The LRP Toolkit User's Manual

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1 Introduction

The LRP Toolkit[1] is a powerful set of utilities to simplify the development of new algorithms based on the Linear Repetitive Process (LRP) settings. The Toolkit is ment to be used within the Scilab environment (see `www.scilab.org` for details). Before exploring the Toolkit functions it is necessary to introduce some theoretical background.

1.1 The Linear Repetitive Process — theory

Repetitive processes are a distinct class of 2D systems (i.e. information propagation in two independent directions) of both system theoretic and applications interest. The essential unique characteristic of such a process is a series of sweeps, termed passes, through a set of dynamics, defined over a fixed finite duration known as the pass length (denoted by $\alpha < +\infty$). On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. This, in turn, leads to the unique control problem in that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction. Hence these processes propagate information in two separate directions, i.e. from pass-to-pass and along a pass respectively.

Physical examples include long-wall coal cutting and metal rolling operations (see, for example, [RO92]). Also, in recent years applications have arisen where adopting a repetitive process setting for analysis has distinct advantages over alternatives. For example, they can be used to analyze an important class of iterative learning control (ILC) schemes [OARF00]. More recently another application has arisen in the context of self-servo writing in disk drives [MCA03] and there are as yet unexploited links with one approach to the analysis of spatially interconnected systems [DD03]. Attempts to control these using standard ("classical" 1D) systems theory/algorithms fail (except in a few very restrictive special cases) precisely because such approaches ignore the inherent 2D structure of repetitive processes, i.e. information propagation occurs in two distinct directions. Here we consider so-called discrete linear repetitive processes which can arise either from direct modelling of a physical process or as a result of sampling the dynamics of a differential process in the along the pass direction.

The state space model

The state space model [RO92] of such a process has the following form over $0 \leq p < \alpha$, $k \geq 0$ where k denotes the pass number or index

$$\begin{aligned} x_{k+1}(p+1) &= Ax_{k+1}(p) + Bu_{k+1}(p) + B_0 y_k(p) \\ y_{k+1}(p) &= Cx_{k+1}(p) + Du_{k+1}(p) + D_0 y_k(p) \end{aligned} \tag{1.1}$$

[1] This manual covers the LRP toolkit version 0.32, built on 14:54:52, 2009-05-07

Here, on pass k, $x_k(p) \in \mathbb{R}^n$ denotes the state vector, $y_k(p) \in \mathbb{R}^m$ denotes the pass profile vector, and $u_k(p) \in \mathbb{R}^r$ denotes the vector of control inputs.

Boundary conditions

To complete the process description, it is necessary to specify the boundary conditions, i.e. the initial state vector at the beginning of each pass and the initial pass profile. Here, no loss of generality arises from assuming

$$\begin{aligned} x_{k+1}(0) &= d_{k+1},\ k \geq 0, \\ y_0(p) &= f(p), \end{aligned} \tag{1.2}$$

where the $n \times 1$ vector d_{k+1} contains known constant entries and $f(p)$ is an $m \times 1$ vector whose entries are known functions of p.

1.1.1 Stability

The stability theory [RO92] for linear repetitive processes consists of two basic concepts, termed asymptotic stability and stability along the pass respectively. Noting again the unique control problem, these properties demand that bounded sequences of inputs produce bounded sequences of pass profiles where 'bounded' is defined in term of the norm on the underlying function space. Asymptotic stability guarantees this property over the finite and fixed pass length and as a consequence there exists the so-called limit pass profile, i.e. after the sufficient number of passes the process dynamics can be replaced by those of a 1D discrete linear system. The fact that the pass length is finite, however, means that this limit profile could be unstable (as a 1D discrete linear system). (Over a finite duration even an unstable 1D linear system is guaranteed to produce a bounded output). Stability along the pass prevents this situation from arising by demanding the boundedness property uniformly, i.e. independent of the pass length. (It is easy to see that asymptotic stability is a necessary condition for stability along the pass.)

Many sets of necessary and sufficient conditions for these properties are known and some of them can be tested by direct application of 1D linear systems stability tests, e.g. Nyquist plots. A major drawback, however, is that these do not provide a basis on which to also address the question of control law design for stability and/or performance. This has led in recent years to the use of Linear Matrix Inequality (LMI) techniques (see e.g. [BGFB94]) and there now exists a large volume of results on the design of physically implementable control laws (for the detailed description refer e.g. to [Sul06, GLR$^+$03] and references therein.)

The unique control problem for the Linear Repetitive Processes is that the output sequence of pass profiles y_k, $k \geq 0$ can contain oscillations which can increase in amplitude in the from pass to pass direction (k). Hence a natural definition of stability is to request that bounded input sequences produce bounded output (pass profiles) sequences.

Stability theory (Rogers and Owens (1992)) for LRPs is based on an abstract model of process dynamics in a Banach space (here denoted by E_α) of the form

$$y_{k+1} = L_\alpha y_k + b_{k+1}, k \geq 0 \tag{1.3}$$

In this model, $y_k \in E_\alpha$ denotes the pass profile on the pass k, L_α is a bounded linear operator which maps E_α into itself and $b_{k+1} \in W_\alpha$, where

W_α is a linear subspace of E_α. Also, the term $L_\alpha y_k$ describes the contribution of the pass k to the pass $k+1$, and b_{k+1} represents the inputs and other effects which enter on the current pass.

Definition 1 *[RO92, Ben00, Sul06] Suppose that $||\cdot||$ denotes the norm on E_α. Then the so-called asymptotic stability holds provided there exist real numbers $M_\alpha > 0$ and $\lambda_\alpha \in (0,1)$ such that $||L_\alpha^k|| \leq M_\alpha \lambda_\alpha^k$, $k \geq 0$ (where $||\cdot||$ is also used to denote the induced operator norm).*

Asymptotic stability

Theorem 1 *[RO92] The linear repetitive process described by (1.1) is asymptotically stable if, and only if*

$$\mathrm{r}(D_0) < 1$$

where r *denotes the spectral radius.*

The equivalent theorem using the Linear Matrix Inequalities (LMI) techniques can be formulated as

Asymptotic stability — LMI

Theorem 2 *[SGRO05] The linear repetitve process of (1.1) is asymptotically stable if, and only if there exists a matrix $Q \succ 0$ of appropriate dimensions such that the following LMI holds:*

$$D_0^T Q D_0 - Q \prec 0$$

The asymptotic stability guarantees the existence of a limit profile but it does not guarantee that this limit profile treated as a 1D system (under the assumption that $\alpha \to \infty$) is stable. The reason for that is the fact that asymptotic stability does not concern dynamics along the pass (along the p dimension).

Mostly, the cases where the limit pass profile is unstable as a 1D linear system are not acceptable. Hence a stronger concept of stability, i.e., stability along the pass must be used. This stronger stability demands the BIBO property to hold independently of dynamics, i.e., in the direction along the pass (p) and from pass to pass (k).

Introduce the formal definition of stability along the pass as follows:

Definition 2 *[RO92, Ben00, Sul06] In terms of the abstract model of (1.3), stability along the pass holds provided that there exist real numbers $M_\infty > 0$ and $\lambda_\infty \in (0,1)$, which are independent of α such that $||L_\alpha^k|| \leq M_\infty \lambda_\infty^k$, $k \geq 0$.*

Theorem 3 *[GPS⁺03] Discrete linear repetitive process described by (1.1) and (2.1) is stable along the pass if, and only if, the 2D characteristic polynomial*

$$C(z_1, z_2) := \det \begin{bmatrix} I_n - z_1 A & -z_1 B_0 \\ -z_2 C & I_m - z_2 D_0 \end{bmatrix} \neq 0 \qquad \forall (z_1, z_2) \in \bar{U}^2 \tag{1.4}$$

where $\bar{U}^2 = \mathrm{diag}\,(z_1, z_2) : |z_1| \leq 1, |z_2| \leq 1$.

Theorem 1.4 is very difficult to use in practice. Note that it requires to check an infinite number of values. To overcome this problem it is recommended to use the following theorem.

Stability along the pass

Theorem 4 *[GPS+03] Discrete linear repetitive process described by (1.1) and (2.1) are stable along the pass if there exist matrices $P = P^T \succ 0$ and $Q = Q^T \succ 0$ satisfying the following LMI*

$$\begin{bmatrix} \widehat{A}_1^T P \widehat{A}_1 + Q - P & \widehat{A}_1^T P \widehat{A}_2 \\ \widehat{A}_2^T P \widehat{A}_1 & \widehat{A}_2^T P \widehat{A}_2 - Q \end{bmatrix} \prec 0 \tag{1.5}$$

The proof can be found in e.g. [GLR+03]. Note that theorem 4 is only sufficient condition.

In addition to the most important stability definitions (asymptotic and along the pass), the Toolkit includes the tests for the so-called practical stability. This notion lies between the asymptotic and along the pass stability. The conditions for this type of stability to hold can be summarized as the following theorem

Practical stability

Theorem 5 *[Gra99] The linear repetitive process described by (1.1) is practically stable if, and only if*

$$\begin{aligned} \mathrm{r}(D_0) &< 1 \\ \mathrm{r}(A) &< 1 \end{aligned}$$

where r *denotes the spectral radius.*

The LMI version of this theorem can be formulated as

Theorem 6 *The linear repetitive process described by (1.1) is practically stable if, and only if there exist matrices $P \succ 0$ and $Q \succ 0$ of appropriate dimensions such that the following LMI is feasible*

$$\begin{bmatrix} A^T P A - P & 0 \\ 0 & D_0^T Q D_0 \end{bmatrix} \prec 0$$

This notion of stability is seldom used and hence will not be discussed in full detail — see [Gra99]

1.1.2 STABILISATION

If the system described by (1.1) is unstable along the pass, it is necessery to stabilise it by means of an appropriate control action. One of the control laws considered to date for discrete linear repetitive processes has the following structure (for the background see, for example, [GLR+03] and the relevant cited references)

The control law

$$u_{k+1}(p) = K_1 x_{k+1}(p) + K_2 y_k(p) = K \begin{bmatrix} x_{k+1}(p) \\ y_k(p) \end{bmatrix}, \tag{1.6}$$

where K_1 and K_2 are matrices to be computed. Currently the only effective approach for the computation of the controller matrices is through the use of LMIs. First, define

$$\hat{A}_1 = \begin{bmatrix} A & B_0 \\ 0 & 0 \end{bmatrix}$$

and

$$\hat{A}_2 = \begin{bmatrix} 0 & 0 \\ C & D_0 \end{bmatrix}$$

In the Toolkit the following methods of obtaining K_1 and K_2 are implemented:

Theorem 7 *[GLR+03] The linear repetitive process of (1.1) with the applied control law of (1.6) is stable along the pass if there exist matrices $Y = Y^T \succ 0$, $Z = Z^T \succ 0$ and N such that the following LMI is feasible*

$$\begin{bmatrix} Z - Y & 0 & Y * \hat{A}_1^T + N^T * \hat{B}_1^T \\ 0 & -Z & Y * \hat{A}_2^T + N^T * \hat{B}_2^T \\ \hat{A}_1 * Y + \hat{B}_1 * N & \hat{A}_2 * Y + \hat{B}_2 * N & -Y \end{bmatrix} \succ 0$$

where

$$\hat{B}_1 = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

$$\hat{B}_2 = \begin{bmatrix} 0 \\ D \end{bmatrix}$$

If the LMI is feasible, the controller K is given by

$$K = NY^{-1} \tag{1.7}$$

Proof of this theorem can be found in ([GLR+03]) In the Toolkit this approach is implemented as `LMIAlongThePass1`.

Another variant of this method is to use

Theorem 8 *[GLR+03] The linear repetitive process of (1.1) with the applied control law of (1.6) is stable along the pass if there exist matrices $P_1 = P_1^T \succ 0$, $P_2 = P_2^T \succ 0$, $P = P^T = \text{diag}\{P_1, P_2\} \succ 0$, N_1 and N_2 such that the following LMI is feasible*

$$\begin{bmatrix} -P & \Phi P + RN \\ P\Phi^T + N^T R^T & -P \end{bmatrix} \succ 0$$

where

$$\Phi = \hat{A}_1 + \hat{A}_2$$

$$N = \begin{bmatrix} N_1 & N_2 \\ N_1 & N_2 \end{bmatrix}$$

If the above LMI is feasible, the controller K is given by

$$K = NP^{-1} \tag{1.8}$$

Again, the proof can be found in [GLR+03]. This method is implemented as `LMIAlongThePass2`

2 THE LRP TOOLKIT

2.1 Alternatives

A main problem encountered during the control related analysis of repetitive processes is how to visualize the process dynamics. This problem has been considered previously (see e.g. [Gra99, GGGR05]) but the resulting software was diffucult to extend and/or based on commercial environments. Some of them can only be used for educational purposes. The detailed discussion about available software packages for the linear repetitive processes is given in [HCG+06].

2.2 The LRP Toolkit

The above arguments confirm that there is no high quality reliable software to support the analysis and design of control laws for use with, for example, experimental facilities, [RHL+05] where ILC control laws designed in a repetitive process setting could be experimentally verified. Moreover the currently available tools do not allow easy inclusion of new algorithms. Also there are no software packages for analysis and synthesis based on open source software (the Java-based toolkit [GGGR05] cannot be used for this purpose due to its limitations). This causes licensing problems and reduces the areas of potential use (e.g. by students).

Why Scilab?

To overcome those limitations a development of a new Toolkit has been initiated. To enhance the usefulness of this tool, the Scilab [SG] environment has been chosen. The main advantage of this option over other packages is the open-source license of the Scilab and a rapidly growing number of users (including PSA Peugeot Citroen, Renault, Dassault Aviation and many others). The introduction to the Scilab environment is given in [GDB+98] and [CCN06]. The LMI component, used in the Toolkit, is described in [NDG].

The main functions of the Toolkit

The main functions of the implemented toolkit include:

- Visualization of the process dynamics: 3D plots and 2D plots (both along the pass and pass-to-pass),
- Stability analysis — asymptotic and along the pass, using both classic (i.e. Nyquist diagram based) and LMI settings,
- Control law design based on the use of LMIs [Sul06, GLR+03],
- Adding new analysis and control law design methods easily. The new methods are automatically supported by the GUI.

2.3 Installation

2.3.1 Requirements

The LRP Toolkit requires

- Scilab (www.scilab.org). Recommended version is Scilab-4.1 or later
- To use graphics user interface, the Scilab version with Tcl/Tk support (almost all versions have this feature)
- About 4MB (megabyte; apprx. 1 million bytes) free disc space
- For LaTeX support, any LaTeX distribution must be installed. For Windows, the MiKTeX (www.miktex.org) is recommended.
- The `gui` directory of the Toolkit must be granted the read and write rights.

The Toolkit is distributed in two forms — as a Windows-only installer or as a multi-platform ZIP file. After installation both files yield identical results. It is possible to move the Toolkit `.sci` files between platforms.

2.3.2 Windows

The LRP Toolkit can be installed either manually or automatically.

2.3.3 Automatic installation

It is recommended to use this method to install the Toolkit. If after deinstallation the main directory of the Toolkit is empty, it will be deleted as well. The installer uses the highly-reliable Nullsoft Installation System (NSIS) **??**. For your convinience, the installer will add an Uninstall information into the Windows registry.

2.3.4 Automatic deinstallation

To uninstall the Toolkit, run the `uninstall.exe` program. After using the provided uninstaller all Windows registry information is no longer needed and is removed. This program removes the Toolkit, without leaving any leftover files. Note that if you change anything in any Toolkit file (except `.bin` and `html` files, which can be easily recreated), during uninstallation phase you will be asked if you want to remove such file. This protects you from accidently deleting your custom functions.

2.3.5 Manual installation

To install the Toolkit manually, unpack the ZIP file into any directory. The Scilab team recommends the `contrib` subdirectory of the Scilab. After copying all the files, modify the `config` file with any text editor. In this file, set the LRPHomeDir to point to the directory where the Toolkit is created. Modify other entries in this file as well.

The `config` file does not have any extension

STOP If you do not modify the `config` file, the Toolkit will not work properly.

2.3.6 Manual deinstallation

To uninstall the Toolkit, simply delete its directory. Do not forget to save the files you modified, if you need them.

2.4 Creating the LRP structure

The LRP structure used by the Toolkit is quite complex. To simplify creation of new structures, you can use three different methods (given from the simplest to the most complex):

- use the graphics user interface
- use the `createLRPModel` function
- use the `createStubLRP` function and fill all the fields manually

Each of those options will be covered in the corresponding subsections.

2.4.1 Creating new LRP structure by using the graphics user interface

This method is the preferred method of creating the LRP structure. It is the simplest and least error-prone approach. To create the structure, run the graphics user interface (fig. 2.1) by selecting the `Start GUI` from the LRP menu, shown as the item in the Scilab main menu. If there are no structure named `lrp`, it will be automatically created.

The created structure is named `lrp`. Any other structure of this name is deleted. If you need to preserve the old system, assingn it a different name, e.g. `myLrp = lrp;`

If you cannot find the LRP menu, close the Scilab and run the Toolkit again. If it does not work or you do not want to close the Scilab session, run the `install.sci` script file by typing

```
global LRP_OPTIONS;
cd(LRP_OPTIONS.path);
exec('install.sci');
```

The first two lines set the Toolkit directory. If you have problems with them, use the `cd(PATH_TO_LRP_TOOLKIT)`, where `PATH_TO_LRP_TOOLKIT` is the directory where you installed the LRP Toolkit — if you are unsure about it, try the `contrib/lrp` directory in the main Scilab directory.

You will now see the LRP Toolkit user interface, shown on fig. 2.1 To create a new LRP system, click `Create a new system` button, positioned in the upper-left corner of the GUI. This will start the new system creator. You will need to give the following information:

- the number of states (n), inputs (r) and outputs (m) in the system
- all the system matrices (A, B, $B0$, C, D, $D0$)
- the pass length α and the number of passes to simulate β.

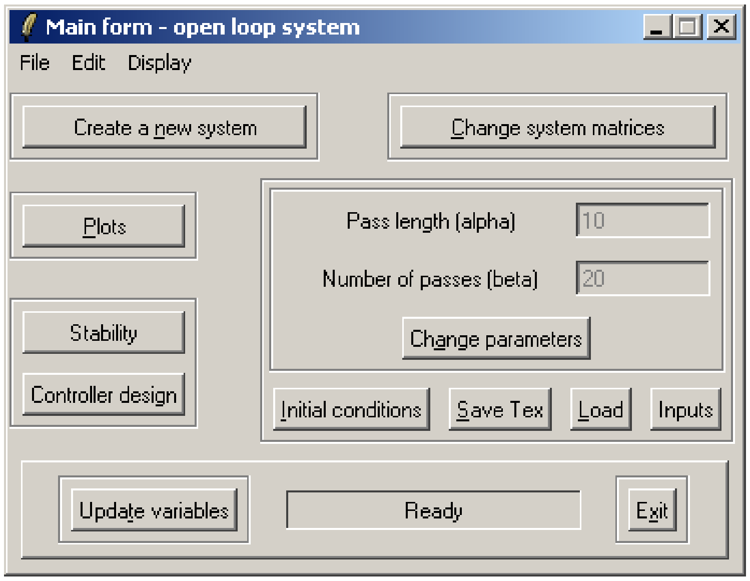


Figure 2.1: The main Toolkit GUI

To simplify entering the matrix' values click on the ellipsis button. Alternatively you may use the Scilab functions that return an appropriately-dimensioned matrix.

STOP If you use functions, you cannot use the ellipsis button. Moreover you are responsible for checking the dimensions of matrices.

Standard boundary conditions

Note that the resulting process will have zero input and standard boundary conditions

$$\begin{aligned} x_{k+1}(0) &= 0,\ k \geq 0, \\ y_0(p) &= 1, \end{aligned} \tag{2.1}$$

You may change them using the `Initial conditions` button.

To change the pass length and the number of simulated passes click the `Change parameters` button.

If any of the values were changed outside the Toolkit GUI, click `Update matrices` button to feed the proper values into the GUI.

To close the GUI, click `Exit`.

2.4.2 Creating new LRP structure by using `createLRPModel`

This function is especially useful in scripts. This function can be called with many different arguments. For detailed description type `help createLRPModel` in the Scilab window.

To create a sample system use `lrp = createLRPModel();` The created system has 2 states, 3 inputs and 4 outputs. The system matrices are random from 0 to 1. The pass length $\alpha = 10$ and number of simulated passes $\beta = 15$. The external input u equals zero. To change the boundary conditions, see section 2.4.4.

2.4.3 Creating new LRP structure by using `createStubLRP`

If you need greater control over the creation process, use the `createStubLRP` function to create the necessary fields. Then fill all the stub values with data.

If you do not fill all the stub values the Toolkit will not function correctly. To check the correctness of the LRP structure, use the `checkLRP` function.

Do not forget to set the `lrp.controllers` field!

2.4.4 Setting the boundary conditions

To set the boundary conditions of the LRP structure named `lrp`, use the appropriate generator function:

For state x

- `lrp = boundCond_x1(lrp)` for $x_k(0) = 1$
- `lrp = boundCond_xsin(lrp)` for $x_k(0)$ of a sine wave of unit height and a period of one.
- `lrp = boundCond_xsin(lrp,T)` for $x_k(0)$ of a sine wave of unit height and a period of `T`.

For output y

- `lrp = boundCond_y1(lrp)` for $y_0(p) = 1$
- `lrp = boundCond_ysin(lrp)` for $y_0(p)$ of a sine wave of unit height and a period of one.
- `lrp = boundCond_ysin(lrp,T)` for $y_0(p)$ of a sine wave of unit height and a period of `T`.

2.5 Visualization of the process dynamics

The Toolkit can be used to visualise the process dynamics. All the standard methods of visualisation are available:

2.5.1 Along the pass plots

This plot is useful for showing the dynamics of the whole pass (along the p dimension). It is also very important tool for comparing the shape of a few passes.

This plot emphasizes the p direction. To draw this plot, the user must first select the pass k for which the plot is to be made. Upto 32 passes can be shown simultanously. On the X points p are shown, the value is given on the Y axis. The example of such plot is shown on figure 2.2.

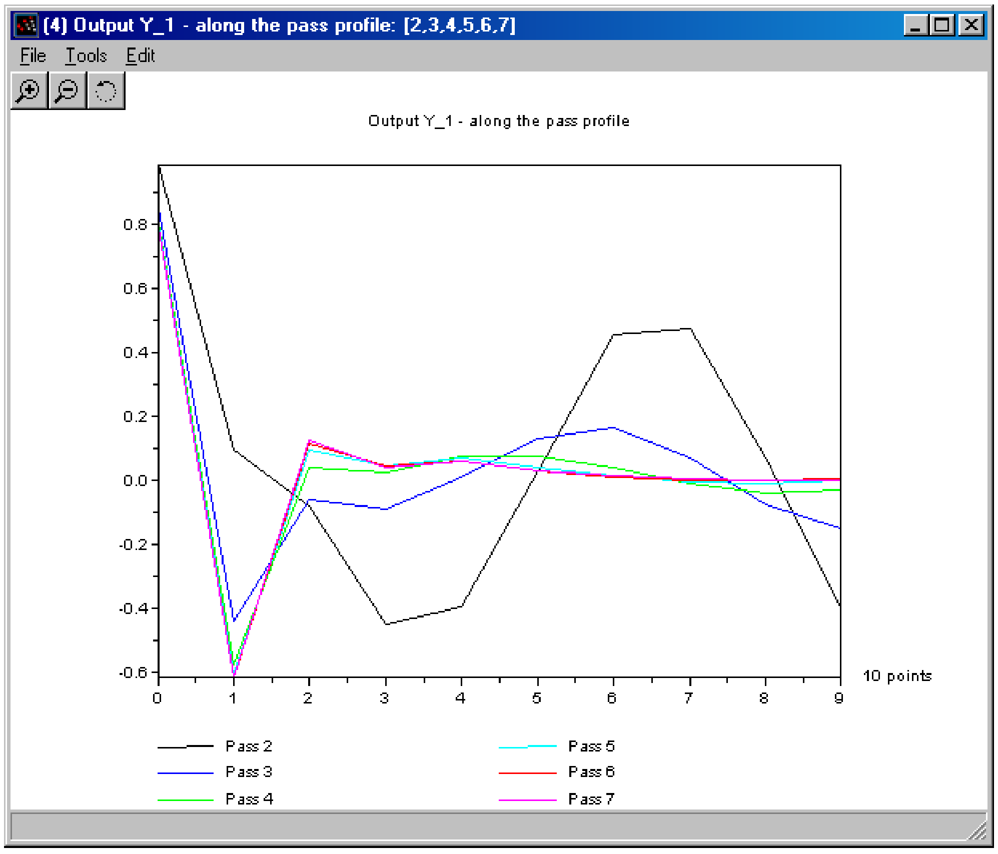


Figure 2.2: Along the pass plot example

2.5.2 Pass to pass plot

This plot is useful for showing the change of dynamics over all passes for a selected point p.

This plot emphasizes the k direction. To draw this plot, the user must first select the point p for which the plot is to be made. Upto 32 points can be shown simultanously. On the X passes k are shown, the value is given on the Y axis. The example of such plot is shown on figure 2.3.

2.6 IMPROVEMENTS

Since the release of the first version of the LRP Toolkit (see [HCG$^+$06]) a vast number of bugs have been fixed. Currently the new release of the Toolkit is available. The most important change is that the functions that do not depend on the considered model structure have been rewritten to accept a much more general parameters. This makes extension of the Toolkit much easier in practice.

After the initial release it became obvious that much stronger type checks are required. This is motivated by a fact that the linear repetitive process model contains many variables that are error-prone. This has lead to a development of new functions for dealing with this task.

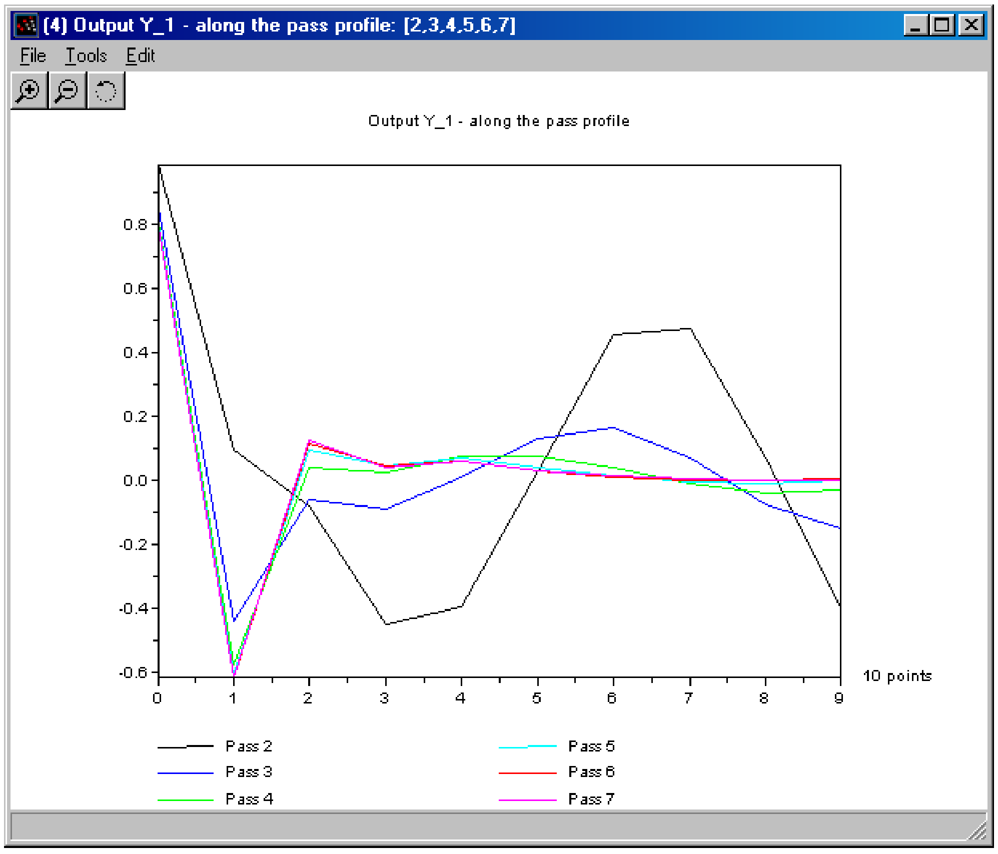


Figure 2.3: Pass to pass plot example

Currently the great effort is made towards new, vastly improved help system. This is based on standard Scilab templates, but contains many more illustrative examples. Moreover a number of potential pitfalls is explained. All such cases have a suggested, valid solution.

Very much attention has been paid to presentation of the results. In the new version of the Toolkit, the drawing engine has been rewritten to allow much easier use in scripts — all the functions have a much clearer syntax. Moreover, the plotting routines have been extended to handle the degenerate cases (like a plot of single point on a single pass). Due to readibility, a maximum of 32 passes can be drawn simultaneously using different colors for clarity. Additionally, due to the extended LATEX support, creating multiple plots is much faster (from O(n) to O(1) calculations of plot surfaces), moreover the plot surface is calculated only for data required for plotting — when user requests an "along the pass" plot for points 7..18 for $\alpha = 100$ only the first 18 points are calculated. If necessary, it can be requested to calculate the entire surface. Compared to the previous version, the 3D plots are now made in full color to better visualize the range of values in the plot.

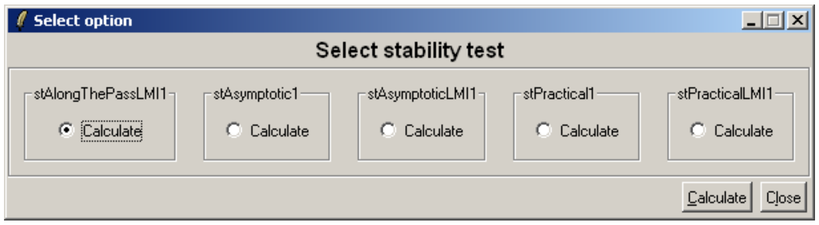


Figure 2.4: Stability analysis window

2.6.1 STABILITY ANALYSIS

In terms of stability analysis for discrete linear repetitive processes of the form considered here, asymptotic stability (and hence the construction of the resulting limit profile described by a 1D discrete linear systems state space model) is simple to check by means of Theorem 1, as it requires that all eigenvalues of the matrix D_0 in (1.1) have modulus strictly less than unity (this task is much harder for the case when the pass initial condition is an explicit function of the previous pass profile.) Asymptotic stability test is implemented in the Toolkit as the `stAsymptotic1` test. Its LMI counterpart of Theorem 2 is also implemented. This function is termed `stAsymptoticLMI1`.

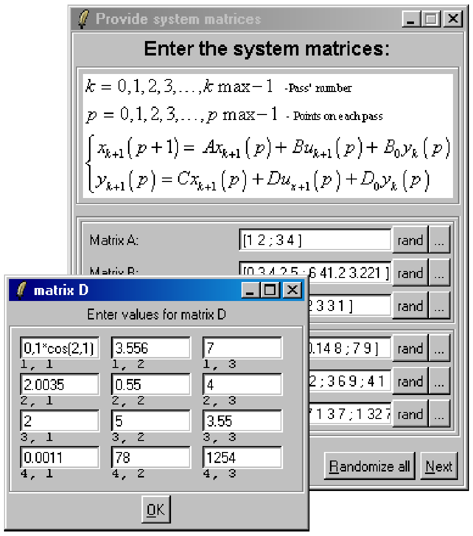
Stability along the pass, however, is a much more challenging task in this respect. In which context, it has been noted in the introduction that a 1D Nyquist test can be used but it has not proved a suitable basis to undertake control law design (in contrast to the 1D case). This has led to the use of LMI based tests (see Theorem 4) which are sufficient but not necessary but can be executed using computations with constant entry matrices and, crucially, provide a basis for control law design. To test the stability along the pass in the Toolkit by means of Theorem 4, the `stAlongThePassLMI1` test can be utilised.

In addition to the most important stability definitions (asymptotic and along the pass), the Toolkit includes the tests for the so-called practical stability. This notion lies between the asymptotic and along the pass stability. The tests implemented according to Theorems 5 and 6 are available as `stPractical1` and `stPracticalLMI1`, respectively.

Both stability properties can be investigated using the LMI technique. One of the reasons of selecting the Scilab as the host platform for the Toolkit was the excellent LMI solver available for this platform as "LMITOOL: a Package for LMI Optimization in Scilab" [NDG]. Note also that provision is available to easily include existing or newly developed tests by simply implementing a single Scilab function with no need to change the GUI, which is shown on fig. 2.4.

2.6.2 CONTROL LAW DESIGN

As outlined in the Background section, the stabilisation of the linear repetive process is not always easy. In developing new techniques it is beneficial to compare the newly developed method with the ones already known. One approach would be to use the LMI tools already available in Scilab and



$k = 0,1,2,3,\dots,k\max-1$

$p = 0,1,2,3,\dots,p\max-1$

$$\begin{cases} x_{k+1}(p+1) = Ax_{k+1}(p) + Bu_{k+1}(p) + B_0 y_k(p) \\ y_{k+1}(p) = Cx_{k+1}(p) + Du_{k+1}(p) + D_0 y_k(p) \end{cases}$$

Matrix A: [1 2 ; 3 4]

Matrix B: [0.3 4 2.5 ; 6 41.2 3.221]

0,1*cos(2,1)	3.556	7
1, 1	1, 2	1, 3
2.0035	0.55	4
2, 1	2, 2	2, 3
2	5	3.55
3, 1	3, 2	3, 3
0.0011	78	1254
4, 1	4, 2	4, 3

Figure 2.5: Matrix wizard windows: main and the matrix entry

code the methods by hand. However, this approach requires substantial knowledge about the internal data structures used by the LMI solver and this is not ideal. Consequently a set of ready-to-use control law design templates have been incorporated into the toolkit. Moreover, expanding it by adding additional control law design methods is a simple task.

2.6.3 LaTeX EXPORT

Presentation of simulation results can be a time consuming task, especially when considering large matrices which are often encountered in this area. To simplify this tedious task the toolkit has been equipped with LaTeX export capabilities. An essential novelty is the fact that the user needs to write the LaTeX file, adding the tags that will be replaced by the simulation results, instead of using a complicated syntax of the previous Toolkit version. It is possible to include any number of plots, both 2D and 3D. On each plot any number of points/passes can be displayed, which is an essential difference with the "interactive" plots discussed earlier. Note however that 32 points/passes on each plot can be displayed in unique colors — a great improvement over the previous version. The process of plot selection is simplified by the use of an interactive wizard. It is also possible to export simulation data from the script.

2.6.4 USABILITY ENCHANCEMENTS

One of the design goals was to make this new toolkit as user friendly as possible. To achieve this, a "new system wizard" for entering various process parameters has been implemented. Since most of the model parameters are matrices, the basic method to define these is the SCILAB convention for entering matrices (exactly the same as in the MATLAB). To simplify this process, it is also possible to enter the matrix element-by-element (see Fig. 2.5).

To make the end product available for a broader audience, the Windows operating system version employs an easy to use multilingual (currently Polish and English) installer based on *Nullsoft Install System* (*NSIS*) [Nul]. This system is widely regarded to be a very reliable, free solution that produces a small overhead code. Additionally, the *NSIS* can package and verify all the files included into the prepared compilation. Moreover, as an additional safety measure, for each installed file the Message-Digest Algorithm 5 (better known as MD5) checksum is calculated using the *MD5 library* [pL]. This value is used when upgrading and uninstalling the toolkit — if any change is detected, the user can choose to leave the file intact. Essentially, this feature provides protection against accidental deletion of manual changes.

During the installation phase, an existing LaTeX installation is automatically detected. Currently, the most popular *MiKTeX* distribution is supported by the installer, but any standard LaTeX can be used.

2.7 IMPLEMENTATION DETAILS

The toolkit consists of a TCL/TK GUI frontend and a number of SCILAB script files and functions. All the basic process parameters are included in the `lrp` structure which is implemented as a new type based on the `tlist` (typed list; native SCILAB datatype) with the following fields:

- `lrp_Sys_Cla` — (`string`), name of the data type
- The `lrp.mat` field of type `lrp_Sys_Cla_Mat` used for storing the model matrices — see (1.1). Currently it contains the values of `A`, `B`, `B0`, `C`, `D`, `D0` —
- The `lrp.dim` field of type `lrp_Sys_Cla_Dim` used for storing the model dimensions. Currently it has the following fields
  - `alpha` — (`positive integer`), pass length (number of points on each pass), denoted α in (1.1),
  - `beta` — (`positive integer`), number of passes over which the simulation will run,
  - `n`, `r`, `m` — (`positive integers`), dimensions of state, input and output vectors respectively,
- The `lrp.ini` field of type `lrp_Sys_Cla_Ini` used for storing the initial conditions. Currently it has two fields:
- `x0`, `y0` — (`real matrices`), boundary conditions, see (2.1). Note that $x_{k+1}(0)$ in (2.1) is here denoted by `x0` for simplicity,
- `controller` — (`list`), list of known control laws for the process; see below,
- `indController` — (`positive integer`), index of current control law. This field contains the index of currently active control law. If `indController=1` then no control law is applied.
- `stability` — (`list`), list of performed stability tests; see below.

Note that the model of (1.1) does not impose any constrains on the number of passes and hence to simulate the process response it is necessary to bound it by some finite value selected by the user — hence the parameter `beta` in the `lrp` structure.

2.7.1 THE lrp.controller FIELD

The `controller` field of the `lrp_Sys_Cla` datatype is a dynamically increasing `list` that contains a number of `tlist` structures. Each element holds the results of control law calculations. By design the first element of the `controller` list (i.e `lrp.controller(1)`) is a copy of all the system matrices. This ,,controller" is necessary for retaining the matrices for the open-loop system.

When new control law matrices are computed, a new `tlist` is added to the `lrp.controller` field. This field is defined by the user and the toolkit does not enforce any constraints on its structure. The only requirement is that there must be a field `solutionExists` of the `boolean` type which informs whether or not it is possible to obtain control law matrices for the example under consideration by the design method being considered.

In order to introduce a new control law the user must 1) give it a name (e.g. `controllerExample`) and 2) implement a set of 3 functions and one `.tex` file:

- `controllerExample` (the same name as a control law name) — the main function used for calculating the (constant) control law matrices given a process state space model. This approach allows faster calculations but also imposes an important drawback — it is not possible (without changes to the core Toolkit files) to simulate controlled process to asses the effects of varying the control law matrix (K). (LMI designs produce a family of such K). The addition of this feature is a subject for future work.
- `setcontrollerExample` (`set`+controller name) — change the system matrices to depict the controlled process (e.g. replace the A matrix into $A + B \cdot K$ matrix where K is the calculated (constant) control law matrix).
- `writecontrollerExample` (`write`+controller name) — a SCILAB function for exporting the results (e.g. controller matrices, parameters etc.) to LaTeX, can be blank if no export is required
- `describecontrollerExample.tex` (`describe`+controller name) — introductory text in LaTeX to be inserted when using the LaTeX export capabilities, can be left blank.

It must be stressed that the Toolkit files written by the user must be placed in the appropriate directories — this is explained in the help file. Note also that the SCILAB enforces a maximum function name length of 25 characters.

2.7.2 THE lrp.stability FIELD

The `stability` field of the `lrp_Sys_Cla` datatype is a dynamically increasing `list` that contains a number of `tlist` structures. Each element represents a preformed stability test.

When the user checks a new stability condition, a new `tlist` is added to the `lrp.stability` field. This field is defined by the programmer and the toolkit does not enforce any constrains on its structure. The only requirement is that there must be a field `solutionExists` of integer type (note here the difference between `lrp.stability` and `lrp.controller` field where a `boolean` type is used instead) which informs the user whether the system is stable, unstable or the method used does not provide the answer. The `solutionExists` field can have the following values

- -1 - test is inconclusive. In this case the only alternative is to use another test.
- 0 - the process is unstable,
- 1 - the process is stable.

Obviously the SCILAB works in finite precision arithmetic and hence numerical errors can influence the results.

To implement a stability test the programmer should write a function that returns an `lrp` structure and is given one as the only argument. Note here that a better solution would be to use a "by variable" (or by pointer) passing method but this is not implemented in SCILAB. This function must be placed in the `stability` directory of the toolkit.

To conserve memory, the `lrp.stability` field is dynamically created. If the user does not complete any stability tests then this field does not exist.

2.7.3 THE BOUNDARY CONDITIONS

The boundary conditions of (2.1) can be entered either as a set of values, or by providing a function that returns the appropriate value given the pass k and point number p.

By default, if the user does not supply the boundary conditions, they are assumed as

$$\begin{aligned} x_{k+1}(0) &= 0 \\ y_0(p) &= 1 \end{aligned}$$

2.8 THE COOLMATRIX TYPE

The conducted usability tests have shown that, due to differences in indexing between the theoretical results and the Scilab requirements, the implementation of any function can very easily lead to the famous "fencepost" error — for example, if the user wants to see pass number 3 he has to enter the index 4 in the Scilab matrix. While this problem may seem simple, it can lead to many difficult-to-detect errors, especially for non-standard LRP systems (like the "wave" processes, where negative indices are often used).

To overcome this difficulty, the new version of the Toolkit includes the $CoolMatrix$ matrix type that allows the user to index the arrays as required (from 0 or any other value, including negative number). The functions used for this type are designed for fast prototyping and hence provide a strict error checking — any attempt to use a wrong index causes an error. Great effort has been made to make this type compatible with the standard Scilab matrix type. The disadvantage of this addition to the Toolkit is the overhead caused by this type. This fact does not very significant, as the type is aimed at the prototyping phase, where a small to medium problems are tested. Moreover, the experience shows that the efficiency of the Toolkit is greatly dependent on the LMI solver, which has the greatest impact in the practical applications.

To create the CoolMatrix variable, the user must provide the allowed range of indices. Note here that the Scilab notation of ”extending” the size of the matrix when called with index larger than its former size is not supported, as this is very error-prone.

3 Example

To illustrate the LRP Toolkit, consider the special case of (1.1) defined by the following matrices

$$A = \begin{bmatrix} 0.19 & 0.07 & 0.19 \\ 0.02 & 0.85 & 0.49 \\ 0.84 & 0.01 & 0.75 \end{bmatrix}, B = \begin{bmatrix} 0.8 \\ 0.7 \\ 0.3 \end{bmatrix}, B_0 = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.1 \\ 0.6 & 0.2 \end{bmatrix},$$
$$C = \begin{bmatrix} 0.5 & 0.3 & 0.1 \\ 0.2 & 0.4 & 0.1 \end{bmatrix}, D = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, D_0 = \begin{bmatrix} 0.44 & 0.26 \\ 0.08 & 0.07 \end{bmatrix}.$$

with pass length $\alpha = 10$ and boundary conditions

$$y_0(p) = \begin{bmatrix} \sin(p) & \sin(p) \end{bmatrix}^T \quad \text{for} \quad 0 \le p < \alpha,$$
$$x_k(0) = \begin{bmatrix} 0.9 & 0.9 & 0.9 \end{bmatrix}^T \quad \text{for} \quad k \ge 0.$$

(Note that the argument of the sin function is given in radians).

By using the stability analysis options offered by the toolkit we conclude that the process is asymptotically stable but unstable along the pass. The plots of the pass profiles for $k = 3,\ 5,\ 7,\ 12,\ 15,\ 20$ confirm this (note the amplitudes of pass profile vectors increase significantly (see Fig. 3.1) as the process dynamics evolve).

Suppose now that the task is to design a control law of the form (1.6) to ensure stability along the pass. This can be undertaken using the toolkit stabilizing control law design options, as detailed in Section 2.6.2. For demonstration purposes we consider the case (in actual fact the theory behind this design produces a family of possible choices for K) when the control law matrix K of (1.6) is given by

$$K = \begin{bmatrix} -0.4887 & -0.5174 & -0.6102 & -0.4543 & -0.2739 \end{bmatrix}.$$

Note here that the user does not have to execute any commands to compute the resulting controlled process state space model — this is automatically done. The resulting controlled stable along the pass process state space model is of the form (1.1) with

$$A = \begin{bmatrix} -0.201 & -0.3439 & -0.2982 \\ -0.3221 & 0.4878 & 0.06287 \\ 0.6934 & -0.1452 & 0.5669 \end{bmatrix}, B = \begin{bmatrix} 0.8 \\ 0.7 \\ 0.3 \end{bmatrix},$$
$$B_0 = \begin{bmatrix} -0.2634 & -0.01911 \\ -0.01802 & -0.09172 \\ 0.4637 & 0.1178 \end{bmatrix}, D_0 = \begin{bmatrix} 0.3491 & 0.2052 \\ -0.1017 & -0.03956 \end{bmatrix},$$
$$C = \begin{bmatrix} 0.4023 & 0.1965 & -0.02204 \\ 0.004519 & 0.193 & -0.1441 \end{bmatrix}, D = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}.$$

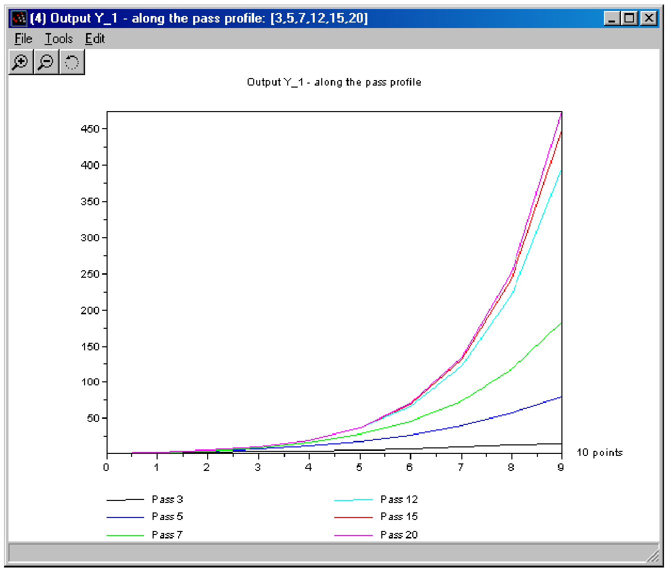


Figure 3.1: Instability along the pass — output 1 along the pass

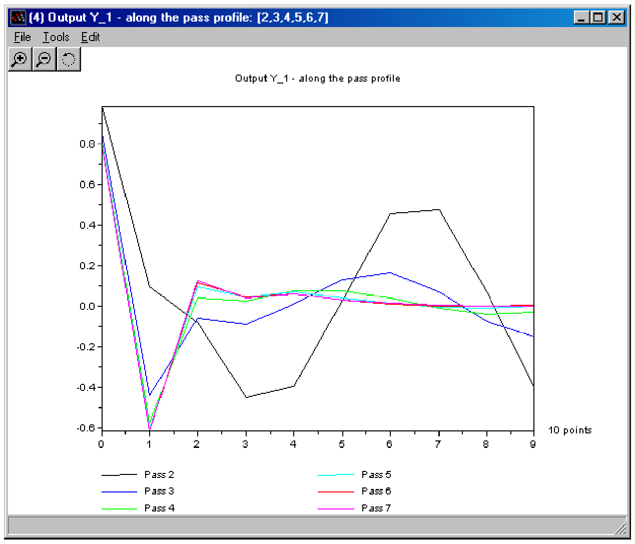


Figure 3.2: Stability along the pass — controlled process output 1 along the pass

All the matrices obtained during the control law computations are available to the user by using the options supplied and they can be also exported to the LaTeX-based source file.

A representative plot of the evolution of the pass profile sequence for the controlled process is given in Fig. 3.1 and the stability along the pass property is evident by inspection.

3.1 CONCLUSIONS AND FUTURE WORK

The Scilab toolkit whose development has been described in this paper has already been useful in the analysis and control law design for discrete linear repetitive processes of the form considered here. Its basic functions include the simulation engine and the stability analysis/control law design abilities. A key point to note is that this toolkit removes the limitations present in others. Moreover, the user is able to export the results to the valid LaTeX compatible format text file.

Software development for this toolkit is an ongoing process; there are many options which remain to be implemented. A representative sample of on going development work is the following topics:

- support for new classes of processes (such as ”wave” or semi-linear) and for a differential process model where the dynamics along the pass are governed by a linear matrix differential equation couple along with discretization methods,
- addition new control law design algorithms,
- control law design for stability and performance (e.g. ensuring that a reference signal is tracked, minimization of the influence of the external disturbances),
- handling of processes with uncertainty in the state space model,
- an installer for the Linux/Unix version.

Results from this work will be reported in due course.

4 Appendix

4.1 Abbreviations and notation

Throughout the manual the following abbreviations and notations are used

Notation	Meaning
GUI	Graphics user interface
LRP	Linear Repetitive Process or
	type of a structure used by the Toolkit to represent such process
$\succ$	positive definitiveness of a matrix
$\prec$	negative definiteness of a matrix
$r(M)$	the spectral radius, the largest module of an eigenvalue of matrix M

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