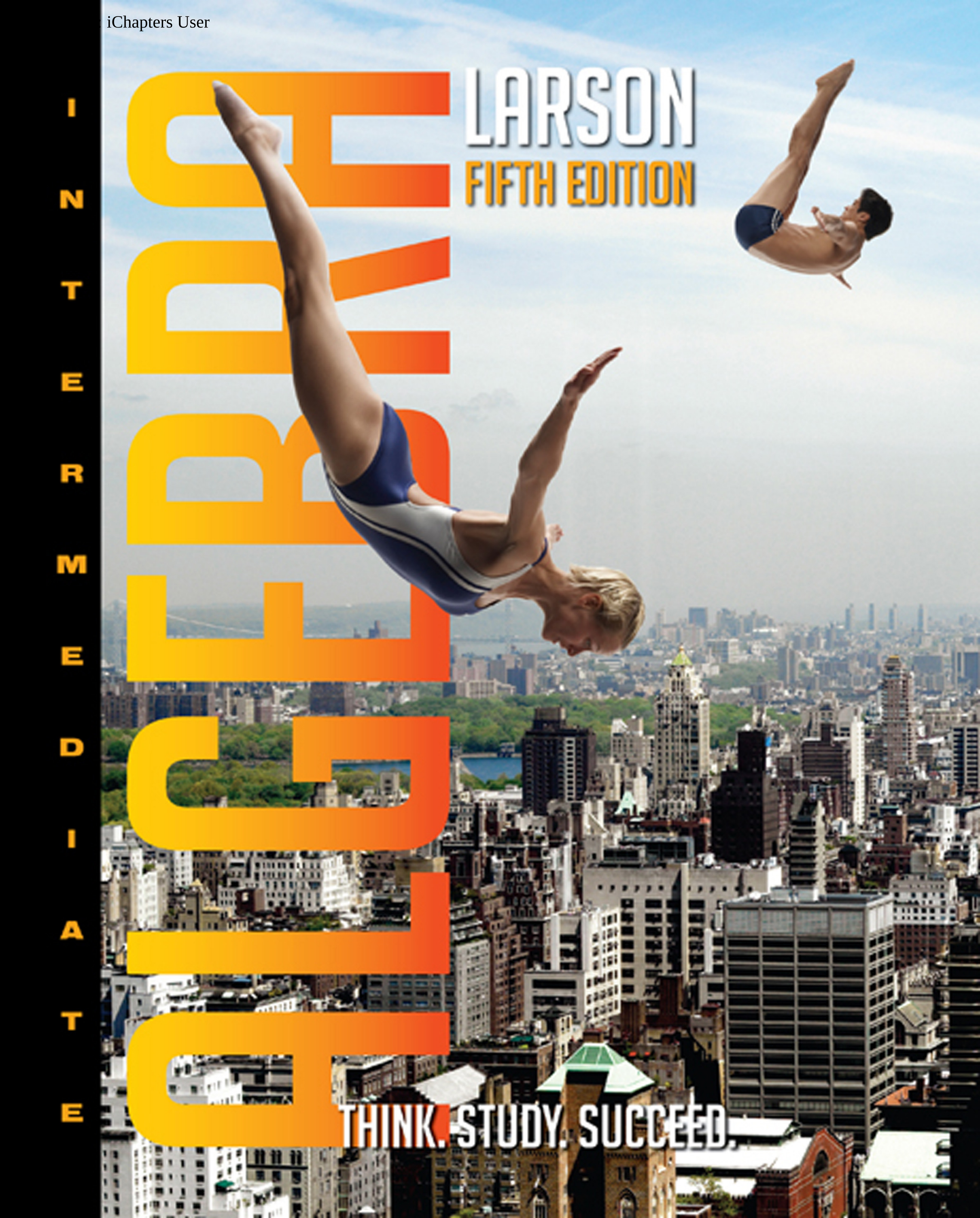
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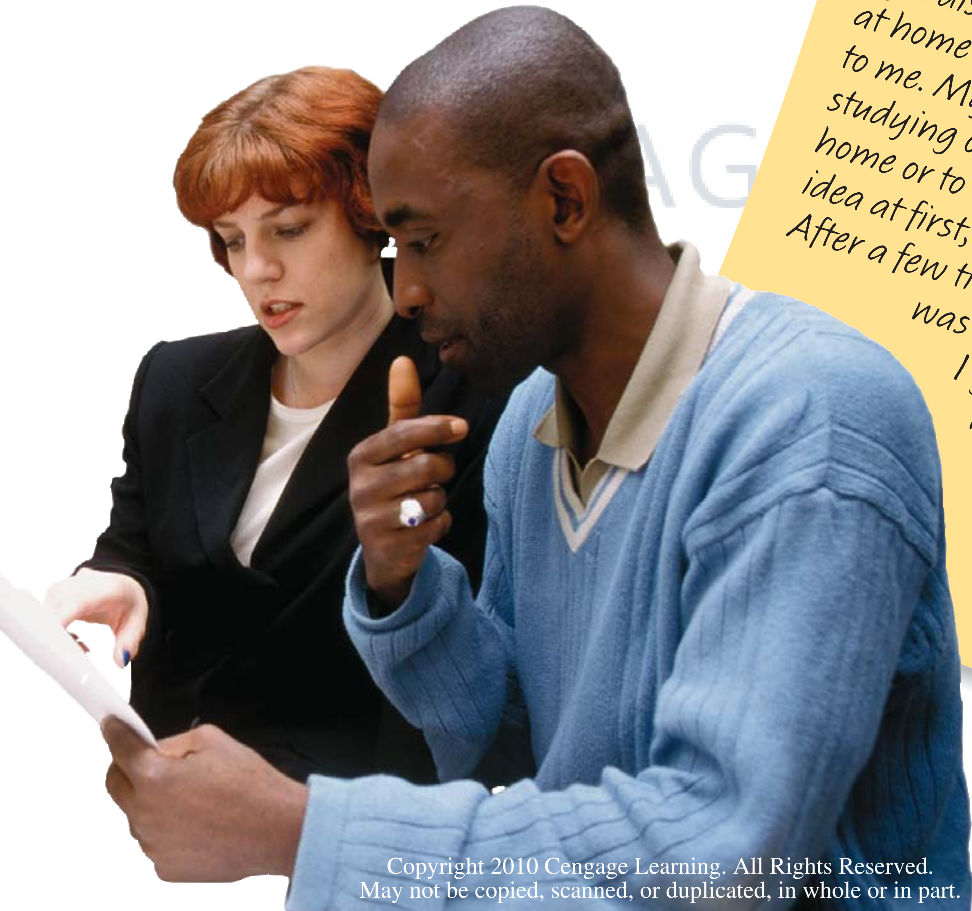
# Chapter 1

# Fundamentals of Algebra





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**Caleb**
Music

# 1.1 The Real Number System


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## What You Should Learn

1 ▶ Understand the set of real numbers and the subsets of real numbers.
2 ▶ Use the real number line to order real numbers.
3 ▶ Use the real number line to find the distance between two real numbers.
4 ▶ Determine the absolute value of a real number.

### Why You Should Learn It

Inequality symbols can be used to represent many real-life situations, such as bicycling speeds (see Exercise 87 on page 10).

1 ▶ Understand the set of real numbers and the subsets of real numbers.

## Sets and Real Numbers

This chapter introduces the basic definitions, operations, and rules that form the fundamental concepts of algebra. Section 1.1 begins with real numbers and their representation on the real number line. Sections 1.2 and 1.3 discuss operations and properties of real numbers, and Sections 1.4 and 1.5 discuss algebraic expressions.

The formal term that is used in mathematics to refer to a collection of objects is the word **set.** For instance, the set

$\{1, 2, 3\}$ A set with three members

contains the three numbers 1, 2, and 3. Note that the members of the set are enclosed in braces { }. Parentheses ( ) and brackets [ ] are used to represent other ideas.

The set of numbers that is used in arithmetic is called the set of **real numbers.** The term *real* distinguishes real numbers from *imaginary* or *complex* numbers—a type of number that you will study later in this text.

If all members of a set *A* are also members of a set *B*, then *A* is a **subset** of *B*. One of the most commonly used subsets of real numbers is the set of **natural numbers** or **positive integers.**

$\{1, 2, 3, 4, . . .\}$ The set of positive integers

Note that the three dots indicate that the pattern continues. For instance, the set also contains the numbers 5, 6, 7, and so on.

Positive integers can be used to describe many quantities in everyday life. For instance, you might be taking four classes this term, or you might be paying 240 dollars a month for rent. But even in everyday life, positive integers cannot describe some concepts accurately. For instance, you could have a zero balance in your checking account. To describe such a quantity, you need to expand the set of positive integers to include zero, forming the set of **whole numbers.** To describe a quantity such as $-5°$, you need to expand the set of whole numbers to include **negative integers.** This expanded set is called the set of **integers.** The set of integers is also a *subset* of the set of real numbers.

$$\{\underbrace{. . . , -3, -2, -1}_{\text{Negative integers}}, \overbrace{0, \underbrace{1, 2, 3, . . .}_{\text{Positive integers}}}^{\text{Whole numbers}}\}$$ The set of integers

### Study Tip

In this text, whenever a mathematical term is formally introduced, the word will appear in boldface type. Be sure you understand the meaning of each new word–it is important that each word become part of your mathematical vocabulary. It may be helpful to keep a vocabulary journal.

> **Technology: Tip**
>
> You can use a calculator to round decimals. For instance, to round 0.2846 to three decimal places on a scientific calculator, enter
>
> FIX 3 .2846 = .
>
> On a graphing calculator, enter
>
> *round* (.2846, 3) ENTER .
>
> Consult the user's manual for your graphing calculator for specific keystrokes or instructions. Then, use your calculator to round 0.38174 to four decimal places.

Even with the set of integers, there are still many quantities in everyday life that you cannot describe accurately. The costs of many items are not in whole dollar amounts, but in parts of dollars, such as \$1.19 and \$39.98. You might work $8\frac{1}{2}$ hours, or you might miss the first *half* of a movie. To describe such quantities, you can expand the set of integers to include **fractions.** The expanded set is called the set of **rational numbers.** Formally, a real number is **rational** if it can be written as the ratio $p/q$ of two integers, where $q \neq 0$ (the symbol $\neq$ means **does not equal**). Here are some examples of rational numbers.

$$2 = \frac{2}{1}, \quad \frac{1}{3} = 0.333 \ldots, \quad \frac{1}{8} = 0.125, \quad \text{and} \quad \frac{125}{111} = 1.126126 \ldots$$

The decimal representation of a rational number is either **terminating** or **repeating.** For instance, the decimal representation of $\frac{1}{4} = 0.25$ is terminating, and the decimal representation of

$$\frac{4}{11} = 0.363636 \ldots = 0.\overline{36}$$

is repeating. (The overbar symbol over 36 indicates which digits repeat.) A real number that cannot be written as a ratio of two integers is **irrational.** For instance,

$$\sqrt{2} = 1.4142135 \ldots \quad \text{and} \quad \pi = 3.1415926 \ldots$$

are irrational.

The decimal representation of an irrational number neither terminates nor repeats. When you perform calculations using decimal representations of nonterminating, nonrepeating decimals, you usually use a decimal approximation that has been **rounded** to a certain number of decimal places. The rounding rule used in this text is to round up if the succeeding digit is 5 or more, or to round down if the succeeding digit is 4 or less. For example, to one decimal place, 7.35 would *round up* to 7.4. Similarly, to two decimal places, 2.364 would *round down* to 2.36. Rounded to four decimal places, the decimal approximations of the rational number $\frac{2}{3}$ and the irrational number $\pi$ are

$$\frac{2}{3} \approx 0.6667 \quad \text{and} \quad \pi \approx 3.1416.$$

The symbol $\approx$ means **is approximately equal to.** Figure 1.1 shows several commonly used subsets of real numbers and their relationships to each other.



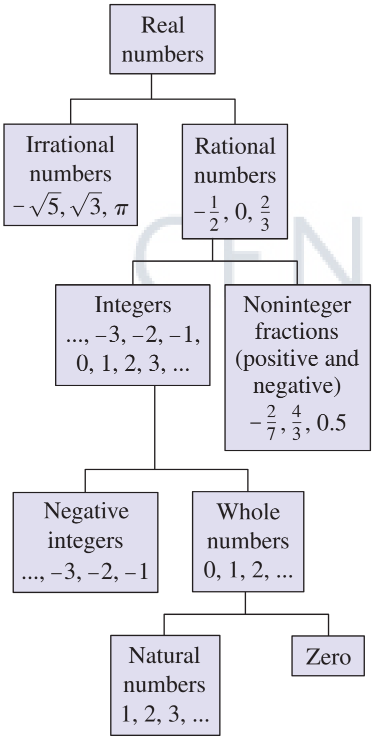



**Figure 1.1** Subsets of Real Numbers

## EXAMPLE 1 Classifying Real Numbers

Which of the numbers in the set $\left\{-7, -\sqrt{3}, -1, -\frac{1}{5}, 0, \frac{3}{4}, \sqrt{2}, \pi, 5\right\}$ are (a) natural numbers, (b) integers, (c) rational numbers, and (d) irrational numbers?

### Solution

**a.** Natural numbers: $\{5\}$

**b.** Integers: $\{-7, -1, 0, 5\}$

**c.** Rational numbers: $\left\{-7, -1, -\frac{1}{5}, 0, \frac{3}{4}, 5\right\}$

**d.** Irrational numbers: $\left\{-\sqrt{3}, \sqrt{2}, \pi\right\}$

**CHECKPOINT** *Now try Exercise 1.*

**2** ▶ Use the real number line to order real numbers.

## The Real Number Line

The picture that represents the real numbers is called the **real number line.** It consists of a horizontal line with a point (the **origin**) labeled 0. Numbers to the left of zero are **negative** and numbers to the right of zero are **positive,** as shown in Figure 1.2.

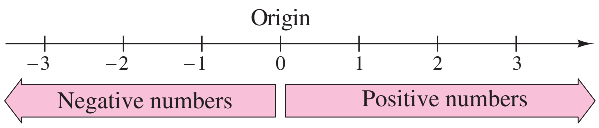
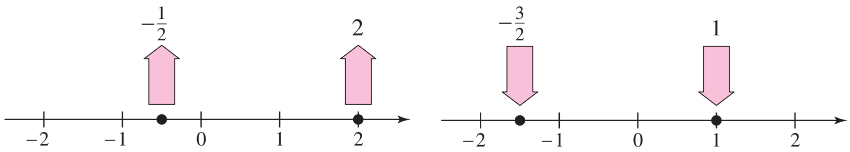



**Figure 1.2** The Real Number Line

Zero is neither positive nor negative. So, to describe a real number that might be positive or zero, you can use the term **nonnegative real number.**

Each point on the real number line corresponds to exactly one real number, and each real number corresponds to exactly one point on the real number line, as shown in Figure 1.3. When you draw the point (on the real number line) that corresponds to a real number, you are **plotting** the real number.




Each point on the real number line corresponds to a real number.

Each real number corresponds to a point on the real number line.

**Figure 1.3**

### EXAMPLE 2 Plotting Points on the Real Number Line

Plot the real numbers on the real number line.

**a.** $-\dfrac{5}{3}$ **b.** 2.3 **c.** $\dfrac{9}{4}$ **d.** −0.3

#### Solution

All four points are shown in Figure 1.4.

**a.** The point representing the real number $-\frac{5}{3} = -1.666 \ldots$ lies between −2 and −1, but closer to −2, on the real number line.

**b.** The point representing the real number 2.3 lies between 2 and 3, but closer to 2, on the real number line.

**c.** The point representing the real number $\frac{9}{4} = 2.25$ lies between 2 and 3, but closer to 2, on the real number line. Note that the point representing $\frac{9}{4}$ lies slightly to the left of the point representing 2.3.

**d.** The point representing the real number −0.3 lies between −1 and 0, but closer to 0, on the real number line.

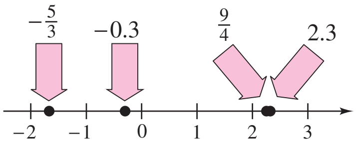



**Figure 1.4**

✓ ***CHECKPOINT*** *Now try Exercise 13.*

The real number line provides a way of comparing any two real numbers. For instance, if you choose any two (different) numbers on the real number line, one of the numbers must be to the left of the other. You can describe this by saying that the number to the left is **less than** the number to the right, or that the number to the right is **greater than** the number to the left, as shown in Figure 1.5.

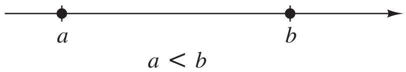



**Figure 1.5** *a* is to the left of *b*.

### Order on the Real Number Line

If the real number $a$ lies to the left of the real number $b$ on the real number line, then $a$ is **less than** $b$**,** which is written as

$$a < b.$$

This relationship can also be described by saying that $b$ is **greater than** $a$ and writing $b > a$. The expression $a \le b$ means that $a$ is **less than or equal to** $b$, and the expression $b \ge a$ means that $b$ is **greater than or equal to** $a$. The symbols $<$ , $>$ , $\le$ , and $\ge$ are called **inequality symbols.**

When asked to **order** two numbers, you are simply being asked to say which of the two numbers is greater.

### EXAMPLE 3 Ordering Real Numbers

Place the correct inequality symbol ($<$ or $>$) between each pair of numbers.

**a.** $-4 \;\square\; 0$ **b.** $-3 \;\square\; -5$ **c.** $\frac{1}{5} \;\square\; \frac{1}{3}$ **d.** $-\frac{1}{4} \;\square\; -\frac{1}{2}$

**Solution**

**a.** Because $-4$ lies to the left of 0 on the real number line, as shown in Figure 1.6, you can say that $-4$ is *less than* 0, and write $-4 < 0$.

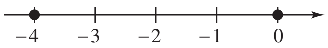



**Figure 1.6**

**b.** Because $-3$ lies to the right of $-5$ on the real number line, as shown in Figure 1.7, you can say that $-3$ is *greater than* $-5$, and write $-3 > -5$.

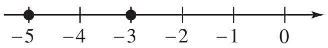



**Figure 1.7**

**c.** Because $\frac{1}{5}$ lies to the left of $\frac{1}{3}$ on the real number line, as shown in Figure 1.8, you can say that $\frac{1}{5}$ is *less than* $\frac{1}{3}$, and write $\frac{1}{5} < \frac{1}{3}$.

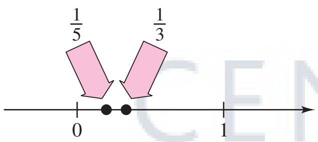



**Figure 1.8**

**d.** Because $-\frac{1}{4}$ lies to the right of $-\frac{1}{2}$ on the real number line, as shown in Figure 1.9, you can say that $-\frac{1}{4}$ is *greater than* $-\frac{1}{2}$, and write $-\frac{1}{4} > -\frac{1}{2}$.

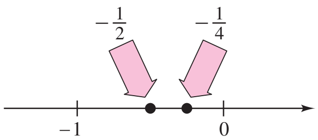



**Figure 1.9**

**CHECKPOINT** *Now try Exercise 19.*

One effective way to order two fractions such as $\frac{5}{12}$ and $\frac{9}{23}$ is to compare their decimal equivalents. Because $\frac{5}{12} = 0.41\overline{6}$ and $\frac{9}{23} \approx 0.391$, you can write

$$\frac{5}{12} > \frac{9}{23}.$$

**3** ▶ Use the real number line to find the distance between two real numbers.

## Distance on the Real Number Line

Once you know how to represent real numbers as points on the real number line, it is natural to talk about the **distance between two real numbers.** Specifically, if $a$ and $b$ are two real numbers such that $a \leq b$, then the distance between $a$ and $b$ is defined as $b - a$.

### Distance Between Two Real Numbers

If $a$ and $b$ are two real numbers such that $a \leq b$, then the **distance between *a* and *b*** is given by

Distance between $a$ and $b = b - a$.

Note from this definition that if $a = b$, the distance between $a$ and $b$ is zero. If $a \neq b$, then the distance between $a$ and $b$ is positive.

### EXAMPLE 4 Finding the Distance Between Two Real Numbers

Find the distance between each pair of real numbers.

**a.** $-2$ and 3 **b.** 0 and 4 **c.** $-4$ and 0 **d.** 1 and $-\frac{1}{2}$

#### Solution

**a.** Because $-2 \leq 3$, the distance between $-2$ and 3 is

$$3 - (-2) = 3 + 2 = 5.$$ See Figure 1.10.

**b.** Because $0 \leq 4$, the distance between 0 and 4 is

$$4 - 0 = 4.$$ See Figure 1.11.

**c.** Because $-4 \leq 0$, the distance between $-4$ and 0 is

$$0 - (-4) = 0 + 4 = 4.$$ See Figure 1.12.

**d.** Because $-\frac{1}{2} \leq 1$, let $a = -\frac{1}{2}$ and $b = 1$. So, the distance between 1 and $-\frac{1}{2}$ is

$$1 - \left(-\frac{1}{2}\right) = 1 + \frac{1}{2} = 1\frac{1}{2}.$$ See Figure 1.13.

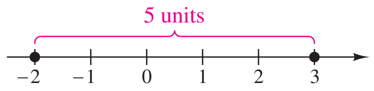



**Figure 1.10**

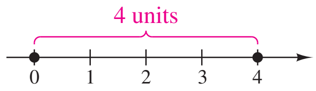



**Figure 1.11**

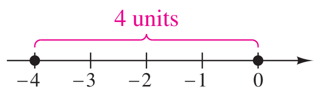



**Figure 1.12**

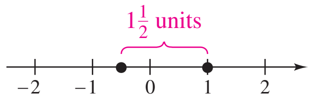



**Figure 1.13**

✔ **CHECKPOINT** *Now try Exercise 29.*

**Study Tip**

Recall that when you subtract a negative number, as in Example 4(a), you add the opposite of the second number to the first. Because the opposite of $-2$ is 2, you add 2 to 3.

4 ▶ Determine the absolute value of a real number.

## Absolute Value

Two real numbers are called **opposites** of each other if they lie the same distance from, but on opposite sides of, 0 on the real number line. For instance, $-2$ is the opposite of 2 (see Figure 1.14).

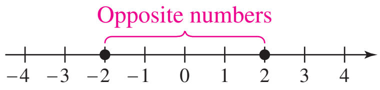



**Figure 1.14**

The opposite of a negative number is called a **double negative** (see Figure 1.15).

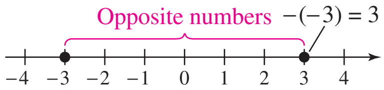



**Figure 1.15**

Opposite numbers are also referred to as **additive inverses** because their sum is zero. For instance, $3 + (-3) = 0$. In general, you have the following.

### Opposites and Additive Inverses

Let $a$ be a real number.

**1.** $-a$ is the opposite of $a$.

**2.** $-(-a) = a$ Double negative

**3.** $a + (-a) = 0$ Additive inverse



The distance between a real number $a$ and 0 (the origin) is called the **absolute value** of $a$. Absolute value is denoted by double vertical bars $|\ \ |$. For example,

$$|5| = \text{“distance between 5 and 0”} = 5$$

and

$$|-8| = \text{“distance between } -8 \text{ and 0”} = 8.$$

**Study Tip**

Because *opposite* numbers lie the same distance from 0 on the real number line, they have the same absolute value. So, $|5| = 5$ and $|-5| = 5$.

Be sure you see from the following definition that the absolute value of a real number is never negative. For instance, if $a = -3$, then $|-3| = -(-3) = 3$. Moreover, the only real number whose absolute value is zero is 0. That is, $|0| = 0$.

### Definition of Absolute Value

If $a$ is a real number, then the **absolute value** of $a$ is

$$|a| = \begin{cases} a, & \text{if } a \ge 0 \\ -a, & \text{if } a < 0 \end{cases}.$$

## EXAMPLE 5 Finding Absolute Values

**a.** $|-10| = 10$ — The absolute value of $-10$ is 10.

**b.** $\left|\frac{3}{4}\right| = \frac{3}{4}$ — The absolute value of $\frac{3}{4}$ is $\frac{3}{4}$.

**c.** $|-3.2| = 3.2$ — The absolute value of $-3.2$ is 3.2.

**d.** $-|-6| = -(6) = -6$ — The opposite of $|-6|$ is $-6$.

Note that part (d) does not contradict the fact that the absolute value of a number cannot be negative. The expression $-|-6|$ calls for the *opposite* of an absolute value, and so it must be negative.

**CHECKPOINT** *Now try Exercise 41.*

For any two real numbers $a$ and $b$, exactly one of the following orders must be true: $a < b$, $a = b$, or $a > b$. This property of real numbers is called the **Law of Trichotomy.** In words, this property tells you that if $a$ and $b$ are any two real numbers, then $a$ is less than $b$, $a$ is equal to $b$, or $a$ is greater than $b$.

## EXAMPLE 6 Comparing Real Numbers

Place the correct symbol ($<$, $>$, or $=$) between each pair of real numbers.

**a.** $|-2| \; \square \; 1$  **b.** $|-4| \; \square \; |4|$  **c.** $|12| \; \square \; |-15|$

**d.** $|-3| \; \square \; -3$  **e.** $2 \; \square \; -|-2|$  **f.** $-|-3| \; \square \; -3$

### Solution

**a.** $|-2| > 1$, because $|-2| = 2$ and 2 is greater than 1.

**b.** $|-4| = |4|$, because $|-4| = 4$ and $|4| = 4$.

**c.** $|12| < |-15|$, because $|12| = 12$, $|-15| = 15$, and 12 is less than 15.

**d.** $|-3| > -3$, because $|-3| = 3$ and 3 is greater than $-3$.

**e.** $2 > -|-2|$, because $-|-2| = -2$ and 2 is greater than $-2$.

**f.** $-|-3| = -3$, because $-|-3| = -3$ and $-3$ is equal to $-3$.

**CHECKPOINT** *Now try Exercise 55.*

When the distance between the two real numbers $a$ and $b$ was defined as $b - a$, the definition included the restriction $a \le b$. Using absolute value, you can generalize this definition. That is, if $a$ and $b$ are *any* two real numbers, then the distance between $a$ and $b$ is given by

Distance between $a$ and $b$ $= |b - a| = |a - b|$.

For instance, the distance between $-2$ and 1 is given by

$|-2 - 1| = |-3| = 3.$ — Distance between $-2$ and 1

You could also find the distance between $-2$ and 1 as follows.

$|1 - (-2)| = |3| = 3$ — Distance between $-2$ and 1

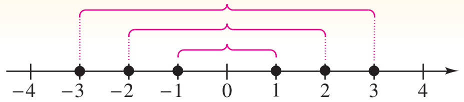
**Smart Study Strategy**

Go to page xxii for ways to *Create a Positive Study Environment.*

## Concept Check

**1.** Two real numbers are plotted on the real number line. How can you tell which number is greater?

**2.** How are the numbers connected by each brace related?




**3.** Is the number 7 a rational number? Explain why or why not.

**4.** The distance between a number $b$ and 0 is 6. Explain what you know about the number $b$.

## 1.1 EXERCISES

Go to pages 50–51 to record your assignments.

## Developing Skills

In Exercises 1–4, which of the real numbers in the set are (a) natural numbers, (b) integers, (c) rational numbers, and (d) irrational numbers? ***See Example 1.***

**1.** $\left\{-6, -\sqrt{6}, -\frac{4}{3}, 0, \frac{5}{8}, 1, \sqrt{2}, 2, \pi, 6\right\}$

**2.** $\left\{-\frac{10}{3}, -\pi, -\sqrt{3}, -1, 0, \frac{2}{5}, \sqrt{3}, \frac{5}{2}, 5, 101\right\}$

**3.** $\left\{-4.2, \sqrt{4}, -\frac{1}{9}, 0, \frac{3}{11}, \sqrt{11}, 5.\overline{5}, 5.543\right\}$

**4.** $\left\{-\sqrt{25}, -\sqrt{6}, -0.\overline{1}, -\frac{5}{3}, 0, 0.85, 3, 110\right\}$

In Exercises 5–8, use an overbar symbol to rewrite the decimal using the smallest number of digits possible.

**5.** 0.2222 . . .  **6.** 1.5555 . . .

**7.** 2.121212 . . .  **8.** 0.436436436 . . .

In Exercises 9–12, list all members of the set.

**9.** The integers between $-5.8$ and $3.2$

**10.** The even integers between $-2.1$ and $10.5$

**11.** The odd integers between $\pi$ and 10

**12.** The prime numbers between 4 and 25

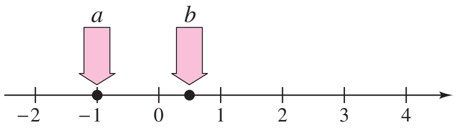
In Exercises 13 and 14, plot the real numbers on the real number line. ***See Example 2.***

**13.** (a) 3 (b) $\frac{5}{2}$ (c) $-\frac{7}{2}$ (d) $-5.2$

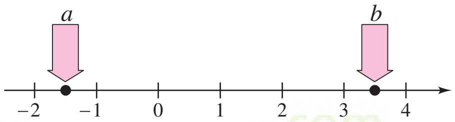
**14.** (a) 8 (b) $\frac{4}{3}$ (c) $-6.75$ (d) $-\frac{9}{2}$

In Exercises 15–18, approximate the two numbers and order them.

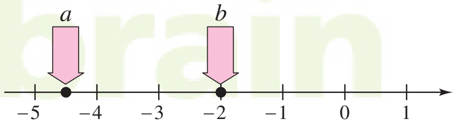
**15.**



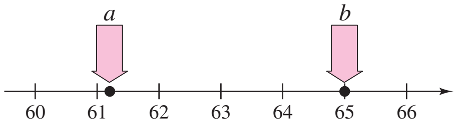

**16.**




**17.**




**18.**




In Exercises 19–28, place the correct inequality symbol (< or >) between the pair of numbers. ***See Example 3.***

**19.** $\frac{4}{5}$ ☐ $1$  **20.** $2$ ☐ $\frac{5}{3}$

**21.** $-5$ ☐ $2$  **22.** $9$ ☐ $-1$

**23.** $-5$ ☐ $-2$  **24.** $-8$ ☐ $-3$

**25.** $\frac{5}{8}$ ☐ $\frac{1}{2}$  **26.** $\frac{3}{2}$ ☐ $\frac{5}{2}$

**27.** $-\frac{2}{3}$ ☐ $-\frac{10}{3}$  **28.** $-\frac{5}{3}$ ☐ $-\frac{3}{2}$

In Exercises 29–40, find the distance between the pair of real numbers. ***See Example 4.***

**29.** 4 and 10
**30.** 75 and 20
**31.** $-12$ and 7
**32.** $-54$ and 32
**33.** 18 and $-32$
**34.** 14 and $-6$
**35.** $-8$ and 0
**36.** 0 and 125
**37.** 0 and 35
**38.** $-35$ and 0
**39.** $-6$ and $-9$
**40.** $-12$ and $-7$

In Exercises 41–54, evaluate the expression. ***See Example 5.***

**41.** $|10|$
**42.** $|62|$
**43.** $|-225|$
**44.** $|-14|$
**45.** $-|-85|$
**46.** $-|-36.5|$
**47.** $-|16|$
**48.** $-|-25|$
**49.** $-\left|-\frac{3}{4}\right|$
**50.** $-\left|\frac{3}{8}\right|$
**51.** $-|3.5|$
**52.** $|-1.4|$
**53.** $|-\pi|$
**54.** $-|\pi|$

In Exercises 55–62, place the correct symbol ($<$, $>$, or $=$) between the pair of real numbers. ***See Example 6.***

**55.** $|-6| \quad \square \quad |2|$
**56.** $|-2| \quad \square \quad |2|$
**57.** $|47| \quad \square \quad |-27|$
**58.** $|150| \quad \square \quad |-310|$
**59.** $|-1.8| \quad \square \quad |1.8|$
**60.** $|12.5| \quad \square \quad -|-25|$
**61.** $\left|-\frac{3}{4}\right| \quad \square \quad -\left|\frac{4}{5}\right|$
**62.** $-\left|-\frac{7}{3}\right| \quad \square \quad -\left|\frac{1}{3}\right|$

In Exercises 63–72, find the opposite and the absolute value of the number.

**63.** 34
**64.** 225
**65.** $-160$
**66.** $-52$
**67.** $-\frac{3}{11}$
**68.** $\frac{7}{32}$
**69.** $\frac{5}{4}$
**70.** $\frac{4}{3}$
**71.** 4.7
**72.** $-0.4$

In Exercises 73–82, plot the number and its opposite on the real number line. Determine the distance of each from 0.

**73.** $-7$
**74.** $-4$
**75.** 5
**76.** 6
**77.** $-\frac{3}{5}$
**78.** $\frac{7}{4}$
**79.** $\frac{5}{3}$
**80.** $-\frac{3}{4}$
**81.** $-4.25$
**82.** 3.5

In Exercises 83–90, write the statement using inequality notation.

**83.** $x$ is negative.
**84.** $y$ is more than 25.
**85.** $u$ is at least 16.
**86.** $x$ is nonnegative.

**87.** A bicycle racer's speed $s$ is at least 16 miles per hour and at most 28 miles per hour.

**88.** The tire pressure $p$ is at least 30 pounds per square inch and no more than 35 pounds per square inch.

**89.** The price $p$ is less than $225.

**90.** The average $a$ will exceed 5000.



In Exercises 91–94, find two possible values of *a*.

**91.** $|a| = 4$
**92.** $-|a| = -7$
**93.** The distance between $a$ and 3 is 5.
**94.** The distance between $a$ and $-1$ is 6.

## Explaining Concepts

***True or False?*** In Exercises 95 and 96, decide whether the statement is true or false. Explain your reasoning.

**95.** Every real number is either rational or irrational.

**96.** The distance between a number $b$ and its opposite is equal to the distance between 0 and twice the number $b$.

**97.** Describe the difference between the rational numbers 0.15 and $0.\overline{15}$.

**98.** Is there a difference between saying that a real number is positive and saying that a real number is nonnegative? Explain your answer.

# 1.2 Operations with Real Numbers

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### Why You Should Learn It

Real numbers can be used to represent many real-life quantities, such as the net profits for Columbia Sportswear Company (see Exercise 136 on page 21).

### What You Should Learn

1 ▶ Add, subtract, multiply, and divide real numbers.
2 ▶ Write repeated multiplication in exponential form and evaluate exponential expressions.
3 ▶ Use order of operations to evaluate expressions.
4 ▶ Evaluate expressions using a calculator and order of operations.

## Operations with Real Numbers

1 ▶ Add, subtract, multiply, and divide real numbers.

There are four basic operations of arithmetic: addition, subtraction, multiplication, and division.

The result of adding two real numbers is the **sum** of the two numbers, and the two real numbers are the **terms** of the sum. The rules for adding real numbers are as follows.

### Addition of Two Real Numbers

1. To **add** two real numbers with *like signs*, add their absolute values and attach the common sign to the result.
2. To **add** two real numbers with *unlike signs*, subtract the smaller absolute value from the greater absolute value and attach the sign of the number with the greater absolute value.

### EXAMPLE 1 Adding Integers

**a.** $-84 + 14 = -(84 - 14)$ — Use negative sign.

$= -70$ — Subtract absolute values.

**b.** $-138 + (-62) = -(138 + 62)$ — Use common sign.

$= -200$ — Add absolute values.

**CHECKPOINT** *Now try Exercise 7.*

### EXAMPLE 2 Adding Decimals

**a.** $-26.41 + (-0.53) = -(26.41 + 0.53)$ — Use common sign.

$= -26.94$ — Add absolute values.

**b.** $3.2 + (-0.4) = +(3.2 - 0.4)$ — Use positive sign.

$= 2.8$ — Subtract absolute values.

**CHECKPOINT** *Now try Exercise 9.*

The result of subtracting two real numbers is the **difference** of the two numbers. Subtraction of two real numbers is defined in terms of addition, as follows.

### Subtraction of Two Real Numbers

To **subtract** the real number $b$ from the real number $a$, add the opposite of $b$ to $a$. That is, $a - b = a + (-b)$.

### EXAMPLE 3 Subtracting Integers

Find each difference.

**a.** $9 - 21$ **b.** $-15 - 8$

**Solution**

**a.** $9 - 21 = 9 + (-21)$ Add opposite of 21.

$= -(21 - 9) = -12$ Use negative sign and subtract absolute values.

**b.** $-15 - 8 = -15 + (-8)$ Add opposite of 8.

$= -(15 + 8) = -23$ Use common sign and add absolute values.

**CHECKPOINT** *Now try Exercise 11.*

### EXAMPLE 4 Subtracting Decimals

Find each difference.

**a.** $-2.5 - (-2.7)$ **b.** $-7.02 - 13.8$

**Solution**

**a.** $-2.5 - (-2.7) = -2.5 + 2.7$ Add opposite of $-2.7$.

$= +(2.7 - 2.5) = 0.2$ Use positive sign and subtract absolute values.

**b.** $-7.02 - 13.8 = -7.02 + (-13.8)$ Add opposite of 13.8.

$= -(7.02 + 13.8) = -20.82$ Use common sign and add absolute values.

**CHECKPOINT** *Now try Exercise 13.*

### EXAMPLE 5 Evaluating an Expression

Evaluate $-13 - 7 + 11 - (-4)$.

**Solution**

$-13 - 7 + 11 - (-4) = -13 + (-7) + 11 + 4$ Add opposites.

$= -20 + 15$ Add two numbers at a time.

$= -5$ Add.

**CHECKPOINT** *Now try Exercise 19.*

To add or subtract fractions, it is useful to recognize the equivalent forms of fractions, as illustrated below.

$$\frac{a}{b} = \frac{-a}{-b} = -\frac{-a}{b} = -\frac{a}{-b}$$ All are positive.

$$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b} = -\frac{-a}{-b}$$ All are negative.

## Addition and Subtraction of Fractions

**1.** *Like Denominators:* The sum and difference of two fractions with like denominators ($c \neq 0$) are:

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c} \qquad \frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

**2.** *Unlike Denominators:* To add or subtract two fractions with unlike denominators, first rewrite the fractions so that they have the same denominator and then apply the first rule.

### Study Tip

Here is an alternative method for adding and subtracting fractions with unlike denominators ($b \neq 0$ and $d \neq 0$).

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

$$\frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

For example,

$$\frac{1}{6} + \frac{3}{8} = \frac{1(8)+6(3)}{6(8)} = \frac{8+18}{48} = \frac{26}{48} = \frac{13}{24}.$$

Note that an additional step is needed to simplify the fraction after the numerators have been added.

To find the **least common denominator (LCD)** for two or more fractions, find the **least common multiple (LCM)** of their denominators. For instance, the LCM of 6 and 8 is 24. To see this, consider all multiples of 6 (6, 12, 18, 24, 30, 36, 42, 48, . . .) and all multiples of 8 (8, 16, 24, 32, 40, 48, . . .). The numbers 24 and 48 are common multiples, and the number 24 is the smallest of the common multiples. To add $\frac{1}{6}$ and $\frac{3}{8}$, proceed as follows.

$$\frac{1}{6} + \frac{3}{8} = \frac{1(4)}{6(4)} + \frac{3(3)}{8(3)} = \frac{4}{24} + \frac{9}{24} = \frac{4+9}{24} = \frac{13}{24}$$

## EXAMPLE 6 Adding and Subtracting Fractions

**a.** $\frac{5}{17} + \frac{9}{17} = \frac{5+9}{17}$ Add numerators.

$= \frac{14}{17}$ Simplify.

**b.** $\frac{3}{8} - \frac{5}{12} = \frac{3(3)}{8(3)} - \frac{5(2)}{12(2)}$ Least common denominator is 24.

$= \frac{9}{24} - \frac{10}{24}$ Simplify.

$= \frac{9-10}{24}$ Subtract numerators.

$= -\frac{1}{24}$ Simplify.

**CHECKPOINT** *Now try Exercise 21.*

**Study Tip**

A quick way to convert the mixed number $1\frac{4}{5}$ into the fraction $\frac{9}{5}$ is to multiply the whole number by the denominator of the fraction and add the result to the numerator, as follows.

$$1\frac{4}{5} = \frac{1(5) + 4}{5} = \frac{9}{5}$$

## EXAMPLE 7 Adding Mixed Numbers

Find the sum of $1\frac{4}{5}$ and $\frac{11}{7}$.

**Solution**

$1\frac{4}{5} + \frac{11}{7} = \frac{9}{5} + \frac{11}{7}$ — Write $1\frac{4}{5}$ as $\frac{9}{5}$.

$= \frac{9(7)}{5(7)} + \frac{11(5)}{7(5)}$ — Least common denominator is 35.

$= \frac{63}{35} + \frac{55}{35}$ — Simplify.

$= \frac{63 + 55}{35} = \frac{118}{35}$ — Add numerators and simplify.

**CHECKPOINT** *Now try Exercise 29.*

**Multiplication** of two real numbers can be described as *repeated addition*. For instance, $7 \times 3$ can be described as $3 + 3 + 3 + 3 + 3 + 3 + 3$. Multiplication is denoted in a variety of ways. For instance, $7 \times 3$, $7 \cdot 3$, $7(3)$, and $(7)(3)$ all denote the product "7 times 3." The result of multiplying two real numbers is their **product,** and each of the two numbers is a **factor** of the product.

### Multiplication of Two Real Numbers

1. To multiply two real numbers with *like signs,* find the product of their absolute values. The product is *positive*.
2. To multiply two real numbers with *unlike signs,* find the product of their absolute values, and attach a minus sign. The product is *negative.*
3. The product of zero and any other real number is zero.



## EXAMPLE 8 Multiplying Integers

**a.** $-6 \cdot 9 = -54$ (Unlike signs) — The product is negative.

**b.** $(-5)(-7) = 35$ (Like signs) — The product is positive.

**c.** $5(-3)(-4)(7) = 420$ (Like signs; Like signs) — The product is positive.

**CHECKPOINT** *Now try Exercise 45.*

**Study Tip**

To find the product of two or more numbers, first find the product of their absolute values. If there is an *even* number of negative factors, as in Example 8(c), the product is positive. If there is an *odd* number of negative factors, as in Example 8(a), the product is negative.

**Study Tip**

When operating with fractions, you should check to see whether your answers can be simplified by dividing out factors that are common to the numerator and denominator. For instance, the fraction $\frac{4}{6}$ can be written in simplified form as

$$\frac{4}{6} = \frac{\overset{1}{\cancel{2}} \cdot 2}{\underset{1}{\cancel{2}} \cdot 3} = \frac{2}{3}.$$

Note that dividing out a common factor is the division of a number by itself, and what remains is a factor of 1.

**Multiplication of Two Fractions**

The product of the two fractions $a/b$ and $c/d$ is given by

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}, \quad b \neq 0, \quad d \neq 0.$$

## EXAMPLE 9 Multiplying Fractions

Find the product.

$$\left(-\frac{3}{8}\right)\left(\frac{11}{6}\right)$$

**Solution**

$$\left(-\frac{3}{8}\right)\left(\frac{11}{6}\right) = -\frac{3(11)}{8(6)} \qquad \text{Multiply numerators and denominators.}$$

$$= -\frac{\cancel{3}(11)}{8(2)(\cancel{3})} \qquad \text{Factor and divide out common factor.}$$

$$= -\frac{11}{16} \qquad \text{Simplify.}$$

**CHECKPOINT** *Now try Exercise 57.*

The **reciprocal** of a nonzero real number $a$ is defined as the number by which $a$ must be multiplied to obtain 1. For instance, the reciprocal of 3 is $\frac{1}{3}$ because

$$3\left(\frac{1}{3}\right) = 1.$$

Similarly, the reciprocal of $-\frac{4}{5}$ is $-\frac{5}{4}$ because

$$-\frac{4}{5}\left(-\frac{5}{4}\right) = 1.$$

In general, the reciprocal of $a/b$ is $b/a$. Note that the reciprocal of a positive number is positive, and the reciprocal of a negative number is negative.

**Study Tip**

**Division** by 0 is not defined because 0 has no reciprocal. If 0 had a reciprocal value $b$, then you would obtain the *false* result

$$\frac{1}{0} = b \qquad \text{The reciprocal of zero is } b.$$

$$1 = b \cdot 0 \qquad \text{Multiply each side by 0.}$$

$$1 = 0. \qquad \text{False result, } 1 \neq 0$$

**Division of Two Real Numbers**

To divide the real number $a$ by the nonzero real number $b$, multiply $a$ by the reciprocal of $b$. That is,

$$a \div b = a \cdot \frac{1}{b}, \quad b \neq 0.$$

The result of dividing two real numbers is the **quotient** of the numbers. The number $a$ is the **dividend** and the number $b$ is the **divisor.** When the division is expressed as $a/b$ or $\frac{a}{b}$, $a$ is the **numerator** and $b$ is the **denominator.**

**EXAMPLE 10** Division of Real Numbers

**a.** $-30 \div 5 = -30 \cdot \frac{1}{5}$ Invert divisor and multiply.

$= -\frac{30}{5}$ Multiply.

$= -\frac{6 \cdot \cancel{5}}{\cancel{5}}$ Factor and divide out common factor.

$= -6$ Simplify.

**b.** $\frac{5}{16} \div 2\frac{3}{4} = \frac{5}{16} \div \frac{11}{4}$ Write $2\frac{3}{4}$ as $\frac{11}{4}$.

$= \frac{5}{16} \cdot \frac{4}{11}$ Invert divisor and multiply.

$= \frac{5(4)}{16(11)}$ Multiply.

$= \frac{5}{44}$ Simplify.

**CHECKPOINT** *Now try Exercise 71.*

**2** ▶ Write repeated multiplication in exponential form and evaluate exponential expressions.

## Positive Integer Exponents

*Repeated multiplication* can be written in what is called **exponential form.**

| *Repeated Multiplication* | | *Exponential Form* |
|---|---|---|
| $\underbrace{7 \cdot 7 \cdot 7 \cdot 7}_{\text{4 factors of 7}}$ | = | $7^4$ |
| $\underbrace{\left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right)\left(-\frac{3}{4}\right)}_{\text{3 factors of } -\frac{3}{4}}$ | = | $\left(-\frac{3}{4}\right)^3$ |

**Technology: Discovery**

When a negative number is raised to a power, the use of parentheses is very important. To discover why, use a calculator to evaluate $(-4)^4$ and $-4^4$. Write a statement explaining the results. Then use a calculator to evaluate $(-4)^3$ and $-4^3$. If necessary, write a new statement explaining your discoveries.

### Exponential Notation

Let $n$ be a positive integer and let $a$ be a real number. Then the product of $n$ factors of $a$ is given by

$$a^n = \underbrace{a \cdot a \cdot a \cdot \cdot \cdot a}_{n \text{ factors}}.$$

In the exponential form $a^n$, $a$ is the **base** and $n$ is the **exponent.** Writing the exponential form $a^n$ is called **"raising $a$ to the $n$th power."**

When a number, say 5, is raised to the *first* power, you would usually write 5 rather than $5^1$. Raising a number to the *second* power is called **squaring** the number. Raising a number to the *third* power is called **cubing** the number.

**EXAMPLE 11** Evaluating Exponential Expressions

**a.** $(-3)^4 = (-3)(-3)(-3)(-3) = 81$ — Negative sign is part of the base.

**b.** $-3^4 = -(3)(3)(3)(3) = -81$ — Negative sign is not part of the base.

**c.** $\left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right) = \frac{8}{125}$

**d.** $(-5)^3 = (-5)(-5)(-5) = -125$ — Negative raised to odd power.

**e.** $(-5)^4 = (-5)(-5)(-5)(-5) = 625$ — Negative raised to even power.

**CHECKPOINT** *Now try Exercise 91.*

In parts (d) and (e) of Example 11, note that when a negative number is raised to an *odd* power, the result is *negative,* and when a negative number is raised to an *even* power, the result is *positive.*

3 ▶ Use order of operations to evaluate expressions.

## Order of Operations

One of your goals in studying this book is to learn to communicate about algebra by reading and writing information about numbers. One way to help avoid confusion when communicating algebraic ideas is to establish an **order of operations.** This is done by giving priorities to different operations. First priority is given to exponents, second priority is given to multiplication and division, and third priority is given to addition and subtraction. To distinguish between operations with the same priority, use the *Left-to-Right Rule.*

**Study Tip**

The order of operations for multiplication applies when multiplication is written with the symbol $\times$ or $\cdot$ . When multiplication is implied by parentheses, it has a higher priority than the Left-to-Right Rule. For instance,

$$8 \div 4(2) = 8 \div 8 = 1$$

but

$$8 \div 4 \cdot 2 = 2 \cdot 2 = 4.$$

### Order of Operations

To evaluate an expression involving more than one operation, use the following order.

1. First do operations that occur within symbols of grouping.
2. Then evaluate powers.
3. Then do multiplications and divisions from left to right.
4. Finally, do additions and subtractions from left to right.

**EXAMPLE 12** Order of Operations Without Symbols of Grouping

**a.** $20 - 2 \cdot 3^2 = 20 - 2 \cdot 9$ — Evaluate power.

$= 20 - 18 = 2$ — Multiply, then subtract.

**b.** $5 - 6 - 2 = (5 - 6) - 2$ — Left-to-Right Rule

$= -1 - 2 = -3$ — Subtract.

**c.** $8 \div 2 \cdot 2 = (8 \div 2) \cdot 2$ — Left-to-Right Rule

$= 4 \cdot 2 = 8$ — Divide, then multiply.

**CHECKPOINT** *Now try Exercise 105.*

When you want to change the established order of operations, you must use parentheses or other symbols of grouping. Part (d) in the next example shows that a fraction bar acts as a symbol of grouping.

### EXAMPLE 13 Order of Operations with Symbols of Grouping

**a.** $7 - 3(4 - 2) = 7 - 3(2)$ — Subtract within symbols of grouping.

$= 7 - 6 = 1$ — Multiply, then subtract.

**b.** $4 - 3(2)^3 = 4 - 3(8)$ — Evaluate power.

$= 4 - 24 = -20$ — Multiply, then subtract.

**c.** $1 - [4 - (5 - 3)] = 1 - (4 - 2)$ — Subtract within symbols of grouping.

$= 1 - 2 = -1$ — Subtract within symbols of grouping, then subtract.

**d.** $\dfrac{2 \cdot 5^2 - 10}{3^2 - 4} = (2 \cdot 5^2 - 10) \div (3^2 - 4)$ — Rewrite using parentheses.

$= (50 - 10) \div (9 - 4)$ — Evaluate powers and multiply within symbols of grouping.

$= 40 \div 5 = 8$ — Subtract within symbols of grouping, then divide.

**CHECKPOINT** *Now try Exercise 109.*

**4 ▶** Evaluate expressions using a calculator and order of operations.

## Calculators and Order of Operations

When using your own calculator, be sure that you are familiar with the use of each of the keys. Two possible keystroke sequences are given in Example 14: one for a standard *scientific* calculator, and one for a *graphing* calculator.

**Technology: Tip**

Be sure you see the difference between the change sign key [+/−] and the subtraction key [−] on a scientific calculator. Also notice the difference between the negation key [(−)] and the subtraction key [−] on a graphing calculator.

**Technology: Discovery**

To discover if your calculator performs the established order of operations, evaluate $7 + 5 \cdot 3 - 2^4 \div 4$ exactly as it appears. If your calculator performs the established order of operations, it will display 18.

### EXAMPLE 14 Evaluating Expressions on a Calculator

**a.** To evaluate the expression $7 - (5 \cdot 3)$, use the following keystrokes.

| *Keystrokes* | *Display* | |
|---|---|---|
| 7 [−] [(] 5 [×] 3 [)] [=] | −8 | Scientific |
| 7 [−] [(] 5 [×] 3 [)] [ENTER] | −8 | Graphing |

**b.** To evaluate the expression $(-3)^2 + 4$, use the following keystrokes.

| *Keystrokes* | *Display* | |
|---|---|---|
| 3 [+/−] [x²] [+] 4 [=] | 13 | Scientific |
| [(] [(−)] 3 [)] [x²] [+] 4 [ENTER] | 13 | Graphing |

**c.** To evaluate the expression $5/(4 + 3 \cdot 2)$, use the following keystrokes.

| *Keystrokes* | *Display* | |
|---|---|---|
| 5 [÷] [(] 4 [+] 3 [×] 2 [)] [=] | 0.5 | Scientific |
| 5 [÷] [(] 4 [+] 3 [×] 2 [)] [ENTER] | .5 | Graphing |

**CHECKPOINT** *Now try Exercise 125.*

## Concept Check

**1.** Is the reciprocal of every nonzero integer an integer?

**2.** Can the sum of two real numbers be less than either number? If so, give an example.

**3.** Explain how to subtract one real number from another.

**4.** If $a > 0$, state the values of $n$ such that $(-a)^n = -a^n$.

## 1.2 EXERCISES

Go to pages 50–51 to record your assignments.

### Developing Skills

In Exercises 1–38, evaluate the expression. ***See Examples 1–7.***

**1.** $13 + 32$

**2.** $16 + 84$

**3.** $-8 + 12$

**4.** $-5 + 9$

**5.** $-6.4 + 3.7$

**6.** $-5.1 + 0.9$

**7.** $13 + (-6)$

**8.** $12 + (-10)$

**9.** $12.6 + (-38.5)$

**10.** $10.4 + (-43.5)$

**11.** $-8 - 12$

**12.** $-3 - 17$

**13.** $-21.5 - (-6.3)$

**14.** $-13.2 - 9.6$

**15.** $4 - (-11) + 9$

**16.** $-17 + 6 - (-24)$

**17.** $5.3 - 2.2 - 6.9$

**18.** $46.08 - 35.1 - 16.25$

**19.** $15 - 6 + 31 + (-18)$

**20.** $6 + 26 - 17 + (-10)$

**21.** $\frac{3}{8} + \frac{7}{8}$

**22.** $\frac{5}{6} + \frac{7}{6}$

**23.** $\frac{3}{4} - \frac{1}{4}$

**24.** $\frac{5}{9} - \frac{1}{9}$

**25.** $\frac{3}{5} + \left(-\frac{1}{2}\right)$

**26.** $\frac{6}{7} + \left(-\frac{3}{7}\right)$

**27.** $\frac{5}{8} + \frac{1}{4} - \frac{5}{6}$

**28.** $\frac{3}{10} - \frac{5}{2} + \frac{1}{5}$

**29.** $3\frac{1}{2} + 4\frac{3}{8}$

**30.** $5\frac{3}{4} + 7\frac{3}{8}$

**31.** $10\frac{5}{8} - 6\frac{1}{4}$

**32.** $8\frac{1}{2} - 4\frac{2}{3}$

**33.** $85 - |-25|$

**34.** $-36 + |-8|$

**35.** $-(-11.325) + |34.625|$

**36.** $|-16.25| - 54.78$

**37.** $-\left|-6\frac{7}{8}\right| - 8\frac{1}{4}$

**38.** $-\left|-15\frac{2}{3}\right| - 12\frac{1}{3}$

In Exercises 39–44, write the expression as a multiplication problem.

**39.** $9 + 9 + 9 + 9$

**40.** $(-15) + (-15) + (-15) + (-15)$

**41.** $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

**42.** $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$

**43.** $\left(-\frac{1}{5}\right) + \left(-\frac{1}{5}\right) + \left(-\frac{1}{5}\right) + \left(-\frac{1}{5}\right)$

**44.** $\left(-\frac{5}{22}\right) + \left(-\frac{5}{22}\right) + \left(-\frac{5}{22}\right)$

In Exercises 45–62, find the product. ***See Examples 8 and 9.***

**45.** $5(-6)$

**46.** $3(-9)$

**47.** $(-8)(-6)$

**48.** $(-4)(-7)$

**49.** $2(4)(-5)$

**50.** $3(-7)(10)$

**51.** $(-1)(12)(-3)$

**52.** $(-2)(-6)(4)$

**53.** $\left(-\frac{5}{8}\right)\left(-\frac{4}{5}\right)$

**54.** $\left(-\frac{4}{7}\right)\left(-\frac{4}{5}\right)$

**55.** $-\frac{3}{2}\left(\frac{8}{5}\right)$

**56.** $\left(\frac{10}{13}\right)\left(-\frac{3}{5}\right)$

**57.** $\frac{1}{2}\left(\frac{1}{6}\right)$

**58.** $\frac{1}{3}\left(\frac{2}{3}\right)$

**59.** $-\frac{9}{8}\left(\frac{16}{27}\right)\left(\frac{1}{2}\right)$

**60.** $\frac{2}{3}\left(-\frac{18}{5}\right)\left(-\frac{5}{6}\right)$

**61.** $\frac{1}{3}\left(-\frac{3}{4}\right)(2)$

**62.** $\frac{2}{5}(-3)\left(\frac{10}{9}\right)$

In Exercises 63–68, find the reciprocal.

**63.** $6$

**64.** $4$

**65.** $\frac{2}{3}$

**66.** $\frac{9}{5}$

**67.** $-\frac{9}{7}$

**68.** $-\frac{2}{13}$

In Exercises 69–82, evaluate the expression. ***See Example 10.***

**69.** $\dfrac{-18}{-3}$

**70.** $-\dfrac{30}{-15}$

**71.** $-48 \div 16$

**72.** $-72 \div 12$

**73.** $63 \div (-7)$

**74.** $-27 \div (-9)$

**75.** $-\frac{4}{5} \div \frac{8}{25}$

**76.** $-\frac{11}{12} \div \frac{5}{24}$

**77.** $\left(-\frac{1}{3}\right) \div \left(-\frac{5}{6}\right)$

**78.** $\left(-\frac{3}{8}\right) \div \left(-\frac{4}{3}\right)$

**79.** $-4\frac{1}{4} \div -5\frac{5}{8}$

**80.** $-3\frac{5}{6} \div -2\frac{2}{3}$

**81.** $4\frac{1}{8} \div 4\frac{1}{2}$

**82.** $26\frac{2}{3} \div 10\frac{5}{6}$

In Exercises 83–88, write the expression using exponential notation.

**83.** $(-7) \cdot (-7) \cdot (-7)$

**84.** $(-4)(-4)(-4)(-4)(-4)(-4)$

**85.** $\left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right)$

**86.** $\left(\frac{5}{8}\right) \cdot \left(\frac{5}{8}\right) \cdot \left(\frac{5}{8}\right) \cdot \left(\frac{5}{8}\right)$

**87.** $-(7 \cdot 7 \cdot 7)$

**88.** $-(5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5)$

In Exercises 89–102, evaluate the exponential expression. ***See Example 11.***

**89.** $2^5$

**90.** $5^3$

**91.** $(-2)^4$

**92.** $(-3)^3$

**93.** $-4^3$

**94.** $-6^4$

**95.** $\left(\frac{4}{5}\right)^3$

**96.** $\left(\frac{2}{3}\right)^4$

**97.** $-\left(-\frac{1}{2}\right)^5$

**98.** $\left(-\frac{3}{4}\right)^3$

**99.** $(0.3)^3$

**100.** $(0.2)^4$

**101.** $5(-0.4)^3$

**102.** $-3(0.8)^2$

In Exercises 103–124, evaluate the expression. ***See Examples 12 and 13.***

**103.** $16 - 6 - 10$

**104.** $18 - 12 + 4$

**105.** $24 - 5 \cdot 2^2$

**106.** $18 + 3^2 - 12$

**107.** $28 \div 4 + 3 \cdot 5$

**108.** $6 \cdot 7 - 6^2 \div 4$

**109.** $14 - 2(8 - 4)$

**110.** $21 - 5(7 - 5)$

**111.** $17 - 5(16 \div 4^2)$

**112.** $72 - 8(6^2 \div 9)$

**113.** $5^2 - 2[9 - (18 - 8)]$

**114.** $8 \cdot 3^2 - 4(12 + 3)$

**115.** $5^3 + |-14 + 4|$

**116.** $|(-2)^5| - (25 + 7)$

**117.** $\dfrac{6 + 8(3)}{7 - 12}$

**118.** $\dfrac{9 + 6(2)}{3 + 4}$

**119.** $\dfrac{4^2 - 5}{11} - 7$

**120.** $\dfrac{5^3 - 50}{-15} + 27$

**121.** $\dfrac{6 \cdot 2^2 - 12}{3^2 + 3}$

**122.** $\dfrac{7^2 - 2(11)}{5^2 + 8(-2)}$

**123.** $\dfrac{3 + \frac{3}{4}}{\frac{1}{8}}$

**124.** $\dfrac{6 - \frac{2}{3}}{\frac{4}{9}}$

In Exercises 125–130, evaluate the expression using a calculator. Round your answer to two decimal places. ***See Example 14.***

**125.** $5.6[13 - 2.5(-6.3)]$

**126.** $6.9[6.1(-4.2) + 16]$

**127.** $5^6 - 3(400)$

**128.** $300(1.09)^{10} + (-156.24)$

**129.** $\dfrac{500}{(1.055)^{20}}$

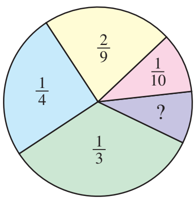
**130.** $5(100 - 3.6^4) \div 4.1$



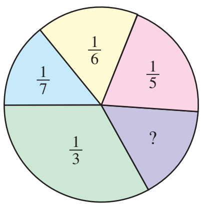
## Solving Problems

***Circle Graphs*** In Exercises 131 and 132, find the unknown fractional part of the circle graph.

**131.**




**132.**



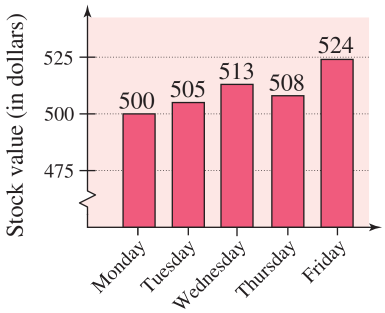

**133.** ***Account Balance*** During one month, you made the following transactions in your non-interest-bearing checking account. Find the balance at the end of the month.

| NUMBER OR CODE | DATE | TRANSACTION DESCRIPTION | PAYMENT AMOUNT | ✓ | FEE | DEPOSIT AMOUNT | BALANCE $2618.68 |
|---|---|---|---|---|---|---|---|
| | 3/1 | Pay | | | | $1236 45 | |
| 2154 | 3/3 | Magazine | $ 25 62 | | | | |
| 2155 | 3/6 | Insurance | $455 00 | | | | |
| | 3/12 | Withdrawal | $ 125 00 | | | | |
| 2156 | 3/15 | Mortgage | $ 715 95 | | | | |

Figure for 133

**134.** ***Profit*** The midyear financial statement of a clothing company showed a profit of $1,345,298.55. At the close of the year, the financial statement showed a profit for the year of $867,132.87. Find the profit (or loss) of the company for the second 6 months of the year.

**135.** ***Stock Values*** On Monday you purchased \$500 worth of stock. The values of the stock during the remainder of the week are shown in the bar graph.

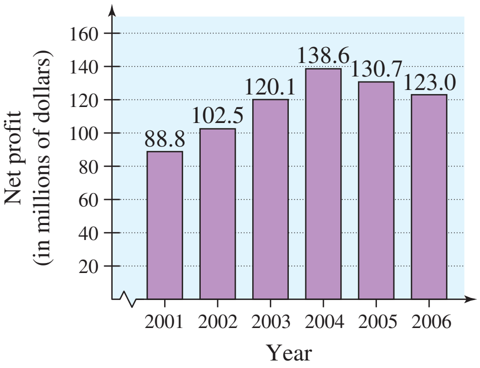



(a) Use the graph to complete the table.

| Day | Daily gain or loss |
|---|---|
| Tuesday | |
| Wednesday | |
| Thursday | |
| Friday | |

(b) Find the sum of the daily gains and losses. Interpret the result in the context of the problem. How could you determine this sum from the graph?

**136.** ***Net Profit*** The net profits for Columbia Sportswear (in millions of dollars) for the years 2001 to 2006 are shown in the bar graph. Use the graph to create a table that shows the yearly gains or losses. (Source: Columbia Sportswear Company)




**137.** ***Savings Plan***

(a) You save \$50 per month for 18 years. How much money has been set aside during the 18 years?

(b) If the money in part (a) is deposited in a savings account earning 4% interest compounded monthly, the total amount in the account after 18 years will be

$$50\left[\left(1+\frac{0.04}{12}\right)^{216}-1\right]\left(1+\frac{12}{0.04}\right).$$

Use a calculator to determine this amount.

(c) How much of the amount in part (b) is earnings from interest?

**138.** ***Savings Plan***

(a) You save \$60 per month for 30 years. How much money has been set aside during the 30 years?

(b) If the money in part (a) is deposited in a savings account earning 3% interest compounded monthly, the total amount in the account after 30 years will be

$$60\left[\left(1+\frac{0.03}{12}\right)^{360}-1\right]\left(1+\frac{12}{0.03}\right).$$

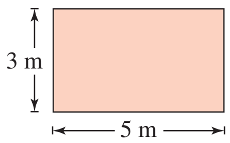
Use a calculator to determine this amount.

(c) How much of the amount in part (b) is earnings from interest?

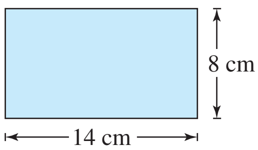


***Geometry*** In Exercises 139–142, find the area of the figure. (The area $A$ of a rectangle is given by $A = \text{length} \cdot \text{width}$, and the area $A$ of a triangle is given by $A = \frac{1}{2} \cdot \text{base} \cdot \text{height}$.)

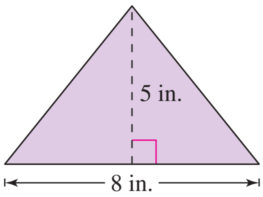
**139.**



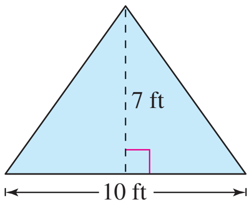

**140.**




**141.**




**142.**



***Volume*** In Exercises 143 and 144, use the following information. A bale of hay is a rectangular solid weighing approximately 50 pounds. It has a length of 42 inches, a width of 18 inches, and a height of 14 inches. (The volume $V$ of a rectangular solid is given by $V = \text{length} \cdot \text{width} \cdot \text{height}$.)

**143.** Find the volume of a bale of hay in cubic feet if 1728 cubic inches equals 1 cubic foot.

**144.** Approximate the number of bales in a ton of hay. Then approximate the volume of a stack of baled hay in cubic feet that weighs 12 tons. (2000 lb = 1 ton)

## Explaining Concepts

***True or False?*** In Exercises 145–149, determine whether the statement is true or false. Justify your answer.

**145.** The reciprocal of every nonzero rational number is a rational number.

**146.** The product of two fractions is the product of the numerators over the LCD.

**147.** If a negative real number is raised to the 12th power, the result will be positive.

**148.** If a negative real number is raised to the 11th power, the result will be positive.

**149.** $a \div b = b \div a$

**150.** Are the expressions $(2^2)^3$ and $2^{(2^3)}$ equal? Explain.

**151.** In your own words, describe the rules for determining the sign of the product or the quotient of two real numbers.

**152.** In your own words, describe the established order of operations for addition and subtraction. Without these priorities, explain why the expression $6 - 5 - 2$ would be ambiguous.

**153.** Decide which expressions are equal to 27 when you follow the standard order of operations. For the expressions that are not equal to 27, see if you can discover a way to insert symbols of grouping that make the expression equal to 27. Discuss the value of symbols of grouping in mathematical communication.

(a) $40 - 10 + 3$

(b) $5^2 + \frac{1}{2} \cdot 4$

(c) $8 \cdot 3 + 30 \div 2$

(d) $75 \div 2 + 1 + 2$

***Error Analysis*** In Exercises 154–156, describe and correct the error.

**154.** $\frac{2}{3} + \frac{3}{2} = \frac{2+3}{3+2} = 1$

**155.** $\frac{5+12}{5} = \frac{\cancel{5}+12}{\cancel{5}} = 12$

**156.** $3 \cdot 4^2 = 12^2$

# 1.3 Properties of Real Numbers

### What You Should Learn

1 ▶ Identify and use the properties of real numbers.

2 ▶ Develop additional properties of real numbers.

### Why You Should Learn It

Understanding the properties of real numbers will help you to understand and use the properties of algebra.

1 ▶ Identify and use the properties of real numbers.

## Basic Properties of Real Numbers

The following list summarizes the basic properties of addition and multiplication. Although the examples involve real numbers, these properties can also be applied to algebraic expressions.

### Properties of Real Numbers

Let $a$, $b$, and $c$ represent real numbers, variables, or algebraic expressions.

| *Property* | *Example* |
|---|---|
| Commutative Property of Addition: | |
| $a + b = b + a$ | $3 + 5 = 5 + 3$ |
| Commutative Property of Multiplication: | |
| $ab = ba$ | $2 \cdot 7 = 7 \cdot 2$ |
| Associative Property of Addition: | |
| $(a + b) + c = a + (b + c)$ | $(4 + 2) + 3 = 4 + (2 + 3)$ |
| Associative Property of Multiplication: | |
| $(ab)c = a(bc)$ | $(2 \cdot 5) \cdot 7 = 2 \cdot (5 \cdot 7)$ |
| Distributive Property: | |
| $a(b + c) = ab + ac$ | $4(7 + 3) = 4 \cdot 7 + 4 \cdot 3$ |
| $(a + b)c = ac + bc$ | $(2 + 5)3 = 2 \cdot 3 + 5 \cdot 3$ |
| $a(b - c) = ab - ac$ | $6(5 - 3) = 6 \cdot 5 - 6 \cdot 3$ |
| $(a - b)c = ac - bc$ | $(7 - 2)4 = 7 \cdot 4 - 2 \cdot 4$ |
| Additive Identity Property: | |
| $a + 0 = 0 + a = a$ | $9 + 0 = 0 + 9 = 9$ |
| Multiplicative Identity Property: | |
| $a \cdot 1 = 1 \cdot a = a$ | $-5 \cdot 1 = 1 \cdot (-5) = -5$ |
| Additive Inverse Property: | |
| $a + (-a) = 0$ | $3 + (-3) = 0$ |
| Multiplicative Inverse Property: | |
| $a \cdot \frac{1}{a} = 1, \quad a \neq 0$ | $8 \cdot \frac{1}{8} = 1$ |

### Study Tip

The operations of subtraction and division are not listed at the right because they do not have many of the properties of real numbers. For instance, subtraction and division are not commutative or associative. To see this, consider the following.

$4 - 3 \neq 3 - 4$

$15 \div 5 \neq 5 \div 15$

$8 - (6 - 2) \neq (8 - 6) - 2$

$20 \div (4 \div 2) \neq (20 \div 4) \div 2$

### EXAMPLE 1 Identifying Properties of Real Numbers

Identify the property of real numbers illustrated by each statement.

**a.** $4(a + 3) = 4 \cdot a + 4 \cdot 3$ **b.** $6 \cdot \frac{1}{6} = 1$

**c.** $-3 + (2 + b) = (-3 + 2) + b$

**d.** $(b + 8) + 0 = b + 8$

#### Solution

**a.** This statement illustrates the Distributive Property.

**b.** This statement illustrates the Multiplicative Inverse Property.

**c.** This statement illustrates the Associative Property of Addition.

**d.** This statement illustrates the Additive Identity Property, where $(b + 8)$ is an algebraic expression.

**CHECKPOINT** *Now try Exercise 5.*

The properties of real numbers make up the third component of what is called a **mathematical system.** These three components are a *set of numbers* (Section 1.1), *operations* with the set of numbers (Section 1.2), and *properties* of the operations with the numbers (Section 1.3).

Set of Numbers → Operations with the Numbers → Properties of the Operations

Note that the properties of real numbers can be applied to variables and algebraic expressions as well as to real numbers.

### EXAMPLE 2 Using the Properties of Real Numbers



Complete each statement using the specified property of real numbers.

**a.** Multiplicative Identity Property: $(4a)1 = \square$

**b.** Associative Property of Addition: $(b + 8) + 3 = \square$

**c.** Additive Inverse Property: $0 = 5c + \square$

**d.** Distributive Property: $7 \cdot b + 7 \cdot 5 = \square$

#### Solution

**a.** By the Multiplicative Identity Property, $(4a)1 = 4a$.

**b.** By the Associative Property of Addition, $(b + 8) + 3 = b + (8 + 3)$.

**c.** By the Additive Inverse Property, $0 = 5c + (-5c)$.

**d.** By the Distributive Property, $7 \cdot b + 7 \cdot 5 = 7(b + 5)$.

**CHECKPOINT** *Now try Exercise 21.*

To help you understand each property of real numbers, try stating the properties in your own words. For instance, the Associative Property of Addition can be stated as follows: *When three real numbers are added, it makes no difference which two are added first.*

**2** ▶ Develop additional properties of real numbers.

## Additional Properties of Real Numbers

Once you have determined the basic properties (or *axioms*) of a mathematical system, you can go on to develop other properties. These additional properties are **theorems,** and the formal arguments that justify the theorems are **proofs.**

### Additional Properties of Real Numbers

Let $a$, $b$, and $c$ be real numbers, variables, or algebraic expressions.

**Properties of Equality**

| | |
|---|---|
| Addition Property of Equality: | If $a = b$, then $a + c = b + c$. |
| Multiplication Property of Equality: | If $a = b$, then $ac = bc$. |
| Cancellation Property of Addition: | If $a + c = b + c$, then $a = b$. |
| Cancellation Property of Multiplication: | If $ac = bc$ and $c \neq 0$, then $a = b$. |

**Properties of Zero**

| | |
|---|---|
| Multiplication Property of Zero: | $0 \cdot a = 0$ |
| Division Property of Zero: | $\frac{0}{a} = 0, \quad a \neq 0$ |
| Division by Zero Is Undefined: | $\frac{a}{0}$ is undefined. |

**Properties of Negation**

| | |
|---|---|
| Multiplication by $-1$: | $(-1)(a) = -a$, $(-1)(-a) = a$ |
| Placement of Negative Signs: | $(-a)(b) = -(ab) = (a)(-b)$ |
| Product of Two Opposites: | $(-a)(-b) = ab$ |

**Study Tip**

When the properties of real numbers are used in practice, the process is usually less formal than it would appear from the list of properties on this page. For instance, the steps shown at the right are less formal than those shown in Examples 5 and 6 on page 27. The importance of the properties is that they can be used to justify the steps of a solution. They do not always need to be listed for every step of the solution.

In Section 2.1, you will see that the properties of equality are useful for solving equations, as shown below. Note that the Addition and Multiplication Properties of Equality can be used to subtract the same nonzero quantity from each side of an equation or to divide each side of an equation by the same nonzero quantity.

| | |
|---|---|
| $5x + 4 = -2x + 18$ | Original equation |
| $5x + 4 - 4 = -2x + 18 - 4$ | Subtract 4 from each side. |
| $5x = -2x + 14$ | Simplify. |
| $5x + 2x = -2x + 2x + 14$ | Add $2x$ to each side. |
| $7x = 14$ | Simplify. |
| $\frac{7x}{7} = \frac{14}{7}$ | Divide each side by 7. |
| $x = 2$ | Simplify. |

Each of the additional properties in the list on the preceding page can be proved by using the basic properties of real numbers.

## EXAMPLE 3 Proof of the Cancellation Property of Addition

Prove that if $a + c = b + c$, then $a = b$. (Use the Addition Property of Equality.)

### Solution

Notice how each step is justified from the preceding step by means of a property of real numbers.

| | |
|---|---|
| $a + c = b + c$ | Write original equation. |
| $(a + c) + (-c) = (b + c) + (-c)$ | Addition Property of Equality |
| $a + [c + (-c)] = b + [c + (-c)]$ | Associative Property of Addition |
| $a + 0 = b + 0$ | Additive Inverse Property |
| $a = b$ | Additive Identity Property |

**CHECKPOINT** *Now try Exercise 59.*

## EXAMPLE 4 Proof of a Property of Negation

Prove that $(-1)a = -a$. (You may use any of the properties of equality and properties of zero.)

### Solution

At first glance, it is a little difficult to see what you are being asked to prove. However, a good way to start is to consider carefully the definitions of the three numbers in the equation.

$a$ = given real number

$-1$ = the additive inverse of 1

$-a$ = the additive inverse of $a$

Now, by showing that $(-1)a$ has the same properties as the additive inverse of $a$, you will be showing that $(-1)a$ must be the additive inverse of $a$.

| | |
|---|---|
| $(-1)a + a = (-1)a + (1)(a)$ | Multiplicative Identity Property |
| $= (-1 + 1)a$ | Distributive Property |
| $= (0)a$ | Additive Inverse Property |
| $= 0$ | Multiplication Property of Zero |

Because $(-1)a + a = 0$, you can use the fact that $-a + a = 0$ to conclude that $(-1)a + a = -a + a$. From this, you can complete the proof as follows.

| | |
|---|---|
| $(-1)a + a = -a + a$ | Shown in first part of proof |
| $(-1)a = -a$ | Cancellation Property of Addition |

**CHECKPOINT** *Now try Exercise 61.*

The list of additional properties of real numbers on page 25 forms a very important part of algebra. Knowing the names of the properties is useful, but knowing how to use each property is extremely important. The next two examples show how several of the properties can be used to solve equations. (You will study these techniques in detail in Section 2.1.)

## EXAMPLE 5 Applying the Properties of Real Numbers

In the solution of the equation

$$b + 2 = 6$$

identify the property of real numbers that justifies each step.

**Solution**

| Solution Step | Property |
|---|---|
| $b + 2 = 6$ | Original equation |
| $(b + 2) + (-2) = 6 + (-2)$ | Addition Property of Equality |
| $b + [2 + (-2)] = 6 - 2$ | Associative Property of Addition |
| $b + 0 = 4$ | Additive Inverse Property |
| $b = 4$ | Additive Identity Property |

**CHECKPOINT** *Now try Exercise 63.*

## EXAMPLE 6 Applying the Properties of Real Numbers

In the solution of the equation

$$3x = 15$$

identify the property of real numbers that justifies each step.

**Solution**

| Solution Step | Property |
|---|---|
| $3x = 15$ | Original equation |
| $\left(\frac{1}{3}\right)3x = \left(\frac{1}{3}\right)15$ | Multiplication Property of Equality |
| $\left(\frac{1}{3} \cdot 3\right)x = 5$ | Associative Property of Multiplication |
| $(1)(x) = 5$ | Multiplicative Inverse Property |
| $x = 5$ | Multiplicative Identity Property |

**CHECKPOINT** *Now try Exercise 65.*

## Concept Check

1. Does every real number have an additive inverse? Explain.
2. Does every real number have a multiplicative inverse? Explain.
3. Are subtraction and division commutative? If not, show a counterexample.
4. Are subtraction and division associative? If not, show a counterexample.

## 1.3 EXERCISES

Go to pages 50–51 to record your assignments.

### Developing Skills

In Exercises 1–20, identify the property of real numbers illustrated by the statement. *See Example 1.*

1. $18 - 18 = 0$
2. $5 + 0 = 5$
3. $\frac{1}{12} \cdot 12 = 1$
4. $52 \cdot 1 = 52$
5. $13 + 12 = 12 + 13$
6. $(-4 \cdot 10) \cdot 8 = -4(10 \cdot 8)$
7. $3 + (12 - 9) = (3 + 12) - 9$
8. $(5 + 10)(8) = 8(5 + 10)$
9. $(8 - 5)(10) = 8 \cdot 10 - 5 \cdot 10$
10. $7(9 + 15) = 7 \cdot 9 + 7 \cdot 15$
11. $10(2x) = (10 \cdot 2)x$
12. $1 \cdot 9k = 9k$
13. $10x \cdot \dfrac{1}{10x} = 1$
14. $0 + 4x = 4x$
15. $2x - 2x = 0$
16. $4 + (3 - x) = (4 + 3) - x$
17. $3(2 + x) = 3 \cdot 2 + 3x$
18. $3(6 + b) = 3 \cdot 6 + 3 \cdot b$
19. $(x + 1) - (x + 1) = 0$
20. $6(x + 3) = 6 \cdot x + 6 \cdot 3$

In Exercises 21–28, complete the statement using the specified property of real numbers. *See Example 2.*

21. Commutative Property of Multiplication: $15(-3) =$ ______
22. Associative Property of Addition: $6 + (5 + y) =$ ______
23. Distributive Property: $5(6 + z) =$ ______
24. Distributive Property: $(8 - y)(4) =$ ______
25. Commutative Property of Addition: $25 + (-x) =$ ______
26. Additive Inverse Property: $13x + (-13x) =$ ______
27. Multiplicative Identity Property: $(x + 8) \cdot 1 =$ ______
28. Additive Identity Property: $(8x) + 0 =$ ______

In Exercises 29–40, give (a) the additive inverse and (b) the multiplicative inverse of the quantity.

29. $10$
30. $18$
31. $-19$
32. $-37$
33. $\frac{1}{2}$
34. $\frac{3}{4}$
35. $-\frac{5}{8}$
36. $-\frac{1}{5}$
37. $6z, \quad z \neq 0$
38. $2y, \quad y \neq 0$
39. $x - 2, \quad x \neq 2$
40. $y - 7, \quad y \neq 7$

In Exercises 41–44, rewrite the expression using the Associative Property of Addition or the Associative Property of Multiplication.

**41.** $32 + (4 + y)$

**42.** $15 + (3 - x)$

**43.** $9(6m)$

**44.** $11(4n)$

In Exercises 45–50, rewrite the expression using the Distributive Property.

**45.** $20(2 + 5)$

**46.** $-3(4 - 8)$

**47.** $(x + 6)(-2)$

**48.** $(z - 10)(12)$

**49.** $-6(2y - 5)$

**50.** $-4(10 - b)$

In Exercises 51–54, use the Distributive Property to simplify the expression.

**51.** $7x + 2x$

**52.** $8x - 6x$

**53.** $\frac{7x}{8} - \frac{5x}{8}$

**54.** $\frac{3x}{5} + \frac{x}{5}$

In Exercises 55–58, the right side of the statement does not equal the left side. Change the right side so that it *does* equal the left side.

**55.** $3(x + 5) \neq 3x + 5$

**56.** $4(x + 2) \neq 4x + 2$

**57.** $-2(x + 8) \neq -2x + 16$

**58.** $-9(x + 4) \neq -9x + 36$



In Exercises 59–62, use the basic properties of real numbers to prove the statement. ***See Examples 3 and 4.***

**59.** If $ac = bc$ and $c \neq 0$, then $a = b$.

**60.** If $a + c = b + c$, then $a = b$.

**61.** $a = (a + b) + (-b)$

**62.** $a + (-a) = 0$

In Exercises 63–66, identify the property of real numbers that justifies each step. ***See Examples 5 and 6.***

**63.**
$$\begin{aligned} x + 5 &= 3 \\ (x + 5) + (-5) &= 3 + (-5) \\ x + [5 + (-5)] &= -2 \\ x + 0 &= -2 \\ x &= -2 \end{aligned}$$

**64.**
$$\begin{aligned} x - 8 &= 20 \\ (x - 8) + 8 &= 20 + 8 \\ x + (-8 + 8) &= 28 \\ x + 0 &= 28 \\ x &= 28 \end{aligned}$$

**65.**
$$\begin{aligned} 2x - 5 &= 6 \\ (2x - 5) + 5 &= 6 + 5 \\ 2x + (-5 + 5) &= 11 \\ 2x + 0 &= 11 \\ 2x &= 11 \\ \tfrac{1}{2}(2x) &= \tfrac{1}{2}(11) \\ \left(\tfrac{1}{2} \cdot 2\right)x &= \tfrac{11}{2} \\ 1 \cdot x &= \tfrac{11}{2} \\ x &= \tfrac{11}{2} \end{aligned}$$

**66.**
$$\begin{aligned} 3x + 4 &= 10 \\ (3x + 4) + (-4) &= 10 + (-4) \\ 3x + [4 + (-4)] &= 6 \\ 3x + 0 &= 6 \\ 3x &= 6 \\ \tfrac{1}{3}(3x) &= \tfrac{1}{3}(6) \\ \left(\tfrac{1}{3} \cdot 3\right)x &= 2 \\ 1 \cdot x &= 2 \\ x &= 2 \end{aligned}$$

***Mental Math*** In Exercises 67–72, use the Distributive Property to perform the arithmetic mentally. For example, you work in an industry in which the wage is \$14 per hour with "time and a half" for overtime. So, your hourly wage for overtime is

$$14(1.5) = 14\left(1 + \frac{1}{2}\right) = 14 + 7 = \$21.$$

**67.** $16(1.75) = 16\left(2 - \frac{1}{4}\right)$

**68.** $15\left(1\frac{2}{3}\right) = 15\left(2 - \frac{1}{3}\right)$

**69.** $7(62) = 7(60 + 2)$

**70.** $5(51) = 5(50 + 1)$

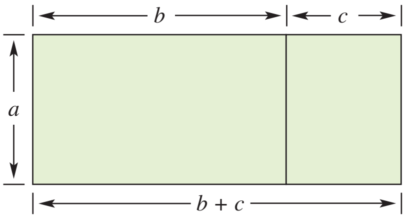
**71.** $9(6.98) = 9(7 - 0.02)$

**72.** $12(19.95) = 12(20 - 0.05)$

## Solving Problems

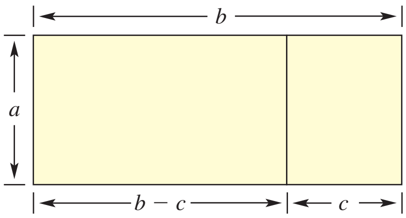
**73.** *Geometry* The figure shows two adjoining rectangles. Demonstrate the Distributive Property by filling in the blanks to write the total area of the two rectangles in two ways.

$\square(\square + \square) = \square + \square$



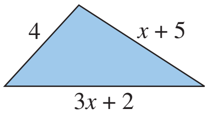

**74.** *Geometry* The figure shows two adjoining rectangles. Demonstrate the "subtraction version" of the Distributive Property by filling in the blanks to write the area of the left rectangle in two ways.

$\square(\square - \square) = \square - \square$

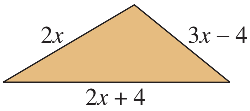



*Geometry* In Exercises 75 and 76, write the expression for the perimeter of the triangle shown in the figure. Use the properties of real numbers to simplify the expression.

**75.**

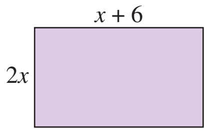



**76.**

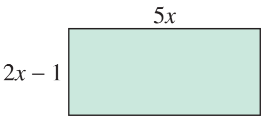



*Geometry* In Exercises 77 and 78, write and simplify the expression for (a) the perimeter and (b) the area of the rectangle.

**77.**




**78.**




## Explaining Concepts

**79.** What is the additive inverse of a real number? Give an example of the Additive Inverse Property.

**80.** What is the multiplicative inverse of a real number? Give an example of the Multiplicative Inverse Property.

**81.** In your own words, give a verbal description of the Commutative Property of Addition.

**82.** Explain how the Addition Property of Equality can be used to allow you to subtract the same number from each side of an equation.

**83.** You define a new mathematical operation using the symbol $\odot$. This operation is defined as $a \odot b = 2 \cdot a + b$. Give examples to show that this operation is neither commutative nor associative.

**84.** You define a new mathematical operation using the symbol $\ddagger$. This operation is defined as $a \ddagger b = a - (b + 1)$. Give examples to show that this operation is neither commutative nor associative.

## Mid-Chapter Quiz

**Take this quiz as you would take a quiz in class. After you are done, check your work against the answers in the back of the book.**

**In Exercises 1 and 2, plot the two real numbers on the real number line and place the correct inequality symbol (< or >) between the two numbers.**

**1.** $-4.5 \quad \square \quad -6$

**2.** $\frac{3}{4} \quad \square \quad \frac{3}{2}$

**In Exercises 3 and 4, find the distance between the two real numbers.**

**3.** $-15$ and $7$

**4.** $-8.75$ and $-2.25$

**In Exercises 5 and 6, evaluate the expression.**

**5.** $|-7.6|$

**6.** $-|9.8|$

**In Exercises 7–16, evaluate the expression. Write fractions in simplest form.**

**7.** $32 + (-18)$

**8.** $-12 - (-17)$

**9.** $\frac{3}{4} + \frac{7}{4}$

**10.** $\frac{2}{3} - \frac{1}{6}$

**11.** $(-3)(2)(-10)$

**12.** $\left(-\frac{4}{5}\right)\left(\frac{15}{32}\right)$

**13.** $\frac{7}{12} \div \frac{5}{6}$

**14.** $\left(-\frac{3}{2}\right)^3$

**15.** $3 - 2^2 + 25 \div 5$

**16.** $\dfrac{18 - 2(3 + 4)}{6^2 - (12 \cdot 2 + 10)}$

**In Exercises 17 and 18, identify the property of real numbers illustrated by each statement.**



**17.** (a) $8(u - 5) = 8 \cdot u - 8 \cdot 5$ (b) $10x - 10x = 0$

**18.** (a) $(7 + y) - z = 7 + (y - z)$ (b) $2x \cdot 1 = 2x$

**19.** During one month, you made the following transactions in your non-interest-bearing checking account. Find the balance at the end of the month.

| NUMBER OR CODE | DATE | TRANSACTION DESCRIPTION | PAYMENT AMOUNT | ✓ | FEE | DEPOSIT AMOUNT | BALANCE $1406.98 |
|---|---|---|---|---|---|---|---|
| 2103 | 1/5 | Car Payment | $375 03 | | | | |
| 2104 | 1/7 | Phone | $ 59 20 | | | | |
| | 1/8 | Withdrawal | $225 00 | | | | |
| | 1/12 | Deposit | | | | $320 45 | |

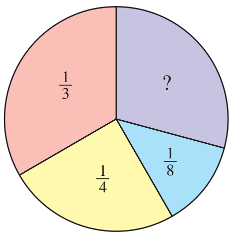



Figure for 21

**20.** You deposit $45 in a retirement account twice each month. How much will you deposit in the account in 8 years?

**21.** Determine the unknown fractional part of the circle graph at the left. Explain how you were able to make this determination.

# 1.4 Algebraic Expressions


William Thomas Cain/Getty Images

### Why You Should Learn It

Many real-life quantities can be determined by evaluating algebraic expressions. For instance, in Example 9 on page 36, you will evaluate an algebraic expression to find yearly revenues of gambling industries.

### What You Should Learn

1 ▶ Identify the terms and coefficients of algebraic expressions.

2 ▶ Simplify algebraic expressions by combining like terms and removing symbols of grouping.

3 ▶ Evaluate algebraic expressions by substituting values for the variables.

## Algebraic Expressions

1 ▶ Identify the terms and coefficients of algebraic expressions.

One of the basic characteristics of algebra is the use of letters (or combinations of letters) to represent numbers. The letters used to represent the numbers are called **variables,** and combinations of letters and numbers are called **algebraic expressions.** Here are some examples.

$$3x, \quad x + 2, \quad \frac{x}{x^2 + 1}, \quad 2x - 3y, \quad 2x^3 - y^2$$

### Algebraic Expression

A collection of letters (called **variables**) and real numbers (called **constants**) combined using the operations of addition, subtraction, multiplication, or division is called an **algebraic expression.**

### Study Tip

It is important to understand the difference between a *term* and a *factor*. Terms are separated by addition, whereas factors are separated by multiplication. For instance, the expression $4x(x + 2)$ has three factors: 4, $x$, and $(x + 2)$.

The **terms** of an algebraic expression are those parts that are separated by *addition*. For example, the algebraic expression $x^2 - 3x + 6$ has three terms: $x^2$, $-3x$, and 6. Note that $-3x$ is a term, rather than $3x$, because

$$x^2 - 3x + 6 = x^2 + (-3x) + 6.$$ Think of subtraction as a form of addition.

The terms $x^2$ and $-3x$ are the **variable terms** of the expression, and 6 is the **constant term.** The numerical factor of a term is called the **coefficient.** For instance, the coefficient of the variable term $-3x$ is $-3$, and the coefficient of the variable term $x^2$ is 1. Example 1 identifies the terms and coefficients of three different algebraic expressions.

### EXAMPLE 1 Identifying Terms and Coefficients

| *Algebraic Expression* | *Terms* | *Coefficients* |
|---|---|---|
| **a.** $5x - \frac{1}{3}$ | $5x, \quad -\frac{1}{3}$ | $5, \quad -\frac{1}{3}$ |
| **b.** $4y + 6x - 9$ | $4y, \quad 6x, \quad -9$ | $4, \quad 6, \quad -9$ |
| **c.** $\frac{2}{x} + 5x^4 - y$ | $\frac{2}{x}, \quad 5x^4, \quad -y$ | $2, \quad 5, \quad -1$ |

**CHECKPOINT** *Now try Exercise 1.*

**2** ▶ Simplify algebraic expressions by combining like terms and removing symbols of grouping.

## Simplifying Algebraic Expressions

In an algebraic expression, two terms are said to be **like terms** if they are both constant terms or if they have the same variable factor. For example, $2x^2y$, $-x^2y$, and $\frac{1}{2}(x^2y)$ are like terms because they have the same variable factor $x^2y$. Note that $4x^2y$ and $-x^2y^2$ are not like terms because their variable factors $x^2y$ and $x^2y^2$ are different.

One way to **simplify** an algebraic expression is to combine like terms.

### EXAMPLE 2 Combining Like Terms

Simplify each expression by combining like terms.

**a.** $2x + 3x - 4$ **b.** $-3 + 5 + 2y - 7y$ **c.** $5x + 3y - 4x$

**Solution**

**a.** $2x + 3x - 4 = (2 + 3)x - 4$ Distributive Property

$= 5x - 4$ Simplest form

**b.** $-3 + 5 + 2y - 7y = (-3 + 5) + (2 - 7)y$ Distributive Property

$= 2 - 5y$ Simplest form

**c.** $5x + 3y - 4x = 3y + 5x - 4x$ Commutative Property

$= 3y + (5x - 4x)$ Associative Property

$= 3y + (5 - 4)x$ Distributive Property

$= 3y + x$ Simplest form

**CHECKPOINT** *Now try Exercise 25.*

**Study Tip**

As you gain experience with the rules of algebra, you may want to combine some of the steps in your work. For instance, you might feel comfortable listing only the following steps to solve Example 2(c).

$$5x + 3y - 4x = 3y + (5x - 4x)$$
$$= 3y + x$$



### EXAMPLE 3 Combining Like Terms

Simplify each expression by combining like terms.

**a.** $7x + 7y - 4x - y$ **b.** $2x^2 + 3x - 5x^2 - x$

**c.** $3xy^2 - 4x^2y^2 + 2xy^2 + x^2y^2$

**Solution**

**a.** $7x + 7y - 4x - y = (7x - 4x) + (7y - y)$ Group like terms.

$= 3x + 6y$ Combine like terms.

**b.** $2x^2 + 3x - 5x^2 - x = (2x^2 - 5x^2) + (3x - x)$ Group like terms.

$= -3x^2 + 2x$ Combine like terms.

**c.** $3xy^2 - 4x^2y^2 + 2xy^2 + x^2y^2$

$= (3xy^2 + 2xy^2) + (-4x^2y^2 + x^2y^2)$ Group like terms.

$= 5xy^2 - 3x^2y^2$ Combine like terms.

**CHECKPOINT** *Now try Exercise 33.*

Another way to simplify an algebraic expression is to remove symbols of grouping. Remove the innermost symbols first and combine like terms. Repeat this process as needed to remove all the symbols of grouping.

A set of parentheses preceded by a *minus* sign can be removed by changing the sign of each term inside the parentheses. For instance,

$$3x - (2x - 7) = 3x - 2x + 7.$$

This is equivalent to using the Distributive Property with a multiplier of $-1$. That is,

$$3x - (2x - 7) = 3x + (-1)(2x - 7) = 3x - 2x + 7.$$

A set of parentheses preceded by a *plus* sign can be removed without changing the signs of the terms inside the parentheses. For instance,

$$3x + (2x - 7) = 3x + 2x - 7.$$

### EXAMPLE 4 Removing Symbols of Grouping

Simplify each expression.

**a.** $3(x - 5) - (2x - 7)$ **b.** $-4(x^2 + 4) + x^2(x + 4)$

**Solution**

**a.** $3(x - 5) - (2x - 7) = 3x - 15 - 2x + 7$ Distributive Property

$= (3x - 2x) + (-15 + 7)$ Group like terms.

$= x - 8$ Combine like terms.

**b.** $-4(x^2 + 4) + x^2(x + 4) = -4x^2 - 16 + x^2 \cdot x + 4x^2$ Distributive Property

$= -4x^2 - 16 + x^3 + 4x^2$ Exponential form

$= x^3 + (4x^2 - 4x^2) - 16$ Group like terms.

$= x^3 + 0 - 16$ Combine like terms.

$= x^3 - 16$ Additive Identity Property

**CHECKPOINT** *Now try Exercise 53.*

**Study Tip**

The exponential notation described in Section 1.2 can also be used when the base is a variable or an algebraic expression. For instance, in Example 4(b), $x^2 \cdot x$ can be written as

$x^2 \cdot x = x \cdot x \cdot x$

$= x^3.$ 3 factors of $x$



### EXAMPLE 5 Removing Symbols of Grouping

**a.** $5x - 2x[3 + 2(x - 7)] = 5x - 2x(3 + 2x - 14)$ Distributive Property

$= 5x - 2x(2x - 11)$ Combine like terms.

$= 5x - 4x^2 + 22x$ Distributive Property

$= -4x^2 + 27x$ Combine like terms.

**b.** $-3x(5x^4) + 2x^5 = -15x \cdot x^4 + 2x^5$ Multiply.

$= -15x^5 + 2x^5$ Exponential form

$= -13x^5$ Combine like terms.

**CHECKPOINT** *Now try Exercise 65.*

3 ▶ Evaluate algebraic expressions by substituting values for the variables.

## Evaluating Algebraic Expressions

To **evaluate** an algebraic expression, substitute numerical values for each of the variables in the expression. Note that you must substitute the value for *each* occurrence of the variable.

### EXAMPLE 6 Evaluating Algebraic Expressions

Evaluate each algebraic expression when $x = -2$.

**a.** $5 + x^2$ **b.** $5 - x^2$

#### Solution

**a.** When $x = -2$, the expression $5 + x^2$ has a value of

$$5 + (-2)^2 = 5 + 4 = 9.$$

**b.** When $x = -2$, the expression $5 - x^2$ has a value of

$$5 - (-2)^2 = 5 - 4 = 1.$$

**CHECKPOINT** *Now try Exercise 73.*

### EXAMPLE 7 Evaluating Algebraic Expressions

Evaluate each algebraic expression when $x = 2$ and $y = -1$.

**a.** $|y - x|$ **b.** $x^2 - 2xy + y^2$

#### Solution

**a.** When $x = 2$ and $y = -1$, the expression $|y - x|$ has a value of

$$|(-1) - (2)| = |-3| = 3.$$

**b.** When $x = 2$ and $y = -1$, the expression $x^2 - 2xy + y^2$ has a value of

$$2^2 - 2(2)(-1) + (-1)^2 = 4 + 4 + 1 = 9.$$

**CHECKPOINT** *Now try Exercise 81.*

### EXAMPLE 8 Evaluating an Algebraic Expression

Evaluate $\frac{2xy}{x + 1}$ when $x = -4$ and $y = -3$.

#### Solution

When $x = -4$ and $y = -3$, the expression $(2xy)/(x + 1)$ has a value of

$$\frac{2(-4)(-3)}{-4 + 1} = \frac{24}{-3} = -8.$$

**CHECKPOINT** *Now try Exercise 85.*

## EXAMPLE 9 Using a Mathematical Model

The yearly revenues (in billions of dollars) for gambling industries in the United States for the years 1999 to 2005 can be modeled by

$$\text{Revenue} = 0.157t^2 - 1.19t + 11.0, \quad 9 \le t \le 15$$

where $t$ represents the year, with $t = 9$ corresponding to 1999. Create a table that shows the revenue for each of these years. (Source: 2005 Service Annual Survey)

### Solution

To create a table of values that shows the revenues (in billions of dollars) for the gambling industries for the years 1999 to 2005, evaluate the expression $0.157t^2 - 1.19t + 11.0$ for each integer value of $t$ from $t = 9$ to $t = 15$.

| **Year** | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 |
|---|---|---|---|---|---|---|---|
| ***t*** | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| **Revenue** | 13.0 | 14.8 | 16.9 | 19.3 | 22.1 | 25.1 | 28.5 |

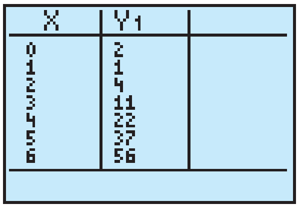
***CHECKPOINT*** *Now try Exercises 101 and 102.*

## Technology: Tip

Most graphing calculators can be used to evaluate an algebraic expression for several values of $x$ and display the results in a table. For instance, to evaluate $2x^2 - 3x + 2$ when $x$ is 0, 1, 2, 3, 4, 5, and 6, you can use the following steps.

1. Enter the expression into the graphing calculator.
2. Set the minimum value of the table to 0.
3. Set the table step or table increment to 1.
4. Display the table.

The results are shown below.




Consult the user's guide for your graphing calculator for specific instructions. Then complete a table for the expression $-4x^2 + 5x - 8$ when $x$ is 0, 1, 2, 3, 4, 5, and 6.

## Concept Check

1. Explain the difference between terms and factors in an algebraic expression.
2. Explain how you can use the Distributive Property to simplify the expression $5x + 3x$.
3. Explain how to combine like terms in an algebraic expression. Give an example.
4. Explain the difference between simplifying an algebraic expression and evaluating an algebraic expression.

## 1.4 EXERCISES

Go to pages 50–51 to record your assignments.

### Developing Skills

In Exercises 1–14, identify the terms and coefficients of the algebraic expression. ***See Example 1.***

**1.** $10x + 5$
**2.** $4 + 17y$
**3.** $12 - 6x^2$
**4.** $-16t^2 + 48$
**5.** $-3y^2 + 2y - 8$
**6.** $9t^2 + 2t + 10$
**7.** $1.2a - 4a^3$
**8.** $25z^3 - 4.8z^2$
**9.** $4x^2 - 3y^2 - 5x + 21$
**10.** $7a^2 + 4a - b^2 + 19$
**11.** $xy - 5x^2y + 2y^2$
**12.** $14u^2 + 25uv - 3v^2$
**13.** $\frac{1}{4}x^2 - \frac{3}{8}x + 5$
**14.** $\frac{2}{3}y + 8z + \frac{5}{6}$

In Exercises 15–20, identify the property of algebra illustrated by the statement.

**15.** $4 - 3x = -3x + 4$
**16.** $(10 + x) - y = 10 + (x - y)$
**17.** $-5(2x) = (-5 \cdot 2)x$
**18.** $(x - 2)(3) = 3(x - 2)$
**19.** $(5 - 2)x = 5x - 2x$
**20.** $7y + 2y = (7 + 2)y$

In Exercises 21–24, use the indicated property to rewrite the expression.

**21.** Distributive Property
$5(x + 6) =$ ______
**22.** Distributive Property
$6x + 6 =$ ______
**23.** Commutative Property of Multiplication
$5(x + 6) =$ ______
**24.** Commutative Property of Addition
$6x + 6 =$ ______

In Exercises 25–40, simplify the expression by combining like terms. ***See Examples 2 and 3.***

**25.** $3x + 4x$
**26.** $18z + 14z$
**27.** $-2x^2 + 4x^2$
**28.** $20a^2 - 5a^2$
**29.** $7x - 11x$
**30.** $-23t + 11t$
**31.** $9y - 5y + 4y$
**32.** $8y + 7y - y$
**33.** $3x - 2y + 5x + 20y$
**34.** $-2a + 4b - 7a - b$
**35.** $7x^2 - 2x - x^2$
**36.** $9y + y^2 - 6y$
**37.** $-3z^4 + 6z - z + 8 + z^4 - 4z^2$
**38.** $-5y^3 + 3y - 6y^2 + 8y^3 + y - 4$
**39.** $x^2 + 2xy - 2x^2 + xy + y$
**40.** $3a - 5ab + 9a^2 + 4ab - a$

In Exercises 41–52, use the Distributive Property to simplify the expression.

**41.** $4(2x^2 + x - 3)$

**42.** $8(z^3 - 4z^2 + 2)$

**43.** $-3(6y^2 - y - 2)$

**44.** $-5(-x^2 + 2y + 1)$

**45.** $-(3x^2 - 2x + 4)$

**46.** $-(-5t^2 + 8t - 10)$

**47.** $x(5x + 2)$

**48.** $y(-y + 10)$

**49.** $3x(17 - 4x)$

**50.** $5y(2y - 1)$

**51.** $-5t(7 - 2t)$

**52.** $-6x(9x - 4)$

In Exercises 53–72, simplify the expression. ***See Examples 4 and 5.***

✓ **53.** $10(x - 3) + 2x - 5$

**54.** $3(x + 1) + x - 6$

**55.** $x - (5x + 9)$

**56.** $y - (3y - 1)$

**57.** $5a - (4a - 3)$

**58.** $7x - (2x + 5)$

**59.** $-3(3y - 1) + 2(y - 5)$

**60.** $5(a + 6) - 4(2a - 1)$

**61.** $-3(y^2 - 2) + y^2(y + 3)$

**62.** $x(x^2 - 5) - 4(4 - x)$

**63.** $x(x^2 + 3) - 3(x + 4)$

**64.** $5(x + 1) - x(2x + 6)$

✓ **65.** $9a - [7 - 5(7a - 3)]$

**66.** $12b - [9 - 7(5b - 6)]$

**67.** $3[2x - 4(x - 8)]$

**68.** $4[5 - 3(x^2 + 10)]$

**69.** $8x + 3x[10 - 4(3 - x)]$

**70.** $5y - y[9 + 6(y - 2)]$

**71.** $2[3(b - 5) - (b^2 + b + 3)]$

**72.** $5[3(z + 2) - (z^2 + z - 2)]$

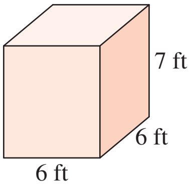
In Exercises 73–90, evaluate the expression for the specified values of the variable(s). If not possible, state the reason. ***See Examples 6–8.***

| | *Expression* | *Values* |
|---|---|---|
| ✓ **73.** | $5 - 3x$ | (a) $x = \frac{2}{3}$ |
| | | (b) $x = 5$ |
| **74.** | $\frac{3}{2}x - 2$ | (a) $x = 6$ |
| | | (b) $x = -3$ |
| **75.** | $10 - 4x^2$ | (a) $x = -1$ |
| | | (b) $x = \frac{1}{2}$ |
| **76.** | $3y^2 + 10$ | (a) $y = -2$ |
| | | (b) $y = \frac{1}{2}$ |
| **77.** | $y^2 - y + 5$ | (a) $y = 2$ |
| | | (b) $y = -2$ |
| **78.** | $2x^2 + 5x - 3$ | (a) $x = 2$ |
| | | (b) $x = -3$ |
| **79.** | $\frac{1}{x^2} + 3$ | (a) $x = 0$ |
| | | (b) $x = 3$ |
| **80.** | $5 - \frac{3}{x}$ | (a) $x = 0$ |
| | | (b) $x = -6$ |
| ✓ **81.** | $3x + 2y$ | (a) $x = 1, \quad y = 5$ |
| | | (b) $x = -6, \quad y = -9$ |
| **82.** | $6x - 5y$ | (a) $x = -2, \quad y = -3$ |
| | | (b) $x = 1, \quad y = 1$ |
| **83.** | $x^2 - xy + y^2$ | (a) $x = 2, \quad y = -1$ |
| | | (b) $x = -3, \quad y = -2$ |
| **84.** | $y^2 + xy - x^2$ | (a) $x = 5, \quad y = 2$ |
| | | (b) $x = -3, \quad y = 3$ |
| ✓ **85.** | $\frac{x}{y^2 - x}$ | (a) $x = 4, \quad y = 2$ |
| | | (b) $x = 3, \quad y = 3$ |
| **86.** | $\frac{x}{x - y}$ | (a) $x = 0, \quad y = 10$ |
| | | (b) $x = 4, \quad y = 4$ |
| **87.** | $\lvert y - x\rvert$ | (a) $x = 2, \quad y = 5$ |
| | | (b) $x = -2, \quad y = -2$ |
| **88.** | $\lvert x^2 - y\rvert$ | (a) $x = 0, \quad y = -2$ |
| | | (b) $x = 3, \quad y = -15$ |
| **89.** | Distance traveled: $rt$ | (a) $r = 40, \quad t = 5\frac{1}{4}$ |
| | | (b) $r = 35, \quad t = 4$ |
| **90.** | Simple interest: $Prt$ | (a) $P = \$7000, \quad r = 0.065, \quad t = 10$ |
| | | (b) $P = \$4200, \quad r = 0.07, \quad t = 9$ |

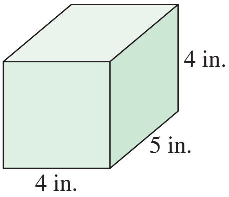
## Solving Problems

***Geometry*** In Exercises 91–94, find the volume of the rectangular solid by evaluating the expression $lwh$ for the dimensions given in the figure.

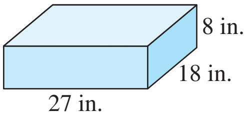
**91.**



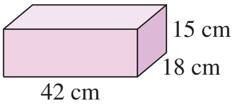

**92.**




**93.**




**94.**




In Exercises 95–98, evaluate the expression $0.01p + 0.05n + 0.10d + 0.25q$ to find the value of the given number of pennies $p$, nickels $n$, dimes $d$, and quarters $q$.

**95.** 11 pennies, 7 nickels, 3 quarters

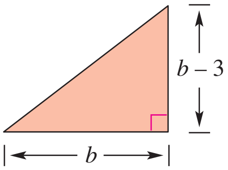
**96.** 8 pennies, 13 nickels, 6 dimes

**97.** 43 pennies, 27 nickels, 17 dimes, 15 quarters

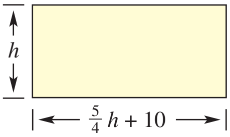
**98.** 111 pennies, 22 nickels, 2 dimes, 42 quarters

***Geometry*** In Exercises 99 and 100, write and simplify an expression for the area of the figure. Then evaluate the expression for the given value of the variable.

**99.** $b = 15$

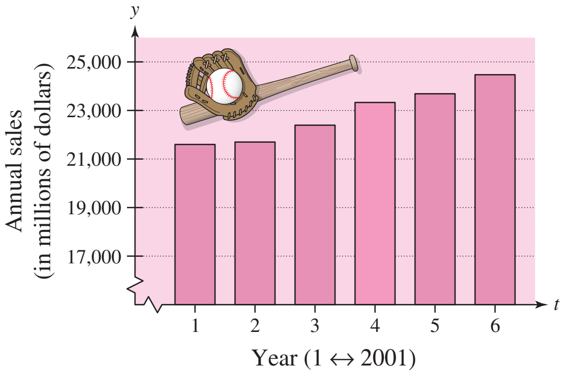



**100.** $h = 12$




***Using a Model*** In Exercises 101 and 102, use the following model, which approximates the annual sales (in millions of dollars) of sports equipment in the United States from 2001 to 2006 (see figure), where $t$ represents the year, with $t = 1$ corresponding to 2001. (Source: National Sporting Goods Association)

$$\text{Sales} = 607.6t + 20{,}737, \quad 1 \le t \le 6$$




**101.** Graphically approximate the sales of sports equipment in 2005. Then use the model to confirm your estimate algebraically.

**102.** Use the model and a calculator to complete the table showing the sales from 2001 to 2006.

| **Year** | 2001 | 2002 | 2003 |
|---|---|---|---|
| **Sales** | | | |

| **Year** | 2004 | 2005 | 2006 |
|---|---|---|---|
| **Sales** | | | |

***Using a Model*** In Exercises 103 and 104, use the following model, which approximates the total yearly disbursements (in billions of dollars) of Federal Family Education Loans (FFEL) in the United States from 1999 to 2005 (see figure), where $t$ represents the year, with $t = 9$ corresponding to 1999. (Source: U.S. Department of Education)

$$\text{Disbursements} = 0.322t^2 - 3.75t + 27.6, \quad 9 \le t \le 15$$

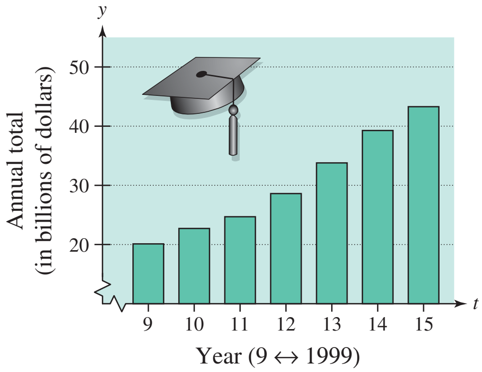



Figure for 103 and 104

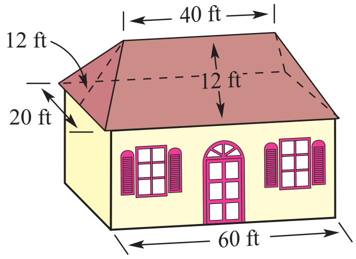
**103.** Graphically approximate the total amount of FFEL disbursements in the year 2000. Then use the model to confirm your estimate algebraically.

**104.** Use the model and a calculator to complete the table showing the total yearly disbursements (in billions of dollars) from 1999 to 2005. Round each amount to the nearest tenth.

| Year | 1999 | 2000 | 2001 | 2002 |
|---|---|---|---|---|
| Amount | | | | |

| Year | 2003 | 2004 | 2005 |
|---|---|---|---|
| Amount | | | |

**105.** *Geometry* The roof shown in the figure is made up of two trapezoids and two triangles. Find the total area of the roof. [For a trapezoid, area $= \frac{1}{2}h(b_1 + b_2)$, where $b_1$ and $b_2$ are the lengths of the bases and $h$ is the height.]




**106.** *Exploration*

(a) A convex polygon with $n$ sides has

$$\frac{n(n-3)}{2}, \; n \geq 4$$

diagonals. Verify the formula for a square, a pentagon, and a hexagon.

(b) Explain why the formula in part (a) will always yield a natural number.



## Explaining Concepts

**107.** Is it possible to evaluate the expression

$$\frac{x+2}{y-3}$$

when $x = 5$ and $y = 3$? Explain.

**108.** Is it possible to evaluate the expression $3x + 5y - 18z$ when $x = 10$ and $y = 8$? Explain.

**109.** State the procedure for simplifying an algebraic expression by removing a set of parentheses preceded by a minus sign, such as the parentheses in $a - (b + c)$. Then give an example.

**110.** How can a factor be part of a term in an algebraic expression? Explain and give an example.

**111.** How can an algebraic term be part of a factor in an algebraic expression? Give an example.

**112.** You know that the expression $180 - 10x$ has a value of 100. Is it possible to determine the value of $x$ with this information? Explain and find the value if possible.

**113.** You know that the expression $8y - 5x$ has a value of 14. Is it possible to determine the values of $x$ and $y$ with this information? Explain and find the values if possible.

# 1.5 Constructing Algebraic Expressions

Clive Brunskill/Getty Images

## What You Should Learn

1 ▶ Translate verbal phrases into algebraic expressions, and vice versa.

2 ▶ Construct algebraic expressions with hidden products.

## Why You Should Learn It

Translating verbal phrases into algebraic expressions enables you to model real-life problems. For instance, in Exercise 73 on page 49, you will write an algebraic expression that models the area of a soccer field.

## Translating Phrases

1 ▶ Translate verbal phrases into algebraic expressions, and vice versa.

In this section, you will study ways to *construct* algebraic expressions. When you translate a verbal sentence or phrase into an algebraic expression, watch for key words and phrases that indicate the four different operations of arithmetic.



### Translating Key Words and Phrases

| *Key Words and Phrases* | *Verbal Description* | *Algebraic Expression* |
|---|---|---|
| **Addition:** | | |
| Sum, plus, greater than, increased by, more than, | The sum of 5 and $x$ | $5 + x$ |
| exceeds, total of | Seven more than $y$ | $y + 7$ |
| **Subtraction:** | | |
| Difference, minus, less than, decreased by, | $b$ is subtracted from 4. | $4 - b$ |
| subtracted from, reduced by, the remainder | Three less than $z$ | $z - 3$ |
| **Multiplication:** | | |
| Product, multiplied by, twice, times, percent of | Two times $x$ | $2x$ |
| **Division:** | | |
| Quotient, divided by, ratio, per | The ratio of $x$ and 8 | $\frac{x}{8}$ |

### EXAMPLE 1 Translating Verbal Phrases

| *Verbal Description* | *Algebraic Expression* |
|---|---|
| **a.** Seven more than three times $x$ | $3x + 7$ |
| **b.** Four less than the product of 6 and $n$ | $6n - 4$ |
| **c.** The quotient of $x$ and 3, decreased by 6 | $\frac{x}{3} - 6$ |

✓ **CHECKPOINT** *Now try Exercise 1.*

## EXAMPLE 2 Translating Verbal Phrases

| Verbal Description | Algebraic Expression |
|---|---|
| **a.** Eight added to the product of 2 and $n$ | $2n + 8$ |
| **b.** Four times the sum of $y$ and 9 | $4(y + 9)$ |
| **c.** The difference of $a$ and 7, all divided by 9 | $\frac{a - 7}{9}$ |

**CHECKPOINT** *Now try Exercise 7.*

In Examples 1 and 2, the verbal description specified the name of the variable. In most real-life situations, however, the variables are not specified and it is your task to assign variables to the *appropriate* quantities.

## EXAMPLE 3 Translating Verbal Phrases

| Verbal Description | Label | Algebraic Expression |
|---|---|---|
| **a.** The sum of 7 and a number | The number $= x$ | $7 + x$ |
| **b.** Four decreased by the product of 2 and a number | The number $= n$ | $4 - 2n$ |
| **c.** Seven less than twice the sum of a number and 5 | The number $= y$ | $2(y + 5) - 7$ |

**CHECKPOINT** *Now try Exercise 19.*

A good way to learn algebra is to do it forward and backward. For instance, the next example translates algebraic expressions into verbal form. Keep in mind that other key words could be used to describe the operations in each expression.

## EXAMPLE 4 Translating Expressions into Verbal Phrases

Without using a variable, write a verbal description for each expression.

**a.** $5x - 10$

**b.** $\frac{3 + x}{4}$

**c.** $2(3x + 4)$

**d.** $\frac{4}{x - 2}$

### Solution

**a.** 10 less than the product of 5 and a number

**b.** The sum of 3 and a number, all divided by 4

**c.** Twice the sum of 3 times a number and 4

**d.** Four divided by a number reduced by 2

**CHECKPOINT** *Now try Exercise 25.*

### Study Tip

When you write a verbal model or construct an algebraic expression, watch out for statements that may be interpreted in more than one way. For instance, the statement "The sum of $x$ and 1 divided by 5" is ambiguous because it could mean

$$\frac{x + 1}{5} \text{ or } x + \frac{1}{5}.$$

Notice in Example 4(b) that the verbal description for

$$\frac{3 + x}{4}$$

contains the phrase "all divided by 4."

**2** ▶ Construct algebraic expressions with hidden products.

## Constructing Mathematical Models

Translating a verbal phrase into a mathematical model is critical in problem solving. The next four examples will demonstrate three steps for creating a mathematical model.

1. Construct a verbal model that represents the problem situation.
2. Assign labels to all quantities in the verbal model.
3. Construct a mathematical model (algebraic expression).

**Study Tip**

Most real-life problems do not contain verbal expressions that clearly identify the arithmetic operations involved in the problem. You need to rely on past experience and the physical nature of the problem in order to identify the operations hidden in the problem statement. Watch for hidden products in Examples 5 and 6.

### EXAMPLE 5 Constructing a Mathematical Model

A cash register contains $x$ quarters. Write an algebraic expression for this amount of money in dollars.

**Solution**

*Verbal Model:* [Value of coin] · [Number of coins]

*Labels:* Value of coin $= 0.25$ (dollars per quarter)
Number of coins $= x$ (quarters)

*Expression:* $0.25x$ (dollars)

**CHECKPOINT** *Now try Exercise 41.*

### EXAMPLE 6 Constructing a Mathematical Model

A cash register contains $n$ nickels and $d$ dimes. Write an algebraic expression for this amount of money in cents.

**Solution**

*Verbal Model:* [Value of nickel] · [Number of nickels] + [Value of dime] · [Number of dimes]

*Labels:* Value of nickel $= 5$ (cents per nickel)
Number of nickels $= n$ (nickels)
Value of dime $= 10$ (cents per dime)
Number of dimes $= d$ (dimes)

*Expression:* $5n + 10d$ (cents)

**CHECKPOINT** *Now try Exercise 45.*

In Example 6, the final expression $5n + 10d$ is measured in cents. This makes "sense" as described below.

$$\frac{5 \text{ cents}}{\text{nickel}} \cdot n \text{ nickels} + \frac{10 \text{ cents}}{\text{dime}} \cdot d \text{ dimes}$$

Note that the nickels and dimes "divide out," leaving cents as the unit of measure for each term. This technique is called *unit analysis*, and it can be very helpful in determining the final unit of measure.

## EXAMPLE 7 Constructing a Mathematical Model

A person riding a bicycle travels at a constant rate of 12 miles per hour. Write an algebraic expression showing how far the person can ride in $t$ hours.

### Solution

For this problem, use the formula Distance = (Rate)(Time).

*Verbal Model:* Rate · Time

*Labels:* Rate = 12 (miles per hour)
Time = $t$ (hours)

*Expression:* $12t$ (miles)

**CHECKPOINT** *Now try Exercise 47.*

Using unit analysis, you can see that the expression in Example 7 has *miles* as its unit of measure.

$$12\frac{\text{miles}}{\cancel{\text{hour}}}\cdot t\ \cancel{\text{hours}}$$

When translating verbal phrases involving percents, be sure you write the percent *in decimal form*.

| *Percent* | *Decimal Form* |
|---|---|
| 4% | 0.04 |
| 62% | 0.62 |
| 140% | 1.40 |
| 25% | 0.25 |

Remember that when you find a percent of a number, you multiply. For instance, 25% of 78 is given by

$$0.25(78) = 19.5. \quad \text{25\% of 78}$$



## EXAMPLE 8 Constructing a Mathematical Model

A person adds $k$ liters of fluid containing 55% antifreeze to a car radiator. Write an algebraic expression that indicates how much antifreeze was added.

### Solution

*Verbal Model:* Percent antifreeze · Number of liters

*Labels:* Percent of antifreeze = 0.55 (in decimal form)
Number of liters = $k$ (liters)

*Expression:* $0.55k$ (liters)

Note that the algebraic expression uses the decimal form of 55%. That is, you compute with 0.55 rather than 55%.

**CHECKPOINT** *Now try Exercise 51.*

When assigning labels to *two* unknown quantities, hidden operations are often involved. For example, two numbers add up to 18 and one of the numbers is assigned the variable $x$. What expression can you use to represent the second number? Let's try a specific case first, then apply it to a general case.

*Specific Case:* If the first number is 7, the second number is $18 - 7 = 11$.

*General Case:* If the first number is $x$, the second number is $18 - x$.

The strategy of using a *specific* case to help determine the general case is often helpful in applications. Observe the use of this strategy in the next example.

## EXAMPLE 9 Using Specific Cases to Model General Cases

**a.** A person's weekly salary is $d$ dollars. Write an expression for the person's annual salary.

**b.** A person's annual salary is $y$ dollars. Write an expression for the person's monthly salary.

### Solution

**a.** *Specific Case:* If the weekly salary is \$300, the annual salary is 52(300) dollars.

*General Case:* If the weekly salary is $d$ dollars, the annual salary is $52 \cdot d$ or $52d$ dollars.

**b.** *Specific Case:* If the annual salary is \$24,000, the monthly salary is $24{,}000 \div 12$ dollars.

*General Case:* If the annual salary is $y$ dollars, the monthly salary is $y \div 12$ or $y/12$ dollars.

**CHECKPOINT** *Now try Exercise 57.*

**Study Tip**

You can check that your algebraic expressions are correct for even, odd, or consecutive integers by substituting an integer for $n$. For instance, by letting $n = 5$, you can see that $2n = 2(5) = 10$ is an even integer, $2n - 1 = 2(5) - 1 = 9$ is an odd integer, and $2n + 1 = 2(5) + 1 = 11$ is an odd integer.

In mathematics, it is useful to know how to represent certain types of integers algebraically. For instance, consider the set $\{2, 4, 6, 8, \ldots\}$ of *even* integers. Because every even integer has 2 as a factor,

$$2 = 2 \cdot 1, \quad 4 = 2 \cdot 2, \quad 6 = 2 \cdot 3, \quad 8 = 2 \cdot 4, \ldots$$

it follows that any integer $n$ multiplied by 2 is sure to be the *even* number $2n$. Moreover, if $2n$ is even, then $2n - 1$ and $2n + 1$ are sure to be *odd* integers.

Two integers are called **consecutive integers** if they differ by 1. For any integer $n$, its next two larger consecutive integers are $n + 1$ and $(n + 1) + 1$ or $n + 2$. So, you can denote three consecutive integers by $n$, $n + 1$, and $n + 2$. These results are summarized below.

### Labels for Integers

Let $n$ represent an integer. Then even integers, odd integers, and consecutive integers can be represented as follows.

1. $2n$ denotes an *even* integer for $n = 1, 2, 3, \ldots$.
2. $2n - 1$ and $2n + 1$ denote *odd* integers for $n = 1, 2, 3, \ldots$.
3. $\{n, n + 1, n + 2, \ldots\}$ denotes a set of *consecutive* integers.

### EXAMPLE 10 Constructing a Mathematical Model

Write an expression for the following phrase.

"The sum of two consecutive integers, the first of which is $n$"

#### Solution

The first integer is $n$. The next consecutive integer is $n + 1$. So the sum of two consecutive integers is

$$n + (n + 1) = 2n + 1.$$

**CHECKPOINT** *Now try Exercise 59.*

Sometimes an expression may be written directly from a diagram using a common geometric formula, as shown in the next example.

### EXAMPLE 11 Constructing a Mathematical Model

Write expressions for the perimeter and area of the rectangle shown in Figure 1.16.

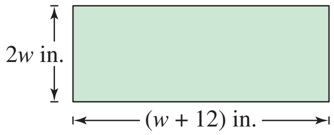



**Figure 1.16**

#### Solution

For the perimeter of the rectangle, use the formula

Perimeter = 2(Length) + 2(Width).



*Verbal Model:* 2 · Length + 2 · Width

*Labels:* Length = $w + 12$ (inches)
Width = $2w$ (inches)

*Expression:* $2(w + 12) + 2(2w) = 2w + 24 + 4w$
$= 6w + 24$ (inches)

For the area of the rectangle, use the formula

Area = (Length)(Width).

*Verbal Model:* Length · Width

*Labels:* Length = $w + 12$ (inches)
Width = $2w$ (inches)

*Expression:* $(w + 12)(2w) = 2w^2 + 24w$ (square inches)

**CHECKPOINT** *Now try Exercise 69.*

## Concept Check

1. The phrase *reduced by* implies what operation?
2. The word *ratio* indicates what operation?
3. A car travels at a constant rate of 45 miles per hour for $t$ hours. The algebraic expression for the distance traveled is $45t$. What is the unit of measure of the algebraic expression?
4. Let $n$ represent an integer. Is the expression $n(n + 1)$ even or odd? Explain.

## 1.5 EXERCISES

Go to pages 50–51 to record your assignments.

## Developing Skills

In Exercises 1–24, translate the verbal phrase into an algebraic expression. ***See Examples 1–3.***

1. The sum of 23 and a number $n$
2. Twelve more than a number $n$
3. The sum of 12 and twice a number $n$
4. The total of 25 and three times a number $n$
5. Six less than a number $n$
6. Fifteen decreased by three times a number $n$
7. Four times a number $n$ minus 10
8. The product of a number $y$ and 10 is decreased by 35.
9. Half of a number $n$
10. Seven-fifths of a number $n$
11. The quotient of a number $x$ and 6
12. The ratio of $y$ and 3
13. Eight times the ratio of $N$ and 5
14. Fifteen times the ratio of $x$ and 32
15. The number $c$ is quadrupled and the product is increased by 10.
16. The number $u$ is tripled and the product is increased by 250.
17. Thirty percent of the list price $L$
18. Twenty-five percent of the bill $B$
19. The sum of a number and 5, divided by 10
20. The sum of 7 and twice a number $x$, all divided by 8
21. The absolute value of the difference between a number and 8
22. The absolute value of the quotient of a number and 4
23. The product of 3 and the square of a number is decreased by 4.
24. The sum of 10 and one-fourth the square of a number

In Exercises 25–40, write a verbal description of the algebraic expression without using the variable. ***See Example 4.***

25. $t - 2$
26. $5 - x$
27. $y + 50$
28. $2y + 3$
29. $2 - 3x$
30. $7y - 4$
31. $\frac{z}{2}$
32. $\frac{y}{8}$
33. $\frac{4}{5}x$
34. $\frac{2}{3}t$
35. $8(x - 5)$
36. $(y + 6)4$
37. $\frac{x + 10}{3}$
38. $\frac{3 - n}{9}$

**39.** $y^2 - 3$

**40.** $x^2 + 2$

In Exercises 41–64, write an algebraic expression that represents the specified quantity in the verbal statement, and simplify if possible. ***See Examples 5–10.***

**41.** The amount of money (in dollars) represented by $n$ quarters

**42.** The amount of money (in dollars) represented by $x$ nickels

**43.** The amount of money (in dollars) represented by $m$ dimes

**44.** The amount of money (in dollars) represented by $y$ pennies

**45.** The amount of money (in cents) represented by $m$ nickels and $n$ dimes

**46.** The amount of money (in cents) represented by $m$ dimes and $n$ quarters

**47.** The distance traveled in $t$ hours at an average speed of 55 miles per hour

**48.** The distance traveled in 5 hours at an average speed of $r$ miles per hour

**49.** The time required to travel 320 miles at an average speed of $r$ miles per hour

**50.** The average rate of speed when traveling 320 miles in $t$ hours

**51.** The amount of antifreeze in a cooling system containing $y$ gallons of coolant that is 45% antifreeze

**52.** The amount of water in $q$ quarts of a food product that is 65% water

**53.** The amount of wage tax due for a taxable income of $I$ dollars that is taxed at the rate of 1.25%

**54.** The amount of sales tax on a purchase valued at $L$ dollars if the tax rate is 5.5%

**55.** The sale price of a coat that has a list price of $L$ dollars if it is a "20% off" sale

**56.** The total bill for a meal that cost $C$ dollars if you plan on leaving a 15% tip

**57.** The total hourly wage for an employee when the base pay is \$8.25 per hour plus 60 cents for each of $q$ units produced per hour

**58.** The total hourly wage for an employee when the base pay is \$11.65 per hour plus 80 cents for each of $q$ units produced per hour

**59.** The sum of a number $n$ and five times the number

**60.** The sum of three consecutive integers, the first of which is $n$

**61.** The sum of three consecutive odd integers, the first of which is $2n + 1$

**62.** The sum of three consecutive even integers, the first of which is $2n$

**63.** The product of two consecutive even integers, divided by 4

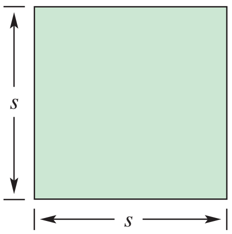
**64.** The absolute value of the difference of two consecutive integers, divided by 2



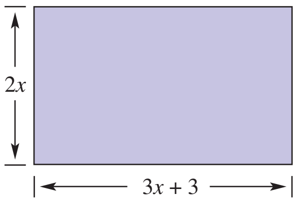
## Solving Problems

*Geometry* In Exercises 65–68, write an expression for the area of the figure. Simplify the expression.

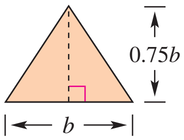
**65.**



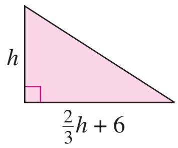

**66.**




**67.**

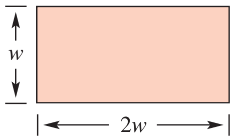



**68.**

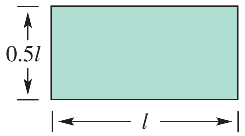


***Geometry*** In Exercises 69–72, write expressions for the perimeter and area of the region. Simplify the expressions. ***See Example 11.***

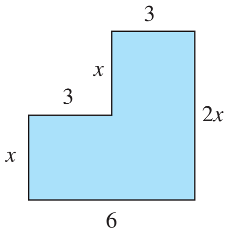
**69.**



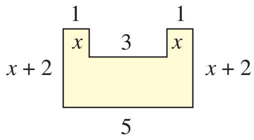

**70.**



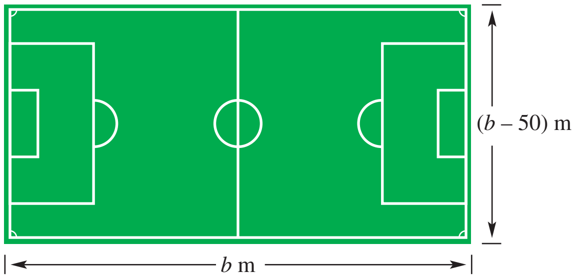

**71.**




**72.**




**73.** ***Geometry*** Write an expression for the area of the soccer field shown in the figure. What is the unit of measure for the area?




**74.** ***Geometry*** Write an expression for the area of the advertising banner shown in the figure. What is the unit of measure for the area?


**75.** ***Finding a Pattern*** Complete the table. The third row contains the differences between consecutive entries of the second row. Describe the pattern of the third row.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $5n - 3$ | | | | | | |
| **Differences** | | | | | | |

**76.** ***Finding a Pattern*** Complete the table. The third row contains the differences between consecutive entries of the second row. Describe the pattern of the third row.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $3n + 1$ | | | | | | |
| **Differences** | | | | | | |

**77.** ***Finding a Pattern*** Using the results of Exercises 75 and 76, guess the third-row difference that would result in a similar table if the algebraic expression were $an + b$.

**78.** ***Think About It*** Find $a$ and $b$ such that the expression $an + b$ would yield the following table.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $an + b$ | 3 | 7 | 11 | 15 | 19 | 23 |



## Explaining Concepts

**79.** Which are equivalent to $4x$?

(a) $x$ multiplied by 4

(b) $x$ increased by 4

(c) the product of $x$ and 4

(d) the ratio of 4 and $x$

**80.** If $n$ is an integer, how are the integers $2n - 1$ and $2n + 1$ related? Explain.

**81.** When a statement is translated into an algebraic expression, explain why it may be helpful to use a specific case before writing the expression.

**82.** When each phrase is translated into an algebraic expression, is order important? Explain.

(a) $y$ multiplied by 5

(b) 5 decreased by $y$

(c) $y$ divided by 5

(d) the sum of 5 and $y$

# What Did You Learn?

Use these two pages to help prepare for a test on this chapter. Check off the key terms and key concepts you know. You can also use this section to record your assignments.

## Plan for Test Success

**Date of test:** / /

**Study dates and times:** / / at : A.M./P.M.

/ / at : A.M./P.M.

**Things to review:**

- ☐ Key Terms, *p. 50*
- ☐ Key Concepts, *pp. 50–51*
- ☐ Your class notes
- ☐ Your assignments
- ☐ Study Tips, *pp. 2, 6, 7, 13, 14, 15, 17, 23, 25, 32, 33, 34, 42, 43, 45*
- ☐ Technology Tips, *pp. 3, 18, 36*
- ☐ Mid-Chapter Quiz, *p. 31*
- ☐ Review Exercises, *pp. 52–54*
- ☐ Chapter Test, *p. 55*
- ☐ Video Explanations Online
- ☐ Tutorial Online

## Key Terms

- ☐ set, *p. 2*
- ☐ subset, *p. 2*
- ☐ real numbers, *p. 2*
- ☐ natural numbers, *p. 2*
- ☐ positive integers, *p. 2*
- ☐ whole numbers, *p. 2*
- ☐ negative integers, *p. 2*
- ☐ integers, *p. 2*
- ☐ fractions, *p. 3*
- ☐ rational numbers, *p. 3*
- ☐ irrational numbers, *p. 3*
- ☐ real number line, *p. 4*
- ☐ origin, *p. 4*
- ☐ nonnegative real number, *p. 4*
- ☐ inequality symbols, *p. 5*
- ☐ opposites, *p. 7*
- ☐ additive inverses, *p. 7*
- ☐ absolute value, *p. 7*
- ☐ sum, *p. 11*
- ☐ difference, *p. 12*
- ☐ least common denominator, *p. 13*
- ☐ least common multiple, *p. 13*
- ☐ product, *p. 14*
- ☐ factor, *p. 14*
- ☐ reciprocal, *p. 15*
- ☐ quotient, *p. 15*
- ☐ dividend, *p. 15*
- ☐ divisor, *p. 15*
- ☐ numerator, *p. 15*
- ☐ denominator, *p. 15*
- ☐ exponential form, *p. 16*
- ☐ base, *p. 16*
- ☐ exponent, *p. 16*
- ☐ order of operations, *p. 16*
- ☐ variables, *p. 32*
- ☐ algebraic expressions, *p. 32*
- ☐ variable terms, *p. 32*
- ☐ constant term, *p. 32*
- ☐ coefficient, *p. 32*
- ☐ like terms, *p. 33*
- ☐ simplify, *p. 33*
- ☐ evaluate, *p. 35*
- ☐ consecutive integers, *p. 45*

## Key Concepts

### 1.1 The Real Number System

**Assignment:** ____________ **Due date:** ______

☐ **Use the real number line to order real numbers.**

If the real number $a$ lies to the left of the real number $b$ on the real number line, then $a$ is less than $b$, which is written as $a < b$. This relationship can also be described by saying that $b$ is greater than $a$ and writing $b > a$.

☐ **Use the real number line to find the distance between two real numbers.**

If $a$ and $b$ are two real numbers such that $a \le b$, then the distance between $a$ and $b$ is given by

$b - a$.

☐ **Use properties of opposites and additive inverses.**

Let $a$ be a real number.

1. $-a$ is the opposite of $a$.
2. $-(-a) = a$ (The double negative of $a$ is $a$.)
3. $a + (-a) = 0$ ($a$ and $-a$ are additive inverses.)

☐ **Determine the absolute value of a real number.**

If $a$ is a real number, then the absolute value of $a$ is

$$|a| = \begin{cases} a, & \text{if } a \ge 0 \\ -a, & \text{if } a < 0 \end{cases}.$$

## 1.2 Operations with Real Numbers

**Assignment:** ______ **Due date:** ______

☐ **Perform operations on real numbers.**

1. *To add with like signs:* Add the absolute values and attach the common sign.
2. *To add with unlike signs:* Subtract the smaller absolute value from the greater absolute value and attach the sign of the number with the greater absolute value.
3. *To subtract $b$ from $a$:* Add $-b$ to $a$.
4. *To multiply two real numbers:* With like signs, the product is positive. With unlike signs, the product is negative. The product of zero and any other real number is zero.
5. *To divide $a$ by $b$:* Multiply $a$ by the reciprocal of $b$.
6. *To add fractions:* Write the fractions so that they have the same denominator. Add the numerators over the common denominator.
7. *To multiply fractions:* Multiply the numerators and the denominators.
8. *To raise $a$ to the nth power ($n$ is an integer):*

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}}$$

☐ **Use order of operations to evaluate expressions.**

1. First do operations within symbols of grouping.
2. Then evaluate powers.
3. Then do multiplications and divisions from left to right.
4. Finally, do additions and subtractions from left to right.

## 1.3 Properties of Real Numbers

**Assignment:** ______ **Due date:** ______

☐ **Use properties of real numbers.**

Let $a$, $b$, and $c$ represent real numbers, variables, or algebraic expressions.

*Commutative Properties:*

$a + b = b + a$ $\qquad ab = ba$

*Associative Properties:*

$(a + b) + c = a + (b + c)$ $\qquad (ab)c = a(bc)$

*Distributive Properties:*

$a(b + c) = ab + ac$ $\qquad a(b - c) = ab - ac$

$(a + b)c = ac + bc$ $\qquad (a - b)c = ac - bc$

*Identity Properties:*

$a + 0 = 0 + a = a$ $\qquad a \cdot 1 = 1 \cdot a = a$

*Inverse Properties:*

$a + (-a) = 0$ $\qquad a \cdot \frac{1}{a} = 1,\ a \neq 0$

See page 25 for additional properties of real numbers.

## 1.4 Algebraic Expressions

**Assignment:** ______ **Due date:** ______

☐ **Identify the terms and coefficients of algebraic expressions.**

The *terms* of an algebraic expression are those parts that are separated by addition.

The *coefficient* of a term is its numerical factor.

☐ **Simplify algebraic expressions.**

To simplify an algebraic expression, remove the symbols of grouping and combine like terms.

☐ **Evaluate algebraic expressions.**

To evaluate an algebraic expression, substitute numerical values for each of the variables in the expression and simplify.

## 1.5 Constructing Algebraic Expressions

**Assignment:** ______ **Due date:** ______

☐ **Translate verbal phrases into algebraic expressions, and vice versa.**

*Addition:* sum, plus, greater than, increased by, more than, exceeds, total of

*Subtraction:* difference, minus, less than, decreased by, subtracted from, reduced by, the remainder

*Multiplication:* product, multiplied by, twice, times, percent of

*Division:* quotient, divided by, ratio, per

☐ **Write labels for integers.**

The following expressions are useful ways to denote integers.

1. $2n$ denotes an even integer for $n = 1, 2, 3, \ldots$.
2. $2n - 1$ and $2n + 1$ denote odd integers for $n = 1, 2, 3, \ldots$.
3. $\{n, n + 1, n + 2, \ldots\}$ denotes a set of consecutive integers.

# Review Exercises

## 1.1 The Real Number System

**1** ▶ Understand the set of real numbers and the subsets of real numbers.

**In Exercises 1 and 2, which of the real numbers in the set are (a) natural numbers, (b) integers, (c) rational numbers, and (d) irrational numbers?**

1. $\left\{\frac{3}{5}, -4, 0, \sqrt{2}, 52, -\frac{1}{8}, \sqrt{9}\right\}$

2. $\left\{98, -141, -\frac{7}{8}, 3.99, -\sqrt{12}, -\frac{54}{11}\right\}$

**In Exercises 3 and 4, list all members of the set.**

3. The natural numbers between $-2.3$ and $6.1$

4. The even integers between $-5.5$ and $2.5$

**2** ▶ Use the real number line to order real numbers.

**In Exercises 5 and 6, plot the real numbers on the real number line.**

5. (a) $4$ (b) $-3$ (c) $\frac{3}{4}$ (d) $-2.4$
6. (a) $-9$ (b) $7$ (c) $-\frac{3}{2}$ (d) $5.25$

**In Exercises 7–10, place the correct inequality symbol ($<$ or $>$) between the numbers.**

7. $-5$ ☐ $3$
8. $-2$ ☐ $-8$
9. $-\frac{8}{5}$ ☐ $-\frac{2}{5}$
10. $8.4$ ☐ $-3.2$

**3** ▶ Use the real number line to find the distance between two real numbers.

**In Exercises 11–14, find the distance between the pair of real numbers.**

11. $11$ and $-3$
12. $4$ and $-13$
13. $-13.5$ and $-6.2$
14. $-8.4$ and $-0.3$

**4** ▶ Determine the absolute value of a real number.

**In Exercises 15–18, evaluate the expression.**

15. $|-5|$
16. $|6|$
17. $-|-7.2|$
18. $|-3.6|$

## 1.2 Operations with Real Numbers

**1** ▶ Add, subtract, multiply, and divide real numbers.

**In Exercises 19–40, evaluate the expression. If it is not possible, state the reason. Write all fractions in simplest form.**

19. $15 + (-4)$
20. $-12 + 3$
21. $340 - 115 + 5$
22. $-154 + 86 - 240$
23. $-63.5 + 21.7$
24. $14.35 - 10.3$
25. $\frac{4}{21} + \frac{7}{21}$
26. $\frac{21}{16} - \frac{13}{16}$
27. $-\frac{5}{6} + 1$
28. $3 + \frac{4}{9}$
29. $8\frac{3}{4} - 6\frac{5}{8}$
30. $-2\frac{9}{10} + 5\frac{3}{20}$
31. $-7 \cdot 4$
32. $9 \cdot (-5)$
33. $120(-5)(7)$
34. $(-16)(-15)(-4)$
35. $\frac{3}{8} \cdot \left(-\frac{2}{15}\right)$
36. $\frac{5}{21} \cdot \frac{21}{5}$
37. $\dfrac{-56}{-4}$
38. $\frac{85}{0}$
39. $-\frac{7}{15} \div \left(-\frac{7}{30}\right)$
40. $-\frac{2}{3} \div \frac{4}{15}$

**In Exercises 41 and 42, write the expression as a repeated addition problem.**

**41.** $7(-3)$

**42.** $5\left(\frac{2}{3}\right)$

**In Exercises 43 and 44, write the expression as a multiplication problem.**

**43.** $8 + 8 + 8 + 8 + 8 + 8 + 8 + 8$

**44.** $(-5) + (-5) + (-5) + (-5) + (-5)$

**2** ▶ Write repeated multiplication in exponential form and evaluate exponential expressions.

**In Exercises 45 and 46, write the expression using exponential notation.**

**45.** $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

**46.** $\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)$

**In Exercises 47–52, evaluate the exponential expression.**

**47.** $(-6)^4$

**48.** $-(-3)^4$

**49.** $-4^2$

**50.** $2^5$

**51.** $-\left(-\frac{1}{2}\right)^3$

**52.** $-\left(\frac{2}{3}\right)^4$

**3** ▶ Use order of operations to evaluate expressions.

**In Exercises 53–56, evaluate the expression.**

**53.** $120 - (5^2 \cdot 4)$

**54.** $45 - 45 \div 3^2$

**55.** $8 + 3[6^2 - 2(7 - 4)]$

**56.** $2^4 - [10 + 6(1 - 3)^2]$

**4** ▶ Evaluate expressions using a calculator and order of operations.

**In Exercises 57 and 58, evaluate the expression using a calculator. Round your answer to two decimal places.**

**57.** $7(408.2^2 - 39.5 \div 0.3)$

**58.** $-59[4.6^3 + 5.8(-13.4)]$

**59.** *Total Charge* You purchased an entertainment system and made a down payment of \$395 plus nine monthly payments of \$45 each. What is the total amount you paid for the system?

**60.** *Savings Plan* You deposit \$80 per month in a savings account for 10 years. The account earns 2% interest compounded monthly. The total amount in the account after 10 years will be

$$80\left[\left(1 + \frac{0.02}{12}\right)^{120} - 1\right]\left(1 + \frac{12}{0.02}\right).$$

Use a calculator to determine this amount.

## 1.3 Properties of Real Numbers

**1** ▶ Identify and use the properties of real numbers.

**In Exercises 61–70, identify the property of real numbers illustrated by the statement.**

**61.** $13 - 13 = 0$

**62.** $7\left(\frac{1}{7}\right) = 1$

**63.** $7(9 + 3) = 7 \cdot 9 + 7 \cdot 3$

**64.** $15(4) = 4(15)$

**65.** $5 + (4 - y) = (5 + 4) - y$

**66.** $6(4z) = (6 \cdot 4)z$

**67.** $(u - v)(2) = 2(u - v)$

**68.** $xy \cdot 1 = xy$

**69.** $8(x - y) = 8 \cdot x - 8 \cdot y$

**70.** $xz - yz = (x - y)z$

**In Exercises 71–74, rewrite the expression by using the Distributive Property.**

**71.** $-(-u + 3v)$

**72.** $-5(2x - 4y)$

**73.** $-a(8 - 3a)$

**74.** $x(3x + 4y)$

## 1.4 Algebraic Expressions

**1 ▶ Identify the terms and coefficients of algebraic expressions.**

**In Exercises 75–78, identify the terms and coefficients of the algebraic expression.**

**75.** $4y^3 - y^2 + \frac{17}{2}y$

**76.** $\frac{x}{3} + 2xy^2 + \frac{1}{5}$

**77.** $52 - 1.2x^3 + \frac{1}{x}$

**78.** $2ab^2 + a^2b^2 - \frac{1}{a}$

**2 ▶ Simplify algebraic expressions by combining like terms and removing symbols of grouping.**

**In Exercises 79–88, simplify the expression.**

**79.** $6x + 3x$ **80.** $10y - 7y$

**81.** $3u - 2v + 7v - 3u$

**82.** $9m - 4n + m - 3n$

**83.** $5(x - 4) + 10$

**84.** $15 - 7(z + 2)$

**85.** $3x - (y - 2x)$

**86.** $30x - (10x + 80)$

**87.** $3[b + 5(b - a)]$

**88.** $-2t[8 - (6 - t)] + 5t$

**3 ▶ Evaluate algebraic expressions by substituting values for the variables.**

**In Exercises 89–92, evaluate the algebraic expression for the specified values of the variable(s). If not possible, state the reason.**

| | *Expression* | *Values* |
|---|---|---|
| **89.** | $x^2 - 2x - 3$ | (a) $x = 3$ (b) $x = 0$ |
| **90.** | $\frac{x}{y + 2}$ | (a) $x = 0,\ y = 3$ (b) $x = 5,\ y = -2$ |
| **91.** | $y^2 - 2y + 4x$ | (a) $x = 4,\ y = -1$ (b) $x = -2,\ y = 2$ |
| **92.** | $\lvert 2x - x^2 \rvert - 2y$ | (a) $x = -6,\ y = 3$ (b) $x = 4,\ y = -5$ |

## 1.5 Constructing Algebraic Expressions

**1 ▶ Translate verbal phrases into algebraic expressions, and vice versa.**

**In Exercises 93–96, translate the verbal phrase into an algebraic expression.**

**93.** Twelve decreased by twice the number $n$

**94.** One hundred increased by the product of 15 and a number $x$

**95.** The sum of the square of a number $y$ and 49

**96.** Three times the absolute value of the difference of a number $n$ and 3, all divided by 5

**In Exercises 97–100, write a verbal description of the algebraic expression without using the variable.**

**97.** $2y + 7$

**98.** $5u - 3$

**99.** $\frac{x - 5}{4}$

**100.** $4(a - 1)$

**2 ▶ Construct algebraic expressions with hidden products.**

**In Exercises 101–104, write an algebraic expression that represents the quantity in the verbal statement, and simplify if possible.**

**101.** The amount of income tax on a taxable income of $I$ dollars when the tax rate is 18%

**102.** The distance traveled when you travel 8 hours at the average speed of $r$ miles per hour

**103.** The area of a rectangle whose length is $l$ units and whose width is 5 units less than the length

**104.** The sum of two consecutive integers, the first of which is $n$

## Chapter Test

**Take this test as you would take a test in class. After you are done, check your work against the answers in the back of the book.**

**1.** Place the correct inequality symbol (< or >) between each pair of numbers.
(a) $\frac{5}{2}$ ☐ $|-3|$  (b) $-\frac{2}{3}$ ☐ $-\frac{3}{2}$

**2.** Find the distance between $-4.4$ and 6.9.

**In Exercises 3–10, evaluate the expression.**

**3.** $-14 + 9 - 15$

**4.** $\frac{2}{3} + \left(-\frac{7}{6}\right)$

**5.** $-2(225 - 150)$

**6.** $(-3)(4)(-5)$

**7.** $\left(-\frac{7}{16}\right)\left(-\frac{8}{21}\right)$

**8.** $\frac{5}{18} \div \frac{15}{8}$

**9.** $\left(-\frac{3}{5}\right)^3$

**10.** $\frac{4^2 - 6}{5} + 13$

**11.** Identify the property of real numbers illustrated by each statement.
(a) $(-3 \cdot 5) \cdot 6 = -3(5 \cdot 6)$
(b) $3y \cdot \frac{1}{3y} = 1$

**12.** Rewrite the expression $-6(2x - 1)$ using the Distributive Property.

**In Exercises 13–16, simplify the expression.**

**13.** $3x^2 - 2x - 5x^2 + 7x - 1$

**14.** $x(x + 2) - 2(x^2 + x - 13)$

**15.** $a(5a - 4) - 2(2a^2 - 2a)$

**16.** $4t - [3t - (10t + 7)]$



**17.** Explain the meaning of "evaluating an algebraic expression." Evaluate the expression $7 + (x - 3)^2$ for each value of $x$.
(a) $x = -1$  (b) $x = 3$

**18.** An electrician wants to divide 102 inches of wire into 17 pieces with equal lengths. How long should each piece be?

**19.** A *cord* of wood is a pile 4 feet high, 4 feet wide, and 8 feet long. The volume of a rectangular solid is its length times its width times its height. Find the number of cubic feet in 5 cords of wood.

**20.** Translate the phrase into an algebraic expression.

"The product of a number $n$ and 5, decreased by 8"

**21.** Write an algebraic expression for the sum of two consecutive even integers, the first of which is $2n$.

**22.** Write expressions for the perimeter and area of the rectangle shown at the left. Simplify the expressions and evaluate them when $l = 45$.

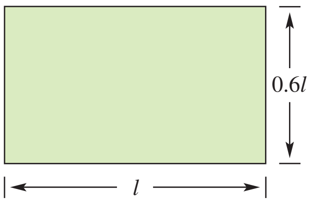


Figure for 22

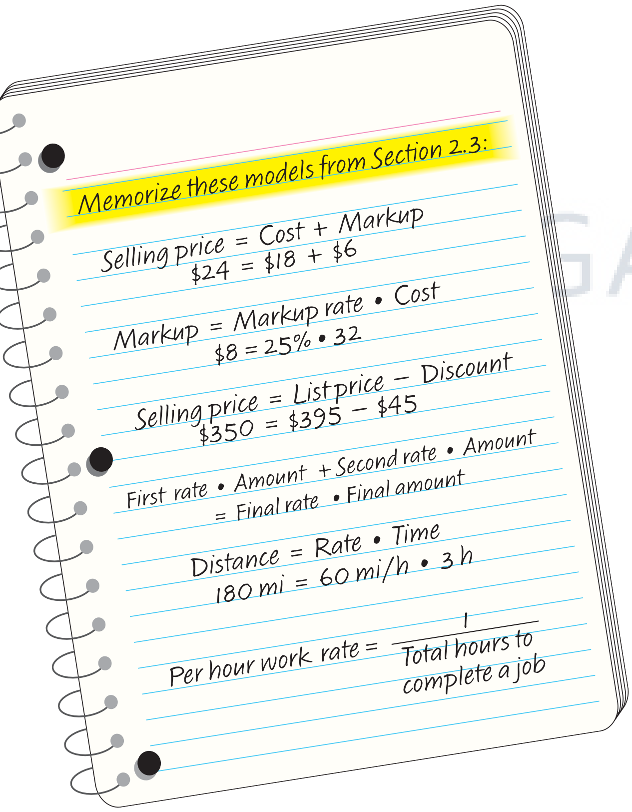
# Study Skills in Action

## Improving Your Memory

Have you ever driven on a highway for ten minutes when all of a sudden you kind of woke up and wondered where the last ten miles had gone? The car was on autopilot. The same thing happens to many college students as they sit through back-to-back classes. The longer students sit through classes on autopilot, the more likely they will "crash" when it comes to studying outside of class on their own.

While on autopilot, you do not process and retain new information effectively. Your memory can be improved by learning how to focus during class and while studying on your own.

Kimberly Nolting
VP, Academic Success Press
expert in developmental education




### Smart Study Strategy

## Keep Your Mind Focused

**During class**

- When you sit down at your desk, get all other issues out of your mind by reviewing your notes from the last class and focusing just on math.
- Repeat in your mind what you are writing in your notes.
- When the math is particularly difficult, ask your instructor for another example.

**While completing homework**

- Before doing homework, review the concept boxes and examples. Talk through the examples out loud.
- Complete homework as though you were also preparing for a quiz. Memorize the different types of problems, formulas, rules, and so on.

**Between classes**

- Review the concept boxes and check your memory using the checkpoint exercises, Concept Check exercises, and the What Did You Learn? section.

**Preparing for a test**

- Review all your notes that pertain to the upcoming test. Review examples of each type of problem that could appear on the test.

# Answers to Odd-Numbered Exercises, Quizzes, and Tests

## CHAPTER 1

### Section 1.1 *(page 9)*

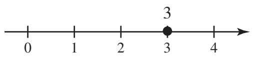
**1.** (a) $\{1, 2, 6\}$ (b) $\{-6, 0, 1, 2, 6\}$
(c) $\left\{-6, -\frac{4}{3}, 0, \frac{5}{8}, 1, 2, 6\right\}$ (d) $\left\{-\sqrt{6}, \sqrt{2}, \pi\right\}$

**3.** (a) $\left\{\sqrt{4}\right\}$ (b) $\left\{\sqrt{4}, 0\right\}$
(c) $\left\{-4.2, \sqrt{4}, -\frac{1}{9}, 0, \frac{3}{11}, 5.\overline{5}, 5.543\right\}$ (d) $\left\{\sqrt{11}\right\}$

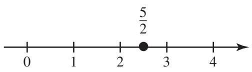
**5.** $0.\overline{2}$ **7.** $2.\overline{12}$

**9.** $-5, -4, -3, -2, -1, 0, 1, 2, 3$ **11.** 5, 7, 9

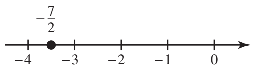
**13.** (a)



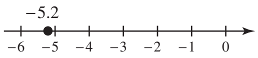

(b)




(c)




(d)




**15.** $-1 < \frac{1}{2}$ **17.** $-\frac{9}{2} < -2$ **19.** $<$ **21.** $<$

**23.** $<$ **25.** $>$ **27.** $>$ **29.** 6 **31.** 19

**33.** 50 **35.** 8 **37.** 35 **39.** 3 **41.** 10

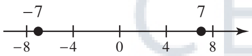
**43.** 225 **45.** $-85$ **47.** $-16$ **49.** $-\frac{3}{4}$

**51.** $-3.5$ **53.** $\pi$ **55.** $>$ **57.** $>$

**59.** $=$ **61.** $>$ **63.** $-34, 34$ **65.** 160, 160

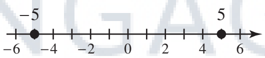
**67.** $\frac{3}{11}, \frac{3}{11}$ **69.** $-\frac{5}{4}, \frac{5}{4}$ **71.** $-4.7, 4.7$

**73.**



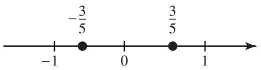

7

**75.**



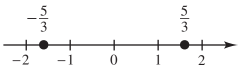

5

**77.**



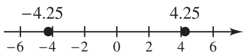

$\frac{3}{5}$

**79.**




$\frac{5}{3}$

**81.**




4.25

**83.** $x < 0$ **85.** $u \geq 16$ **87.** $16 \leq s \leq 28$

**89.** $p < 225$ **91.** $-4, 4$ **93.** $-2, 8$

**95.** True. If a number can be written as the ratio of two integers, it is rational. If not, the number is irrational.

**97.** 0.15 is a terminating rational number and $0.\overline{15}$ is a repeating rational number.

### Section 1.2 *(page 19)*

**1.** 45 **3.** 4 **5.** $-2.7$ **7.** 7 **9.** $-25.9$

**11.** $-20$ **13.** $-15.2$ **15.** 24 **17.** $-3.8$

**19.** 22 **21.** $\frac{5}{4}$ **23.** $\frac{1}{2}$ **25.** $\frac{1}{10}$ **27.** $\frac{1}{24}$

**29.** $\frac{63}{8}$ **31.** $\frac{35}{8}$ **33.** 60 **35.** 45.95 **37.** $-\frac{121}{8}$

**39.** $4 \cdot 9$ **41.** $6\left(\frac{1}{4}\right)$ **43.** $4\left(-\frac{1}{5}\right)$ **45.** $-30$

**47.** 48 **49.** $-40$ **51.** 36 **53.** $\frac{1}{2}$ **55.** $-\frac{12}{5}$

**57.** $\frac{1}{12}$ **59.** $-\frac{1}{3}$ **61.** $-\frac{1}{2}$ **63.** $\frac{1}{6}$ **65.** $\frac{3}{2}$

**67.** $-\frac{7}{9}$ **69.** 6 **71.** $-3$ **73.** $-9$ **75.** $-\frac{5}{2}$

**77.** $\frac{2}{5}$ **79.** $\frac{34}{45}$ **81.** $\frac{11}{12}$ **83.** $(-7)^3$ **85.** $\left(\frac{1}{4}\right)^4$

**87.** $-7^3$ **89.** 32 **91.** 16 **93.** $-64$ **95.** $\frac{64}{125}$

**97.** $\frac{1}{32}$ **99.** 0.027 **101.** $-0.32$ **103.** 0

**105.** 4 **107.** 22 **109.** 6 **111.** 12

**113.** 27 **115.** 135 **117.** $-6$ **119.** $-6$

**121.** 1 **123.** 30 **125.** 161 **127.** 14,425

**129.** 171.36 **131.** $\frac{17}{180}$ **133.** \$2533.56

**135.** (a)

| Day | Daily gain or loss |
|---|---|
| Tuesday | \$5 |
| Wednesday | \$8 |
| Thursday | $-$\$5 |
| Friday | \$16 |

(b) The stock gained \$24 in value during the week. Find the difference between the first bar (Monday) and the last bar (Friday).



**137.** (a) \$10,800 (b) \$15,832.22 (c) \$5032.22

**139.** 15 square meters **141.** 20 square inches

**143.** 6.125 cubic feet

**145.** True. A nonzero rational number is of the form $\frac{a}{b}$, where $a$ and $b$ are integers and $a \neq 0, b \neq 0$. The reciprocal will be $\frac{b}{a}$, which is also rational.

**147.** True. When a negative number is raised to an even power, the result is positive.

**149.** False. $6 \div 3 = 2 \neq \frac{1}{2} = 3 \div 6$

**151.** If the numbers have like signs, the product or quotient is positive. If the numbers have unlike signs, the product or quotient is negative.

**153.** (a) $40 - (10 + 3) = 27$

(b) $5^2 + \frac{1}{2} \cdot 4 = 27$

(c) $(8 \cdot 3 + 30) \div 2 = 27$

(d) $75 \div (2 + 1) + 2 = 27$

**155.** Only common factors (not terms) of the numerator and denominator can be divided out.

## Section 1.3 *(page 28)*

**1.** Additive Inverse Property

**3.** Multiplicative Inverse Property

**5.** Commutative Property of Addition

**7.** Associative Property of Addition

**9.** Distributive Property

**11.** Associative Property of Multiplication

**13.** Multiplicative Inverse Property

**15.** Additive Inverse Property **17.** Distributive Property

**19.** Additive Inverse Property

**21.** $-3(15)$ **23.** $5 \cdot 6 + 5 \cdot z$

**25.** $-x + 25$ **27.** $x + 8$

**29.** (a) $-10$ (b) $\frac{1}{10}$ **31.** (a) 19 (b) $-\frac{1}{19}$

**33.** (a) $-\frac{1}{2}$ (b) 2 **35.** (a) $\frac{5}{8}$ (b) $-\frac{8}{5}$

**37.** (a) $-6z$ (b) $\dfrac{1}{6z}$

**39.** (a) $-(x - 2)$ or $-x + 2$ (b) $\dfrac{1}{x - 2}$

**41.** $(32 + 4) + y$ **43.** $(9 \cdot 6)m$

**45.** $20 \cdot 2 + 20 \cdot 5$ **47.** $x(-2) + 6(-2)$ or $-2x - 12$

**49.** $-6(2y) + (-6)(-5)$ or $-12y + 30$

**51.** $(7 + 2)x = 9x$ **53.** $\dfrac{x}{8}(7 - 5) = \dfrac{x}{4}$

**55.** $3x + 15$ **57.** $-2x - 16$

**59.** Answers will vary. **61.** Answers will vary.

**63.** $x + 5 = 3$

Original equation

$(x + 5) + (-5) = 3 + (-5)$

Addition Property of Equality

$x + [5 + (-5)] = -2$

Associative Property of Addition

$x + 0 = -2$

Additive Inverse Property

$x = -2$

Additive Identity Property

**65.** $2x - 5 = 6$

Original equation

$(2x - 5) + 5 = 6 + 5$

Addition Property of Equality

$2x + (-5 + 5) = 11$

Associative Property of Addition

$2x + 0 = 11$

Additive Inverse Property

$2x = 11$

Additive Identity Property

$\frac{1}{2}(2x) = \frac{1}{2}(11)$

Multiplication Property of Equality

$\left(\frac{1}{2} \cdot 2\right)x = \frac{11}{2}$

Associative Property of Multiplication

$1 \cdot x = \frac{11}{2}$

Multiplicative Inverse Property

$x = \frac{11}{2}$

Multiplicative Identity Property

**67.** 28 **69.** 434 **71.** 62.82

**73.** $a(b + c) = ab + ac$

**75.** $(4) + (x + 5) + (3x + 2)$; $4x + 11$

**77.** (a) $2(x + 6) + 2(2x)$; $6x + 12$

(b) $(x + 6)(2x)$; $2x^2 + 12x$

**79.** The additive inverse of a real number is its opposite. The sum of a number and its additive inverse is the additive identity zero. For example, $-3.2 + 3.2 = 0$.

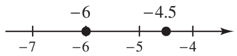
**81.** Given two real numbers $a$ and $b$, the sum $a$ plus $b$ is the same as the sum $b$ plus $a$.

**83.** Sample answer: $4 \odot 7 = 15 \neq 18 = 7 \odot 4$

$3 \odot (4 \odot 7) = 21 \neq 27 = (3 \odot 4) \odot 7$

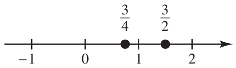
## Mid-Chapter Quiz *(page 31)*

**1.**




$-4.5 > -6$

**2.**




$\frac{3}{4} < \frac{3}{2}$

**3.** 22 **4.** 6.5 **5.** 7.6 **6.** $-9.8$ **7.** 14

**8.** 5 **9.** $\frac{5}{2}$ **10.** $\frac{1}{2}$ **11.** 60 **12.** $-\frac{3}{8}$

**13.** $\frac{7}{10}$ **14.** $-\frac{27}{8}$ **15.** 4 **16.** 2

**17.** (a) Distributive Property

(b) Additive Inverse Property

**18.** (a) Associative Property of Addition

(b) Multiplicative Identity Property

**19.** \$1068.20 **20.** \$8640

**21.** $\frac{7}{24}$; The sum of the parts of the circle graph is equal to 1.

## Section 1.4 *(page 37)*

**1.** $10x, 5;\ 10, 5$ **3.** $-6x^2, 12;\ -6, 12$

**5.** $-3y^2, 2y, -8;\ -3, 2, -8$ **7.** $-4a^3, 1.2a;\ -4, 1.2$

**9.** $4x^2, -3y^2, -5x, 21;\ 4, -3, -5, 21$

**11.** $-5x^2y, 2y^2, xy;\ -5, 2, 1$ **13.** $\frac{1}{4}x^2, -\frac{3}{8}x, 5;\ \frac{1}{4}, -\frac{3}{8}, 5$

**15.** Commutative Property of Addition

**17.** Associative Property of Multiplication

**19.** Distributive Property

**21.** $5x + 30$ or $5x + 5 \cdot 6$

**23.** $(x + 6)5$ **25.** $7x$ **27.** $2x^2$ **29.** $-4x$

**31.** $8y$ **33.** $8x + 18y$ **35.** $6x^2 - 2x$

**37.** $-2z^4 - 4z^2 + 5z + 8$ **39.** $-x^2 + 3xy + y$

**41.** $8x^2 + 4x - 12$ **43.** $-18y^2 + 3y + 6$

**45.** $-3x^2 + 2x - 4$ **47.** $5x^2 + 2x$

**49.** $-12x^2 + 51x$ **51.** $10t^2 - 35t$ **53.** $12x - 35$

**55.** $-4x - 9$ **57.** $a + 3$ **59.** $-7y - 7$

**61.** $y^3 + 6$ **63.** $x^3 - 12$ **65.** $44a - 22$

**67.** $-6x + 96$ **69.** $12x^2 + 2x$ **71.** $-2b^2 + 4b - 36$

**73.** (a) 3 (b) $-10$ **75.** (a) 6 (b) 9

**77.** (a) 7 (b) 11

**79.** (a) Not possible; undefined (b) $\frac{28}{9}$

**81.** (a) 13 (b) $-36$ **83.** (a) 7 (b) 7

**85.** (a) Not possible; undefined (b) $\frac{1}{2}$

**87.** (a) 3 (b) 0 **89.** (a) 210 (b) 140

**91.** 252 $\text{ft}^3$ **93.** 3888 $\text{in.}^3$ **95.** \$1.21

**97.** \$7.23 **99.** $\frac{1}{2}b^2 - \frac{3}{2}b$; 90

**101.** \$23,500 million; \$23,775 million

**103.** \$22 billion; \$22.3 billion **105.** 1440 square feet

**107.** No. When $y = 3$, the expression is undefined.

**109.** To remove a set of parentheses preceded by a minus sign, distribute $-1$ to each term inside the parentheses. For example: $13 - (-10 + 5) = 13 + 10 - 5 = 18$.

**111.** A factor can consist of a sum of terms; the term $x$ is part of the sum $x + y$, which is a factor of $(x + y) \cdot z$.

**113.** No. There are an infinite number of values of $x$ and $y$ that would satisfy $8y - 5x = 14$. For example, $x = 10$ and $y = 8$ would be a solution, and so would $x = 2$ and $y = 3$.

## Section 1.5 *(page 47)*

**1.** $23 + n$ **3.** $12 + 2n$ **5.** $n - 6$ **7.** $4n - 10$

**9.** $\frac{1}{2}n$ **11.** $\frac{x}{6}$ **13.** $8 \cdot \frac{N}{5}$ **15.** $4c + 10$

**17.** $0.30L$ **19.** $\frac{n + 5}{10}$ **21.** $|n - 8|$ **23.** $3x^2 - 4$

**25.** A number decreased by 2

**27.** A number increased by 50

**29.** Two decreased by three times a number

**31.** The ratio of a number and 2

**33.** Four-fifths of a number

**35.** Eight times the difference of a number and 5

**37.** The sum of a number and 10, divided by 3

**39.** The square of a number, decreased by 3

**41.** $0.25n$ **43.** $0.10m$ **45.** $5m + 10n$

**47.** $55t$ **49.** $\frac{320}{r}$ **51.** $0.45y$

**53.** $0.0125I$ **55.** $L - 0.20L = 0.80L$

**57.** $8.25 + 0.60q$ **59.** $n + 5n = 6n$

**61.** $(2n + 1) + (2n + 3) + (2n + 5) = 6n + 9$

**63.** $\frac{2n(2n + 2)}{4} = n^2 + n$ **65.** $s^2$

**67.** $\frac{1}{2}b(0.75b) = 0.375b^2$

**69.** Perimeter: $2(2w) + 2w = 6w$; Area: $2w(w) = 2w^2$

**71.** Perimeter: $6 + 2x + 3 + x + 3 + x = 4x + 12$

Area: $3x + 6x = 9x$ or $6(2x) - 3(x) = 9x$

**73.** $b(b - 50) = b^2 - 50b$; square meters

**75.**

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $5n - 3$ | $-3$ | 2 | 7 | 12 | 17 | 22 |
| **Differences** | 5 | 5 | 5 | 5 | 5 | |

The differences are constant.

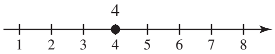
**77.** $a$ **79.** a and c

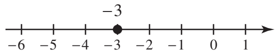
**81.** Using a specific case may make it easier to see the form of the expression for the general case.

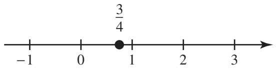
## Review Exercises *(page 52)*

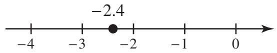
**1.** (a) $\{52, \sqrt{9}\}$ (b) $\{-4, 0, 52, \sqrt{9}\}$
(c) $\{\frac{3}{5}, -4, 0, 52, -\frac{1}{8}, \sqrt{9}\}$ (d) $\{\sqrt{2}\}$

**3.** $\{1, 2, 3, 4, 5, 6\}$

**5.** (a)



(b)



(c)



(d)



**7.** $<$ **9.** $<$ **11.** 14 **13.** 7.3 **15.** 5

**17.** $-7.2$ **19.** 11 **21.** 230 **23.** $-41.8$

**25.** $\frac{11}{21}$ **27.** $\frac{1}{6}$ **29.** $\frac{17}{8}$ **31.** $-28$

**33.** $-4200$ **35.** $-\frac{1}{20}$ **37.** 14 **39.** 2

**41.** $(-3) + (-3) + (-3) + (-3) + (-3) + (-3) + (-3)$

**43.** $8(8)$ **45.** $6^7$ **47.** 1296 **49.** $-16$ **51.** $\frac{1}{8}$

**53.** 20 **55.** 98 **57.** 1,165,469.01 **59.** \$800

**61.** Additive Inverse Property **63.** Distributive Property

**65.** Associative Property of Addition

**67.** Commutative Property of Multiplication

**69.** Distributive Property **71.** $u - 3v$ **73.** $-8a + 3a^2$

**75.** $4y^3, -y^2, \frac{17}{2}y; 4, -1, \frac{17}{2}$

**77.** $-1.2x^3, \frac{1}{x}, 52; -1.2, 1, 52$

**79.** $9x$ **81.** $5v$ **83.** $5x - 10$ **85.** $5x - y$

**87.** $-15a + 18b$ **89.** (a) 0 (b) $-3$

**91.** (a) 19 (b) $-8$ **93.** $12 - 2n$ **95.** $y^2 + 49$

**97.** The sum of two times a number and 7

**99.** The difference of a number and 5, divided by 4

**101.** $0.18I$ **103.** $l(l - 5) = l^2 - 5l$

## Chapter Test *(page 55)*

**1.** (a) $<$ (b) $>$ **2.** 11.3 **3.** $-20$

**4.** $-\frac{1}{2}$ **5.** $-150$ **6.** 60 **7.** $\frac{1}{6}$

**8.** $\frac{4}{27}$ **9.** $-\frac{27}{125}$ **10.** 15

**11.** (a) Associative Property of Multiplication

(b) Multiplicative Inverse Property

**12.** $-12x + 6$ **13.** $-2x^2 + 5x - 1$ **14.** $-x^2 + 26$

**15.** $a^2$ **16.** $11t + 7$

**17.** Evaluating an expression is solving the expression when values are provided for its variables.
(a) 23 (b) 7

**18.** 6 inches **19.** 640 cubic feet **20.** $5n - 8$

**21.** $2n + (2n + 2) = 4n + 2$

**22.** Perimeter: $2l + 2(0.6l) = 3.2l$, 144;
Area: $l(0.6l) = 0.6l^2$, 1215

This page contains answers for this chapter only.