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51. Mass Balance Module

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51.1. Where do we need mass balancing?

Mass balancing is a common practice in metallurgy. The mass balance of a circuit is needed for several reasons:

1. To estimate the metallurgical performance of the circuit
2. To locate process bottlenecks and for circuit diagnosis
3. To create models of the processing stages
4. To simulate the process.

The following steps are often required to simulate a process:

1. Collecting experimental data (experimental work, sampling, sample preparation, assaying)
2. Mass balancing and data reconciliation of the experimental data
3. Model building
4. Simulation.

In HSC Chemistry® 8, you can do all the steps in one program: *HSC Sim* with the *Mass Balance* tool. The work flow in the HSC Sim and Mass Balance tool starts from a flowsheet drawing, followed by importing experimental data, performing mass balancing, model fitting & building (model fitting will be available later on) and simulation (**Fig. 1**).
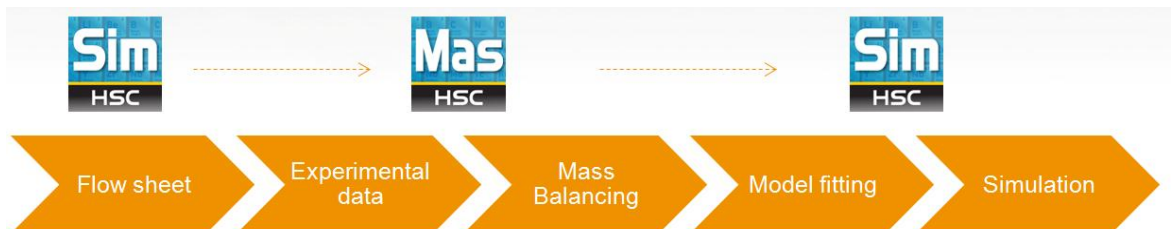

Fig. 1. From flowsheet with data through mass balancing to modeling and simulation.

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51.2. Mass balancing capabilities in HSC Chemistry® 8

HSC 8 allows the user to solve the following mass balance problems (**Table 1**). There are three different possibilities for the solution:

- Solids
- Assays only
- Solids and water independently

Liquid flow rates and %solids are solved only if Solve solids and water independently is chosen. If Assays only is chosen, there must be a Total solids measurements for all the flows to be balanced.

1. Unsized
    - Balance Total Solids flow rates. At least one total solids flow rate measurement must be given
    - Balance Total Water/liquid flow rate. This is done by first balancing the Total solids flow rates and after that the Total Water/liquid flow rate independently
    - Balance %solids
    - Balance Solids component assays/Mineral assay. At least one solids component assay or mineral assay must exist

2. Sized
    - Balance Total Solids flow rates for bulk and all the size fractions. At least one total solids flow rate measurement for bulk must be given
    - Balance size fraction wt-% for size fractions
    - Balance Total Water/liquid flow rate for bulk. This is done by first balancing the Total solids flow rates and after that Total Water/liquid flow rate independently
    - Balance %solids for bulk
    - Balance Solids component assays/Mineral assay for bulk. If no solids component assays/mineral assays exist, the solids component assays/mineral assays for bulk are not balanced

3. Sized by assay
    - Balance Total Solids/Slurry flow rates for bulk and all the size fractions. At least one total solids/slurry flow rate measurement must be given
    - Balance size fraction wt-% for size fractions
    - Balance Total Water/liquid flow rate for bulk. This is done by first balancing the Total solids flow rates and after that Total Water/liquid flow rate independently
    - Balance %solids for bulk
    - Balance Solids component assays/Mineral assay for bulk and the size fractions

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Table 1. Mass balance cases that can be solved with HSC Sim.

Measured or estimated values	Unsized Components	Sized	Sized by Assays
Total solids flow rates	X	X	X
Water/Liquid flow rates	X	X	X
Solids component Assay/Mineral Assay bulk	X	X	X
Solid Component Assays/Mineral assays Size Fractions	X		X
%Solids	X	X	X
Fraction m%		X	X
Total Solids Flow rate size fractions		X	X

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51.3. Overview of the HSC Sim 8 Mass Balancing tool

The HSC Mass Balance tool is started from the HSC 8 Main Menu dialog or from HSC Sim Menu: Tools → Mass Balance. The window layout consists of: Balancing Navigator, Working Area, Property Panel, and Upper Buttons.
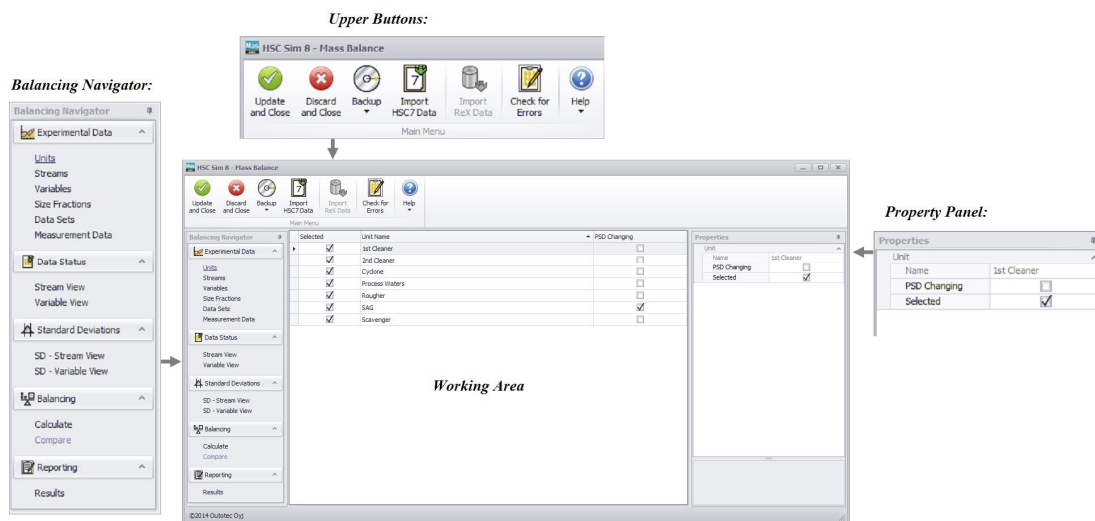

Fig. 2. Main components of the Mass Balance window.

51.4.   Step-by-step example: data reconciliation and mass balancing in HSC 8

This step-by-step example shows how to do mass balancing for unsized data, consisting of the solids flow rates and assayed element (Cu, Ni, S) for a rougher-scavenger flotation bank. The work starts from HSC Sim 8 by drawing the process flowsheet. The Mass Balance tool is started from the HSC Sim Tools menu. The steps to import the data and to balance it follow the left side Balancing Navigator panel (**Fig. 2**) from top to bottom. More examples are found in Chapter 52.

51.4.1.   Drawing the flowsheet

First the flowsheet is prepared in HSC Sim 8. In this rougher-scavenger flotation example it looks as shown in **Fig. 3**.

You can draw all the streams and units. HSC will create the mass balance equations according to the available data; therefore, there is no need to draw a flowsheet for mass balancing only or a new flowsheet every time for different kinds of mass balance problems.

When naming the streams, use identical names to those in your analysis lists. Before proceeding, please check the stream connections and check the flowsheet for possible errors.
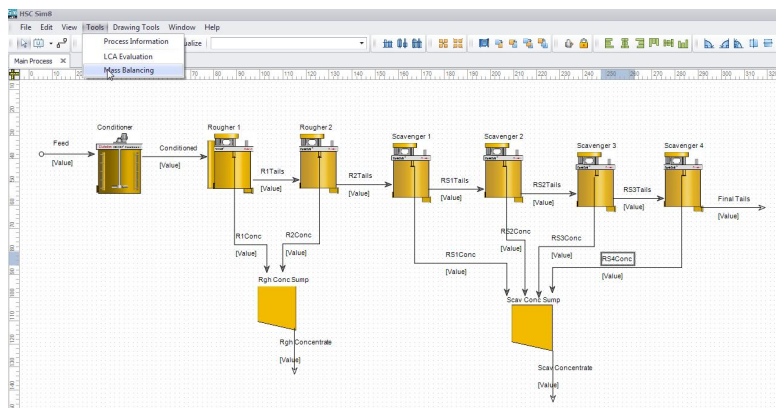

Fig. 3. HSC Sim flowsheet drawing of a flotation process with rougher and scavenger cells.

51.4.2.   Importing the experimental data

When the flowsheet is ready (i.e. all streams are named properly and connections have been checked), you can import your experimental data. The following subsections will concentrate on how to import the data for mass balancing and data reconciliation.

1.   Select units

The units are listed based on the flowsheet drawing figure, but they cannot be edited here. In this view you can:

- Select or deselect the units to be included in the balancing calculation.
- Set whether the unit will change the particle size distribution of the solids, e.g. grinding mills. This selection is needed in sized balancing, to indicate that the fraction balance will not be held over those units.

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Fig. 4. Experimental Data – Units.

2.   Select streams

The streams are listed based on the flowsheet drawing figure, but they cannot be edited here.

Here you can:

- Select and deselect the streams to be included in the balancing calculation.
- Change the stream type:
  - o   Unknown
  - o   Solids/Slurry
  - o   Liquid/Water

By default it is unknown, since the data has not yet been imported. When importing the data, the type is detected by pressing Detect stream types, but it can be changed here.

Fig. 5. Experimental Data – Streams.

3.   Add variables

Next, the measured variables are added / removed from the upper bar buttons. The variable name and unit on the list can be edited, and the variable can also be unselected from balancing if desired. The variable types are listed in **Table 2**.
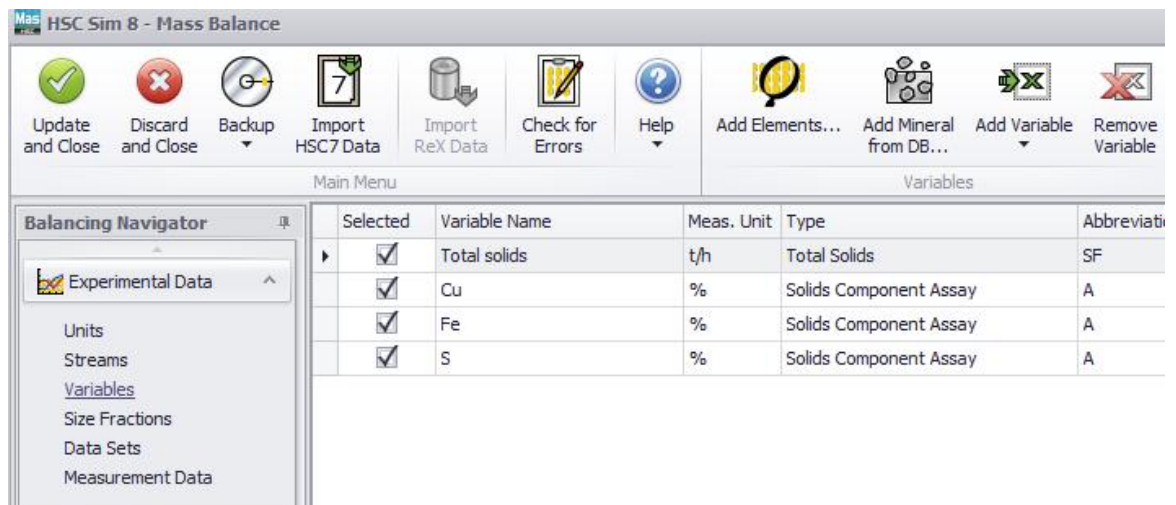
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Fig. 6. Experimental Data – Variables.

Table 2. Data types of the variables.

Data type	Abbreviation	Examples
Total solids flow rate	SF	Total t/h, g
Solids component flow rate	SC	Iron t/h, plastic t/h, g
Solids component assay	A	Cu%, P2O5%
Mineral assay	M	Ccp%, Py%, Qtz%
Size Fraction wt %	SA	0-20um %, 20-45um %
Solids percentage	SP	35%
Not included	NA	Column with comments, extra data (temperature), etc.

4.   Add size fractions

By default, one size fraction exists: 'Bulk', which cannot be removed. More size fractions can be added to /removed from the list by clicking the buttons on the upper bar. Fraction names can be given.
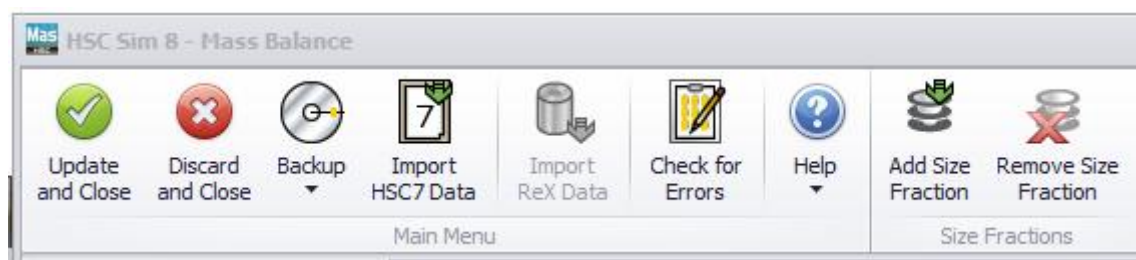

Fig. 7. Experimental Data – Size Fractions.

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5. Add dataset(s)

By default, one dataset exists, which can be renamed here. In addition, more datasets can be added and selected one at a time to carry out data reconciliation.
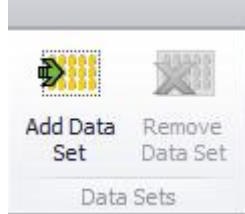

Fig. 8. Experimental Data – Datasets.

6. Import measurement data

In this view, a stream variable template is automatically generated and displayed. The data can be entered by:

a) Typing manually in the table

b) Copying an empty data template → organizing the data e.g. in Excel → select 'Paste Experimental Data'. This will automatically place the data in the correct rows and columns based on the clipboard table content. Note: if the data is horizontal (stream names in rows) there must exist a column labelled Fraction that contains fraction names. If the data is vertical (stream names in columns) there must exist a row labelled Fraction containing the fraction names.

c) Importing the old HSC7 Analyses.xls mass balance file

Each dataset is presented in a separate table tab.

Fig. 9. Experimental Data – Measurement Data.

51.4.3. Reviewing and complementing the data

In this part, the idea is to inspect the data and get an understanding of what values will be available after balancing. Also, the data status before balancing is reviewed here and can be changed. Thus changes are reflected in the status after balancing.

The status indications are:

- Stream:
  - Missing: no data
  - Complete: all variables have data
  - Partial: some variables have data

- Variable:
  - Missing: no data
  - Measured: data existing
  - Guesstimated: data are a user-given guesstimation; high uncertainty is set automatically for this The guesstimation is given in the property **User value**
  - Fixed: the user sets including the value that you do not wish to change during the balancing. However, the fixed value may change during balancing because the fixing is done by setting the uncertainty to small. The fixed value is given in the property **User value**
  - By Equation: stream total solids can be set to be a multiple of another stream
  - Excluded: data will not be included in the calculation

- After Balancing:
  - Balanced: solved using data reconciliation
  - Calculated: calculated based on unit material balance
  - Non Available: data are also missing after balancing

The status indications can be changed by clicking the data table cell (dropdown menu) or from the property panel on the right.

The balance equations for the units can be reviewed by clicking the upper bar button shown.
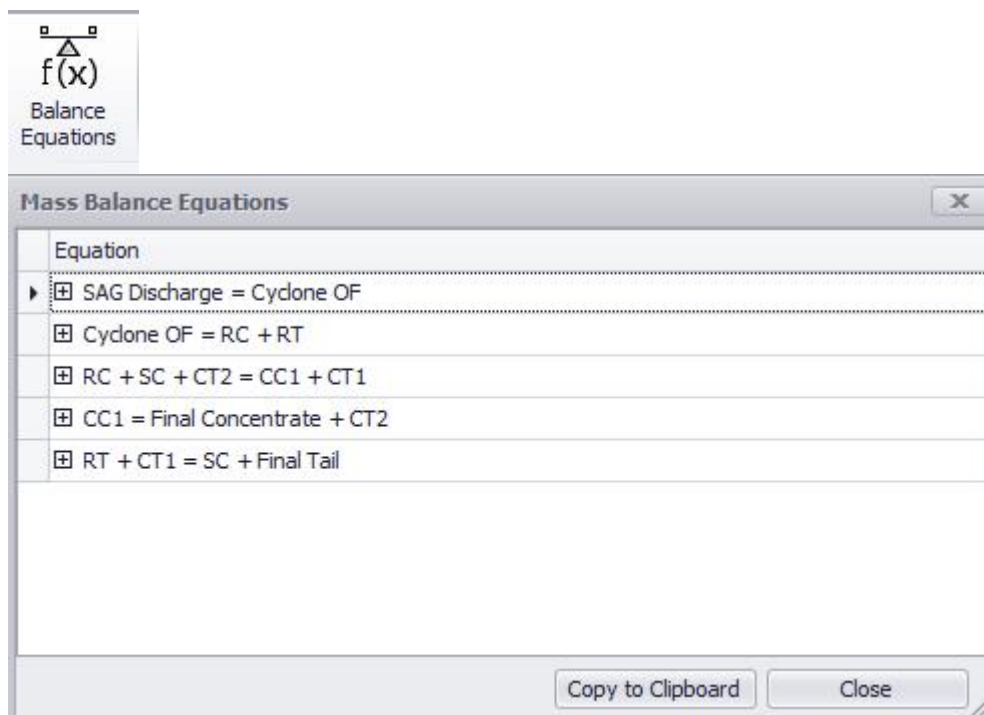

Fig. 10. Opening and reviewing the balance equations.

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1.   View by streams

The data are presented in a pivot table where the <u>stream</u> is the major column. The data status is shown in pie charts.
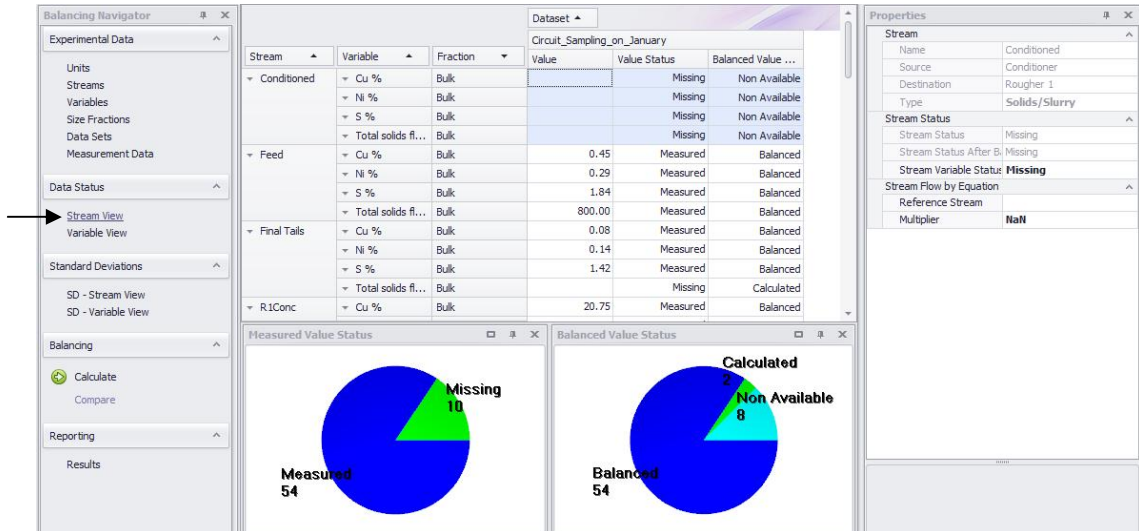

Fig. 11. Data Status – Stream View

2.   View by variables

The data is presented in a pivot table where the <u>variable</u> is the major column. The data status is shown in pie charts.
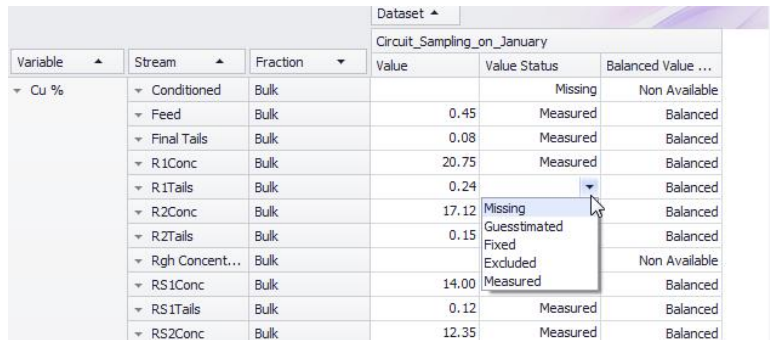

Fig. 12. Data Status – Stream View.

51.4.4.   Setting the measurement accuracies

Each assay and piece of raw data is subject to errors. Mass balancing and data reconciliation is meant for adjusting unreliable values, whereas reliable values should be adjusted only a little, if at all. Therefore, the user has to give a value of how reliable each item of raw data is. This is done by defining the error model to give a <u>standard deviation</u> value for the measurement data.

In the HSC Sim 8 Mass Balance tool, the standard deviation can be given using user-friendly pre-settings for the error models.

To define the standard deviations you need to select:

Table 3. Descriptions of error model functions.

Error Model	Parameters	Example
Fixed	Fixed standard deviation as a plain number or fixed relative standard deviation as a negative number or in parentheses	0.1 -5, (5)
X.x = X%+0.x	The integer is the relative standard deviation; the decimal is the detection limit	10.1
X%+0.1	Given parameter is the relative standard deviation; 0.1 is the detection limit	6
10%+X	Parameter is the detection limit; relative standard deviation is fixed at 10%	0.05
a;b;c = MIN(a%+b;c)	a is the relative standard deviation, b is the detection limit, and c is the maximum standard deviation	10;0.01;0.5
a;b;c=MAX(a%+b;c)	a is the relative standard deviation, b is the detection limit, and c is the minimum standard deviation	10;0.02;0.2
a;b;c=%;MIN;MAX	a is the relative standard deviation, MIN is the minimum standard deviation, and MAX is the maximum standard deviation	10;
Manual	Standard deviation is given in the data individually for each stream	

1.   Standard deviations by streams

The data are presented in a pivot table where the <u>stream</u> is the major column. Standard deviation settings of all variables for that stream can be easily set at once.
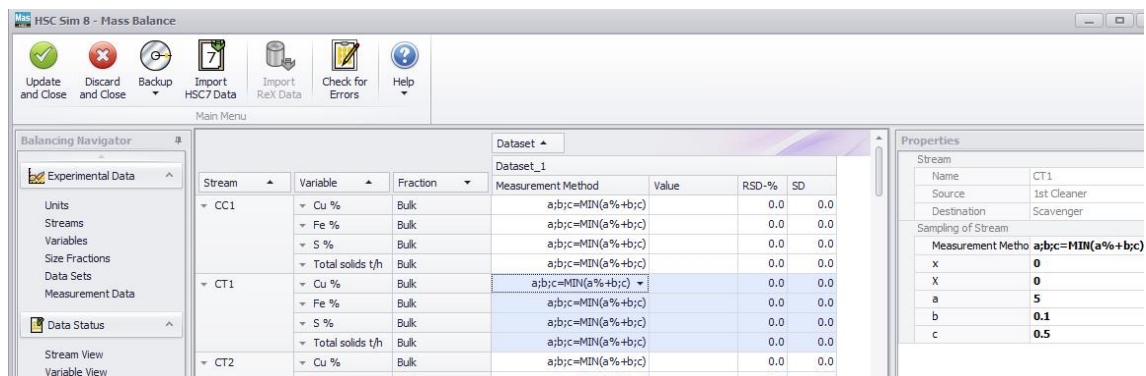

Fig. 13. Standard Deviations – SD – Stream View.

2.   Standard deviations by variables

The data are presented in a pivot table where the <u>variable</u> is the major column. Standard deviation settings of all streams for that variable can be easily set at once.
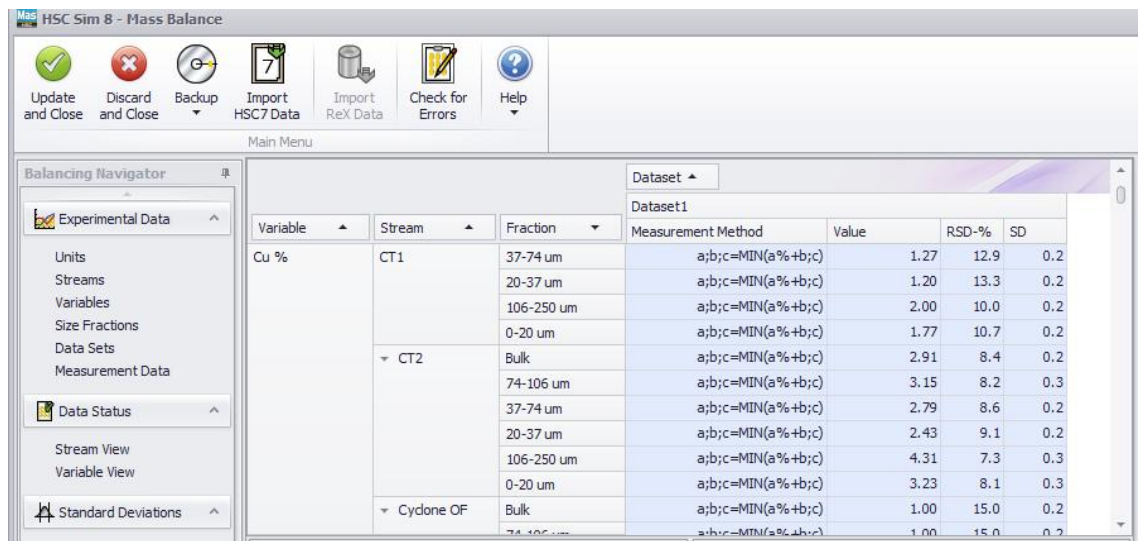
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Fig. 14. Standard Deviations – SD – Variable View.

51.4.5. Mass balancing

In the mass balancing upper bar, the following can be selected:

- Sum = 100: if the mineral component sum is required to be 100%.
- Method: LS, NNLS, CLS
- Data to Balance: Assays only, Solids and assays, Solids and water
- PSD Balance: Unsized, Sized, Sized by Assay
- Calculate: runs the data reconciliation

The balancing results can be viewed graphically with, see **Fig. 15**:

- Balance Convergence
- Parity Chart

To solve a mass balance problem, the following mathematical methods are available in the Balance/Report Options on the right-hand side:

- Least Squares Solution (LS)
- Non-negative Least Squares Solution (NNLS)
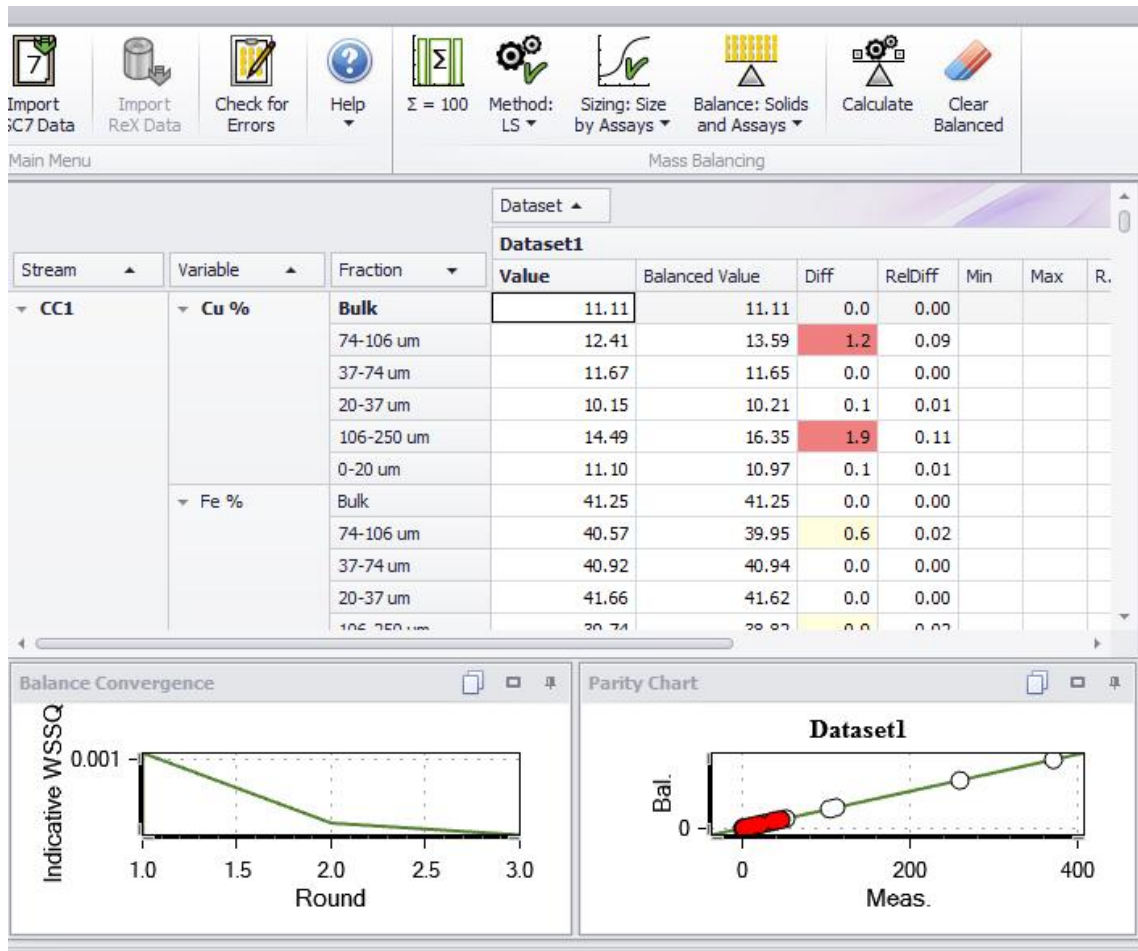- Constrained Least Squares Solution (CLS)

Fig. 15. Balancing – Calculate.

Mass balance problems are solved in two stages: firstly, the total mass flow rates are solved and then the assays are reconciled. In solving the assays, the least squares solution finds the best solution by minimizing the weighted sum of squares, i.e.

$$WSSQ = \sum_{j=1}^{k} \sum_{i=1}^{n} \frac{(a_{ij} - b_{ij})^2}{s_{ij}^2}$$ (1)

where j refers to the stream, k is the number of streams, i refers to the components (analyses), n is the number of components, *a* is the measured value, *b* is the balanced value, and s is the standard deviation.

In non-negative least squares, all '*a*'s are subject to being non-negative.

In constrained least squares, all '*a*'s are subject to being between the min. and max.

By clicking dataset you can see the solution parameters Balance tolerance, Max iter and Estimate of null SD in the properties window. Balance tolerance is the condition that defines when the iterations stop and Max iter is the maximum number of iterations. If you don't get reasonable balance you can try to change Estimate of null SD greater.

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51.4.6.   Reporting and reviewing the results

Fig. 16. Stream Summary

	A	B	C	D	E	F
1	Variable	WSSQ	DiffTot	RelDif	AVG_SD	AVG_RSD
2	Total Solids t/h	0.000	0.000	0.000	0.500	0.002
3	Cu %	0.045	0.037	0.136	0.250	0.028
4	Fe %	6.681	1.556	6.429	0.450	0.056
5	S %	0.102	0.222	0.401	0.490	0.027
6						
7	Stream	Sum WSSQ	DiffSum	RelDiffSum	RelDiffAvg	
8	ROM	4.044	0.907	0.393	0.007	
9	SAG Discharge	0.000	0.000			
10	Cyclone UF	0.000	0.000			
11	Cyclone OF	0.000	0.000			
12	RC	0.000	0.000			
13	CC1	0.000	0.000			
14	Final Concentrate	0.002	0.026	0.033	0.019	
15	RT	0.000	0.000			
16	SC	0.000	0.000			
17	Final Tail	2.781	0.882	6.354	0.070	
18	CT2	0.000	0.000			
19	CT1	0.000	0.000			
20	Mill Water	0.000	0.000			
21	Mill Sump Water	0.000	0.000			

Fig. 17. Reporting – Results: Goodness.

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	A	B	C	D	E	F	G
1	Units	Equations	Stream	Total Solids	Cu %	Fe %	S %
2	SAG + Cyclo	ROM = Final Concentrate + Final Tail					
3		Inputs	ROM	215.000	1.874	14.142	17.653
4		Outputs	Final Conce	6.076	1.586	0.679	2.418
5			Final Tail	208.924	0.288	13.463	15.235
6		Balance		0.00E+00	8.73E-13	-3.62E-11	-5.13E-12
7							

Fig. 18. Reporting – Unit Balance.

51.4.7. Importing HSC7 Excel files

The Excel files to be imported may contain a sheet with flowsheet information (**Fig. 19**). If this sheet is named Flowsheet, the program automatically detects the sheet where the flowsheet information is located. If the name is something else or there is no sheet containing flowsheet information, the name must be specified (**Fig. 23**, Select Flowsheet)

Fig. 19. An Excel sheet containing flowsheet data.

The measurement data is read from a different sheet of the same Excel file. The first column must be named Streams (Cell A1), the second Source (Cell A2), and the third Destination (cell A3) if the data are horizontal (**Fig. 21**). If the data are vertical, the first row must be named Streams (A1), the second row must be named Source (A2), and the third Destination (A3) (**Fig. 20**). Imported vertical data can only be unsized or sized (without analyses).

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Fig. 20. Vertical sized data (without analyses).

Fig. 21. Horizontal unsized data.

The program automatically detects a sheet named Streams as the sheet where the measurement data can be found. If the name of the sheet containing the measurement data is not Streams, the user should tell the program where the measurement data can be found (**Fig. 23**, Analyses)

The following screenshots show how an HSC7 file is imported. Importing an HSC7 file is started by clicking Tools-Mass Balancing and then clicking the Import HSC7 Data button (**Fig. 22**). After that a dialog will open (**Fig. 23**).
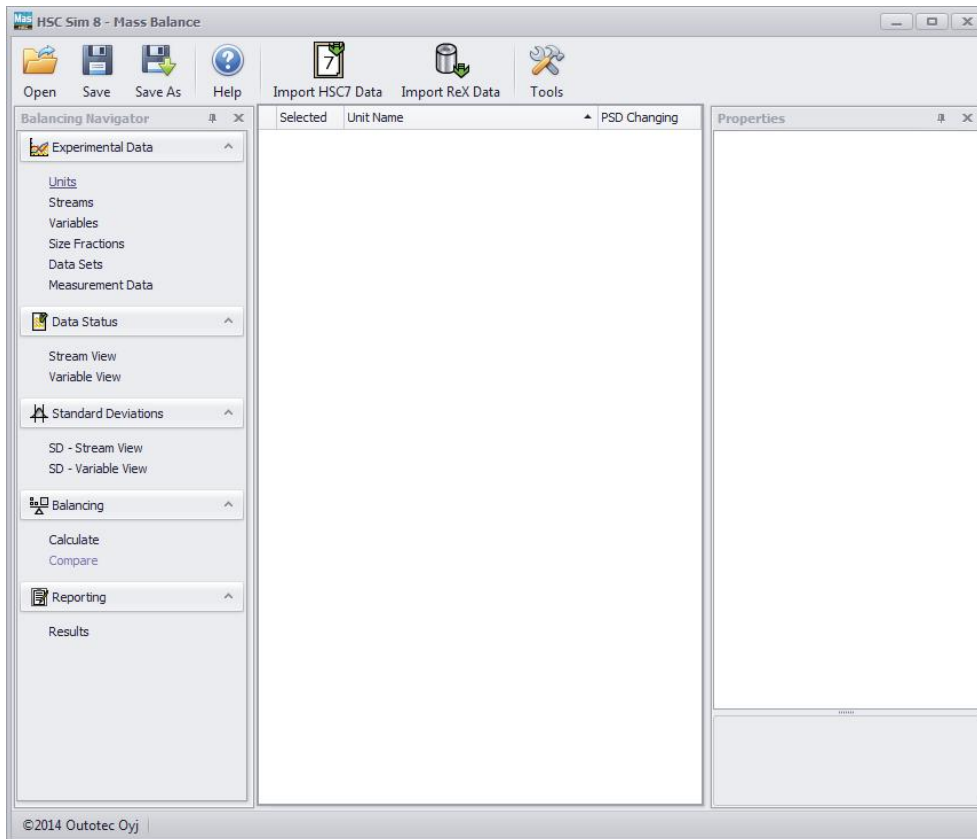
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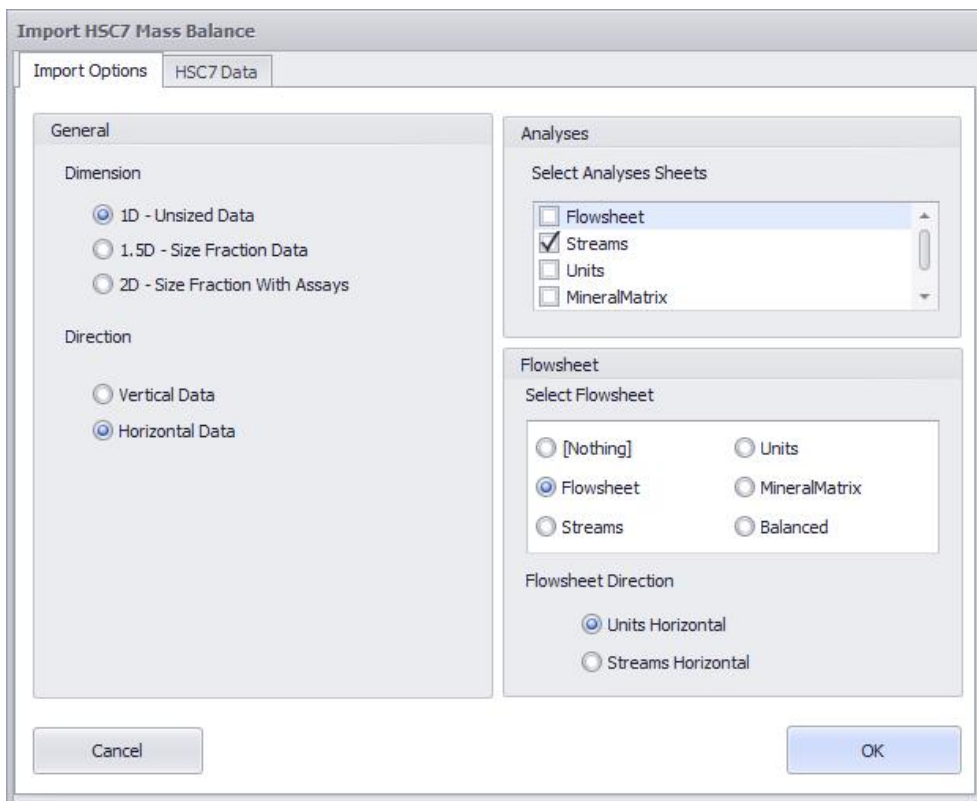January 27, 2015

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Fig. 22. Import HSC7 Data.

Fig. 23. Import HSC7 Data Dialog.

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51.5. Mass balance buttons and dropdowns

Update and Close	Update and save changes to HSC Sim flowsheet and close
Discard and Close	Discard changes and close HSC mass balance
Backup	Open/Save backup (HSCMas file) of the mass balance
Import HSC7 Data	Import existing HSC7 mass balance data (analyses.xls)
Check for Errors	Error check
Help	Help and examples
Detect Stream Types	Detects stream types automatically for Solid/slurry and Liquid based on the data. If there are streams with no data the stream type is set as Unknown. The user should either uncheck these Unknown streams or set the types manually
Add Elements...	Select elements from periodic table
Add Mineral from DB...	Select minerals from HSC database
Add Variable	Add new variable
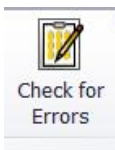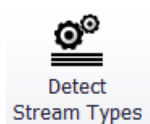
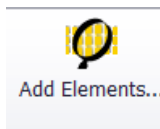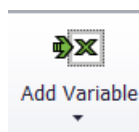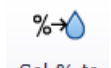|	
Remove Variable	Remove the selected variable
Add Size Fraction	Add a new sieve size
Add Data Set	Add a new data set
Remove Data Set	Remove the selected data set
Flip Table	Transpose streams and variables
Copy	Copy the whole table
Paste Experimental Data	Paste data to correct places based on the stream and variable names
Element to Mineral...	Element to mineral conversion using HSC Geo
View Mineral Matrix	View minerals and their compositions
Sol-% to Water	Converts solids percentage measurements to water flow rates
Back Calculate Missing Values	Back calculate missing data if possible
Clear Back Calculated Values	Clear back calculated missing data
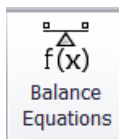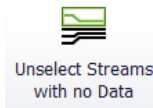
Icon	Description
Balance Equations	View the mass balance equations
Unselect Streams with no Data	Unselect the streams without data
Exclude Elements	Exclude all elements (A) and do balancing with minerals (M)
Σ = 100	Set component sum=100 when balancing minerals
Method: LS ▾	Select calculation method. The available methods are LS, NNLS, and CLS
Sizing: Unsized ▾	PSD balance: Possible selections are Unsized, Sized and Sized by Assays
Balance: Solids and Assays ▾	Select the data to be balanced
Calculate	Calculates mass balance
Clear Balanced	Clears the mass balancing results
Stream Tables ▾	Create or update HSC Sim stream tables
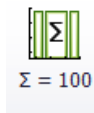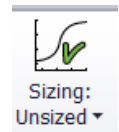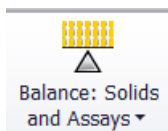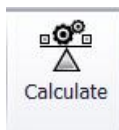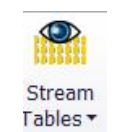
Outotec

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51.6.   Error check messages

1. Dataset: No Active dataset found
   - Please contact the developers

2. Units: No selected units found
   - At least one unit should be selected

3. Streams: No selected streams found
   - Some of the streams should be selected

4. Units: No input stream
   - Found a unit with no input streams

5. Units: No output stream
   - Found a unit with no output streams

6. Streams: Source and destination missing
   - Found a stream with no source and destination

7. Fractions: Bulk not found
   - Please contact the developers

8. Data: No data found
   - Add data in the measurement data view

9. Variables: Total solids measurement missing
   - There should be exactly one selected variable of the Total Solids type

10. Variables: Too many variables of the type total solids
    - There should be exactly one selected variable of the Total Solids type

11. Variables: No selected assays found
    - There should be at least one selected variable of the Mineral Assay or Solids Component Assay type

12. Streams: All stream types unknown
    - Go to Streams and press Detect Stream Types button or set the stream types manually

13. User Values: UserValues missing
    - ValueStatus of some data is set to Fixed or Questimated but the UserValue is not given

14. Stream Types: Unknown found
    - Go to Streams and press Detect Stream Types button or set the stream types manually

15. Variables: Size Fraction wt % variable missing
    - There should be exactly one variable of the type Size Fraction wt %

16. Variables: Too many variables of the type Size Fraction wt-%
    - There should be exactly one variable of the type Size Fraction wt %

17. Units: No combined units found
    - There can be several reasons why combined units cannot be formed. There may be too many missing measurements or there may be errors in the flowsheet.

18. Notification: Min or Max values detected: It is recommended to use the CLS method

51.7. Mathematics and algorithms

In this section, the main algorithms and methods for data reconciliation in HSC Chemistry®
8 are briefly summarized.

Definitions

N_{F1}	=	number of flows, unsized
N_{F2}	=	number of flows, sized, sized by assay
N_U	=	number of units
N_{SRU}	=	number of size-reducing units (PSD changing unit)
N_{SF}	=	number of sub-flows
N_E	=	number of chemical elements
G	=	grade of chemical element
G^M	=	measured grade of chemical element
F	=	solids flow rate
F^M	=	measured solids flow rate
W	=	water flow rate
W^M	=	measured water flow rate
Pc	=	%solids = $F*100/(F+W)$
Pc^M	=	measured %solids
M	=	fraction m% = $F_{flow,subflow}*100/F_{flow}$
M^M	=	fraction m% measurements

$$e_{flow}^{unit} = \begin{cases} 1, & the\ flow\ enters\ the\ unit \\ -1, & the\ flow\ exits\ the\ unit \\ 0, & otherwise \end{cases}$$

51.7.1. Unsized (bulk) mass balance

In the *unsized (bulk)* mass balance solution, solids and water <u>flow rates</u> are solved first.
After that the bulk <u>analyses</u> are solved. During the calculation: 1) the solids flow rates are
solved first and after that 2) the water flow rates are solved.

1. Bulk flow rates

The equations for solving the bulk solids flow rate are:

mass balance Equations (2)

$$\sum_{flow=1}^{N_{F1}} e_{flow}^{unit} * F_{flow,tot} = 0 \qquad unit = 1,..,N_U \qquad (2)$$

analyses Equation (3)

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$$\sum_{flow=1}^{N_{F1}} e_{flow}^{unit} * G_{flow,tot,chemical\_element}^{M} * F_{flow,tot} \approx 0 \quad unit = 1,...,N_U$$

(3)

$$chemical\_element = 1,...,N_E$$

solids flow rate measurements (4)

$$F_{flow,tot} \approx F_{flow,tot}^{M} \qquad flow = 1,...,N_{F1}$$

(4)

The equations for solving the bulk water flow rate are:

mass balance Equations (5)

$$\sum_{flow=1}^{N_{F1}} e_{flow}^{unit} * W_{flow,tot} = 0 \qquad unit = 1,..,N_U$$

(5)

water flow rate measurements Equation (6)

$$W_{flow,tot} \approx W_{flow,tot}^{M} \qquad flow = 1,..,N_{F1}$$

(6)

% solids measurements (7)

$$(Pc^M/100) * W_{flow,tot} \approx F_{flow,tot} - (Pc^M/100) * F_{flow,tot} \qquad flow = 1,..,N_{F1}$$

(7)

Mass balance Equations (2) and (5) are the equality constraints for the solutions. The solution method used is element-wise weighted total least squares[2]. The weights are standard deviations of the solids flow rate measurements F^M, the analyses G^M, the water flow rate measurements W^M, and the %solids measurements Pc^M. The flow rates can be solved without any constraints (LS), subject to non-negativity constraints (NNLS)[3] ,and subject to simple bounds

$lb1 \le F \le ub1$

$lb2 \le W \le ub2$

(CLS)[4].

2.   Bulk analyses

Let G and G_M be N_{F1} x N_E matrices. The operator *vec* stacks the matrix columns into a vector.

The equations for the analyses solution are:

measurements (8)

$$vec(G) \approx vec(G^M)$$

(8)

mass balance Equations (9) for the analyses:

$$vec(B * G) = 0$$

(9)

where the N_U x N_{F1} matrix B is defined:

$$B_{unit,flow} = e_{unit}^{flow} * F_{flow,tot} \tag{10}$$

If the option minerals sum=100 is selected, the following Equations (11) are included:

$$\sum_{chemicalelement=1}^{N_E} G_{flow,tot,chemical\_element} = 100 \qquad flow = 1,...,N_{F1} \tag{11}$$

Equations (9) and (11) are the equality constraints for the solution. The solution method used is weighted least squares 1). If Equations (11) are included in Equations (9), the matrix B is $(N_U - 1) \times N_{F1}$ to avoid linear dependency of equality constraints.

As before, the analyses can be solved without any constraints (LS), subject to non-negativity constraints (NNLS), and subject to simple bounds (CLS). If there are no constraints, the minimal maximum norm solution can be calculated (LS, MinMax)[5].

51.7.2. Sized mass balance (without sized analyses)

Sized mass balance differs from *unsized (bulk)* solution in that the fraction m% is solved and the fraction m% measurement is used in the solution of total flows. *Sized* differs from *Size by Assay* in that the analyses are not given and the number of flows is the same as in the *unsized* case.

1.   Total flow mass balances of the streams

Firstly, the total flows are solved. The Equations are:

mass balance Equations (12)

$$\sum_{flow=1}^{N_{F1}} e_{flow}^{unit} * F_{flow,tot} = 0 \qquad unit = 1,..,N_U \tag{12}$$

fraction m% measurements (13)

$$\sum_{flow=1}^{N_{F1}} e_{flow}^{unit} * M_{flow,subflow}^{M} * F_{flow,tot} \approx 0 \qquad unit = 1,...,N_U - N_{SRU} - 1 \tag{13}$$

$$subflow = 1,..,N_{SF}$$

Units are indexed up to $N_U - N_{SRU} - 1$ to avoid linear dependency of equality constraints.

Flow measurements (14)

$$F_{flow,tot} \approx F_{flow,tot}^{M} \qquad flow = 1,...,N_{F1} \tag{14}$$

If 1D analyses are given, the following Equations are included:

analyses (15)

$$\sum_{flow=1}^{N_{F1}} e_{flow}^{unit} * G_{flow,tot,chemical\_element}^{M} * F_{flow,tot} \approx 0 \quad unit = 1,...,N_U \tag{15}$$

$$chemical\_element = 1,...,N_E$$

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Equations (12) are equality constraints for the solution. The solution method used is element-wise weighted total least squares. The weights are standard deviations of the solids flow rate measurements F^M, the analyses G^M and fraction m% M^M.

As before, the flow rates can be solved without any constraints (LS), subject to non-negativity constraints (NNLS), and subject to simple bounds (CLS). If there are no constraints, the minimal maximum norm solution can be calculated (LS, MinMax)[5].

2. Size fraction flow mass balance

Then the %m values of the sub-flows are solved. The equations are:

fraction m% measurements (16)

$$M_{flow,subflow} \approx M^M_{flow,subflow} \tag{16}$$

mass balance equations (17)

$$\sum_{flow=1}^{N_{F1}} e^{unit}_{flow} * M_{flow,subflow} * F_{flow,tot} = 0 \qquad unit = 1,...,N_U \quad unit\ not\ size\ reducing \tag{17}$$
$$subflow = 1,..,N_{SF}$$

sum fraction m% is a hundred (18)

$$\sum_{i=1}^{N_{SF}} M_{flow,subflow} = 100 \tag{18}$$

Equations (17) and (18) are equality constraints. The solution method used is element-wise total least squares and %m values can be solved without any constraints (LS), subject to non-negativity constraints (NNLS), and subject to simple bounds (CLS).

3. Bulk analyses

If the unsized bulk analyses are given, the balanced analyses are calculated as described above (see: unsized mass balance).

51.7.3. Sized by assay mass balance

Before a *sized by assay* mass balance solution, the *unsized* or *sized* mass balance must be solved first. The results $F_{flow,tot}$ of the *unsized* or *sized* solution are used in the *sized by assay* solution. In a *sized by assay* solution, the size fraction sub-flows are calculated first and after that the analyses are solved.

1. Sized by assay fraction sub-flows

The equations for size by assay sub-flow solutions are:

mass balance Equations (19) for each unit that is not size reducing

$$\sum_{flow=1}^{N_{F2}} e^{unit}_{flow} * F_{flow,subflow} = 0 \qquad unit = n_1,...,n_{NU-1} \quad unit\ not\ size\ reducing \tag{19}$$
$$subflow = 1,..,N_S$$

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Above n_i is the index of the ith unit that is not size reducing. Units are indexed up to N_U - 1 to avoid linear dependency of equality constraints.

Sum of sub-flows is the total flow (20)

$$\sum_{subflow=1}^{NS} F_{flow,subflow} = F_{flow,tot} \qquad flow = 1,...,N_{F2}$$ (20)

Analyses for each unit that are not size reducing (21)

$$\sum_{flow=1}^{N_{F2}} e_{flow}^{unit} * G_{flow,chemical\_element,sublow} * F_{flow,subflow} \approx 0$$

$$unit = n_1,...,n_{NU} \quad unit \ not \ size \ reducing$$ (21)
$$chemical\_element = 1,...,N_E$$
$$subflow = 1,..,N_S$$

Fraction %m measurements (22)

$$F_{flow,subflow} \approx M_{flow,subflow}^{M} * F_{flow,tot} / 100$$
$$flow = 1,...,N_{F2}$$ (22)
$$subflow = 1,..,N_S$$

or alternatively

flow measurements (23)

$$F_{flow,subflow} \approx F_{flow,subflow}^{M}$$
$$flow = 1,...,N_{F2}$$ (23)
$$subflow = 1,...,N_S$$

The solution method used is element-wise weighted least squares. The equations can be solved without any constraints (LS), subject to non-negativity constraints (NNLS), and subject to simple bounds (CLS).

2.   Sized by assay fraction analyses

Let G and G^M be N_{F2*NS} x N_E matrices. The operator *vec* stacks the matrix columns into a vector.

The equations for the analyses solution are:

$$vec(G) \approx vec(G^M)$$ (24)

mass balance equations (25) for the analyses

$$vec(B*G) = 0$$ (25)

where the (N_S - 1)*(N_U - N_{SRU})x(N_{F2}*(N_S - 1)) matrix B is defined

$$B = diag(B^{subflow}) \qquad sublow = 1,..,N_S - 1$$ (26)

Sub-flows are indexed up to N_S - 1 to avoid linear dependency of equality constraints. Where the operator *diag* adds matrices $B^{subflow}$ at the diagonal of the matrix B and

$$B_{unit,flow}^{sublow} = e_{flow}^{unit} * F_{flow,subflow} \tag{27}$$

the sum of sub-flows is the total flow (28)

$$\sum_{subflow=1}^{NS} F_{flow,subflow} * G_{flow,chemical\_element,subflow} = F_{flow,tot} * G_{flow,tot,chemical\_element}$$

$$flow = 1,...,N_{F2}$$

$$chemical\_element = 1,...,N_E \tag{28}$$

If the option minerals sum=100 is selected, the following Equations (29) are included:

$$\sum_{chemicalelement=1}^{N_E} G_{flow,chemical\_element,subflow} = 100 \qquad flow = 1,...,N_{F1}$$

$$subflow = 1,...,N_S \tag{29}$$

If Equations (29) are included, B is $(N_S - 1)*(N_U - N_{SRU} - 1)x(N_{F2}*(N_S - 1))$
to avoid linear dependencies.

The solution method used is weighted least squares. The equations can be solved without any constraints (LS), subject to non-negativity constraints (NNLS), and subject to simple bounds (CLS).

51.8. References

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