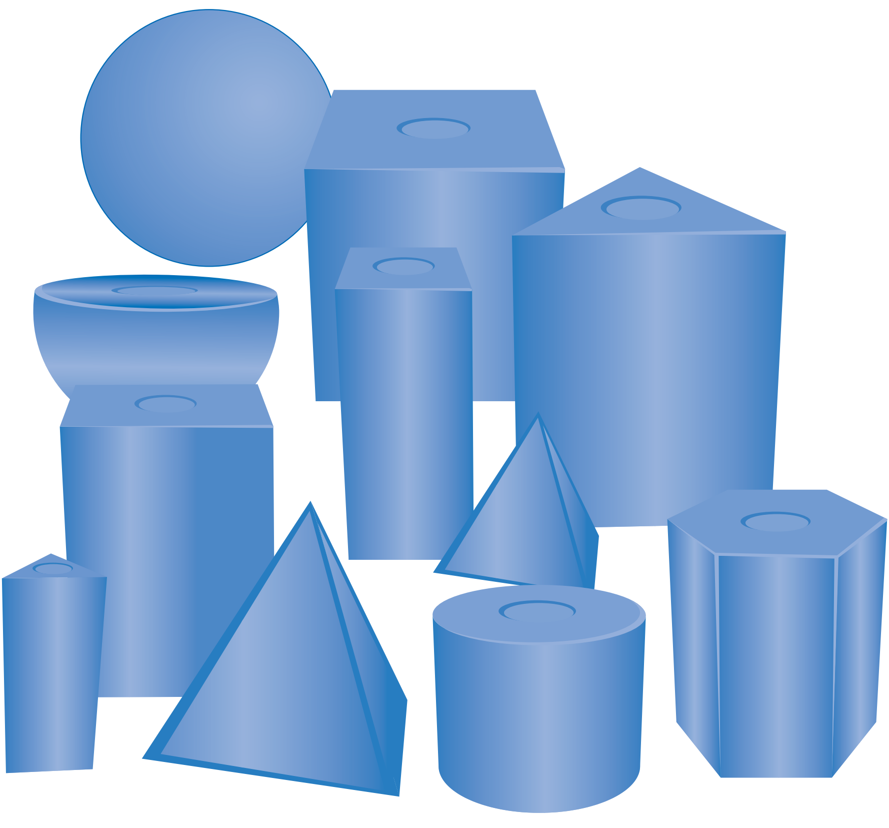
LEARNING RESOURCES®

LER 7630

POWER SOLIDS™

ACTIVITY GUIDE

A Hands-on Approach to Learning About Area and Volume



Power Solids™ Volume Table

Large Square Prism	$V = A \times H$	$V = (s \times s) \times H$	$V = (4.6)(4.6)(4.6)$	$V = 97.3\ cm^3$
Small Rectangular Prism	$V = A \times H$	$V = (1 \times w) \times H$	$V = (2 \times 2) \times 4.6$	$V = 18.4\ cm^3$
Large Rectangular Prism	$V = A \times H$	$V = (1 \times w) \times H$	$V = (2 \times 4.6) \times 4.6$	$V = 42.3\ cm^3$
Hexagonal Prism	$V = A \times H$	$V = (w \times 3/2 \times s) \times H$	$V = (4 \times 3/2 \times 2.3) \times 4.6$	$V = 63.5\ cm^3$
Large Triangular Prism	$V = A \times H$	$V = (1/2 \times b \times h) \times H$	$V = 1/2 \times (4.2 \times 3.6) \times 4.6$	$V = 34.8\ cm^3$
Small Triangular Prism	$V = A \times H$	$V = (1/2 \times b \times h) \times H$	$V = 1/2 \times (1.5 \times 1.3) \times 4.6$	$V = 4.5\ cm^3$
Square Pyramid	$V = 1/3\ A \times H$	$V = 1/3\ (1 \times w) \times H$	$V = 1/3\ (4.6 \times 4.6) \times 4.6$	$V = 32.5\ cm^3$
Triangular Pyramid	$V = 1/3\ A \times H$	$V = 1/3\ (1/2 \times b \times h) \times H$	$V = 1/3\ (1/2 \times 4.3 \times 3.8) \times 4.6$	$V = 12.5\ cm^3$
Sphere	$V = 4/3\ \pi \times r^3$	$V = 4/3\ \pi \times r^3$	$V = 4/3\ \pi \times 2.33$	$V = 50.9\ cm^3$
Cone	$V = 1/3\ A \times H$	$V = 1/3(\pi \times r^2) \times H$	$V = 1/3 \times \pi \times (2.3)^2 \times 4.6$	$V = 25.5cm^3$
Cylinder	$V = A \times H$	$V = (\pi \times r^2) \times H$	$V = \pi \times (2.3)^2 \times 4.6$	$V = 76.4\ cm^3$
Hemisphere	$V = 1/2 \times (4/3\ \pi \times r^3)$	$V = 1/2 \times (4/3\ \pi \times r^3)$	$V = 1/2 \times (4/3\ \pi \times 2.3^3)$	$V = 25.5\ cm^3$

Introduction

The transparent Power Solids™ set includes 12 plastic three-dimensional shapes that allow for hands-on study of volume. Power Solids can be integrated easily with daily math lessons for introducing, teaching, and reviewing math concepts effectively. They allow students to make concrete connections between geometric shapes and their associated formulas for volume, and to observe volumetric relationships between the geometric shapes as well.

Most shapes in this set are variations of a **prism** or a **pyramid**, both of which are **polyhedrons**. Polyhedrons are solid figures with flat sides, or **faces**. Faces may meet at a point, called a **vertex**, or at a line, called an **edge**. A **prism** has two congruent bases; the remaining faces are rectangles. A **pyramid** has one base and the remaining faces are triangles.

Three shapes in this set have curved faces rather than flat ones; the cylinder, cone, and sphere. Technically, they are not polyhedrons. Even so, a cylinder can be thought of as a circular prism: a figure with congruent circular bases and a single, rectangular face. A cone can be thought of as a pyramid with a circular base and a face that is a wedge. A sphere is a unique shape with no parallel to prisms or pyramids.

At the outset, learning formulas for the volume of more than a dozen geometric shapes may seem daunting to your students. Formulas become much easier to remember when students recognize that only the method for calculating the area of a base changes from formula to formula; the other variables are calculated the same way, regardless of shape.

Getting Started With Power Solids™

Allow students to become familiar with the manipulatives before beginning directed activities. You may want to explore prisms and pyramids on separate days. Encourage students to handle, observe, and discuss the Power Solids. Ask them to write down their observations as they make the following comparisons: How are the shapes similar? (All shapes have the same height. They are all three-dimensional. They all have empty spaces inside them.) How are the different? (Some have flat sides; some have curved sides. Some are box-shaped; some are round, and some are triangle-shaped.) Where have students seen these shapes in the world around them? (Great Pyramids of Egypt, traffic pylons, film canisters, soccer balls, pieces of chalk, boxes, lipstick tubes, and so on.)

Introduce and identify the following terms: **face**, **edge**, **vertex** or **corner**, and **base**. Mention to students that the base of each Power Solid can be identified by the hole in the face.

Ask students how they might organize the shapes into categories based on their features. Write students' answers on the board. Then, define **pyramids** and **prisms**. Hold up an example of a prism and a pyramid for the class. Encourage students to organize the Power Solids again based on this information. Discuss and explain the cylinder, sphere, and cone as exceptions.

Work with students to create a table like this one to record their observations.

Power Solids™	Number of Bases	Shape of Base(s)	Number of Faces	Number of Edges	Number of Vertices
Large Square Prism					
Small Rectangular Prism					
Large Rectangular Prism					
Hexagonal Prism					
Triangular Prism					
Square Pyramid					
Triangular Pyramid					
Sphere					
Cylinder					
Cone					

Show students a cardboard box. Ask if the box is a prism or a pyramid. (prism) Have a student volunteer identify the box's bases, faces, edges, and vertices. Have another student do the same for an oatmeal container. You may need to cut the container to make identification easier.

This would be a good time for your students to make constructions of the various models. You can construct models of toothpicks and gumdrops, straws and yarn, or pipe cleaners. As you go through formulas, encourage students to refer to their models to visualize why the formulas work.

Introducing Volume

Volume, or the capacity of an object, is sometimes confused with surface area. At first glance, the formulas appear somewhat similar. A helpful way to compare the concepts is to explain surface area as the amount of room on the *outside* of a shape, and volume as the amount of space *inside* a shape. Discuss the value of measuring volume, giving such examples as knowing how much water a pool will hold, how much air a SCUBA tank will hold, or how much cement a cement mixer will hold. Ask students for other examples.

Students will benefit from practice with building, measuring, and filling containers to understand volume. The Power Solids have a removable base and can be filled with water, sand, rice, or other materials. By filling one Power Solid and pouring its contents into another Power Solid, students can explore volume relationships between shapes. If you intend to have students perform exact measurements using a graduated cylinder, be sure they are comfortable reading the bottom edge of the water level, or **meniscus**.

Students can measure volume by reading sand levels in a graduated cylinder before and after filling a Power Solid. Have your students take the average of three trials to eliminate some errors. First, fill a large graduated cylinder nearly to the top and take a reading. Use the sand in the cylinder to fill the Power Solid. Take a reading for the sand remaining in the cylinder and subtract it from the starting quantity. The difference is the volume of sand poured into the Power Solid.

Challenge students to order the Power Solids from largest to smallest volume by estimation. You may want to allow them to fill their Power Solids or use cube models to make more accurate estimations. As you introduce the formulas for finding the volume of each shape, encourage students to refer to their Power Solids for reference. You also may wish to distribute copies of the table on page 2 for reference. Once you have finished your discussion, students can mathematically calculate the volume of each Power Solid to confirm the accuracy of their initial estimations.

Explain to students that the thickness of the plastic takes away from the volume each shape can hold. Therefore, students must measure from inside edge to inside edge rather than from outside edge to outside edge when computing what the shape can hold. Also, explain that the shapes are slightly larger at the opening so they can slip out of the mold during manufacturing. This will cause slight variations in the measurements. Tell them that the standard height is 4.6 cm, and the other measurements are derived from this to keep the shapes in relationship to each other.

Volume Formulas

Prism

Finding the volume of a general prism is a matter of multiplying the area of the base times the height of the prism:

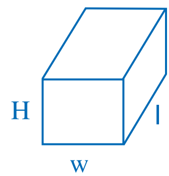
Volume$_{\text{general prism}}$ **= A** x **H**

Identify the variables:

A = Area of the base

H = Height of the prism

The formula for the area of the base of the prism depends upon the shape of the base.

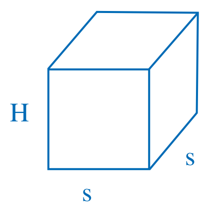


Rectangular Prism

Volume$_{\text{rectangular prism}}$ **= A** x **H**

= (l x **w)** x **H**

Square Prism



Volume$_{\text{square prism}}$ **= A** x **H**

= (s x **s)** x **H**

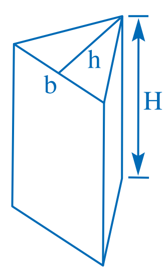
Identify the variables:

A = Area of the square base

H = Height of the prism

s = Length of the side

Triangular Prism



Volume$_{\text{triangular prism}}$ **= A** x **H**

= ($^1/_2$ b x **h)** x **H**

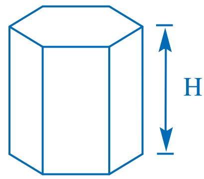
Identify the variables:

A = Area of the triangle base ($^1/_2$ b x h)

h = Altitude, or **height**, of the triangle

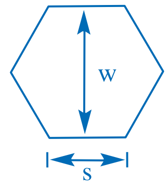
H = Height of the prism

Hexagonal Prism



Volume$_{\text{hexagonal prism}}$ **= A** × **H**

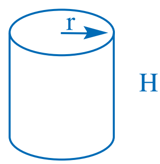
Identify the variables:
A = Area of the hexagonal base
H = Height of the prism



Explain that the area for a hexagon is calculated as follows:
$A = w \times {}^{3}/_{2}\, s$

Identify the variables:
w = Width of hexagon as shown
s = Length of side

Cylinder



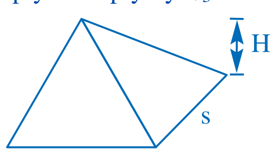
Volume$_{\text{cylinder}}$ **= A** × **H**
= $(\pi r^2) \times H$

Pyramid

Introduce the general formula for finding the volume of a pyramid:

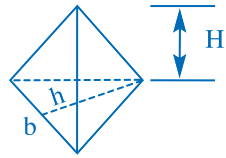
Volume$_{\text{pyramid}}$ **=** $^{1}/_{3}\ A \times H$

Ask students to identify the difference between this general formula and the one for the prism. (There is one more variable: $^{1}/_{3}$.) If students remember a volume formula for a prism, it is easy to remember the volume formula for a pyramid with the same-size base and height: simply multiply by $^{1}/_{3}$. You can demonstrate this concept by pouring three filled pyramids into the corresponding prism in the Power Solid set.



Square Pyramid

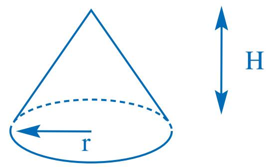
Volume$_{\text{square pyramid}}$ **=** $^{1}/_{3}\ A \times H$
= $^{1}/_{3}\,(s \times s) \times H$



Triangular Pyramid

Volume$_{\text{triangular pyramid}}$ $= {}^{1}/_{3}\ A \times H$

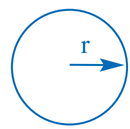
$= {}^{1}/_{3}\,(b \times h) \times H$



Cone

Volume$_{\text{cone}}$ $= {}^{1}/_{3}\ A \times H$

$= {}^{1}/_{3}\,(\pi r^2) \times H$



Sphere

Volume$_{\text{sphere}}$ $= {}^{4}/_{3}\ \pi r^2$

Also from Learning Resources®:

- LER 7631 Investigating with Power Solids™
- LER 7633 Geometry Template

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