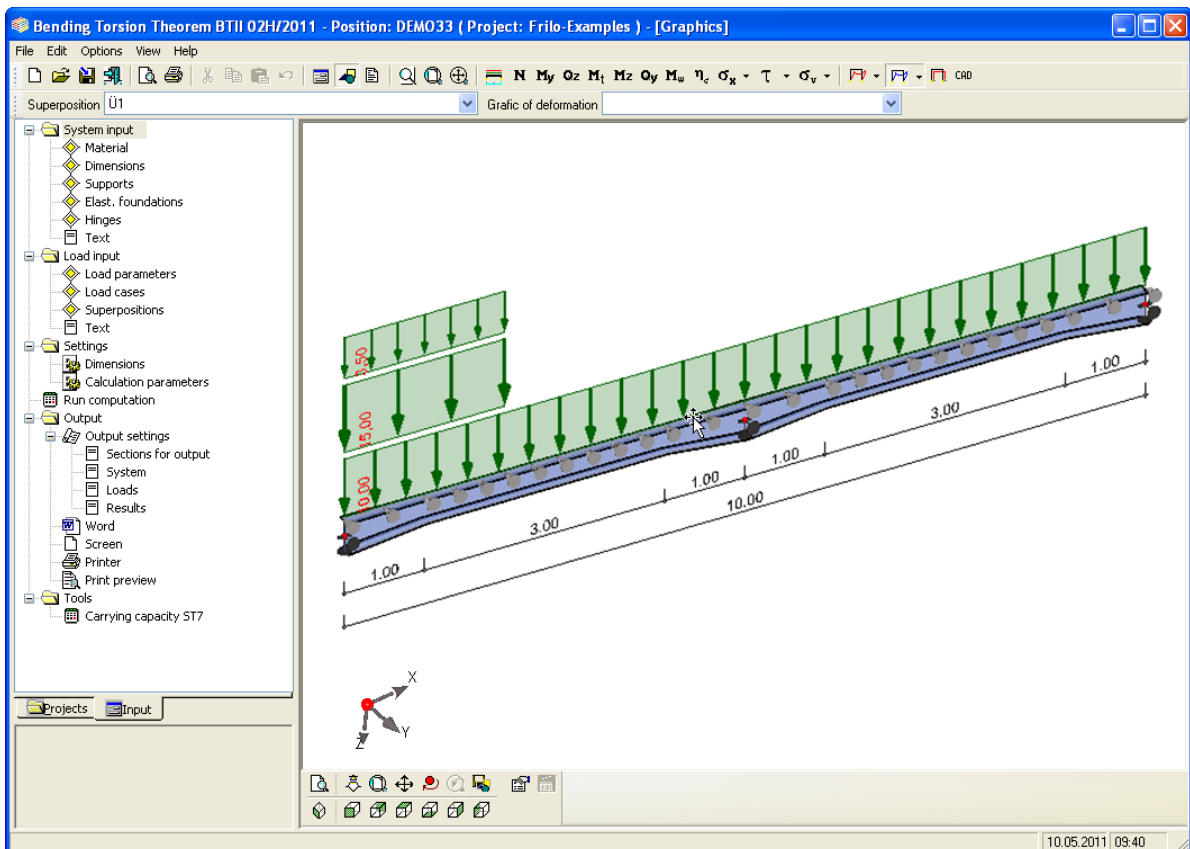


Lateral torsional buckling analysis BTII

User manual for Frilo design applications



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Application options

General scope of application

The *BTII* application allows you to perform analyses of the ultimate and serviceability limit states of steel bar systems with any types of supports. The steel bars may have open or closed cross sections with thin-walled members.

The most important features of *BTII* are the following:

- Calculation of *internal* forces, elastic deformations and axial and shear stresses on uniformly or three-dimensionally loaded beam systems with consideration to warping torsion in second order buckling torsion analyses.
- Calculation of the ideal bifurcation loads for the lateral buckling and lateral torsional buckling failure modes as well as determination of the slenderness ratios and the reduction factors for stability analyses in accordance with the equivalent bar method.
- Optional definition of moving loads to examine crane runways, for instance, in the ultimate and serviceability limit states.
- Calculation of secondary flange bending stress considered as local beam loading due to eccentric loading on the lower flange.

Special applications

- Purlins supported by the roof skin with or without pin-joints
- Ledgers supported by purlins or trapezoidal steel sections
- Columns supported by the wall lining and/or bracing
- Stability verifications of craneway beams with or without horizontal bracing
- Determination of the ideal bifurcation loads for the calculation of buckling slenderness ratios in concrete and timber construction.

Design standards

The *BTII* application performs structural safety analyses in accordance with DIN 18800 and EN 1993-1-1 and takes the corresponding National Appendix into account. The following National Appendices are available:

- DIN EN 1993-1-1/NA
- ÖNORM B 1993-1-1
- NA to BS EN 1993-1-1
- NEN EN 1993-1-1/NB
- NBN EN 1993-1-1 ANB
- CSN EN 1993-1-1/NA.

System definition

The *BTII* application allows you to define any bar system composed of straight bars including

- cross sectional jumps and/or haunches;
- simple- and double-symmetrical T-sections with and w/o top flange angles, U-sections, thin-walled closed sections and any type of thin-walled open cross sections;
- discrete three-dimensional supporting conditions with a distance to the shear centre; definable as rigid supports or supports with discrete spring stiffness;
- continuous three-dimensional supporting conditions such as elastic foundation or shear field foundation also with a distance to the shear centre;
- beam sections connected with shear force joints and moment joints.

Special notes concerning the system definition:

- The material shows elastic behaviour.
- The modulus of elasticity and the shear modulus are constant over the total beam.
- The z-axis is the symmetry axis for single-symmetrical cross sections.
- The finite elements have constant cross sections. Haunches are calculated by approximation.

Loads, load cases, superpositions and deformations

In *BTII*, linearly variable line loads and point loads in direction of or around the y/z-axis as well as bending, torsional and warping moments can optionally be defined. Loads that produce axial forces cannot be put out directly. To compensate for this restriction, you can define constant or linear variable axial force curves. Additional bending moments that result from an offset of the centre of gravity must be defined explicitly by the user. The loads are assigned to load cases. All loads that are member of the same load case are considered to act always simultaneously. The load case defines the action that produces the loads and indicates in addition, how it is to be handled in the automatic generation of the load case combinations.

The user can generate load case combinations either automatically with the help of an assistant or define them manually on the basis of typical design practices.

For second order analyses, imperfections are taken into account. To include them in the form of initial bow or initial sway imperfections, you simply need to specify the zero-points and the amplitudes of the sinusoidal or parabolic half-waves.

Moving loads

You can optionally define node loads in the form of a load train.

Local beam loading

When overhead underslung cranes travel along the beam on rails or wheels, crane wheel loads or trolley loads apply eccentrically to the beam web. Therefore, secondary flange bending stresses occur in the proximity of the load application point in two directions. The application calculates the stresses resulting from the local load introduction and superimposes these stresses with the global beam stresses. Local beam loading due to the operation of underslung overhead cranes is considered for double-T beams with or w/o top flange angles.

Calculation and analysis

- Verification of the cross sectional bearing capacity on the basis of elastic or plastic cross section values.
- The calculation in accordance with DIN 18800-1 verifies the b/t-ratio on which the e-e or e-p verification is based. In the calculation in accordance with EN 1993-1-1, the cross sections are classified.
- Verification of the system's bearing capacity in a second order buckling torsion analysis or in a structural safety analysis on the basis of bifurcation loads for the failure modes lateral buckling, torsional bucking and lateral torsional buckling.

The analyses are subject to the following restrictions:

The *BTII* application only considers cross sections of class 1 to 3. The verification of a cross section of class 4 is not implemented in the current version.

Interfaces to BTII

A number of FRILO applications support the *BTII* interface, which provides for the transfer of the system and the loads to *BTII*.

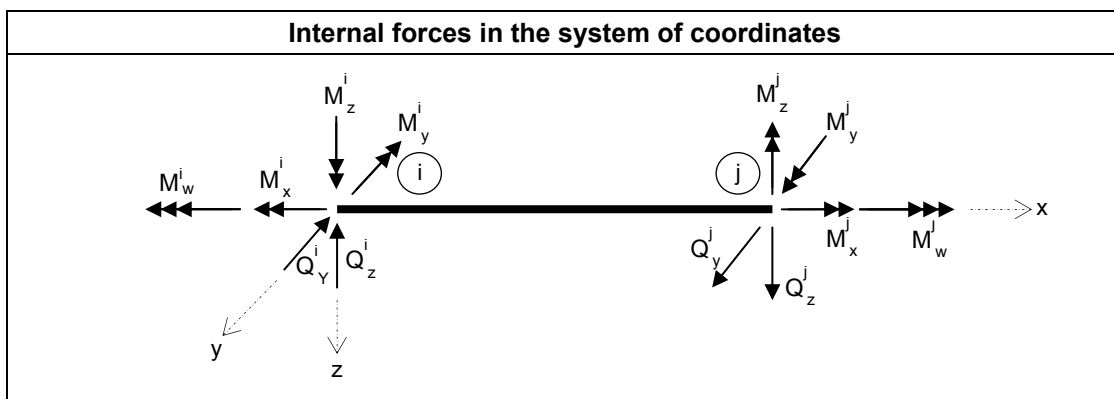
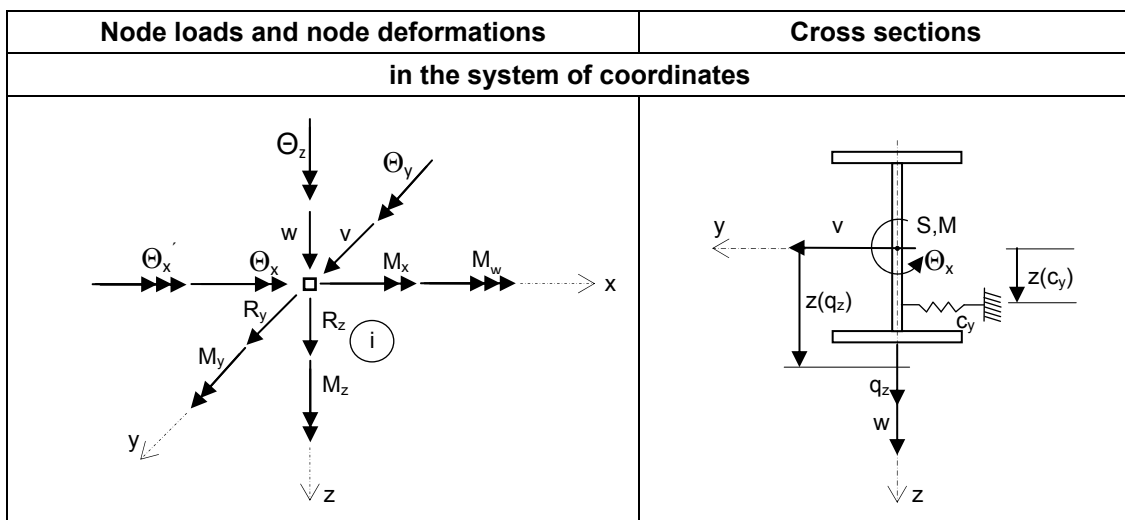
Basis of calculation

The theoretical fundamentals of this application are described in detail in the reference literature, particularly in reference [7] and [10].

In the majority of the cases relevant in practice, you cannot describe the exact deformations of the general buckling torsion problem in a single closed system. Therefore, the beam is verified in accordance with the finite elements method, which means that it is divided into a number of sections of different lengths (finite elements). The number of sections is pre-set by the user.

The state of deformation within an element is described with the help cubic polynomials for the shift perpendicular to the bar axis and the distortion. The elements are linked via the nodes in-between them. The elements have 6 degrees of freedom each at the left and right node.

- Shift v and w in the y/z -direction
- Torsion $\Theta_x, \Theta_y, \Theta_z$ around the $x/y/z$ -axis.
- Warping Θ_x'



General notes concerning BTII

Moving loads

You can optionally define node loads in the form of a load train. The limit load positions for the first wheel in travelling direction must be specified by the user. You can select among various criteria to define the target for the decisive load position.

For each load position, a linear or non-linear calculation of the beam is performed depending on the defined target. If you select the maximum axial stress as a target value for the decisive load position, you can choose among two alternative criteria. These are either the absolute maximum axial beam stress or the greatest absolute stress considering the load introduction stresses described below. The application calculates automatically deformations, internal forces and stresses for the decisive load position in first and second order analyses.

Local beam loading

When underslung overhead cranes travel along the beam on wheels or trolleys, the crane wheel loads or trolley loads apply eccentrically to the beam web. Therefore, secondary flange bending stresses occur in the proximity of the load application point in two directions. The application calculates local load introduction stresses on the basis of [1] and superimposes them with the global beam stresses in accordance with the von-Mises yield criterion.

The experimental and theoretical examinations by Hannover and Reichwald form the basis for the consideration of local beam loading caused by the operation of underslung overhead cranes in the *BTII* application. In the current version, this type of calculation can be performed on double-T beam cross section types.

Coordinates for supports, springs and concentrated point loads

The locations of supports, discrete springs, concentrated loads, element borders as well as zero-points of imperfection half-waves are defined by specifying the x-coordinate. Internally, the application generates nodes at supports, springs, loads and deformation zero-points. If the distance of a node to the relevant point is smaller than the minimum element extension specified by the user, no node is generated and the support, spring or individual load is displaced to the next closest node. If this displacement is not acceptable for the analysis, the user must subsequently adjust the minimum element extension accordingly and perform a new calculation.

Variable cross sections

The application allows you to couple asymmetric cross sections. You should note in this connection that the relative location of the centre of gravity and that of the shear centre do not coincide if different shapes of cross sections are used. Since internal forces and deformations refer partly to the centre of gravity and partly to the shear centre, the principle of equilibrium in the strict sense is violated in the nodes. This problem can be neglected with haunched beams, however.

Thin-walled open sections

This option allows the user to define any open cross section in a freely selectable local system of coordinates.

Bearing capacity analysis (E-P) IAW DIN 18800-1

This option allows you to utilize plastic bearing capacity reserves in a second order analysis. First, *BTII* performs a calculation of the three-dimensionally pre-deformed system. After this, the values of the yield function are calculated at each element end in accordance with DIN 18800 P.1 Eq. (41) and Eq. (42) using the extension for warping moments specified in [6]. In addition to this, the limiting values for the referenced shear forces $v_y = V_y/V_{pl,y}$ and $v_z = V_z/V_{pl,z}$ are determined as required by DIN 18800 P1 Element (757).

Settings

The menu item ▶ Edit ▶ Settings displays a dialog for the adjustment of general settings and calculation parameters. You can access this dialog also by double-clicking on the menu item ▶ Settings ▶ Dimensions. This dialog allows you to set dimensions and calculation parameters (discretisation, analyses) and adjust output settings.

Dimensions

This option allows you to set the desired units for dimensions, forces etc. The number of decimal places is set by the application according to selected unit. The selected units apply also to the output.

Calculation parameters

This option allows you to set the parameters for the discretisation of the system and the verification of the cross sectional bearing capacity.

Output profile

In this section, the user defines the profile for the output of the sections, the system, the loads and the results. The output profile allows you to define the scope of the output.

System inputs

Material and calculation parameters

Design standard

Allows you to select the design standard that constitutes the basis of the structural safety analysis. When you select a national version of EN 1993-1-1, also the corresponding National Appendix is used.

Material

Steel grade

Allows you to select the steel grade. The following steel grades are currently implemented.

- Hot-rolled, non-alloy structural steel
- Hot-rolled structural steel, normalized
- Hot-rolled structural steel, thermo-mechanically rolled
- Hot-rolled, weatherproof structural steel
- High-temperature steel
- Hot-finished hollow sections
- User-defined steels

Steel quality

Allows you to select the steel quality depending on the selected steel grade.

User-defined

If you have selected "User-defined steels" among the steel grade options, a dialog for the definition of the user-defined parameters is displayed.

Analysis of the system's bearing capacity

1st order analyses

Internal forces, deformations and stresses are calculated in first order analyses.

The bearing capacity of the system cannot be verified in this type of analysis!

2nd order analyses

Internal forces, deformations and stresses are calculated in second order buckling torsion analyses whereby the imperfections are taken into account. Evidence of the system's bearing capacity is established via the verification of the cross sectional bearing capacity.

Equivalent bar method

When using the equivalent bar method for the examination of the lateral buckling and lateral torsional buckling behaviour, *BTII* performs an eigenvalue calculation by applying the linear subspace method. The resulting ideal bifurcation loads $N_{ki,y}$, $N_{ki,z}$ and $M_{ki,y}$ are used to calculate the corresponding effective slenderness ratios. These ratios allow the calculation of the relevant reduction factors for the bearing resistances, which are required for the stability analysis.

Analysis of the cross sectional bearing capacity

Elastic - elastic (e-e)

The design values of the internal forces calculated in accordance with the theory of elasticity are used to determine the axial and shear stresses acting on the cross section in accordance with the mechanics of materials. These stresses are compared to the design value of the yield strength. The structural safety of the cross section is ensured when the loading in all cross sectional parts is smaller or in, the worst case, equal to the design values of the resistances. The plastic bearing reserves are not taken into account.

When the calculation is performed as per EN 1993-1-1, the equations are based on the elastic cross sectional values. This method is suitable for cross section classes 1 to 3.

Elastic - plastic (e-p)

The internal forces and deformations are calculated on the basis of the theory of elasticity. The resistances are determined with utilization of the plastic bearing capacity. The structural safety of the cross section is ensured when the design values of the internal forces do not exceed the limit internal forces in the plastic state.

There are three methods available for this verification:

1. Analysis as per DIN 18800-1, Para. 7.5.3

DIN 18800-1 describes in paragraph 7.5.3. a verification method for the structural safety of double-symmetrical I-sections. This method is extended by the terms for torsion and warping. However, it is limited to the analysis of standardised or user-defined double-symmetrical I-sections.

2. Analysis as per EN 1993-1-1, Para. 6

The structural safety verification of cross sections is stipulated in EN 1993-1-1 Para. 6. The terms elastic-elastic method and elastic-plastic method are not included in EN 1993-1-1. The related verification equations take the classification of the cross sections into account and refer to the elastic or plastic cross sectional values that are determined by the class of the cross section (classes 1 to 4). When you select the elastic-plastic verification method, the verification equations are based on the plastic cross sectional values. This method is suitable for cross sections of the classes 1 and 2.

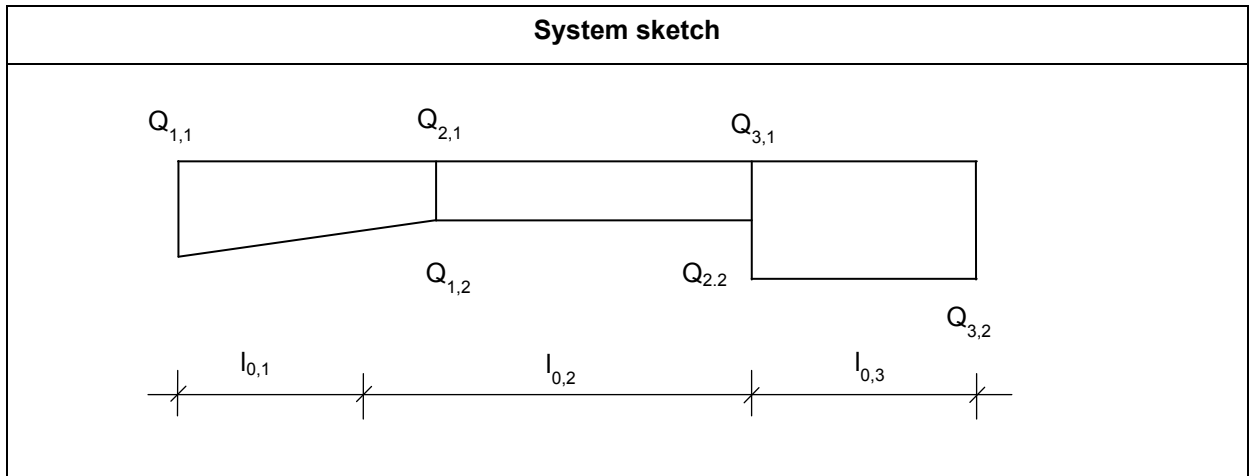
3. Partial internal forces method according to Kindmann

The partial internal forces method by Kindmann allows you to verify any type of hvh beam. hvh refers in this connection to a cross section of the horizontal-vertical-horizontal type. This means that any type of cross section with two or three limbs (flanges and webs) perpendicular to each other can be verified on the basis of plastic limit internal forces.

Dimensions

System definition

Definition of the beam sections



Option	Description
From x	X-coordinate at the beginning of the beam section, measured from the left beam edge
To x	X-coordinate of the end of the beam section, measured from the left beam edge
Q ₁	Index referring to the cross section assigned to the left edge of the beam section
Q ₂	Index referring to the cross section assigned to the right edge of the beam section
l ₀	Length of the beam section

You can divide the beam into several sections. This allows you to describe cross sectional jumps and haunches. Haunches can be defined for the following section types:

- Standard sections
- User-defined double-symmetrical I-sections
- User-defined single-symmetrical I-sections
- User-defined single-symmetrical I-sections with top flange angles
- User-defined U-sections

Functions available via the tool bar

Symbol	Shortcut	Description
	-	Adds a beam section at the end of the list.
	Ctrl-I	Inserts a beam section above the active list item
	Ctrl-D	Deletes the active beam section
	-	Deletes all beam sections
	F5	Displays the cross section selection dialog

Cross sections





Defining new cross sections

Click into the next empty table cell (in the column "Name"). Press the F5 key or type in a name for the cross section. The dialog "Select/edit cross section" is displayed offering several options to select or define a cross section.

Editing cross sections

Click onto the name of the cross section that you like to edit. Press the F5 key. The dialog "Select/edit cross section" is displayed allowing you to edit a previously defined cross section.

Functions available via the tool bar

Symbol	Shortcut	Description
	-	Adds a cross section at the end of the list.
	Ctrl-I	Inserts a cross section above the active list item
	Ctrl-D	Deletes the active cross section
	-	Deletes all cross sections

Selecting/editing a cross section

The dialog allows you to enter a new cross section or edit an existing one. You can access the editing dialog via the cross section dialog (F5 key).

In the left area of the screen, the available input options are displayed:

- F + L section file
- Steel dimensions
- Structural values (single- or double-symmetrical)

You can display the hidden submenu of the option "F+L section file" for instance by clicking on the + symbol or the 1 key.

In the right area of the screen, you can either select the desired section or enter the dimensions or the structural values.

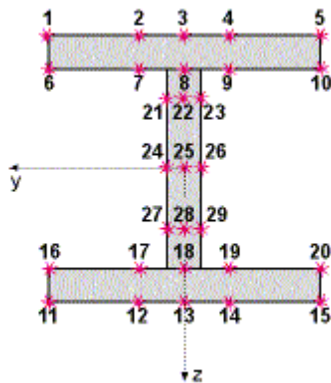
Clicking "OK" confirms the entered values and closes the section selection window.

Reading/writing cross sections

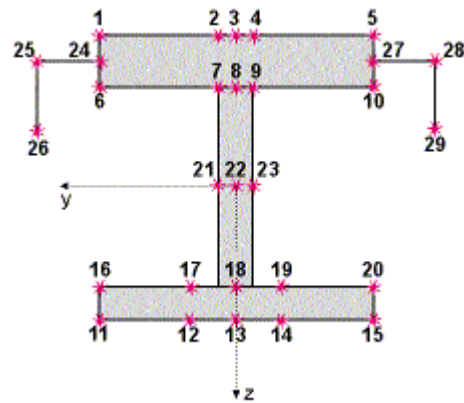
You can save a cross section that you have defined via its dimensions, for instance, in an ASCII file by pressing the "Write" button and load this file by pressing the "Read" button. The storage path is freely selectable. The "Name" button activates the name field and you can edit the name of the corresponding cross section.

Stress points on the cross section

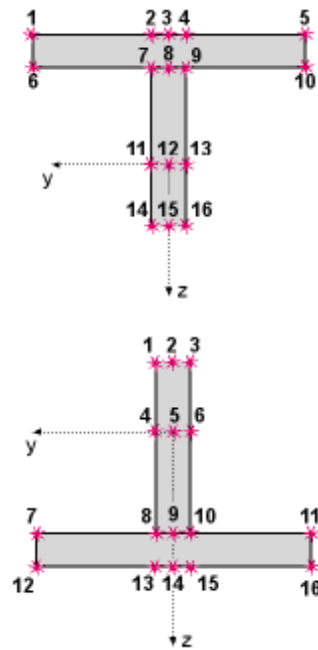
Double-symmetrical and single-symmetrical I-section



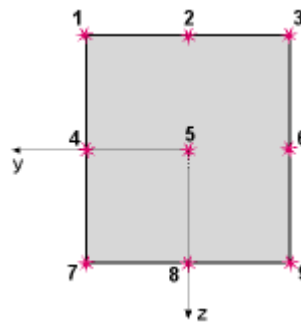
Double-T with top flange angles



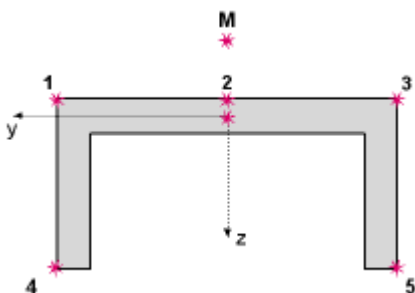
T-sections



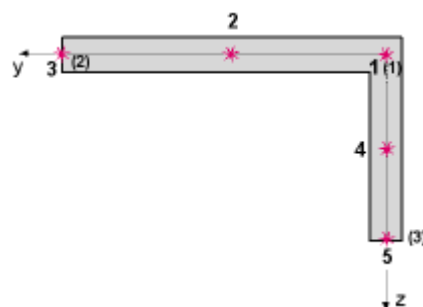
Solid cross section



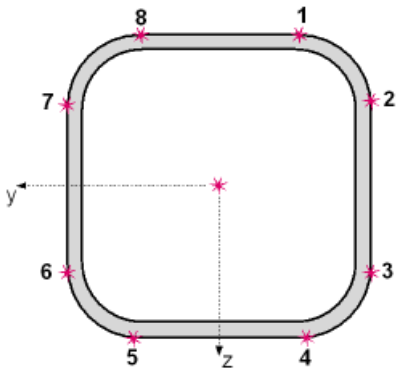
U-section as per DIN 1026 and user-defined U-section



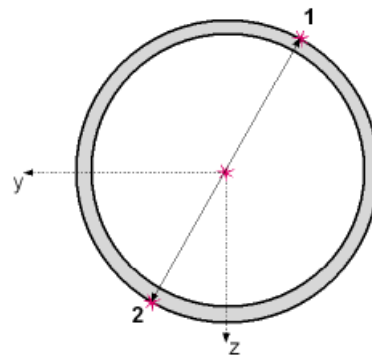
Thin-walled open section



Square hollow section



Round hollow section



Reference points on the cross section

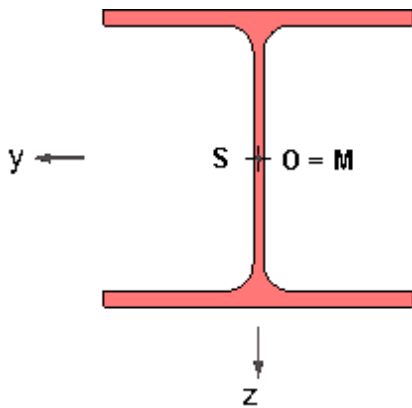
If the shear centre of a standard section is known, it is always the reference point of this section.

The following rules apply to user-defined cross sections:

- The reference point of single-symmetrical I- and T-sections and single-symmetrical I-sections with top flange angles is always the centre of the clear web height.
- The reference point of user-defined U-sections is the shear centre in horizontal direction and the centre of the clear flange height in vertical direction.
- The reference point of thin-walled open sections is given by the zero-point of the coordinate system which is implicitly defined when the user enters the cross section.

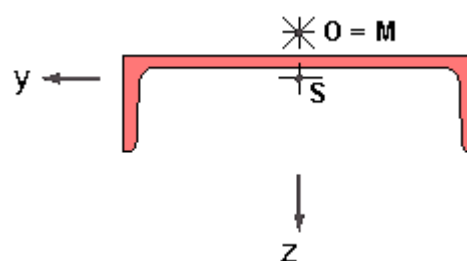
Double-T-section

The reference point is the shear centre.



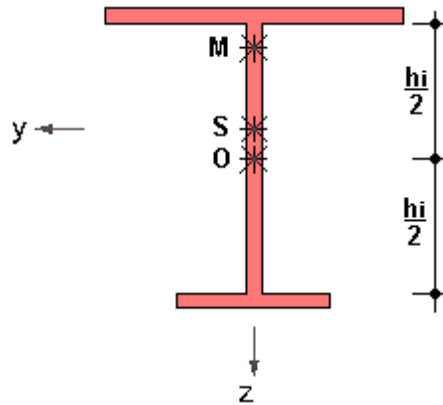
U-section as per DIN 1026

The reference point is the shear centre.



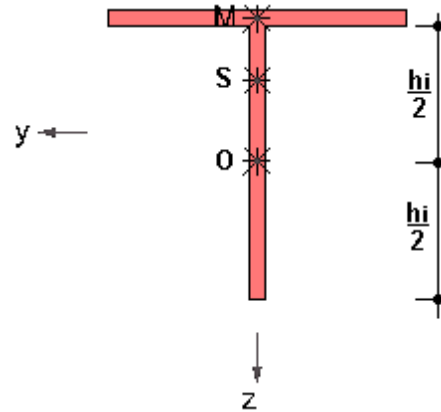
Single-symmetrical I-section

The reference point is the centre of the clear web height.



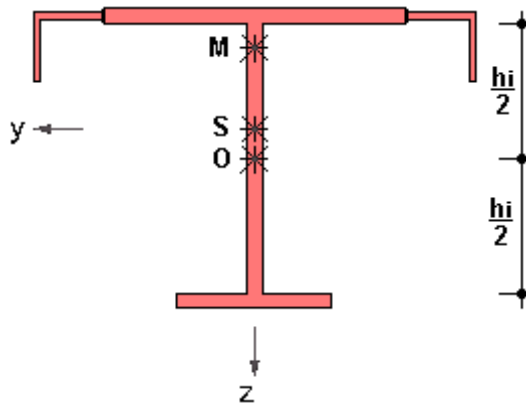
T-section

The reference point is the centre of the clear web height.



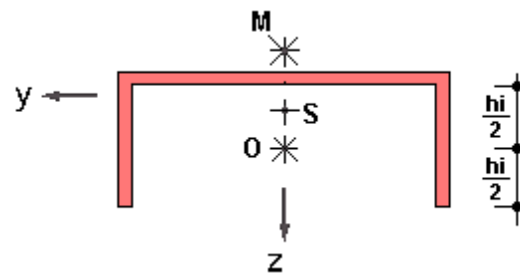
Double T with top flange angles

The reference point is the centre of the clear web height.



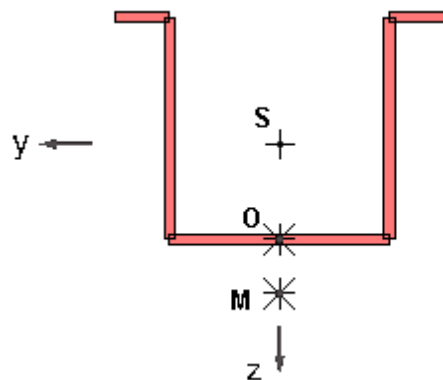
User-defined U-section

The reference point is the centre of the clear web height.

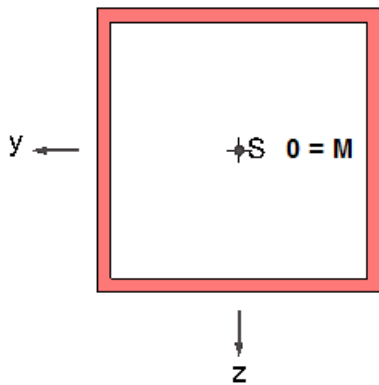


Thin-walled open section

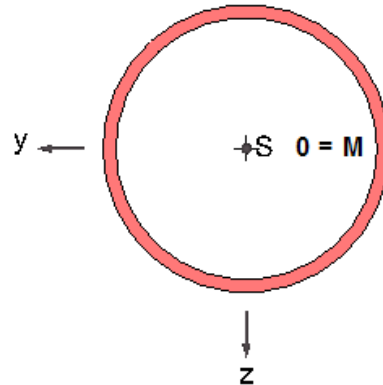
The reference point is given by the zero-point of the implicitly defined system of coordinates.



Square hollow section



Round hollow section



Steel dimensions

This menu item allows you to define a cross section via its dimensions. Select first the type of cross section (double-T, U, square etc). The corresponding input fields for the dimensions are displayed.

The resistances are calculated by the application and displayed in the lower half of the screen.

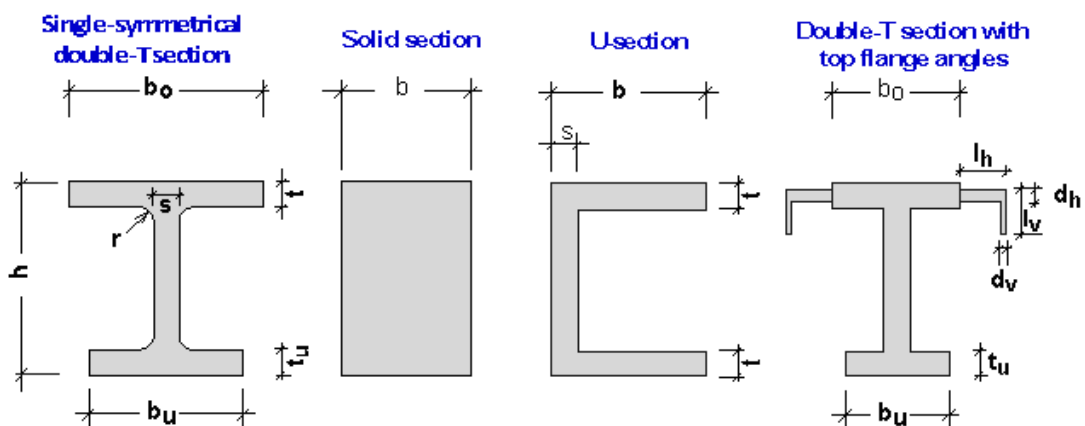
Name Clicking on this button enables the Name field and you can edit the name of the cross section.

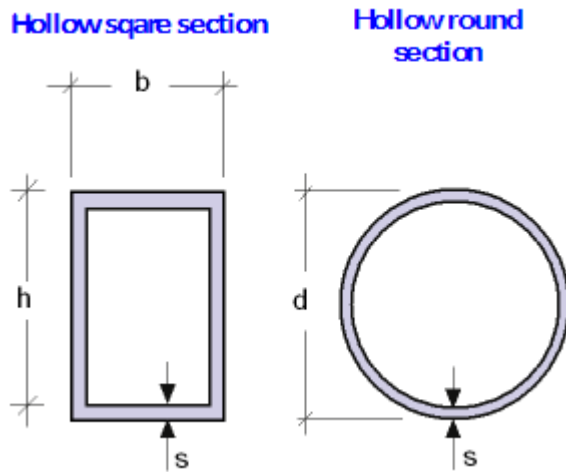
Read Clicking on this button loads a section definition file that you have previously saved with via the Write button.

Write Clicking on this button saves section definition data in an ASCII file.

Display all section values

The option displays a window showing the structural, design and geometry values.





Structural values (I, A, W)

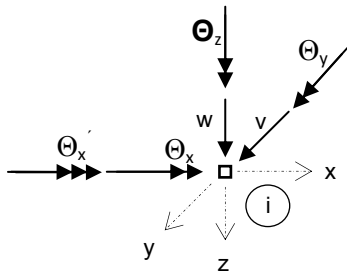
This menu allows the user to define the cross section via the values for the structural calculation ($I_{y,z}$, I_t , A , $A_{qy,z}$, b_y , h_z) and the values for the stress analysis (W_y top, W_z left, W torsion, $A_{ty,z}$). The decisive resistance moments must also be specified. If no values are entered, no stress analysis is performed for the corresponding cross section.

Supports

Definition of discrete supporting conditions

Supports in this connection refer to discrete supporting conditions that are realised as rigid or elastic translational or torsional fixities. Rigid supports in direction of the global degrees of freedom are defined by specifying "-1" in the corresponding columns. For elastic supports, the absolute value of the spring stiffness must be entered.

Input values



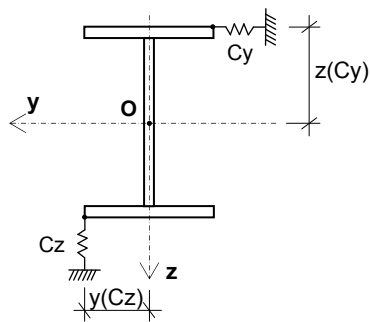
Value	Description
At x	Distance of the support from the left beam edge
Cy	Supporting condition for shift in y-direction
Cz	Supporting condition for shift in z-direction
theta x	Supporting condition for torsion around the x-axis
theta y	Supporting condition for torsion around the y-axis
theta z	Supporting condition for torsion around the z-axis
theta xy	Supporting condition for warping
Distances	Definition of the distances to the reference point

Functions available via the tool bar

Symbol	Shortcut	Description
	-	Adds a supporting condition at the end of the list.
	Ctrl-I	Inserts a supporting condition above the active list item
	Ctrl-D	Deletes the active supporting condition
	-	Deletes all supporting conditions
	F5	Displays a dialog for the definition of distances

Distances of discrete supporting conditions

Click on the "Edit" button to display the dialog for the definition of the distances.



Value	Description
$y(Cz)$	Distance of the elastic support to the reference point in y-direction
$z(Cy)$	Distance of the elastic support to the reference point in z-direction

This input option is only available if a spring value was defined in direction of the corresponding translational degree of freedom.

Notes concerning the input of spring stiffnesses

Discrete spring stiffnesses describe the stiffnesses of the components connected to the examined beam (e.g. purlins on top of beams, horizontal beams on top of wall columns, tension rods for purlins, etc) by approximation. The application allows also the eccentric location of springs to provide fixity against lateral shift in y- or z-direction. The locations are defined via their distance to the reference point. The reference point depends on the defined section type, however. See [Reference points of the cross sections](#).

The application converts the distances to the shear centre. You can also use eccentric discrete springs to provide fixity against lateral shifts in the y- or z-direction at any point of the cross section. For this purpose, you must define high, but not too high, spring stiffnesses. As a rule, stiffness should be $< 10^{16}$. In order to ensure the numerical stability of the calculation, discrete stiffnesses intended as shift fixities should not be greater than strictly necessary. You can check this by verifying the kinematic constraint conditions in the cross section.

Foundations







Definition of foundation regions

The term foundation in this context refers to continuous supporting conditions. Foundation regions must be located inside the beam and must not overlap. As with discrete elastic supporting conditions, you can define a distance of the foundation region to the reference point.

Input values

Value	Description
From x	First coordinate of the foundation region
To x	Last coordinate of the foundation region
Type	Type of foundation 0 = elastic foundation 1 = shear field stiffness
cy	Foundation modulus for translational foundation in y-direction with type = 0: elastic foundation with type = 1: shear field stiffness
Flag	Control parameter for the specification of the distance of the application point of the cy-foundation to the reference point 0 = absolute distance to the reference point 1 = factor to be multiplied with the section height
z(cy)	Distance of the cy-foundation to the reference point in z-direction or factor for this distance
cz	Foundation modulus for translational foundation in z-direction with type = 0: elastic foundation in z-direction with type = 1: no specification
Flag	Control parameter for the specification of the distance of the application point of the cz-foundation to the reference point 0 = absolute distance to the reference point 1 = factor to be multiplied with the top flange width
y(cz)	Distance of the cz-foundation to the reference point in y-direction or factor for this distance
ctheta	Foundation modulus for the rotational foundation around the x-axis
Distances	Click on the "Edit" button to display the dialog for the definition of the distances to the reference point with graphic support.

Functions available via the tool bar

Symbol	Shortcut	Description
	-	Adds a foundation region at the end of the list.
	Ctrl-I	Inserts a foundation region above the active list item
	Ctrl-D	Deletes the active foundation region
	-	Deletes all foundation regions
	F5	Displays a dialog for the definition of the distances
		Launches the <i>FLS13</i> application if correctly installed and licensed. <i>FLS13</i> allows you to calculate the values for translational, shear field and rotational foundation.

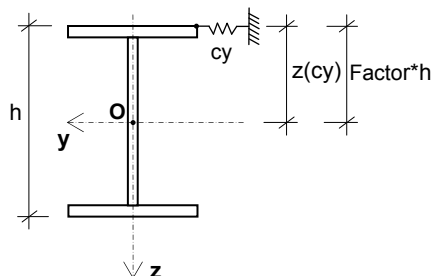
Distances of foundations

Foundation regions extend over a particular area of the beam. The distances are defined uniformly over the total region. If required, several foundation regions must be defined.

Graphic representation

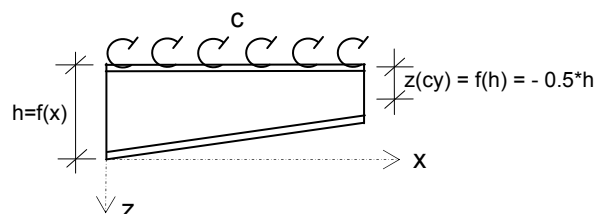
The graphic representation of the selected foundation region either shows the total region or a discrete point on the beam. Tick the option **x-coordinate** to display the representation at the point x_0 . Otherwise, the entire foundation region is shown.

Distance in z-direction

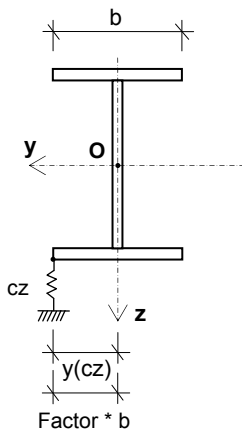


Value	Description
Absolute distance	The distance to the reference point is defined via an absolute value
Relative distance	The distance to the reference point is defined via a factor to be multiplied with the height of the cross section.
$z(cy)$	The distance of the cy -foundation to the reference point in z -direction
Factor	The factor for the distance to the reference point in z -direction

Illustration



Distance in y-direction



Value	Description
Absolute distance	The distance to the reference point is defined via an absolute value
Relative distance	The distance to the reference point is defined via a factor to be multiplied with the height of the cross section.
$y(cz)$	The distance of the cy-foundation to the reference point in y-direction
Factor	The factor for the distance to the reference point in y-direction

Notes concerning the input of foundations

Defining a fixed axis of rotation

The problem of lateral torsional buckling with a fixed axis of rotation at a distance z from the shear centre often occurs in practice. You can describe it in *BTII* as follows:

Define an elastic translational foundation in y-direction with the stiffness 10^8 to 10^{10} at the distance z to the shear centre. The resulting shift and torsion in regard to the centre of gravity and the shear centre are equal to zero along the pre-set fixed axis of rotation.

Calculation of the foundation constants

If the application *ST13 - Shear Field Stiffness* is installed on your computer, you can launch it by pressing the F5 key in either of the input fields $c_{\theta}/c_y/c_z$. *ST13* allows you to calculate the foundation constants for trapezoidal sheet metal structures.

The application calculates the torsion spring c_{θ} [kNm/m], the ideal shear stiffness S [kN] as well as the translational foundation c_y [kN/m²]. These values allow you to take the stabilising effect of trapezoidal steel sheet sections into account. In addition to this, the application verifies the fixity against lateral shift and torsion. If the verification is not successful, an additional lateral stability verification is required. In practice, the verification whether the rotational foundation is sufficient is hardly ever successful. A lateral torsional buckling analysis is required in most cases. The spring constants calculated by *ST13* can be transferred to the relevant applications such as *BTII*.

You should note in this connection, however, that the calculation in *ST13* is based on the simplified assumptions of DIN 18800-2, El. (308) and El. (309).

These assumptions presume a constant double-T cross section and beam supports on both sides of the beam. Foundation regions extending over different cross sections must be divided accordingly. When you transfer data from *BTII* to *ST13*, the cross section in the middle of the foundation region is considered to be relevant. You can edit the cross section in *ST13* to modify it subsequently.





Pinned joints

BTII allows you to define shear force and moment joints. A degree of freedom is treated as a pinned joint if you set a check mark in the corresponding column. To define a moment joint for M_y , for instance, tick the check box in the theta column.

Input values

Value	Description
At x	Distance of the joint to the left beam edge
v	Shear force joint in y-direction
w	Shear force joint in z-direction
theta x	Moment joint around the x-axis
theta y	Moment joint around the y-axis
theta z	Moment joint around the z-axis
theta xy	Warping joint

Functions available via the tool bar

Symbol	Shortcut	Description
	-	Adds a joint at the end of the list.
	Ctrl-I	Inserts a joint above the active list item
	Ctrl-D	Deletes the active joint
	-	Deletes all joints

Input of loads

Load parameters

Specifications concerning the safety concept

Consequence classes

Allows you to define the consequence class the safety concept should be based on: CC1, CC2 or CC3. (Only in combination with the analysis as per NEN EN 1993-1-1 and NBN EN 1993-1-1)

Equation for the permanent and the transient design situation

Specifies which equation should be used for the structural safety analysis in the permanent or transient design situation. (Only in combination with the analysis as per NEN EN 1993-1-1 and NBN EN 1993-1-1.)

Specifications concerning the load introduction

Load introduction stresses due to loading on the lower flange

When underslung overhead cranes travel along the beam on wheels or trolleys, the crane wheel loads or trolley loads apply eccentrically to the beam web. Therefore, secondary flange bending stresses occur in the proximity of the load application point in two directions. The application calculates the local load introduction stresses for double-T sections on the basis of the experimental and theoretical examinations of Hannover and Reichwald and superimposes these stresses with global axial beam stresses in accordance with the von-Mises yield criterion. The following options are available for this calculation:

- **No calculation**
A point load in z-direction is considered as a force entity.
- **Calculation w/o consideration of the load position**
The decisive load position underneath the travelling crane is calculated without consideration of the secondary flange bending stresses. A point load in z-direction is interpreted as two force entities, one acting on the left and the other on the right lower flange with a distance e_y to the outer flange edge.
- **Calculation with consideration of the load position**
The decisive load position underneath the travelling crane is calculated with consideration of the secondary flange bending stresses. The decisive load position results from the superposition of the axial beam stresses with the load introduction stresses in x-direction. Point loads in z-direction are handled as described above.

Type of point load

If you have selected the option "Moving load", you must set the desired criterion of the decisive load position in the selection window. The available criteria for the decisive load position are the minimum or maximum internal forces or the greatest absolute axial stress or comparison stress.

If you select the maximum axial stress as a criterion for the decisive load position, you can choose among two additional options: These are either the absolute maximum axial beam stress or the greatest absolute stress considering the load introduction stresses, if the corresponding option was selected.

If you select the comparison stress as a criterion for the decisive load position, this criterion also includes the secondary flange bending stress in x-direction, if applicable.

Load cases

Loads are always defined in combination with load cases. This means that all loads assigned to the same load case are always caused by the same action and always considered to act simultaneously.






The load cases are not taken into account directly They are included in the calculation via superposition factors that correspond to the partial safety factors on the action side.

Load case definition

Input options

Value	Description
Load case	Consecutive number of the load case
Designation	Name of the load case
Type	Allows to specify whether the load case should contain the lower or the upper values or the difference of the upper and lower values of the loads: 1 = lower value, 2 = upper value, 3 = upper - lower values.
Action	Index of the action that is assigned to the load case.
Alternative	Alternative group, loads in an alternative group exclude each other,
Self-weight	Tick the corresponding check box to include the self-weight automatically together with this load case.
Type of point load	0 = static point load 1 = moving point load Load cases can include static and moving point loads, independently of each other, if the load parameters have been set accordingly.
Loads	Allows you to edit the loads assigned to the load case.

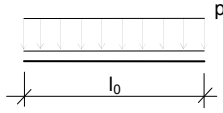
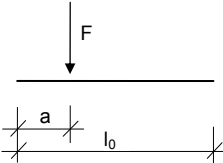
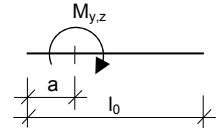
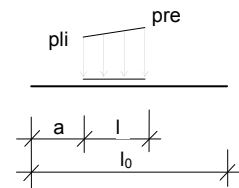
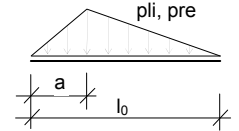
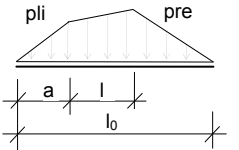
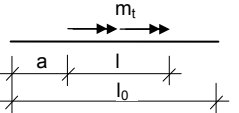
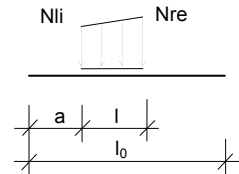
Functions available via the tool bar

Symbol	Shortcut	Description
	-	Adds a load case at the end of the list.
	Ctrl-I	Inserts a load case above the active list item
	Ctrl-D	Deletes the active load case
	-	Deletes all load cases
	F5	Displays a dialog for the definition of the loads.






This input option is only available if the inclusion of moving loads was set in the load parameter section.

Loads

Value	Description
Type	Load types 1 to 8
Dir.	Load direction: 2 = in direction of the y-axis 3 = in direction of the z-axis
Ple	Left load ordinate
a	Distance to the left beam edge
Pri	Right load ordinate
l	Length of a load section.
Flag	Specifies how the distance to the load application point is defined. See load application points
ey	Distance of the load application point to the reference point in y-direction
ez	Distance of the load application point to the reference point in z-direction
Distances	Click on the "Edit" button to display the dialog for the definition of the distances to the reference point with graphic support.
Remark	Comments concerning the load

Load type	Description	System sketch
0 Cancel	The option displays the insert row for a load. If you have already defined a load in this row, it is deleted when you select this option.	
1 Uniformly distributed load	A linear load that applies constantly over the total length of the beam.	
2 Concentrated load (point load)	A concentrated load applying at the distance a from the left beam edge.	
3 Concentrated moment (point moment)	A moment applying at a distance a from the left beam edge	
4 Trapezoidal load	A linear load linearly variable over the length l applying at a distance a from the left beam edge	
5 Triangular load	A triangular load variable over the total length of the beam.	
6 Trapezoidal load over l_0	A trapezoidal load variable over the total length of the beam.	
7 Torsional region moment	A torsional region moment applying over a length l at a distance a from the left beam edge	
8 Axial force	An axial force linearly variable over the length l applying at a distance a from the left beam edge	

Functions available via the tool bar

Symbol	Shortcut	Description
	-	Adds a load at the end of the list.
	Ctrl-I	Inserts a load above the active list item
	Ctrl-D	Deletes the active load
	-	Deletes all loads
	F5	Displays a dialog for the input of the distances

Definition of the distances to the reference point

Some loads extend over a particular area of the beam. The distances are defined consistently over the total area. If required, several loads must be entered.

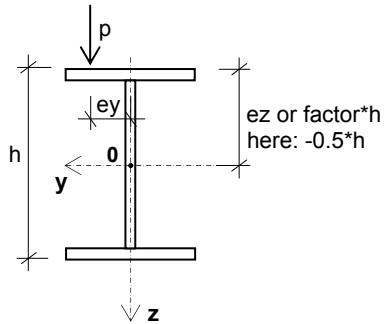
The following rules apply to the input of distances:

1. Linear loads extending in both directions (y- and z-direction, types 1, 4, 5, 6) can be defined via a distance in the y-direction and in the z-direction. These kinds of loads can produce a torsional moment.
2. Point loads can be defined only via one distance in the load direction. Therefore, point loads cannot produce torsional moments except when applying to open polygonal cross sections.
3. Point loads in z-direction can be included as wheel loads acting on the lower flange. Half of the value is assigned to the right flange and half of the value to the left flange. The corresponding option must be set in the load parameters.

Note:

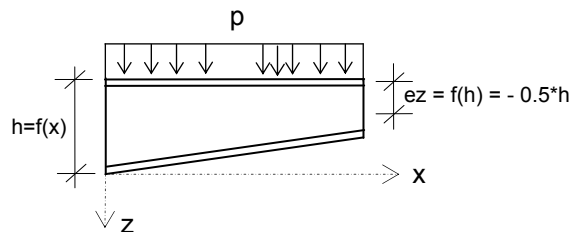
If a beam is composed of different cross sections, you should be aware that the reference points may vary and with them the decisive distances to the centre of gravity or the shear centre of the cross section in question.

Load distances without loading on the lower flange

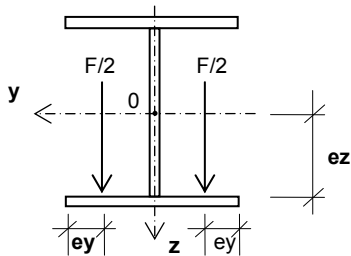


Value	Description
Absolute distance	The distance to the reference point is defined via an absolute value
Relative distance	The distance to the reference point is defined via a factor to be multiplied with the height of the cross section.
e_z	Distance of the load to the reference point in the z-direction, flag = 0
Factor	Factor for the distance of the load to the reference point in the z-direction, flag = 1
e_y	The load applies at a distance e_y to the reference point of the cross section.

Illustration



Load distances with loading on the lower flange



Value	Description
e_z = the distance to the reference point	The distance to the reference point is defined via an absolute value
e_y = the distance to the outer edge of the flange	The load is defined as lower flange loading with $F/2$ on each flange side and applies at a distance e_y to the outer edges of the flange sides.

Superpositions

The calculation of the system is based on load case combinations. They are generated in accordance with the combination rules stipulated by EN 1990. Load case combinations include the load cases via coefficients into the calculation. These coefficients correspond to the partial safety factors for the actions.

Load case combinations are assigned to a design situation and a limit state independently of each other. The partial safety factors for the actions are determined via the load combination factors. The partial safety factors for the resistances are determined by the design situation and the limit state.







Load case combinations

Value	Description
No.	Consecutive number of the superposition
Designation	Name of the superposition
min x	Smallest coordinate for the trolley travel
max x	Highest coordinate for the trolley travel
Criterion	Criterion for the decisive load position
Design	Design situation as per EN 1990
Limit state	Limit state as per EN 1990
Type of...	Imperfections 1 = parabolic 2 = sinusoidal
Imperfections	Displays a dialog for the input of the imperfections.
Factors	Displays a dialog for the input of the superposition factors.

Notes concerning moving loads

When the "Moving load" option was selected in the "Load parameter" menu, areas where the wheel loads move must be defined on the beam for each superposition. To define such a travelling area, the smallest and the greatest permissible x-coordinate must be specified for the front wheel in travelling direction. Limit load positions may be defined in such a manner that individual wheels are beyond the beam.

Functions available via the tool bar

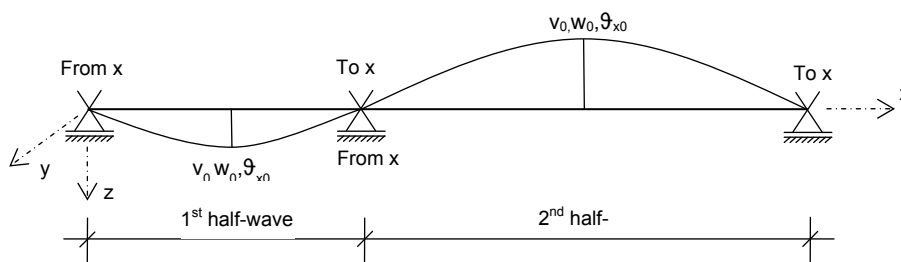
Symbol	Shortcut	Description
	-	Adds a load case combination at the end of the list.
	Ctrl-I	Inserts a load case combination above the active list item
	Ctrl-D	Deletes the active load case combination.
	-	Deletes all load case combinations.
		Displays a dialog for the input of the imperfections
		Displays a dialog for the automatic generation of the load case combinations.

Imperfections

General

BTII allows you to define imperfections in the directions of the both cross sectional major axes y and z as well as initial sway imperfections around the longitudinal axis of the bar. In order to reduce input work in connection with the inclusion of imperfections, you simply need to specify the zero-points of the half-waves and their amplitudes. On the basis of these specifications, the application calculates the magnitude of the imperfections in all node points in-between the zero-passages of the half-waves. The equivalent imperfection loads required for the 2nd order analysis result from the multiplication of the imperfections with the geometric stiffness matrices.

Definition of imperfection half-waves



Value	Description
Area	Consecutive number of the imperfection half-wave
Dir.	Direction of the amplitude of the imperfection half-wave 0: Cancellation of the input 1: Pre-distortion around the x-axis 2: Imperfection in y-direction 3: Imperfection in z-direction
From x	Coordinate of the front end of the imperfection region from the left beam edge
To x	Coordinate of the rear end of the imperfection region from the left beam edge
Amplitude y (v_0)	Amplitude of the imperfection half-wave in the centre of the imperfection region in y-direction
Amplitude z (w_0)	Amplitude of the imperfection half-wave in the centre of the imperfection region in z-direction
Amplitude theta x (ϑ_{x0})	Amplitude of the imperfection half-wave in the centre of the imperfection region around the x-axis

Inclusion of imperfections in the second order analyses only

If geometric and structural imperfections should be calculated in second order analyses, geometric equivalent imperfections must be taken into account. These are initial sway imperfections caused by angles of bar rotation for sway systems and initial bow imperfections in the form of sinusoidal or parabolic half-waves for non-sway systems. Even-though geometric equivalent imperfections are not defined in the form of an imperfect system geometry in design practice but, for reasons of simplification, via static equivalent loads, *BTII* allows the inclusion of imperfection half-waves.





Notes concerning the course of imperfection half-waves

The course of the imperfection half-waves should correspond to the lowest mode shape of lateral buckling or lateral torsional buckling. According to [6] you can alternatively define the imperfection in such a manner that the modal component is sufficiently great to achieve an approximation of the load deformation curve to the 1st eigenvalue.

Notes concerning verification as per DIN 18800-2

The amplitudes should be determined on the basis of the buckling curves a, b, c, d as per DIN 18800 Part 2 and the direction of deflection (y or z). You should note in connection with the elastic-elastic verification method as per DIN 18800 P.2, that the imperfection may be reduced to 2/3 according to table 3. In addition to this, you may reduce the initial bow imperfections by 50 % in the lateral stability verification according to Element (202).

Functions available via the tool bar

Symbol	Shortcut	Description
	-	Adds a imperfection half-wave at the end of the list.
	Ctrl-I	Inserts a imperfection half-wave above the active list item
	Ctrl-D	Deletes the active imperfection half-wave
	-	Deletes all imperfection half-waves

Superposition factors

No.	Consecutive number of the load case
Load case	Name of the load case
Factor	The factor for the inclusion of the load case in the load case combination

See also [Automatic generation of load case combinations](#).

Note: The superposition factor corresponds to the partial safety factor for actions in accordance with the partial safety concept.

Generating load case combinations

General

Under normal conditions, superposition factors are defined in accordance with typical design practices. In addition to this, the user can benefit from the automatic generation of load case combinations. This dialog allows you to generate load case combinations for all design situations and limit states in accordance with the combination rules of EN 1990. The settings concerning the safety concept in the "Load parameter" menu are taken globally into consideration for the generation of the load case combinations.

Design situation

Select the design situation for which you like to generate the load case combinations.

Limit states

Select the limit state for which you like to generate the load case combinations.

Automatic generation of load case combinations

The following combination rules as per EN 1990 apply to the combination of design situation and limit state:

Design situation	Limit state		
	EQU	ULS	SLS
Permanent (p)	Eq. (6.10)	Eq. (6.10) or Eq.(6.10a) and Eq. (6.10b)	-
Transient (T)	Eq. (6.10)	Eq. (6.10) or Eq.(6.10a) and Eq. (6.10b)	-
Accidental (A)	Eq.(6.11a) or Eq. (6.11b)	Eq.(6.11a) or Eq. (6.11b)	-
Earthquake (E)	Eq. (6.12)	Eq. (6.12)	-
Fire (F)	Eq.(6.10a) and Eq. (6.10b)	Eq.(6.10a) and Eq. (6.10b)	-
Infrequent (char)	-	-	Eq. (6.14b)
Frequent (frequ)	-	-	Eq. (6.15b)
Quasi-permanent (qp)	-	-	Eq. (6.16b)

The user can generate combinations of design situation and limit state that are not implemented via an equation. These combinations do not comply with the combination rules of EN 1990, however.

Design and analysis

Calculation parameters

The menu option ▶ Settings ▶ Calculation parameters allows you to set the parameters for the discretisation of the system and the analysis procedure.

Discretisation

Minimum element extension (min dx)

Specifies the minimum length of a finite element in the discretisation of the system. The user can control the number of elements by specifying minimum and maximum element extensions.

Minimum number of elements on the beam (ne)

Specifies the minimum number of finite elements in the discretisation of the system. The real number of elements could be considerably higher. The quotient of the beam length and the minimum number of elements gives orientation for the element length in the beam sections.

As a rule, the user should define between **5 and 15** elements in order to ensure that, with average shift gradients, the difference in the deformations is less than 5 % compared to the exact solution. The **number of required elements** depends on the gradient of the bending curve. With steep gradients such as those of point loads, individual springs and stiffness jumps and with elastic foundation in combination with stability-critical loading, the number of elements must be increased. If you are unsure about the number of required elements simply perform a new calculation with refined elements. If the results differ considerably perform another calculation with even more refined elements.

Discretisation of the system

The beam is described by its areas and one or more cross sections. This allows the user to define cross section jumps and haunches. For the calculation, however, the beam must be divided into sections with constant cross sections. Haunches are represented by a suitable number of similar cross sections with gradually increasing sizes.

In the discretisation process, the node mesh is first generated from the front and rear end coordinates of the beam regions. Due to the fact that *BTII* displaces node loads, supporting conditions, region borders as well as the zero-points of the imperfection half-waves automatically to the next-closest node, additional section borders have to be defined in these points. This will only happen, however, if the distance between the section border to be inserted and the existing section borders exceeds the specified minimum size. The minimum size is defined via the minimum element extension.

Types of analyses

Elastic ultimate resistance

This option allows you to specify whether the shear stresses resulting from Saint Venant's torsion and warping torsion should be considered in the calculation of the comparison stress.

Plastic ultimate resistance IAW DIN 18800-1

DIN 18800-1, El.(755) specifies that limit bending moments in the plastic state should be limited to the 1.25-fold value of the elastic limit bending moment. This reduction can be dispensed with if the system consists of single-span beams and continuous beams with constant cross sections over their total length.

Partial internal forces method according to Kindmann

See "Plastic ultimate resistance IAW DIN 18800-1"

Equivalent bar method IAW DIN 18800-2

This option allows you select among two different methods for the verification of the lateral stability against torsional buckling with biaxial loading either including or not axial forces. You can also select both verification methods.

Method of calculation

Second order analysis

The **second order analysis is based on iteration**. The first step in each calculation is a first order analysis. The resulting internal forces form the basis of the next iteration step to calculate the geometric stiffness matrix describing the non-linear behaviour. As typical in civil engineering, the modification of the internal forces is not considered in the following iteration steps for the generation of the geometric matrix ("Disregard of the modification of the main deflection"). This corresponds to the freezing of the axial bar forces after the first iteration step in connection with the two-dimensional stability problem. The iteration ends with the 2nd iteration step. If the defined **loads** are greater than the lateral buckling or lateral torsional buckling loads, the load-deformation problem can be solved but the equilibrium becomes unstable in this state. The determinant of the system stiffness matrix is negative in this case. Therefore, *BTII* aborts the calculation and displays a corresponding message.

If a load level was defined that is only slightly below the load level of the lowest eigenvalue (=smallest torsional buckling load), deformations increase considerably. In this case, the results are useful only under certain conditions because the theoretical basis still describes the equilibrium of the deformed system but assumes only small deformations.

The **forces and moments** calculated in the 2nd order analysis are already referenced to the major axis system. Therefore, no transformation is required for the subsequent stress examination.

Warping torsion

Torsional loading on thin-walled open sections is distributed via Saint-Venant's torsion M_{tp} (primary torsional moment) and warping torsion M_{ts} (secondary torsional moment). The larger the fixity against cross section warping the larger the portion that is distributed via warping torsion and vice versa. The fixity depends on the shape of the cross section and the behaviour of the torsional moments. With solid cross sections and circular hollow cross sections, for instance, warping fixity is low. The same applies to the area of torsional moments with constant behaviour. Accordingly, the load distribution via Saint-Venant's torsion prevails. In contrast to this, the distribution via warping torsion is predominant particularly at jumps in the torsional moments behaviour and at warping restraints.

Axial warping stresses in the longitudinal direction of the bar and warping moments, also referred to as bending moments, occur due to warping fixity. In stress analyses on open cross sections, warping stresses resulting from warping torsion must therefore be considered in addition to the axial stresses caused by the axial force and the bending moments.

The equation for the total axial stresses is as follows:

$$\sigma_x = \frac{N}{A} + \frac{M_y}{W_y} + \frac{M_z}{W_z} + \frac{M\omega}{I_\omega} \cdot \omega$$

N axial force

M_y, M_z bending moments around the y- or z-axis on the deformed cross section

M warping moment, also indicated with M_w or B

A cross sectional area

W_y, W_z section moduli around the y- or z-axis

I_ω warping moment of inertia, also indicated with I_w or C

ω standard main warping, also indicated with w_M

Equivalent bar analyses

When using the equivalent bar method for the stability examination, *BTII* performs an **eigenvalue calculation** by applying the linear subspace method.





Output

Output of the results on the screen

Selection of the results to be shown in graphics

The calculation results are displayed for the load case combinations not the individual load cases. You can select the relevant load case combinations via the tool bar.

Another tool bar allows you to select the presentation of the deformation diagrams.

	System
N	Normal force behaviour
M_y	Moment around the y-axis
Q_z	Shear force in z-direction
M_t	Torsional moment
M_z	Moment around the z-axis
Q_y	Shear force in z-direction
M_w	Warping moment
η_c	Structural safety according to the selected verification method
σ_x ▼	Axial force behaviour in the selected point of the cross section
τ ▼	Shear force behaviour in the selected point of the cross section
σ_v ▼	Comparison stress behaviour in the selected point of the cross section
 ▼	Deformations: The selectable options are shift, distortion, warping and 3D-representation
 ▼	Imperfections: The selectable options are imperfection, initial sway imperfection and 3D-representation
	Kinematics figure

Load cases

The loads assigned to the generated load cases are only displayed on the screen if the load case input dialog is active. If so, the loads associated to the currently active entry in the load case table are shown. The displayed load values are those entered by the user.

Superpositions

The loads assigned to the generated load case combinations are only displayed on the screen if the superposition input dialog is active. If so, the loads associated to the currently active entry in the superposition table are shown. The displayed load values are the ones used in the superposition.

Imperfections

The imperfections assigned to the generated load case combinations are only displayed on the screen if the superposition input dialog is active. The imperfections associated to the currently active entry in the superposition table are shown.

You can define the scope of the output via the output profile.

Output of the system and the results for documentation

Output profile

The output profile offers comprehensive adjustment options that allow the user to control the scope of the documentation.

Output sections

The application determines automatically the verification points at which the design values, stresses and structural safety verifications are put out. The user can define additional output sections by specifying the corresponding x-coordinates.

System

This option allows you to specify the parameters that should be put out for the system.

Loads

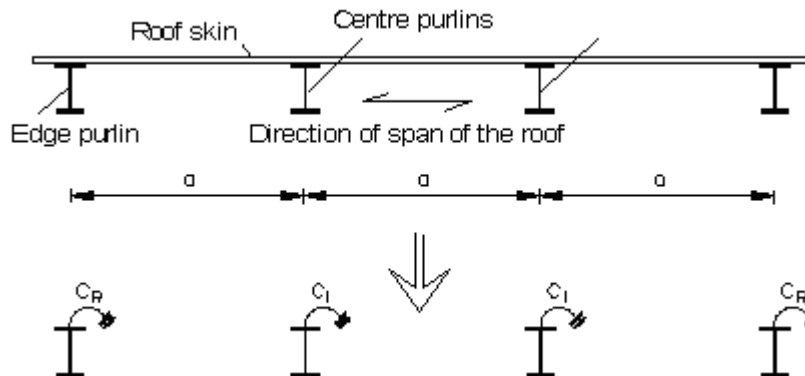
This option allows you to specify the loads that should be put out and select whether they should be put out as text or graphic.

Results

This option allows you to set the desired output parameters separately for each superposition.

Notes concerning practical applications

Purlins with torsionally elastic support by the roof skin



ED: modulus of elasticity of the roof skin

ID: moment of inertia of the roof skin per length unit

The transfer of the moment between the purlin and the roof due to contact or loading on the connectors is to be verified. See also Vogel/Heil [13].

If the moment to be transferred exceeds the contact moment (= created by the drift of the load application point to the flange edge), the compliance of the connection between the purlin and the trapezoidal sheet must be taken into account in addition. See also Lindner [5].

Trusses with torsionally elastic support by purlins

The spring stiffnesses are calculated as described above. It must be distinguished between centre trusses and edge trusses.

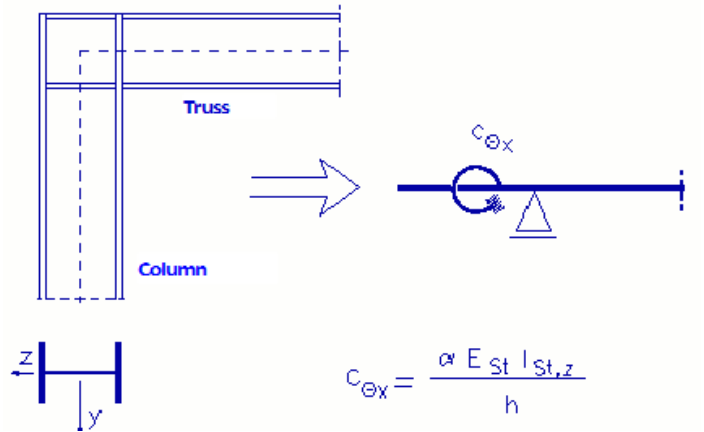
Trusses with elastic translational support at the top chord by purlins

The stiffness of the horizontal equivalent spring results from the compliance of the horizontal roof structure in the edge spans. If required, also the slip in the connectors must be taken into account.

For more information concerning the calculation of equivalent stiffnesses in different types of structural framework, see Rubin/Vogel [12], for instance.

Trusses with elastic torsional support by columns

- Est modulus of elasticity of the column
- I_{st,z} moment of inertia of the column around the z-axis
- h height of the column section
- α restraint value depending on the support of the column base around the weak axis.
α = 4: restrained
α = 3: pinned



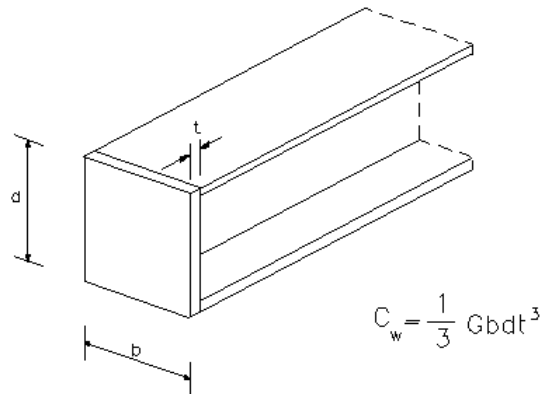
The supporting effect is low under normal conditions.

Beam with elastic warping support

The free warping fixity increases the torsional stiffness of beams with thin-walled open cross sections. In the following, we are going to give you some information about the calculation of discrete warping springs C_w in three frequent cases of warping fixity.

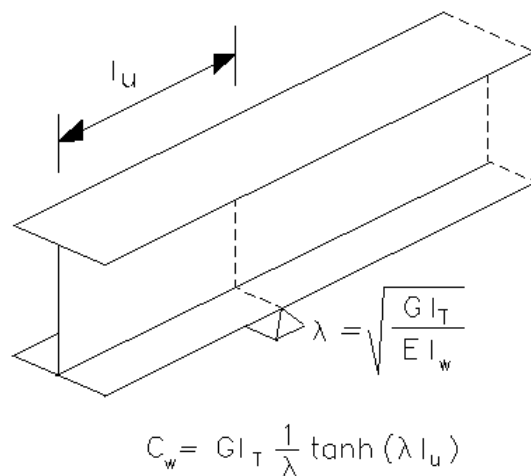
a) End plate

- G shear modulus of the end plate's material



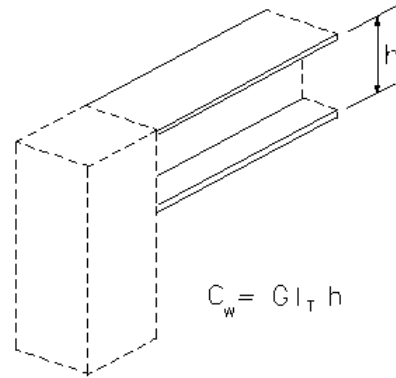
b) Projection (can directly be modelled!)

- I_T Saint-Venant's torsional moment of inertia
- I_ω warping moment of inertia
- I_u projection
- E modulus of elasticity
- G shear modulus



c) Column connection

- h beam height (distance between the centres of gravity of the flanges)
- I_T Saint-Venant's torsional moment of inertia of the column



Open sections

$$I_T = \sum \left(\frac{1}{3} \cdot s_i \cdot t_i^3 \right)$$

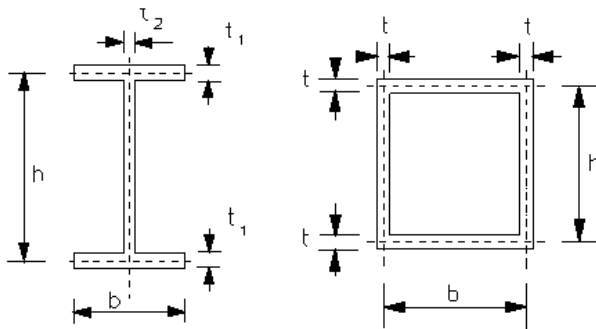
Closed sections

$$I_T = 4 \cdot \frac{A_m^2}{\sum \frac{s_i}{t_i}}$$

The variables refer to:

- s_i length of the i^{th} rectangle
- t_i width of the i^{th} rectangle
- A_m surface of the cross section enclosed by the section centre line

Examples



$$I_T = \frac{1}{3} (2bt_1^3 + ht_2^3)$$

$$I_T = 4b^2 h^2 t / (2(b+h))$$

Beam with shear field support

Purlins under roof plates are supported rigidly or elastically against lateral shift through the shear field stiffness at the height of the top chord. In the current version, *BTII* does not provide any options to describe the shear field effect exactly.

An approach by approximation can be achieved by converting the shear field stiffness S^* to an equivalent elastic foundation with the stiffness C_y applying at the top chord \bar{c}_y

The conversion is obtained by setting the virtual work of the elastic foundation equal to that of the shear field.

$$\int S^* v_o' \delta v_o' dx = \int \bar{c}_y v_o \delta v_o dx$$

When assuming a sinusoidal horizontal shift of the top chord with n half-waves over the length of the beam

$$v_o = \bar{v}_o \sin \frac{n \pi x}{l}$$

it follows

$$\bar{c}_y = S^* \left(\frac{n \pi}{l} \right)^2$$

First, perform the calculation of the elastic foundation \bar{c}_y with $n = 1$. Verify subsequently the elastic foundation on the basis of the shift of the top chord caused through this and/or repeat the calculation with $n > 1$.

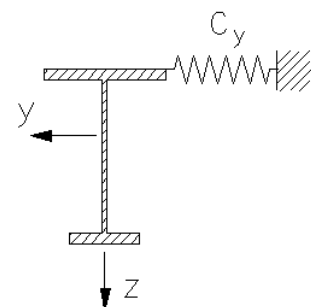
This approximation is sufficient in many cases of practical application.

Lateral torsional buckling with a fixed axis of rotation

The problem of lateral torsional buckling with a fixed axis of rotation at a distance z from the shear centre often occurs in practice. You can describe it in *BTII* as follows:

Define an elastic translational foundation in y -direction with a *stiffness* $10E+8$ to $10E+10$ at the distance z_0 from the centre of gravity. The resulting shift and torsion in regard to the centre of gravity and the shear centre are equal to zero along the pre-set fixed axis of rotation.

You can also use eccentric discrete springs to provide fixity against lateral shift in the y - or z -direction at any point in the cross section. For this purpose, you ought to define *high but not too high spring stiffnesses*. As a rule, *the stiffnesses should be* $< 10E+16$. In order to ensure the numerical stability of the calculation, discrete stiffnesses intended as shift fixities should not be greater than strictly necessary. You can check this by verifying the kinematic constraint conditions in the cross section.



Torsion with solid cross sections

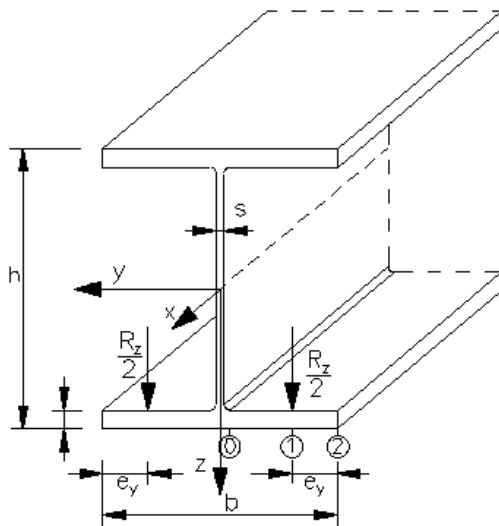
The calculation of beams with solid cross sections such as glued laminated girders or pre-stressed concrete girders requires particular attention in regard to the load distribution of torsional loading.

On thin-walled open cross sections, typical in steel construction, the load is distributed via Saint-Venant's torsion and warping torsion (cross sectional warping fixity), whereby the distribution depends mainly on the beam length and the type of loading.

On solid cross sections, typical in reinforced concrete and timber construction, the load portion distributed via warping torsion is very low and can therefore be neglected. In *BTII* you can take this fact into account by setting the warping moment of inertia of the cross sections to zero, when entering the data. In this case, the warping bimoments determined in the calculation of the internal forces acting on the bar end are equal to zero.

Stresses due to local beam loading

When flange bending stresses apply due to the operation of underslung overhead cranes, the global stresses calculated in accordance with the beam bending theory must be superimposed with the local bending stresses. *BTII* handles this calculation as described in the notes concerning the calculation in reference [1].



The superposition of global and local stresses is limited to the node points where the point loads apply to the flanges. It is analysed separately for each flange side, the top and bottom edge of the flange at the web/flange transition (points 3/4), at the load application points (points 2/5) as well as at the flange edges (points 1/6).

The variable flange thickness of cross sections with inclined flanges (section type 5) can optionally be taken into account. According to [1], local stresses in the length direction of the beam σ_x can be reduced by 75 % before superimposing them with the axial beam stresses. The comparison stresses in the specified points in accordance with the von-Mises yield criterion can optionally be calculated with or without consideration of shear stresses resulting from the Saint-Venant's torsional moment portion.

Lateral buckling of frame systems

Problem

The equivalent bar method as per DIN 18800-2 is an alternative calculation method for the verification of load-bearing systems in second order analyses with inclusion of deformations. This simplified verification is based on ideal bifurcation loads, which are calculated on the straight beam in *BTII*. The calculation of the ideal bifurcation loads is performed separately for each of the failure modes lateral buckling and lateral torsional buckling.

This approach has proven its worth for beam and column systems that typically comply with the Euler buckling modes. Where frame systems are concerned, the calculation is often based on second order analyses. The second order analysis provides for lateral buckling in the plane of the frame under normal conditions. The lateral torsional buckling failure mode must be examined separately, however. This examination is based on a simplified verification in accordance with the equivalent bar method.

Equivalent bars for lateral torsional buckling analyses

In order to verify a bar in a sway or non-sway frame system in accordance with the equivalent bar method, you have to extract it from the total system. A single-span beam with fork supports is assumed for the examination of the lateral torsional buckling failure mode. The bar end moments, which result from the calculation of the basic frame in a first or second order analysis, are applied to the single-span beam in accordance with the behaviour of the internal forces. The span moments can be calculated in first-order analyses. The load bifurcation factor is calculated numerically for the structural system generated this way producing the basic value M_{kiy} for the equivalent bar method.

Equivalent bars for lateral buckling examinations

Under normal conditions, the verification of the lateral stability of frame systems is included in the second order calculation of the internal forces. The simplified verification in accordance with the equivalent bar method is much more difficult in this case because you have to adjust the structural system of the equivalent bar via load conditions in such a manner that the effective length corresponds to that of the entire system. To do this, you have to calculate corresponding spring stiffnesses. The equivalent bar verification for lateral buckling requires a system modified in the described way.

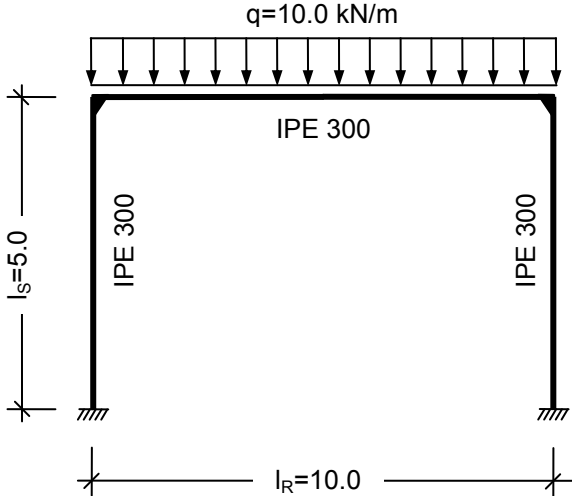
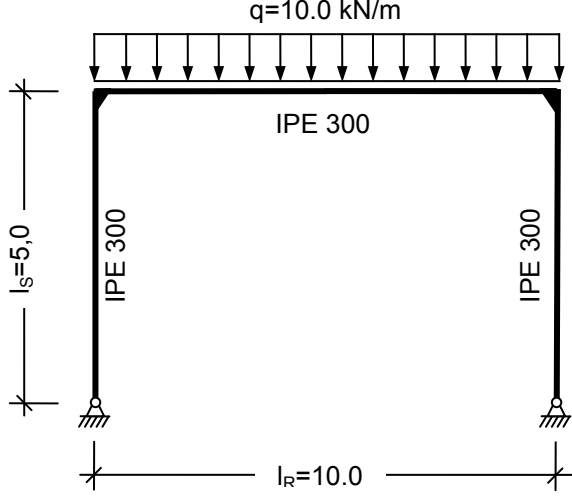
It is particularly difficult to determine the corresponding rotational and translations springs and requires the consultation of expert literature. This verification method and the preparative work involved are quite elaborate in comparison to the calculation of the frame system in second order analyses.

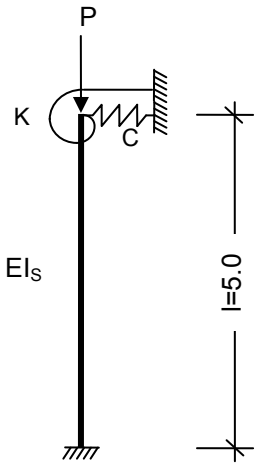
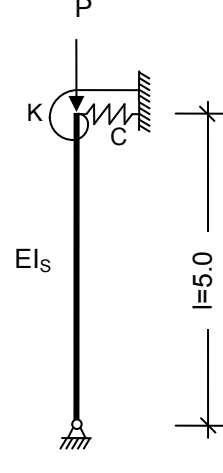
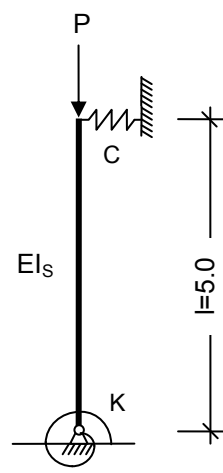
In the following examples, the lateral buckling stability of the frame columns will be examined. You will learn how to calculate the torsion spring constant and which structural system you have to enter in *BTII*.

Example: pinned and restrained frame

Taken from Petersen: Statik und Stabilität der Baukonstruktionen, 2nd edition, 1982, Publ. Vieweg-Verlag, p. 340, table 5.3.

In the present example, the lateral buckling stability of the frame column is examined.

	
<p>$C = 0 (\delta = 0)$ single-storey sway frames with restrained column bases.</p>	<p>$C = 0 (\delta = 0)$ single-storey sway frames with pinned column bases.</p>

		
<p>Parameters</p> $\delta = \frac{Cl^3}{EI_S}; \gamma = \frac{K \cdot l_S}{EI_S}$	<p>Effective length</p> $s_K = \beta \cdot l_S$	
<p>Formulas</p> $\frac{1}{\gamma} = \frac{1}{6} \cdot \frac{EI_S}{EI_R} \cdot \frac{l_R}{l_S} \quad \frac{EI_S}{K \cdot l_S} = \frac{1}{6} \cdot \frac{EI_S}{EI_R} \cdot \frac{l_R}{l_S} \quad \frac{K \cdot l_S}{EI_S} = \frac{6 \cdot EI_R}{EI_S} \cdot \frac{l_S}{l_R} \quad Kl_S = \frac{6 \cdot EI_R}{l_R}$		
<p>Torsion spring constant</p> $K = \frac{6 \cdot 21000 \cdot 8360}{1000} = \frac{3 \cdot 21000 \cdot 8360}{500} = 10533 \text{ KNm / rad}$		

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- [13] U. Vogel, W. Heil: *Traglast-Tabellen*. Published by Beratungsstelle für Stahlverwendung, Stahleisen GmbH, Düsseldorf 1981.