The model below shows a linear influence of the values of the roots and the number of duplicated roots on the errors. In addition, the model has a cross product term that includes both the root values and the number of duplicates.

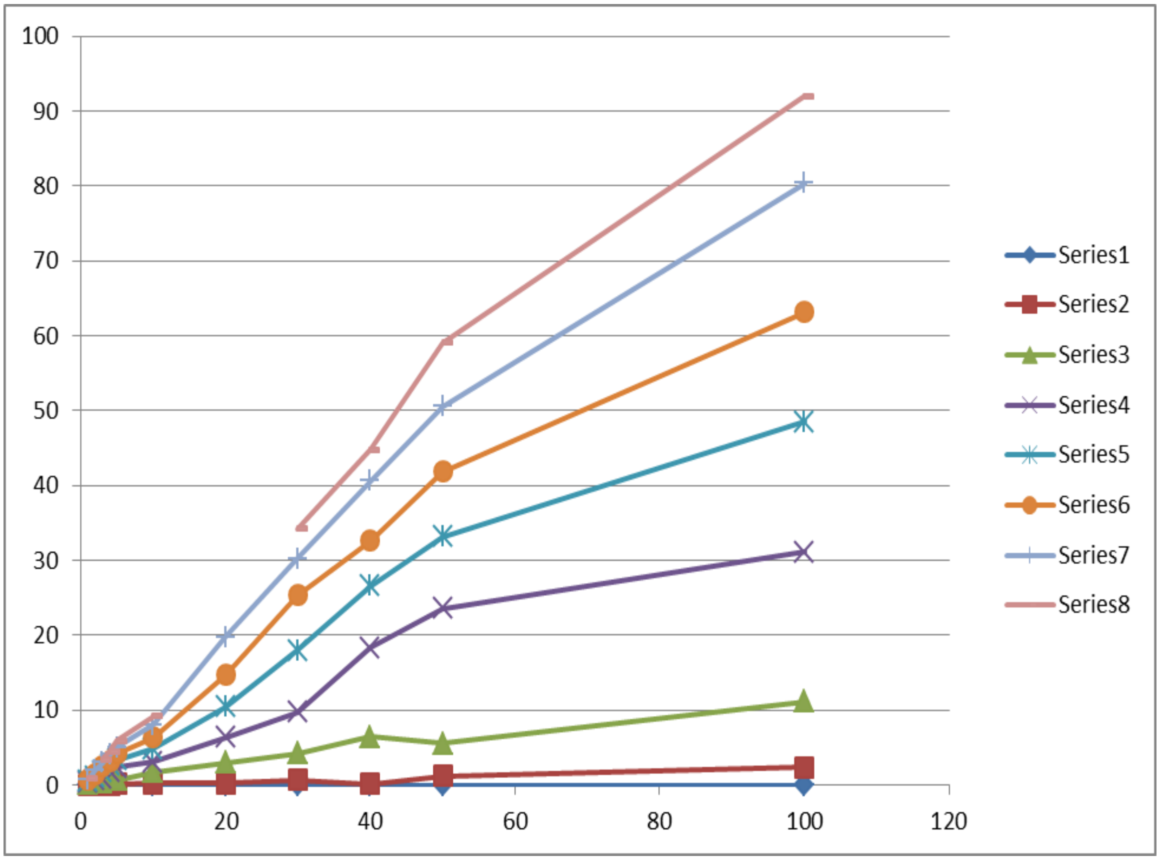

Fig. 4 - A linear plot for the results.

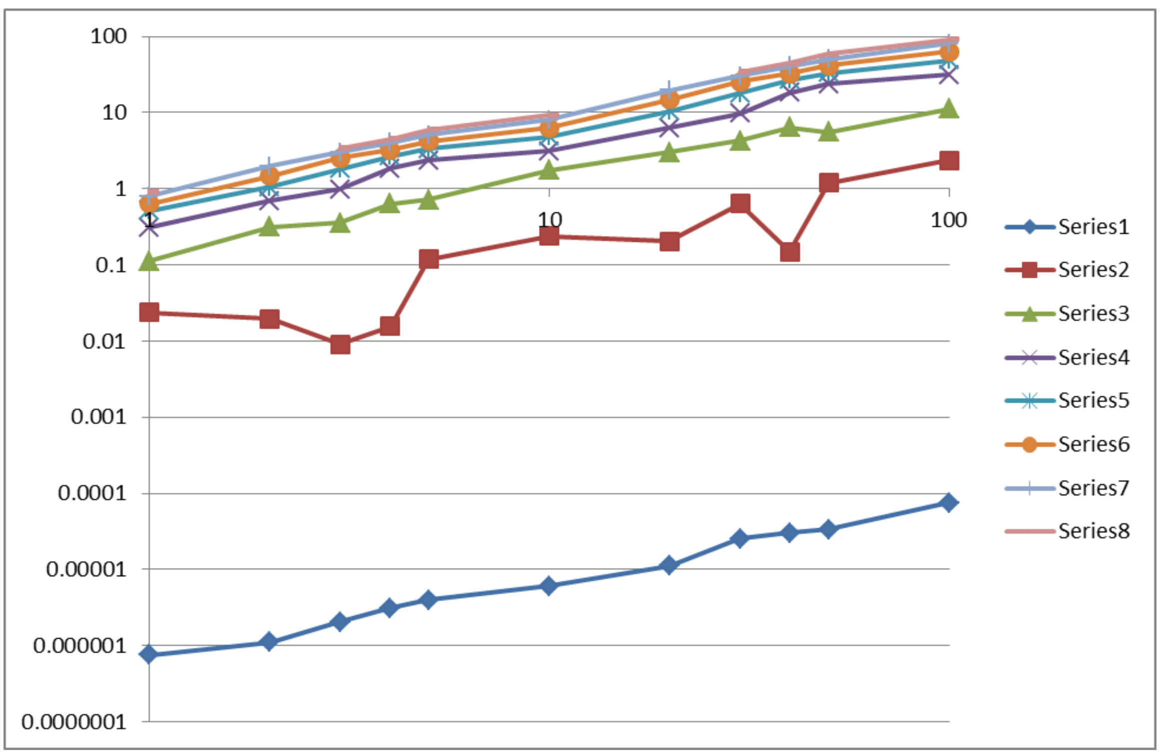

Fig. 5 - A log-log plot for the results.

The regression ANOVA table for the above model indicates that it is possible to drop the term for the number of duplicates without significant loss of correlation. Doing so, yields the following model:

Number of observations = 88
Adjusted R-Square = 0.96853
F statistic = 1339.59
NormErr = 0.984344257243684 – 0.272775553843399 * Root + 0.0314431520006403 * Root * N

Finally, I performed a search for the best model that relates the error with the values of the roots and the number of duplicated roots. The search employed linearized multiple regression and covered a wide variety of popular transformations. Such transformations include functions like the logarithm, reciprocal, square, cube, reciprocal square, reciprocal cube, square root, and reciprocal square root. The best model that I obtained is:

Number of observations = 88
F statistic = 4229.83
Adjusted R-Square = 0.98982
ln(NormErr) = –0.112444241487657 + 1.01931043509749 * ln(Root) – 357.981953542852 /N^2

Figure 5 agrees with the above model as it shows logarithmic lines that are somewhat straight.

Comparing PROOT with the Matlab roots Function

How do the results of PROOT stack up against other polynomial solving tools? I ran a similar analysis on Matlab's roots function and obtained the data in Table 2.

Table. 2- The resulting square roots of the mean squared errors (NormErr) for Matlab's roots function.

		Number	Of	Duplicate	Roots			
Root	5	10	15	20	25	30	35	40
1	0.00110017	0.046654747	0.190355923	0.315265272	0.460383822	0.620049177	0.766216523	0.901217991
2	0.002200341	0.088936771	0.382060543	0.652304998	0.955636607	1.235873769	1.465711962	1.775271906
3	0.003448921	0.14421786	0.543322189	0.997975469	1.461086619	1.870331984	2.22172822	2.633223623
4	0.004400682	0.175477828	0.764121085	1.274225415	1.868542293	2.400188501	2.997573651	3.550543811
5	0.00590331	0.261612309	0.77531048	1.590484639	2.272848794	3.087419409	3.748654527	4.440941079
10	0.01180662	0.523224619	1.550620961	3.180969278	4.974854445	6.174838817	7.564532863	8.881882158
20	0.02361324	1.047740343	3.101241921	6.361938555	8.925339062	12.42682286	15.12906573	17.76376432
30	0.030521903	1.247066378	5.337837594	9.428758046	14.14962977	18.93328928	22.55593546	26.86599005
40	0.047226479	2.095480686	6.202483842	12.72387711	18.18279036	24.85364572	30.25813145	35.34012844
50	0.0604143	2.253174036	8.700136175	16.37453381	23.88436412	29.93058001	37.49176674	43.07384657
100	0.1208286	4.506348071	17.40027235	32.74906762	48.02155301	59.86116002	73.79690739	86.07878242

Comparing the data in tables 1 and 2, you can see that the errors increase with the values of the roots and the number of their duplicates. You can also notice that the errors in PROOT for 5, 10, and 15 duplicates are less than that of Matlab's roots function. For 20 duplicates, the errors for both functions PROOT and roots are close to each other. In the case of 25, 30, 35, and 40 duplicates, the errors of PROOT exceed those of Matlab's function roots—the difference is not overwhelming. So you can say that function PROOT stacks up well against a powerful tool like Matlab's roots function.

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I was also able to obtain similar models for the errors with Matlab's roots function. The first two models match the ones for the PROOT function. Here is the first model:

Number of observations = 88
Adjusted R-Square = 0.99577
F statistic = 6827.06
NormErr = – 0.1354891060389 – 0.182209523189305* Root + 0.00807518594388061* N + 0.0260245276550454 * Root * N

The second model is:

Number of observations = 88
Adjusted R-Square = 0.99580
F statistic = 10313.99
NormErr = 0.0462025776984127– 0.185304881350049 * Root + 0.0261620991288563 * Root * N

The third model is:

Number of observations = 88
Adjusted R-Square = 0.99953
F Statistic = 92299.51
ln(NormErr) = 0.783443710801719 + 0.99705546254637* ln(Root) – 37.8606384516461/N

The above model is somewhat similar to the third model for the PROOT errors. The term for the duplicate count is the reciprocal of the count, instead of the reciprocal squared.

Conclusion

The results indicate that the results of function PROOT deteriorate as the number of duplicates and the values of the duplicate roots increases. It is worth pointing out that the deterioration is relatively small for duplicate roots up to 20. To better handle duplicate roots one must use specialized algorithms. The question is how much effort do we need to employ to hunt for duplicate roots, when such roots don't occur very frequently in real world problems? Of course when they do, they can be a headache.

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About the Author

Namir Shammas is a native of Baghdad, Iraq. He resides in Richmond, Virginia, USA. Namir graduated with a degree in Chemical Engineering. He received a master degree in Chemical engineering from the University of Michigan, Ann Arbor. He worked for a few years in the field of water treatment before focusing for 17 years on writing programming books and articles. Later he worked in corporate technical documentation. He is a big fan of HP calculators and collects many vintage models. His hobbies also include traveling, music, movies (especially French movies), chemistry, cosmology, Jungian psychology, mythology, statistics, and math. As a former PPC and CHHU member, Namir enjoys attending the HHC conferences. *Email me at: nshammas@aol.com*

Repurposing the HP 20b/30b Calculator Platform

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Repurposing the HP 20b/30b Calculator Platform

Jake Schwartz

Introduction

In June of 2008, Hewlett-Packard released the new HP 20b business/financial calculator, which represented the initial iteration of a new calculator hardware platform. At the HHC2008 calculator conference in Corvallis, Oregon in September of that year, HP's Cyrille de Brebisson gave a 20b presentation where he explained that the hardware had been designed for repurposing by third parties and he demonstrated firmware-support tools for that purpose. Soon after, a few basic attempts were made to replace the firmware with alternate function sets, including one to duplicate the capabilities of the venerable HP45 scientific (non-programmable) calculator from the early 1970s. Also at the 2008 conference, the committee made sure to spotlight a lengthy proceedings paper[(1)], submitted by Walter Bonin from Germany, containing various suggestions for usage of existing HP calculator industrial design to create followon scientific machines boasting powerful capabilities of the great high-end non-graphing Scientifics of the past such as the HP41 series (from 1979-1990) and the HP42S (from 1988-1995). With HP's ongoing focus on the financial segment of the market, and the HP 35S from 2007 representing merely a continuation of the scientific "mid range" (as compared to the 41/42 caliber), highlighting suggestions for extending this category of machine seemed relevant, especially with Hewlett-Packard folks in the audience.

Then in the following year, the 20b platform bore additional fruit with the HP 30b advanced business calc, with its basic "macro" programming capability and more-complete suite of financial functions. And true to form, in October at the HHC2009 conference in Fort Collins, Colorado, attendees were treated to HP presentations detailing the 30b's functional and programming capabilities. What was not yet apparent, however, was that Walter had teamed up with fellow HP enthusiast Paul Dale (from Australia) to embark on a very sophisticated repurposing project for the 20b/30b hardware whose goal was to provide just the type of high-end scientific machine he had proposed the year before. As 2010 rolled around, Walter and Paul were using the HP Museum Forum at http://www.hpmuseum.org (curated by Dave Hicks and independent of HP) to announce their fledgling project (now known as "WP 34S"), show early keyboard layouts and to solicit input from the community for suggesting improvements. By the summer, the design was taking solid shape and attention was beginning to build. There was an emulator which would run under the Linux operating system, but nothing yet for Windows, nor had they achieved a way to transfer their code to real hardware for testing. As the season flew by and interest in this important project seemed to be waning, it was thought that making the HHC2010 conference attendees aware of it might reinvigorate the community. As a result, I presented a synopsis of the capabilities, included an abridged copy of Walter's detailed manual in the conference proceedings and made a plea to the group to anyone with the appropriate expertise to offer assistance in order to give the project the push needed to succeed.

In early 2011, Marcus von Cube (also in Germany) jumped in with some crucial assistance and by March, a Windows version of the WP 34S emulator had been devised and the participation from users who frequented the HP Museum Forum increased steadily. Soon after, the first version actually made it to the 20b/30b hardware and with HP making serial-interface PC firmware-transfer cables[(2)] available by request, the number of beta testers using real calculators began to take off. Copies of the 34S firmware along with a complete user manual are available for free download at http://sourceforge.net/projects/wp34s/files /. One final piece of the puzzle was related to the fact that the entire calculator keyboard was redefined. A method was therefore needed to re-label all the keys as well as the "land" around them to mark the shifted functions. Based on life-size keyboard images posted by Walter on the HP Museum, people began fashioning their own rudimentary keyboard overlays as well as stick-on key covers. Users posted images of their efforts and virtually all these attempts remained crude until Eric Rechlin[(3)] purchased a cutting machine and began to offer vinyl, preprinted, adhesive-backed overlays and peel-off adhesive key covers. An image of the HP 20b transformed into the 34S is shown, in the center, with its "peers" in Figure 1 below.

Figure 1. (Left to right :) HP 35s scientific; WP-34S scientific repurposed from an HP 20b; HP 30b business calculator. The 34S utilizes one of Eric Rechlin's professionally-made keyboard overlays and individual key covers.

An Amalgam of Many Predecessors

In Walter's manual for the WP 34S, it is mentioned that the functionality attempts to encompass the HP-42S' scientific functions, the fraction mode of the HP-32SII, integer and bit manipulation from the HP-16C and probability distributions from the HP-21S. In reality, this firmware goes much further in several areas, such as: (1) expanding the complex-arithmetic functionality of the 42S or 15C as well as offering an optional 8-level RPN stack mode; (2) enhancing the integer capabilities of the 16C by providing conversions to and math in *any* integer base between 2 and 16 (being not restricted solely to binary, octal, decimal and hexadecimal); (3) offering over 50 constants plus 80 unit conversions; and (4) expanding programming features in many areas. Also, it should be stressed that due to the high-speed CPU in the 20b/30b, programs run on the order of 50 to 100 times as fast as the HP 42S (and even moreso compared to the 35S). With its 111 storage registers, 100 flags and 506 program steps in its basic memory (as well as 3 additional 506-step program banks available for storage in flash memory), this powerhouse of capability is something no serious calculator user of science and/or math should overlook. It has recently been announced that Marcus will be attending and speaking at the HHC2011 calculator conference this September in San Diego, California to present his "insider's perspective" on the project and we very much look forward to meeting and hearing him. Activities have begun with respect to creating user programs to augment the built-in 34S capabilities, especially in the matrix-manipulation area. The natural starting point for these has been to adapt and update the "M1" through "M5" matrix routines from the PPC ROM[4] for the HP41 series. Since the programming languages are very similar, this adaptation should be fairly straightforward, and it would be expected that the PPC ROM code plus the example programs in the PPC ROM manual will begin to take on new relevance thirty years after their original creation. Kudos go out to Hewlett-Packard for making the 20b/30b hardware open for repurposing and it is hoped that the continued success of the WP 34S project might spur others to attempt to carry this calculating platform even further in the near future.

Observations and Conclusions

Fig. 2 – P1 of 10 pages of designs.

HP considers its graphing machines to be its high end scientific calculator entry - if you consider there being an entry level model, mid-range model, and high end model. Many users, however, do not need the size or added complexity of a graphing machine and only wished that the scientific product range included one with additional capability. Long time HP creative calculator user Walter Bonin gave the product line a lot of thought, and with each new model he made his own keyboard arrangement with the functions that he thought should be there. Many of his "designs" may be seen in the HHC 2008 conference proceedings. His ten full page keyboard layouts build on the HP-42S, and HP-15C.

Walter Bonin (Germany) teamed up with Paul Dale (Australia) and Marcus von Cube (Germany) to implement his concepts into the 20b/30b platform dubbing it the WP-34s. The prefix letter choice should be obvious.

About the Author

Jake Schwartz has been an HP calculator fan since 1971 after first trying the HP9810A desktop RPN machine at a co-op job at RCA in New Jersey. He has owned most of the scientific top-of-the-line handhelds since the HP35A in the early 1973, joined the PPC Calculator Club in 1977 and contributed to many of the clubs since, including serving as Peripheral Routines coordinator for the PPC ROM project for the HP41 in 1980. Currently working at Lockheed-Martin as a software engineer, Jake has been presenting at and videotaping the annual U.S. HP calculator conferences for more than two decades.

Notes

(1) A copy of the HHC2008 conference proceedings which includes Walter's 10-page paper, "Imagine: A Development Since 1988", along with proceedings for 35 conferences between 1979 and 2010 is available as part of the 22,300+-page PPC DVD HP-calculator historical reference disk. For more information, check http://www.pahhc.org/ppccdrom.htm.

(2) Gene Wright is the current point of contact at present for obtaining an HP serial-transfer cable. Contact Gene at: genewright143@hotmail.com

(3) Contact Eric Rechlin at http://commerce.hpcalc.org/overlay.php to order overlays. If that link is unavailable, check his main commerce page at http://commerce.hpcalc.org/.

(4) *The PPC ROM was a custom plug-in module containing 122 routines in many disciplines, and developed for the HP41 series by members of the PPC club in 1980-81. Copies of the complete 500-page user manual (including scannable program barcode) and the 32-page pocket guide appear in the PPC DVD reference disk mentioned earlier.*

HP's Calculator Manuals

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HP's Calculator Manuals

Richard J. Nelson

Note: Most of the opinions expressed here are my own and they reflect my obsessive belief that examples and a high index rating are vital to every technical manual. Experience has shown that providing them, however, is very very difficult.

Introduction

Legacy users of HP's Calculators usually remember the manuals that came with their machines in a positive way. Popular opinion seems to suggest that newer manuals are not as good as the older manuals, and certainly every manual user, old or new, seems to complain that there are never enough examples[(1)].

Most commonly the manual that describes the machine and how to use it is called the owner's manual. This is a generic term for a manual that comes with your car, refrigerator, or calculator. The owner's manual is assumed to contain the important information related to the product and how to use it.

HP didn't start out calling their calculator manuals, "Owner's Manuals", and they don't call them by that name today. The HP-35A (1972) came with an "*HP-35 operating manual*" (36 pp). The lower case is HP's choice. The HP-65A (1974) came with an "*HP-65 Owner's Handbook*" (107 pp.) The HP-67A (1976) came with an "*HP-67 Owner's Handbook and Programming Guide*" (353 pp.). The HP-41C/41CV came with an "*Operating manual, A Guide for the Experienced User*" (71 pp.). The HP48G Series machines (1993) came with a "User's Guide" (592pp.). All of these manuals were printed, most spiral bound, and heavy. User's Guide makes sense because it is simple, short, and descriptive.

After 16 years of calculator manuals the eleven Pioneer Series of machines (1988-1991) came with an "*Owner's Manual*" (larger size, ~250 pp.). More recent machines now come with a "*User's Guide*" or "*Quick Start Guide*." The name has changed from manual to handbook to guide.

Another often-remembered "feature" of HP's manuals is the use of humor when illustrating examples that include people. Especially memorable are the unusual names for people used in the example. One reason that users tend to remember these things is that they desperately need examples in manuals. The "*PPC ROM Manual*" demonstrated this need by providing extensive examples for every one of the 153 routines described in the 500 page 8-1/2"x 11" (US letter size) 2-1/2 pound (1.1 Kilo) tome.

Why is this completely obvious aspect of the user's need not emphasized by HP when it produces its manuals? After 35 years of writing about this issue, especially after producing the *PPC ROM Manual*, I believe that I know the answer. The resources required to include lots of examples for any publication is simply beyond the resources that are available. The *PPC ROM Manual* was able to provide the much-needed examples because of the donated 100+ man years – 876,528 man hours – which the User Community put into the project. Can you imagine HP, or any calculator manufacturer, spending these kinds of resources on any similar project? At a value of $20/hr. for the time, that would be $17.5 million dollars or $35,000 dollars per document page to produce the content. Even the world's largest technology company does not have that level or resources for a calculator product.

I mention these things because of the "chatter" often seen on the various HP web sites that users complain about HP's manuals. I am sensitive to these complaints because I have been writing HP's manuals for over 30 years. I have probably met with (in person, face to face) the majority of the manual writers at HP for most of their machines.

Back in the days when HP had all of their operations – engineering, marketing, and manufacturing – in one location in Corvallis, I would visit the "factory" for three days meeting with many different teams dedicated to calculators. I especially remember one meeting with the documentation group (at a mutual request because of some of the things I had written[2]) that mentioned that they thought that their manuals were some of the best in the industry. After all, they had won awards for their manuals. Of course I was coming from a perspective of what is desired from the user's perspective and these experiences drove the decision to include extensive examples (and a formal format of section headings, extensive index, etc.) in the *PPC ROM Manual*.

The Ideal Manual

Manuals for computer related products have always, and will always, fall short from the user's perspective simply because of resources, complexity, cost, and time. The needs of the user don't change, it is the manufacturer's continuous attempts to competitively meet those needs that changes.

"If resources were unlimited", is a prerequisite that every complainer should use when writing about manuals. From the user's perspective I personally would like to see a manual approach similar to the following.

1. A **photograph** of the keyboard with each and every key identified with a description of what the notations and symbols mean. This should be a straight-on photo not one at an unreadable angle.

2. A **key response description** of what happens when any key is pressed. Presenting this matrix of key responses will require that the user understand that keys change in their meanings depending on the mode or environment at the moment the key is pressed.

3. A **reference organization** that recognizes that the manual is used at least five times more frequently for reference than it is used for explanation (initial) reading.

4. **Each page is numbered**[2] sequentially. Bill Wickes addressed the page numbering issue of section numbering vs. sequential page numbering in his famous trilogy of HP48 books by using both.

5. **Many Examples –** practical, real world, timely, and meaningful.

6. **Consistent Writing Style**. Manuals should be written following a published style[3] that dictates what must be covered in an owner's manual, handbook, or guide.

7. A **two tier index**[4]. While examples are omitted because of necessity, the sparse indexes in most calculator manuals are a classic example of inconsistency. Using modern computer software it is an easy task to produce an index with a minimum index rating of 3 or higher (Index rating = number of index entries divided by the number of text pages). Certainly every technical document that is to be used as a reference has at least three words, terms, or important ideas per page. Simply write down these index items with the page number as the proofing is being done. Word and most other document software have an index capability.

8. **Format.** Most of HP's recent User's Guides follow a "standard" format. Section one is usually "Getting Started" and the last part is "Appendixes and References." If the manual is printed it has a soft cover using a "perfect bound" style of binding.

HP calculator teams have reinvented themselves during the 39 years of calculators. How should the manual issue be viewed using today's highly competitive market place? Many of you reading this have watched HP up close and personal during their active participation at HHC's since 2002 when Fred Valdez broke the mold for HP User Community relationships. Attendees of HHC 2007 witnessed the manual changes that are taking place. A new calculator GM, Wing, first met at HHC demonstrated that HP has changed in ways that brings a smile to the face of the legacy user. Product indicators of this very positive change are the HP50g and the HP35s. Regarding the latter, the very nice HP 35s manual is available to be down loaded at:

http://h10010.www1.hp.com/wwpc/pscmisc/vac/us/product_pdfs/user_guide.pdf

Printed manuals is still an HP challenge and there have been several solutions proposed to solve this "problem." Still, having a manual in a useful downloadable form as illustrated at the link above is most helpful. If you must have a printed copy you may print it yourself, or have it done at a copy store, and bind it using the method described in my HHC 2004 paper "Personal Low Cost Binding system." It is clear that manuals will become ever smaller and be more of a Quick Start Guide. The primary User's guide is available on a CD/DVD and/or downloaded from HP's web site.

Observations and Conclusions

Manuals will always be a topic of discussion by users, young and old, newbie and experienced. What is most important, however, is to understand the issues involved, sharing your desires with HP, and being prepared to reach ever deeper into your wallet for printed good quality manuals, handbooks, or guides.

Most calculator owners use their manual mostly in a reference mode. How does HP's more recent manuals stack up? A quick look at the HP35s manual index shows 1152 page references for the 338 indexed pages for an Index rating of 3.4. This is great!

Paper is heavy and manuals take valuable packaging space so the trend of putting the manual on a CD and making it available for free down load is really a win-win solution. Electronic searching is facilitated, production time is saved, cost is reduced, and if you really need a bound paper version you can do it yourself. See one method of low cost binding at:

http://www.pahhc.org/2004/HHC2004/Low_Cost_Book_Binding_R4_dist.pdf

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Notes:

(1). Technology changes the way we use information. Manuals are provided on CD's. Examples, as mentioned here, are usually associated with a manual; printed, downloaded, or provided by HP on a CD. What is an example? It is a step by step procedure, process, or algorithm that shows you how to solve a problem or use a process. Today HP is using video technology in the form of Training Guides for their machines The Training Guides may be found on their web site, and they are being produced by experienced users of their machines.

(2). One of the comments I had written over 20 years ago regarding HP documentation was the fact that at least the pages should be sequentially numbered on all documents over four pages in length. The writers were surprised that I should mention this until I showed them a 72 page example that had obviously slipped through the cracks. Perhaps I am overly sensitive to page numbering because I so often reference page numbers in review articles.

(3). When I asked if an HP Style Manual existed most of the people in the room were hesitant to talk about it. They implied that they had one, but when I asked if I could get a copy I received a blank look. Privately a writer later explained that each writer used his or her own references, and that they did not have an official HP Style Manual. This older HP confident writer explained that there was an official HP Style manual, but it hadn't been updated in many years – at that time – and the younger writers didn't know about it. That was over twenty years ago. Today technology and language change so fast that corporate style manuals are seldom used.

(4). In the spirit of the famous numerical calculation philosophy of Prof. William Kahn there are avoidable errors and unavoidable errors. Perhaps the lack of examples could be justified to be in the latter category, but having a poor index is most certainly in the former category. An index value of three or higher is most desirable and it greatly extends the usefulness of the User's Guide. One reason that many manuals have a sparse index is that the Index is often done last, the project is often behind schedule, and not going back to add to the index entries is easily justified.

Fundamentals of Applied Math Series #7

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Average Mean Median & Mode

Richard J. Nelson

Introduction – What is the difference?

Fig. 1 – HP35s $\overline{X}$ *key.*

Everyone remembers (from school) what an average is. The average cost for a student's five text books is calculated by adding the five prices, e.g. $83, $52, $72, $44, & $ 70 = $ 321, and dividing the sum by the total number of items being averaged = 321/5 = $ 64.20. What does the average text book cost, asks the parent thinking of budget planning. For the example above it is $ 64.20, but is this the same for all students? Is this the same for the "average" student?

Sometimes we hear about a news report that mentions a median value when the expected term is the average value. A similar situation may also apply to the mean or mode. How are these different?

Like many "mathematical" concepts there are sometimes different levels to a meaning. You may think of this situation as the "normal" meaning and the purist's meaning. If you can't depend on mathematics as being exact what is? Human nature even influences mathematics.

Data Sets and Statistics

Previous calculator functions reviewed in this series were simple unary functions, e.g. given x, calculate LN x; given n calculate the cube root of n, etc. When multiple data, called sets, are involved, the calculated answer provides perspective and is a form of analysis of the data. The calculations are more involved and in many cases they are quite complex with the calculated values providing a comparative measure or a statistic of the data set.

The average value of a data set provides a means of using the set without knowing each value in the set.

Let's imagine the problem of clearing out the rocks in a length of a stream flowing through your property. If you know how many rocks there are, and the average weight for a rock, you could calculate the size of the truck you need to rent. Let's say that the average rock weighs 35 pounds and there are 73 rocks total.

The previous owner was going to do the job and he gave you the numbers. The total weight (35 x 73) is 2,555 pounds. You might possibly be able to lift and load a 35 pound rock, but the analysis, average and total, is such that you may actually have a bunch of small rocks with a few rocks that weigh hundreds of pounds. In fact, no rock may weigh 35 pounds and you really don't have enough information to get the job done. In the real world a personal visual inspection of the stream would tell you that you may need more than just a truck.

How may we calculate the average of a data set on our calculator? The HP35s is our "typical" calculator we are using for this series. The average of a single data set is usually represented as $\overline{X}$.

We could just key in the values and add as we go. This would be the approach used for most simple calculators and the approach you would use for a calculator you are not familiar with. In general terms more advanced scientific (and graphing) calculators provide a means of inputting the data set, storing the complete set, and then calculating a wide range of functions on the data.

The category of functions, of which the average is only one, is statistical functions. The HP35s uses what may be called accumulated sums based statistics.

A more advanced machine such as the HP50g will use list based statistics. Usually more advanced calculators have more memory and they will handle larger sets of data. In the early days of calculators memory was much more expensive than it presently is and the data was stored as sums. There are usually six statistics registers, for two data sets designated as x and y (see Fig. 1), and each time you keyed a data point the data went into the six dedicated statistics registers. These are: (1) number of data points or ordered pairs, (2) sum of x, (3) sum of y, (4) sum of x squared, (5) sum of y squared, and (6) sum of the product of x & y.

More advanced calculators will store the data points into a list which is only limited by the total memory of the machine. The user then performs the desired operations and the individual data is always preserved. This is in sharp contrast to accumulated sums based statistics which essentially destroys each data point when it is entered.

Each method has its advantages and disadvantages. The accumulated sums method is simpler, faster, and easier to use. The data list method is busier, requires more thought, allows data entry verifying, and is more suitable for multiple uses such as calculating the median and mode of large sets of data.

Fig. 2 - HP35s ∑+ key.

The HP 35s key we need to calculate the average is shown in fig. 1, but this key is used for only half of the problem. Fig. 2 shows another key located just to the left that is needed to enter the data. Assuming that the statistics registers are clear you key the first data point and press the ∑+ key. The data is processed and the display will show 1 to indicate that the first data point (or x y pair) is input. Continue keying in the values and the ∑+ key pressings will count your data points. When you are finished you will then press the left shifted (gold) $\overline{X}$ key to calculate the average. See you User's Guide for the details of clearing, and calculating all of the statistics values.

Suppose you are a teacher and the principal asks you for the average grade for all of your five classes. Without thinking you may think that since you have the average grade for each class that you could just add these five averages and divide the total by five to a get the average grade for all of your students.

This would not be absolutely correct however, because the average of a group of averages is not the average of all members of each group. Let's illustrate this with some real data. See Table 1. The grades are given values as 1 = A+, 2 = A, 3 = A-, 4 = B+, 5 = B, 6 = B-, 7 = C+, 8 = C, 9 = C-, 10 = D+, 11 = D, 12 = D-, and F = 13 and is failing. There are 15 students in three classes, and 17 students in two classes.

The average grade if the all the grades are averaged is (439/79 = 5.56) B-, but if the average of the averages (27.06/5 = 5.41) is used the grade is B. The two may be quite close, but that is because the data is closely grouped.

You may calculate what is called the weighted average of the five classes as follows without having to deal with all 49 values of the data as follows.

Weighted average of the 5 classes
= [17(5.23) + 15(4.27) + 17(7.88) + 15(6.27) + 15(3.87)] / [17 + 15 + 17 + 15 + 15]
= 439.02 / 79 = 5.56

Table 1 – Student Class Grades

#	Class 1	Class 2	Class 3	Class 4	Class 5
1	8	5	5	9	3
2	5	4	7	5	5
3	3	6	5	9	5
4	8	1	9	3	9
5	10	2	6	7	2
6	4	4	7	5	5
7	3	5	10	2	1
8	4	2	4	9	4
9	2	3	11	6	1
10	6	4	9	6	4
11	3	7	9	6	6
12	7	2	7	8	2
13	4	8	8	2	8
14	2	2	10	8	2
15	7	9	3	9	1
16	4	—	6	—	—
17	9	—	11	—	—
TOT	89	64	134	94	58
AVG	5.23	4.27	7.88	6.27	3.87
Grd	B	B+	C	B-	A-

Average vs. Mean[(1)], Median, and Mode

Mean: In normal life people use the terms average and mean interchangeably. Statisticians, however, will consider the mean to be only one form of the average. Technical language will usually use the term mean where casual English will use the term average.

Statistics deals with sets of data and the term average is more of a broad term that describes a data set as a whole whereas mean, median, and mode are subtle (more technical) versions of expressing the average of a data set. Depending on the distribution of the data, mean, median, and mode may provide a more mathematically accurate description of the average of the data[(2)].

Median: This is the central point of the data set. In order to determine, vs. calculate, the median of the data set you must first sort the data from low value to high value. The sorted values are divided into two equal groups with the value in the middle the value of the mean. If the groups are equal the bottom value of the top group is averaged with the top value of the bottom group. One difference between the mean and the median is that the mean (average or arithmetic mean) may be a calculated value whereas the median is usually an actual data value[(3)].

Mode: This is the value that occurs most frequently in a data set.

The average terms may be briefly summarized in table 2.

Table 2 – Comparison of Common Averages (2)

Examples values { 1, 2, 2, 3, 4, 7, 9 }

Type	**Description**	**Example**	**Result**
Arithmetic mean	Sum divided by number of values: $\bar{x}=\frac{1}{n}\sum_{i=1}^{n} x_i=\frac{1}{n}(x_1+\cdots+x_n)$	(1+2+2+3+4+7+9) / 7	**4**
Median	Middle value separating the greater and lesser halves of a data set	1, 2, 2, **3**, 4, 7, 9	**3**
Mode	Most frequent value in a data set	1, **2**, **2**, 3, 4, 7, 9	**2**

Observations and Conclusions

The basics of Average, Mean, Median, and Mode have been discussed. One of the misunderstandings by many students and technical users is how they relate to each other as the latter three terms are detailed technical terms and average is a casual English term. Mean, while a technical term, implies average in that it is usually understood to be the arithmetic mean which is the same as average. Table 2 provides a comparison with examples of each of the terms.

This topic approaches the very broad topic of statistics which will have to wait for a future article in this series. One aspect that is part of a future article is data that occurs multiple times, frequency. This aspect along with "weighted data" is the frequency of occurrence of each value and how this is handled by our calculators.

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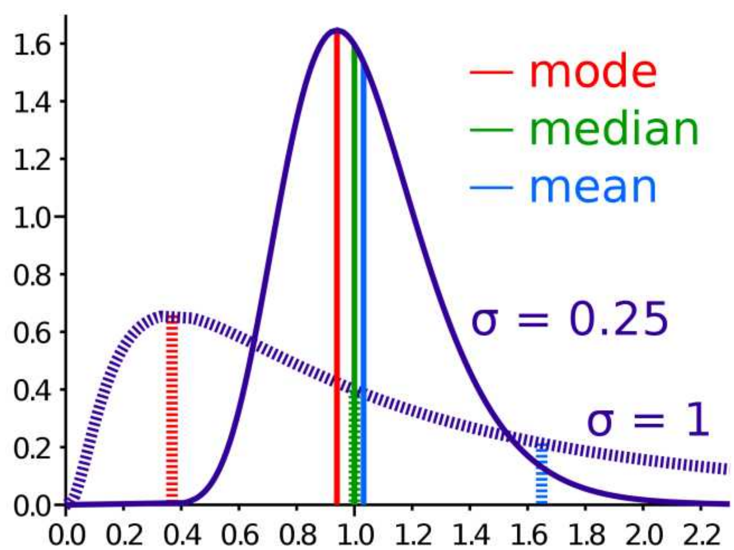
(1) *Mean as used here is the arithmetic mean (average) - vs. geometric, quadratic, or harmonic mean.*

(2) *For an excellent distinction of these subtleties' see* http://en.wikipedia.org/wiki/Mode_%28statistics%29
Table 2 is taken from that text. This text will also provide the mathematical details of the three differences which are illustrated from the following plot from that article.



(3) *The mode value is an obvious choice for an odd number of (sample) data values. If the number of data is even it is customary to use certain rules to determine which one to use. A data set as a distribution may be bimodal or multimodal. See the link above in (2).*

From The Editor

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From The Editor – Issue 24

Summer is expressing itself, 113 degrees worth, here in the Sonoran Desert where I write these words. In case you haven't noticed ***HP Solve*** is quarterly this year. Most ***HP Solve*** readers are spending their days outdoors, but here in the desert being outdoors, from June to August, is where you do not want to be. Triple digit temperatures for 100 plus days in a row during the summer is normal. It will start to cool off in September when students go back to school and the HP Hand Held Conference is held. See additional comments below.

Community News

HP calculator user activity as reported in issue 23 continues to build. This should make HHC 2011 an especially exciting conference with so many new machines (5?) to talk about. If you have never attended an HHC this year's planned presentations will make it exceptional. You will find additional information at: hhuc.us. BTW the cost of attending an HHC is very low and you will go home with more than you came with in all aspects.

HP has always inspired its users to build on its leadership machines with new applications ideas, and even new hardware. The most unbelievable inspiration is one user redesigning the HP nut microprocessor. Monte Darymple not only cloned the nut microprocessor used for the famous HP-41 series calculators, but he actually had it made. He then redesigned the HP-41 CPU circuit board and built a replacement board that has been shipped to 21 early adopters.

This unique group of HP calculator users have been testing the resulting machine called the HP-41CL. This board swap brings the 1979 HP calculator up to a typical 2011 calculator capability in terms of speed, ROM, and RAM capacity. Not only is the HP-41CL 50 times faster, but it is a system compatible with more I/O options, more accessories, and more software than any other calculator on the planet past or present! When you install the 41CL circuit board you also install more than 100 of the best software ROM modules programmed for the HP-41 system. At a bargain basement price of $25 each – many of them sell for two or three times this price on eBay - that is $2,500 worth of software at your command with a $235 upgrade. Follow Monte and this exciting project at: http://systemyde.com/hp41/

Another topic being presented at HHC 2011 is a new HP calculator restoration book by Geoff Quickfall. ***HP Solve*** readers will remember that Geoff was interviewed in ***HP Solve*** issue 12, and the book was previewed in issue 19, page 23. The extensive photography provided in this book may also be sampled with illustrations of installing the 41CL board at

http://www.hpmuseum.org/cgi-sys/cgiwrap/hpmuseum/archv020.cgi?read=186300#186300

Yet another very exciting project uses one of HP's business calculators to create a new calculator dubbed the WP-34s. In this case the very recent HP 30b's feature of being repurposed is used to create a new high end scientific calculator. Jake Schwartz delves into some of the features of this community project created by an HP user in Germany. This project will be discussed in detail and machines can be repurposed at HHC 2011. The WP-34s is a true RPN machine, not an RPL based RPN machine. The PPC ROM manual, based on the HP-41, is an excellent resource for programs. Matrix operations are an example of functions not included in the WP-34s, but the PPC ROM routines provides a rich resource for easily adding them.

Third party news

A new business book dealing with the HP10BII+ is now in type setting. Written by John Webber it is titled Get Rich Slow. In addition to being an author John is a lecturer and his books and other activities may be found at: www.webbertext.com. You may email John at: jwebber@olympuspub.com

Technology News

One of the most technologically complex consumer electronics devices is the smart phone. The sales volume of these handheld marvels attracts an incredible amount of research resources and leading edge technological advancements excel if they are used in a smart phone. Antenna, battery, display, key, radio, semiconductor, and sensor technology advances as used in smart/cell phones could eventually impact calculators because high volume manufacturing lowers cost. Color displays, for example, are common in cell phones. When a color display costs the same as a monochromic display the possibility of a color displays being used in a calculator increases. Touch screens, when low enough in cost, will eventually be used in calculators. As I mention in the Customization article, "The basic idea is that a calculator has a key per function, i.e. a calculator has to have as efficient a user interface as possible." Efficiency is vital for a calculator.

One technology to watch is haptic touch displays and this topic was discussed by Jake Schwartz in issue 19. This technology allows you to feel what you touch so that a flat screen may also become an effective keyboard. One of the more sensitive issues with HP calculator users is the feel of the keyboard. Does this technology offer a solution for calculators? A recent article, Haptics Adds New Dimensions to Touchscreens, in BusinessWeek provided an update and based on the list of very large companies investigating the technology the promise looks interesting.

[http://www.businessweek.com/magazine/content/11_26/b4234043549730.htm]

Fig. 1 – Original ad for demonstrator.

Educational Aids news

Do you understand the basics of computers? Do you know how bits are processed by a computer to perform the mathematical operations that we use daily with our calculators? Does your school have a means to demonstrate these concepts in a dramatic, interesting, and fun way?

Here is a news item that caught my eye because it would be a neat demonstrator for any school to have. Computers were very new in the 60's and many companies developed machines to demonstrate the underlying digital concepts of computers.

Fig. 1 shows one such machine, the DigiComp II, and Fig. 2 shows a recent version of the same computer. I think of it as the "8 Ball Computer." Watch the video demonstrate the multiplication problem 3 x 13 = 39 at the link below. No batteries are required, just gravity. It is a complete automatic binary computer that is big enough for classroom use.

Giant Digi-Comp II by Evil Mad Scientist Laboratories is a oversize recreation of the Digi-Comp II, a mechanical educational computer from the 1960s that used marbles to perform simple calculations. Giant Digi-Comp II was on display at Maker Faire Bay Area 2011. Kits will be available for purchase at Evil Mad Science this summer.
http://laughingsquid.com/giant-digi-comp-ii-recreates-mechanical-computer-from-1960s/

Fig. 2 – Pinball machine-like computer.

http://laughingsquid.com/giant-digi-comp-ii-recreates-mechanical-computer-from-1960s/

Here is the content of this issue.

S01 – Back to School Students are one of the largest group of HP calculator users and HP has great news for students – a special sale.

S02 – MCL kits School learning is more than ever "hands on" with integrated measurement, data recording, and analysis using HP calculators. HP's DataStream compatibility is now further improved.

S03 – Customization One of HP's greatest contributions to calculators is the feature of customization as exemplified by the HP-41 series of machines. This vital feature is an HP calculator first. You may also read about other HP calculator firsts at:

http://h20331.www2.hp.com/Hpsub/cache/392617-0-0-225-121.html

S04 – PROOT with Duplicate Roots by Namir Shammas who is a familiar author to ***HP Solve*** readers. He has written articles for ***HP Solve*** and given papers at HHCs on Solvers. The solver first appeared as a feature on any calculator with the 1973 introduction of HP-80. Namir examines the HP-50g command PROOT.

S05 – Repurposing the HP 20b/30b Calculator Platform by Jake Schwartz. As mentioned in the Customization article repurposing is a form of customizing your calculator. Jake provides an updated overview of this interesting user community repurposing project.

S06 – HPs Manuals Every user has an opinion on what a calculator manual should look like, how it should be organized, and what its content should be. This overview of HPs manual covers the entire history of what HP has done to provide the user with what he or she needs to use an HP calculator. A list of the features of an "ideal" manual is included with a cost perspective of such a manual using the PPC ROM Manual as an example.

S07 – #7 in Fundamentals of Applied Math Series – Average Mean Median & Mode A review of the math related to a general purpose calculator average function that uses a more complex operation, data analysis.

S08 – Regular Columns This is a collection of news items and repeating/regular columns.

- **From the editor.** This column provides feedback and commentary from the editor.
- **One Minute Marvels.** This OMM provides three routines to provide a rounding function for numbers. The method used by our calculators to round numbers is explained and the Astronomers Rule for statistically rounding is provided. A reader challenge is to write a program to statistically round.

S09 – How Fast is Fast? Calculator speed is a calculator technical attribute that is often discussed by users. As you might expect most serious users want more speed. This article discusses calculator speed examining "how fast is fast" and "how fast is fast enough." A table of typical calculator speeds is provided.

That is it for this issue. I hope you enjoy it. If not, tell me!

Also tell me what you liked, and what you would like to read about.

X < > Y, Richard Email me at: hpsolve@hp.com

HP 48 One Minute Marvel No. 11 – Rounding

One Minute Marvels are short, efficient, unusual, and fun HP 48 programs that may be entered into your machine in a minute or less. These programs were developed on the HP 48, but they will usually run on the HP 49 and HP 50 as well. Note that the byte count is for the program only.

Rounding is an important numerical practice and is used for just about every numerical problem you solve. HP's calculators provide display settings that will automatically round the answer to the display setting without changing the number itself. Here are three rounding programs by Joseph K. Horn that provide additional rounding scenarios. A second input is required in addition to the number being rounded. These special cases allow the rounding to be up or down as desired. The number being rounded is obj2 and the basis for the rounding is obj1.

1. Round obj2 "up" to a multiple of obj1: **'UP'** << SWAP OVER / CEIL * >>
   Examples: 153 25 **'UP'** ⇒ 175
   167 25 **'UP'** ⇒ 175
2. Round obj2 "down" to a multiple of obj1: **'DOWN'** << OVER SWAP MOD - >>
   Examples: 153 25 **'DOWN'** ⇒ 150
   167 25 **'DOWN'** ⇒ 150
3. Round obj2 to the "closest" multiple of obj1: **'NEAR'** << SWAP OVER / 0 RND * >>
   Examples: 153 25 **'NEAR'** ⇒ 150
   167 25 **'NEAR'** ⇒ 175

Rounding is an interesting statistical issue and the use of calculators with their "industry standard" rounding practices have "clouded" the issue of rounding. Suppose you need to calculate a financial value and the result is put into a document. The dollars and cents value will need to be rounded to the nearest cent. The rounded number needs to be to two decimal places, e.g. $ 245.735952341 becomes $ 245.74.

Rounding is a simple concept. Round the desired digit to the value that is closest to the longer value being rounded. If the digits after the rounded place are 1 – 4 they are dropped and if the digits are 6 – 9 the rounded place is increased by 1.

The issue is the middle digit of 5 following the rounded place digit. The vast majority of calculators in the world use the rule of adding 1 if the digits are 5 – 9. From a statistics perspective this means that statistically many rounded numbers are too high. I learned what is called the Astronomers Rule of rounding. You only round up one digit if the following digit is 5 if it will make the rounded number even.

In the cents rounding case 245.735…is rounded to 245.74 as given above, but if the number was 245.765… it would be rounded to 245.76 - not adding 1 - because "6" is already even and the following digits are dropped. Statistically the rounded numbers are balanced in the special "5" case.

Reader challenge: How would you write a rounding program to implement the statistically more accurate Astronomers Rule of rounding? Should calculators have this option?

How Fast is Fast?

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How Fast is Fast?

Richard J. Nelson

Introduction

One of the more powerful aspects of English is its ability to convey an idea without being specific. If you listen to advertisements, for example, and you really hear the words, you will be amazed at the ability of the language to convey an image without being exact or precise. What does a low price really mean?

Calculator users are technical people. They want to know and express ideas in very specific terms to avoid confusion and misunderstanding. The normal way of saying, “the small nut will not fit on the large screw” might be technically expressed as, “the 6-32 nut will not fit on an 8-32 screw.” In terms of an advertised price the relative positioning of one product with respect to another is instantly defined if a number is given.

Numbers are used for precision and included in the use of numbers is what calculator designers call dynamic range – the range of numbers that the calculator is able to use. The dynamic range for the first scientific calculator, the HP-35A in 1972, was described on the first text page, i, of the 36 page operating manual[1] as: “It will handle numbers as small as 10^{-99} and up to 10^{99} …”

If you have ever had to calculate the factorial of a number[2] – just for fun – you quickly discovered the dynamic range limit of your scientific or graphing calculator. The dynamic range of 10^{-99} to 10^{99} was considered more than adequate[3] when the HP-35A was introduced. The HP 35s extends this dynamic range as described in the User’s Guide: “The smallest number available on the calculator is -9.99999999999 x 10^{-499}, while the largest number is 9.99999999999 x 10^{499}.”[4]

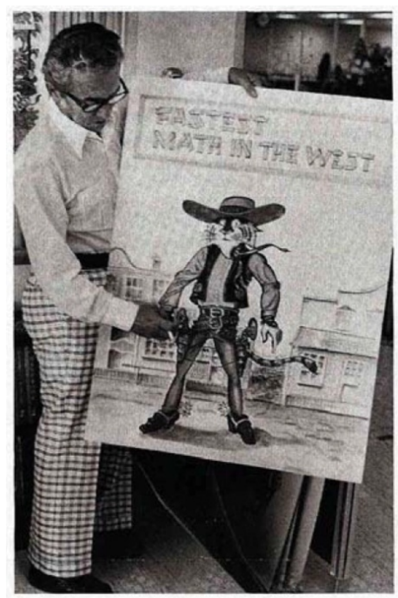
One of the lesser advertised aspects of HP’s calculators are that all functions work over the full dynamic range. Many competing machines have restrictions in their manuals that provide limits for various operations and functions so that even though the calculator may work with a specified dynamic range, the algorithms they use are not robust enough to always work – and be accurate/correct.

Speed numbers

Calculator speed was a major specification for the HP-35A and the early literature even listed the numbers: Addition, subtraction, multiplication, and division were 60 to 100 milliseconds. The slowest functions were Trigonometric functions at 500 milliseconds. Half a second is considered a noticeable amount of time for the user to wait.

The newer models are programmable and how fast the calculator runs becomes increasingly important as elaborate programs perform very complex operations and analysis.

The question is: How fast is fast?



Bob Schaeffer shows one of the early tiger posters extolling the virtues of the HP-35.

Fig. 1 – Calculator speed has been of concern since the very beginning.

The popular press has reported in the past that Bill Hewlett, the internal champion of HP Calculators, insisted that all operations or functions should take less than one second to provide an answer. The HP-35A cut that specification in half. The solver in HP's financial calculators uses an iterative method that may take a noticeable much longer amount of time to arrive at an answer. The question is, How fast is fast?

<u>How fast is fast enough?</u>

From a practical point of view perhaps the question should really be, "How fast is fast enough?" The business calculator user might realize that it would take him or her half an hour to solve the IRR problem by hand and that five seconds is fast enough. If you simply can't solve the problem any other way five seconds is certainly fast enough.

Calculator enthusiasts are similar to race car enthusiasts and the technical challenge of going ever faster is what makes the competition interesting. This also applies to all things electronic in which the smaller the circuits are the faster they respond. A faster response means that more is done in less time and as the old adage says, time is money.

The obsession for speed hasn't changed in the 39 years that personal handheld calculators have been in use. Many calculators are programmable – an essential feature for serious and very convenient problem solving – and using a common program to measure speed allows the technical user to compare machines.

The HP 30b calculator is one of HP's most recent technological advancements. There are three aspects that make it especially stand out: (1) it is fast, (2) it is customizable, and (3) it is repurposeable. All of these advancements are discussed in this issue. The first of these outstanding technological accomplishments are being discussed here by trying to answer: How fast is fast?

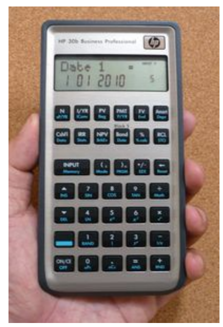


Fig. 2 – HP 30b.

If you are technologically interested you will want to know two things. How fast is the HP 30b, and how does it stack up with other machines?

The HP User Community has addressed this issue by publishing[(5)] benchmark speed numbers for a full gamut of machines. Table 1 provides a few of those HP calculator comparisons. When attempting to answer "How fast is fast?" it is very important to compare machines under similar conditions.

Many machines will run in several "modes" using various "languages." The two most common are user code and assembly code. Assembly code programs will almost always run faster than user code programs.

All machines in the HP Museum data base are solving the same problem called the N-Queens problem. Not all machines are able to run programs in a similar manner and the details including program listings are provided as well.

If you study the values in Table 1 you will notice that the 30b is 360 times faster than the 12C. As a point of interest the 30b repurposing capability will most likely produce a "version" or program that puts it near the bottom of the table. And, as Internet rumor has it, we haven't seen the last of the 15C either.

Table 1 – Typical HP Calculator Speed Comparisons

Model	Conditions	Speed (Time in decreasing order)	Ratio
HP-15C	User code	1:19:10 = 4,750 sec.	REF
HP-12C	User Code	55:52 = 3,352 sec.	1.42x
HP-41C	User Code	17:35 = 1,055 sec.	4.5x
HP 35s	User Code	2:49 = 169 sec.	28.1x
HP-50g	User RPL	1:07 = 67 sec.	70.9x
HP 30b	User Code	9.31 sec.	510x
HP-41CX	Assembly	11 sec.	431x
HP-50g	Assembly, Arm 9.75 MHz.	0.000404 sec. (404 µs.)	11,757,426x
HP-50g	Assembly, Fast mode 2.7x	0.000150 sec. (150 µs.)	31,666,667x

<u>Observations and Conclusions</u>

The word fast doesn't convey very much technical information. To say a 75.001 MHz clock is fast when the nearest competitor is 75.000 MHz is accurate, but not very informative. This is especially true when you consider that engineers consider a significant change as twice as much. Calculator users are technical people who need numeric values in order to reach conclusions. The first scientific calculator provided an answer in 500 ms. or less, and this was considered fast enough. Today's machines are orders of magnitude faster and the user cannot detect the difference between 50 ms. and 5 ms. in terms of the answer calculation time.

Programmable calculators, however, need to be much faster because complex programs demand it. Data analysis involving large data sets are common problems being worked on by today's programmable calculators. The HP 30b is one of HP's fastest calculators available and the reader could argue that it is fast enough. Because it is programmable, however, it cannot (ever) be fast enough.

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(1) The HP-35A was called a pocket calculator. A shirt pocket was implied and it was intended to replace a slide rule, the only other computational device that average person could hope town and use on a personal basis. The <u>operating manual</u> (all lower case letters were used by HP) was also a shirt pocket size of 3-11/16" wide, 6" high, and very thin. I checked the shirt I was wearing while writing this and it fits.

(2) Calculating the factorial (!) of a number is a problem (function) that challenges the calculator for accuracy, dynamic range, and speed. The factorial of a number, n, is the product of each integer from n to 1, e.g. 9! = 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 362,880. A single digit input produces a six digit answer.

(3) One HP internal publication describing the HP-35A (pocket calculator news'n notes) article used the example of calculating the number of cubic inches of a "Cosmic Cube" a million light years on an edge to illustrate that the dynamic range of 10^{-99} *to* 10^{99} *covered all practical calculations. Today the calculator is used as much for analysis as for problem solving and an even wider dynamic range is desirable.*

(4) There appears to be an error in the HP25s User's guide in that the negative number is missing a "-" sign for the smallest number exponent in several places- 4-15, 4-16.

(5) See the HP Museum site for the data (as of February 2007).
http://www.hpmuseum.org/cgi-sys/cgiwrap/hpmuseum/articles.cgi?read=700